Link to HW2 code

1 Task 1

The numerical calculus library can be found in **num_calc.py** and contains a symmetrical derivative calculation as well as Riemann (left, right, and midpoint), trapezoid, and Simpson's rule integration methods. All the methods take in a function as well as an array of points at which the function is to be evaluated (not necessarily evenly spaced).

2 Task 2

Figure 1 shows the enclosed mass of the galaxy given parameters $r_{200} = 230$ kpc, $v_{200} = 160$ km/s and $c = 15$, as used in the homework example. The slope of the enclosed mass function indicates that most of the mass is concentrated within ~ 10 kpc and begins to flatten out at larger distances.

The total mass was found by evaluating M_{enc} at a distance of 300 kpc which yielded a total galaxy mass of $1.545922 \times 10^{12} M_{\odot}$ which is very close to the approximate mass of the Milky Way galaxy.

Figures 2 and 3 are very similar in that they both represent the mass found in radial bins of size $(r + \Delta r) - (r - \Delta r) = 2\Delta r$. The difference is that Figure 3 also takes into account the physical value of the separation between radial bins so as to convey a mass change as a function of distance rather than just the total enclosed mass in subsequent radial bins.

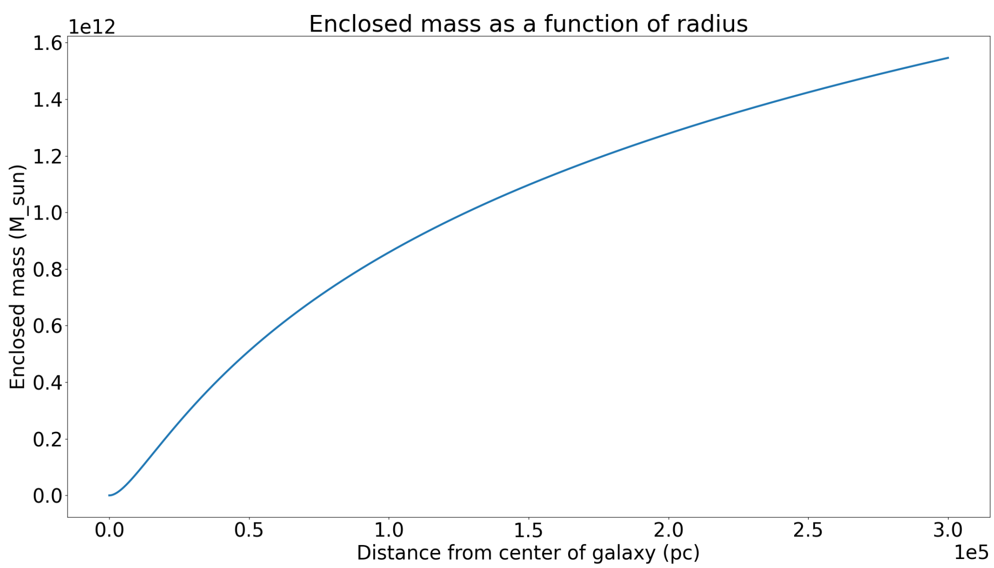

Figure 1: Enclosed mass of the galaxy as a function of the radial distance from the galaxy center. Logically, as the distance increases, more mass is enclosed by the sphere of radius r. The slope of the enclosed mass function begins to flatten out as distance grows larger which is expected since matter density decreases as a function of distance.

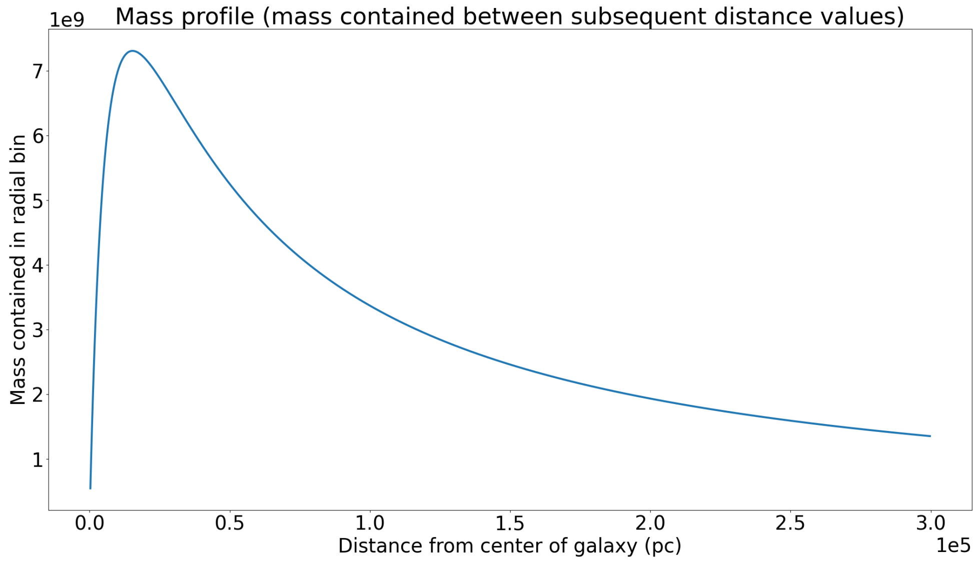

Figure 2: Plot showing how much mass is enclosed within a small shell around $r \pm \Delta r$, simply expressed as $M_{enc}(r + \Delta r) - M_{enc}(r - \Delta r)$. The difference between this figure and Figure 3 is that this figure represents only the mass in the shell, not the mass scaled by the distance $2\Delta r$ as with the derivative.

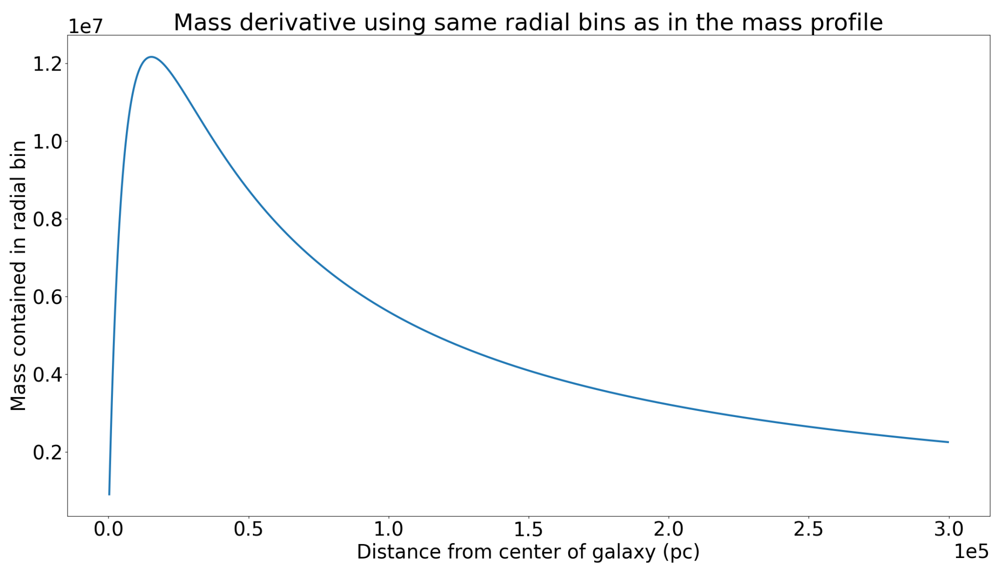

Figure 3: The derivative of $M(r)$ with a matching profile as that in Figure 2 except with slightly decreased values resulting from the division of the enclosed mass in the shell by the distance $2\Delta r$ between the maximum and minimum distance.

2.1 Fixed v_{200}, varied c

By increasing c, the concentration, the halo will contain more mass and thus it is expected that the mass derivative will have higher values. Likewise, it is expected that since the halo is closer to the center of the galaxy, the mass distribution will shift to be predominantly present near the center of the galaxy. To exaggerate the effect, two values of $c = 100$ and $c = 1$ were used with v_{200} staying fixed at 160 km/s.

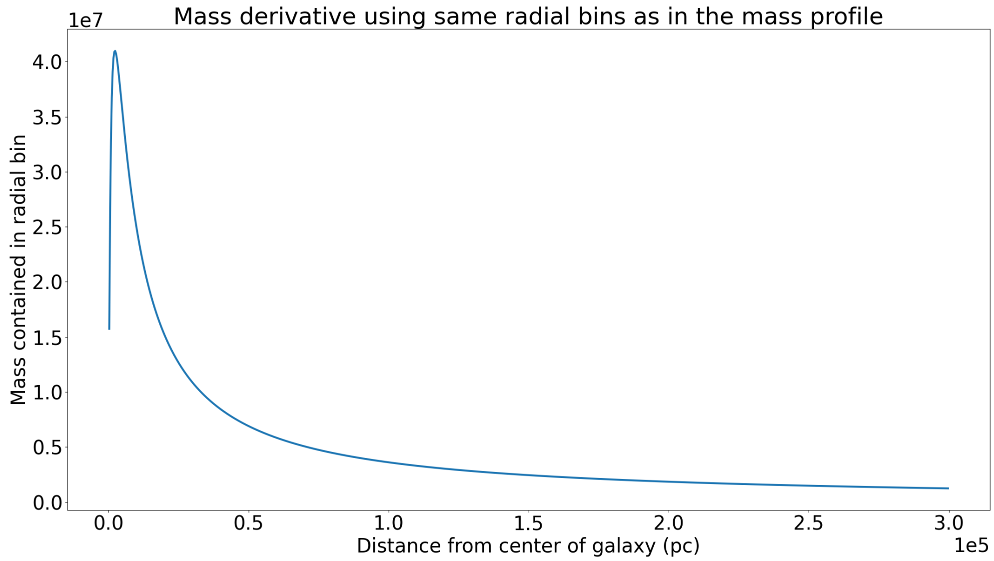

Figure 4: Same as Figure 3 but with $c = 100$, a sharper peak and larger mass fraction per radial bin.

Figure 4 shows our expectations were correct, with a more sharply peaked mass derivative which also reached a higher peak mass fraction value compared to Figure 3. Likewise, in Figure 5, we see the opposite effect, with the mass fraction much more diffuse, and the galaxy spreading out to much larger distances than in either of the previous cases.

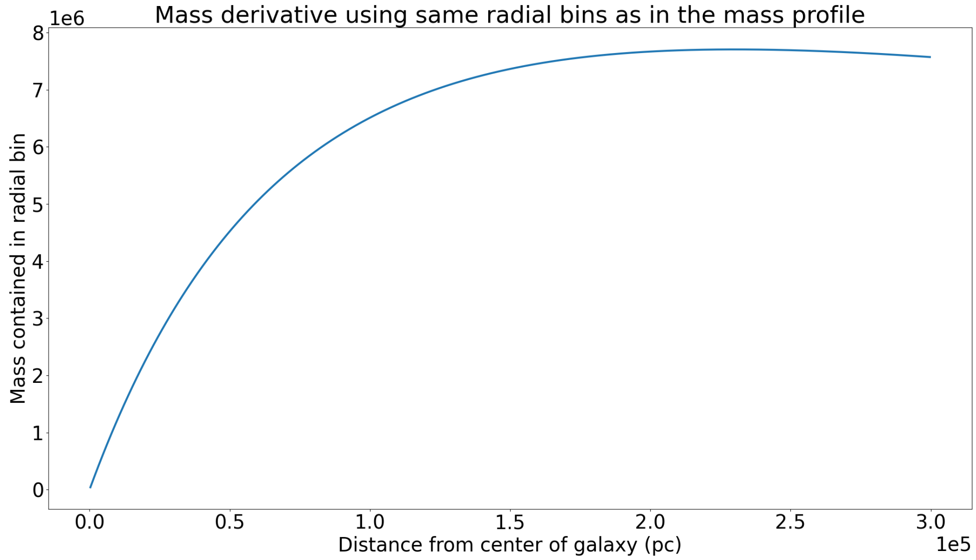

Figure 5: Same as Figure 3 but with $c = 1$, a flatter peak and lower mass fraction per radial bin.

2.2 Fixed c, varied v_{200}

Since the circular velocity is related to the mass interior to the given radius by equating the gravitational force with the centripetal force, it immediately follows that higher velocities will result in larger enclosed masses. This makes sense, as a larger mass will produce a larger gravitational force which requires a faster radial velocity to produce a strong enough counteracting centripetal force. Thus, by increasing our v_{200}, we expect to find a much more massive representation of the galaxy. Again, for exaggeration, values of $v_{200} = 1000$ km/s and $v_{200} = 1$ km/s were used.

As expected, we see the predicted behavior in Figures 6 and 7, with a shockingly low mass fraction per radial bin for the latter $v_{200} = 1$ km/s case.

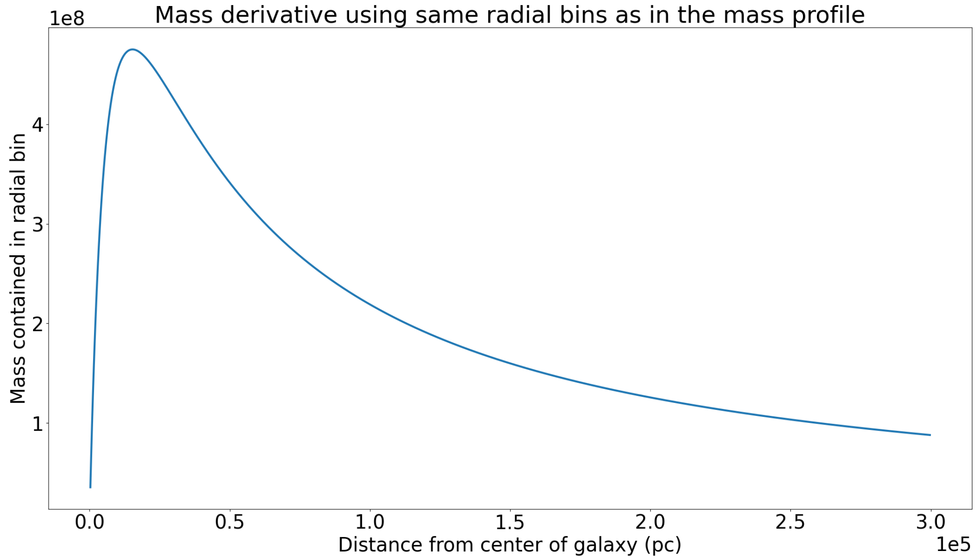

Figure 6: Same as Figure 3 but with $v_{200} = 1000$ and much larger mass fraction per radial bin.

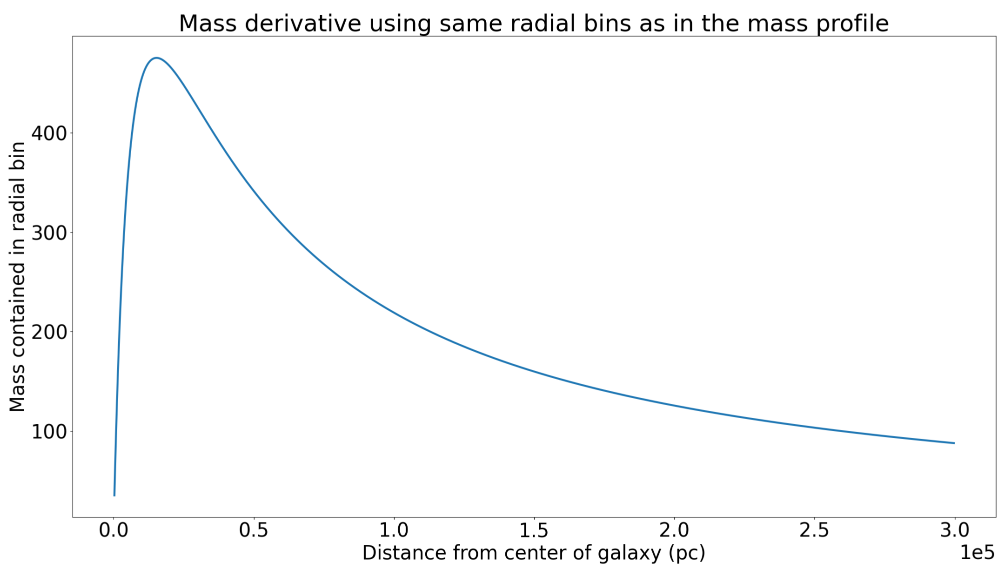

Figure 7: Same as Figure 3 but with $v_{200} = 1$ and much lower mass fraction per radial bin.

3 Task 3

The matrix class can be found in **mat.py**. All the functions work generally except for the determinant, which works only in convenient cases because it uses the existence of the inverse to calculate the determinant from the row-echelon form of the matrix acquired during Gaussian elimination. An ugly representation of test cases for all but the determinant are presented in **test.py**.