Lecture 2: Fuzzy Logic Control

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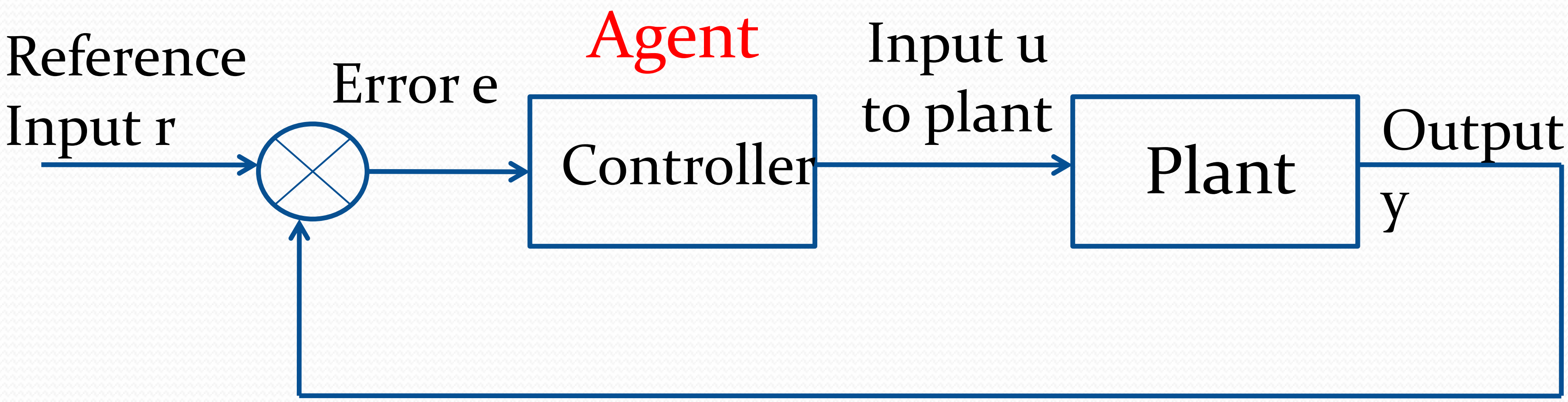
Agenda

- Motivation (Why is fuzzy logic control useful?)
- Basic concept and principle in fuzzy controller
- How a fuzzy controller works
- General knowledge about fuzzy sets

Closed-Loop Control System



- Error e = r-y
- Controller: decide the plant input *u* in order to reduce the error
- Agent: decide the action to be executed on the plant such that the output behaves as specified by the reference input

Conventional Controller Design

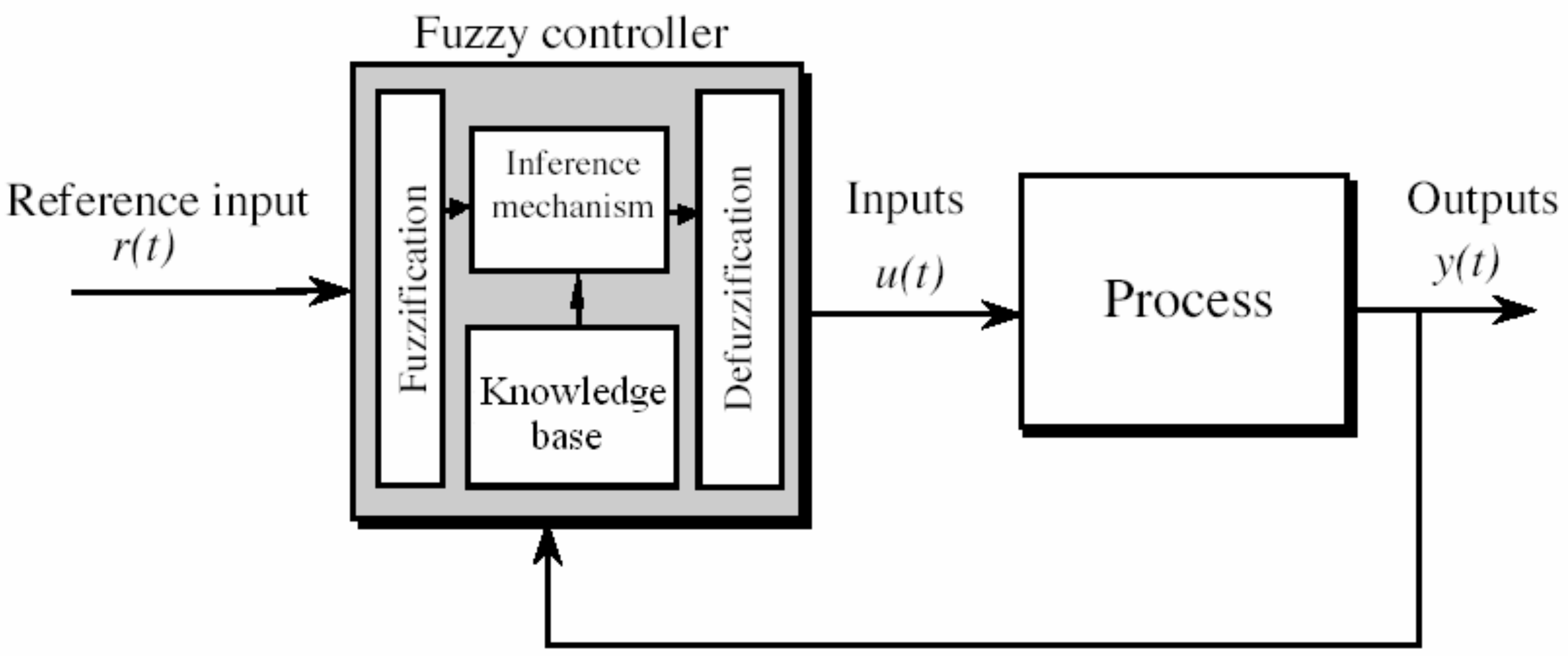
1. Develop a mathematical model of the dynamics of the plant
2. Use the mathematical model, or a simple version of it, to design a controller
3. Perform mathematical or simulation-based analysis of control performance using the model of the closed-loop system
4. Implement the controller via, for example, a microprocessor, and evaluate the performance of the closed-loop system in real operation

Limitation of Conventional Scheme

- Design and analysis largely depends on the accuracy of the mathematic model
- The model is usually imprecise in representing the plant
- The model very likely simplifies the complex plant.
- Heuristics are required to tune a controller in implementation. Smart heuristic tuning plays a crucial role for success.

Fuzzy Control

Fuzzy control aims to directly exploit human's heuristic Knowledge about how to control a process/plant



Four Elements of Fuzzy Controller

- **Knowledge-base**: human linguistic description of how to deal with the plant in form of if-then rules

- **Inference mechanism**: emulate human reasoning in applying the rules from the knowledge base

- **Fuzzification**: convert (crisp) controller inputs into information that can easily be used by the (fuzzy) inference mechanism

- **Defuzzification**: convert conclusions of the fuzzy inference mechanism into actual and crisp input to the process (plant)

Fuzzy Control of Inverted Pendulum

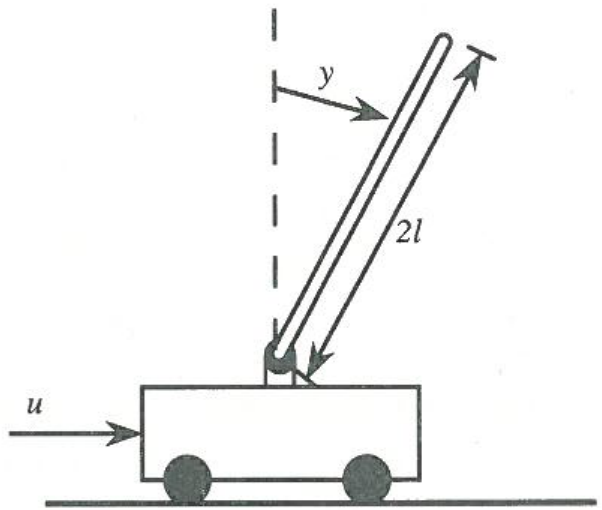
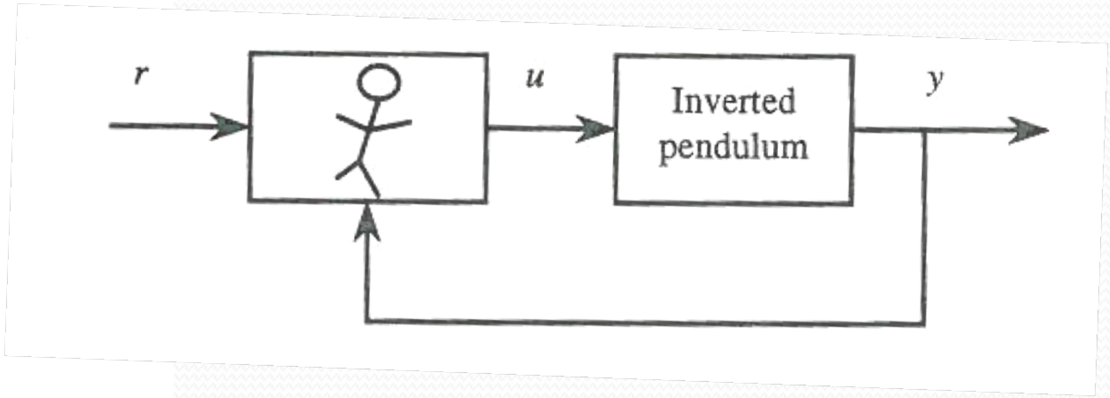
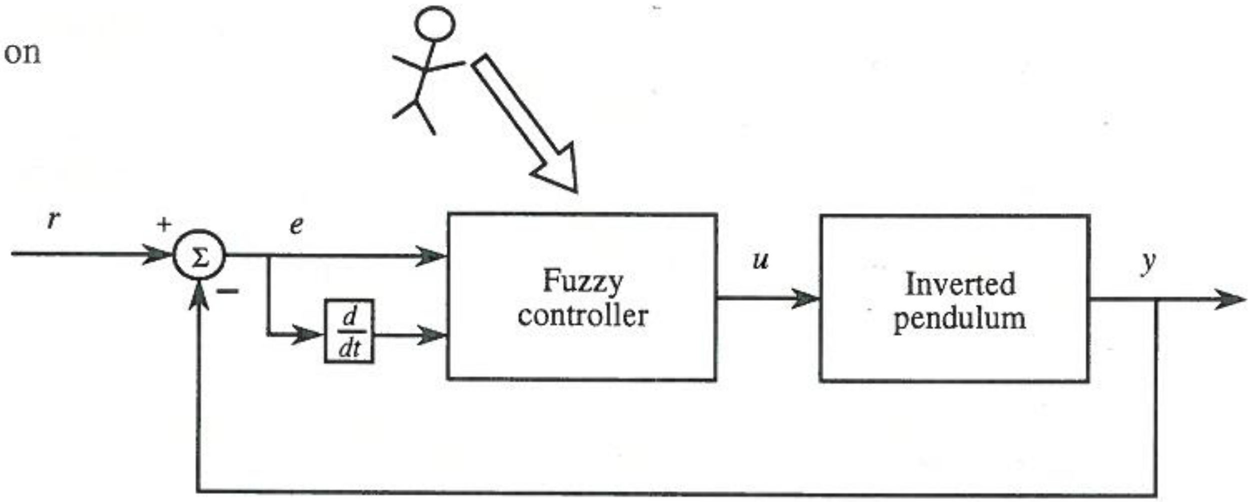


FIGURE 2.2 Inverted pendulum on a cart.





Two inputs to fuzzy controller:

1) error ***e=r-y***;

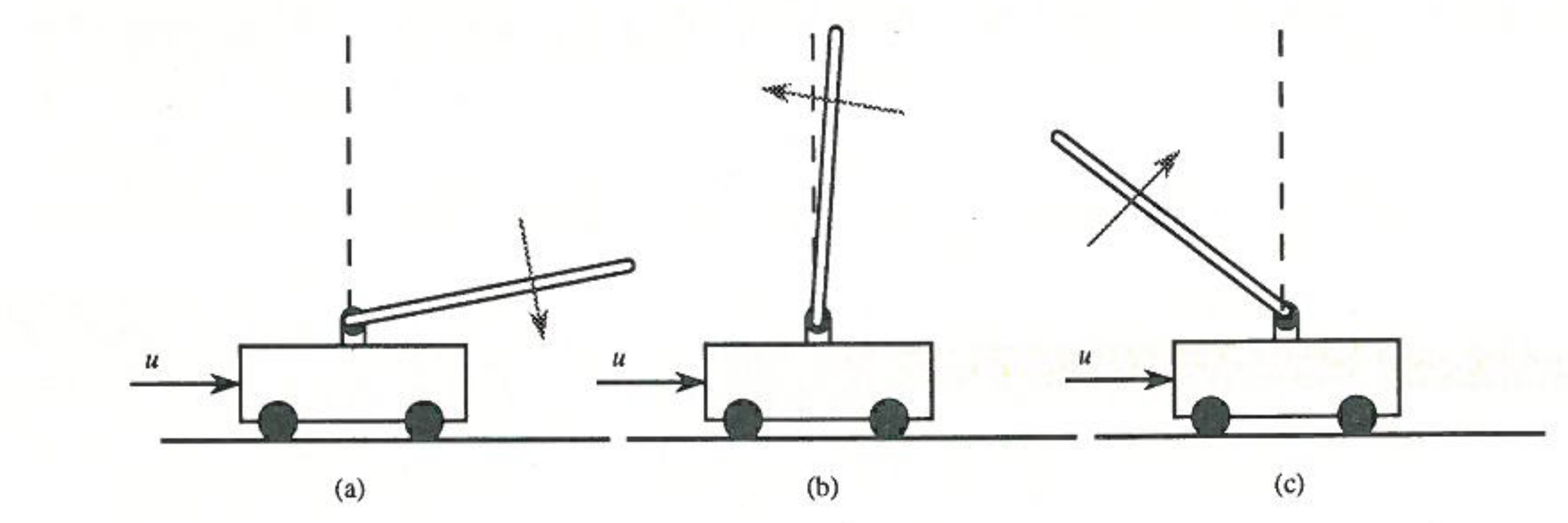
2) change-in-error ***de/dt***

Linguistic Descriptions

The inputs and output of the fuzzy controller are all described by five linguistic terms:

NL: negative large
NS: negative small
Z: zero
PS: positive small
PL: positive large

Linguistic Rules



a) **If** the error is NL **and** change-in-error is NL **Then** force is PL

b) **If** the error is Z **and** change-in-error is PS **Then** force is NS

c) **If** the error is PL **and** change-in-error is NS **Then** force is NS

Linguistic Rule Base

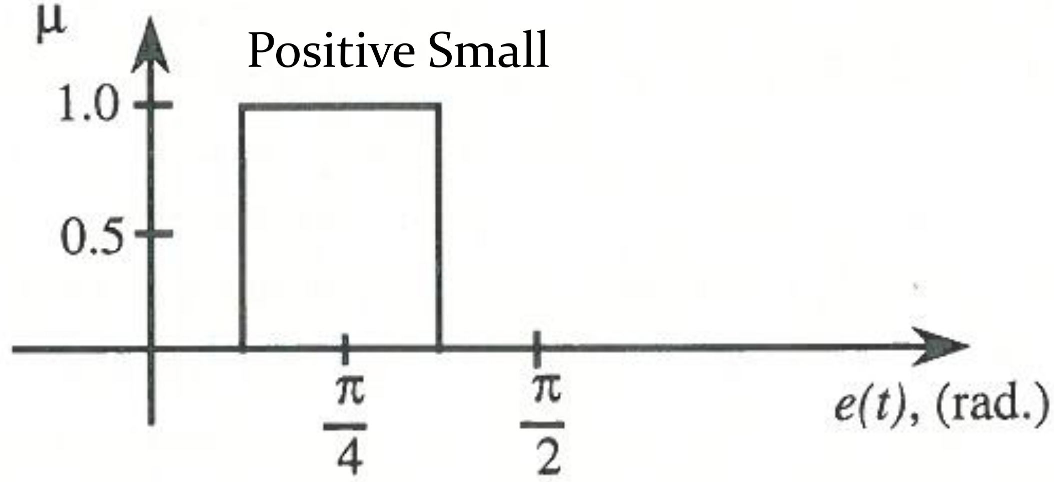
Describes the strategy of what to do in various situations in terms of error and change-in-error

”force” u		”change-in-error” de/dt				
		NL	NS	Z	PS	PL
”error” e	NL	PL	PL	PL	PS	Z
	NS	PL	PL	PS	Z	NS
	Z	PL	PS	Z	NS	NL
	PS	PS	Z	NS	NL	NL
	PL	Z	NS	NL	NL	NL

Quantification of Knowledge

The meaning of the linguistic terms have to be defined as a pre-step in order to use linguistic rules for decision making

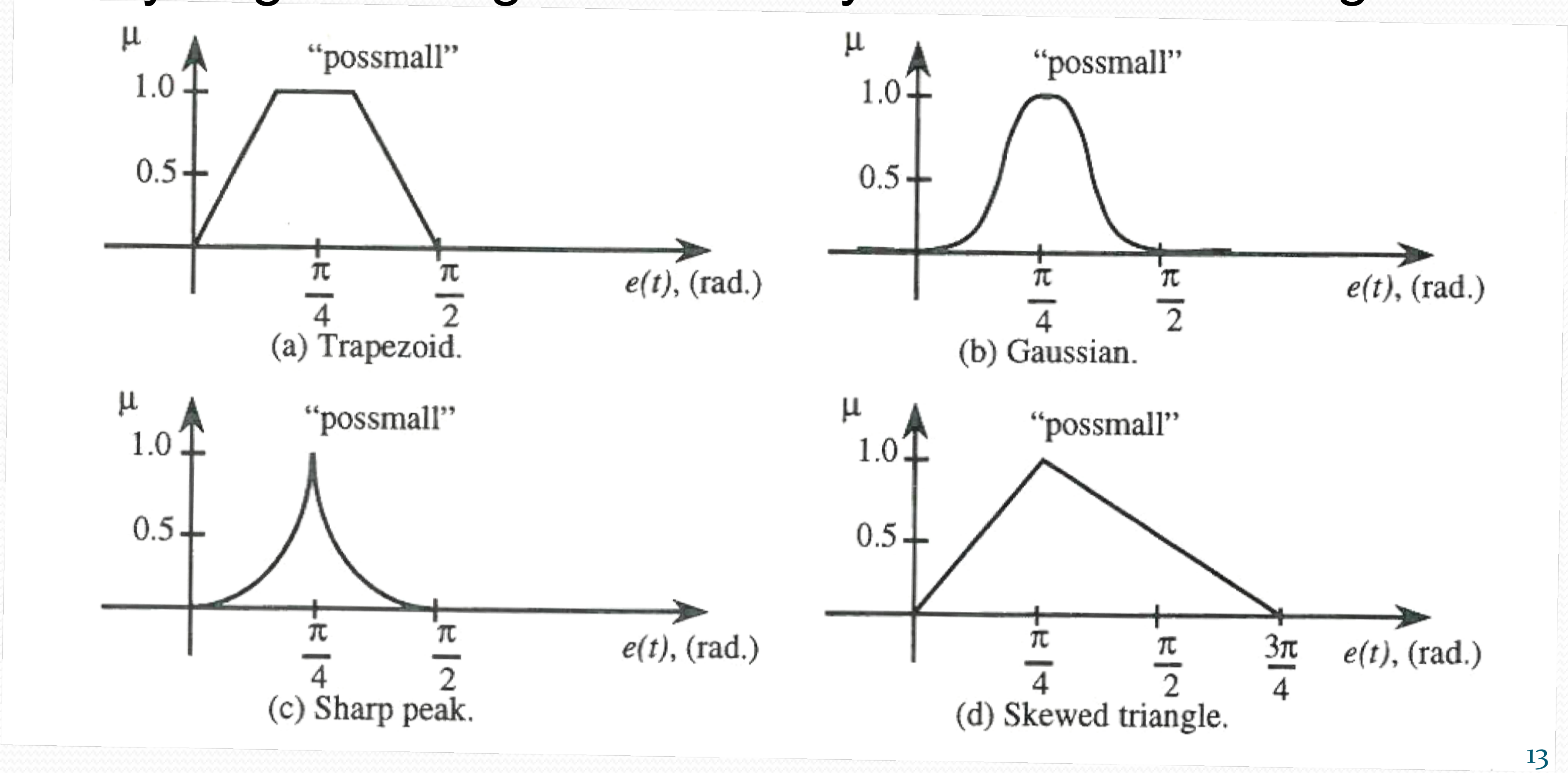
Traditional definition as a crisp set with sharp boundary



Conventional expert systems and rule-based reasoning

Fuzzy Quantification

- Use a membership function to specify the meaning of PS
- PS is treated as a fuzzy subset
- Every angle belongs to the fuzzy subset PS with a grade



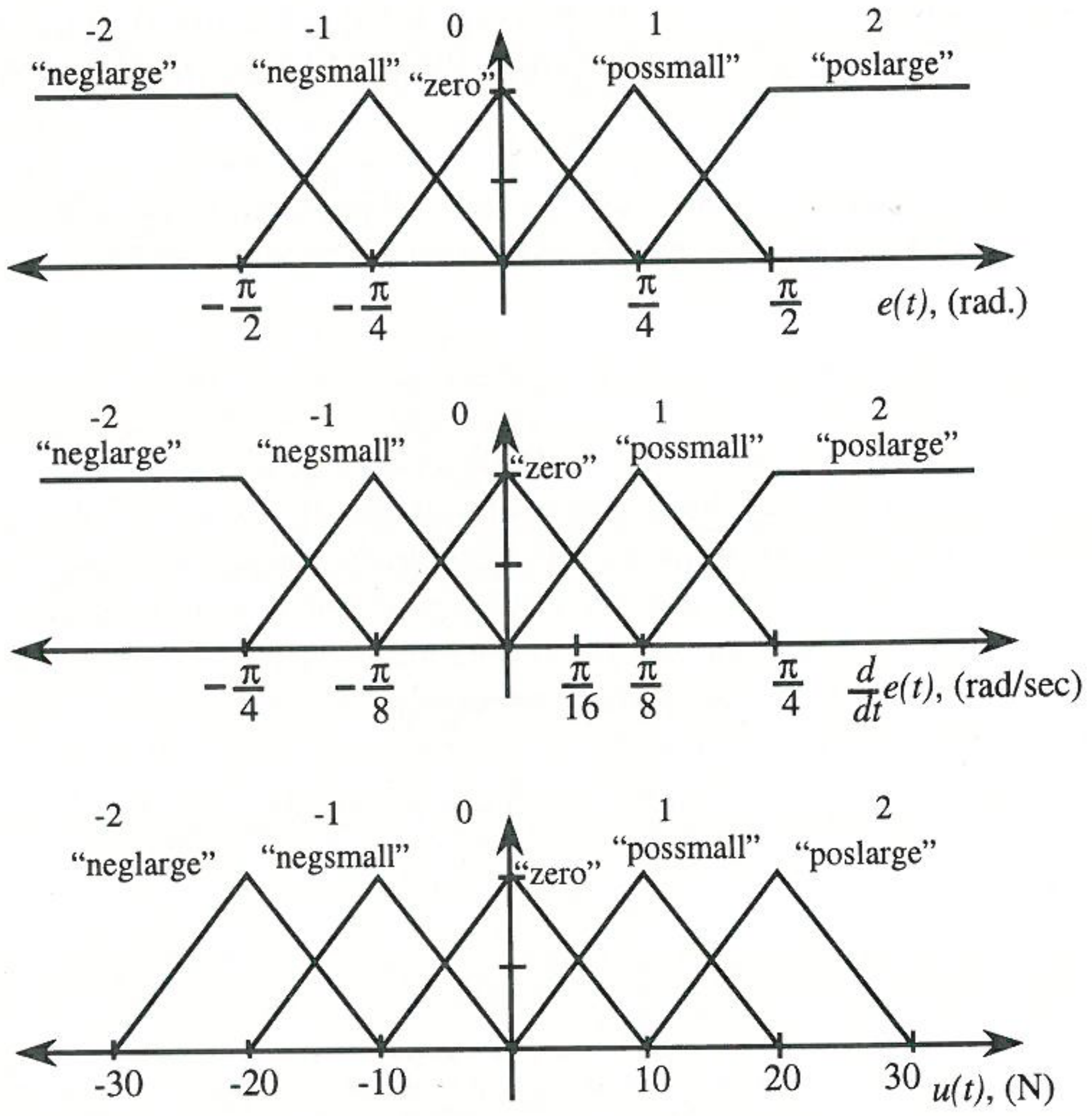
(a) Trapezoid.

(b) Gaussian.

(c) Sharp peak.

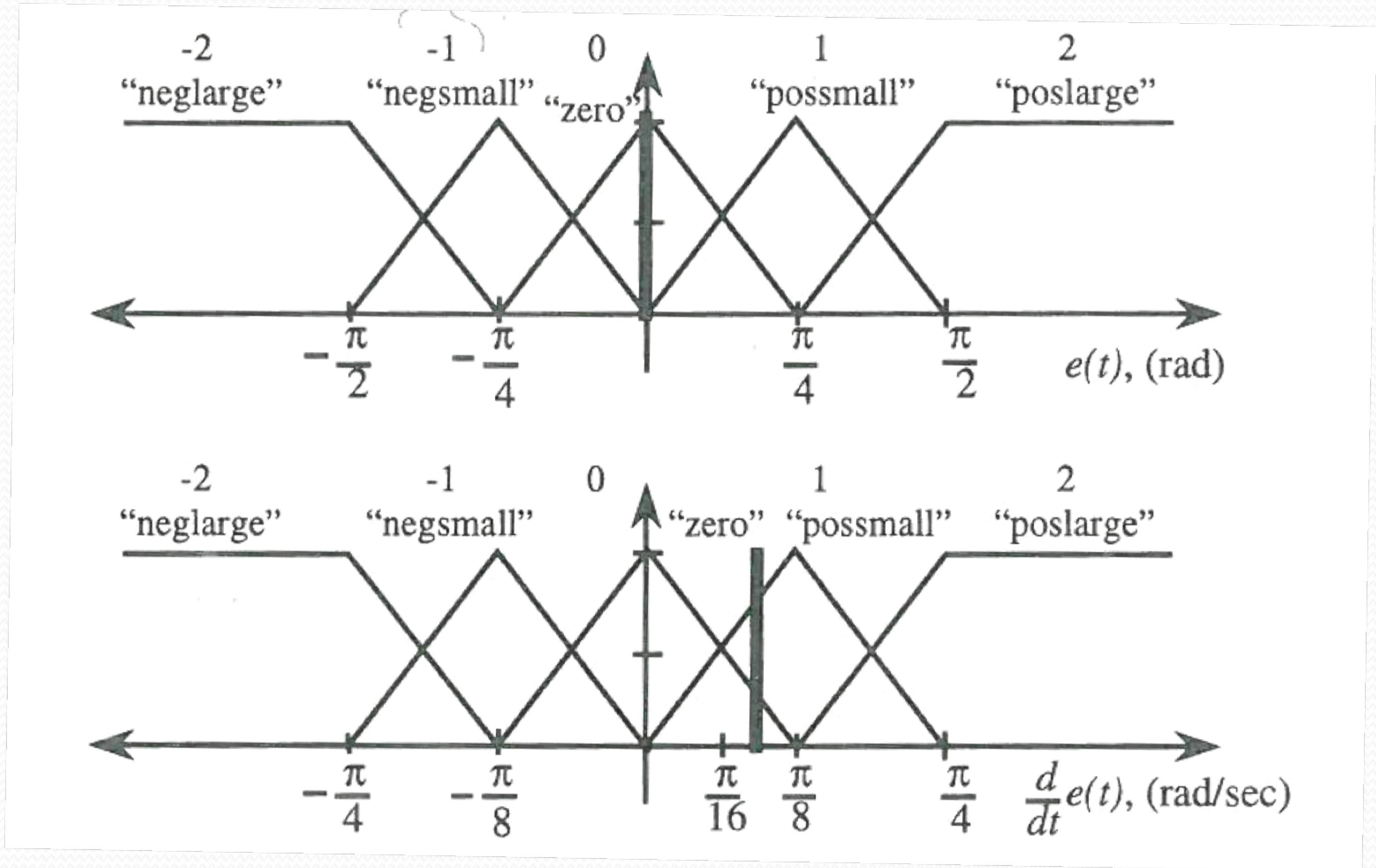
(d) Skewed triangle.

Membership Functions Used



Fuzzification

Calculate the membership degrees of every input with respect to the linguistic terms



$e=0$, $e'=3\pi/32$

$\mu_Z(e)=1$

$\mu_Z(e')=0.25$

$\mu_{PS}(e')=0.75$

Rule Matching

Decide to which degree the condition part of each rule is satisfied in the current situation. This degree is termed as **firing strength** of the rule

$e=0$, $e'=3\pi/32$, $\mu_Z(e)=1$, $\mu_Z(e')=0.25$, $\mu_{PS}(e')=0.75$

If the error is Z **and** change-in-error is Z **Then** force is Z

Two ways to quantify the "**AND**" operation in fuzzy sense

1) Minimum: firing strength $t=\min\{\mu_Z(e), \mu_Z(e')\}=0.25$

2) Product: firing strength $t=\mu_Z(e)\bullet\mu_Z(e')=0.25$

Which Rules Are Effective?

$e=0$, $e'=3\pi/32$, $\mu_Z(e)=1$, $\mu_Z(e')=0.25$, $\mu_{PS}(e')=0.75$

"force" u		"change-in-error" de/dt				
		NL	NS	Z	PS	PL
"error" e	NL	PL	PL	PL	PS	Z
	NS	PL	PL	PS	Z	NS
	Z	PL	PS	Z	NS	NL
	PS	PS	Z	NS	NL	NL
	PL	Z	NS	NL	NL	NL

Fuzzy Implication

R1: If the error is Z and change-in-error is Z Then force is Z

R2: If the error is Z and change-in-error is PS Then force is NS

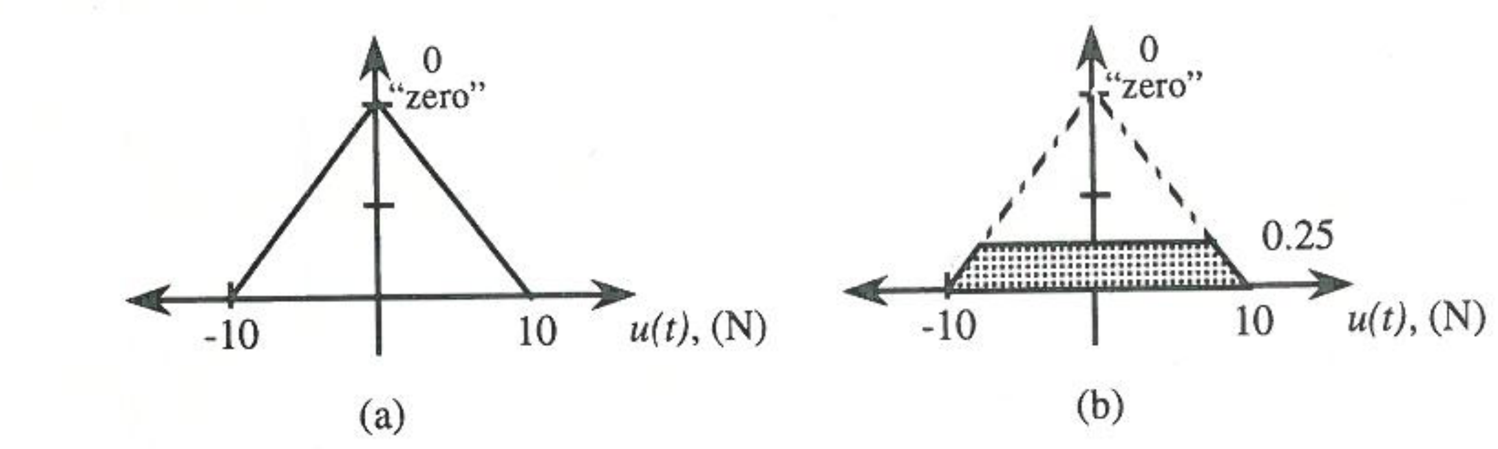
Now we want to derive the recommendations of the effective rules.

- The recommendation of a rule is actually an implied fuzzy set
- The implied fuzzy set shows the degrees of recommendation that the controller output should take on various values

Recommendation of Rule 1

R1: If the error is Z and change-in-error is Z Then force is Z

$t_1=\min\{\mu_Z(e), \mu_Z(e')\}=0.25$



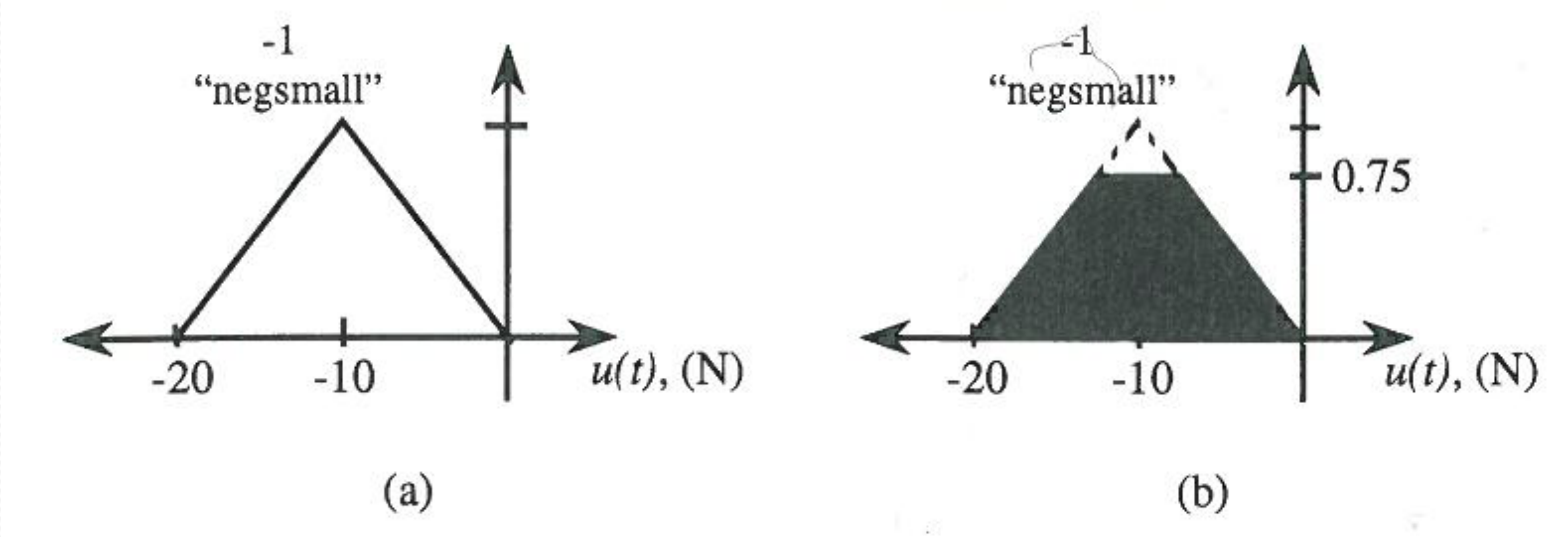
The recommendation is an implied fuzzy set defined by

$$\mu_{(1)}(u)=\min\{t_1,\mu_Z(u)\}=\min\{0.25,\mu_Z(u)\}$$

Recommendation of Rule 2

R2: If the error is Z and change-in-error is PS Then force is NS

$t_2=\min\{\mu_Z(e),\ \mu_{PS}(e')\}=0.75$

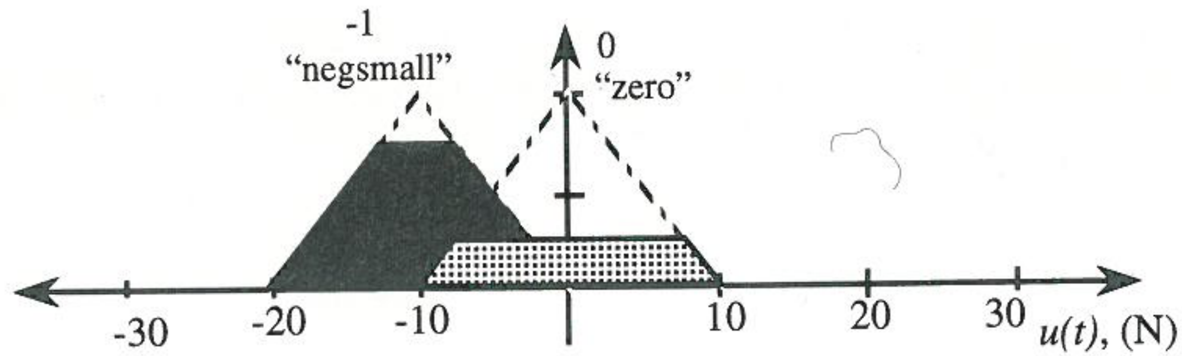


The implied fuzzy set from rule 2 as recommendation:

$$\mu_{(2)}(u)=\min\{t_2,\mu_{NS}(u)\}=\min\{0.75,\mu_{NS}(u)\}$$

Defuzzification

Combining the implied fuzzy sets from the rules into a crisp value for execution

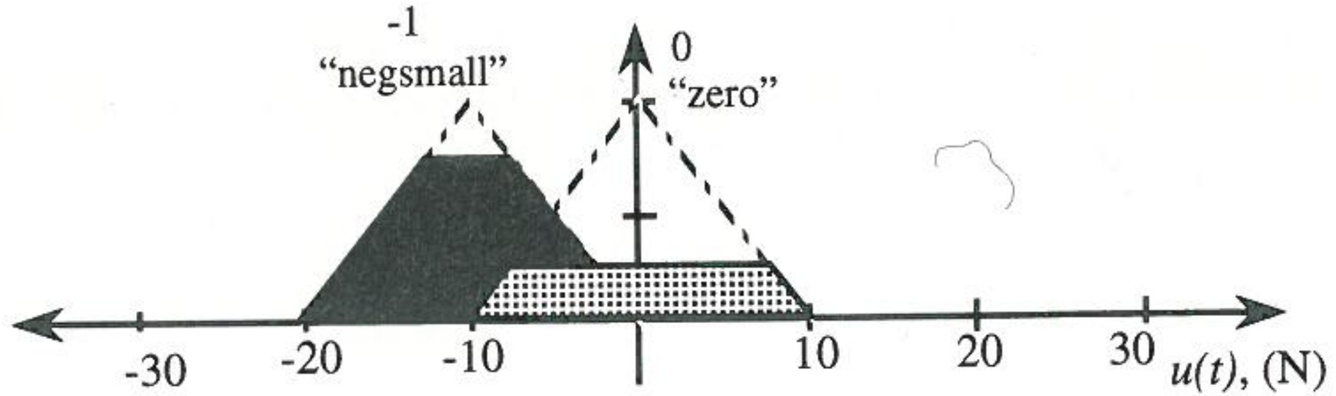


Center of Gravity (COG)

$$u^{crisp} = \frac{\sum_i b_i \int \mu_{(i)}(u)du}{\sum_i \int \mu_{(i)}(u)du}$$

where b_i denotes the center of the membership function of the conclusion of effective rule i

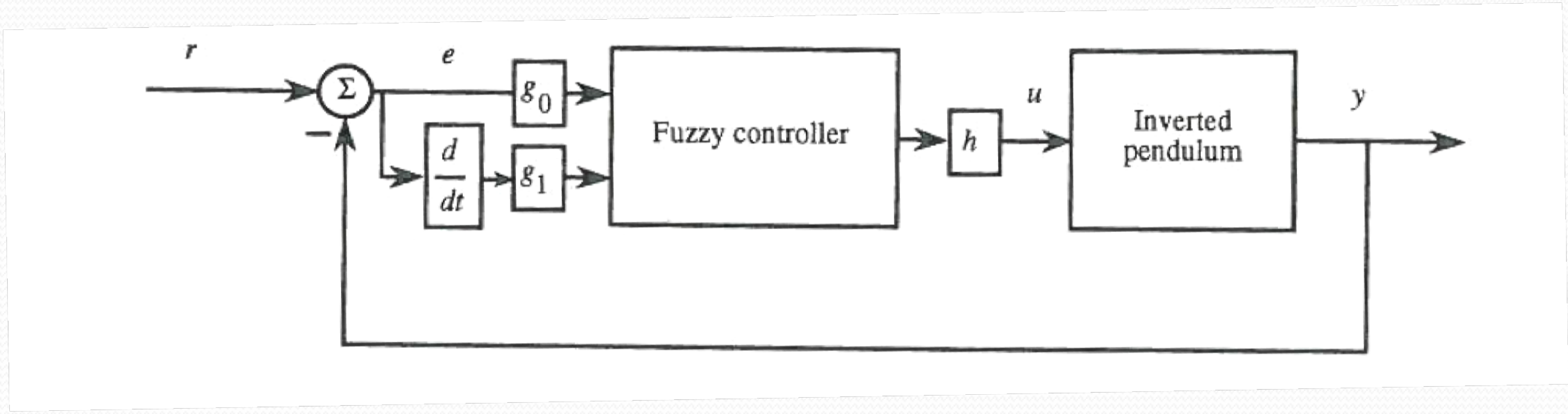
Center of Gravity: Example



b1=0, b2=-10

$$u^{crisp} = \frac{0 \cdot 4.375 + (-10) \cdot 9.375}{4.375 + 9.375} = -6.81$$

Tuning Fuzzy Controller via Scaling



- Introduce scaling factors g_0, g_1, and h for inputs and controller output respectively

- Change the scaling factors can cause different system behavior

Effect of Varying Scaling Factor

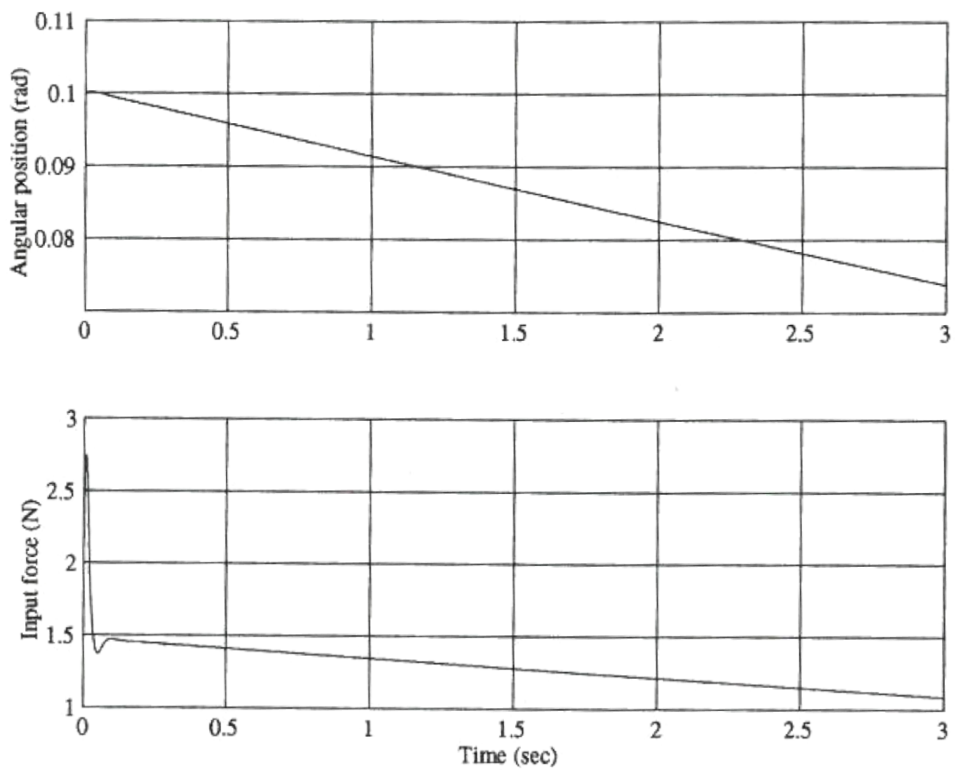


FIGURE 2.25 Fuzzy controller balancing an inverted pendulum, first design. $g_0=g_1= h=1$

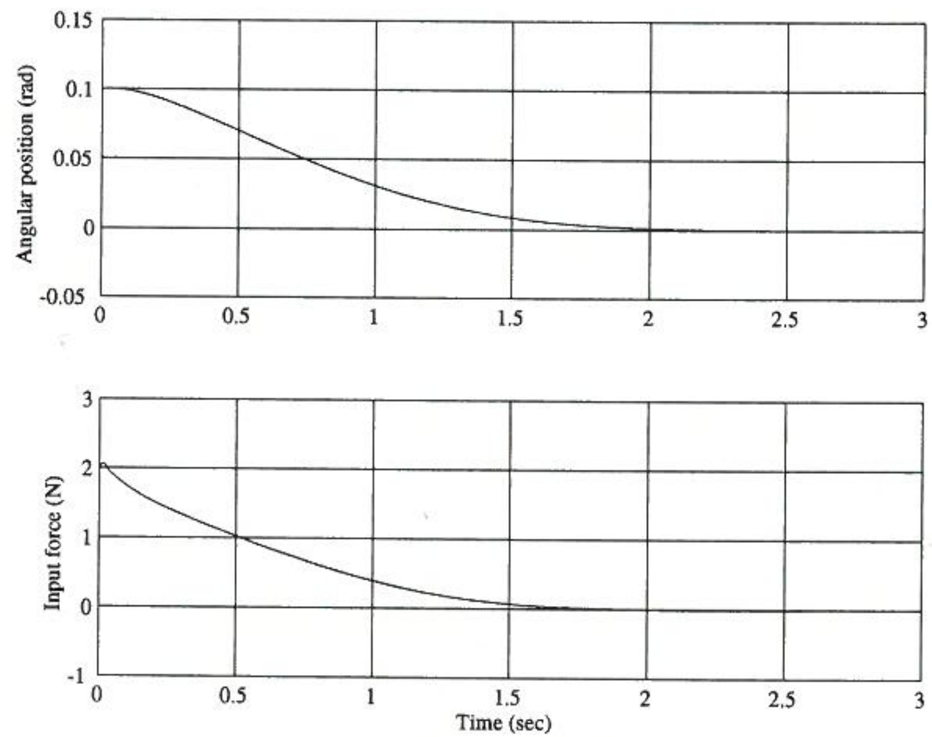


FIGURE 2.27 Fuzzy controller balancing an inverted pendulum with $g_0 = 1$, $g_1 = 0.1$, and $h = 1$.

Nonlinear Control Surface

Inherently fuzzy controller implements a nonlinear relation between controller inputs and controller output. Such nonlinearity is often called control surface

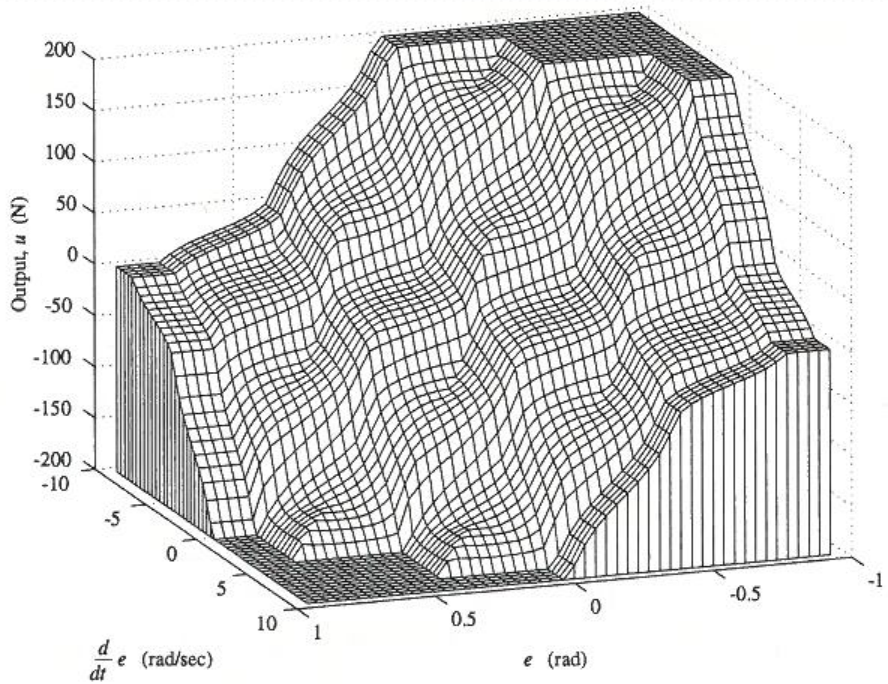


FIGURE 2.35 Control surface of the fuzzy controller for $c^i = 5hsign(i)i^2$, $h = 10.0$, $g_0 = 2.0$, and $g_1 = 0.1$.

Significance of Fuzzy Set Theory

Fuzzy logic and fuzzy theory aim to represent and manimulate linguistic, vague and uncertaion information in order to create systems that are much closer in spirit to human thinking and reasoning

Fuzzy vs. Crisp Sets

Fuzzy set indicates how much (to which extent) an element belongs to it.

Crisp set indicates whether an element belongs to it

Fuzzy Set

Definition :

A fuzzy set F in a universe of discourse U is characterized by membership function $\boldsymbol{\mu}_{F,}$ which takes values in the interval [0,1], i.e., $\boldsymbol{\mu}_F$: U →[0,1]

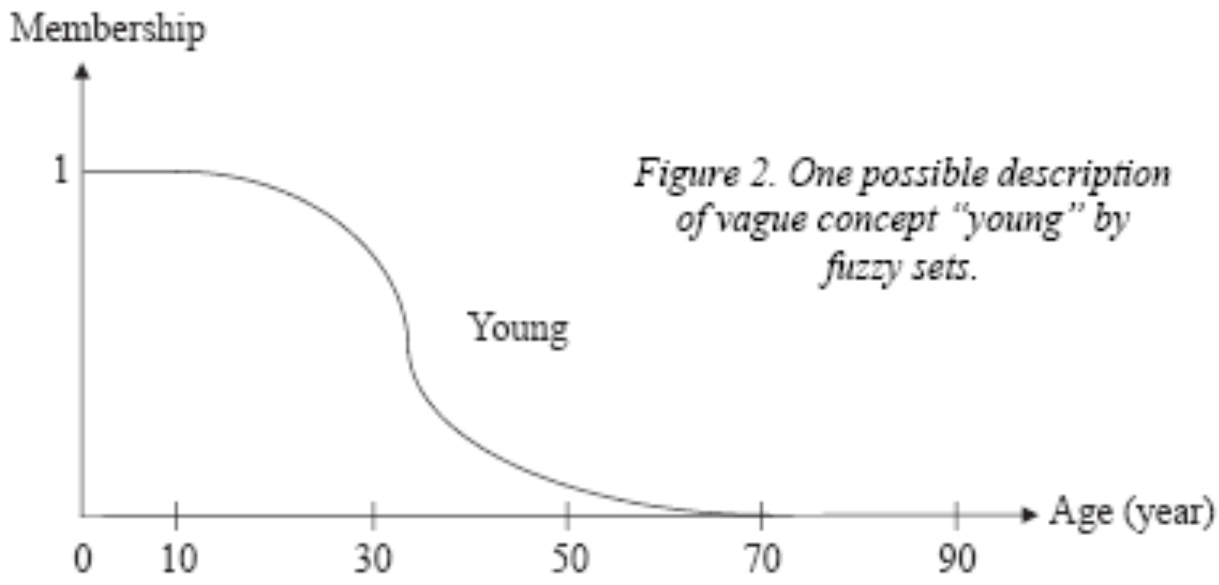
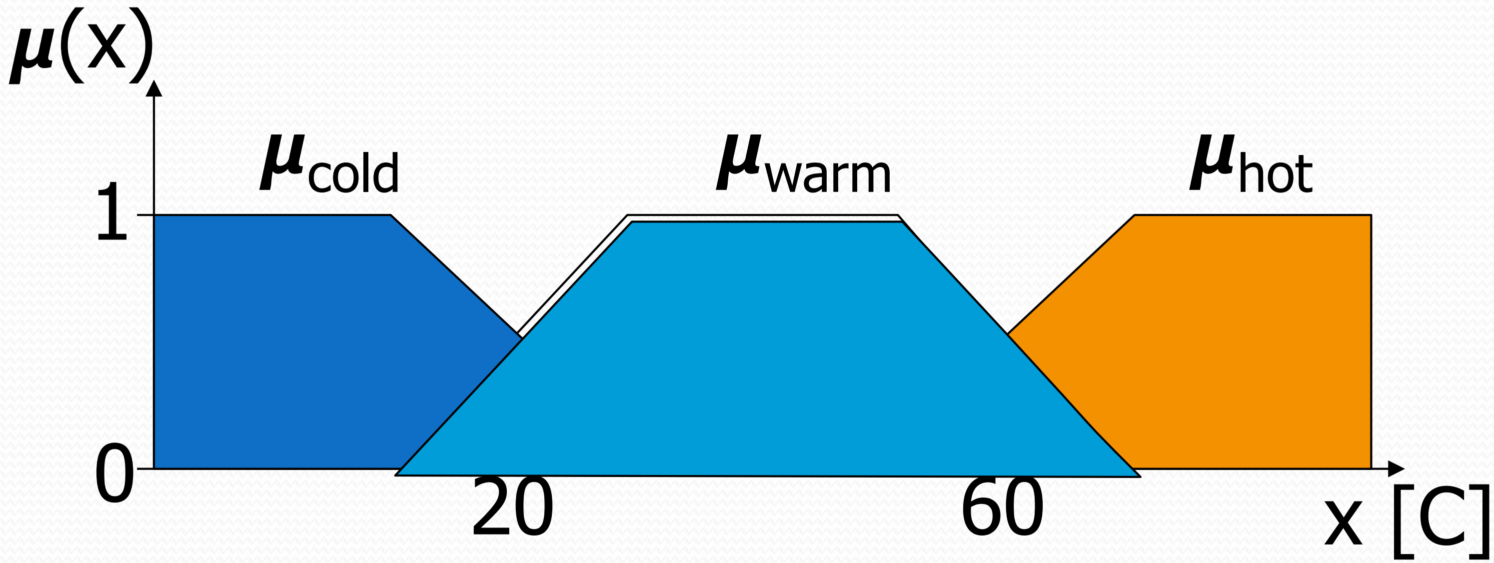


Figure 2. One possible description of vague concept "young" by fuzzy sets.

Linguistic Variables

A *linguistic variable has a set of linguistic terms/values*. Each linguistic value corresponds to a fuzzy set and explained with its membership function

linguistic variable : temperature
linguistics terms (fuzzy sets) : { cold, warm, hot }



Fuzzy Rules

- A fuzzy rule is a linguistic expression of causal dependencies between linguistic variables in form of if-then statement
- General form:
  If <antecedent> then <consequence>
- Example:
  if temperature is cold and oil price is cheap
  then heating is high

linguistic variables

linguistic values/terms (fuzzy sets)

Recommendation for Reading

- It is important to carefully study and understand the content in the slides
- Popular article for fuzzy control:
  "Fuzzy Systems Technology: A Brief Overview", available in the blackboard
- Deeper knowledge on fuzzy control:
  "Fuzzy logic in control systems: fuzzy logic control – Part I", available in the blackboard for selective reading