

# Automatic Implementation of Equipment Models For Transient Stability Simulation by Symbolic Manipulations

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**Abstract**—One of the most important tasks in developing transient stability simulation software is to maintain a library of power system equipment models. This paper discusses the process of automatic implementation of custom equipment transient stability models described in graphical and textual forms or their combination. The implementation process is based on symbolic manipulation and does not require programming skills from users.

**Index Terms**—Power System Transient Stability Simulation, Power System Equipment modeling, Symbolic Manipulations, Computer Algebra Systems, Compilation.

## I. INTRODUCTION

SINCE the beginning of the 2000s, the ability of engineers to create their own custom mathematical models of power system equipment has become one of the major requirements for transient stability simulation software. The intensive development of FACTS technology, the elaboration of control systems and technologies, and the growing demands for energy efficiency increase lead to the fact that dozens of new equipment units appear on the market every year. Often, the transient properties of such equipment differ significantly from the properties of the built-in simulation models. The productivity of the approach to keeping the library of models built into the software up to date by the developers seems to be moderate, since the main task of the developers is to develop the software itself. The professional community of software users is immersed in equipment progress trends and is motivated to obtain accurate simulation results. Therefore, if there are convenient and reliable tools for creating custom models, the process of updating the library with higher quality and at lower costs can be provided directly by users. Modern communication technologies allow the exchange of model development results and unlimited access to the current library online.

Despite the fact that for almost all engineering areas in which some kind of simulation is required, there is at least one software package (and most often several options are available), a unified approach to describing user models has not been developed. One of the universal environments that can be considered a de facto standard is Matlab/Simulink [1]. Model development in Simulink is focused on graphical interface in the form of a block diagram. Another widely used environment, Modelica [2], also claims to have the maximum coverage of engineering areas, but offers an approach to

model using a special object-oriented language. In parallel with universal modeling environments, specialized software developed for particular areas is quite successfully developing. The choice of such software is obviously conditioned by the fact that it considers the features and established practice of the engineering discipline, and also allows to effectively use the features of the mathematical description of the problem. In most cases, specialized software provides an acceptable level of functionality without the need to create custom models by tuning to the of built-in ones, and allows to focus directly on the analysis of the results.

Specialized simulation software also includes products for simulation and analysis of transient stability. Almost every product has tools for creating custom models, but even in this relatively narrow niche, a unified approach to their presentation has not been formed. In particular, EUROSTAG [3] and PSS@SINCAL [4] use graphical interface, while DigSILENT PowerFactory [5] offers a programming language in textual format. If we consider in more detail the process of implementing a custom model, that is, its transformation from a representation in which it is developed by an engineer into a representation suitable for embedding in the software, it turns out that the graphical and textual approaches to model development at some point in the implementation process should give identical results. Thus, there is no need to focus on one approach or another, and moreover, they can be combined. Users with little development experience tend to prefer graphical interface block-building tools as they are more visual. As models become more complex, it is often more convenient to use a textual representation, since graphical block diagrams of relatively simple algorithmic expressions can be cumbersome. With major changes in the model, efforts will be required to maintain the visibility of its graphical representation. Since model editing is most often done to increase detail by adding new blocks to existing links, a significant part of the graphic image has to be rebuilt. This paper discusses the process of implementing custom models that allows to perform symbolic transformations that are invariant to the representation method of the initial mathematical description of the model and allows to combine graphical and textual approaches to the model development.

## II. REQUIREMENTS FOR CUSTOM MODELS

Before defining the requirements for the custom models implementation, it is necessary to give a brief description of

the problem being solved. Transient stability simulation is in fact is the process of solving a differential-algebraic system of equations (DAE) of the form:

$$\begin{cases} \dot{y} = f(x(t), y(t), t) \\ 0 = g(x(t), y(t), t) \end{cases} \quad (1)$$

$$x(t_0) = x_0, y(t_0) = y_0$$

where  $g$  and  $f$  are smooth vector functions. The solution of (1) is formally to find  $x(t)$  and  $y(t)$  for  $t \in [t_0; T_{end}]$ . Since the analytical solution of (1) is impossible in most cases, a numerical integration is used which involves replacing differential part of (1) with finite-difference equations and their sequential solution with some integration method. The integration method builds sequence of approximations  $z_n(t_n) = [y_n(t_n), x_n(t_n)]^T$ , satisfying the conditions:

$$\|z(t_n) - z_n(t_n)\| \leq \epsilon \quad (2)$$

where  $z(t_n)$  is analytical solution and  $z_n(t_n)$  is approximated solution. Since (1) is usually stiff, mostly implicit integration methods are used, requiring the solution of a nonlinear system of equations at each integration step  $n$ .

Transient trajectories are not smooth and subject to discontinuities at particular time instants due to switching and limiting of state variables. Thus  $f$  and  $g$  are not continuous in  $t \in [t_0; T_{end}]$  but only piece-wise continuous, as well as their derivatives. This requires a restart integration method at time instants of discontinuities  $t_d$  with new initial conditions  $z_d(t_d)$ . In some cases structure and dimension of (1) also subject to change. Discontinuities are associated with events that can be divided into time events and state events. Time events are unconditional and has a known time of occurrence. They can be applied to solution at scheduled time instants. State events have only conditions of occurrence and their times must be located during solution.

Based on the above characteristics of the problem, to solve system (1), the following minimum set of procedures should be implemented:

- evaluation of initial conditions at  $t_0$ ;
- evaluation of residuals of equations;
- construction of a submatrix of partial derivatives;
- time location of the state events  $t_d$ ;
- evaluation of switched (1) initial conditions at the discontinuities  $t_d$ .

The custom model is a subsystem of (1), therefore, a set of procedures identical to the set for the general system must be implemented for it.

When developer is asked for creation of equipment model, he starts by analyzing the given block diagram, forms a system of equations, and then implements a program with functions that ensure the interaction of the model with the core software. The result of that work is an integral part of the software. To implement a custom model, a similar process is required with two differences: the developer does not participate in the process, and the result of the work is not included directly in the software, but operates as external module. The latter is usually, a native machine code executable module in the

operating system format. (\*.dll for Windows or \*.so for Unix systems). Implementing custom model functions in native code eliminates technical differences between custom and built-in models and ensures maximum performance.

The generation of native code in itself is the complex task, since it is necessary to take into account the system architecture features. Therefore, when implementing custom models, to generate native code, standard systems for compiling executable modules are used, for which the source text of the custom model program is preliminarily created in one of the general-purpose programming languages. In this case, such a programming language is called intermediate. In practice, C/C++ and Fortran compilers are often used. Automatically generated programs in these languages are the penultimate stage of the implementation of the custom model, preceding the compilation of the executable module in native code.

### III. MODEL REPRESENTATION IN AST-FORM

Consider the model representation in block diagram and textual forms. As an example, the simple model of automatic voltage regulator is used. Textual form of representation is

```
Uf = limited_lag(Usum, Trv, Ufmin, Ufmax)
Usum = Ku * Vc + Klu * derlag(Vc, Tlu) + lag(Vs, Tbch) - Klif * derlag(If, Tlif)
Vs = Kf * Tf * derlag(Su, Tf) + Klif * derlag(Su, Tlf)
Vc = Vref - Vg + Ig * Xc
```

Fig. 1. Sample AVR model pseudocode representation

shown in Fig. 1 and block diagram in Fig. 2.

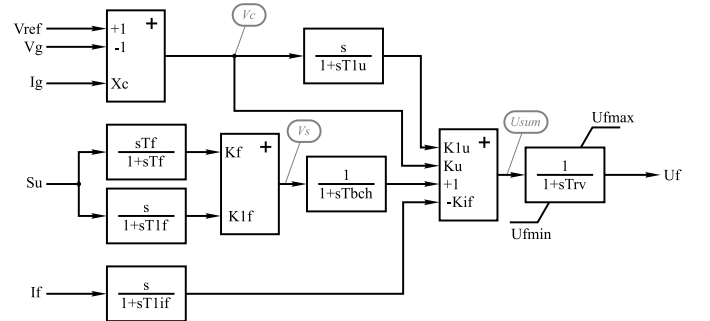


Fig. 2. Sample AVR model block diagram

Both representations use elementary functional blocks: lag, derivative lag and limited lag. Ready-made elementary functional blocks are included in custom model toolbox to simplify model creation and enable efficient implementation by core software instead of letting the user build these blocks from individual differential equations. In addition, the implementation of the elementary blocks in the core software allows to use common procedures for discrete events location and processing and hide these details from the user. If necessary, differential equations can be explicitly specified using the integrator block.

Formally, the block diagram of the model corresponds to a directed graph whose vertices are elementary blocks, and edges are defined by links. Compared to a conventional graph, the order in which the edges in a node are listed matters, because elementary blocks have positional arguments that





Fig. 4. Compression algorithm.

Fig. 5. Decompression algorithm.

Therefore, the predictor function should consider the most of the features of the data being processed. A hash table-based predictor suits for general floating point periodic data. It automatically adjusts to the processed sequence and, if a period can be distinguished in the data, it provides high accuracy of the forecast. However, the use of a hash table comes with a major disadvantage of such a predictor, since its storage requires memory (about 16MB for optimal prediction quality at transient simulation duration over 60s). In the considered problem of compression of the results of transients simulation, an individual hash table must be created for each state variable.

It was noted above that simulation results in most cases are approximations to piecewise smooth functions. This property gives option to use Lagrange polynomial extrapolation for prediction:

$$b_i = b_i(t_i) = \sum_{m=1}^{np+1} a(t_{i-m})l_m(t_i), \quad (3)$$

with basis polynomials:

$$l_m(t_i) = \prod_{k=1, k \neq m}^{np+1} \frac{t_i - t_{i-k}}{t_{i-m} - t_{i-k}} \quad (4)$$

where  $np$  is the order of the polynomial, which also the order of predictor. The Lagrange polynomial does not require  $t_i$  to be equally spaced. This property allows to output the results from integration method directly without dense output processing.

The prediction method used for encoding is similar to the prediction method for DAE integration. For such a predictor to work, it is sufficient to have a vector of previous values with a depth equal to the order of the predictor. The order can be changed, if necessary, at virtually no additional cost. Increasing the order of the predictor improves the quality of prediction for high-order components of transient. Considering also that all values of the results are related to common time values, the basis polynomials 4 can be calculated once for all state variables that going to output at the current step  $t_i$ . Thus, for the results, it is optimal to choose a predictor with Lagrange extrapolation, which requires a negligible amount of memory and makes it possible to eliminate redundant computations.

The order of the predictor polynomial can vary from zero to some  $n_{Pmax}$ . For the first step of transient stability simulation, the value of the DAE initial conditions is used as the predictor value. The next step gives a linear extrapolation, and so on. It has been found empirically that the best compression ratio gives the value  $n_{Pmax} = 4$ . As the order increases prediction worsens. Presumably, the reason for this is the Runge phenomenon, since most of the results are stored with a step  $h_P$ , with slight deviations.

As noted above, for discrete changes, the time instants  $t_d^-$  and  $t_d^+$  are preserved. Since  $t_d^- \neq t_d^+$ , the proposed

compression method remains operational, but at the instants of discrete changes it can degrade the compression ratio, as the function undergoes a discontinuity and polynomial extrapolation cannot give a qualitative prediction. In the instants of discrete changes, the integration stops and new initial conditions are calculated for  $t_d^+$ , after which the integration resumes. The order of the predictor after calculating the new initial conditions is reset to zero and increases to  $n_{Pmax}$  as new integration results are accumulated. This approach to processing discrete changes allows to maintain an acceptable compression ratio, since it takes into account the discontinuity of the function of the simulation result.

With an exact match between the predicted and stored value of the simulation result, the encoding scheme allows to reduce the representation to 1 byte - F0 (in hexadecimal), while  $K_c = 0.125$ . Such prediction accuracy is unlikely for time-varying parameters, but for piecewise constant parameters such as outputs of discrete elements, or parameters that are at the limits, this is easily achievable. Taking into account the fact that the time intervals during which the values of such parameters do not change can be long, it is possible to further improve the compression ratio by using Run Length Encoding (RLE). Before saving, the results compressed by the proposed method are accumulated in a fixed size buffer with a repetition counter. If the buffer is not empty and the value of the current calculation result is the same as the previous one, the current value is not written to the buffer, but instead the repeat counter is incremented. Thus, a series of repeating consecutive values is encoded by the given value and the number of its repetitions. If the repeat counter has a non-zero value, but the previous and current values of the calculation result do not match, the contents of the buffer are stored, after which the buffer size and the repeat counter are reset to zero values.

The simulation results when using additional run-length encoding are stored in the form of blocks. The block is marked with a repeat counter. If the repetition counter is zero, then the block contains non-repeating data compressed by the proposed method. Otherwise, the block contains a series of equal values that can be recovered by decoding the single value and copying it in sequence. The buffer size within 50-100 samples does not lead to a significant memory consumption, but allows I/O operations to process more data, which increases write speed when saving results to file and the speed of access when reading and restoring results for viewing and analysis. File I/O operations, both read and write, are sequential.

The proposed compression method provides exact data recovery, that is, it is a lossless compression method. This property of the algorithm is valuable, but it is redundant for storing the results of transient stability simulation, as it is carried out with the finite accuracy of the integration method. The local error is controlled by predictor-corrector integration scheme:

$$d_i = C \frac{e_i}{|y_i|Rtol + Atol} \leq 1 \quad (5)$$

where

$e_i$  is the corrector equation error with respect to the state variable  $y_i$ ;

$C$  is a constant of the integration method;

$Rtol$  is the relative error tolerance;  
 $Atol$  is the absolute error tolerance.

Obviously, limiting the accuracy while storing the simulation results to the value of  $Atol$  will not lead to a deterioration in the quality of the transient stability analysis. Therefore, a preliminary filter can be applied, which rounds values to  $Atol$ . Filtering by itself does not provide compression, but allows to get the result in the form of a piecewise constant function and use run-length encoding for intervals in which the change in the result value does not exceed  $Atol$ , as shown on Fig. 6. Filtering does not introduce an additional error into the simulation result and does not affect the quality of the analysis of the simulation result, but improves compression ratio.

Fig. 6. Pre-compression results filtering for RLE in range of  $Atol$ .

In addition, filtering allows to eliminate the effect that may occur when the result values oscillate around zero. The IEEE-754 representation of a double-precision number contains sign information in the high bit. Slight oscillations in the result values around zero can lead to a sign change. The relatively small absolute prediction error, but affecting the sign, will lead to the fact that when encoding  $Z_b$ , counted from the most significant bits, will be equal to zero, and instead of reducing the size of data, the algorithm will increase it, since 4 extra bits will be required for  $Z_b$  encoding. Filtering eliminates the sign change caused by oscillations in the results with amplitudes not exceeding the  $Atol$ , and allows to maintain an acceptable compression ratio. When  $Atol$  values of  $10^{-4}$  or more are acceptable, which is usually adequate for transient stability simulation for post-contingency control task, a single precision real number format requiring only 32 bits can be used to store the results. This will halve the size of results at the cost of accuracy, but at the same time retain the ability to use the proposed compression method with minimal changes in the encoding scheme.

## VI. IMPLEMENTATION AND COMPARISON WITH MAINSTREAM COMPRESSION ALGORITHMS

To study proposed compression method implementation, series of tests were carried out to store transient stability simulation results for the model with the following parameters:

TABLE III  
TEST MODEL FEATURES

Equipment model	Count
Bus	842
Branch	1189
Generator(simplified Park's)	149
Excitation system (Exciter+AVR+DEC)	145

The tests were performed on a PC with an Intel Core i7-4770 CPU 3.4GHz and 32GB of RAM. A 4GB disk drive was emulated in RAM, to make file data files I/O times negligible.

The total number of state variables in the model was 6892. The Transient with a duration of 100s and a step for the results output  $H_p = 10^{-2}s$  in this model was simulated for two cases.

In the first case (Case 1), the frequency stabilization channels of the AVR were switched off, due to which the transient process decayed much more slowly compared to the second case (Case 2), in which the AVR stabilization channels were in operation. Absolute integration error tolerance were  $Atol = 10^{-7}$ . Comparison of the simulation results of the slip of one of the generators of the model is shown in the figure 7:

Fig. 7. Transient cases with different decays.

For Case 2, due to faster decay, the integration method increased the step at the final stage of the simulation over  $H_p$ , which can be seen from the location of the markers in the figure 8. For that case, therefore, a significantly smaller amount of the results were saved than for Case 1. The resulting sizes were compressed by the proposed method during the simulation. After simulation, the results were restored and compressed by two mainstream archiving programs: WinRAR 5.5 and 7zip 16.02 with maximum compression settings. The results of the experiment are shown in the table IV. The best results are in bold.

Fig. 8. Integration step impact on result output step.

TABLE IV  
COMPRESSION PERFORMANCE COMPARISON

	Case 1	Case 2
Proposed method		
Samples count	8052	5298
State vars count	6892	6892
Uncompressed, MB	423.39	278.58
Time, s	<b>16</b>	<b>10</b>
RAM, GB	<b>0.003</b>	<b>0.003</b>
Compressed, MB	<b>169.47</b>	102.24
Ratio, %	<b>40.03</b>	36.70
WinRAR		
Time, s	19	10
Time 1 core, s	69	38
RAM, GB	0.82	0.82
Compressed, MB	175.20	<b>98.16</b>
Ratio, %	41.38	<b>35.24</b>
7zip		
Time, s	38	25
Time 1 core, s	120	62
RAM, GB	2.94	1.86
Compressed, MB	197.57	106.52
Ratio, %	46.66	38.24

During the comparison, the sizes of the compressed data were measured and the compression ratios were determined, as well as the time spent on the compression. The compression ratio of the proposed method turns out to be comparable with the compression ratios of mainstream archivers. For a larger amount of results, the proposed method gives the best compression ratio. For a smaller size of results, the proposed method is slightly inferior to WinRAR.

It should be noted that in the comparison, the proposed compression method was run in parallel with the process of the transient stability simulation on one core of an eight-core processor. The load on the processor from the compression method, thus, could not exceed 12.5%. The WinRAR and 7zip

archivers were run with no threading restrictions. The average load on the processor when running WinRAR was about 65%, and when running 7zip — 53%. In order to compare the time required by that archivers to process data using only one processor core, they were artificially restricted by the operating system. The execution time of archivers on one processor core is also shown in the table. The amount of memory required by the proposed method is negligible compared to the amount required by archivers that use general purpose compression methods.

The "Compressed" row shows the full size of the results compressed by the proposed method, which, in addition to these results, also includes auxiliary data, for example, a list of state variables with their names and units, a list of model elements, etc., as the compressed size for archivers, is only the size of result data. At the end of the comparison, the results compressed by the proposed method was additionally compressed by WinRAR. The compression ratio for Case 1 turned out to be 96%, for Case 2 — 94%, which can be explained by a better compression of auxiliary data. However, compression of the auxiliary data for the result will require its full decompression when reading for analysis. Considering that the additional possible compression is negligible, the auxiliary data is stored without using compression to speed up the reading of results.

## VII. CONCLUSION

The proposed method for compressing the results of transient stability simulation makes it possible to obtain a compression ratio close to the compression ratio of mainstream archiving programs that implement general purpose algorithms, with significantly less computational efforts. The amount of memory required for the method is several orders of magnitude smaller than the amount required for general purpose compression algorithms. The method allows to compress the result of the simulation for each state variable separately, therefore, to restore a subset of the data required for analysis, it is not necessary to restore the entire amount of data. The method supports a lossy compression option with a normalized error not exceeding the absolute integration error tolerance. The results obtained are achieved through the use of specific properties of the data of the results of the transient stability simulation. The compression method is implemented as a component of the developed software for transient stability simulation.

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## VIII. BIOGRAPHY SECTION



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