1. $f: X \to [-\infty, \infty]$, $X \subset \mathbb{R}^n$.

$\Rightarrow$ epigraph is given by $epi(f) = \{(x,w) \mid x \in X, w \in \mathbb{R}, f(x) \leq w\}$

2. $f: X \to [-\infty, \infty]$, $\Rightarrow$ effective domain is given by

$$dom(f) = \{x \in X \mid f(x) < \infty\}$$

3. $dom(f) = \{x \mid \exists w \in \mathbb{R}, s.t., (x,w) \in epi(f)\}$

proof: ① $x \in dom(f)$, $\therefore \exists w = f(x) \in \mathbb{R}$, s.t., $f(x) \leq w$.

$\therefore (x,w) \in epi(f)$ $\quad \therefore x \in$ right

② $x \in$ right, $\therefore \exists w \in \mathbb{R}$, s.t., $(x,w) \in epi(f)$

$\therefore f(x) \leq w < \infty$ $\quad \therefore x \in dom(f)$

4. $\therefore dom(f) =$ projection of $epi(f)$ on $\mathbb{R}^n$.

5. $epi(f) = epi(f|_{dom(f)}) = \{(x,w) \mid x \in dom(f), w \in \mathbb{R}, f|_{dom(f)}(x) \leq w\}$

proof: $(x,w) \in epi(f) \Leftrightarrow x \in X, f(x) \leq w < \infty$

$\Leftrightarrow x \in dom(f), f|_{dom(f)}(x) \leq w$

$\Leftrightarrow (x,w) \in epi(f|_{dom(f)})$

6. ~~dom(f)~~ $dom(f|_{dom(f)}) = \{x \in dom(f) \mid f|_{dom(f)}(x) < \infty\}$

$= \{x \in dom(f)\} = dom(f)$

7. let $g(x) = \begin{cases} f(x), & x \in X \\ \infty, & x \notin X \end{cases}$, $\Rightarrow epi(g) = epi(f)$, $dom(g) = dom(f)$

proof: ① $dom(g) = \{x \in \mathbb{R}^n \mid g(x) < \infty\} = \{x \in X \mid g(x) < \infty\}$

$= \{x \in X \mid f(x) < \infty\}$

$= dom(f)$

② $g|_{dom(g)} = g|_{dom(f)} = f|_{dom(f)}$

③ $\therefore epi(g) = epi(g|_{dom(g)}) = epi(f|_{dom(f)}) = epi(f)$