

$$p \notin L,$$

$$\Rightarrow \text{distance}(p, L)$$

$$\text{soln: } \text{Let } x_0 \in L, x_0 \in L^*,$$

$$\text{here } L^* = \{x | ax + c = 0\}$$

$$c = -p \cdot d \in \mathbb{R}.$$

$$L^* \text{ is plane}$$

$$\textcircled{2} \quad \therefore \text{distance}(p, L)$$

$$= \|x_0 - p\|$$

$$\textcircled{3} \quad \therefore ax + c = 0, x_0 \in L$$

$$\therefore \|ax + \lambda\|^2 + c = 0$$

$$\therefore \lambda = -\frac{(p-a)d}{\|a\|^2}$$

$$\Rightarrow x_0 = a + \lambda d$$

$$\Rightarrow \text{dis} = \|x_0 - p\|$$

$$\textcircled{4} \quad L(p)$$

$$= L(p - x_0) + L(x_0) - c$$

$$= L(p - x_0) - c$$

$$= L(-s_0 n) - c$$

$$= -s_0 \|n\|^2$$

$$\therefore \|s_0\| \|n\| = \frac{|L(p)|}{\|n\|}$$

$$\textcircled{5} \quad \therefore \text{dis} = \|p - x_0\|$$

$$= \|-s_0 n\|$$

$$= \frac{|L(p)|}{\|n\|}$$

$$2. \text{ line } L \text{ in } \mathbb{R}^n$$

$$L = \{x | a + \lambda d = x\}$$

$$\boxed{\text{distance}(p, L)}$$

$$1. \text{ plane } L \text{ in } \mathbb{R}^n,$$

$$L = \{x | nx + c = 0\}, p \notin L, c \in \mathbb{R},$$

$$\Rightarrow \text{distance}(p, L)$$

$$\text{soln:}$$

$$\textcircled{1} \text{ let } L(x) = nx + c,$$

$$L^* = \{x | nx = p + s n, \forall s \in \mathbb{R}\}$$

$$\textcircled{2} \text{ let } x_0 \in L^*, x_0 \in L$$

$$\Rightarrow \text{distance}(p, L) = \|p - x_0\|$$

$$\textcircled{3} \quad \therefore \exists s_0, \text{ s.t. } x_0 = p + s_0 n,$$

$$\therefore x_0 \in L, \therefore L(x_0) = 0$$