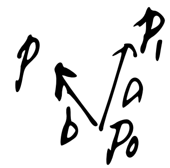
$\boxed{\text{distance}(p, e)}$

1. point p, edge $\overrightarrow{p_0 p_1}$,

$p, p_0, p_1 \in \mathbb{R}^2$

$\Rightarrow$ classify.



solve: let $p_2 = p$, $e = \overrightarrow{p_0 p_1}$

$a = p_1 - p_0$, $b = p_2 - p_0$

∴ $s = a_x b_y - b_x a_y$

(∵ $\begin{pmatrix} a_x \\ a_y \\ 0 \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ 0 \end{pmatrix} = \begin{vmatrix} a_x & a_y & 0 \\ b_x & b_y & 0 \\ i & j & k \end{vmatrix}$

$= \begin{pmatrix} 0 \\ 0 \\ \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix}$)

∴ if $s > 0 \Rightarrow p$ lies to the left of e.

else if $s < 0 \Rightarrow$ right

else if $a_x b_x < 0$ or $a_y b_y < 0 \Rightarrow$ behind

else if $\|a\| < \|b\| \Rightarrow$ beyond

else if $p_2 = p_0 \Rightarrow$ origin

else if $p_2 = p_1 \Rightarrow$ destination

else $\Rightarrow$ between.

2. rotate $e = \overrightarrow{p_0 p_1}$ 90° cw around middle point $\Rightarrow e'$

solve: ∵ rotate axis $\theta \Rightarrow R_\theta = \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}$

∴ rotate point θ clockwise $\Rightarrow R = R_\theta$

$\therefore$ $n = ((p_4 - p_3).y, -(p_4 - p_3).x)^T$

$n^T[(p_1 + t(p_2 - p_1)) - p_3] = 0$

$\therefore$ $n^T p_1 + t n^T (p_2 - p_1) = n^T p_3$

$\therefore$ if $n^T(p_2 - p_1) \neq 0$, type = skew,

$$t = \frac{n^T(p_3 - p_1)}{n^T(p_2 - p_1)}$$

else:

class ← classify (p_1, $\overrightarrow{p_3 p_4}$)

if class = left or right

type ← parallel

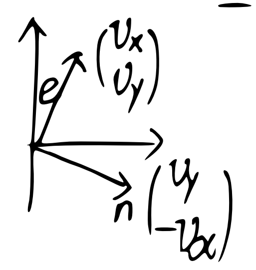
else

type ← collinear

let $v = p_1 - p_0$

$\therefore$ $n = R_{\frac{\pi}{2}} v = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

$= \begin{pmatrix} v_y \\ -v_x \end{pmatrix}$



$\therefore$ $e' = (m - \frac{n}{2}, m + \frac{n}{2})^T$

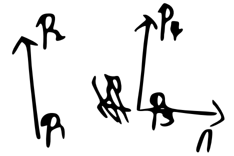
where $m = \frac{p_0 + p_1}{2}$

3. edge $\overrightarrow{p_1 p_2}$, $\overrightarrow{p_3 p_4}$ ⇒ type and t,

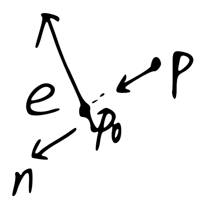
s.t., $p_1 + t(p_2 - p_1) \in$ line $p_3 p_4$

if type is skew.

solve:



4.



Given point p,
edge e,
$\Rightarrow$ distance(p, e)

solve:

$n \leftarrow e$ rotate $270°$ cw,
$n \leftarrow n / \|n\|$
let edge $f = (\overrightarrow{p, p+n})$

intersect$(f, e) \Rightarrow$ type, t

here type must be skew.

$\therefore p_0 = p + tn \in$ line e

$\therefore t > 0$ if p lies to the right of e.
$t < 0$ if p lies to the left.

$\therefore$ distance$(p, e) = |t|$