1. affine function $f(x) = a'x + b$,

$\Rightarrow$ convex

proof: $f(\alpha x + (1-\alpha)y)$

$= a'(\alpha x + (1-\alpha)y) + b$

$= \alpha(a'x + b) + (1-\alpha)(a'y + b)$

$= \alpha f(x) + (1-\alpha) f(y)$ $\therefore$ convex

2. norm $\|\cdot\|$ is convex.

3. f convex $\Rightarrow$ its level set convex.

proof: $A = \{x \mid f(x) \leq \gamma\}$. $x, y \in A \subset C$

$\alpha x + (1-\alpha)y \in A$?

$\because$ C convex, $\therefore$ $\alpha x + (1-\alpha)y \in C$

$\therefore$ $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$

$\leq \gamma$

$\therefore$ $\alpha x + (1-\alpha)y \in A$. $\therefore$ A convex

4. $f(x) = \sqrt{|x|}$ isn't convex

proof: if f convex, $\Rightarrow$ $f(\alpha x) \leq \alpha f(x) + (1-\alpha) f(0)$

$\therefore$ $\sqrt{|\alpha x|} \leq \alpha \sqrt{|x|}$ $\therefore$ $\sqrt{\alpha}\sqrt{|x|} \leq \alpha\sqrt{|x|}$

let $x \neq 0$, $\alpha \neq 0, \alpha \neq 1$, $\therefore$ $\alpha \geq 1$, error.

$\therefore$ f isn't convex

5. $f(x) = \sqrt{|x|}$, $\Rightarrow$ level set $A = \{x \in R \mid f(x) < \gamma\}$ convex

proof: $x, y \in A$, $\therefore$ $f(\alpha x + (1-\alpha)y) = \sqrt{|\alpha x + (1-\alpha)y|}$

$\leq \sqrt{\alpha|x| + (1-\alpha)|y|} \leq \gamma$, $\therefore$ A convex.