実数の
無限は濃すぎて、数えられない.

可算無限個の区間 I_i $i \in N$ に
分けて、$f(x_i)$, $x_i \in I_i$
各区間の代表点 x_i, と 代表値 $f(x_i)$

連続と離散的に捉える

数学関数 と プログラムで実現する

digital

連続関数 $f(x)$, $x \in R$

$$\{(x, f(x)) \mid x \in R\}$$
$$(I) \subset R$$

$$\{(x_i, f(x_i)) \mid i \in N, x_i \in I\}$$

直線 $(x_i, f(x_i))$ $(x_{i+1}, f(x_{i+1}))$

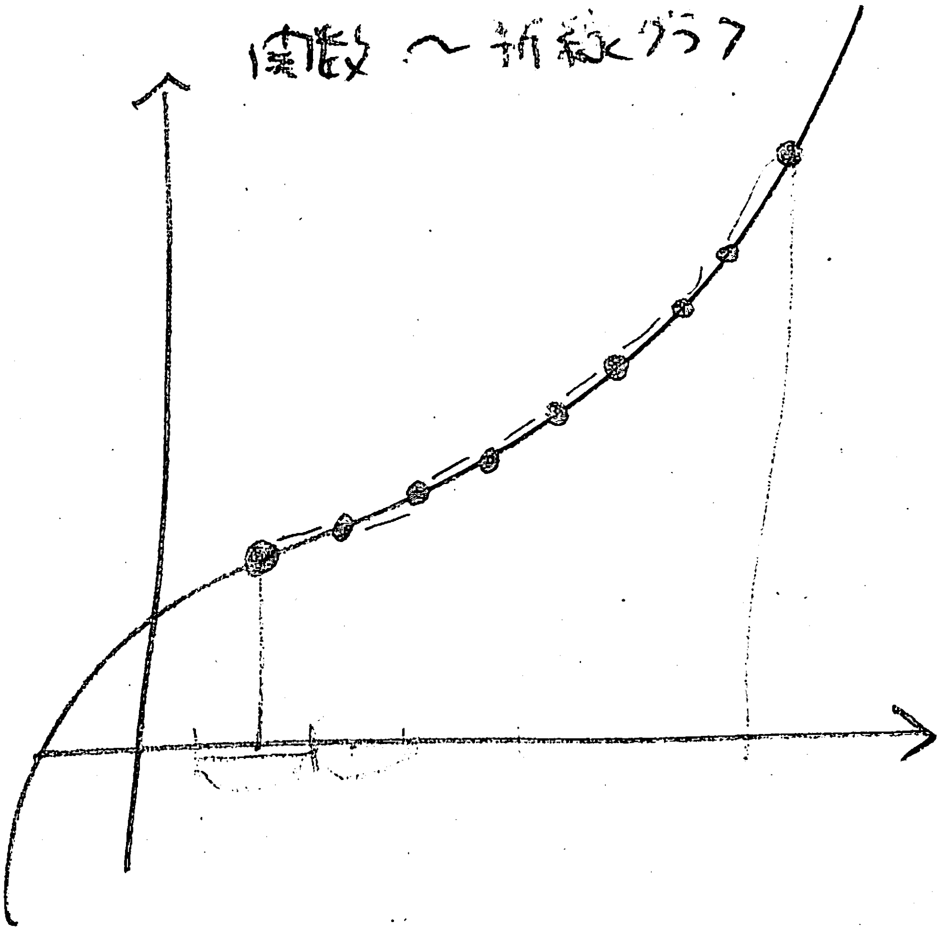
一次関数が可算

連続

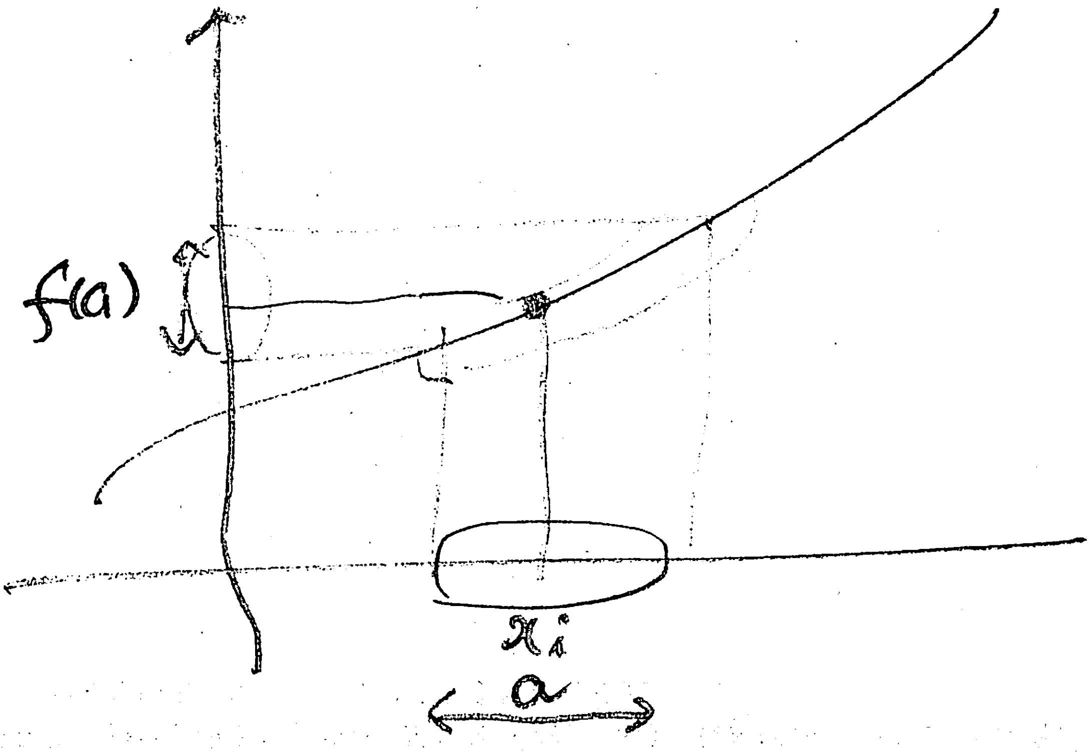
$$\lim_{x \to a} f(x) = f(a)$$

$$f(x) \longrightarrow f(a) \quad (x \to a)$$

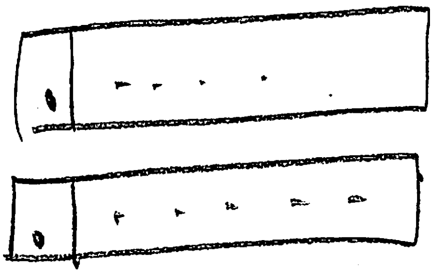
関数 〜 折線グラフ



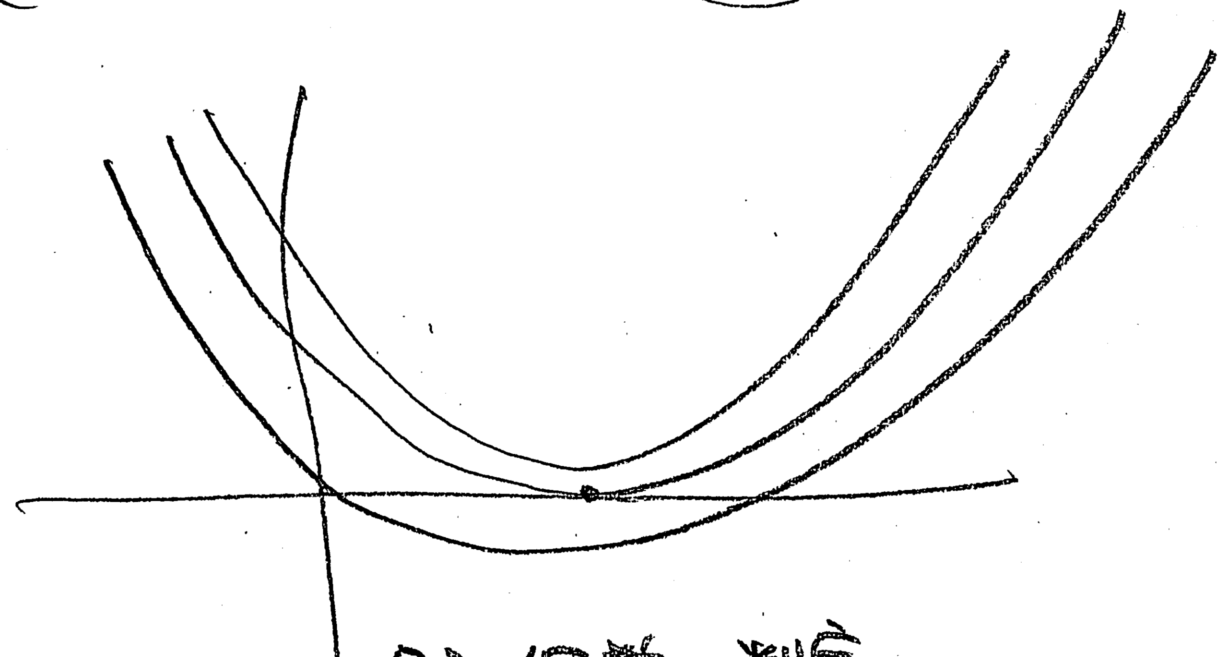
直線



有限精度の計算で
連続関数を扱う



$$\begin{cases} x_{i+1} - x_i = \underline{dx} & \text{精} \\ f(x_{i+1}) - f(x_i) \simeq 0 & \text{桁落ち} \\ \dfrac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} \sim \dfrac{0}{dx} \end{cases}$$



解の個数の判定

悪条件　精度落ち

関数の変化と微分（離散から）

$$f(x+\Delta x) - f(x) \equiv \Delta f$$

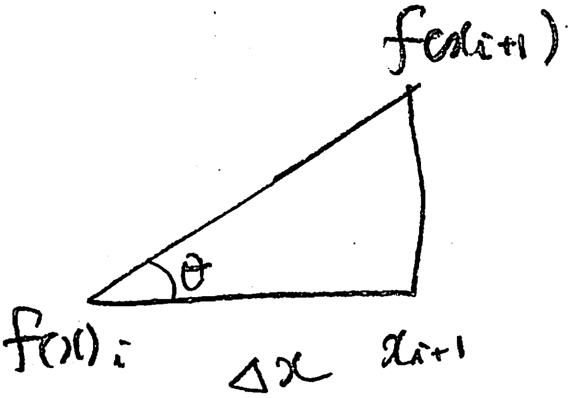
$$\Delta x = \frac{b-a}{n}$$

$\tan\theta$

$$\equiv \underbrace{\frac{f(x+\Delta x)-f(x)}{\Delta x}} \cdot \Delta x$$

$$\simeq f'(x)$$

$$\underline{\tan\theta_i} \cdot \Delta x$$

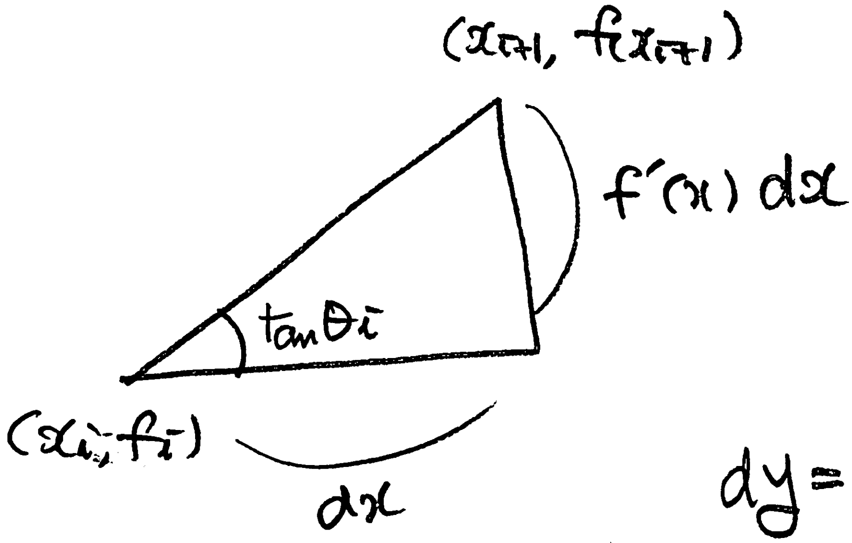


$$\lim_{\Delta x \to dx} \frac{f(x+\Delta x)-f(x)}{\Delta x} \equiv f'(x)$$

$$\lim_{\Delta x \to dx} f(x+\Delta x) - f(x) = f'(x)\,dx$$

d-

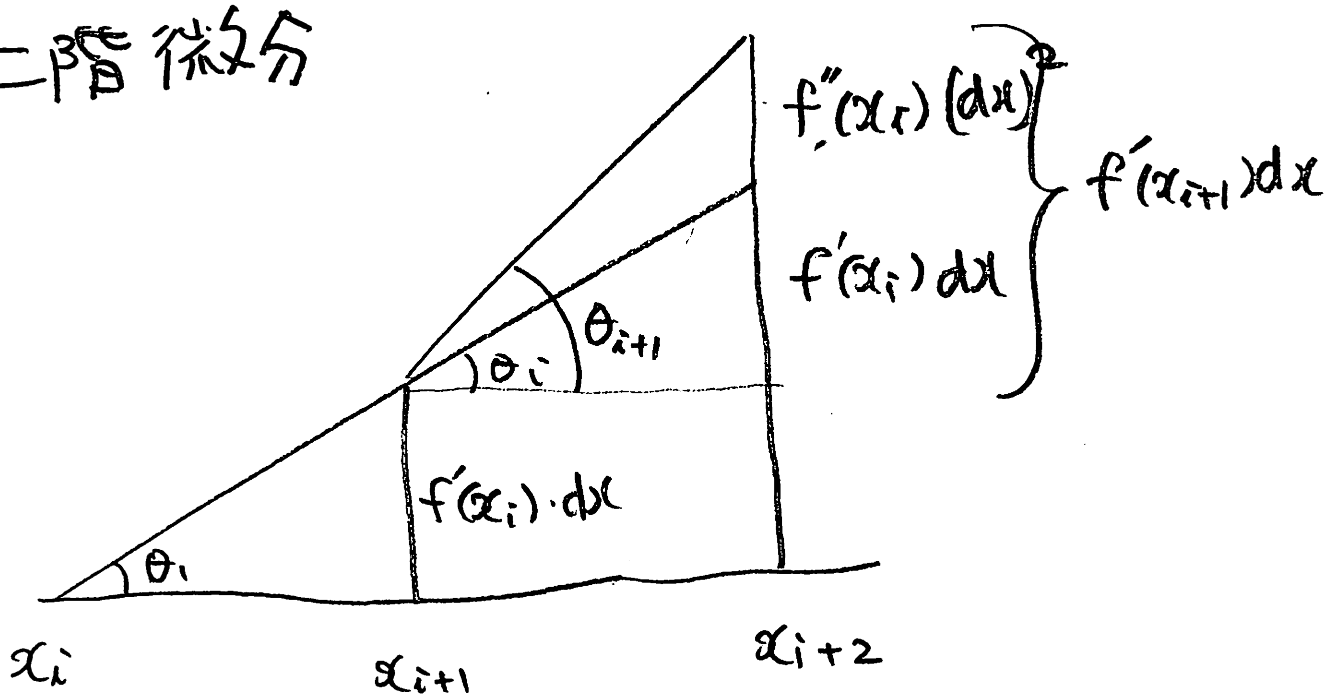
一階微分



$$dy = f'(x)\,dx$$

$$d(y) = d(f(x))$$

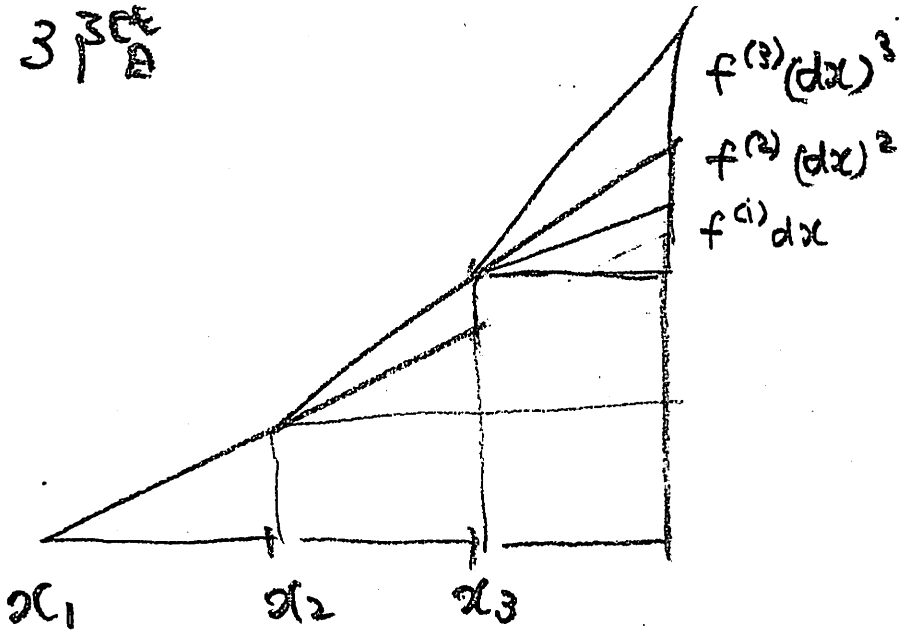
二階微分



$$d(dy) = d(f'(x))\cdot dx$$

$$d^2(y) = f''(x)(dx)^2$$

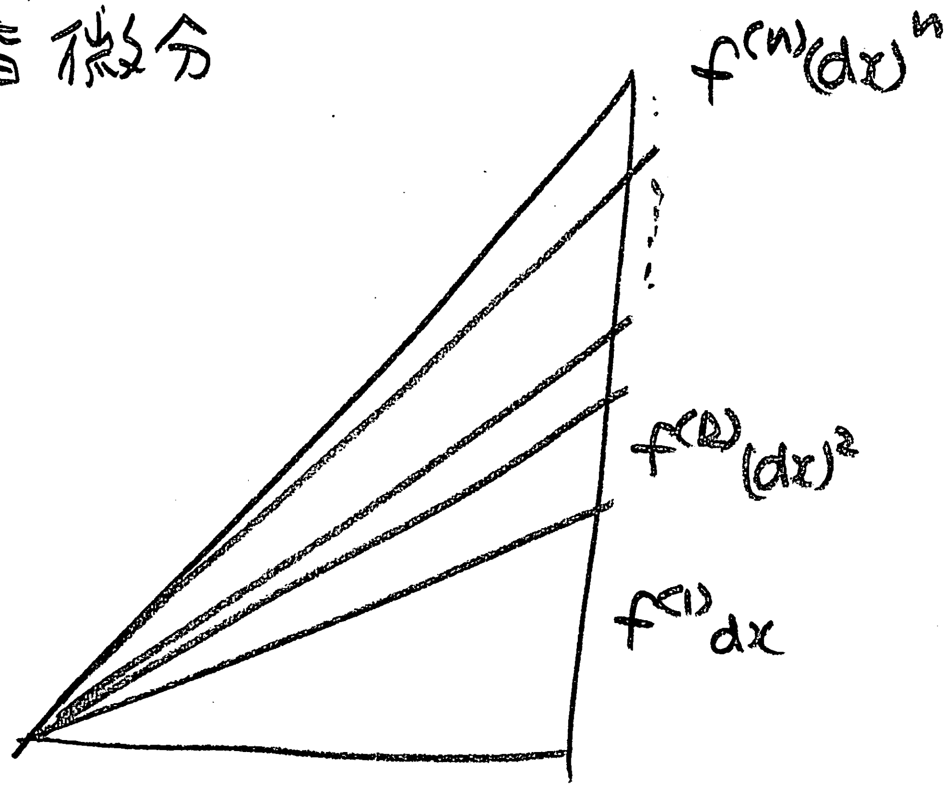
3階



$$d^3(y) = f^{(3)}(x_1)\,(dx)^3$$

$$f(x_4) = f(x_3) + \sum_{i=1}^{3} f^{(i)}(x_1)\,(dx)^i$$

n階微分



$$d^n(y) = f^{(n)}_{(x_1)} \cdot (dx)^n$$

$$\Delta(y) = \sum_{n=0}^{\infty} d^{(n)}(y) = \sum_{n=0}^{\infty} f^{(n)}_{(x_0)} \cdot (dx)^n$$

$$\int_a^b \Delta(y) = y(b) - y(a)$$

テイラー展開

$$\int_a^b \sum f^{(n)}(x_0)\,(dx)^n$$

$$= f(a) + \sum_{i=1}^{\infty} f^{(i)}(a) \cdot (b-a)^i$$

$$\int_a^b (dx)^n = \int_a^b x\,(dx)^{n-1} = \int_a^b \frac{x^2}{2!}(dx)^{n-2}$$

$$= \frac{1}{n!}\left[x^n\right]_a^b$$

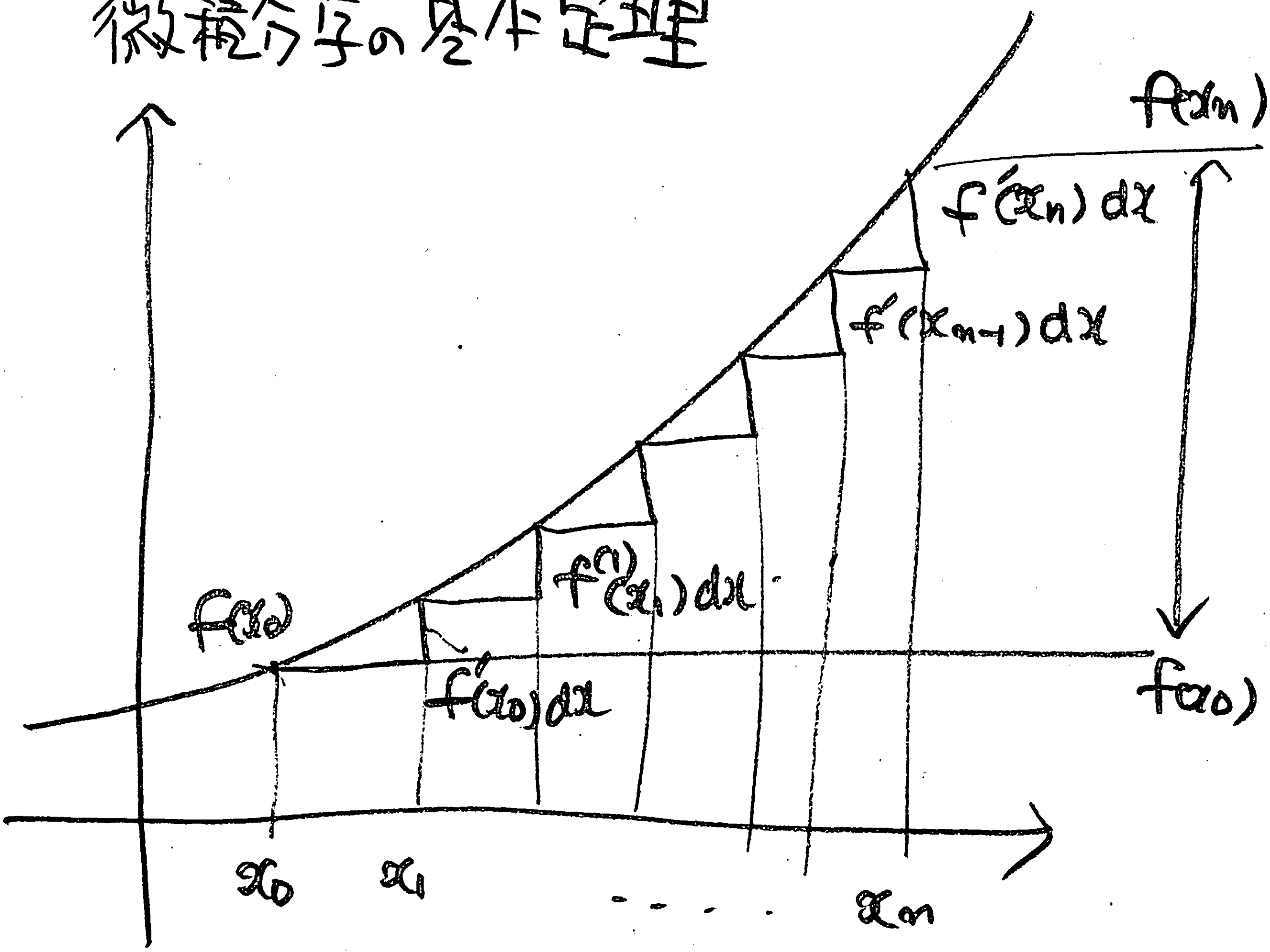
$$z = f(x, y)$$

$$dz = f_x\,dx + f_y\,dy$$

$$d^2z = f_{xx}(dx)^2 + f_{xy}\,dx\,dy + f_{yx}\,dy\,dx + f_{yy}(dy)^2$$

$\vdots$

微積分学の基本定理



$$f(x_n) - f(x_0) = \sum f'(x_i)\,dx$$

$$= \int_{x_0}^{x_{n+1}} f'(x)\,dx$$