Link to HW5 code

1 Task 1

The Barnes-Hut algorithm code is located in **barnes_hut.py**. The code successfully divides the box spanning from 0 to 10 in the x- and y-directions into sub-boxes which iteratively continue dividing until all sub-boxes contain either 0 or 1 particles. The **find_leaves** function finds all nodes which are leaves (contain only one particle) and counts them to verify that all particles are considered. The **return_leaves** function returns an array of nodes containing the single particles.

The second function might seem redundant and it would be in the context of using the Barnes-Hut algorithm correctly. However, I was not able to figure out how to use the center-of-mass of non-leaf nodes to help simplify the calculation. In theory it made sense, but I was not sure how to traverse the tree and identify which nodes to use and not use based on the limiting angle. As such, I instead looped through all the leaf node particles and did a direct force calculation with successful force softening implementation.

The other problem is that I lose some particles over the course of the evolution. I'm not sure if this is expected, as I don't really anticipate any of the particles having strong enough acceleration to go off the edge, but over the course of $\sim 10^5$ years only up to a couple hundred galaxies are lost (what a bizarre statement...).

The relevant parameters used in this galaxy movement simulation are the choice of force softening kernel, the "Plummer" radius, and the time step between simulation iterations. For my simulations, I used a modified Plummer model where instead of using $\frac{1}{\sqrt{r^2+\epsilon^2}}$, I used $\frac{1}{r^2+\epsilon^2}$, with $\epsilon = 10^{-3}$. The time step between simulation iterations was chosen as 10^4 years.

The snapshots below indicate the first, sixth, and final iterations showing the evolution of the galaxies' positions over the course of roughly 10^5 years.

Naively, one would think that the two large clumps would become even more compact over time rather than more diffuse. The interaction I anticipated was the two large clumps combining to form one new large clump at (7.5, 5) or something akin to that with the galaxies at small x slowly drifting towards the right side. Perhaps this is what would happen given a large enough simulation step, but I think it is more likely that I have made a mistake somewhere in my code (using a 10^5 time step as a test case confirms this hypothesis as it yields the same results but with more particles being lost outside the boundaries of the box).

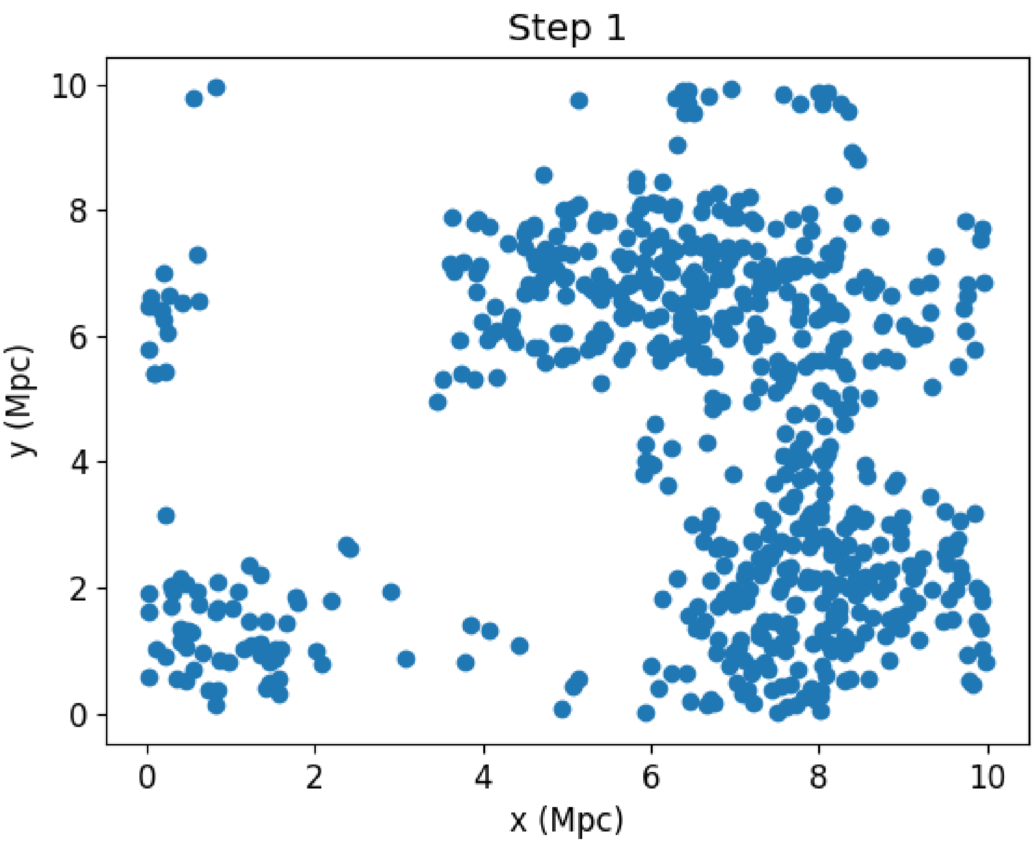

Figure 1: Initial configuration of galaxies, with large clumps centered near (7, 7) and (8, 2).

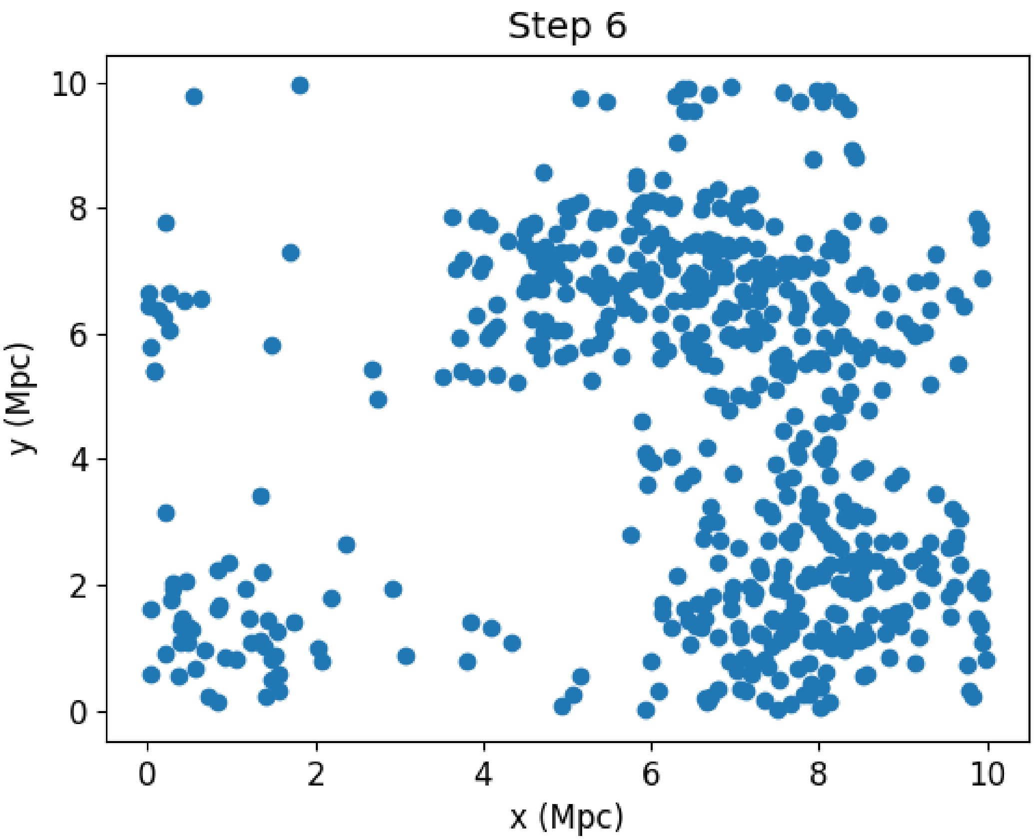

Figure 2: Sixth, and roughly mid-point, configuration of galaxies, with large clumps still present near (7, 7) and (8, 2) and a few galaxies roaming in the space near the middle of the box which was previously unpopulated.

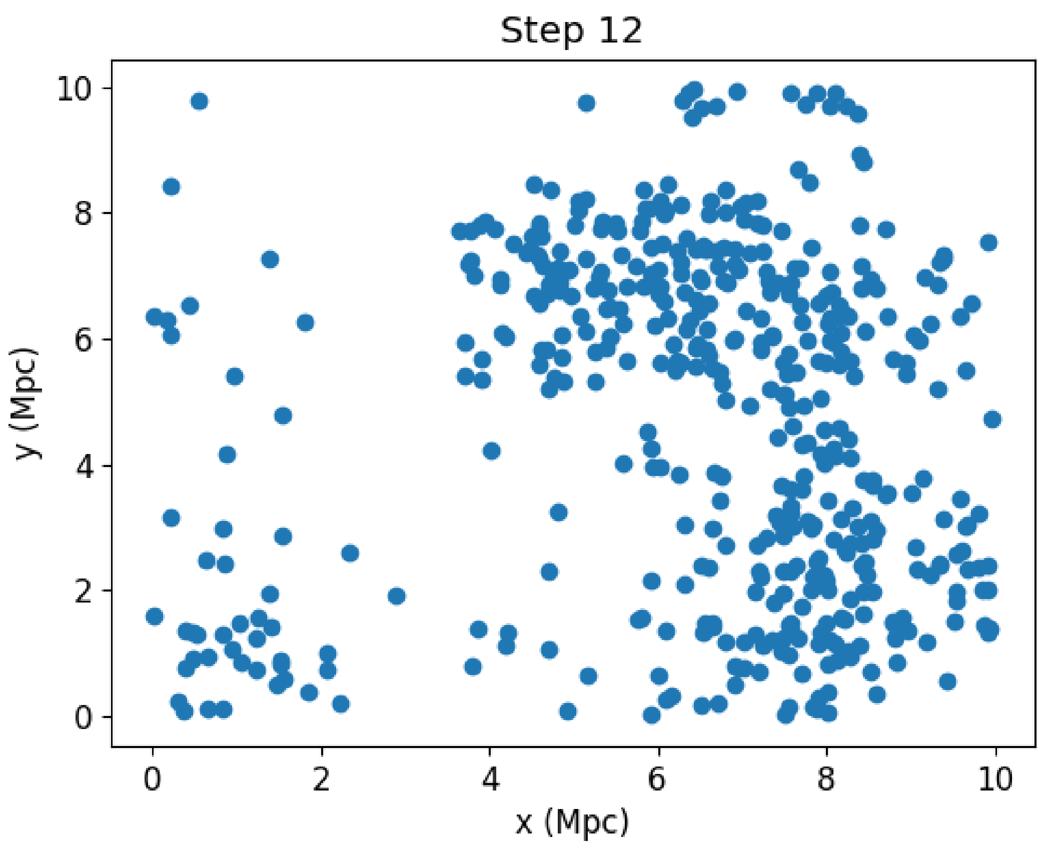

Figure 3: Final configuration of galaxies, with the large clumps near (7, 7) and (8, 2) becoming more sparse and showing signs of spreading out and becoming more diffuse. The clump located in the top left of Figure 1 has almost entirely dissipated by this point.

2 Task 2

For task 2, the potential calculation is found in **tasks.py** and is done on a very rough grid. The extremely unresolved grid was chosen on purpose as there appear artifacts at (7, 2) and (8, 7) which manifest as unphysical peaks. These likely stem from the approach I used to calculate the potential, which is obviously wrong as evidenced by the existence of said artifacts. The potential is also not forced to go to zero at the boundaries as I could not figure out how to do that using my approach. That specific condition seems to imply the use of some method involving boundary conditions. Considering that the potential is the gradient of the force, likely the correct approach involved a finite differencing method using 0 as the boundary conditions for the potential on the edges of the box. Figure 4 shows the contour of the potential using my very rough, incorrect approach.

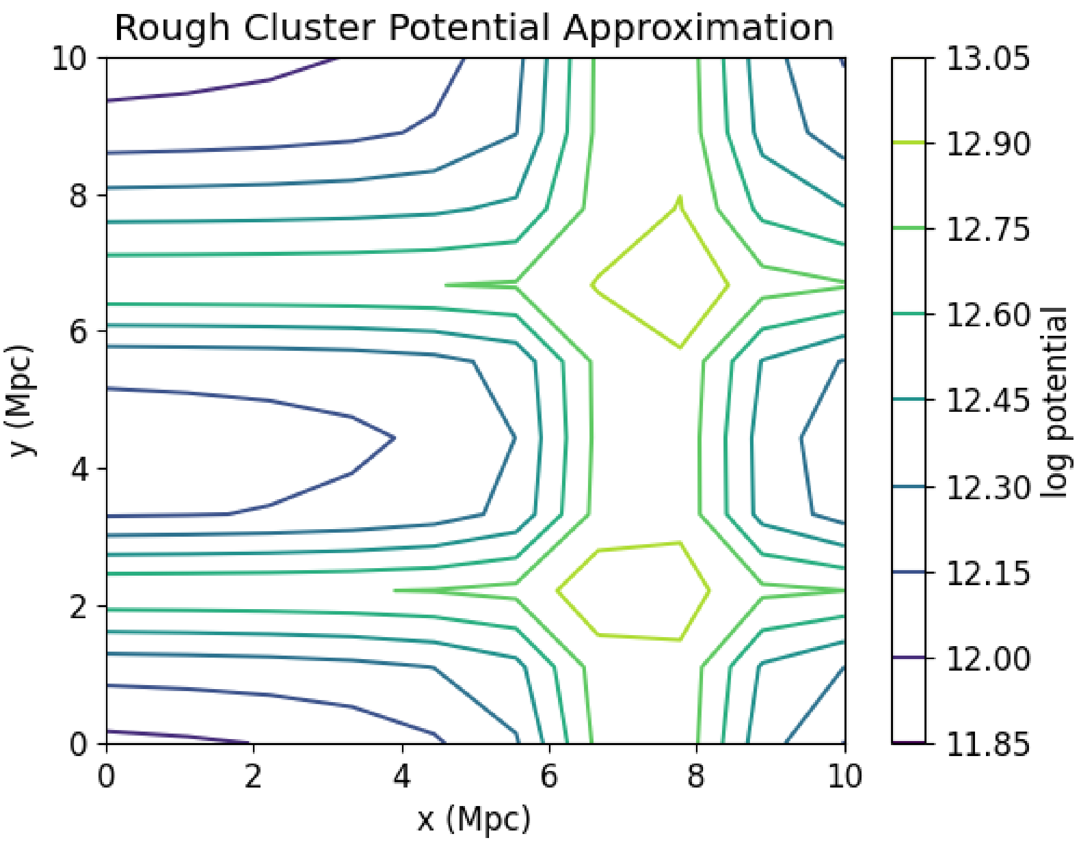

Figure 4: Rough (and incorrect) potential calculation given two clumps of galaxies, one centered at (7, 7) and the other at (8, 2). The low resolution choice shows results that look correct at first glance, but finer resolution reveals artifacts which stem from the simplistic and incorrect approach used.