Link to HW8 code

1 Task 1

This homework involved the use of the Cooley-Tukey algorithm to make a fast Fourier transform (FFT) calculation of simulated gravitational wave strain data. The frequency at which the binary orbited as well as the amplitude of the peak at that frequency were used to determine the total mass M and separation R of the binary. The Cooley-Tukey algorithm starts with the discrete Fourier transform (DFT)

$$H_k = \sum_{n=0}^{N-1} h_n e^{-2\pi ikn/N} \tag{1}$$

where N is the total number of samples in the array being transformed, n is the n-th entry in the array being transformed and k represents the k-th entry in the transform array. The fast part of the FFT comes in the form of a speed-up which recursively breaks up the array into even and odd components until a certain maximum array length is met. At this point, the DFT of the sub-array is computed since it is small enough to be an efficient calculation. In the end, this recursive sub-division of even and odd components of the original array changes the number of calculations from N^2 to $N \log N$.

A test of the Fourier transform of a simple sine wave with $\omega = 6$ for the DFT, FFT, and numpy's FFT routine is shown in Figure 1. All three methods show agreement, and the FFT is considerably faster than the DFT (although the Cooley-Tukey algorithm is much slower than numpy's version, which probably has more complex speed-up routines).

Figure 2 shows the FFT of the strain data with the expected peak occurring at $\mathbf{f_{GW} = 2.2 \times 10{-}3}$ **Hz** which corresponds to an orbital period of roughly 7 minutes. The amplitude of the peak, when normalized by the number of samples in the strain data array, is $\mathbf{h = 2.02 \times 10^{-22}}$. By re-arranging the scaling relations provided in the homework, namely Equations 2 and 3, and using the fact that the distance to the source is 12 pc, we find that the mass and radius in terms of known quantities are

$$\frac{12h}{2.6 \times 10^{-21}} \left(\frac{R}{R_\odot}\right) \approx \left(\frac{M}{M_\odot}\right)^2 \tag{2}$$

and

$$\frac{f_{GW}}{10^{-4}} \left(\frac{R}{R_\odot}\right)^{3/2} \approx \left(\frac{M}{M_\odot}\right)^{1/2}. \tag{3}$$

Re-scaling Equation 2 yields

$$\left(\frac{12h}{2.6 \times 10^{-21}}\right)^{1/4} \left(\frac{R}{R_\odot}\right)^{1/4} \approx \left(\frac{M}{M_\odot}\right)^{1/2} \tag{4}$$

which, when combined with Equation 3, yields

$$\left(\frac{12h}{2.6 \times 10^{-21}}\right)^{1/4} \left(\frac{R}{R_\odot}\right)^{1/4} \approx \frac{f_{GW}}{10^{-4}} \left(\frac{R}{R_\odot}\right)^{3/2}. \tag{5}$$

Solving for the separation R and plugging that back into either Equation 2 or 3 will yield the total mass M which, for the f_{GW} and h obtained from the FFT, were $\mathbf{M = 0.279\ M_\odot}$ and $\mathbf{R = 0.083\ R_\odot}$. These parameters seem extremely small for the given binary, especially the mass. The separation makes some sense considering the 7 minute orbital period, however the mass implies two white dwarves weighing under $0.3 M_\odot$ combined, which is extremely unlikely (impossible?).

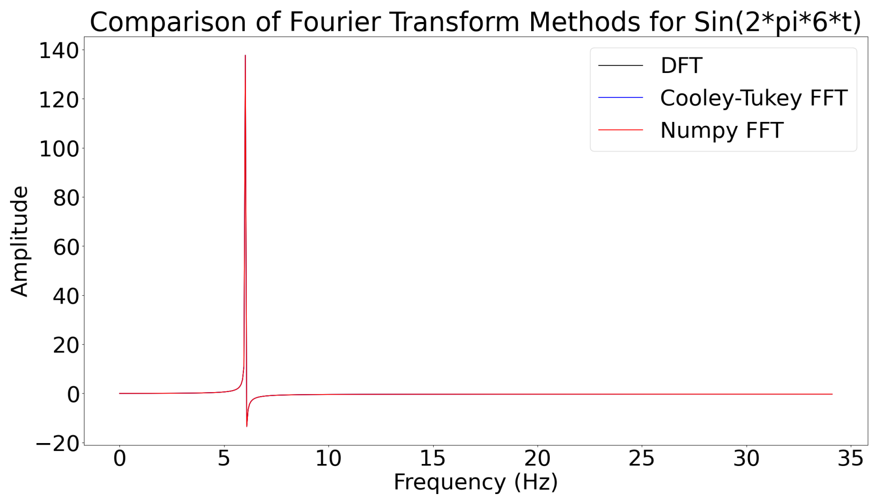

Figure 1: Comparison of DFT, Cooley-Tukey FFT, and numpy FFT routines. All three show near-perfect agreement when a threshold of 1 is used for the maximum array length to use the DFT in the FFT recursion step.

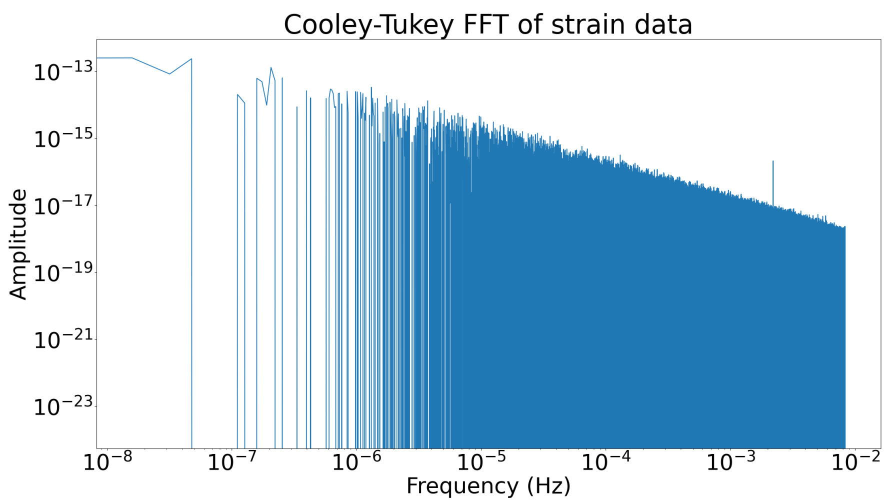

Figure 2: FFT of strain data using the Cooley-Tukey FFT code. This is extremely similar to the output generated by numpy's FFT function.