Metodologia Ecológica

Aula 7

- A super-hipótese nula, resistente a quaquer teste (poder e erro tipo II)

- Comparação entre duas ou mais amostras: princípios de análise de variância (ANOVA)

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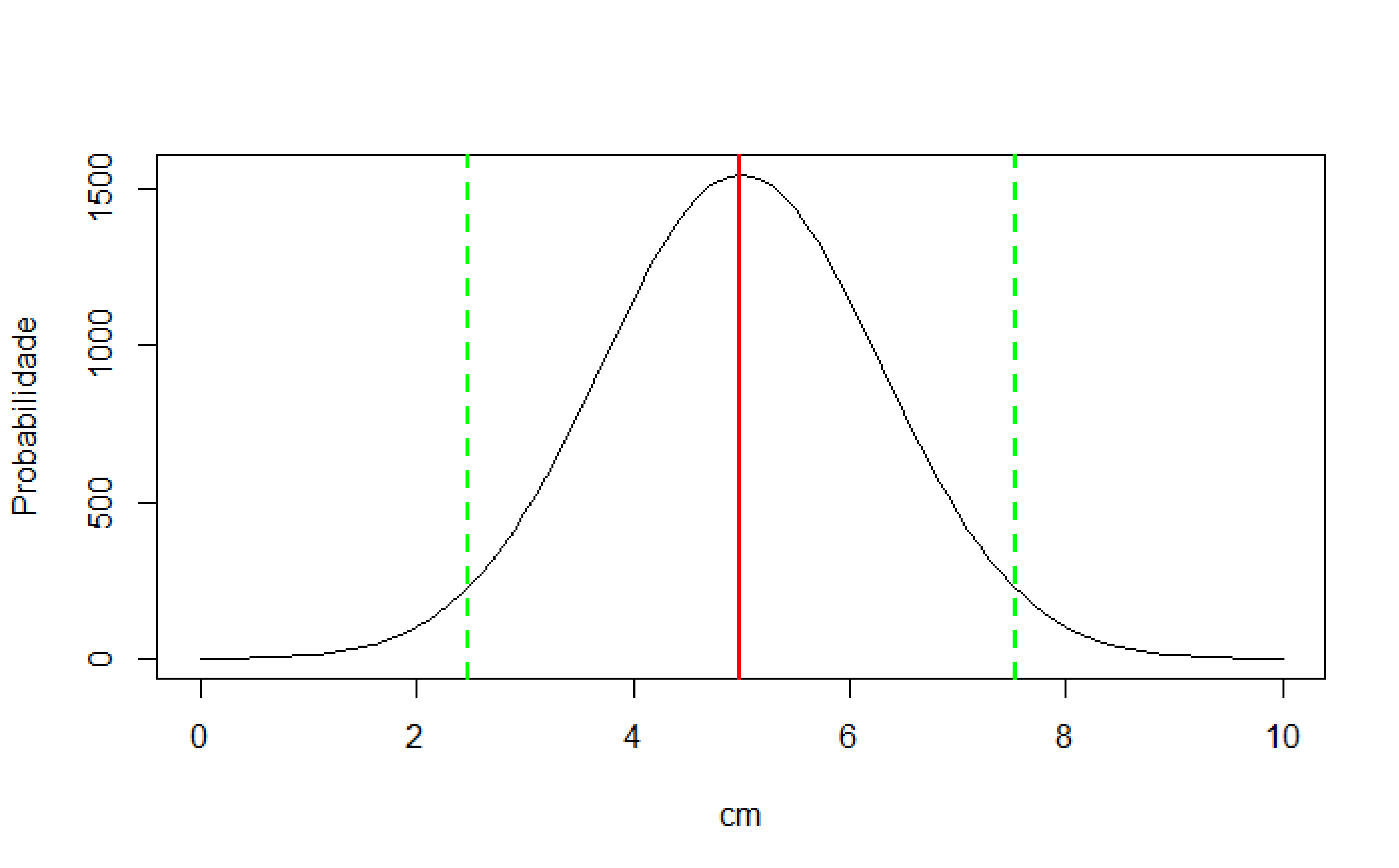
Variável aleatória Normal

- Slide com curva Normal
- Áreas da curva correspondendo a 2,5% e 97,5%
- Curva Normal Padrão e unidades de Z
- Intervalo de Confiança para 95% das repetições do mesmo procedimento
- Caso em que a média é estimada da amostra: forma da curva da distribuição de probabilidades

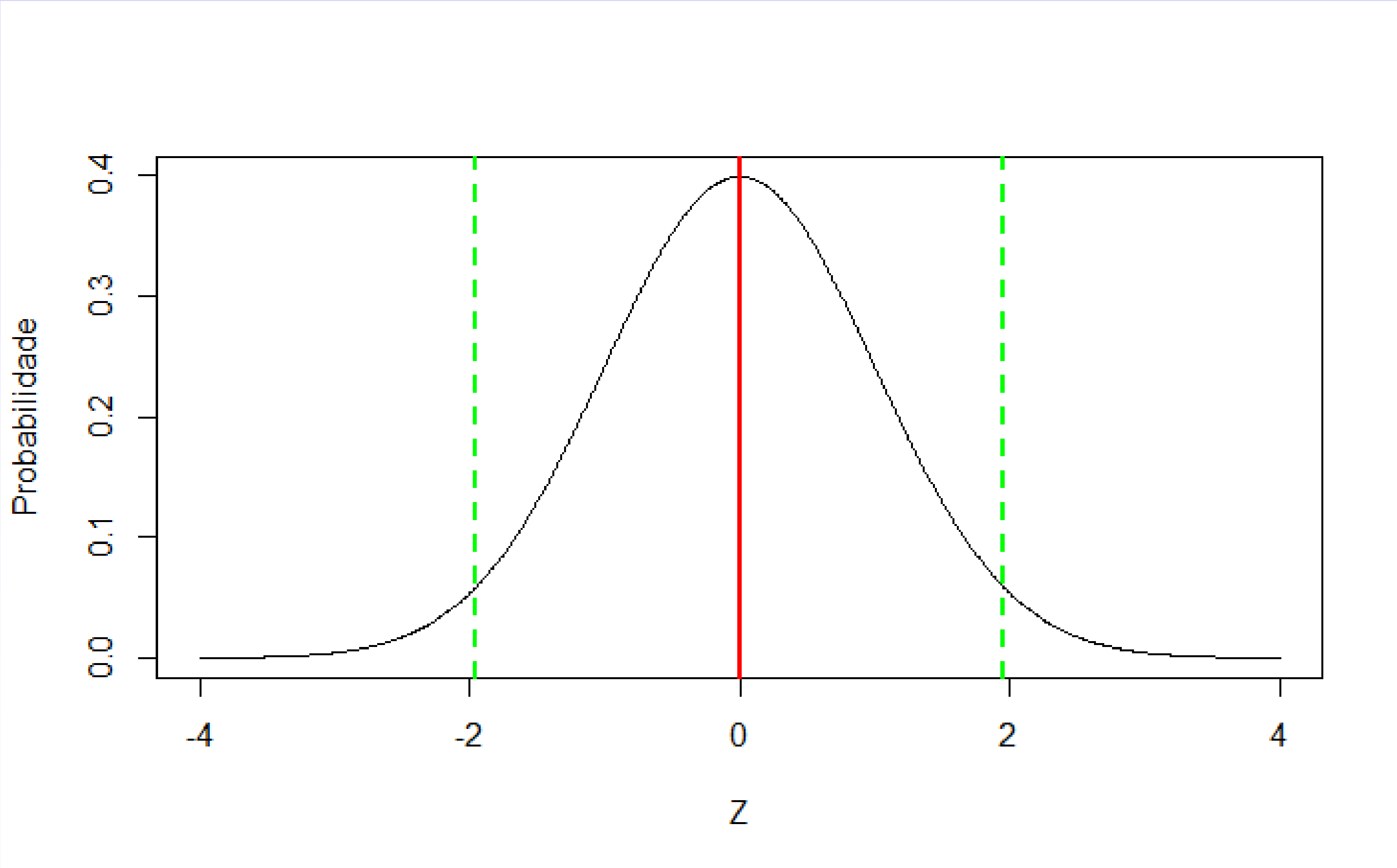
Variável aleatória Normal

média = 5, sd = 1,29



Variável aleatória Normal Padrão

média = 0, sd = 1



Intervalos de confiança para *média*

P(média – 1,96ep ≤ μ ≤ média + 1,96ep) = 0,95

Intervalos de confiança generalizados

$$\bar{x} \pm t \bullet \frac{\sigma}{\sqrt{n}}$$

Distribuição *t* (de Student)

Intervalos de confiança para *média*

$$P\,(-\,1{,}96 \leq Z \leq +\,1{,}96) = 0{,}95$$

Z = raiz(n)*(x1 – xmédia)/σ

– 1,96 ≤ raiz(n)*(média – μ)/σ ≤ + 1,96

– 1,96 ep ≤ média - μ ≤ + 1,96ep

– média – 1,96 ep ≤ - μ ≤ - média + 1,96ep

média – 1,96 ep ≤ μ ≤ média + 1,96ep

$$\bar{x} \pm t \bullet \frac{\sigma}{\sqrt{n}}$$

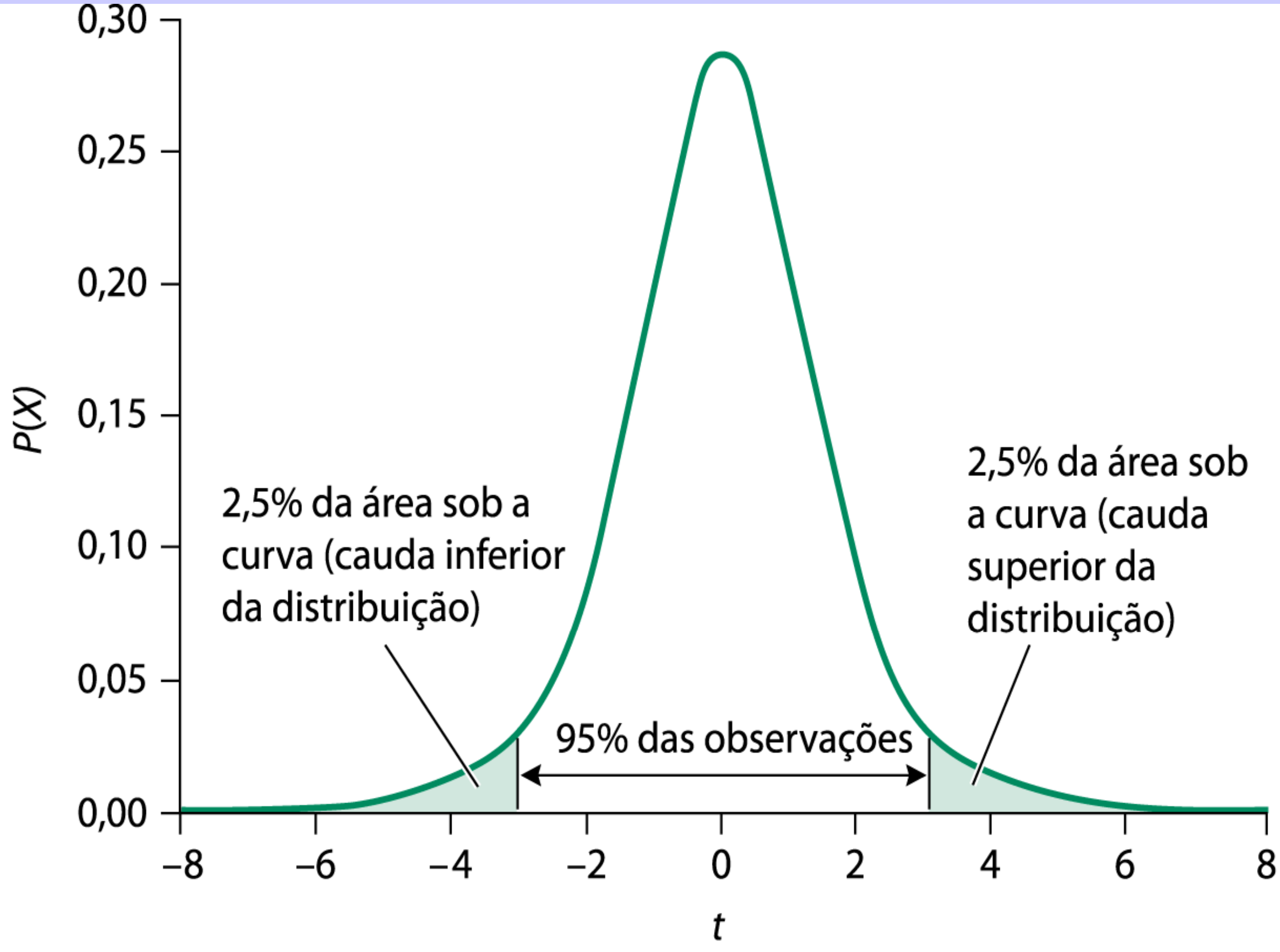


Figura 3.7 Distribuição-*t* ilustrando que 95% das observações, ou massa de probabilidade, caem dentro do percentil de ±1,96 desvio-padrão da média (média = 0). As duas caudas da distribuição contêm cada 2,5% das observações ou massa de probabilidade da distribuição. Elas somam 5% das observações, e a probabilidade $P = 0{,}05$ de que uma observação caia nessas caudas. Esta distribuição é idêntica à distribuição-*t* ilustrada na Figura 3.5.

Erro Tipo I : α

- Probabilidade de rejeitar Ho quando verdadeira
- Probabilidade de falso negativo
- Para calcular depende:

basta especificar Ho

- Teste conservador

Testes de hipótese nula comuns

- Diferença entre médias de duas amostras:
  - Teste *t*
  - Razão *F*
  - Premissas

Testes *t*

para testar Ho de diferença entre duas médias

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$t = \frac{Diferença_{entre_{médias}}}{Erro_{padrão_{da_{diferença_{entre_{médias}}}}}}$$

Erro Tipo II : β

- Probalidade de não rejeitar Ho quando falsa, ou probabilidade de falso positivo, β
- Para calcular depende:
  - Da hipótese alternativa
  - Do tamanho do efeito que se pretende detectar
  - Do tamanho da amostra
  - Do delineamento experimental
- Poder: probabilidade de rejeitar corretamente Ho quando falsa: 1- β
- Princípio de precaução em monitoramento ambiental

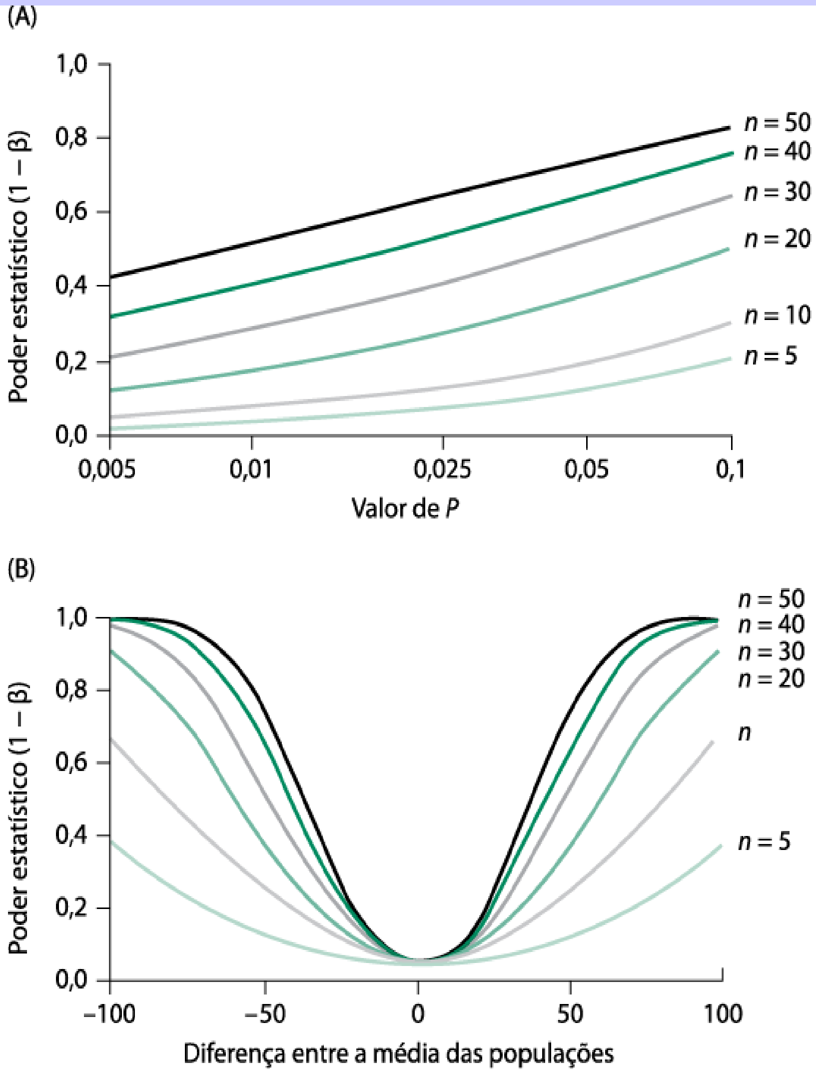


Figura 4.5 Relação entre poder estatístico, valor de *P* e tamanho do efeito observável em função do tamanho amostral. (A) O valor de *P* é a probabilidade de incorretamente rejeitar uma hipótese nula verdadeira, enquanto o poder estatístico é a probabilidade de corretamente rejeitar uma hipótese nula falsa. O resultado geral propõe que, quanto menor o valor de *P* usado para rejeitar a hipótese nula, menor o poder estatístico de corretamente detectar um efeito do tratamento. A um dado valor de *P*, o poder estatístico é maior quando o tamanho amostral é maior. (B) Quanto menor o tamanho do efeito observável do tratamento (i. e., quanto menor a diferença entre o grupo-tratamento e o grupo-controle), maior é o tamanho amostral necessário a um bom poder estatístico para detectar o efeito do tratamento.[21]

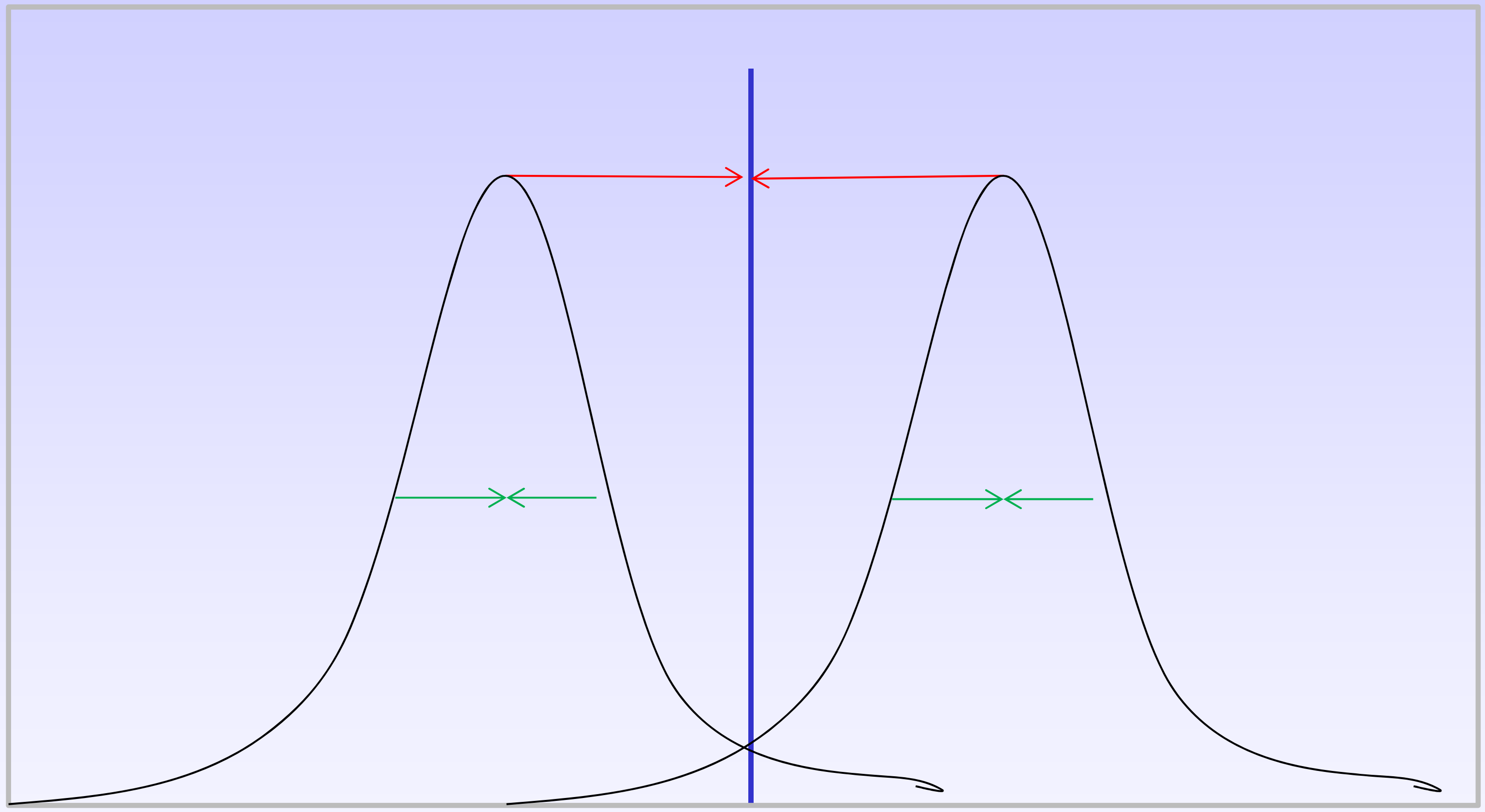


TABLE 9.1 Complete ANOVA table for single factor linear regression

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Regression	1	$SS_{reg} = \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$	$\frac{SS_{reg}}{1}$	$\sigma^2 + \beta_1^2 \sum_{i=1}^{n} X^2$	$\frac{SS_{reg}/1}{RSS/(n-2)}$	Tail of the F distribution with 1, $n-2$ degrees of freedom
Residual	$n-2$	$RSS = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$	$\frac{RSS}{(n-2)}$	σ^2		
Total	$n-1$	$SS_Y = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$	$\frac{SS_Y}{(n-1)}$	σ_Y^2		

TABLE 10.1 Partitioning of the sum of squares in ANOVA

Unmanipulated	Control	Treatment	
10	9	12	
12	11	13	
12	11	15	
13	12	16	
$\overline{Y}_1 = 11.75$	$\overline{Y}_2 = 10.75$	$\overline{Y}_3 = 14.00$	$\overline{Y} = 12.17$
$\sum_{j=1}^{n}(Y_{1j}-\overline{Y}_1)^2 = 4.75$	$\sum_{j=1}^{n}(Y_{2j}-\overline{Y}_2)^2 = 4.75$	$\sum_{j=1}^{n}(Y_{3j}-\overline{Y}_3)^2 = 10.00$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(Y_{ij}-\overline{Y}_i)^2 = 19.50 = SS_{within\ groups}$
$\sum_{j=1}^{n}(\overline{Y}_1-\overline{Y})^2 = 0.68$	$\sum_{j=1}^{n}(\overline{Y}_2-\overline{Y})^2 = 8.08$	$\sum_{j=1}^{n}(\overline{Y}_3-\overline{Y})^2 = 13.40$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(\overline{Y}_i-\overline{Y})^2 = 22.16 = SS_{among\ groups}$
$\sum_{j=1}^{n}(Y_{1j}-\overline{Y})^2 = 5.43$	$\sum_{j=1}^{n}(Y_{2j}-\overline{Y})^2 = 12.83$	$\sum_{j=1}^{n}(Y_{3j}-\overline{Y})^2 = 23.40$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(Y_{ij}-\overline{Y})^2 = 41.66 = SS_{total}$

TABLE 10.2 ANOVA table for one-way layout

Source	Degrees of freedom (df)	Sum of squares (SS)	Mean square (MS)	Expected mean square	F-ratio	P-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+n\sigma_A^2$	$\frac{MS_{among\ groups}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$ $a(n-1)$ degrees of freedom
Within groups (residual)	$a(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(Y_{ij}-\bar{Y}_i)^2$	$\frac{SS_{within\ groups}}{a(n-1)}$	σ^2		
Total	$an-1$	$\sum_{i=1}^{a}\sum_{i=1}^{n}(Y_{ij}-\bar{Y})^2$	$\frac{SS_{total}}{(an-1)}$	σ_Y^2		

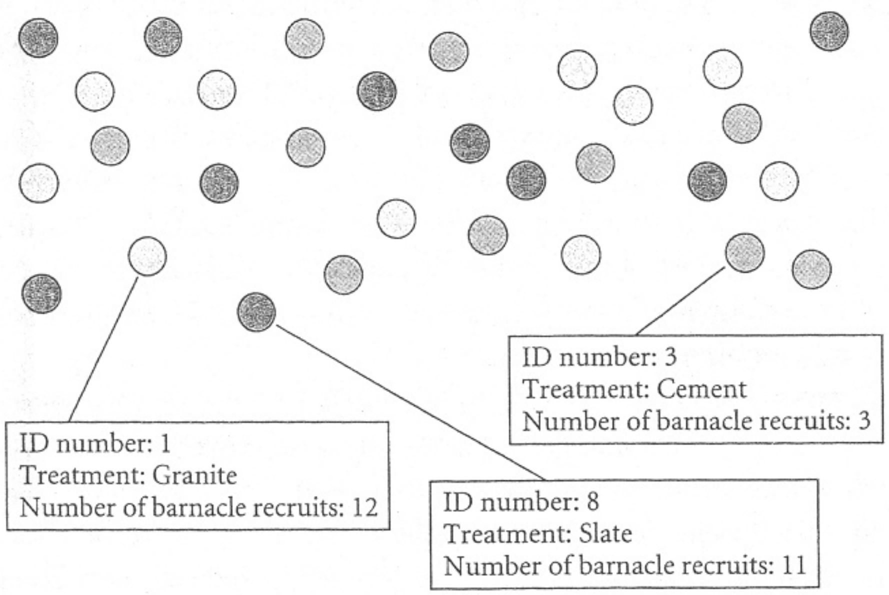
TABLE 10.3 One-way ANOVA table for the hypothetical data in Table 10.1

Source	Degrees of freedom (df)	Sum of squares (SS)	Mean square (MS)	F-ratio	P-value
Among groups	2	22.17	11.08	5.11	0.033
Within groups (residual)	9	19.50	2.17		
Total	11	41.67			

TABLE 10.3 One-way ANOVA table for the hypothetical data in Table 10.1

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	F-ratio	*P*-value
Among groups	2	22.17	11.08	5.11	0.033
Within groups (residual)	9	19.50	2.17		
Total	11	41.67			

ANOVA de um fator



ID number	Treatment	Replicate	Number of barnacle recruits
1	Granite	1	12
2	Slate	1	10
3	Cement	1	3
4	Granite	2	14
5	Slate	2	10
6	Cement	2	8
7	Granite	3	11
8	Slate	3	11
9	Cement	3	7
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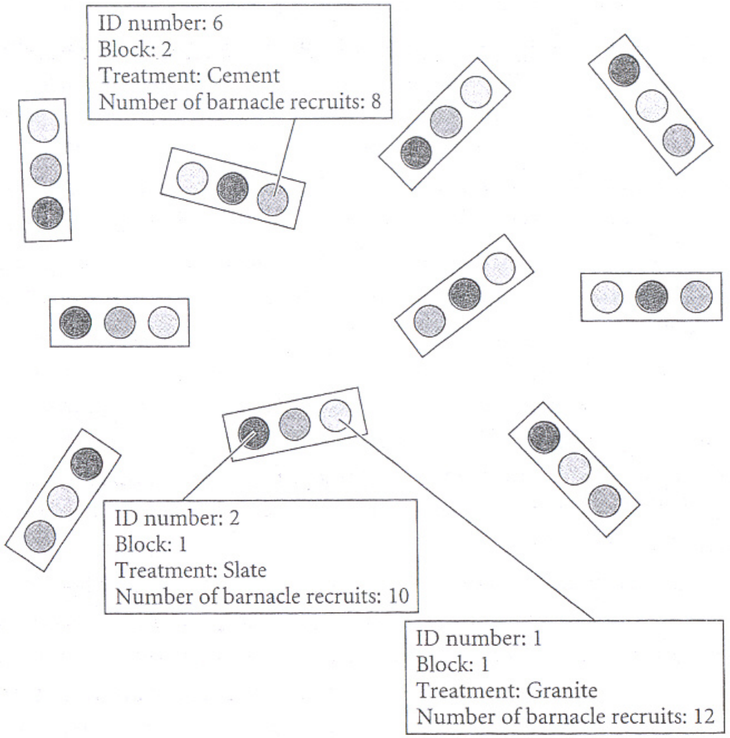
.	.	.	.
30	Cement	10	8

Blocos aleatorizados

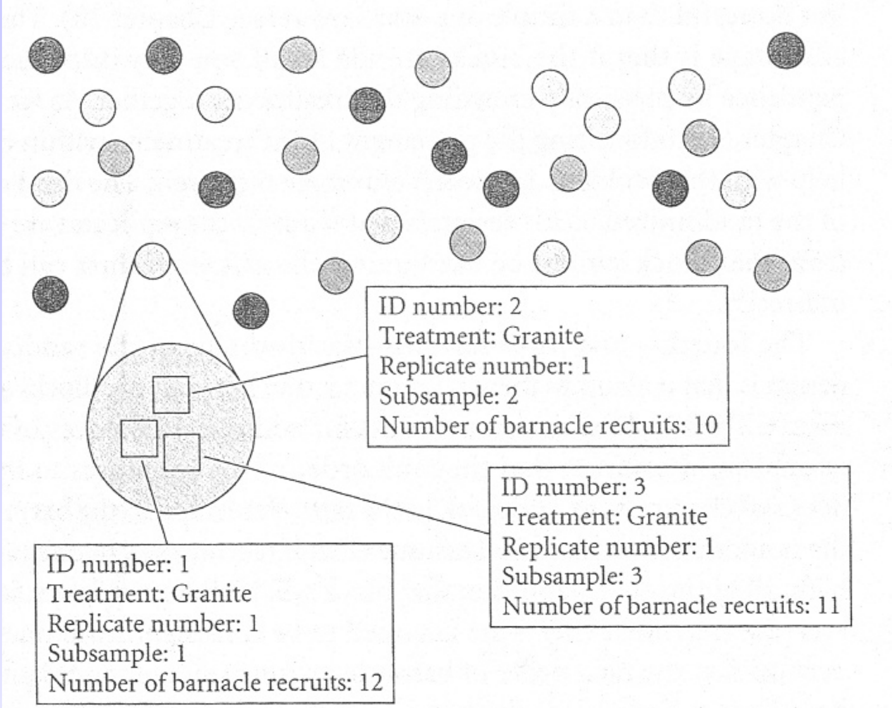


ID number	Treatment	Block	Number of barnacle recruits
1	Granite	1	12
2	Slate	1	10
3	Cement	1	3
4	Granite	2	14
5	Slate	2	10
6	Cement	2	8
7	Granite	3	11
8	Slate	3	11
9	Cement	3	7
.	.	.	.
.	.	.	.
30	Cement	10	8

TABLE 10.4 ANOVA table for randomized block design

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{Y}_i-\overline{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+b\sigma_A^2$	$\frac{MS_{among\ groups}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$, $(a-1)(b-1)$ degrees of freedom
Blocks	$b-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{Y}_j-\overline{Y})^2$	$\frac{SS_{blocks}}{(b-1)}$	$\sigma^2+a\sigma_B^2$	$\frac{MS_{blocks}}{MS_{within\ groups}}$	Tail of the F-distribution with $(b-1)$, $(a-1)(b-1)$ degrees of freedom
Within groups (residual)	$(a-1)(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij}-\overline{Y}_i)^2$	$\frac{SS_{within\ groups}}{(a-1)(b-1)}$	σ^2		
Total	$ab-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij}-\overline{Y})^2$	$\frac{SS_{total}}{(ab-1)}$	σ_Y^2		

ANOVA hierárquica (*Nested*)



ID number	Treatment	Replicate number	Subsample	Number of barnacle recruits
1	Granite	1	1	12
2	Granite	1	2	10
3	Granite	1	3	11
4	Slate	2	1	14
5	Slate	2	2	10
6	Slate	2	3	7
7	Cement	3	1	5
8	Cement	3	2	6
9	Cement	3	3	10
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.	.	.	.	.
90	Cement	30	3	6

TABLE 10.5 ANOVA table for nested design

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+bn\sigma^2_A+n\sigma^2_{B(A)}$	$\frac{MS_{among\ groups}}{MS_{among\ replicates(groups)}}$	Tail of the F-distribution with $(a-1), a(b-1)$ degrees of freedom
Among replicates within groups	$a(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_{j(i)}-\bar{Y}_i)^2$	$\frac{SS_{replicates(groups)}}{a(b-1)}$	$\sigma^2+n\sigma^2_{B(A)}$	$\frac{MS_{among\ replicates(groups)}}{MS_{subsamples}}$	Tail of the F-distribution with $a(b-1), ab(n-1)$ degrees of freedom
Subsamples within replicates (residual)	$ab(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{j(i)})^2$	$\frac{SS_{subsamples}}{ab(n-1)}$	σ^2		
Total	$abn-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abn-1)}$	σ^2_Y		

TABLE 10.5 ANOVA table for nested design

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+bn\sigma_A^2+n\sigma_{B(A)}^2$	$\frac{MS_{among\ groups}}{MS_{among\ replicates(groups)}}$	Tail of the F-distribution with $(a-1)$, $a(b-1)$ degrees of freedom
Among replicates within groups	$a(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_{j(i)}-\bar{Y}_i)^2$	$\frac{SS_{replicates(groups)}}{a(b-1)}$	$\sigma^2+n\sigma_{B(A)}^2$	$\frac{MS_{among\ replicates(groups)}}{MS_{subsamples}}$	Tail of the F-distribution with $a(b-1)$, $ab(n-1)$ degrees of freedom
Subsamples within replicates (residual)	$ab(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{j(i)})^2$	$\frac{SS_{subsamples}}{ab(n-1)}$	σ^2		
Total	$abn-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abn-1)}$	σ_Y^2		

TABLE 10.4 ANOVA table for randomized block design

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+b\sigma_A^2$	$\frac{MS_{among\ groups}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$, $(a-1)(b-1)$ degrees of freedom
Blocks	$b-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{Y}_j-\bar{Y})^2$	$\frac{SS_{blocks}}{(b-1)}$	$\sigma^2+a\sigma_B^2$	$\frac{MS_{blocks}}{MS_{within\ groups}}$	Tail of the F-distribution with $(b-1)$, $(a-1)(b-1)$ degrees of freedom
Within groups (residual)	$(a-1)(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij}-\bar{Y}_i)^2$	$\frac{SS_{within\ groups}}{(a-1)(b-1)}$	σ^2		
Total	$ab-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij}-\bar{Y})^2$	$\frac{SS_{total}}{(ab-1)}$	σ_Y^2		

TABLE 10.5 ANOVA table for nested design

Source	Degrees of freedom (df)	Sum of squares (SS)	Mean square (MS)	Expected mean square	F-ratio	P-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+bn\sigma_A^2+n\sigma_{B(A)}^2$	$\frac{MS_{among\ groups}}{MS_{among\ replicates(groups)}}$	Tail of the F-distribution with $(a-1), a(b-1)$ degrees of freedom
Among replicates within groups	$a(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_{j(i)}-\bar{Y}_i)^2$	$\frac{SS_{replicates(groups)}}{a(b-1)}$	$\sigma^2+n\sigma_{B(A)}^2$	$\frac{MS_{among\ replicates(groups)}}{MS_{subsamples}}$	Tail of the F-distribution with $a(b-1), ab(n-1)$ degrees of freedom
Subsamples within replicates (residual)	$ab(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{j(i)})^2$	$\frac{SS_{subsamples}}{ab(n-1)}$	σ^2		
Total	$abn-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abn-1)}$	σ_Y^2		

TABLE 10.2 ANOVA table for one-way layout

Source	Degrees of freedom (df)	Sum of squares (SS)	Mean square (MS)	Expected mean square	F-ratio	P-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+n\sigma_A^2$	$\frac{MS_{among\ groups}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$ $a(n-1)$ degrees of freedom
Within groups (residual)	$a(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(Y_{ij}-\bar{Y}_i)^2$	$\frac{SS_{within\ groups}}{a(n-1)}$	σ^2		
Total	$an-1$	$\sum_{i=1}^{a}\sum_{i=1}^{n}(Y_{ij}-\bar{Y})^2$	$\frac{SS_{total}}{(an-1)}$	σ_Y^2		

ANOVA de dois fatores

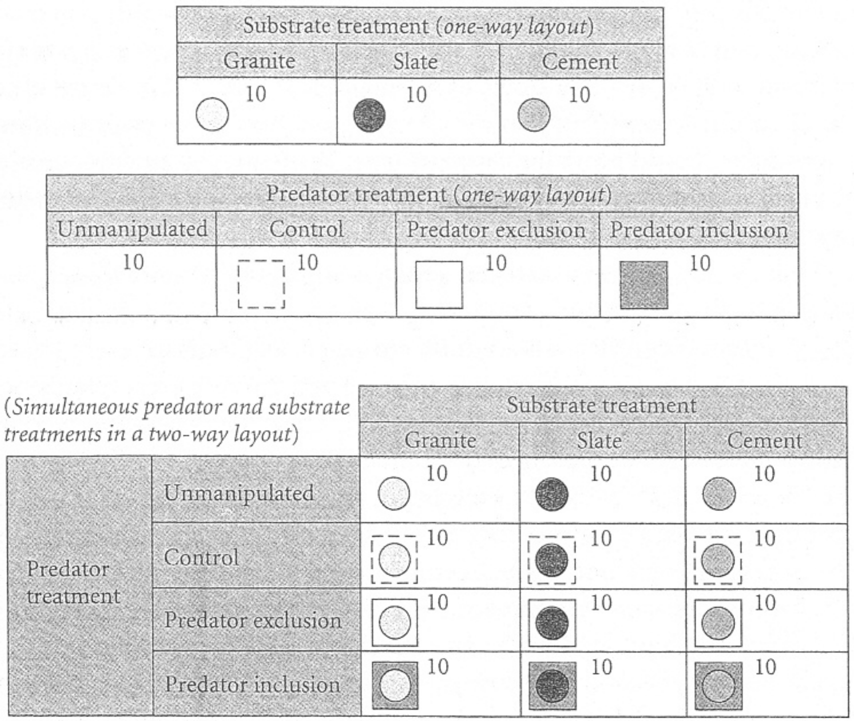
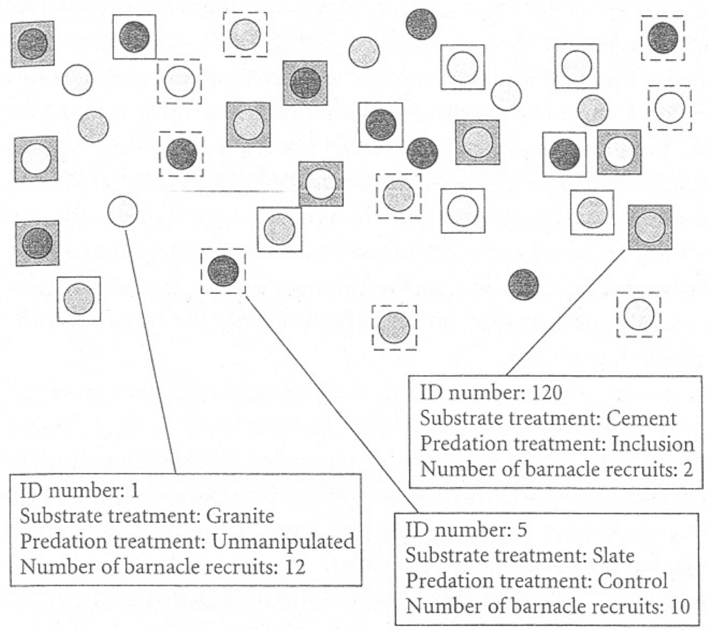


Figure 7.9 Treatment combinations in two single-factor designs and in a fully crossed two-factor design. This experiment is designed to test for the effect of substrate type (Granite, Slate, or Cement) and predation (Unmanipulated, Control, Predator exclusion, Predator inclusion) on barnacle recruitment in the rocky intertidal. The number 10 indicates the total number of replicates in each treatment. The three shades of the circles represent the three substrate treatments, and the patterns of the squares represent the four predation treatments. The two upper panels illustrate two one-way designs, in which only one of the two factors is systematically varied. In the two-factor design (lower panel), the 4 × 3 = 12 treatments represent different combinations of substrate *and* predation. The symbol in each cell indicates the combination of predation and substrate treatment that is applied.

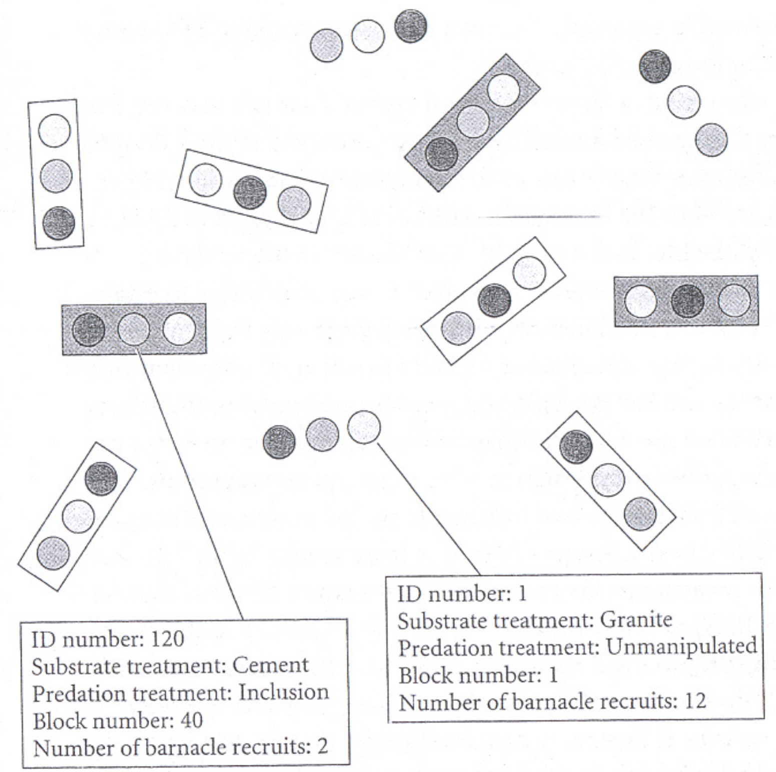


ID number	Substrate treatment	Predation treatment	Number of barnacle recruits
1	Granite	Unmanipulated	12
2	Slate	Unmanipulated	10
3	Cement	Unmanipulated	8
4	Granite	Control	14
5	Slate	Control	10
6	Cement	Control	8
7	Granite	Predator exclusion	50
8	Slate	Predator exclusion	68
9	Cement	Predator exclusion	39
.	.	.	.
.	.	.	.
120	Cement	Predator inclusion	2

TABLE 10.6 **ANOVA table for two-way design**

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Factor *A*	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_A}{(a-1)}$	$\sigma^2+nb\sigma_A^2$	$\frac{MS_A}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$, $ab(n-1)$ degrees of freedom
Factor *B*	$b-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_j-\bar{Y})^2$	$\frac{SS_B}{(b-1)}$	$\sigma^2+na\sigma_B^2$	$\frac{MS_B}{MS_{within\ groups}}$	Tail of the F-distribution with $(b-1)$, $ab(n-1)$ degrees of freedom
Interaction $(A\times B)$	$(a-1)(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_{ij}-\bar{Y}_i-\bar{Y}_j+\bar{Y})^2$	$\frac{SS_{AB}}{(a-1)(b-1)}$	$\sigma^2+n\sigma_{AB}^2$	$\frac{MS_{AB}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)(b-1)$, $ab(n-1)$ degrees of freedom
Within groups (residual)	$ab(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{ij})^2$	$\frac{SS_{within\ groups}}{ab(n-1)}$	σ^2		
Total	$abn-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abn-1)}$	σ_Y^2		

ANOVA
com blocos subdivididos (*split-plot*)



ID number	Substrate treatment	Predation treatment	Block number	Number of barnacle recruits
1	Granite	Unmanipulated	1	12
2	Slate	Unmanipulated	1	10
3	Cement	Unmanipulated	1	8
4	Granite	Control	2	14
5	Slate	Control	2	10
6	Cement	Control	2	8
7	Granite	Predator exclusion	3	50
8	Slate	Predator exclusion	3	68
9	Cement	Predator exclusion	3	39
.	.	.	.	.
.	.	.	.	.
120	Cement	Predator inclusion	40	2

TABLE 10.8 **ANOVA table for split-plot design**

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Factor *A* (whole-plot treatment)	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_A}{(a-1)}$	$\sigma^2+c\sigma_B^2+bc\sigma_A^2$	$\frac{MS_A}{MS_B}$	Tail of the F-distribution with $(a-1)$, $a(b-1)$ degrees of freedom
Factor *B*(*A*) (plots nested within *A*)	$a(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(\bar{Y}_j-\bar{Y})^2$	$\frac{SS_B}{(b-1)}$	$\sigma^2+c\sigma_B^2$		
Factor *C* (within-plot treatment)	$(c-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(\bar{Y}_k-\bar{Y})^2$	$\frac{SS_C}{(c-1)}$	$\sigma^2+\sigma_{BC}^2+ba\sigma_C^2$	$\frac{MS_C}{MS_{B(A)xC}}$	Tail of the F-distribution with $(c-1)$, $a(b-1)(c-1)$ degrees of freedom
$A\times C$ Interaction	$(a-1)(c-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(\bar{Y}_{ik}-\bar{Y}_i-\bar{Y}_k+\bar{Y})^2$	$\frac{SS_{AC}}{(a-1)(c-1)}$	$\sigma^2+\sigma_{BC}^2+b\sigma_{AC}^2$	$\frac{MS_{AC}}{MS_{B(A)xC}}$	Tail of the F-distribution with $(a-1)(c-1)$, $a(b-1)(c-1)$ degrees of freedom
$B(A)\times C$ Interaction	$a(b-1)(c-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(Y_{ijk}-\bar{Y}_{ik})^2$	$\frac{SS_{BC}}{a(b-1)(c-1)}$	$\sigma^2+\sigma_{BC}^2$		
Total	$abc-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abc-1)}$	σ_Y^2		

Possíveis interações numa ANOVA de dois fatores