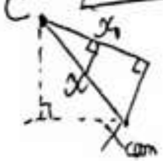
camera, warp image

1. 

$\because \begin{cases} u = \dfrac{f x_{cam}}{z_{cam}} + u_0 \\ v = \dfrac{f y_{cam}}{z_{cam}} + v_0 \end{cases}$

$\therefore \lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & & u_0 \\ & f & v_0 \\ & & 1 \end{pmatrix} \begin{pmatrix} x_{cam} \\ y_{cam} \\ z_{cam} \end{pmatrix}$

$\therefore \lambda x = K X_{cam}$.

where $x = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$, $X = \begin{pmatrix} x_{cam} \\ y_{cam} \\ z_{cam} \end{pmatrix}$

$K = \begin{pmatrix} f & & u_0 \\ & f & v_0 \\ & & 1 \end{pmatrix}$

2. $\lambda x = K R X^{\sim}$

$X + t = s X^{\sim}$

here $x = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$ is image coordinates.

$X = \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix}$ is world coordinates.

$X^{\sim} = \begin{pmatrix} x^{\sim} \\ y^{\sim} \\ 1 \end{pmatrix}$

K, R, t, s. is parameters.

λ is set to make equation hold.

3. $x_0, x_1, x_2, x_3 \Rightarrow T$.

解: ① make sure $x_i(3) = 0$ or 1,

$i = 0, 1, 2, 3$.

② if $x_i(3) = 1$

$x_i(1) \leftarrow x_i(1) - x_0(1)$

$x_i(2) \leftarrow x_i(2) - x_0(2)$

$i = 1, 2, 3$.

③ $x_i(1) x_j(1) + x_i(2) x_j(2) + f_{ij}^2 = 0$

$\Rightarrow f_{ij}$, $i \neq j$, $i, j \in \{1, 2, 3\}$

④ $f = \max\{f_{ij}\}$

⑤ if $x_i(3) \neq 0$

$x_i(3) \leftarrow f$

$i = 1, 2, 3$.

⑥ $x_i \leftarrow \dfrac{x_i}{\|x_i\|}$, $i = 1, 2, 3$.

⑦ $\therefore K = \begin{pmatrix} f & & u_0 \\ & f & v_0 \\ & & 1 \end{pmatrix}$

$R = (x_1, x_2, x_3)$

$T = (KR)^{-1}$

$\therefore \lambda T x = X^{\sim}$

// R should be orthogonal, but here R may not be.

4. $x_a, x_b, l, T \Rightarrow s$

解: $X_a^{\sim} = \lambda T x_a$, $X_b^{\sim} = \lambda T x_b$

$\therefore l = \|X_a - X_b\| = s \|X_a^{\sim} - X_b^{\sim}\|$

$\therefore s = l / \|X_a^{\sim} - X_b^{\sim}\|$

5. use

$X = \lambda s T x - t$

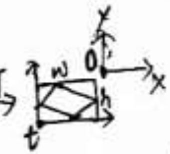
λ is set to make $X(3) = s$,

t can be set to zero.

6. warp image

$\{x \mid x \in \mathbb{Z}^2, x \in [x_1, x_2] \times [y_1, y_2]\}$

$\rightarrow [1, m] \times [1, n]$

solve: ① four vertices $\xrightarrow{T}$ 

② $s = (m/h) \wedge (n/w)$

③ $t(3) \leftarrow 1$, $t \leftarrow s \cdot t$, $X \leftarrow \begin{pmatrix} x_w \\ y_w \\ 0 \end{pmatrix}$

④ $X^{\sim} = (X + t)/s$

$x = \lambda T^{-1} X^{\sim}$

⑤ for every X, we get x.

keep the pair (x, X).

7. $\because X = \lambda s T x - t$.

$\therefore$ we can absorb s in T.

let $S = \begin{pmatrix} s & & \\ & s & \\ & & 1 \end{pmatrix}$.

$T \leftarrow S \cdot T$.

8. we can let $S = \begin{pmatrix} s_x & & \\ & s_y & \\ & & 1 \end{pmatrix}$

and solve s_x, s_y.

9. let $\{x_a, x_b, l\}$ be one point,

then two points, $T \Rightarrow s_x, s_y$.

solve: $\because l^2 = \|X_a - X_b\|^2 = \left\| \begin{pmatrix} s_x x_a^{\sim} \\ s_y y_a^{\sim} \\ 1 \end{pmatrix} - \begin{pmatrix} s_x x_b^{\sim} \\ s_y y_b^{\sim} \\ 1 \end{pmatrix} \right\|^2$

$= s_x^2 (x_a^{\sim} - x_b^{\sim})^2 + s_y^2 (y_a^{\sim} - y_b^{\sim})^2$

$\therefore a_1 = (x_a^{\sim} - x_b^{\sim})^2$, $a_2 = (y_a^{\sim} - y_b^{\sim})^2$

$\therefore$ for point i, we get

$s_x^2 a_1 + s_y^2 a_2 = b_i = l^2$

$\therefore A x = b \Rightarrow x$.

where $x = \begin{pmatrix} s_x^2 \\ s_y^2 \end{pmatrix}$,

10. for 5:

$X = \lambda S T x - t$,

λ is set so that $\tilde{X}(3) = 1$,

so that $X(3) = s$, $t = 0$.

for 7, or 8,

$\because S = \begin{pmatrix} s_x & & \\ & s_y & \\ & & 1 \end{pmatrix}$, not $\begin{pmatrix} s & & \\ & s & \\ & & s \end{pmatrix}$

$\therefore X(3) = \tilde{X}(3) = 1$.

$\therefore \lambda$ is set so that $X(3) = 1$.

11. $U = WX$,

$U = \begin{pmatrix} e \\ n \\ 1 \end{pmatrix}$, $W = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ & & 1 \end{pmatrix}$

$X = \begin{pmatrix} x_w \\ y_w \\ 1 \end{pmatrix}$.

12. $U = WX$, $\cancel{X = \lambda S T x}$

$X = S X^{\sim}$

$X^{\sim} = \lambda T x$, λ is set so that $X^{\sim}(3) = 1$

$\therefore U = \lambda W S T x$

~~AS has form~~

let $B = WS$, $\therefore B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ & & 1 \end{pmatrix}$

$\therefore$ we can set

$U = \lambda W T x$

$\Rightarrow W$

$X^{\sim} = x^{T}$

13. $U = \lambda W T x = W X^{\sim}$

$\Rightarrow W$

$\begin{pmatrix} e \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ & & 1 \end{pmatrix} \begin{pmatrix} x^{\sim} \\ y^{\sim} \\ 1 \end{pmatrix}$

$e = a_{11} x^{\sim} + a_{12} y^{\sim} + a_{13}$

$n = a_{21} x^{\sim} + a_{22} y^{\sim} + a_{23}$

~~given 3 points~~

$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} x_1^{\sim} & y_1^{\sim} & 1 \\ x_2^{\sim} & y_2^{\sim} & 1 \\ x_3^{\sim} & y_3^{\sim} & 1 \end{pmatrix} \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix}$

11, 14, 27.

9,20 10:30 10:15

given 3 points

$\therefore b_e = M \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix}$ $b_e = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$, $M = \begin{pmatrix} x_1^{\sim} & y_1^{\sim} & 1 \\ x_2^{\sim} & y_2^{\sim} & 1 \\ x_3^{\sim} & y_3^{\sim} & 1 \end{pmatrix}$

$\because b_n = M \begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix}$

$\therefore T \leftarrow WT$

14. $\therefore X = \lambda T x$

λ is set so that $X(3) = 1$.

15. $T^{-1} X = \lambda x$

$\therefore$ we can $x = \lambda T^{-1} X$,

λ is set so that $x(3) = 1$.

16. $X_5^{T} = \lambda T x_5^{T}$

$X_5 = \lambda x_5 T'$

λ is set so $X_5 = (\,:\,, :\,, 1)$

~~20 warp image~~.

~~(efficient~~

(~~efficient version of 6~~)

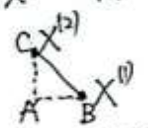
20. ~~get~~.

given

$X^{(1)}, X^{(2)} \to T_2$.

w, h.

solve:



∴ ⇒ A, B, C.

$A \leftrightarrow (0,0)$

$B \leftrightarrow (w,0)$

$C \leftrightarrow (0,h)$

$\Rightarrow T_2$.

21 image → world image

for x_2 in world image

$x_1 = \lambda T^{-1} x_2$

if $x_1 \in$ image

draw x_2 value ← x_1 value.

18 $\overset{x_1}{\text{image}} \to \overset{x_2}{\text{world image}}$.

solve:

$X = \lambda T_1 x_1$, X is world point.

if we know T_2:

$x_2 = T_2 X$

$\therefore x_2 = T_2 X$

$= \lambda T_2 T_1 x_1$

$\therefore x_2 = \lambda T x_1$

$T = T_2 T_1$

19. world image → image

$\because x_2 = \lambda T x_1$,

$\therefore x_1 = \lambda^{-1} T^{-1} x_2$

$\therefore x_1 = \lambda T^{-1} x_2$

$$\because \begin{pmatrix} x_w \\ y_w \\ 1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

given 3 points pair,

$$\therefore \begin{pmatrix} x_w^{(1)} & y_w^{(1)} & 1 \\ x_w^{(2)} & y_w^{(2)} & 1 \\ x_w^{(3)} & y_w^{(3)} & 1 \end{pmatrix} = \begin{pmatrix} x^{(1)} & y^{(1)} & 1 \\ x^{(2)} & y^{(2)} & 1 \\ x^{(3)} & y^{(3)} & 1 \end{pmatrix} \begin{pmatrix} T_{11} & T_{21} & 0 \\ T_{12} & T_{22} & 0 \\ T_{13} & T_{23} & 1 \end{pmatrix}$$

~~$\Rightarrow T' \Rightarrow T$~~

$\therefore X_s = x_s T'$

$\Rightarrow T' \Rightarrow T$

$\because x_s$ is invertible.

$x_s \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\therefore T'(:,3) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

17.3 $\therefore$ give x_s, X_s

$\Rightarrow T \Rightarrow T(3,:) = (0,0,1)$

17, 18.

17. affine transformation.

$x_s, X_s, \Rightarrow T$.

here $X = Tx$.

solve 1: $\because (X_1, X_2, X_3) = T(x_1, x_2, x_3)$

$\therefore X_s^T = T x_s^T$

$\therefore x_s T' = X_s$

$\Rightarrow T' \Rightarrow T$.

solve 2:

let $X = \begin{pmatrix} x_w \\ y_w \\ 1 \end{pmatrix}$, $x = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$, $T = \{T_{ij}\}$

$\therefore x_w = T_{11}x + T_{12}y + T_{13}$

$y_w = T_{21}x + T_{22}y + T_{23}$

here $T(3,:) = 0, 0, 1$.

proof:

let H be the set.

let G be set {transformation of plane}

$\therefore$ G is group

$\because e \in H$

$\forall \alpha \in H, \beta \in H$

$\alpha\beta^{-1} \in H$

$\therefore H < G$,

$\because \alpha\beta = \beta\alpha$

$\therefore$ H is abelian group.

4. let α be reflection of the plane in a line l.

$\Rightarrow H = \{\alpha, e\}$

$\Rightarrow$ H is group.

proof:

$\forall x, y \in H$,

$$xy^{-1} = \begin{cases} e \in H, \\ \alpha \in H, & x \neq y \end{cases}$$

$\therefore H < G$, $\therefore$ H is group.

geometric transformations.

1. $\alpha : S \to S$, α is invertible

$\Leftrightarrow \alpha$ is a transformation of S.

2: set $T := T(S)$

$:= \{\text{transformation of } S\}$

$\Rightarrow$ T is group.

proof: ① $\alpha\beta \in T$.

② $(\alpha\beta)\gamma(x)$

$= \alpha(\beta(\gamma(x)))$

$= \alpha((\beta\gamma)(x))$

$= (\alpha(\beta\gamma))x \quad \forall x \in S$

$\therefore (\alpha\beta)\gamma = \alpha(\beta\gamma)$

③ $e \in T$, $\alpha e = \alpha$, $\forall \alpha \in T$.

④ $\forall \alpha \in T$, $\exists \alpha^{-1} \in T$,

$\alpha \cdot \alpha^{-1} = e$.

3. the set of all translations of the plane is a group.

It's also abelian group.

It is infinite.

3D viewing

1. eye, look, up

$\Rightarrow n = eye - look$

$u = up \times n$

$v = n \times u$

$\therefore n \leftarrow \frac{n}{\|n\|}$, $u \leftarrow \frac{u}{\|u\|}$, $v \leftarrow \frac{v}{\|v\|}$

1.3 v will usually not be aligned with up.

1.4 the user provides up as a suggestion of "upwardness".

2. pitch, heading, yaw, roll.

3. We most often set a camera to have zero roll, and call it a "no-roll" camera.

$\therefore$ the u-axis of a no-roll camera is horizontal, that is, perpendicular to the y-axis of the world.

4. model~~view~~ matrix $= V \cdot M$.

4.3. $V: \begin{cases} eye \to origin \\ u \to i \\ v \to j \\ n \to k \end{cases}$

$\Rightarrow V$.

solve:

$\because X = (u, v, n)$

~~解~~

X: camera axes represented by world axes.

$\therefore V = X^{-1}p + t$

$\because X^{-1} eye + t = 0$

$\therefore t = -X^{-1} eye$

$= -X^T eye$

$= -\begin{pmatrix} u^T \\ v^T \\ n^T \end{pmatrix} eye$

$\therefore V = (X^{-1}, t)$

~~$\therefore V = \begin{pmatrix} u^T & t \\ v^T & \\ n^T & \\ 0 & 1 \end{pmatrix}$~~

$V = \begin{pmatrix} A & t \\ 0 & 1 \end{pmatrix}$

$A = \begin{pmatrix} u^T \\ v^T \\ n^T \end{pmatrix}$

$t = -\begin{pmatrix} u^T eye \\ v^T eye \\ n^T eye \end{pmatrix}$

$\therefore V = \begin{pmatrix} u^T & -u^T eye \\ v^T & -v^T eye \\ n^T & -n^T eye \\ 0 & 1 \end{pmatrix}$

5. $u, v, n \Rightarrow$ roll, pitch, heading

solve:

① heading $=$ ~~$\arctan\left(\frac{-n_z}{-n_x}\right)$~~ $\arctan\left(\frac{n_x}{n_z}\right)$

② pitch $= \sin^{-1}(-n_y)$

③ $\because b$ is horizontal, $b \in$ uv-plane. $b = j \times n$.

$\therefore$ roll $= \cos^{-1} \frac{b \cdot u}{\|b\|}$

$\because b = j \times n = \begin{pmatrix} i & j & k \\ 0 & 1 & 0 \\ n_x & n_y & n_z \end{pmatrix} = \begin{pmatrix} n_z \\ 0 \\ -n_x \end{pmatrix}$

$\therefore$ roll $= \cos^{-1}\left(\frac{u_x n_z - u_z n_x}{n_x^2 + n_z^2}\right)$

6. Camera:

6.2 eye, u, v, n, aspect, viewangle, near, far.

6.3 set(eye, look, up)

eye ← eye.

$n = eye - look$

$u \leftarrow up \times n$.

$n \leftarrow n/\|n\|$, $u \leftarrow \frac{u}{\|u\|}$

$v \leftarrow n \times u$.

setModelViewMatrix:

6.4 setModelViewMatrix.

m(4,4)

$$M = \begin{pmatrix} u^T & -(eye, u) \\ v^T & -(eye, v) \\ n^T & -(eye, n) \\ 0 & 1 \end{pmatrix}$$

setShape(viewAngle, aspect ratio, near distance, far plane distance)

viewAngle ← viewAngle

aspect ← aspect

near ← near

far ← far

setPerspectiveProjectionMatrix.

(θ, aspect, near, far)

6.6. setPerspectiveProjection Matrix.

(viewAngle, aspect ratio, near plane, far plane).

6.7 slide(δ)

$$eye \leftarrow eye + (u, v, n)\begin{pmatrix} \delta u \\ \delta v \\ \delta n \end{pmatrix}$$

∴ $eye \leftarrow eye + \delta_u u + \delta_v v + \delta_n n$ 6.5

setModelViewMatrix

6.8. rotateAxes(u, v, θ)

$$\therefore \begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

∴ $u' \leftarrow c\theta \cdot u + s\theta \cdot v$

$v' \leftarrow -s\theta \cdot u + c\theta \cdot v$

$u \leftarrow u'$

$v \leftarrow v'$

6.9 roll(θ), θ in radius.

rotateAxes(u, v, θ)

setModelViewMatrix

6.10 pitch(θ)

rotateAxes(v, n, θ)

setModelViewMatrix

6.11 yaw(θ)

rotateAxes(n, u, θ)

setModelViewMatrix.

7. $h = 480$, $w = 640$

viewport$(0, 0, \underset{h}{\cancel{480}}, \underset{w}{\cancel{640}})$

camera.set$\left(\begin{pmatrix}4\\4\\4\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right)$

Camera.setShape$(30, \frac{w}{h}, 0.5, 50)$

8. projecting $P = (P_x, P_y, P_z)$ onto the near plane of the camera to a point (x^*, y^*).

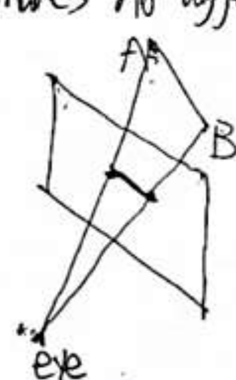
$\because k\begin{pmatrix}x^*\\y^*\\-N\end{pmatrix} = \begin{pmatrix}P_x\\P_y\\P_z\end{pmatrix}$

$\therefore \begin{cases} x^* = -N\frac{P_x}{P_z} \\ y^* = -N\frac{P_y}{P_z} \end{cases}$

8.3 the effect of the near plane distance N is simply to scale the picture. $\because$ we ultimately map this projection to a viewport of a fixed size on the display.

$\therefore$ the size of the projected image makes no difference.

8.4



9. Projecting parallel lines.

9.3 $P(t) = A + ct$.

$\therefore p(t) = \left(-N\frac{P_x(t)}{P_z(t)}, -N\frac{P_y(t)}{P_z(t)}\right)$

$= \left(-N\frac{A_x + c_x t}{A_z + c_z t}, -N\frac{A_y + c_y t}{A_z + c_z t}\right)$

This is parametric form for a straight line. 3

$\therefore$ straight lines project to straight lines.

9.4 if $c_z = 0$

$\Rightarrow p(t) = \left(-N\frac{A_x + c_x t}{A_z}, -N\frac{A_y + c_y t}{A_z}\right)$

$\therefore$ the slope of $p(t)$ is $\frac{c_y}{c_x}$

$\therefore$ If two lines in 3D are parallel to each other and to the viewplane, they project to two parallel lines.

9.5 if $c_z \neq 0$

$\Rightarrow \lim_{t \to +\infty} p(t) = \left(-N\frac{c_x}{c_z}, -N\frac{c_y}{c_z}\right)$

This point is called vanishing point for the $P(t)$.

11.4

A matrix that has values other than (0,0,0,1) for its fourth row does not perform an affine transformation.

It performs perspective transformation. It is a transformation, not a projection.

A projection reduces the dimensionality of a point.

11.5 perspective projection

= perspective transformation

+ orthographic projection.

$\therefore -b$ should be set as large as possible.

$\because -b = \frac{2NF}{F-N} = \frac{2F}{\frac{F}{N}-1}$

$\therefore N$ should be set as large as possible.

11. a matrix will not only be able to perform an affine transformation, it will be able to perform a perspective transformation.

11.3. an affine transformation always has (0,0,0,1) in its fourth row.

10.4

10.5. if $N \ll F$

$\Rightarrow f(P_z) \approx 1 + \frac{2N}{P_z}$

10.6.

$\because f(P_z) = -a - \frac{b}{P_z}$

and f varies more and more slowly as P_z approaches $-F$ from right.

$\therefore$ all parallel lines in 3D share the same vanishing point.

10. adding pseudodepth.

P "projects to"

$$\begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} = \begin{pmatrix} -N\frac{P_x}{P_z} \\ -N\frac{P_y}{P_z} \\ \frac{aP_z+b}{-P_z} \end{pmatrix}$$

10.3 $f(P_z) = \frac{aP_z+b}{-P_z} = -a - \frac{b}{P_z}$

let $\begin{cases} f(-N) = -1 \\ f(-F) = 1 \end{cases}$

$\Rightarrow \begin{cases} -a + \frac{b}{N} = -1 \\ -a + \frac{b}{F} = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{N+F}{N-F} \\ b = \frac{2NF}{N-F} \end{cases}$

12.

$$\begin{pmatrix} N & & & \\ & N & & \\ & & a & b \\ & & -1 & 0 \end{pmatrix}\begin{pmatrix} kP_x \\ kP_y \\ kP_z \\ k \end{pmatrix} = \begin{pmatrix} kNP_x \\ kNP_y \\ k(aP_z+b) \\ -kP_z \end{pmatrix}$$

∴ we obtain : $\begin{pmatrix} -N\frac{P_x}{P_z} \\ -N\frac{P_y}{P_z} \\ -\frac{aP_z+b}{P_z} \end{pmatrix}$

12.3. perspective transformation:

$(P_x, P_y, P_z)^T \to \left(-N\frac{P_x}{P_z}, -N\frac{P_y}{P_z}, -\frac{aP_z+b}{P_z}\right)^T$

12.4. projection:

$\left(-N\frac{P_x}{P_z}, -N\frac{P_y}{P_z}, \frac{aP_z+b}{-P_z}\right)^T \to \begin{pmatrix} -N\frac{P_x}{P_z} \\ -N\frac{P_y}{P_z} \\ 0 \end{pmatrix}$

13. the geometric nature of the perspective transformation.

13.3. the perspective transformation alters 3D point P into another 3D point according to 12.3, to prepare it for projection.

It is useful ~~to~~ think of it as causing a warping of 3D space. ∵

13.4 It preserves straightness and flatness.

It also ~~prese~~ preserves "in-between-ness".

13.5 the camera's view volume is transformed into a parallelepiped.

∵

14. canonical view volume

14.2 a cube that extends from -1 to 1 in each dimension.

14.3.

$$\begin{pmatrix} s_1 & & t_1 \\ & s_2 & t_2 \\ & & 1 \end{pmatrix}\begin{pmatrix} wx \\ wy \\ w \end{pmatrix}$$

$s_1 = \frac{2}{r-l}$, $s_2 = \frac{2}{t-b}$

$t_1 = -\frac{r+l}{r-l}$, $t_2 = -\frac{t+b}{t-b}$

14.4. $(P_x, P_y, P_z)^T$

$\to \left(-N\frac{P_x}{P_z}, -N\frac{P_y}{P_z}, -\frac{aP_z+b}{P_z}\right)$

$\to \begin{pmatrix} -s_1N\frac{P_x}{P_z} + t_1 \\ -s_2N\frac{P_y}{P_z} + t_2 \\ -\frac{aP_z+b}{P_z} \end{pmatrix}$

∴ matrix

$$= \begin{pmatrix} s_1N & & -t_1 & \\ & s_2N & -t_2 & \\ & & a & b \\ & & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2N}{r-l} & & \frac{r+l}{r-l} & \\ & \frac{2N}{t-b} & \frac{t+b}{t-b} & \\ & & \frac{N+F}{N-F} & \frac{2NF}{N-F} \\ & & -1 & 0 \end{pmatrix}$$

14.5. method 2:

$$\text{matrix} = \begin{pmatrix} s_1 & & t_1 & \\ & s_2 & t_2 & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} N & & & \\ & N & & \\ & & a & b \\ & & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} s_1N & & -t_1 & \\ & s_2N & -t_2 & \\ & & a & b \\ & & -1 & 0 \end{pmatrix}$$

17. clipEdge(a, b).

$t_{in}=0$, $t_{out}=1$, t_{hit}.

u: a boundary

v: b boundary.

$u_{2i-1} = a_n + a_i$, $\forall i<n$

$u_{2i} = a_n - a_i$

$v_{2i-1} = b_n + b_i$, $\forall i<n$

$v_{2i} = b_n - b_i$

∴ $u, v \in \mathbb{R}^{(n-1)\times 2}$

$x_i = \begin{cases} 1, & \text{if } u_i>0 \\ 0, & \text{else.} \end{cases}$

$y_i = \begin{cases} 1, & v_i>0 \\ 0, & v_i\le 0 \end{cases}$

$c_i = 1$ 时

if $x \mid y \neq c$
return 0. // reject

if $x \,\&\, y = c$
return 1.

∴ for $v\in\mathbb{R}^n$, $v_n>0$

$v \in$ CVV of $\mathbb{R}^{n-1}$

$\Leftrightarrow v_n \pm v_i > 0$, $\forall i<n$.

16. homogeneous coordinates of line $\in \mathbb{R}^n$.

$v(t) = a + t(c-a)$, $a\in\mathbb{R}^n$, $c\in\mathbb{R}^n$, $t\in\mathbb{R}$.

16.3

$v_i/v_n = 1$

$\Rightarrow a_i + t(c_i - a_i) = a_n + t(c_n - a_n)$

∴ $t = \dfrac{a_n - a_i}{(a_n-a_i)-(c_n-c_i)}$

$v_i/v_n = -1$

$\Rightarrow t = \dfrac{a_n + a_i}{(a_n+a_i)-(c_n+c_i)}$

15. clipping faces against the view volume.

15.3 a point lies inside the camera view $\Leftrightarrow$ its transformed version lies inside the canonical view volume.

15.4 $P = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = v$ (can extend to $\mathbb{R}^n$)

$\frac{x}{w} > -1 \Leftrightarrow w + x > 0$, for $w>0$

$\frac{x}{w} < 1 \Leftrightarrow w - x > 0$, for $w>0$

14.6. matrix 14.5 is known as the projection matrix

It is the matrix setFrustum (left, right, bottom, top, N, F) creates.

14.7.

~~createPerspective(viewAngle,~~

~~setPerspectiveProjectionMatrix~~

setPerspectiveMatrix.
(viewAngle, aspectRatio, N, F)

top = $N\tan(\text{viewAngle}/2)$

bottom = $-$top,

right = top $\cdot$ aspect

left = $-$right

then setFrustum(l, r, b, t, N, F)

21.

$\because$ viewport(a_1, b_1, a_2, b_2)

$$\begin{pmatrix} s_1 & & t_1 \\ & s_2 & t_2 \\ & & 1 \end{pmatrix}$$

$s_1 = \frac{b_1 - a_1}{2}$, $(b_1 > a_1)$

$s_2 = \frac{b_2 - a_2}{2}$, $(b_2 > a_2)$

$t_1 = \frac{a_1 + b_1}{2}$

$t_2 = \frac{a_2 + b_2}{2}$.

$\because$ for z: $[-1,1] \to [0,1]$.

$\therefore$ matrix

$$= \begin{pmatrix} s_1 & & & t_1 \\ & s_2 & & t_2 \\ & & \frac{1}{2} & \frac{1}{2} \\ & & & 1 \end{pmatrix}$$

here $a_2 = a_1 + nr$

$b_2 = b_1 + nc$

aspectRatio $= nc/nr$.

18. We normally set the aspect ratio ~~width~~/height of the viewport to be the same as that of the view volume.

19. the viewport transformation maps ~~pseudodepth~~ pseudodepth from range -1 to 1 into range 0 to 1.

20. pipeline:

P

$\downarrow$

modelview matrix

$\downarrow$

projection matrix

$\downarrow$ clipping

viewport matrix

$\downarrow$

P.

for $i = 1$, $i \leq (n-1) \cdot 2$

if $v_i < 0$. // b is outside // exits

$t_{hit} = \frac{u_i}{u_i - v_i}$

$t_{out} \leftarrow t_{out} \wedge t_{hit}$.

else if $u_i < 0$, // a is outside // enters

$t_{hit} = \frac{u_i}{u_i - v_i}$

$t_{in} \leftarrow t_{in} \vee t_{hit}$

if $t_{in} > t_{out}$ return 0.

if $x \neq c$ // a is out.

$a' \leftarrow a + t_{in}(b - a)$

if $y \neq c$

$b \leftarrow a + t_{out}(b - a)$

$a \leftarrow a'$.

return 1.