1. $\forall x, f(x)=\infty \Leftrightarrow epi(f)=\phi$

proof: $f=\infty \Leftrightarrow dom(f)=\phi$

$\because$ dom(f) is projection of epi(f)

$\therefore dom(f)=\phi \Leftrightarrow epi(f)=\phi$

2. $\exists x\in X$, s.t., $f(x)=-\infty \Leftrightarrow$ epi(f) contains a vertical line.

proof: $\exists x$, s.t., $f(x)=-\infty \Leftrightarrow f(x)\leq w, \forall w\in R$

$\Leftrightarrow$ epi(f) contains a vertical line.

3. f is proper if epi(f) $\neq \phi$ and epi(f) doesn't contain a vertical line.

4. forbidden sum $-\infty+\infty$ may happen in $\alpha f(x)+(1-\alpha)f(y)$

$\Leftrightarrow f$ is improper.

5. $C\subset R^n$ convex, $f: C\to[-\infty,\infty]$ convex if epi(f) convex.

definition:

6. $f: C\to[-\infty,\infty]$ convex $\Rightarrow$ dom(f) convex, level set convex.

~~proof~~

7. $f: C\to(-\infty,\infty)$ convex $\Leftrightarrow$ epi(f) convex

proof: ① $\Rightarrow$, let $(x,u),(y,v)\in epi(f)$, $\therefore f(x)\leq u, f(y)\leq v$

$p=\alpha(x,u)+(1-\alpha)(y,v)\in epi(f)$?

$=(\alpha x+(1-\alpha)y, \alpha u+(1-\alpha)v)$

$\because f(\alpha x+(1-\alpha)y)\leq\alpha f(x)+(1-\alpha)f(y)\leq\alpha u+(1-\alpha)v$, $\therefore p\in epi(f)$

② $\Leftarrow$ let $x,y\in C$, $u=f(x)$, $v=f(y)$. $\therefore (x,u),(y,v)\in epi(f)$

$\because$ epi(f) convex, $\therefore p=\alpha(x,u)+(1-\alpha)(y,v)\in epi(f)$

$\therefore f(\alpha x+(1-\alpha)y)\leq\alpha u+(1-\alpha)v=\alpha f(x)+(1-\alpha)f(y)$