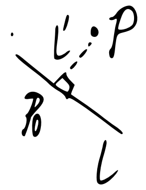
$\boxed{\text{distance}(p, l)}$

3. 

Given $l, p. \Rightarrow p_0$, s.t., $pp_0 \perp l$, $p_0 \in l$.

solve:

① method 1.

$$\left.\begin{matrix} p_0 \in l \\ p_0 p \perp l \end{matrix}\right\} \Rightarrow \begin{cases} p_0^T l = 0 \\ (p_0 \times p) \perp l \end{cases} \Rightarrow \begin{cases} a x_0 + b y_0 + c = 0 \\ a y_0 - b x_0 + c' = 0 \\ c' = bx - ay \end{cases}$$

$$\Rightarrow \begin{cases} x_0 = \dfrac{bc' - ac}{a^2+b^2} \\ y_0 = \dfrac{-ac' - bc}{a^2+b^2} \end{cases}, \quad \text{here } p_0 = \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$$

$$\left(\because p_0 \times p = (p_0)_\times p = \begin{pmatrix} 0 & -1 & y_0 \\ 1 & 0 & -x_0 \\ -y_0 & x_0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} y_0 - y \\ x - x_0 \\ x_0 y - y_0 x \end{pmatrix} \right.$$

$$\left. (p_0 \times p) \perp l \quad \therefore \begin{pmatrix} y_0 - y \\ x - x_0 \end{pmatrix}^T \begin{pmatrix} a \\ b \end{pmatrix} = 0 \right)$$

② method 2: let $l' \perp l$, $p \in l'$, $\therefore p_0 = l \times l'$

$$\because l' = \begin{pmatrix} -b \\ a \\ c' \end{pmatrix}, \quad c' = bx - ay, \quad \therefore p_0 = l_\times l' = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix} \begin{pmatrix} -b \\ a \\ c' \end{pmatrix}$$

$$\therefore p_0 = (bc' - ac, \ -ac' - bc, \ a^2 + b^2)^T$$

1. line $l \perp$ line l', $l = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $l' = \begin{pmatrix} a' \\ b' \\ c' \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix}^T \begin{pmatrix} a' \\ b' \end{pmatrix} = 0$$

2. $l' \perp l$, point $p \in$ line l',

$l = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $p \in P^2$, $p = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

$$\Rightarrow l' = \begin{pmatrix} -b \\ a \\ bx - ay \end{pmatrix}$$

proof: let $l' = \begin{pmatrix} a' \\ b' \\ c' \end{pmatrix}$

$\because l' \perp l$, $\therefore a' = -b$, $b' = a$

$\therefore l' = \begin{pmatrix} -b \\ a \\ c' \end{pmatrix}$. $\quad \because p \in l'$

$\therefore -bx + ay + c' = 0$

$\therefore c' = bx - ay$

② method 2 : (see 232 $dis(p,l)$)

let $p, p_0 \in \mathbb{R}^2$, $l \in \mathbb{R}^3$,

$l = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} n \\ c \end{pmatrix}$, $n = \begin{pmatrix} a \\ b \end{pmatrix}$,

$\because p_0 = p + s_0 n \in l$

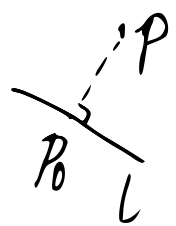
$\therefore \begin{pmatrix} p + s_0 n \\ 1 \end{pmatrix}^T \begin{pmatrix} n \\ c \end{pmatrix} = 0$

$\therefore \begin{pmatrix} p \\ 1 \end{pmatrix}^T \begin{pmatrix} n \\ c \end{pmatrix} + \begin{pmatrix} s_0 n \\ 0 \end{pmatrix}^T \begin{pmatrix} n \\ c \end{pmatrix} = 0$

$\therefore s_0 = -L(p)/\|n\|^2$

here $L(p) = l^T \begin{pmatrix} p \\ 1 \end{pmatrix} = \begin{pmatrix} n \\ c \end{pmatrix}^T \begin{pmatrix} p \\ 1 \end{pmatrix}$

$\therefore dis = \|p - p_0\| = \|s_0 n\| = |L(p)|/\|n\|$

4. 

Given p, l in $\mathbb{R}^2$, $\Rightarrow distance(p,l)$

solve: ① method 1:

$\because 3 \Rightarrow p_0$

$\therefore dis^2 = (x-x_0)^2 + (y-y_0)^2$

$= \frac{a^2(L(p))^2}{(a^2+b^2)^2} + \frac{b^2(L(p))^2}{(a^2+b^2)^2}$

$= \frac{(L(p))^2}{a^2+b^2}$

(here $L(p) = l^T p = ax + by + c$)

$\therefore dis = |L(p)| / \sqrt{a^2+b^2}$