$\boxed{\text{3×3 matrix for transforming coordinates.}}$

1. translate

$x = x_0 + \Delta x$

$y = y_0 + \Delta y$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & & \Delta x \\ & 1 & \Delta y \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

2. rotate

2.1 around origin

$x_0 = rc\alpha$

$y_0 = rs\alpha$

$x = rc(\alpha+\theta) = x_0 c\theta - y_0 s\theta$

$y = rs(\alpha+\theta) = y_0 c\theta + x_0 s\theta$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} c\theta & -s\theta & \\ s\theta & c\theta & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

2.2 around (x_p, y_p)

$\begin{pmatrix} x-x_p \\ y-y_p \\ 1 \end{pmatrix} = \begin{pmatrix} c\theta & -s\theta & \\ s\theta & c\theta & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0-x_p \\ y_0-y_p \\ 1 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & & x_p \\ & 1 & y_p \\ & & 1 \end{pmatrix} \begin{pmatrix} c\theta & -s\theta & \\ s\theta & c\theta & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & -x_p \\ & 1 & -y_p \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} c\theta & -s\theta & * \\ s\theta & c\theta & * \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

3. scale

$x = k_1 x_0$

$y = k_2 y_0$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

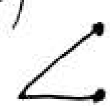
4. $x = x_0 + k_1 y_0$

$y = y_0$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & k_1 & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

4.2 $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & & \\ k_2 & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

$\Leftrightarrow \begin{cases} x = x_0 \\ y = k_2 x_0 + y_0 \end{cases}$



4.3 $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & k_1 & \\ k_2 & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

$\therefore x = x_0 + k_1 y_0$

$y = k_2 x_0 + y_0$

5. symmetry

5.1 x axis

$x = x_0$

$y = -y_0$

$\therefore \begin{pmatrix} x \\ y \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

5.2 y axis

$x = -x_0$

$y = y_0$

$\therefore \begin{pmatrix} x \\ y \\ 2 \end{pmatrix} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

5.3 $y = x$

$x = y_0$

$y = x_0$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

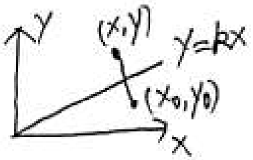
5.4 $y = -x$

$x = -y_0$

$y = -x_0$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} & -1 & \\ -1 & & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

5.5 $y = kx$



$\therefore \begin{cases} (1,k)(x_0-x, y_0-y) = 0 \\ k \cdot \frac{x_0+x}{2} = \frac{y_0+y}{2} \end{cases}$

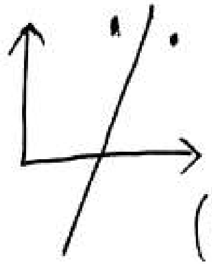
$\therefore y = \frac{1}{k^2+1}\left(2kx_0 + (k^2-1)y_0\right)$

$x = \frac{1}{k^2+1}\left((1-k^2)x_0 + 2ky_0\right)$

$\therefore \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1-k^2}{1+k^2} & \frac{2k}{1+k^2} & \\ \frac{2k}{1+k^2} & \frac{k^2-1}{1+k^2} & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$

5.6

$y=kx+b$



$(0,0)\to(0,b)$

$$\begin{pmatrix}x\\y\\1\end{pmatrix}=\begin{pmatrix}1&&0\\&1&b\\&&1\end{pmatrix}\begin{pmatrix}\frac{1-k^2}{1+k^2}&\frac{2k}{1+k^2}&\\\frac{2k}{1+k^2}&\frac{k^2-1}{1+k^2}&\\&&1\end{pmatrix}\begin{pmatrix}1&&0\\&1&-b\\&&1\end{pmatrix}\begin{pmatrix}x_0\\y_0\\1\end{pmatrix}$$

6. matrix property.

6.1 $\begin{pmatrix}I&A\\&1\end{pmatrix}\begin{pmatrix}B&C\\&1\end{pmatrix}$

$=\begin{pmatrix}B&*\\&1\end{pmatrix}$

6.2 $\begin{pmatrix}B&C\\&1\end{pmatrix}\begin{pmatrix}I&A\\&1\end{pmatrix}$

$=\begin{pmatrix}B&*\\&1\end{pmatrix}$

6.3 $\begin{pmatrix}1&&\Delta x\\&1&\Delta y\\&&1\end{pmatrix}\begin{pmatrix}a&b&\\c&d&\\&&1\end{pmatrix}$

$=\begin{pmatrix}a&b&\Delta x\\c&d&\Delta y\\&&1\end{pmatrix}$