$\boxed{\text{distance(line, curve)}}$

1. curve $f(t)$, $t \in R$, point $p \in R^n$,

$\Rightarrow$ distance(p, f)

solve:

① let $L(t) = \frac{1}{2}\|f(t) - p\|^2$, $y = f(t) - p$

$\therefore \frac{\partial L}{\partial t} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial t} = y^T J$

② $\because L' = \frac{\partial L}{\partial t} = (f(t) - p)^T f'(t)$

$\therefore L'' = f'(t)^T \frac{\partial(f(t) - p)}{\partial t} + (f(t) - p)^T f''(t)$

$= \|f'(t)\|^2 + y^T f''(t)$

③ use Newton's method,

$a = -\frac{L'(t)}{L''(t)}$ $\therefore t_{k+1} = t_k + a$

$\Rightarrow t_{min} = \operatorname{argmin} L(t)$

$\therefore p_0 = f(t_{min})$ $\therefore dis = \|p - p_0\|$

2. curve $f(t)$, line l in R^2.

$\Rightarrow$ distance(l, f)

solve: ① let $l = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $n = \begin{pmatrix} a \\ b \end{pmatrix}$, $l \leftarrow \frac{1}{\|n\|} l$

② let $L(t) = \frac{1}{2}\|\text{distance}(f(t), l)\|^2 \cancel{= \frac{1}{2}}$

$\therefore L(t) = \frac{1}{2}\left\| \begin{pmatrix} f(t) \\ 1 \end{pmatrix}^T l \right\|^2$

③ let $y = \tilde{f}^T l$, $\tilde{f} = \begin{pmatrix} f(t) \\ 1 \end{pmatrix}$, $\therefore L = \frac{1}{2}\|y\|^2$

$\because \tilde{f} = \begin{pmatrix} f \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} f(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\therefore \frac{\partial \tilde{f}}{\partial f} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

$\therefore \frac{\partial L}{\partial t} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial \tilde{f}}\frac{\partial \tilde{f}}{\partial f}\frac{\partial f}{\partial t} = y^T l^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} J = y(a\ b)f'$

④ $L'' = \frac{\partial((a\ b)v)}{\partial t}$, $v = yf'$,

$\therefore L'' = (a\ b)\frac{\partial v}{\partial t} = (a\ b)(yf'' + f'\frac{\partial y}{\partial t})$

$\because \frac{\partial y}{\partial t} = (a\ b)f'$ $\therefore L'' = y(a\ b)f'' + ((a\ b)f')^2$

⑤ like 4 $\Rightarrow t_{min}$ $\therefore p_0 = f(t_{min})$ $\therefore dis = dis(p_0, l) = \left| \begin{pmatrix} p_0 \\ 1 \end{pmatrix}^T l \right|$