8) Bepaal de AO van $(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = 0$

* scheiden v/d veranderlijken : lukt niet !
* homogene DVG ?

$M(x,y) = y^2 e^{xy^2} + 4x^3$ & $N(x,y) = 2xye^{xy^2} - 3y^2$

$\rightarrow M(\lambda x, \lambda y) = \lambda^2 y^2 e^{\lambda^3 xy^2} + 4\lambda^3 x^3$! GEEN homogene functie, want $M(\lambda x, \lambda y) \neq \lambda^n M(x,y)$

$\Rightarrow$ GEEN homogene DVG

* totale DVG ?

$\frac{\partial M(x,y)}{\partial y} = 2y e^{xy^2} + 2xy^3 e^{xy^2} = \frac{\partial N(x,y)}{\partial x} \Rightarrow$ totale DVG (exacte DVG)

DVG : $\underbrace{\boxed{\frac{\partial F(x,y)}{\partial x}}}_{= M(x,y)} dx + \underbrace{\boxed{\frac{\partial F(x,y)}{\partial y}}}_{= N(x,y)} dy = 0$ m.a.w. $dF = 0$ $\Rightarrow$ oplossing: $F(x,y) = C_0$ met C_0 = constante

(1) $\frac{\partial F(x,y)}{\partial x} = M(x,y) = y^2 e^{xy^2} + 4x^3$

$F(x,y) = \int (y^2 e^{xy^2} + 4x^3)\,dx + K(y)$

$F(x,y) = e^{xy^2} + x^4 + K(y)$ (*)

(2) $\frac{\partial F(x,y)}{\partial y} = 2xy e^{xy^2} + \frac{dK(y)}{dy} = N(x,y) = 2xye^{xy^2} - 3y^2$

$\Rightarrow \int dK(y) = \int (-3y^2)\,dy \Rightarrow K(y) = -y^3 + C_1$ (**)

(3) substitutie van (**) in (*):

$F(x,y) = e^{xy^2} + x^4 - y^3 + C_1$

(4) AO: $F(x,y) = C_0$

AO: $\boxed{e^{xy^2} + x^4 - y^3 = C}$

(11) Bepaal de PO door (1,1) van $\left(4x^3y^3+\frac{1}{x}\right)dx+\left(3x^4y^2-\frac{1}{y}\right)dy=0$

[1] AO

* scheiden van de veranderlijken: lukt niet!
* homogene DVG?

$$M(x,y)=4x^3y^3+\frac{1}{x} \quad \& \quad N(x,y)=3x^4y^2-\frac{1}{y}$$

$$\rightarrow M(\lambda x,\lambda y)=4\lambda^6x^3y^3+\frac{1}{\lambda x} \neq \lambda^n M(x,y)$$

! GEEN homogene functie v/d n^{de} graad

$\Rightarrow$ GEEN homogene DVG!

* totale DVG?

$$\frac{\partial M(x,y)}{\partial y}=12x^3y^2=\frac{\partial N(x,y)}{\partial x} \Rightarrow \text{totale DVG (exacte DVG)}$$

DVG: $\underbrace{\boxed{\frac{\partial F(x,y)}{\partial x}}}_{=M(x,y)}dx+\underbrace{\boxed{\frac{\partial F(x,y)}{\partial y}}}_{=N(x,y)}dy=0$ m.a.w. $dF=0 \Rightarrow$ oplossing: $F(x,y)=C_0$ met $C_0=$ constante

(1) $\frac{\partial F(x,y)}{\partial x}=M(x,y)=4x^3y^3+\frac{1}{x}$

$$F(x,y)=\int\left(4x^3y^3+\frac{1}{x}\right)dx+K(y) \Rightarrow \boxed{F(x,y)=x^4y^3+\ln|x|+K(y)} \quad (*)$$

(2) $\frac{\partial F(x,y)}{\partial y}=3x^4y^2+\underline{\frac{dK(y)}{dy}}=N(x,y)=3x^4y^2\underline{-\frac{1}{y}}$

$$\Rightarrow \int dK(y)=\int\left(-\frac{dy}{y}\right) \Rightarrow \boxed{K(y)=-\ln|y|+C_1} \quad (**)$$

(3) substitutie van (**) in (*):

$$F(x,y)=x^4y^3+\ln|x|-\ln|y|+C_1=x^4y^3+\ln\left|\frac{x}{y}\right|+C_1$$

(4) AO: $F(x,y)=C_0$

AO: $x^4y^3+\ln\left|\frac{x}{y}\right|=C$

[2] PO door (1,1).

(1,1) in AO: $1+\underbrace{\ln 1}_{=0}=C \Rightarrow C=1$

$$\Rightarrow \text{PO}: \boxed{x^4y^3+\ln\left|\frac{x}{y}\right|=1}$$

(14) Bepaal de AO van $xy' = y + \sqrt{x^2-y^2}$.

DVG: $\quad x\,dy = \left(y + \sqrt{x^2-y^2}\right)dx \quad (*)$

$\left(y' = \frac{dy}{dx}\right)$

$$\underbrace{\left(y+\sqrt{x^2-y^2}\right)}_{=M(x,y)} dx + \underbrace{(-x)}_{=N(x,y)} dy = 0$$

* scheiden v/d veranderlijken: lukt niet!
* homogene DVG? $(\lambda > 0)$

$$\left.\begin{array}{l} M(\lambda x, \lambda y) = \lambda y + \lambda\sqrt{x^2-y^2} = \lambda M(x,y) \\ N(\lambda x, \lambda y) = -\lambda x = \lambda N(x,y) \end{array}\right\} \Rightarrow$$

$M(x,y)$ en $N(x,y)$ zijn homogene functies van dezelfde graad (1^{ste} graad)

$\Rightarrow$ homogene DVG

(1) substitutie van $y = ux$ in $(*)$:

$$x\left[x\,du + \cancel{u\,dx}\right] = \left(\cancel{ux} + \sqrt{x^2(1-u^2)}\right)dx$$

$$\Rightarrow x^2\,du = |x|\sqrt{1-u^2}\,dx$$

$x \geq 0: \sqrt{x^2} = x$

$x < 0: \sqrt{x^2} = -x$

$$\Rightarrow \frac{du}{\sqrt{1-u^2}} = \frac{dx}{|x|}$$

Geval 1: $x > 0$ $\quad \int \frac{du}{\sqrt{1-u^2}} = \int \frac{dx}{x} \Rightarrow \text{Bgsin}\,u = \ln|x| + C$

Geval 2: $x < 0$ $\quad \int \frac{du}{\sqrt{1-u^2}} = \int \frac{dx}{-x} \Rightarrow \text{Bgsin}\,u = -\ln|x| + C$

$\Rightarrow$ Algemeen: $\text{Bgsin}\,u = \frac{x}{|x|}\ln|x| + C \quad (**)$

(2) substitutie van $u = \frac{y}{x}$ in $(**)$:

$$\boxed{\text{Bgsin}\left(\frac{y}{x}\right) = \frac{x}{|x|}\ln|x| + C}$$

15. Bepaal de PO door $(1, \pi/2)$ van $\left(x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)\right)dx + x\cos\left(\frac{y}{x}\right)dy = 0$

1 AO

* scheiden vd veranderlijken : lukt niet!
* homogene DVG?

$M(x,y) = x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)$ & $N(x,y) = x\cos\left(\frac{y}{x}\right)$

$\to M(\lambda x, \lambda y) = \lambda x \sin\left(\frac{y}{x}\right) - \lambda y \cos\left(\frac{y}{x}\right) = \lambda M(x,y)$

$\to N(\lambda x, \lambda y) = \lambda x \cos\left(\frac{y}{x}\right) = \lambda N(x,y)$

} M(x,y) en N(x,y) zijn homogene functies van dezelfde graad (namelijk de 1ste graad)

⇒ homogene DVG

(1) substitutie van $y = ux$ in DVG:

$(x\sin u - \cancel{ux\cos u})dx + x\cos u[xdu + \cancel{udx}] = 0$

$\Rightarrow x^2\cos u\, du = -x\sin u\, dx \Rightarrow \int \frac{\cos u}{\sin u}du = \int \frac{-dx}{x}$

$\Rightarrow \ln|\sin u| = -\ln|x| + \underbrace{C_1}_{=\ln|C|} = \ln\left|\frac{C}{x}\right| \Rightarrow \sin u = \frac{C}{x} \Rightarrow x\sin u = C$ (*)

(2) substitutie van $u = \frac{y}{x}$ in (*):

AO: $x\sin\left(\frac{y}{x}\right) = C$

2 PO door $(1, \pi/2)$

$(1, \pi/2)$ in AO: $\sin\left(\frac{\pi}{2}\right) = C \Rightarrow C = 1$

$\Rightarrow$ PO: $\boxed{x\sin\left(\frac{y}{x}\right) = 1}$

22. Bepaal de AO van $(x-2)y' = y + 2(x-2)^3$

DVG: $(x-2)\,dy = [y + 2(x-2)^3]\,dx$

$(y' = \frac{dy}{dx})$

$$\underbrace{[y + 2(x-2)^3]}_{=M(x,y)}dx + \underbrace{(2-x)}_{=N(x,y)}dy = 0$$

* scheiden vd veranderlijken : lukt niet!
* homogene DVG?

$M(\lambda x, \lambda y) = \lambda y + 2(\lambda x - 2)^3 \neq \lambda^n M(x,y)$ ⇒ geen homogene functie

⇒ geen homogene DVG

* totale DVG?

$\frac{\partial M(x,y)}{\partial y} = 1 \neq \frac{\partial N(x,y)}{\partial x} = -1$ ⇒ geen totale DVG (geen exacte DVG)

* lineaire DVG?

→ lineaire DVG in y en y'? m.a.w. vd vorm $y' + yP(x) = Q(x)$?

$(x-2)y' = y + 2(x-2)^3 \Rightarrow y' = \frac{y}{x-2} + 2(x-2)^2$

$\Rightarrow y' + y \cdot \underbrace{\left(\frac{-1}{x-2}\right)}_{P(x)} = \underbrace{2(x-2)^2}_{Q(x)}$ (*) lineaire DVG in y en y'.

$y' + y \cdot P(x) = Q(x)$

(1) substitutie van $y = u.v$ in (*):

$u'v + \underbrace{u.v' + u.v\left(\frac{-1}{x-2}\right)}$ $= 2(x-2)^2$ (*)

(2) functie v kiezen zodat coëfficiënt van u nul wordt

$v' + v\left(\frac{-1}{x-2}\right) = 0 \xRightarrow{v' = dv/dx} \int \frac{dv}{v} = \int \frac{dx}{x-2} \Rightarrow \ln|v| = \ln|x-2| + C_0$

→ Kies bv. $C_0 = 0$: $\boxed{v = x-2}$ (**)

(3) substitutie van (**) in (*)

$u'\cancel{(x-2)} = 2(x-2)^{\cancel{2}} \xRightarrow{u' = du/dx} \int du = \int 2(x-2)\,dx$

$\Downarrow$

$\boxed{u = (x-2)^2 + C}$ (***)

(4) substitutie van (**) en (***) in $y = u.v$

AO: $\boxed{y = (x-2)^3 + C(x-2)}$

(26) Bepaal de AO van $y' - y = xy^5$.

DVG: $y' + y.(-1) = y^5.x \underset{y'=dy/dx}{\Rightarrow} dy = (xy^5+y)dx$

$\Rightarrow \underbrace{(xy^5+y)}_{=M(x,y)}dx + \underbrace{(-1)}_{=N(x,y)}dy = 0$

* scheiden vld veranderlijken: lukt niet
* homogene DVG?

$M(\lambda x, \lambda y) = \lambda^6 xy^5 + \lambda y \neq \lambda^n M(x,y) \Rightarrow$ geen homogene functie $\Downarrow$ geen homogene DVG

* totale DVG?

$\frac{\partial M(x,y)}{\partial y} = 5xy^4 + 1 \neq \frac{\partial N(x,y)}{\partial x} = 0 \Rightarrow$ geen totale DVG (geen exacte DVG)

* lineaire DVG?

→ lineaire DVG in y en y'? m.a.w. vld vorm $y' + y\,P(x) = Q(x)$?

$y' + y(-1) = y^5 x$ omwille van y^5 → geen lineaire DVG in y en y'

→ lineaire DVG in x en x'? m.a.w. vld vorm $x' + x\,P(y) = Q(y)$?

$dy = (xy^5+y)dx \underset{x'=\frac{dx}{dy}}{\Rightarrow} \frac{1}{xy^5+y} = x'$ geen lineaire DVG in x en x'.

* DVG van Bernoulli?

→ DVG van Bernoulli in y en y'? m.a.w. vld vorm $y' + y\,P(x) = y^n.Q(x)$? DVG van Bernoulli in y en y'

(*) $y' + y.\underbrace{(-1)}_{} = \underbrace{y^5}_{}.\underbrace{x}_{}$

$y' + y.P(x) = y^n.Q(x)$

(1) DVG (*) delen door y^5 $(=y^n)$: $\frac{y'}{y^5} + \frac{-1}{y^4} = x$ (**)

(2) $z = 1/y^4 = y^{-4}$ $(=y^{1-n})$

$\Rightarrow z' = \frac{dz}{dx} = -4y^{-5}.\frac{dy}{dx} = -4\frac{y'}{y^5} \Rightarrow \frac{y'}{y^5} = \frac{z'}{-4}$

$\Rightarrow$ substitutie in (**):

$\frac{z'}{-4} + z(-1) = x \Rightarrow z' + z.\underbrace{(4)}_{} = \underbrace{-4x}_{}$ lineaire DVG in z en z' (□)

$z' + z.P(x) = Q(x)$

(a) substitutie van $z = u.v$ in (□):

$u'v + \underline{u.v' + u.v(4)} = -4x$ (▷)

(b) functie v kiezen zodat coëfficiënt van u nul wordt:

$v' + 4v = 0 \underset{v'=dv/dx}{\Rightarrow} \int \frac{dv}{v} = \int(-4dx) \Rightarrow \ln|v| = -4x + C_0$

↓ kies bv. $C_0 = 0$

$\boxed{v = e^{-4x}}$ (▷▷)

(PI) $\int f dg = f.g - \int g df$

* $f = -x \to df = -dx$
* $dg = 4e^{4x}dx \to g = e^{4x}$

$\int(-4xe^{4x})dx = -xe^{4x} + \int e^{4x}dx$

(c) substitutie van (▷▷) in (▷):

$u'e^{-4x} = -4x \underset{u'=du/dx}{\Rightarrow} \int du = \int(-4x e^{4x})dx \overset{(PI)}{\Rightarrow} \boxed{u = -xe^{4x} + \frac{e^{4x}}{4} + C}$ (▷▷▷)

(d) substitutie van (▷▷) en (▷▷▷) in $z = u.v$

$z = -x + \frac{1}{4} + Ce^{-4x}$

(3) substitutie van $z = 1/y^4$ in (d): $\boxed{\frac{1}{y^4} = -x + \frac{1}{4} + Ce^{-4x}}$

28 Bepaal de AO van $\left(\frac{x}{y} - x^3\cos y\right)y' = 2$

DVG: $y' = \frac{dy}{dx}$

$$(x - x^3 y\cos y)dy = 2y\,dx \Rightarrow \underbrace{2y}_{=M(x,y)}dx + \underbrace{(x^3 y\cos y - x)}_{=N(x,y)}dy = 0$$

* scheiden v/d veranderlijken: lukt niet
* homogene DVG?

$N(\lambda x, \lambda y) = \lambda^4 x^3 y\cos\lambda y - \lambda x \neq \lambda^n N(x,y) \Rightarrow$ geen homogene functie $\Rightarrow$ geen homogene DVG

* totale DVG?

$\frac{\partial M(x,y)}{\partial y} = 2 \neq \frac{\partial N(x,y)}{\partial x} = 3x^2 y\cos y - 1 \Rightarrow$ geen totale DVG (geen exacte DVG)

* lineaire DVG?

→ lineaire DVG in y en y'? m.a.w. v/d vorm $y' + y.P(x) = Q(x)$?

• $y' = \frac{2}{\frac{x}{y} - x^3\cos y}$ geen lineaire DVG in y en y'

→ lineaire DVG in x en x'? m.a.w. v/d vorm $x' + x.P(y) = Q(y)$?

$\frac{x}{y} - x^3\cos y = \frac{2}{y'} = 2\frac{dx}{dy} \xRightarrow{dx/dy = x'} 2x' + x\left(\frac{-1}{y}\right) = -x^3\cos y$

$\Rightarrow x' + x\left(\frac{-1}{2y}\right) = -\frac{x^3\cos y}{2}$ geen lineaire DVG in x en x'

* DVG van Bernoulli

→ DVG van Bernoulli in y en y'? m.a.w. v/d vorm $y' + yP(x) = y^n Q(x)$?

neen, geen DVG van Bernoulli in y en y' (zie •)

→ DVG van Bernoulli in x en x'? m.a.w. v/d vorm $x' + xP(y) = x^n Q(y)$?

(*) $x' + x\left(-\frac{1}{2y}\right) = x^3\left(-\frac{\cos y}{2}\right)$

$x' + x.P(y) = x^n.Q(y)$

DVG van Bernoulli in x en x'

(1) DVG (*) delen door x^3 $(=x^n)$: $\frac{x'}{x^3} + \frac{1}{x^2}\left(-\frac{1}{2y}\right) = -\frac{\cos y}{2}$ (**)

(2) $z = 1/x^2 = x^{-2}$ $(= x^{1-n})$

$\Rightarrow z' = \frac{dz}{dy} = -2x^{-3}\frac{dx}{dy} = -\frac{2x'}{x^3} \Rightarrow \frac{x'}{x^3} = \frac{z'}{-2}$

$\Rightarrow$ substitutie in (**):

$\frac{z'}{-2} + z\left(-\frac{1}{2y}\right) = -\frac{\cos y}{2} \Rightarrow z' + z\left(\frac{1}{y}\right) = \cos y$ lineaire DVG in z en z' (□)

$z' + z.P(y) = Q(y)$

(a) substitutie van $z = u.v$ in (□):

$u'v + \underbrace{u.v' + u.v\left(\frac{1}{y}\right)} = \cos y$ (▷)

(b) functie v kiezen zodat coëfficiënt van u nul wordt:

$v' + v\left(\frac{1}{y}\right) = 0 \xRightarrow{v' = dv/dy} \int\frac{dv}{v} = \int -\frac{dy}{y} \Rightarrow \ln|v| = -\ln|y| + C_0 = \ln\left|\frac{1}{y}\right| + C_0$

↓ Kies bv. $C_0 = 0$

$\boxed{v = \frac{1}{y}}$ (▷▷)

(c) substitutie van (▷▷) in (▷):

$u'\left(\frac{1}{y}\right) = \cos y \xRightarrow{u' = du/dy} \int du = \int y\cos y\,dy \xRightarrow{PI} \boxed{u = y\sin y + \cos y + C}$ (▷▷▷)

(PI) $\int f dg = fg - \int g df$
* $f = y \rightarrow df = dy$
* $dg = \cos y\,dy \rightarrow g = \sin y$

$\int y\cos y\,dy = y\sin y - \int \sin y\,dy$

(d) substitutie van (▷▷) en (▷▷▷) in $z = u.v$:

$z = \frac{1}{y}(y\sin y + \cos y + C)$

(3) substitutie van $z = 1/x^2$ in (d): $\boxed{\frac{y}{x^2} = y\sin y + \cos y + C}$