$$x' \times Hx = x' \cdot x^T H_{1,vec}$$

2. $x \times y = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$

$= \left(\begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix}, \begin{vmatrix} x_3 & x_1 \\ y_3 & y_1 \end{vmatrix}, \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \right)^T$

$= (x_2y_3 - x_3y_2,\ x_3y_1 - x_1y_3,\ x_1y_2 - x_2y_1)^T$

$= \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} y$

$= x_x y$

3. $x'_{3\times1} \times H_{3\times n} x_{n\times1}$

$= \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \times \begin{pmatrix} h_1^T x \\ h_2^T x \\ h_3^T x \end{pmatrix}, \left(H = \begin{pmatrix} h_1^T \\ h_2^T \\ h_3^T \end{pmatrix} \right)$

1. $s\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = H_{3\times n} x_{n\times1}$

$H = \begin{pmatrix} h_1^T \\ h_2^T \\ h_3^T \end{pmatrix}$

$\therefore\ sx' = h_1^T x$

$sy' = h_2^T x$

$s = h_3^T x$

1,3 $\therefore \begin{cases} x' = \dfrac{h_1^T x}{h_3^T x} \\ y' = \dfrac{h_2^T x}{h_3^T x} \end{cases}$

1,4 $\therefore\ x' h_3^T x = h_1^T x$

$y' h_3^T x = h_2^T x$

$\Rightarrow \begin{pmatrix} x' h_3^T - h_1^T \\ y' h_3^T - h_2^T \end{pmatrix} x = 0$

3.2 $x' \times Hx$

$$= (x_\times \cdot \mathrm{vec}\, x^T).\mathrm{reshape}(3,3n)\, H.\mathrm{vec}$$

$$= x'_\times \cdot x^T H.\mathrm{vec}$$

$$= \begin{pmatrix} 0 & -z' & y' \\ z' & 0 & -x' \\ -y' & x' & 0 \end{pmatrix} \begin{pmatrix} h_1^T x \\ h_2^T x \\ h_3^T x \end{pmatrix}$$

$$= \begin{pmatrix} -z' h_2^T x + y' h_3^T x \\ z' h_1^T x - x' h_3^T x \\ -y' h_1^T x + x' h_2^T x \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -z' x^T & y' x^T \\ z' x^T & 0 & -x' x^T \\ -y' x^T & x' x^T & 0 \end{pmatrix}_{3\times(3n)} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{(3\times n)\times 1}$$

$$= x'_\times \cdot x^T H.\mathrm{vec}$$

4. $(Ga)_\times = |G| G^{-T} a_\times G^{-1}$

$\therefore (Ga)_\times b = (Ga) \times b$

$$= |G| G^{-T} [a \times (G^{-1} b)]$$

5. $\hat{x} = Tx, \quad \hat{x}' = T' x',$

$\therefore \quad \hat{x}' \times \hat{H} \hat{x} = 0$

$\Leftrightarrow (T'x') \times \hat{H} T x = 0$

$\Leftrightarrow |T'| T'^{-T} [x' \times (T'^{-1} \hat{H} T x)] = 0$

$\Leftrightarrow x' \times (T'^{-1} \hat{H} T) x = 0$

(here $\cdot$ means : $A \underset{m\times n}{\cdot} B = (A_{ij} B)_{m\times n}$)