$\boxed{\text{bilinear}(X, Y, A, b),\ \text{dims}(X) \geq 2}$

1. mmadd: $Y = f(A, X, B, \alpha, \beta)$

$$= \alpha AX + \beta B$$

$$\Rightarrow \frac{dL}{dA} = \alpha \frac{dL}{dY} X^T$$

$$\frac{dL}{dX} = \alpha A^T \frac{dL}{dY}$$

$$\frac{dL}{dB} = \beta \frac{dL}{dY}$$

2. bilinear: $Z = f(X, Y, A, b)$

$z_{ij} = x_i^T A_j y_i + b_j$,

$X = \begin{pmatrix} x_1^T \\ x_b^T \end{pmatrix}$ $Y = \begin{pmatrix} y_1^T \\ y_b^T \end{pmatrix}$, $Z = \begin{pmatrix} z_1^T \\ z_b^T \end{pmatrix} = (Z_1 \cdots Z_l)$

$A = \begin{pmatrix} A_1 \\ A_l \end{pmatrix}$, $X \in R^{b\times m}$, $Y \in R^{b\times n}$, $Z \in R^{b\times l}$

$A \in R^{l\times m\times n}$, $b \in R^l$

2.3 forward:

① method1: dims(X)=2 $Z_j = \begin{pmatrix} x_1^T A_j y_1 \\ x_b^T A_j y_b \end{pmatrix} = (XA_j \dot{e} Y).sum(-1)$

$\Rightarrow Z$

$\therefore Z \leftarrow Z + b^T$

② method 1': dims(X) ≥ 2, i.e., $p \geq 1$

$X \in R^{()\times m}$, $Y \in R^{()\times n}$, $Z \in R^{()\times l}$,

$() = b_1 \times b_2 \times \cdots \times b_p$

solve: let $b = b_1 \times \cdots \times b_p$

$X.reshape(1, b, m)$, $Y.reshape(1, b, n)$

$\therefore Z = ((XA) \dot{e} Y).sum(-1).t \in R^{b\times l}$

$Z \leftarrow (Z + b^T).reshape(b_1 \times \cdots b_p \times l)$

③ e.x., $(l, m, n) = (7, 4, 5)$,

∴ $A.\text{shape} = (7\,4\,5)$, $X.\text{shape} := (2, 3, 4)$

∴ $Y.\text{shape}(2, 3, 5)$, $Z.\text{shape} = (2, 3, 7)$

solve: $X.\text{reshape}(1, 6, 4)$, $Y.\text{reshape}(1\,6\,5)$

∴ $(XA \dot{e} Y).\text{shape} = (1\,6\,4) \times (7\,4\,5)\, \dot{e}\, (1\,6\,5)$

$= (7\,6\,5)\, \dot{e}\, (1\,6\,5)$

$= (7\,6\,5)$

$\xrightarrow{\text{sum}(-1)} \text{shape} = (7\,6)$

$\xrightarrow{t} \text{shape} = (6, 7) \xrightarrow{\text{reshape}} \text{shape}(2, 3, 7)$

∴ $Z \leftarrow Z + b.\text{reshape}(1, 1, 7)$

④ method 2: $\text{dims}(X) = 2$

∵ $z_{ij} = x_i^T A_j y_i + b_j$

∴ $\forall j \Rightarrow z_i$, ∴ $\forall i \Rightarrow Z$

∴ $Z \leftarrow Z + b^T$

⑤ method 2': $\text{dims}(X) \geq 2$, i.e., $p \geq 1$. (see ②)

$X.\text{add\_axis}(-2).\text{add\_axis}(-2) \in \mathbb{R}^{()\times 1\times 1\times m}$

$Y.\text{add\_axis}(-2).\text{add\_axis}(-1) \in \mathbb{R}^{()\times 1\times n\times 1}$

$A.\text{reshape}(\overbrace{1, \cdots, 1}^{p}, l, m, n)$

∴ $Z = XAY \in \mathbb{R}^{()\times l\times 1\times 1}$

$Z.\text{remove\_axis}(-1, -2) \in \mathbb{R}^{()\times l}$

$Z \leftarrow Z + b.\text{reshape}(\overbrace{1, \cdots, 1}^{p}, l)$

⑥ e.x., (use ③)

solve: $X.\text{add\_axis}(-2).\text{add\_axis}(-2).\text{shape} = (2\,3\,1\,1\,4)$

$Y.\text{add\_axis}(-2).\text{add\_axis}(-1).\text{shape} = (2\,3\,1\,5\,1)$

$A.\text{reshape}(1\,1\,7\,4\,5)$

∴ $Z = XAY$, $Z.\text{shape} = (2\,3\,1\,1\,4) \times (1\,1\,7\,4\,5) \times (2\,3\,1\,5\,1)$

$= (2\,3\,7\,1\,5) \times (2\,3\,1\,5\,1)$

$= (2\,3\,7\,1\,1)$

$Z.\text{remove\_axis}(-1, -2).\text{shape} = (2\,3\,7)$

$Z \leftarrow Z + b.\text{reshape}(1, 1, 7)$