Project Phase 1
Simulation of rare events: A report

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Introduction – What are Rare Events

- **Rare events** are events that are expected to occur infrequently or, more technically, those that have low probabilities (say, order of 10−3 or less)
- Rare-event simulation involves estimating extremely small but important probabilities.
- Any important measures of performance in communications networks are defined in terms of rare event probabilities.

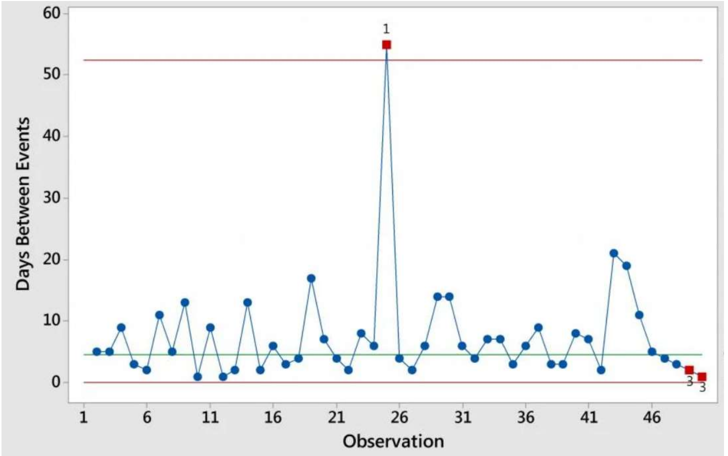

Figure: Rare events

Introduction – What are Rare Events

- Obtaining accurate estimates of such rare event probabilities using computer simulation can require execution times that are prohibitively long.

- There are some simulation techniques that enable generating rare events and predicting performance of a system.

Computer time needed to achieve a specified relative error (standard simulation)

	1 replication per second		1,000 replications per second	
	Desired Relative Error		Desired Relative Error	
γ	100%	10%	100%	10%
10^{-3}	17 min	1 day	1 sec	2 min
10^{-5}	1 day	4 months	2 min	3 hr
10^{-7}	4 months	32 years	3 hr	12 days
10^{-9}	32 years	3,169 years	12 days	3 years

Computer time vs probability

Introduction – Applications

- Rare-event simulation is used in many application areas, including communication and networking where packets loss and data loss, communication systems where the rare event could be the loss of important information, is crucial for system metrics.
- Another example where it is widely used is finance and insurance, where the rare event could be a large financial gain or loss, power systems where the rare event could be a major power outage
- nuclear physics where the rare event could be that particles cross a given protection shield, and aviation safety where the rare event could be an aircraft collision.
- It is also encountered in computer graphics (image synthesis by Monte Carlo methods), computational statistics, computational physics, computational biology, military applications, and so on.

Related Research

- Computation of such rare-event probabilities is challenging. Analytical solutions are usually not available for nontrivial problems, and standard Monte Carlo simulation is computationally inefficient.
- Therefore, much research effort has focused on developing advanced stochastic simulation methods that are more efficient.
- In spite of the advances in computational power, using simulation to obtain rare event probabilities still requires prohibitively long execution times.

Related Research

- W have three methods for facilitating simulation of Rare events:
  - Importance sampling
  - Subset simulation
  - Splitting

Simulation of Rare Events

- Importance Sampling (IS)
- Importance sampling (IS) refers to a collection of Monte Carlo methods where a mathematical expectation with respect to a target distribution is approximated by a weighted average of random draws from another distribution.
- IS-based techniques can be broadly grouped into two categories:
  - techniques where the individual stochastic elements are modified or biased,
  - and those where the global evolution of the system is manipulated.
- A chief issue when using this technique is determining how much the distributions of the random number generators should be modified or biased.
- Modification of stochastic properties - An important category of IS techniques involves modifying the underlying probability distributions of one or more of these random number generators in the simulation mode.

Simulation of Rare Events

- <u>Subset Simulation</u>

- The Subset Simulation (SS) method is an advanced stochastic simulation method for estimating rare events which is based on Markov chain Monte Carlo (MCMC).

- The basic idea behind SS is to represent a very small probability p Ɛ of the rare event E as a product of larger probabilities of “more-frequent” events and then estimate these larger probabilities separately

Simulation of Rare Events

- Splitting
- The idea of splitting is to "split" (or clone) a simulation run into separate runs whenever it gets "near" the rare event of interest. These runs share a common history up to the splitting point, but conditional on that history, they evolve independently of each other. In this way, more computer time is spent on promising runs that are close to the rare event.
- Three ways – Fixed, Fixed effort, Fixed-success splitting and fixed probability of success.

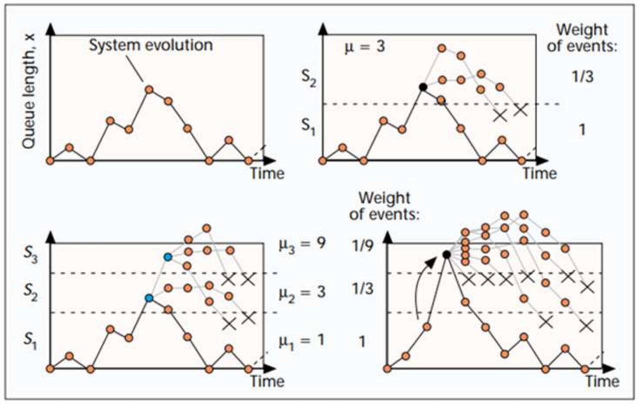

Figure 2. *Splitting increases the probability of reaching rare states by splitting the system trajectory upon entering well-defined subsets of the state-space.*

Figure: Splitting

Simulation of Rare Events

- Splitting

- Fixed -In this approach, any simulation run that reaches a new level is split into a fixed number of independent runs.
  - An advantage of fixed splitting is that it can be implemented recursively in a depth-first manner. This means that the computer only needs to store, at most, a single system state per level, corresponding to the simulation history of the current run.

- Fixed effort- In this approach, a predetermined total number of runs are made at each level. This is different from fixed splitting where the total number of runs at each level is random based on the outcomes of runs at lower levels.
  - The main advantage is that there is no need to know a good splitting factor R in advance, and because the total number of runs at each level is fixed

- Fixed-success splitting and fixed probability of success - In fixed-success splitting, at each level, the total number of trajectories that must reach the next level is fixed; independent replications at the current level continue until the fixed number of successes is achieved.

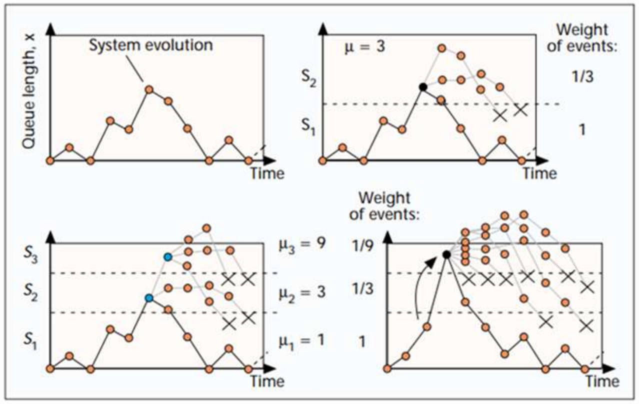

Figure 2. *Splitting increases the probability of reaching rare states by splitting the system trajectory upon entering well-defined subsets of the state-space.*

Figure: Splitting

Further research

- With computer technology advancement, rare events will be easier to evaluate with more complicated network mechanism.
- An approach could be useful for large attribute-rich datasets with multiple rare event types requiring various levels of modifications.
- Such data can be split first into multiple data streams and then we can perform importance sampling individually on the data streams.
- This yields case-specific rare events depending on the splitting and importance sampling priorities.

Conclusion

- There is a trade-off in simulation techniques, pushing the system too hard or altering its parameters, will result in the IS estimator to be much worse than standard simulation and the results may not reflect the reality.
- We hope that as computer technology continues to advance, simulation will be used to evaluate more complicated network mechanisms.
- At the same time, more reliable networks will be characterized by even rarer events.

Thank you!