

1. hyperplane $A = \{x \mid a'x = b\}$, b scalar

$\Rightarrow A$ closed, convex

proof: ① $\bar{A} = A$?

let $y \in \bar{A}$, $\therefore y_n \rightarrow y$, $\{y_n\} \subset A$

$\therefore a'y_n = b, \forall n, \therefore a'y = b \therefore y \in A$.

$\therefore \bar{A} \subset A, \therefore \bar{A} = A, \therefore A$ closed

② $\forall x, y \in A, \therefore a'x = b, a'y = b$

$\therefore a'(\alpha x + (1-\alpha)y) = b$

$\therefore \alpha x + (1-\alpha)y \in A, \therefore A$ convex

2. ~~halfspace~~ halfspace $A = \{x \mid a'x \leq b\}$

$\Rightarrow A$ closed, convex

proof: change $=$ in 1 to \leq .

3. polyhedral $A = \{x \mid a_j'x \leq b_j, j=1, \dots, r\}$

$A \neq \emptyset, \Rightarrow A$ closed, convex

4. subspace \subset polyhedral cone \subset polyhedral set