Optimization in Logistics

Marek Mateusz Narozniak

AfterShip Limited

marek@aftership.net

August 26, 2016

Overview

Introduction - Puzzles

How optimization works?

1. Don't code!
2. Define your problem - create a *model*.
3. Let solver handle it.

Introduction - Puzzles

Sudoku

3			2
	4	1	
	3	2	
4			1

·	a	b	·
c	·	·	d
e	·	·	f
·	g	h	·

Sudoku - model

$$a \in \{1, 2\}$$
$$b \in \{3, 4\}$$
$$c \in \{1, 2\}$$
$$d \in \{3, 4\}$$
$$e \in \{1, 2\}$$
$$f \in \{3, 4\}$$
$$g \in \{1, 2\}$$
$$h \in \{3, 4\}$$

$$a \neq c$$
$$b \neq d$$
$$b \neq h$$
$$c \neq e$$
$$d \neq f$$
$$e \neq g$$
$$f \neq h$$

Sudoku - model

decision variables

$$\begin{aligned} a &\in \{1,2\} \\ b &\in \{3,4\} \\ c &\in \{1,2\} \\ d &\in \{3,4\} \\ e &\in \{1,2\} \\ f &\in \{3,4\} \\ g &\in \{1,2\} \\ h &\in \{3,4\} \end{aligned}$$

constraints

$$\begin{aligned} a &\neq c \\ b &\neq d \\ b &\neq h \\ c &\neq e \\ d &\neq f \\ e &\neq g \\ f &\neq h \end{aligned}$$

domains

Sudoku - Solution

3	1	4	2
2	4	1	3
1	3	2	4
4	2	3	1

$$a = 1$$
$$b = 4$$
$$c = 2$$
$$d = 3$$
$$e = 1$$
$$f = 4$$
$$g = 3$$
$$h = 2$$

Introduction - Puzzles

solver found value for each decision variable that is present in the variable domain and does not violate any of the constraints

3	1	4	2
2	4	1	3
1	3	2	4
4	2	3	1

$$a = 1$$
$$b = 4$$
$$c = 2$$
$$d = 3$$
$$e = 1$$
$$f = 4$$
$$g = 3$$
$$h = 2$$

Overview

Postmen Rates Problem

Postmen Rates Definition

Given n parcels, each of them characterized by w_i, to each of them assign a carrier among with set of price penalties and discounts.

Postmen Rates Input Data

$$
\begin{aligned}
w_p &= \text{Weight of parcel } p \\
w_w^{\max} &= \text{Maximum weight for weight class } w \\
w_w^{\min} &= \text{Minimum weight for weight class } w \\
r_{w,c}^{\text{base}} &= \text{Base rate for shipping item with weight } w \text{ by carrier } c \\
r_{c,d,w}^{\text{dscnt}} &= \text{Price reduction for weight } w \text{ by carrier } c \text{ after applying discount } d
\end{aligned}
$$

Postmen Rates Problem

Postmen Rates Decision Variables

$$w^x_{p,w,c} = \begin{cases} 1 & \text{Parcel } p \text{ takes carrier } c \text{ and weight class } w \\ 0 & \text{Otherwise} \end{cases}$$

$$d_{p,c,d,w} = \begin{cases} 1 & \text{We apply discount } d \text{ to parcel } p \\ 0 & \text{Otherwise} \end{cases}$$

$$f^r_p \in \mathbb{R}^+_0 \quad \text{Final rate applied to parcel } p$$

$$f^d_p \in \mathbb{R}^+_0 \quad \text{Final discount applied to parcel } p$$

Postmen Rates Problem

Postmen Rates Model

$$
\begin{aligned}
\text{minimize} \quad & \sum_{p \in P} f_p^r - f_p^d \\
\text{subject to} \quad & w_p \leq \sum_{c \in C} \sum_{w \in W} w_{p,w,c}^x w_w^{\max}, \quad && p \in P \\
& w_p \geq \sum_{c \in C} \sum_{w \in W} w_{p,w,c}^x w_w^{\min}, \quad && p \in P \\
& \sum_{c \in C} \sum_{w \in W} w_{p,w,c}^x = 1, \quad && p \in P
\end{aligned}
$$

Postmen Rates Problem

Postmen Rates Model (continued)

$$
\begin{aligned}
\text{subject to} \quad & \sum_{d \in D} d_{p,c,d,w} \le w^x_{p,w,c}, && p \in P, w \in W, c \in C \\
& \sum_{w \in W} \sum_{c \in C} d_{p,c,d,w} \le 1, && p \in P, d \in D \\
& f^d_p = \sum_{d \in D} \sum_{c \in C} \sum_{w \in W} d_{p,c,d,w} r^{\text{dscnt}}_{c,d,w}, && p \in P \\
& f^r_p = \sum_{c \in C} \sum_{w \in W} w^x_{p,w,c} r^{\text{base}}_{w,c}, && p \in P
\end{aligned}
$$

Postmen Rates Problem - custom logic

Example 1

Carrier c_1 gives discount d_1 on all the parcels if and only if there are more than 2 parcels such that their weight is greater or equal than $\frac{1}{2}$kg (w').

Example 1 - formalized

$$\text{subject to} \quad \sum_{w \in W} 2d_{p,c_1,d_1,w} \leq \sum_{p' \in P} w^x_{p',w',c_1}, \quad p \in P$$

Overview

Traveling Salesman Problem

Definition

Given n locations, find a path that allows us to visit all locations exactly once and overall cost of the tour is minimal.

Input Data

$$\delta_{i,j} = \text{Cost of moving from location } i \text{ to location } j$$

Decision Variables

$$x_{i,j} = \begin{cases} 1 & \text{The path goes from location } i \text{ to location } j \\ 0 & \text{Otherwise} \end{cases}$$

Traveling Salesman Problem

Traveling Salesman Problem

Model

decision $x_{i,j} \in \{0, 1\}, \quad (i, j) \in \{1, \dots, n\}^2$

minimize $$\sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{i,j} x_{i,j}$$

subject to $$\sum_{\substack{i=1 \\ i \neq j}}^{n} x_{i,j} = 1, \qquad j \in \{1, \dots, n\}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} x_{i,j} = 1, \qquad i \in \{1, \dots, n\}$$

Traveling Salesman Problem

Figure: Total distance 0km

Traveling Salesman Problem

Flow formulation

For each location entering flow and exiting flow are defined. Sum of exiting flows must be equal to sum of entering flows minus one, since part of the flow is left at each location.

Flow Input Data

$$\vartheta \in \{1, \ldots, n\} \quad \text{Starting location}$$

Flow Decision Variables

$$u_{i,j} \in \mathbb{Z} \quad \text{Amount of flow moving from } i \text{ to } j$$

Traveling Salesman Problem

Flow Conservation Constraints

$$
\begin{aligned}
\text{subject to} \quad & u_{i,j} \leq x_{i,j}(n-1), && (i,j) \in \{1,\dots,n\}^2 \\
& \sum_{\substack{j=1 \\ j \neq i}}^{n} u_{j,i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} u_{i,j} + 1, && i \in \{1,\dots,n\} \\
& \sum_{\substack{j=1 \\ j \neq \vartheta}}^{n} u_{j,\vartheta} + n = \sum_{\substack{j=1 \\ j \neq \vartheta}}^{n} u_{\vartheta,j} + 1
\end{aligned}
$$

Traveling Salesman Problem

Figure: Total distance 9107.2km

Overview

Vehicle Routing Problem

Definition

Given n locations to be visited and a fleet of v available vehicles, find a schedule ensures every location is visited by exactly one vehicle and overall cost of all tours is minimal.

Vehicle Routing Problem

Definition

Given n locations to be visited and a fleet of v available vehicles, find a schedule ensures every location is visited by exactly one vehicle and overall cost of all tours is minimal.

Difference from TSP

Instead of one salesman we have multiple salesmans... Concept of *fleet*.

Vehicle Routing Problem

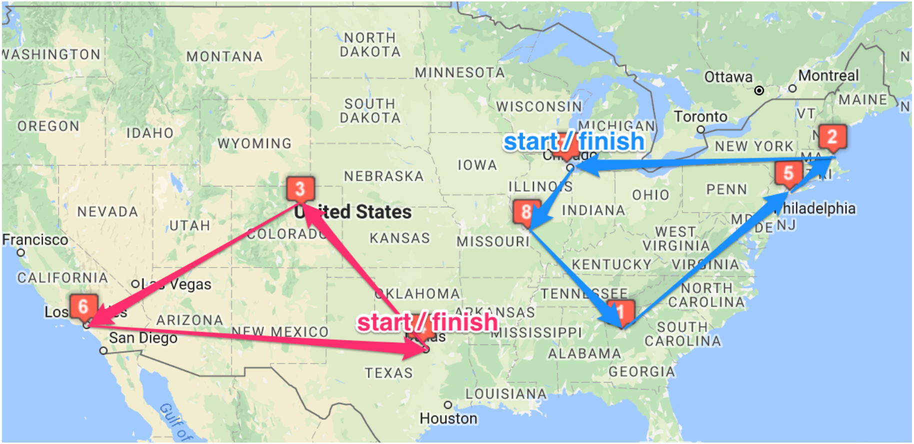

Figure: Blue: 4107.8 km, Red: 4400.4 km

Overview

Stochastic Freight Scheduling

Definition

Given set of scenarios of freight demands with associated probabilities as well as given set of possible routes and available vehicles find a schedule that minimizes the costs whatever the scenario occurs.

Stochastic Freight Scheduling

Definition

Given set of scenarios of freight demands with associated probabilities as well as given set of possible routes and available vehicles find a schedule that minimizes the costs whatever the scenario occurs.

Difficulty

We need to schedule the flight before we know our orders!

Stochastic Freight Scheduling

Model

J. M. Mulvey, and A. Ruszczynski.
A New Scenario Decomposition Method for Large Scale Stochastic Optimization.
Technical Report SOR-91-19, Dept. of Civil Engineering and Operations Research, Princeton Univ. Princeton, N.J. 08544.

References

- GitHub repository with examples used in this presentation
  https://github.com/marekyggdrasil/optilogistics
- Online Mathematical Programming in GNU MathProg
  https://www3.nd.edu/~jeff/mathprog/
- GNU MathProg Reference Handbook
  https://www3.nd.edu/~jeff/mathprog/glpk-4.47/doc/gmpl.pdf
- GLPK Reference Handbook
  http://kam.mff.cuni.cz/~elias/glpk.pdf

Thank you!