Fracture toughness-based models for damage simulation of pultruded GFRP materials

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Abstract

This paper presents a numerical study about the efficiency of implementing fracture toughness (G_c) as an input to the simulation of damage progression in pultruded glass fibre reinforced polymer (GFRP) materials. The G_c properties implemented in numerical modelling were determined through Compact Tension (CT) and Wide Compact Tension (WCT) fracture experiments conducted recently by the authors. The numerical models of those tests were developed in Abaqus software, using both built-in tools and user-defined material (UMAT) subroutines. The sensitivity of different damage parameters was assessed, taking into account the shape of the cohesive law (linear or exponential) and the ultimate transverse tensile stress, which ranged between the material strength, σ_u, determined through mechanical characterization tests, and the cohesive stress, σ_c, determined by assessing the initial slope in WCT fracture toughness results (with respect to the crack tip opening displacement). Validation of the numerical models was performed taking into account the experimental values of ultimate loads, softening slopes and crack growth rates. The experimentally based G_c results provided a good agreement between numerical and experimental results for all pultruded GFRP materials investigated. The best fit between numerical and experimental results was obtained for two sets of properties: (i) linear cohesive law and material strength; and (ii) exponential cohesive law and cohesive stress.

Keywords: A. Pultruded FRP composites; B. Fracture Toughness; C. Damage mechanics; C. Numerical analysis.

1. Introduction

The numerical simulation of damage in composite materials is still a challenging issue [1, 2]. This is particularly the case of pultruded fibre reinforced polymer (FRP) materials used in construction industry. These materials present higher levels of variability [3], lower fibre layup refinement and, furthermore, they have received less attention from the scientific community when compared to composites for automotive and aerospace applications [4]. Information about the intralaminar fracture properties of pultruded glass-FRP (GFRP) profiles is very useful towards the simulation of damage in finite element (FE) models. However, such properties have been assessed only in a very limited number of studies [5-8] due to the inherent complexity in determining fracture properties in an objective manner, in face of R-curve behaviour [4], size effects [9] and the complexity of the experimental methodology itself [10, 11]. In fact, several numerical works had no other option than to implement values found in the literature, often with no experimental basis [2, 12]. This is a significant limitation, given the considerable variability found in the fibre layup of pultruded GFRP profiles [8]. Some numerical studies have pursued a different approach, by calibrating the material fracture properties in order to obtain the best agreement between the test results and the numerical simulation [13-16]. This is a cumbersome and complex task, particularly in cases where different failure modes occur and interact.

Adding to the complexity of experimentally determining the critical energy release rate (G_c, also known as fracture toughness), several authors have reported that the material cohesive law must also be determined, in order to prevent specimen geometry dependencies in numerical simulations [9, 17]. Due to these problems, the authors recently conducted experiments to assess the fracture properties of different pultruded GFRP profiles [7, 8]. These previous experimental investigations are briefly summarized ahead. It should be highlighted that determining the fracture toughness at the laminate level was the main goal of these previous studies; however, the cohesive laws were also assessed, by monitoring the energy release rate (G) with respect to the opening displacement at the crack tip, until the stable propagation stage was reached and $G = G_c$ [11, 18].

The main objective of the present paper is to validate experimentally-based fracture properties by implementing such properties into FE models within software Abaqus [19]. Validation was performed by comparing load *vs*. displacement curves from numerical models and experimental fracture tests reported in [7, 8]. The numerical models developed in this study are based on a continuum damage formulation that implements the Hashin criterion [18] to determine damage initiation and on two alternative cohesive laws for damage evolution: (i) a linear cohesive law, using built-in tools available in Abaqus; and (ii) an exponential cohesive law, implemented with user defined material subroutines (UMAT) in Abaqus. After presenting and discussing the comparison between numerical and experimental results, some conclusions are drawn about the applicability of the developed laws and models.

2. Summary of previous experiments

2.1. Materials

A total of five materials were studied within the scope of this work. The experimental tests are briefly discussed ahead and further detailed in [7, 8]. The materials are described by the type of cross-section (I, U or plate), its height and manufacturer. One I-section profile was provided by Fiberline Composites (FC), another was acquired from Creative Pultrusions (CP) and two others (I- and U-sections) were acquired from STEP, Sociedade Técnica de Estruturas Pultrudidas (ST). Finally, one plate was acquired from Alto Perfis Pultrudidos (AP). Regarding the reinforcement in the transverse direction, these materials can be decomposed into three different fibre layups, determined through burn-off tests: (i) continuous filament mats (CFM) with randomly oriented fibres; (ii) cross-ply materials, with transverse reinforcements made of woven fibres oriented at 0º and 90º (W[0/90]); and (iii) quasi-isotropic layups, with transverse reinforcements consisting of 45º and 90º oriented fibres (Q). These categories are summarized for all materials in Table 1.

One material that had been studied in [8] (I150-AP) was not included in the present paper due to the uncertainties about its elastic and strength properties. The relevant geometric and mechanical properties of all five materials considered herein are summarized in Table 1, including their elastic and strength properties.

2.2. Specimen geometries

The present study addresses three specimen geometries: baseline Compact Tension (CT), scaled-up CT and Wide Compact Tension (WCT) (the DCB test was not considered as it was primarily developed for interlaminar fracture and its implementation to translaminar fracture would present complex challenges regarding the test-setup, requiring also the application of considerably higher loads). The

nominal geometries of these test configurations are displayed in Figure 1, where a_0 is the initial notch length, measured from the centre of the loading hole, and Δa is the crack growth. The initial notch length in baseline CT tests was established as 30 mm (a_0=18 mm), corresponding to half of the specimen width (60 mm), in line with previous research on the CT test configuration [10]. This ratio was also considered for scaled-up CT tests. Finally, a 30 mm initial notch length was also considered for WCT tests, as these were developed to provide additional room for stable crack propagation. In addition to the aforementioned initial notch lengths, higher initial notch lengths were also considered for each test configuration, in order to assess potential geometry dependency issues on fracture toughness results. Each test configuration was tested at least for two different initial notch lengths (see Table 2).
The WCT specimen consists of a baseline CT specimen with the double of the width, whereas the scaled-up CT specimens are proportional to baseline CT tests. These scaled-up tests were only considered for one material, I152-CP, due to the unsuccessful results provided by WCT tests for this specific material. The remainder of the materials were assessed only by using baseline CT and WCT tests.

2.3. Test setup

The experimental tests were conducted under displacement control at a rate of 0.5 mm/min for baseline CT tests and 1.0 mm/min for scaled-up CT and WCT tests. These different displacement rates were considered in order to have similar test periods for all test configurations. The specimens were painted with white matte paint and targets were marked for videoextensometry, using a black marker pen. This methodology was used to monitor the crack mouth opening displacement (CMOD), the crack tip opening displacement (CTOD) and the crack growth (Δa). These measurements are illustrated in Figure 2.

2.4. Summary of experimental results

2.4.1. Ultimate loads

The experimental results reported in previous research conducted by the authors [7, 8] are summarized in Table 2, including the average ultimate loads for each test configuration and initial notch length (a_0). The results summarized in Table 2 reflect a significant diversity across the various materials, with the ultimate load varying from 1.5 to 4.5 kN in baseline CT tests (a_0 =18 mm).

2.4.2. Fracture toughness results

The G *vs.* CTOD results are displayed in Figure 3, taking into consideration the WCT and scaled-up CT tests. For more detailed results of baseline CT tests, see [7, 8].
Figure 3 shows two distinct trends, as four materials reach a stable propagation stage in the WCT tests (horizontal plateaus in Figure 3(a)), whereas the scaled-up CT results show an increasing trend throughout crack propagation (Figure 3(b)). Given these results, the I152-CP materials were modelled considering a conservative value of G_c = 160 N/mm, as several specimens reached this lower bound value. It is noteworthy that this material presents a significantly higher fracture toughness than the remainder, which is in line with its considerably higher ultimate transverse tensile stress (σ_{22}^{+}) – see Table 1. The determination of cohesive laws is illustrated in Figure 4, where the fitting functions (FF) are shown in Figure 4(a) and the resulting cohesive laws are illustrated in Figure 4(b). Three different data reduction methods are presented in Figure 4(b): (i) FE based J-integral (F-int); (ii) compliance calibration (CC); and (iii) modified compliance calibration (MCC).
The critical energy release rate results determined through the WCT test configuration, which were used as input in the numerical models, are presented in Table 3 [7, 8], alongside the cohesive stresses measured from the initial slope of the G *vs.* CTOD curves. The results reported in Table 3 pertain to specimens presenting lower initial notch lengths (a_0=18 mm), which are expected to be less susceptible to size effects, as the initial notch *vs.* specimen width ratio is lower [9].
Regarding the cohesive stress, all methods yield similar results. However, in what concerns G_c, two methods present similar results (J-integral and CC), which are based on visual measurements of crack

growth. In a different approach, MCC is based on measurements of compliance to determine the actual crack front position. All three methods present similar results before the stable propagation plateau, a point from which the MCC yields lower energy release rate results. In fact, Table 3 shows a significant difference between the MCC method and the remainder, with differences of G_c ranging from 20% to 30%, when compared to J-integral results. This is believed to be related to the loading/unloading cycles performed to measure the specimen compliance, which were necessary to perform MCC, but may be affected by pulled-out fibres. This hypothesis is tested in the numerical results section, by using as input G_c values from different methods.
Finally, the scaled-up CT tests yielded a cohesive stress of 184 MPa. It is noteworthy that the thinnest materials (PL300-AP and I152-CP) present the highest differences between material strengths and cohesive stresses, ranging from 50% to 86%, whereas in the other materials such differences range from 13% to 26% (considering the J-integral method). This difference between cohesive stress and material strength is related to the failure modes of each test: (i) the coupons tested to evaluate the material strength present a pure brittle failure (sudden collapse), associated with matrix cracking and delamination, while (ii) the specimens tested to evaluate the cohesive stress present a more progressive failure, governed by fibre bridging.

3. Numerical study

3.1. Overview

The numerical study detailed ahead was performed in three main stages: (i) a preliminary study, which aimed at validating the geometry of the models, the data reduction methods and the damage parameters to be used as input; (ii) a comprehensive study performed for all materials and test configurations, focusing on the comparison of experimental and numerical load *vs.* CMOD curves; and (iii) a more detailed study on damage propagation and numerical stress states, which focused on a lower number of test series. The numerical results section (Section 4) is thus divided into these three stages: (i) Preliminary study; (ii) Load *vs*. CMOD curves; and (iii) Damage propagation.
The models presented ahead are based on two different damage evolution models: (i) Abaqus Standard built-in tools [19], which include a linear damage progression law; and (ii) a UMAT model, in which an exponential cohesive law was implemented. Another parameter that was considered in the analysis was the transverse tensile strength (σ_{22}^{+}). Two different values were considered per material: (i) the material strength measured through mechanical characterization tests; and (ii) the cohesive stress measured through the initial slope of G *vs.* CTOD experimental results [7, 8]. These two sets of properties promote two different numerical analysis, (i) the effect that the shape of the cohesive law can have on the results, for the same ultimate stress level and G_c; and (ii) the effect of considering different ultimate stresses, for the same cohesive law and G_c.
Given these various damage parameters, the following nomenclature was used: (i) linear cohesive law, denoted by “L”; (ii) exponential cohesive law, denoted by “E”; (iii) models based on the material strength, labelled by “σ_u”; and (iv) models based on the cohesive stress, labelled by “σ_c”. Therefore, each numerical model is identified first by the cohesive law and then by the ultimate stress, resulting in a total of four possible combinations (L-σ_u, L-σ_c, E-σ_u and E-σ_c). As an example, WCT-I200-FC-E-σ_c denotes a WCT test of I200-FC material, simulated with a UMAT-based numerical model calibrated with an exponential cohesive law (E) and considering the cohesive stress (σ_c). Additionally, the initial notch length (a_0) may be added to these references in brackets, when relevant.

3.2. Geometry

The geometry considered for each model followed that of specimens used in the experiments, as illustrated in Figure 1. Therefore, the average experimental initial notch length of each experimental series was considered in the corresponding numerical model. This methodology was chosen to ascertain that the differences found between experimental and numerical results could be attributed to the material input parameters and not to geometry discrepancies. The notch tip shape was modelled to present a round shape, after the mesh sensitivity tests performed in the parametric study, as detailed ahead.

3.3. Material properties

The material properties considered in the FE models were those obtained from the mechanical characterization tests, given in Table 1. Being the main focus of this study, the transverse tensile fracture toughness (G_2^+) was based on the experimental results presented in Table 3. The transverse compressive fracture toughness G_2^- was assumed equal to G_2^+, with exception of the I152-CP model; here, this parameter was expected to be relevant, as the experimental tests showed a higher propensity for compressive failure, after significant crack growth lengths [8]. Therefore, the G_2^- value was calibrated as a function of the fitting of numerical and experimental load *vs.* CMOD curves. The longitudinal fracture properties G_1^+ and G_1^- were assumed equal to 100 N/mm (typical value found in the literature [2]), as they are expected to have a low impact on the results.

3.4. Finite element mesh

As a result of the parametric study detailed ahead, the models with built-in tools of CT tests were designed with a 0.5 mm FE size, whereas the remaining models were prepared with an average FE size of 1 mm, in order to reduce computational time. Finally, the models developed for damage evolution assessment also featured a 0.5 mm FE size, in order to improve the accuracy of damage growth tracking. All models were developed with CPS4 plane stress and full integration elements, as the geometry and loading are included in a bi-dimensional plane.

3.5. Boundary conditions

All boundary conditions were imposed at the loading holes through the "Coupling" tool. The centre of each loading hole was rigidly connected to the relevant semi-circle of its perimeter, in order to mimic the hard contact that occurred in experimental tests. One loading hole was horizontally and vertically restrained, whereas the other was horizontally restrained but vertically moved by an imposed displacement. This methodology was validated by comparing load *vs.* CMOD curves obtained from simulations and tests, as displayed ahead in the numerical results section (Section 4). Figure 5 illustrates the assessment of boundary conditions, through the comparison between CMOD *vs.* CTOD curves obtained from numerical analyses and experimental tests.

Figure 5 shows a nearly identical behaviour between the numerical curve and the corresponding experimental curves, with a non-linear trend that corresponds to damage evolution, followed by a more linear slope, corresponding to a stage where the crack tip area is fully damaged. These results validate the geometry of the model, the boundary conditions and the material properties used as input.

3.6. Damage initiation and evolution

3.6.1. Failure initiation criteria

The Hashin criterion [20] was employed to determine damage initiation. This criterion, readily available in commercial software, has been widely adopted in previous studies. The Hashin criterion includes four different damage components: (i) fibre tension, d_f^t; (ii) fibre compression, d_f^c; (iii) matrix tension, d_m^t; and (iv) matrix compression, d_m^c. The failure initiation criterion (F) for each damage component is detailed below,

$$F_f^t = \frac{\hat{\sigma}_1^2}{X_t^2} + \alpha \frac{\hat{\tau}_{12}^2}{S_L^2} < 1.0 \quad if \quad \hat{\sigma}_1 \geq 0 \tag{1}$$

$$F_f^c = \frac{\hat{\sigma}_1^2}{X_c^2} < 1.0 \quad if \quad \hat{\sigma}_1 < 0 \tag{2}$$

$$F_m^t = \frac{\hat{\sigma}_2^2}{Y_t^2} + \frac{\hat{\tau}_{12}^2}{S_L^2} < 1.0 \quad if \quad \hat{\sigma}_2 \geq 0 \tag{3}$$

$$F_m^c = \frac{\hat{\sigma}_2^2}{4S_T^2} + \left(\frac{S_{c,2}^2}{4S_T^2} - 1\right)\frac{\hat{\sigma}_2}{Y_c} + \frac{\hat{\tau}_{12}^2}{S_L^2} < 1.0 \quad if \quad \hat{\sigma}_2 < 0 \tag{4}$$

where X_t and Y_t are the tensile ultimate stresses of the fibre and matrix, respectively; X_c and Y_c are the compressive ultimate stresses of the fibre and matrix, respectively; S_L corresponds to the longitudinal shear ultimate stress; S_T is the transverse shear ultimate stress; finally, α determines the influence of shear stresses in fibre tensile failure. In this study, this interaction is not relevant as longitudinal stresses have no influence in the experimental failure modes, and thus α was set to zero. The effective stress $\hat{\sigma}$ is computed using the linear transformation presented in equation (5),

$$\begin{Bmatrix} \hat{\sigma}_1 \\ \hat{\sigma}_2 \\ \hat{\tau}_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{1-d_f} & 0 & 0 \\ 0 & \frac{1}{1-d_m} & 0 \\ 0 & 0 & \frac{1}{1-d_s} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{5}$$

where d_f, d_m and d_s are damage variables, as described in the following section.

3.6.2. Damage evolution

The damage variables corresponding to the fibre (d_f), matrix (d_m) and shear (d_s) failure are defined in equations (6), (and (8). The shear damage variable is indirectly determined as a function of the remaining damage variables.

$$d_f = \begin{cases} d_f^t & if \quad \hat{\sigma}_1 \geq 0 \\ d_f^c & if \quad \hat{\sigma}_1 < 0 \end{cases} \tag{6}$$

$$d_m = \begin{cases} d_m^t & if \quad \hat{\sigma}_2 \geq 0 \\ d_m^c & if \quad \hat{\sigma}_2 < 0 \end{cases} \tag{7}$$

$$d_s = 1 - (1 - d_f^t)(1 - d_f^c)(1 - d_m^t)(1 - d_m^c) \tag{8}$$

After damage initiation, the elastic constitutive relation is rewritten as follows:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \frac{1}{D} \begin{bmatrix} (1-d_f)E_{11} & (1-d_f)(1-d_m)\nu_{21}E_{22} & 0 \\ (1-d_f)(1-d_m)\nu_{12}E_{11} & (1-d_m)E_{22} & 0 \\ 0 & 0 & (1-d_s)G_{12}D \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \tag{9}$$

$$D = 1 - (1 - d_f)(1 - d_m)\nu_{12}\nu_{21} \tag{10}$$

where, σ and τ represent the stresses, ε represents the strains, E is the elastic modulus in either the longitudinal (1) or transverse (2) directions. Finally, G_{12} and ν_{12}, ν_{21} correspond respectively to shear modulus and Poisson ratios.

Viscous regularization was implemented to facilitate convergence in both conventional and UMAT Abaqus models. A value of 1E-4, validated in previous studies [21, 22], was found to produce adequate results.

3.6.3. Cohesive laws

As mentioned, cohesive laws with two different shapes were considered: (i) linear; and (ii) exponential. The difference between these two formulations is qualitatively illustrated in Figure 6 and is further detailed ahead in equations (11) and (12).

3.6.3.1. Linear Softening

The most straightforward damage evolution law is a linear cohesive law. A typical formulation for such a cohesive law is given below,

$$d_i = \frac{\delta_{eq}^{u}(\delta_{eq}-\delta_{eq}^{0})}{\delta_{eq}(\delta_{eq}^{u}-\delta_{eq}^{0})} \quad \text{if} \quad \delta_{eq}^{0} \leq \delta_{eq} \leq \delta_{eq}^{u} \tag{11}$$

where d_i is a given damage variable, δ_{eq} is the current equivalent displacement, δ_{eq}^{0} is the equivalent displacement at damage initiation, and δ_{eq}^{u} is the equivalent displacement at $d_i = 1$.

3.6.3.2. Exponential Softening

As presented in previous sections, an exponential cohesive law should be more representative of the actual damage propagation in composite materials [9, 17]. As this option is not currently available for orthotropic materials in Abaqus built-in tools, this law was taken into account in the user-defined material (UMAT) subroutines by implementing the following equation,

$$d_i = 1 - \frac{\delta_{eq}^{0}}{\delta_{eq}} e^{\left[-\frac{\sigma_{eq}^{0}}{G_c}(\delta_{eq}-\delta_{eq}^{0})\right]} \quad \text{if} \quad \delta_{eq}^{0} \leq \delta_{eq} \tag{12}$$

where G_c is the critical energy release rate, and σ_{eq}^{0} is the stress level at damage initiation.

4. Numerical results

4.1. Preliminary study

4.1.1. Parametric study

A preliminary parametric study was performed for one of the materials (I200-FC) and one of the test configurations (baseline CT), with the goal of validating the element mesh size and notch tip shape. The mesh size was evaluated by developing L-σ_u models (linear cohesive law with material strength) with FE sizes varying from 0.25 to 1.00 mm. The notch tip was modelled with two different shapes: (i) a square shaped notch, similar to the CT test specimens, and (ii) a semi-circular shape, similar to the WCT test specimens [7, 8]. Figure 7 presents the FE meshes that were considered, as well as the different notch shapes.

Figure 8 presents the load *vs.* CMOD curves based on the meshes and notch tip shapes presented in Figure 7. The models with square shaped notch (Figure 8(a)) were found to have significantly higher mesh sensitivity when compared to those with round notch shape (Figure 8(b)). This sensitivity is visible not only in the difference between post-peak descending branches but also in the maximum load. Based on these results, the numerical models/results presented ahead are all based on round shaped notch tips. A more detailed study on mesh sensitivity of damage models for composite materials can be found in [22].

Figure 9 illustrates the load *vs*. CMOD curves for different cohesive laws and ultimate transverse tensile stresses for the WCT-I150-ST-a_0=28 mm (Fig. 9(a)) and WCT-PL300-AP-a_0=28 mm (Fig. 9(b)) models. It is shown that changing the cohesive law (L *vs*. E) affects the load *vs.* CMOD curves at damage initiation and up to the softening branch. It can also be seen that changing the value of the ultimate transverse tensile stress (σ_c *vs*. σ_u) affects a larger area of the load *vs.* CMOD curve, as damage initiation is also affected by this parameter. In Figure 9(a), the influence of changing between material strength (σ_u=34 MPa) and cohesive stress (σ_c=41 MPa) leads to a slightly lower increase of ultimate load than that caused by changing from exponential (E) to linear (L) cohesive laws. On the contrary, Figure 9(b) shows a case where considering the cohesive stress (σ_c=132 MPa) instead of the material strength (σ_u=71 MPa) has a more significant effect, with the E-σ_c model providing higher loads than the L-σ_u model.

Figure 9 clearly shows that both damage parameters can have a significant impact in the numerical load *vs.* CMOD curves. It is also noteworthy that the influence of these parameters is different across different materials: (i) changing from linear (L) and exponential (E) cohesive laws has a similar effect in different materials; (ii) changing between the material strength (σ_u) and cohesive stress (σ_c) can have significantly different effects, as a function of the σ_c/σ_u ratio (which varies among the materials tested).

4.1.2. Data reduction methods

Before developing the models for all the materials and test configurations, a preliminary assessment was performed in order to select the most suited data reduction method. In this regard, as J-integral and CC results are quite similar, a comparison was only performed for J-integral and MCC results. The results shown in this section were obtained using Abaqus built-in tools and the L-σ_u model. Figure 10 presents a comparison between experimental and numerical load *vs.* CMOD curves, for four different materials.

In general, it is clear that the load *vs.* CMOD curves obtained from numerical analyses agree quite well with those obtained experimentally. Figure 10 also shows that considering the MCC-based critical energy release rate leads to underestimations of the ultimate load, which is particularly noticeable for PL300-AP and I150-ST materials (Figures 10 (b) and (c)). Therefore, the numerical results presented next are based on the FE based J-integral data reduction method. At this time, both the J-integral and CC methods are considered to be the most reliable among those tested in [7, 8], whereas the MCC method has been confirmed to be incompatible with loading/unloading cycles [8, 23].

4.1.3. Ultimate loads

In order to select the damage parameters that produce a better fit between numerical and experimental results, a preliminary study was performed, consisting of running a model for each combination of cohesive law shapes (L and E) and ultimate stresses (σ_u and σ_c). These numerical results were then compared to experimental results, in terms of ultimate loads. Table 4 presents a summary of ratios between numerical and average experimental ultimate loads, for both CT and WCT tests.

The results detailed in Table 4 clearly show that two numerical models stand out in terms of accuracy: (i) the linear cohesive law calibrated with the material strength (L-σ_u model), and (ii) the exponential cohesive law calibrated with the cohesive stress (E-σ_c model). The other two combinations consistently either overestimated (L-σ_c model) or underestimated (E-σ_u model) the experimental results. Given these results, the sections ahead only include numerical results obtained from the two most accurate models, L-σ_u and E-σ_c.

4.2. Load *vs.* CMOD curves

4.2.1. WCT tests

The WCT tests are presented firstly as they were used to determine both the fracture toughness and the cohesive parameters taken as input in the various models. It was thus expected that the numerical models would simulate accurately the experimental WCT tests. Figure 11 presents a summary of numerical and experimental load *vs.* CMOD curves for WCT tests on four different materials.

Figure 11 shows an overall good agreement for both L-σ_u and E-σ_c models; however, for the I200-FC and U150-ST materials, the models underestimate the softening slope (Figures 11 (a) and (d)). It is also noteworthy that the E-σ_c model presents a higher ultimate load than the L-σ_u model only in the simulation of the PL300-AP profile. This is due to the aforementioned higher discrepancy between the values of σ_c and σ_u for this specific GFRP material (132 *vs.* 71 MPa).

4.2.2. Baseline CT tests

The baseline CT tests pose a more relevant challenge to these numerical models, due to their significantly lower width (compared to WCT tests) and subsequent potential influence of size effects [9, 17]. This issue was previously addressed experimentally [7, 8], by comparing baseline CT and WCT predictions of G_c. The experimental results reported in [7, 8] showed that the G_c results obtained from CT tests were considerably higher than those derived from WCT tests, and thus CT tests were considered inaccurate. Figure 12 presents the numerical and experimental load *vs.* CMOD curves for baseline CT tests on our different profiles.

Figure 12 shows a good agreement between numerical and experimental results, especially regarding the ultimate loads. It is noteworthy that, unlike the previously shown WCT results, for some materials the numerical models seem to overestimate the softening slope. This discrepancy between WCT and CT numerical results may indicate some level of geometry dependency of the measured fracture properties.

4.2.3. Scaled-up CT tests

Figure 13 presents experimental and numerical load *vs.* CMOD curves of scaled-up CT tests. As mentioned, this test configuration was only implemented for the I152-CP profile and a conservative G_2^+ value of 160 N/mm was assumed because several test specimens reached this threshold. As the specimens presented a compressive dominated failure, G_2^- was calibrated to provide the best fit to experimental results, resulting in a value of 35 N/mm.

The load *vs.* CMOD results presented in Figure 13 seem to indicate that the value considered for G_2^+ (160 N/mm) may be lower than the actual G_c value, as the numerical ultimate loads are significantly lower than the experimental ones. This is in line with the experimental results reported in [8], which suggest that the G_c value of this material may reach up to 200 N/mm.

4.3. Damage propagation

4.3.1. Damage evolution *vs.* crack growth

In this subsection, a brief study on damage evolution is presented and validated by comparing numerical simulations with experimental crack growth measurements. Figure 14 presents a comparison between damage propagation obtained from simulations and crack growth measured from experimental tests, for different damage thresholds: (i) d_m^t>0, which means that a crack will open at the onset of damage initiation; (ii) d_m^t=1, which corresponds to assuming that a crack will only be visible after the material is fully damaged; and (iii) d_m^t≥0.5, which is an intermediate scenario. As the crack growth (Δa) experimental measurements began when the crack reached 1 mm of length, these thresholds were also considered for a damage propagation length of 1 mm in the numerical results.

Regarding the WCT-U150-ST (a_0=28 mm) series, Figure 14(a) shows crack growth *vs.* CMOD curves obtained from simulations and tests and Figure 14(b) shows damage initiation results plotted over the load *vs.* CMOD curves, also obtained from numerical analyses and experimental tests (representative specimen).

The results presented in Figure 14 indicate that the intermediate damage threshold (d_m^t≥0.5) provides a better fit in terms of crack initiation compared with the other two thresholds (d_m^t>0 and d_m^t=1). However, Figure 14(a) shows that there is a slight discrepancy between the experimental crack growth rate and the numerical damage evolution slope. This discrepancy is within an acceptable margin of error, as the models are homogenized through the thickness and thus do not take into account the heterogeneous layup of the material.

After this initial assessment, a damage threshold of 0.5 was considered in subsequent studies. Figure 15 present a comparison between Δa *vs*. CMOD curves for four different profiles obtained from baseline CT and WCT tests and the corresponding numerical simulations, considering L-σ_u and E-σ_c models.

Figure 15 illustrates a similar qualitative trend (identical Δa *vs.* CMOD slopes) but visible quantitative differences (up to ≈5 mm) between numerical damage evolution and experimental crack growth rates. In all materials, for both types of models considered (L-σ_u and E-σ_c) the numerical simulations slightly overestimate the experimentally determined crack lengths. However, given the simplifications assumed in these numerical models, the differences found can still be considered quite acceptable. Figure 15 also shows a reduced difference between the L-σ_u and E-σ_c models, in respect to damage evolution. However, a general trend can be identified, with E-σ_c models presenting a lower damage evolution rate and thus a better fit to experimental crack growth rates.

4.3.2. Damage and stress evolution ahead of the crack tip

This section presents an assessment of the damage evolution and stress states throughout the crack growth path. This study was performed for a length of 40 mm ahead of the crack tip, as this was the monitored length in fracture tests. Results obtained for the series WCT-P300-AP-a_0=18 mm are presented next, as an example.

Figure 16 presents the damage evolution (parameter d_m^t) throughout the length defined above, for different levels of CMOD. Figure 16 illustrates the difference between considering a linear and an exponential cohesive law (L-σ_u *vs.* E-σ_u), as well as the difference between considering the material strength and the cohesive stress (E-σ_u *vs.* E-σ_c).

In this specific material, the aforementioned significant difference between material strength and cohesive stress leads to a higher damage evolution rate for the L-σ_u model, when compared to the E-σ_c model. This result is in line with Figure 15(c), where the E-σ_c model presents a lower damage evolution rate. It is noteworthy that the different models provide relatively similar load *vs.* CMOD curves (Figure 16(b)), despite the different damage evolution trends (Figure 16(a)). Figure 16 also shows that the E-σ_u model presents the highest damage evolution rate, which is in line with its worse fit to experimental results, as summarized in Table 4.

Figure 17 illustrates the evolution of transverse stress (σ_{22}) with Δa, for the same CMOD levels presented in Figure 16 and obtained from the same models. As expected, the maximum transverse stresses shown in Figure 17(a) closely follow the damage propagation shown in Figure 16(a), in respect to Δa, as the transverse stress is progressively reduced behind the crack tip (damaged area). Figure 17 also shows that despite the similar load *vs.* displacement curves of L-σ_u and E-σ_c models, there can be significant differences in the stress states obtained from numerical analyses, as the E-σ_c results present a significantly higher peak stress, which also leads to a slower progression of damage.

Figure 18 presents, for both L-σ_u and E-σ_c models, the longitudinal (σ_{11}) and transverse (σ_{22}) stress contours near the crack tip area, for CMOD=1.8 mm (i.e., the last CMOD level presented in Figures 16 and 17). The different transverse peak stresses at the crack tip are easily noticeable. Furthermore, the E-σ_c model (Fig. 18(d)) seems to present a higher concentration of transverse stresses in a narrower area around the crack tip, when compared to the L-σ_u model (Figure 18(c)). This difference is in line with the different ultimate stresses considered for each model. Both models were calibrated with the same G_2^+ value (21 N/mm), but the E-σ_c model adopted the highest ultimate stress (132 > 71 MPa). This is why the E-σ_c model exhibits a more brittle response and thus, a narrower distribution of stresses around the crack tip. This result is also in line with the stress profiles depicted in Figure 17.

Among the pultruded GFRP materials tested, the material from the P300-AP profile presents the lowest longitudinal strength (σ_{11}^+=258 MPa) and is the only material where numerical longitudinal stresses have reached the material strength, with model E-σ_c (Figure 18(b)). However, no relevant numerical damage evolution was recorded on the longitudinal direction for this material. Therefore, the longitudinal fracture parameters (G_1^+ and G_1^-) were found to have no impact in these simulations. Furthermore, for all other materials, the longitudinal stresses were found to be considerably lower than the longitudinal material strength.

5. Discussion

The models presented above, calibrated with experimentally based fracture toughness parameters, provided numerical results that showed a good agreement with test results. It should be highlighted that these materials present a wide range of elastic and strength properties, and that no calibration was performed in these numerical simulations other than testing different cohesive laws and alternating between the material strength and the cohesive stress, for σ_{22}^+.

Despite this wide variety of materials, the simplified numerical methodology yielded ultimate loads with relative differences to test data that varied between -10% and +11% for L-σ_u models and between -12% and +15% for E-σ_c models. These results are within typical coefficients of variation exhibited by pultruded GFRP materials in material characterization tests [3]. It is also noteworthy that, despite leading to similar load *vs.* displacement curves, for some materials the L-σ_u and E-σ_c model results presented relevant differences in terms of transverse stress states (σ_{22}). These discrepancies may be used for further validation, by monitoring the strain fields in experimental tests (possibly through digital image correlation measurements) and comparing these data to numerical results.
There are also some less clear trends in the results. Firstly, there is a low impact in changing from a simplified model, calibrated with a linear cohesive law and material strength, to a more complex model with an exponential cohesive law and cohesive stress. The major differences between these models were only visible in terms of damage propagation and stress profiles ahead of the crack tip. Regarding damage propagation, the E-σ_c models showed a better fit to experimental results; however, the difference between L-σ_u and E-σ_c model results was low for the several studied materials.
As previously noted, the exponential cohesive law provided the best fit to experimental load *vs*. displacement curves, when calibrated with the cohesive stress measured through experimental fracture tests. This cohesive stress may fit well with the damage propagation patterns of fracture tests, however, it is less clear that it will correctly simulate cases where damage is more brittle, such as mechanical characterization tests, in which damage develops uniformly in a localized section of the specimen. In that instance, considering the cohesive stress should lead to numerical overestimations of the experimental ultimate load. Therefore, it is still unclear which is the better solution between L-σ_u and E-σ_c models for a generalised case. This discussion leads to the need of a more complex damage initiation and evolution analysis, which accounts for different types of damage, such as matrix cracking or fibre bridging, in respect to different ultimate stresses. This will be an important step towards a robust numerical methodology valid for more complex cases.
Another point of interest in the results pertains to the possible geometry dependency of the experimentally determined fracture properties. Despite the good agreement between numerical and experimental ultimate loads, across different test configurations, a diverging trend can be seen in the results: some models overestimated the softening slope of baseline CT tests and underestimated the softening slopes of WCT tests, for the same material. This issue, which would have been expected to be mitigated by including an exponential cohesive law [9, 17], should be further investigated by testing specimens with a wider range of geometries.

6. Conclusions

This numerical study has shown that the experimental determination of fracture toughness at the laminate level can be directly implemented into FE models, in order to simulate damage evolution in pultruded GFRP profiles. The fracture properties of five different pultruded GFRP materials, previously determined in [7, 8], were used as input and no further calibrations were performed. Despite this simplified approach, a good agreement between numerical and experimental ultimate loads was reached for all pultruded GFRP materials. The numerical results have thus shown that this numerical methodology is valid for simplified cases, such as fracture test specimens.
The numerical models included two cohesive law shapes, linear and exponential, besides two different values for the transverse ultimate tensile stress per material: (i) the material strength, determined through mechanical characterization tests; and (ii) the cohesive stress, measured from the initial slopes of G *vs*. CTOD curves of WCT tests [7, 8]. These evolution parameters led to a total of four different damage evolution sets of properties, of which two models were found to yield numerical ultimate loads with higher accuracy: (i) a model calibrated through a linear cohesive law and the material strength (L-σ_u); and (ii) a model calibrated through an exponential cohesive law and the cohesive stress measured through WCT fracture tests (E-σ_c). The L-σ_u models, in spite of being a more simplified methodology, yielded the overall best fit between numerical and experimental average

ultimate loads; whereas the E-σ_c models yielded the best fit between numerical damage evolution and experimental crack growth, when a numerical damage threshold of d_m^t≥0.5 was considered.
Despite the reported promising results, there were also some relevant questions raised that require further research: (i) the numerical results show a different fit to the softening stage of baseline CT and WCT load *vs.* displacement curves, which may indicate some level of geometry dependency of the experimentally determined fracture toughness; (ii) the applicability of the E-σ_c models to other experimental tests, namely mechanical characterization tests, should yield overestimations of the failure loads, as the cohesive stress was found to be significantly higher than the material strength in some materials.
In order to simulate more generalized and complex cases, these two topics should be addressed in the future, (i) analytically, through a more complex damage formulation, in particular regarding different failure modes connected to transverse tension, such as matrix cracking, delamination or fibre bridging; and (ii) experimentally, by applying this methodology to a wider experimental program in terms of test configurations and geometry ranges.

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