$$\boxed{y=\frac{x}{\|x\|} \Rightarrow \frac{\partial y}{\partial x}}$$

1. $s=\|x\|$

$\Rightarrow \frac{\partial s}{\partial x_i} = \frac{x_i}{s}$

$\therefore \frac{\partial s}{\partial x} = \frac{1}{s}x^T$

2. $s=\|x\|$, $y=\bar{x}=\frac{x}{s}$

$\Rightarrow \frac{\partial y_j}{\partial x_i} = \frac{1}{s}1_{i=j} - \frac{1}{s}y_i y_j$

3. $s=\|x\|$, $y=\frac{x}{s}$

$\Rightarrow \frac{\partial y}{\partial x_i} = \frac{1}{s}e_i - \frac{1}{s}y_i y$

$= \frac{1}{s}(I-yy^T)e_i$

4. $\frac{\partial y}{\partial x} = \frac{1}{s}(I-yy^T)$,

here $s=\|x\|$, $y=\frac{x}{s}$

5. $L=g(y)\in R$, $y=f(x)$,

$y\in R^m$, $x\in R^n$

$\Rightarrow \frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial x}$

proof: $\because \frac{\partial L}{\partial x_i} = \sum_j \frac{\partial L}{\partial y_j}\frac{\partial y_j}{\partial x_i}$

$= \frac{\partial L}{\partial y}\frac{\partial y}{\partial x_i}$

$\therefore \frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial x}$

6. $L=g(y)\in R$, $y=\frac{x}{\|x\|}\in R^n$

$\Rightarrow$

$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial x}$

$= \frac{1}{s}\frac{\partial L}{\partial y}(I-yy^T)$

7. $y=Ax$, $A\in R^{m\times n}$

$\Rightarrow \frac{\partial y}{\partial x}=A$,

proof: $\because y_i = \sum_j A_{ij}x_j$

$\therefore \frac{\partial y_i}{\partial x_j} = A_{ij}$ $\therefore \frac{\partial y}{\partial x} = A$

8. $y=kx$, $k\in R$, $x,y\in R^n$

$\Rightarrow \frac{\partial y}{\partial k} = x$, $\frac{\partial y}{\partial x} = \frac{\partial (kIx)}{\partial x} = kI$

9. $L=g(z)\in R$, $z=ky$, $y=\frac{x}{\|x\|}$

$k\in R$, $x,y,z\in R^n$

$\Rightarrow$ ① $\frac{\partial L}{\partial k} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial k} = \frac{\partial L}{\partial z}y$

② $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}\frac{\partial y}{\partial x}$

$= \frac{\partial L}{\partial z}(kI)\frac{1}{s}(I-yy^T)$

$= \frac{k}{s}\frac{\partial L}{\partial z} - \frac{k}{s}\frac{\partial L}{\partial z}yy^T$

$= \left(\frac{k}{s}\right)\frac{\partial L}{\partial z} - \left(\frac{k}{s^2}\frac{\partial L}{\partial k}\right)x^T$