Link to HW4 code

1 Task 1

In this assignment we consider how the balance between internal pressure and gravitational collapse creates hydrostatic equilibrium. The Newtonian (non-relativistic) equations are

$$\frac{dP(r)}{dr} = -\frac{GM_{enc}(r)\rho(r)}{r^2} \tag{1}$$

$$\frac{dM_{enc}}{dr} = 4\pi r^2 \rho(r). \tag{2}$$

In order to obtain the mass-radius curve, we need to loop over relevant central densities for white dwarfs and see how the radius and mass are affected given the starting central density. In each case below, the cutoff is decided as when the pressure becomes negative, as this is non-physical and a good indicator of a stopping point. For white dwarfs, we scan across $10^4 \leq \rho_c \leq 10^6$ g/cm^3 and record the radii and masses of the star at the time of termination due to non-physical pressure. The relationship between pressure and density is given by a polytropic equation of state of index 3/2 boiling down to

$$P = 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3}. \tag{3}$$

Figure 1 shows the mass-radius curve for white dwarves using the non-relativistic polytropic hydrostatic equilibrium equations.

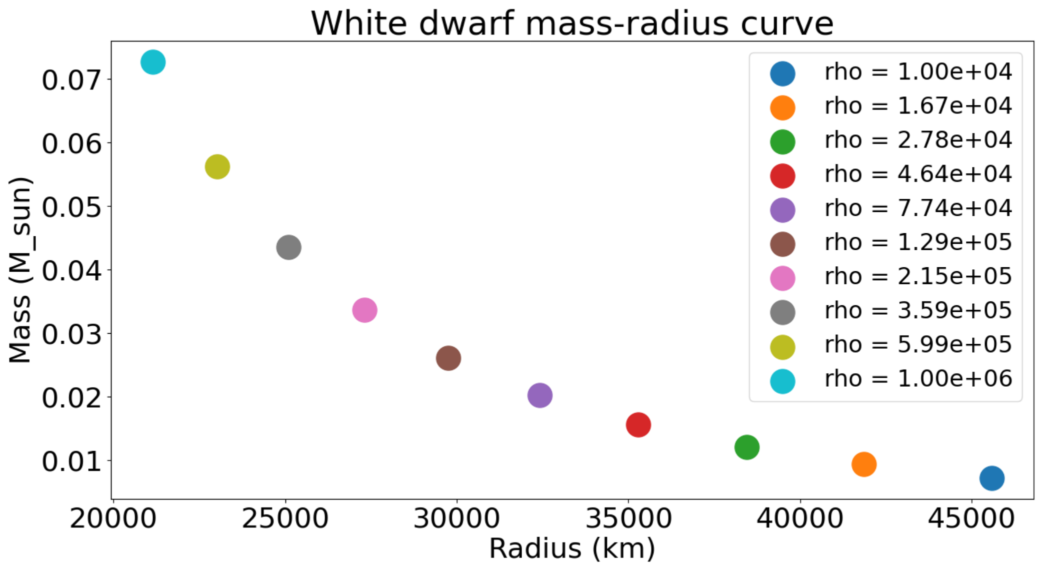

Figure 1: Mass-radius curve for white dwarves, going out to several Earth radii.

2 Task 2

Task 2 introduces the Tolman-Oppenheimer-Volkoff equation of state for neutron stars which considers the relativistic effects that cannot be ignored for such extremely dense systems. With central densities $10^14 \leq \rho_c \leq 10^16$ g/cm^3, the pressure-density relationship changes from that of white dwarfs and is given as

$$P = 5.4 \times 10^9 \rho^{5/3}. \tag{4}$$

Figure 2 shows the mass-radius curve for neutron stars using the relativistic (TOV) hydrostatic equilibrium equations.

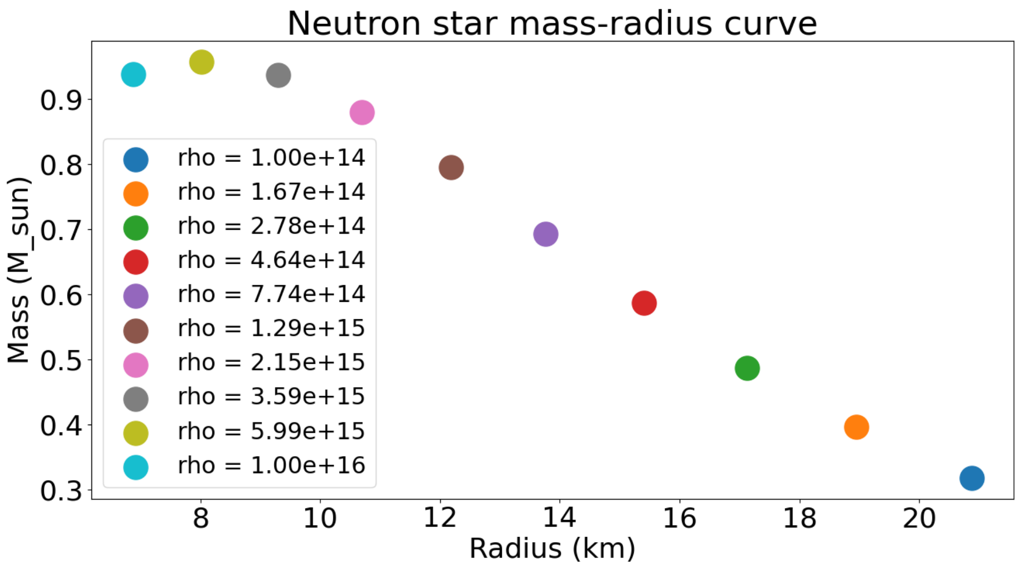

Figure 2: Mass-radius curve for neutron stars going out to a couple tens of kilometers.

3 Task 3

From Figure 2, the 13 km radius falls roughly between the purple and brown points, from which we can infer a starting central density of around 10^{15}. Integrating out to 13 km with this central density yields a neutron star mass of about $0.75 M_\odot$. This seems to be roughly half the expected neutron star mass - the reason for the missing factor of 2 is currently uncertain.