Decision Analysis with Uncertainty

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Agenda

- Review of basic probability theory
- Decision analysis under uncertainty (example)
- Value of information in decision analysis (continued example)
- Technical application of decision analysis

Basic Probability Theory

Basic Properties of Probability

1. Probabilities must lie between 0 and 1, i.e. for any occurance A, we have $0 \le P(A) \le 1$

2 . Sum of probabilities must equal one, i.e. the probabilities of a set of exhaustive and mutually exclusive outcomes must be summed to unity

$$P(A_1)+P(A_2)+.....+P(A_n)=1$$

Production Rule

The probability of a joint outcome is given by

$$P(A \wedge B) = P(A|B)P(B)$$
$$= P(B|A)P(A)$$

If A and B are independent of each other, we have

$$P(A|B)=P(A)$$

$$P(A \wedge B)=P(A)P(B)$$

Disjunction

The probability of A occurring or B occurring is given as

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

Especially when A and B are exclusive, it becomes

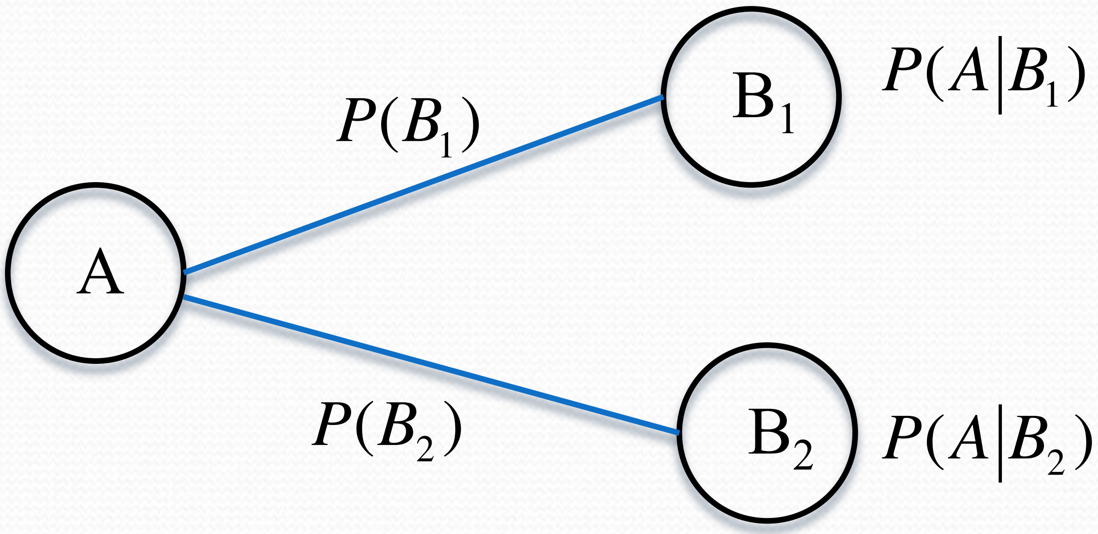
$$P(A \vee B) = P(A) + P(B)$$

Total Probability of an Event

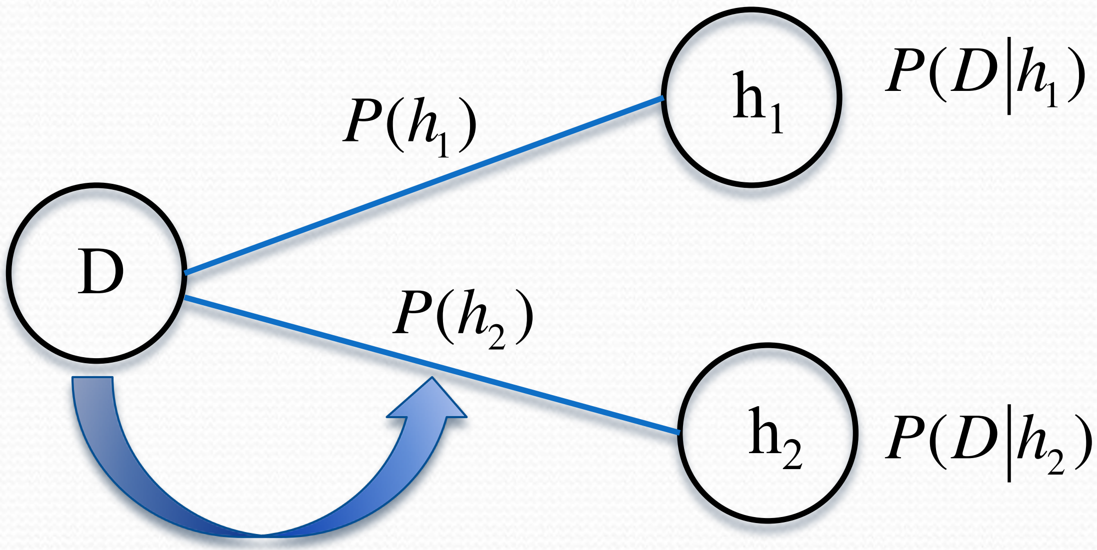
Let B_1, B_2, ..., B_n be a set of exhaustive and mutually exclusive situations, the probability of occurrence of A can be calculated as:

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

Weighted averaging of probabilities in different situations



Bayes Theorem



Given the observation of event of D, revise the beliefs on the hypotheses:

$$P(h_i \mid D) = P(h_i)P(D \mid h_i) / P(D)$$

Bayes Theorem

Let h_i be a hypothesis about something in the real world, the probability of h_i being true given the occurrence of event D is given by

$$P(h_i|D) = \frac{P(D|h_i)P(h_i)}{P(D)} = \frac{P(D|h_i)P(h_i)}{\sum_j P(D|h_j)P(h_j)}$$

P(D) : total probability of the event D (*evidence*)

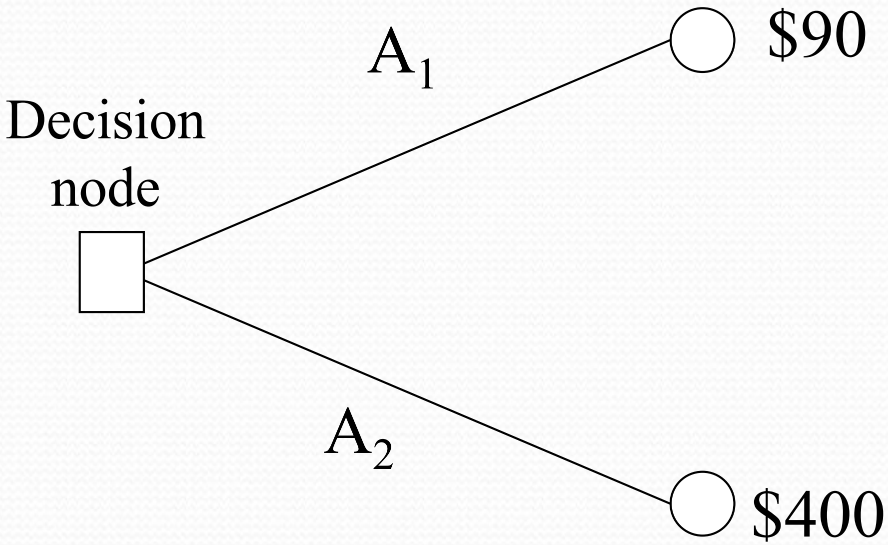
$P(h_i)$: prior probability of the hypothesis h_i

$P(h_i|D)$: posterior probability of the hypothesis given the event D

$P(D|h_i)$: probability of the event D given the hypothesis h_i, likelihood of D given h_i

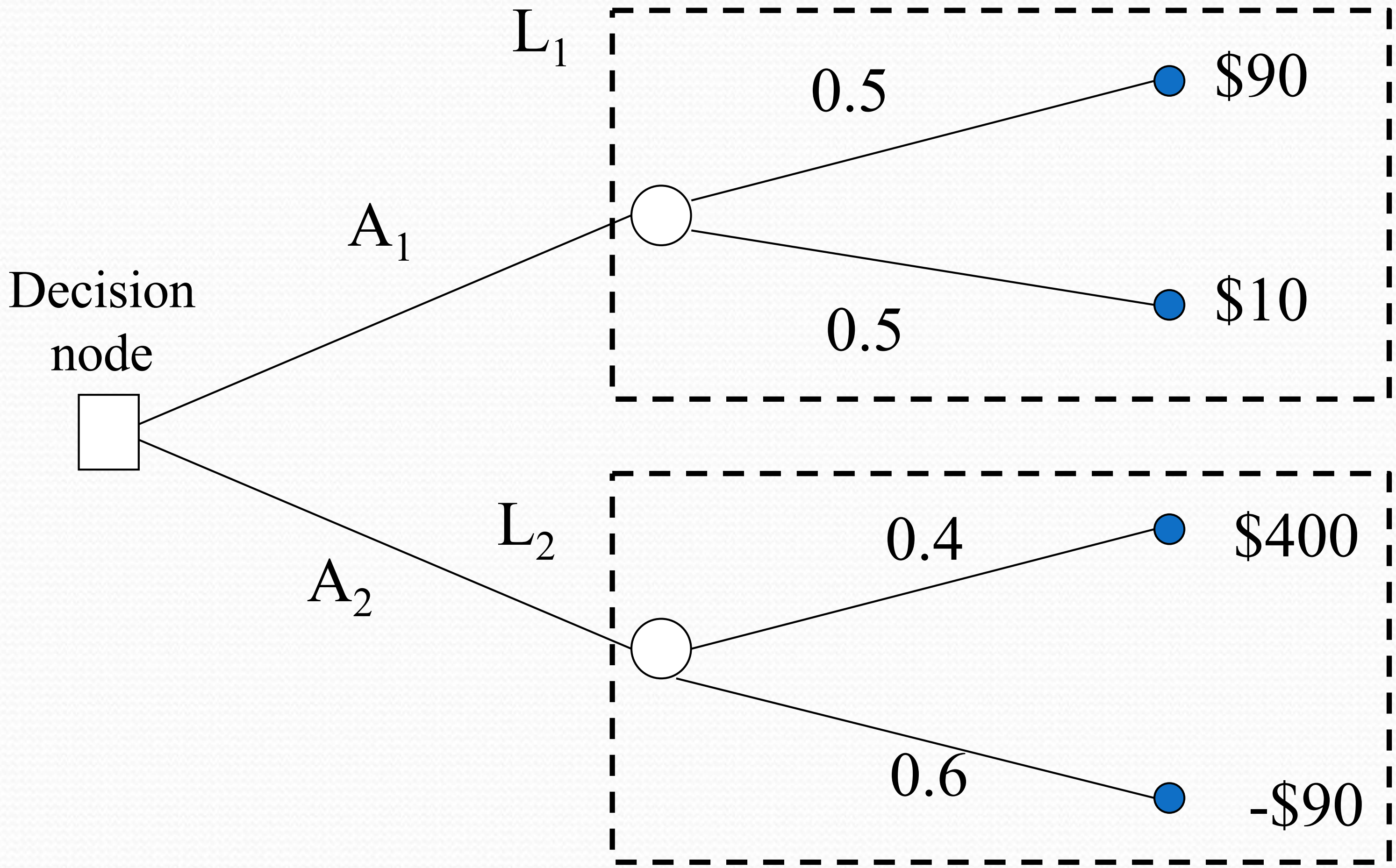
Decision Analysis under Uncertainty (with example)

Deterministic Decision Problem



Very easy decision: A2

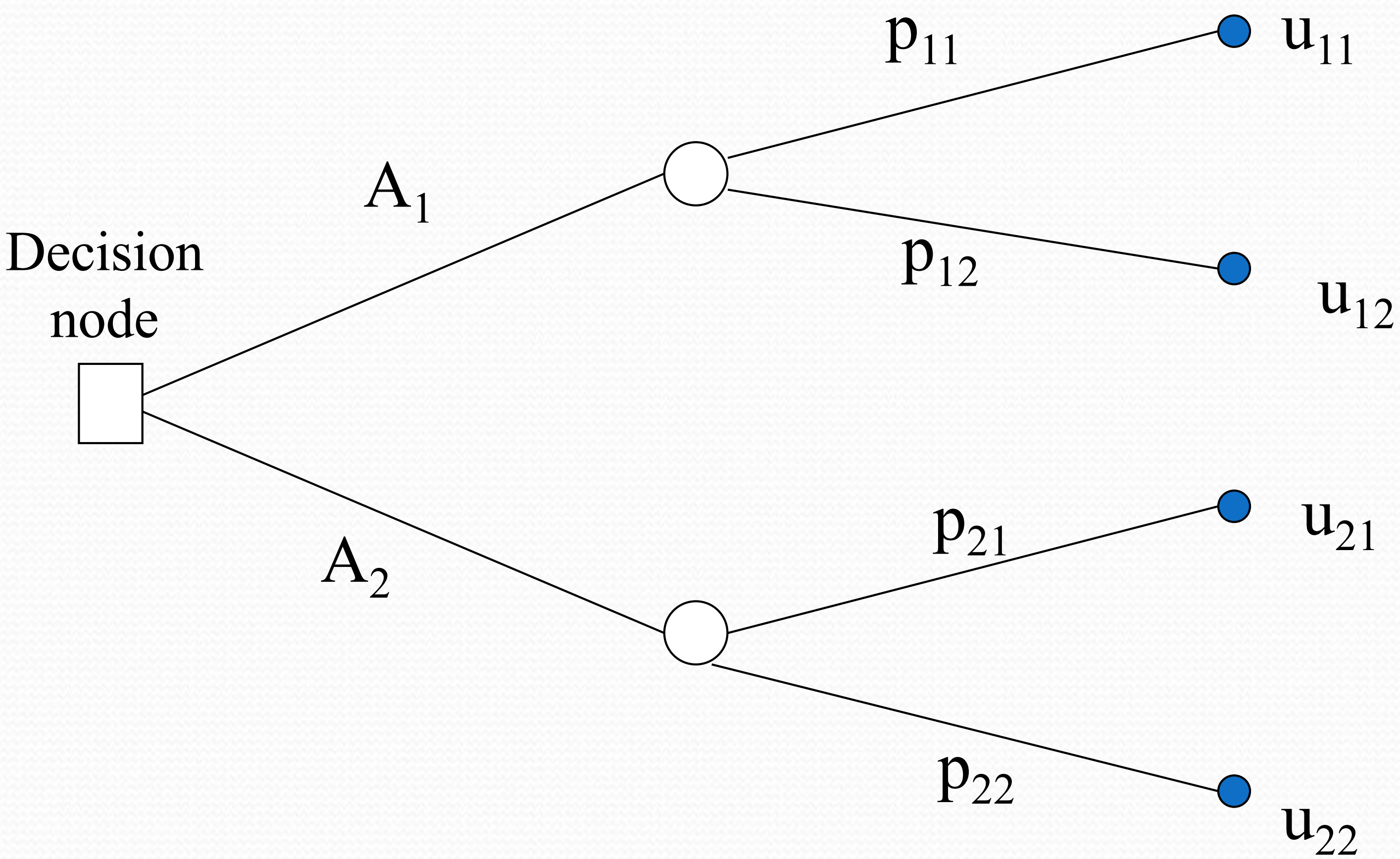
Uncertainty with Outcomes



$EU(A_1)=0.5 \cdot 90+0.5 \cdot 10=50$

$EU(A_2)=0.4 \cdot 400+0.6(-90)=106$

Decision Analysis Problem



Decision node : where decision maker decides the action

Chance node: where chance decides the outcome of an action

Bayesian Decision Theory

The principle of maximizing expected utility (MEU). In a given decision situation the decision maker should prefer the alternative with maximal expected utility

According to the MEU principle, we calculate

$EU(A_1)=u_{11}p_{11}+u_{12}p_{12}$ $EU(A_2)=u_{21}p_{21}+u_{22}p_{22}$

Then we prefer act A1 to act A2 provided that

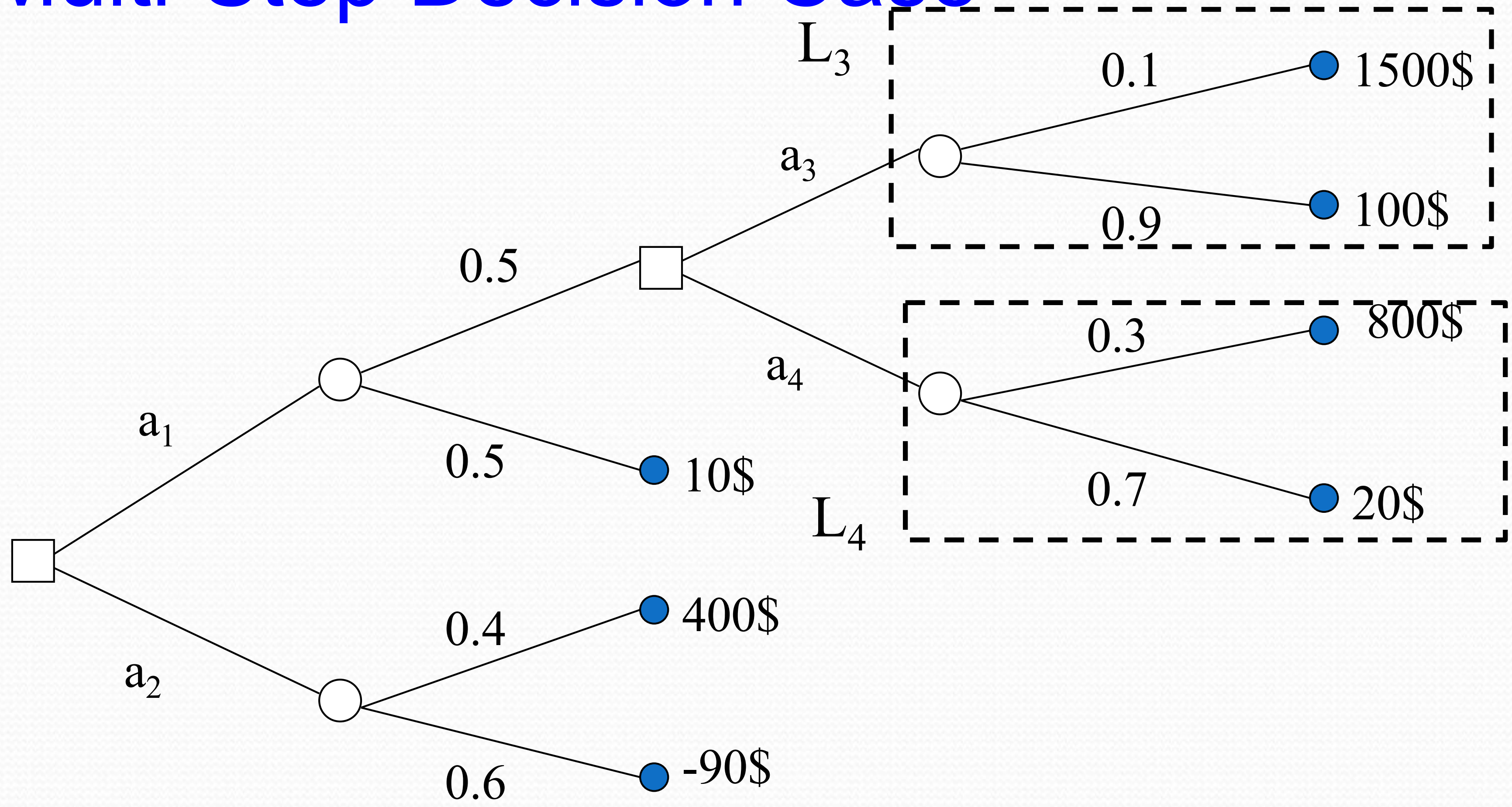
$$EU(A1)>EU(A2)$$

Meaning of the MEU Principle

- The expected utility of an act approximates the average utility achieved by playing the game many times and conducting that act repeatedly.

- The fundamental idea behind the MEU principle is to optimize performance in the long run

Multi-Step Decision Case

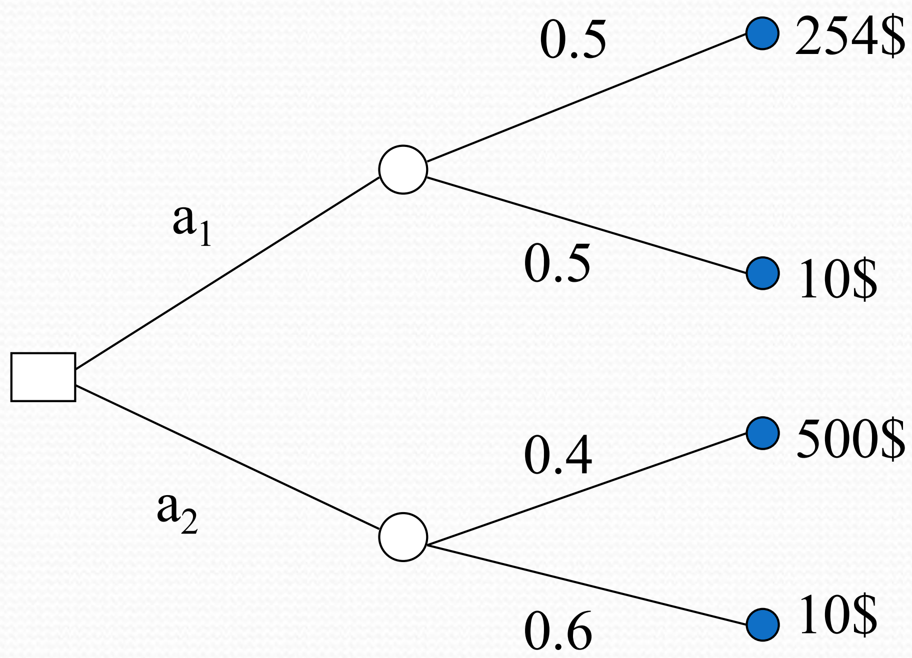


Analysis via folding back the decision tree

EU(a_3)=0.1 ×1500+0.9 × 100, EU(a_4)=0.3 × 800+0.7 × 20=254

The act a_4 will be preferred at the second decosion node

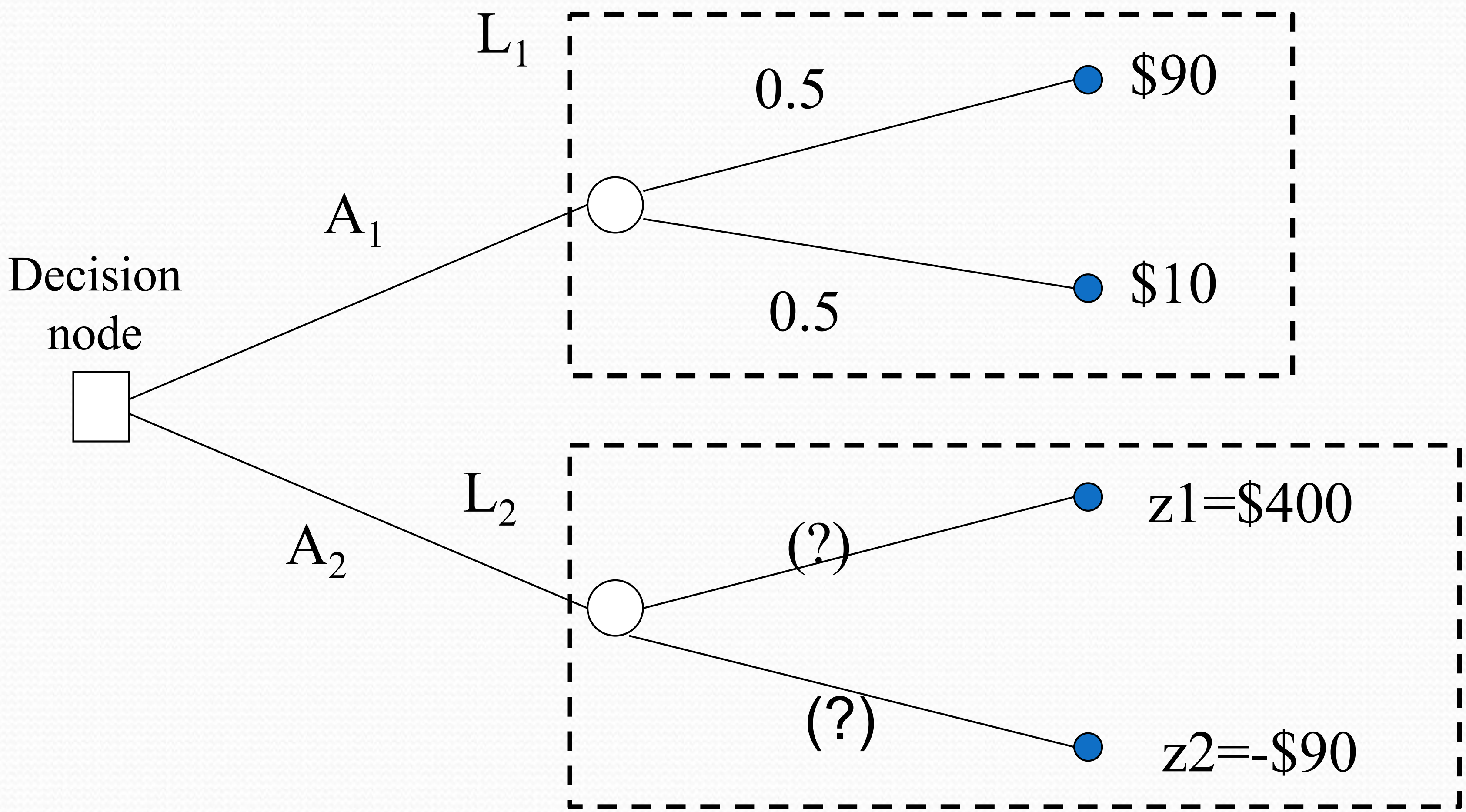
Replacing the Sub-Tree with Maximum Expected Utility



With this folding back scheme we can simplify any complex decision problem to the basic form as shown in the above figure.

Value of information in decision analysis (continued example)

Value of Information



To decide if the current awareness about situation should be improved by buying information

Buying Information?

Suppose the lottery L_1 is public with exactly probabilities for outcomes. Lottery L2 is run by an underground organization with outcomes probabilities decided by the head of the organization. When the head is in good mood, the probabilities are the same as before, otherwise the probability of wining $400 is zero.
A person in the head's family I going to sell the information about he head's mood. Should I buy his information?

P(BadMood)=0.3, P(GoodMood)=0.7

$P(z1)=P(z1|Bad)P(Bad)+P(z1|Good)P(Good)=0\times0.3+0.4\times0.7=0.28$

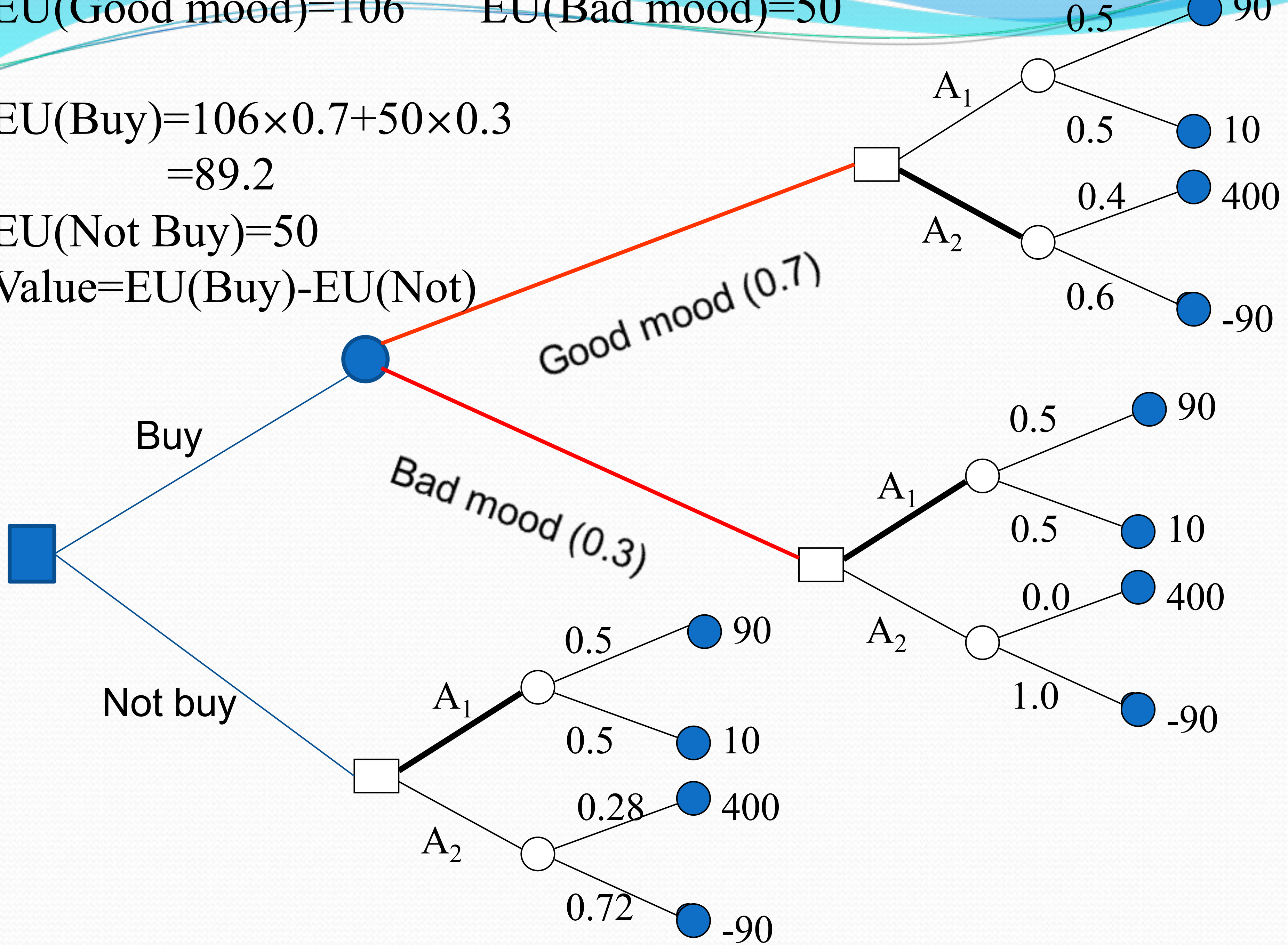
$P(z2)= P(z2|Bad)P(Bad)+P(z2|Good)P(Good)=1\times0.3+ 0.6\times0.7=0.72$

EU(Good mood)=106 EU(Bad mood)=50

EU(Buy)=106×0.7+50×0.3
=89.2

EU(Not Buy)=50

Value=EU(Buy)-EU(Not)



Value of Imperfect Information

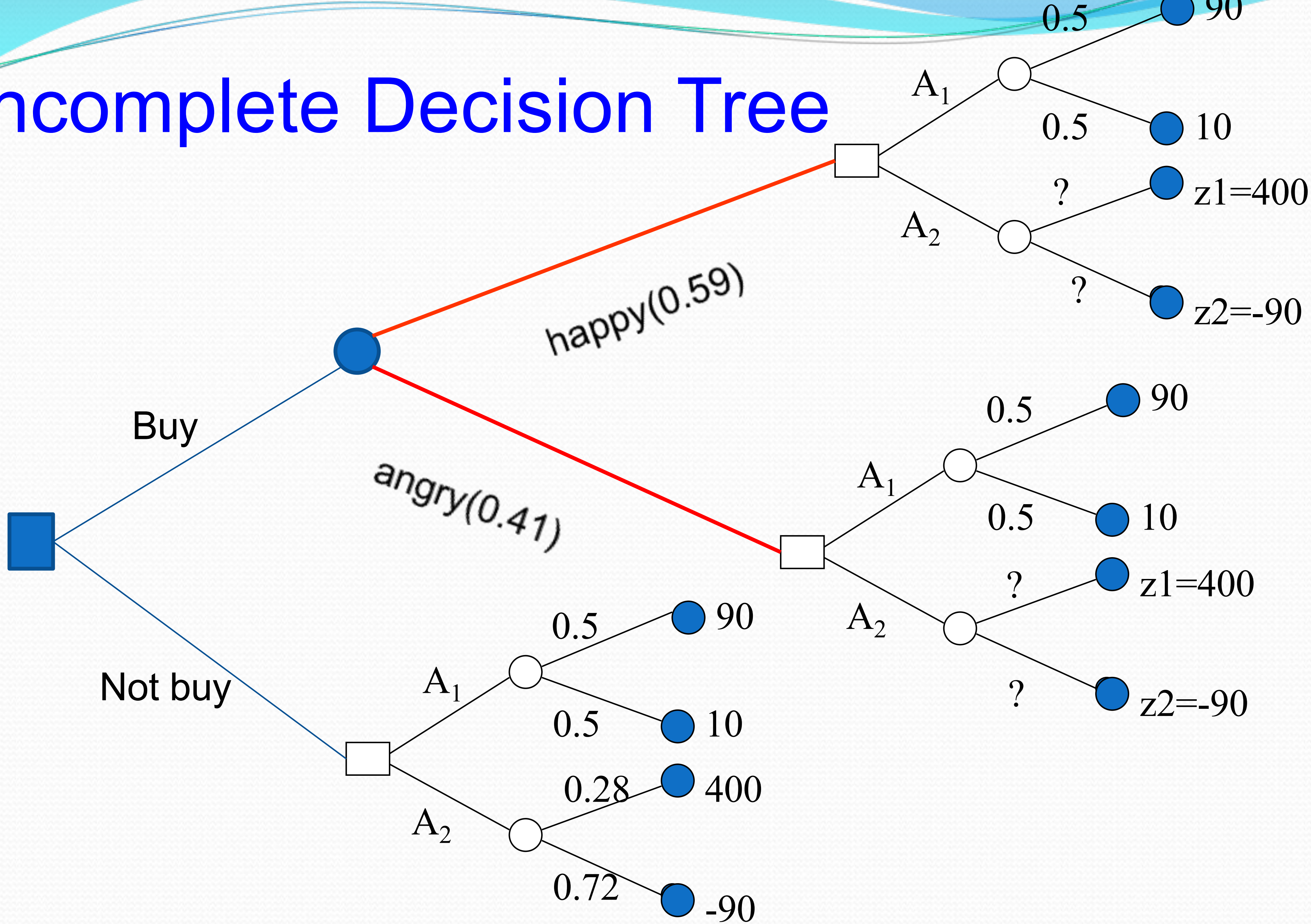
Suppose another person Mr. Loose who is loosely related to the head. He can offer the information about if the head is happy or angry. The conditional probabilities of happy and angry given the moods are shown below:

	good	bad
happy	0.8	0.1
angry	0.2	0.9

$$P(happy)=P(happy|good)P(good)+P(happy|bad)P(bad)$$
$$= 0.8\times0.7+0.1\times0.3=0.59$$

$$P(angry)=1-P(happy)=0.41$$

Incomplete Decision Tree



Revising the Estimates on Mood

	good	bad
happy	0.8	0.1
angry	0.2	0.9

	happy	angry
good	0.95	0.34
bad	0.05	0.66

P(good|happy)=P(good)P(happy|good)/P(happy)
=0.7×0.8/0.59=0.95

P(z2|happy)=P(z2|good)P(good|happy)+P(z2|bad)P(bad|happy)
=0.6×0.95+1×0.05=0.62

P(z1|happy)=0.38;   P(z1|angry)=0.136

P(z2|angry)=0.864

EU(happy)=96.2

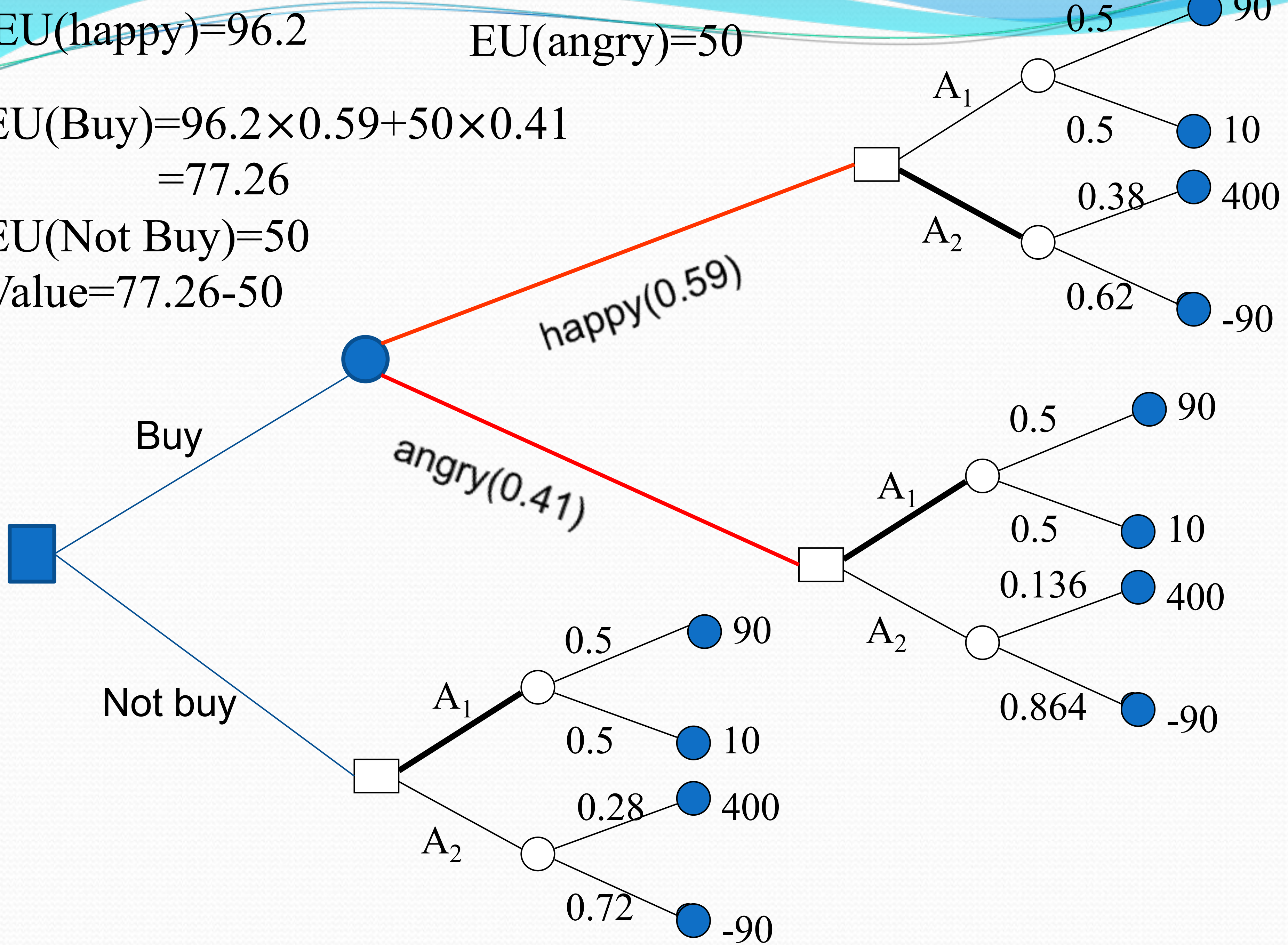
EU(angry)=50

EU(Buy)=96.2×0.59+50×0.41

=77.26

EU(Not Buy)=50

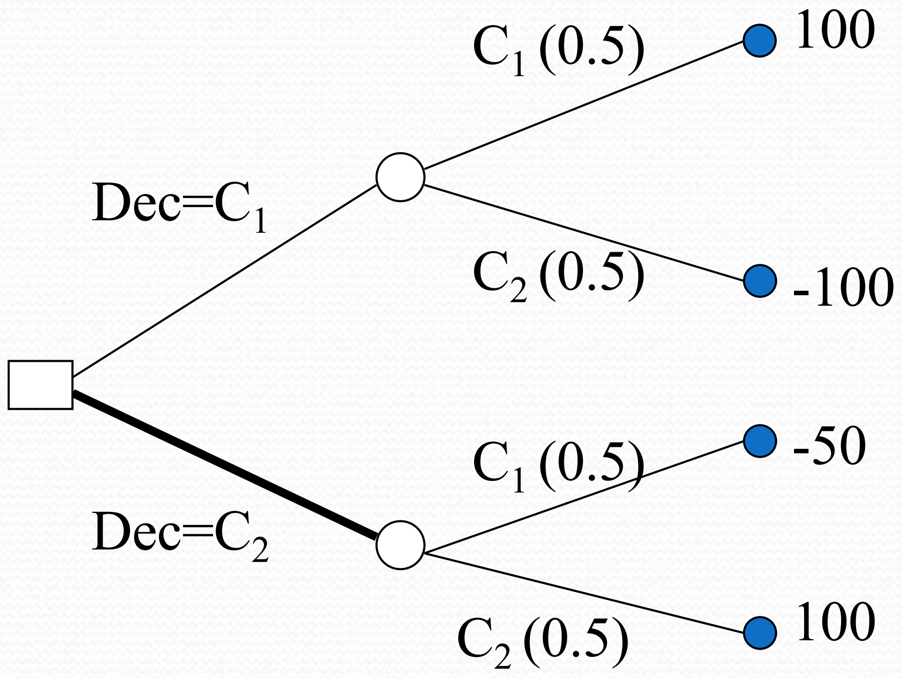
Value=77.26-50



Technical Applications of Decision Analysis

Classification as Decision Analysis

$P(C_1)=P(C_2)=0.5$
$EU(Dec=C_1)=0.5\times100+0.5(-100)=0$
$EU(Dec=C_2)=0.5(-50)+0.5\times100=25$



Considering a Test/Measurement

Assume a test with Positive or Negative result. The conditional Probabilities of results given the class are shown in table

	C_1	C_2
Positive	0.8	0.3
Negative	0.2	0.7

$$P(\text{positive})=P(\text{positive}|C_1)P(C_1)+P(\text{positive}|C_2)P(C_2)$$
$$=0.8\times0.5+0.3\times0.5=0.55$$
$$P(\text{negative})=1-P(\text{positive})=0.45$$

$$P(C_1|\text{positive})=P(C_1)P(\text{positive}|C_1)/P(\text{positive})$$
$$=0.5\times0.8/0.55=0.73$$

$P(C_2|\text{positive})=0.27$ $\quad$ $P(C_1|\text{negative})=0.22$

$P(C_2|\text{negative})=0.78$

EU(Positive)=46

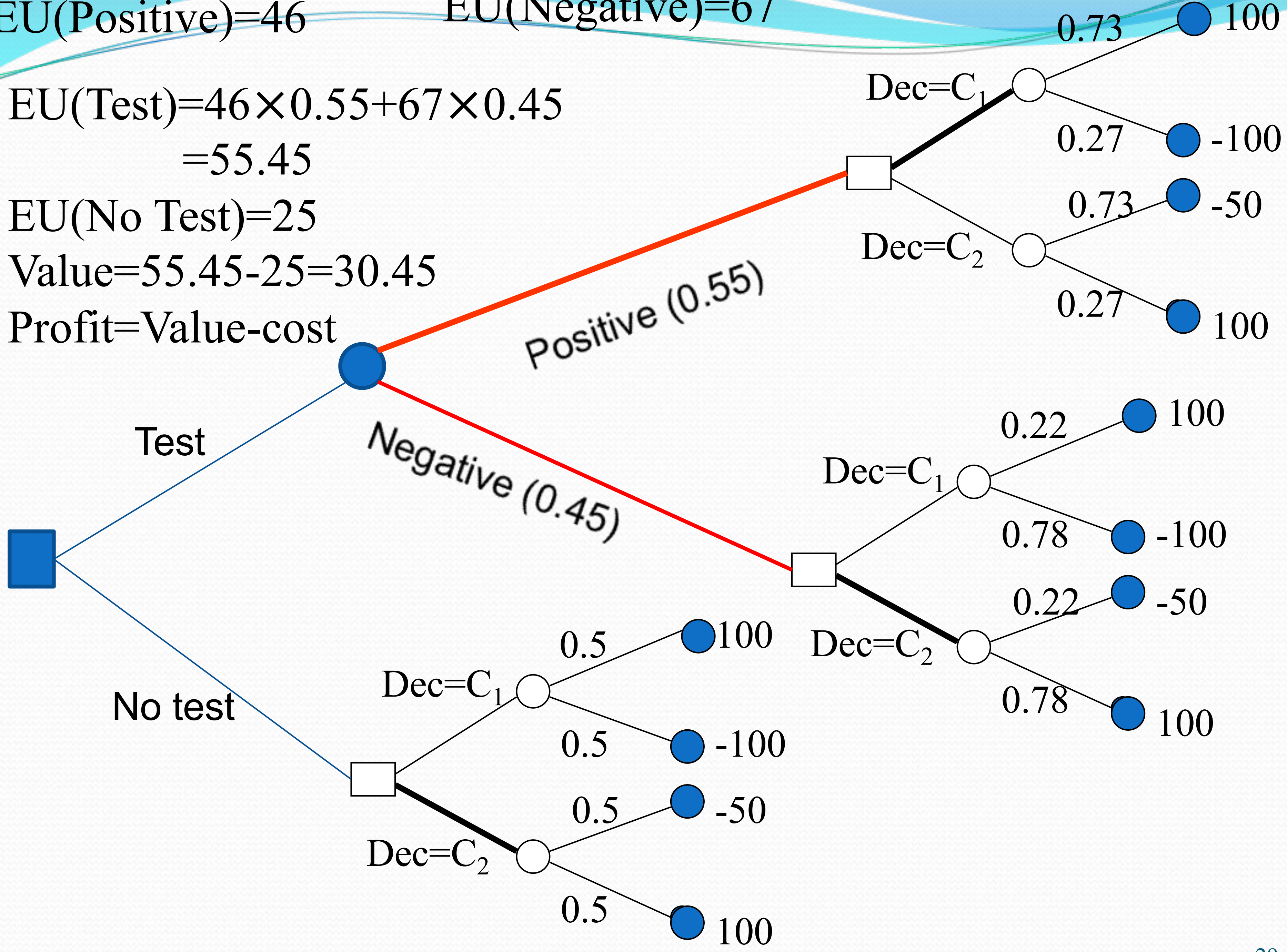
EU(Negative)=67

EU(Test)=46×0.55+67×0.45
=55.45

EU(No Test)=25

Value=55.45-25=30.45

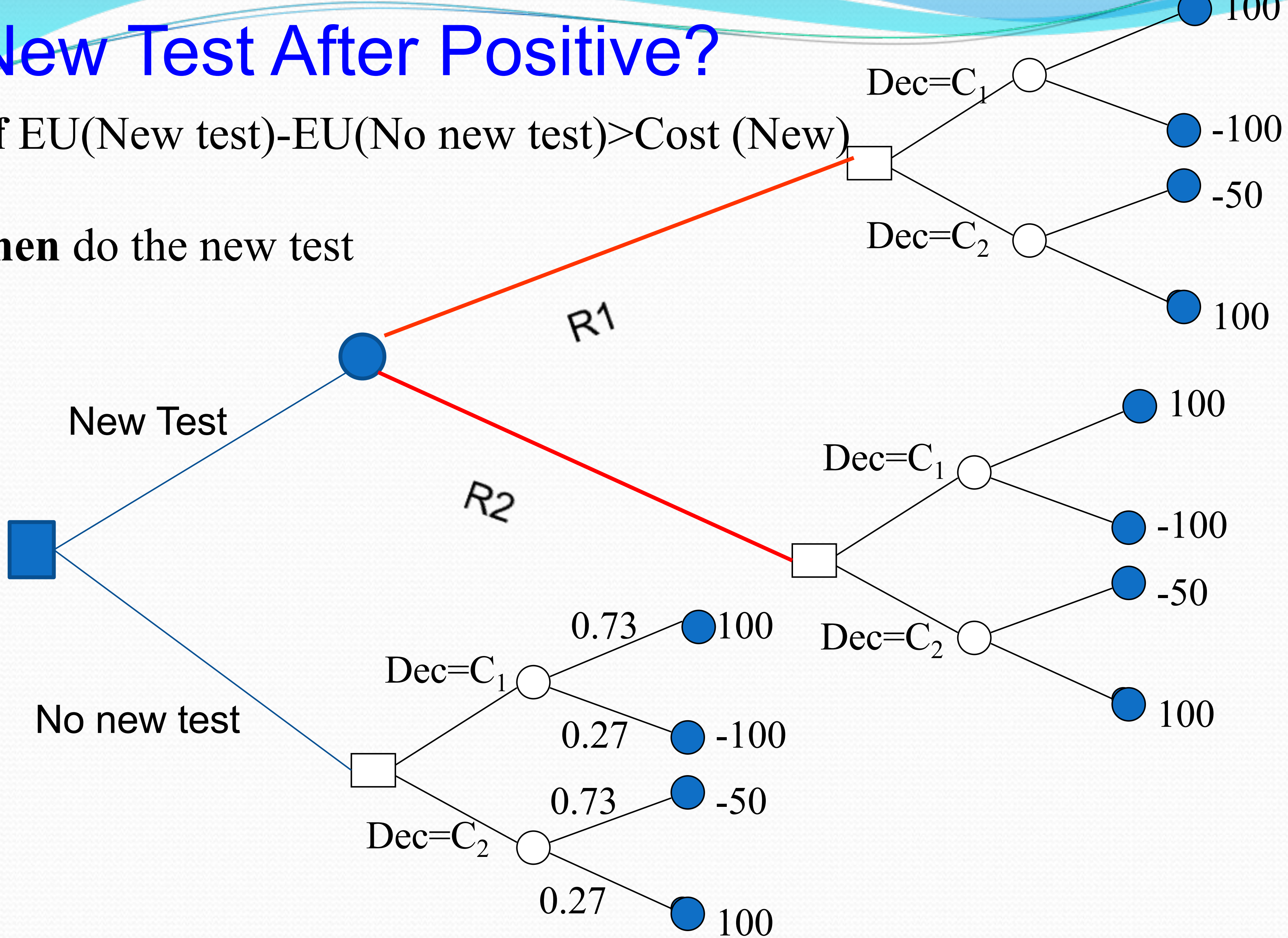
Profit=Value-cost



New Test After Positive?

If EU(New test)-EU(No new test)>Cost (New)

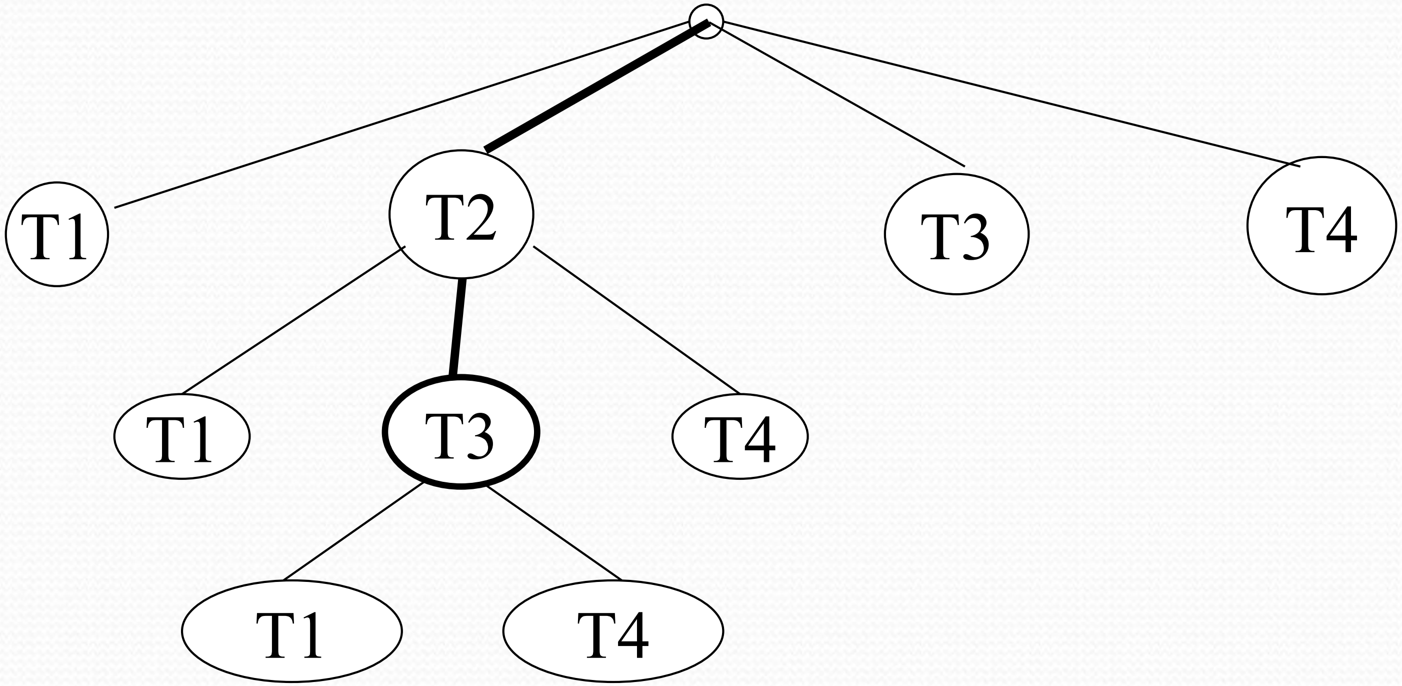
then do the new test



Active Measurements/Tests

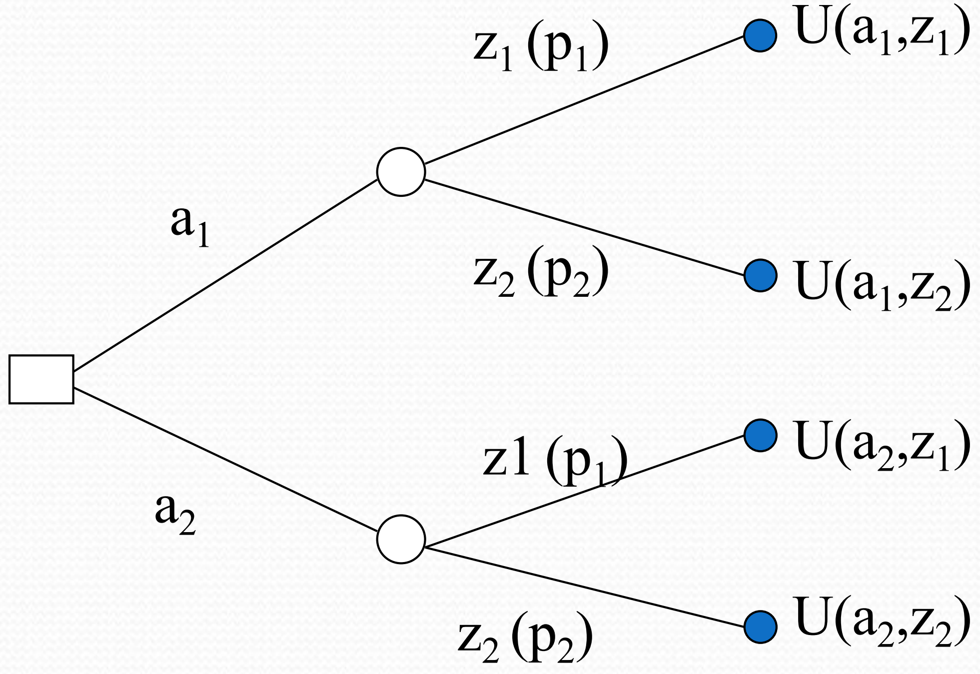
Suppose there are a set measurement alternatives T1, T2, ….., Tk. We can make a sequence of tests, every time choosing the worthy test giving the largest profit, until no test can produce more increase of utility than its own cost

Hill-climbing search



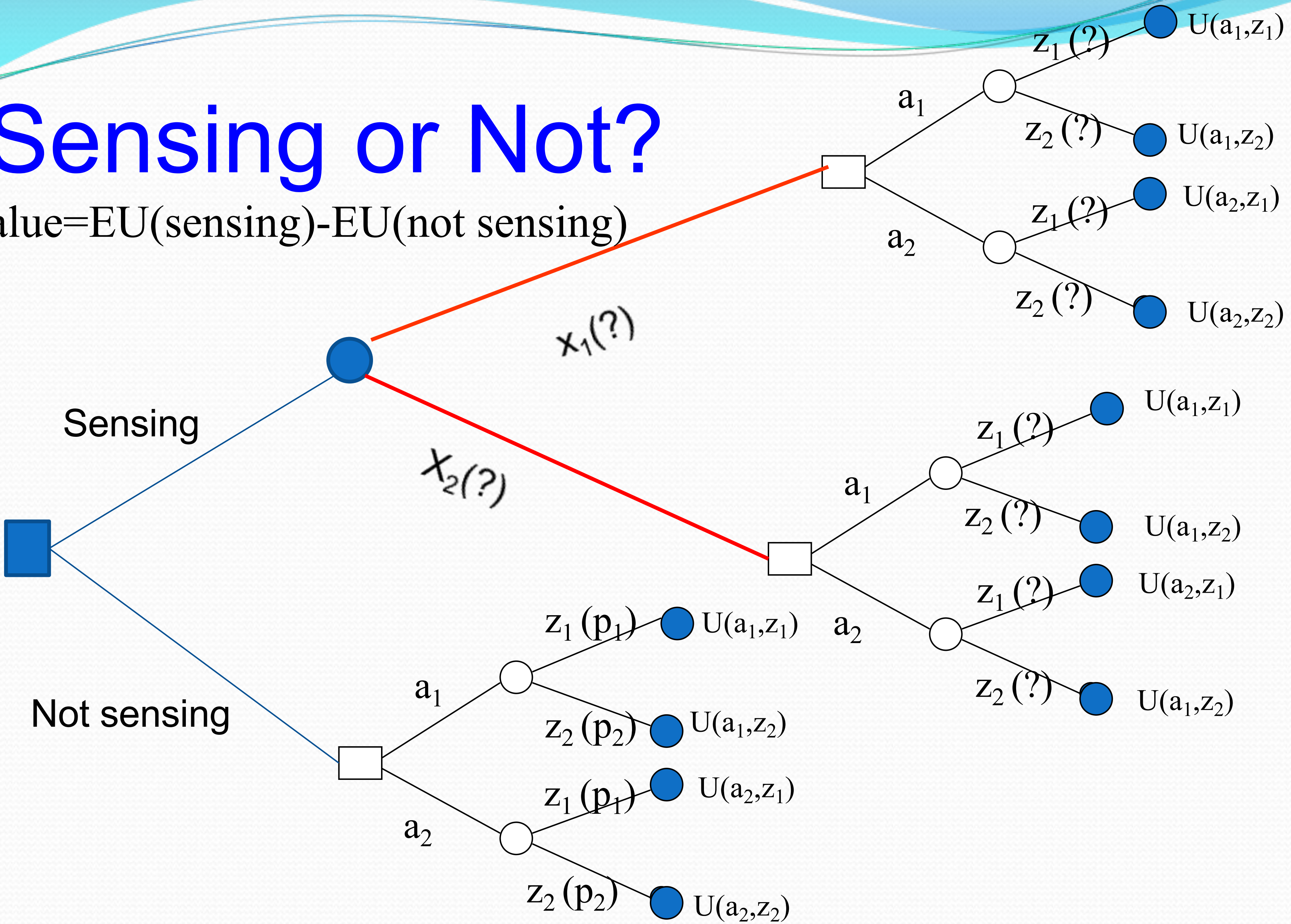
Robot Action Selection

z_i is the state of the environment, $U(a_i, z_j)$ denotes the utility of executing action a_i when state z_j is true

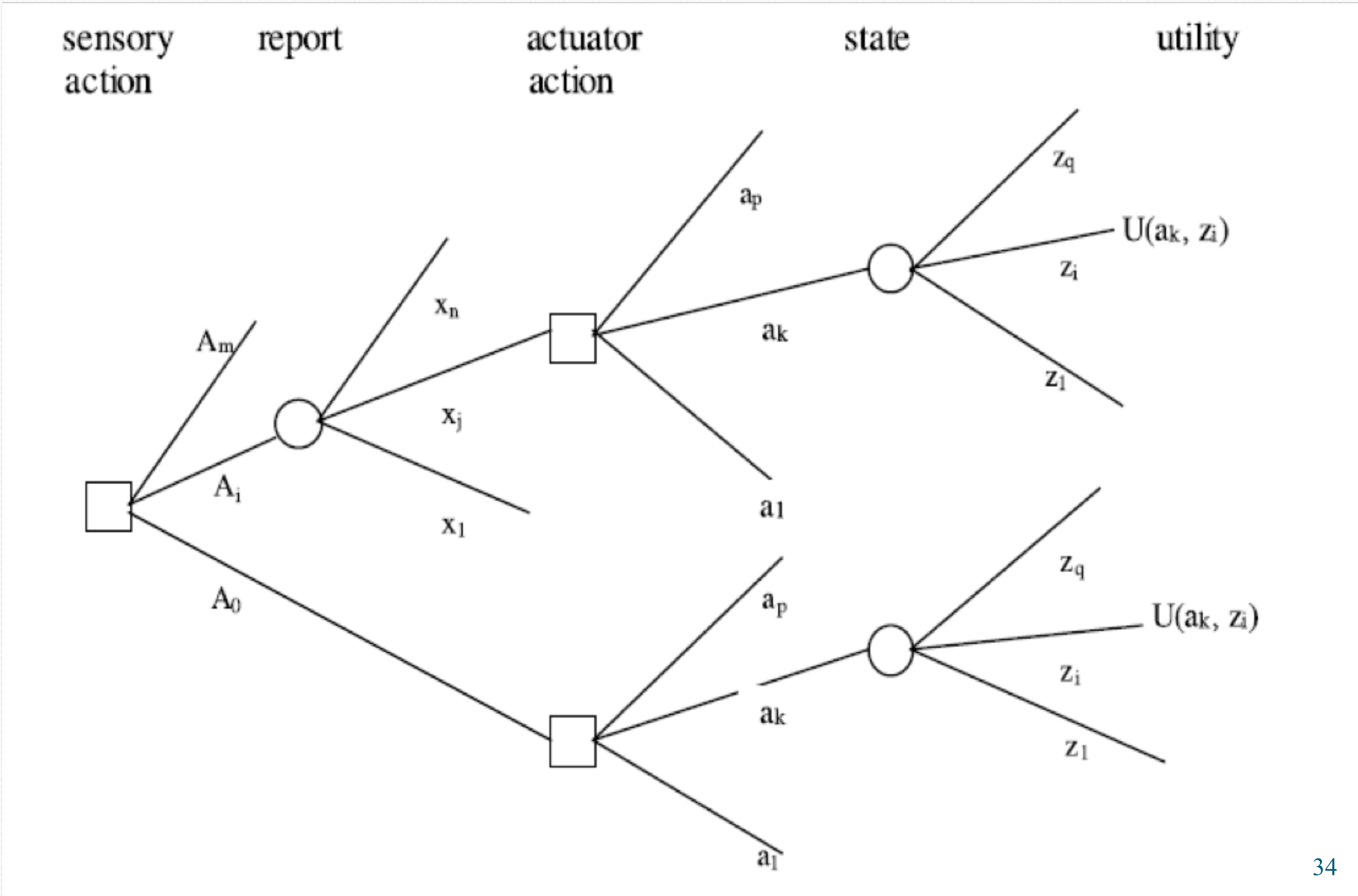


Sensing or Not?

Value=EU(sensing)-EU(not sensing)

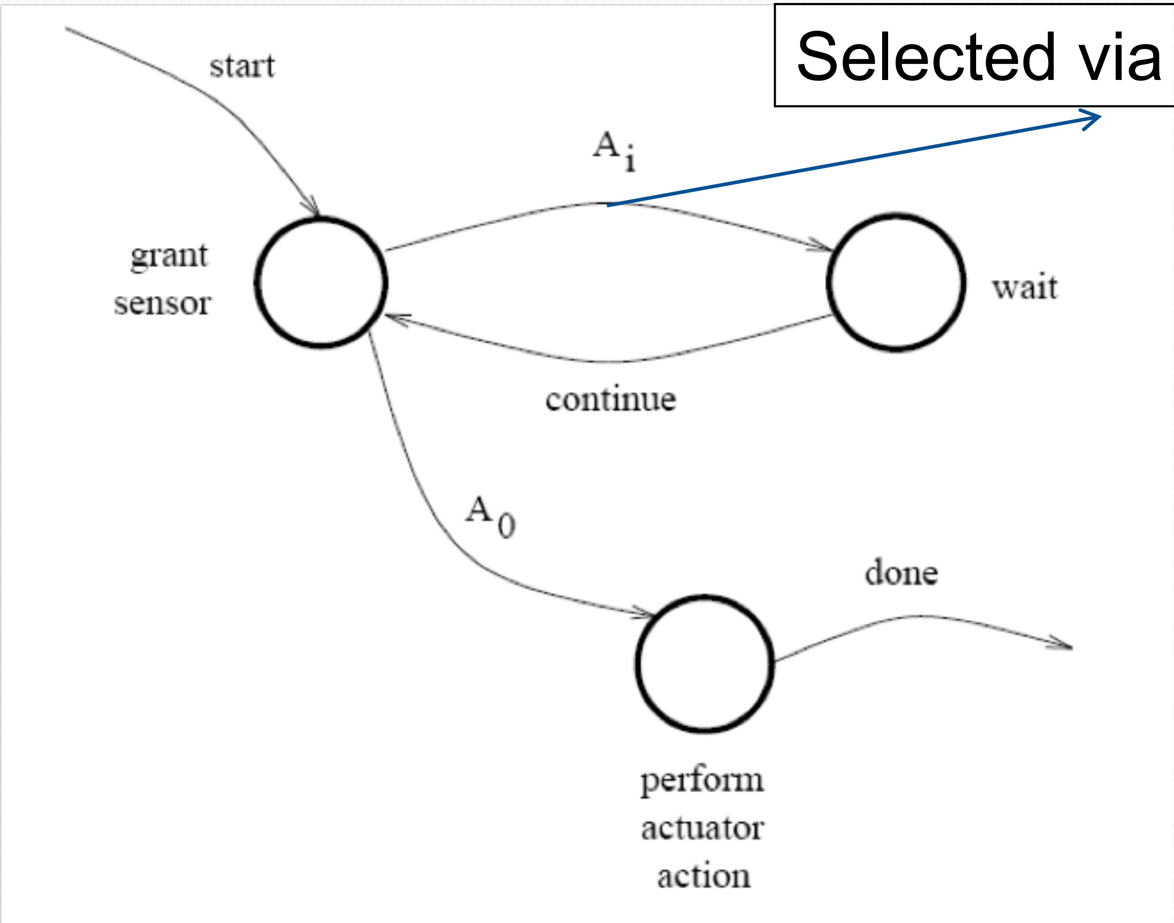


Decision Theoretic Sensor Planning



Continuous Sensing Actions (Active perception)

Selected via decision analysis



Paper " Sensor planning with Bayesian decision theory" in blackboard

Recommendation for Reading

- It is important to carefully study and understand the content in the slides.

- Sections 1.1 and 1.2 in the article “Decision analysis” would be good reference for basics of decision theory, available in the blackboard.

- Read section 2 in the paper “Sensor planning with Bayesian decision theory” to understand how decision analysis can be used for active data acquisition. The students from robotics programme may have interest to read the full paper, which is available in the blackboard.