$$L = \frac{1}{2}\|f(x)\|^2 \Rightarrow L', L''$$

1. $L(x) = \frac{1}{2}\|f(x)\|^2$, $f: \mathbb{R}^n \to \mathbb{R}^m$,
$y = f(x)$, $f = \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$

$$\Rightarrow L' = \frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial x}$$

$$= y^T f'$$

$$= y^T J \in \mathbb{R}^{1\times n}$$

1.3 ,
$$\frac{\partial L}{\partial x} = y^T f' = (y_1\ y_2\ y_m)\begin{pmatrix} \frac{\partial y_1}{\partial x} \\ \frac{\partial y_2}{\partial x} \\ \frac{\partial y_m}{\partial x} \end{pmatrix}$$

$$= \sum_{i=1}^{m} y_i \frac{\partial y_i}{\partial x} = \Sigma y_i f_i'$$

$\left(L = \frac{1}{2}\|f\|^2 = \frac{1}{2}(y_1^2 + y_2^2 + y_m^2) \Rightarrow \frac{\partial L}{\partial x} = \sum_{i=1}^{m} y_i \frac{\partial y_i}{\partial x} \right)$

1、 $L'' = \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial x}\right)$

$$= \sum_{i=1}^{m} \frac{\partial}{\partial x}\left(y_i \frac{\partial y_i}{\partial x}\right)$$

$$= \Sigma \frac{\partial}{\partial x}\left(y_i \left(\frac{\partial y_i}{\partial x}\right)^T\right)$$

$$= \sum_{i=1}^{m}\left[\left(\frac{\partial y_i}{\partial x}\right)^T \frac{\partial y_i}{\partial x} + y_i \frac{\partial}{\partial x}\left(\frac{\partial y_i}{\partial x}\right)^T\right]$$

$$= \left(\frac{\partial y}{\partial x}\right)^T \frac{\partial y}{\partial x} + \sum_{i=1}^{m} y_i \frac{\partial}{\partial x}\left(\frac{\partial y_i}{\partial x}\right)$$

$$= J^T J + \sum_{i=1}^{m} y_i f_i''(x)$$

2. $L(x) = \frac{1}{2}\|f(x)\|^2$, $y = f(x) = Wx + b$,
$W \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$

$$\Rightarrow L' = y^T J = y^T W \in \mathbb{R}^{1\times n}$$

$$L'' = \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial x}\right) = \frac{\partial L'}{\partial y}\frac{\partial y}{\partial x} = W^T W$$

$$= J^T J$$