ES120 Spring 2018 – Section 9 Notes

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Problem 1:

Determine the dimension d so that the aluminum and steel struts will have the same weight, and compute the critical load for each strut.

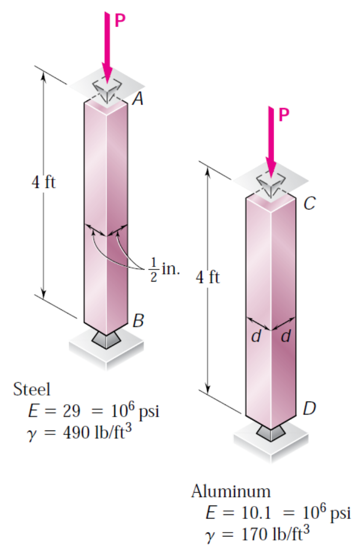


Figure 1

Solution 1

For this problem we must first obtain the total weight of the different beams, namely

$$W = \gamma L d_s^2, \tag{1}$$

for the steel the weight is

$$W_s = (0.2835)(4 * 12)(0.5)^2 = 3.4028 \text{ lb} \tag{2}$$

for the aluminum the weight as a function of the dimension d is

$$W_a = (0.09838)(4 * 12)d^2 = 4.7222d^2 \tag{3}$$

So we want them to be the same weight, thus,

$$W_s = W_a \Rightarrow 3.4028 = 4.7222d^2 \Rightarrow \boxed{d = 0.849 \text{ in}} \tag{4}$$

Now to compute the critical load for each strut we need to solve for the P_{cr} given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{5}$$

For the steel strut, we have that

$$I = \frac{1}{12}d_s^4 = \frac{1}{12}\left(\frac{1}{2}\right)^4 = 5.208 \times 10^{-3}\ \text{in}^4 \tag{6}$$

$$P_{cr} = \frac{\pi^2(29 \times 10^6)(5.2083 \times 10^{-3})}{(4 * 12)^2} = \boxed{647\ \text{lb}} \tag{7}$$

For the aluminum strut, we have that

$$I = \frac{1}{12}d_s^4 = \frac{1}{12}(0.849)^4 = 43.271 \times 10^{-3}\ \text{in}^4 \tag{8}$$

$$P_{cr} = \frac{\pi^2(10.1 \times 10^6)(13.271 \times 10^{-3})}{(4 * 12)^2} = \boxed{1872\ \text{lb}} \tag{9}$$

Problem 2:

Knowing that a factor of safety of 2.6 is required, determine the largest load **P** that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

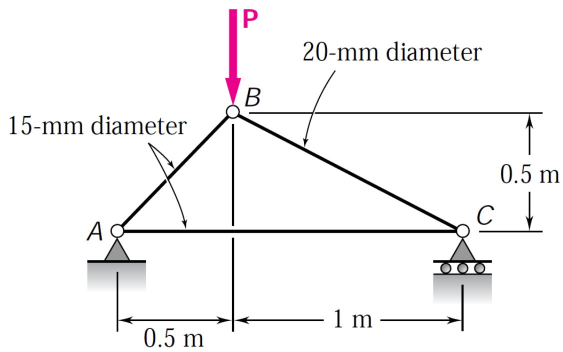


Figure 2

Solution 2

For this problem we must first find the length of the individual members

$$L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180\,\text{m} \tag{10}$$

$$L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711\,\text{m} \tag{11}$$

We can also compute the second moment of inertias as:

$$I_{BC} = \frac{\pi}{64}(20)^4 = 7.854 \times 10^{-9}\,\text{m}^4 \tag{12}$$

$$I_{AB} = \frac{\pi}{64}(15)^4 = 2.485 \times 10^{-9}\,\text{m}^4 \tag{13}$$

We can now compute what the critical load for both members BC and AB using

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{14}$$

$$P_{cr,BC} = \frac{\pi^2(200 \times 10^9)(7.854 \times 10^{-9})}{(1.1180)^2} = 12.403\,\text{kN} \tag{15}$$

$$P_{cr,AB} = \frac{\pi^2(200 \times 10^9)(2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106\,\text{kN} \tag{16}$$

Given our factor of safety FS we can compute the allowable force recalling that

$$F_{all} = \frac{P_{cr}}{FS} \tag{17}$$

So for the different members that becomes

$$F_{all,BC} = \frac{12.403}{2.6} = 4.770\text{kN} \tag{18}$$

$$F_{all,AB} = \frac{9.8106}{2.6} = 3.773\text{kN} \tag{19}$$

Now we can obtain the free body diagram for point B as

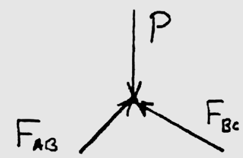


Figure 3

Performing force balance in the horizontal and vertical directions respectively, we obtain

$$\Sigma F_x = 0: \quad \frac{0.5}{0.70711} F_{AB} - \frac{1}{1.1180} F_{BC} = 0 \Rightarrow F_{BC} = 0.7905 F_{AB} \tag{20}$$

$$\Sigma F_y = 0: \quad \frac{0.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} - P = 0 \Rightarrow P = 1.06066 F_{AB} \tag{21}$$

Which combining both we can also obtain a relationship between P and F_{BC}, namely

$$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC} \tag{22}$$

Therefore, solving for the allowable value for P yields,

$$P < 1.06066 F_{all,AB} = (1.06066)(3.773) = 4.0 \text{ kN} \tag{23}$$

$$P < 1.3416 F_{all,BC} = (1.3416)(4.770) = 6.4 \text{ kN} \tag{24}$$

The smallest of the the two gives us the largest load namely

$$\boxed{P_{all} = 4.0 \text{ kN}} \tag{25}$$

Problem 3:

Using the method of work and energy, determine the deflection at point D caused by the load **P**.

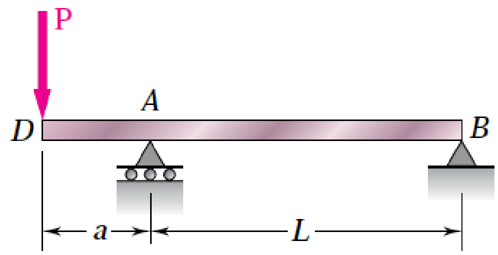
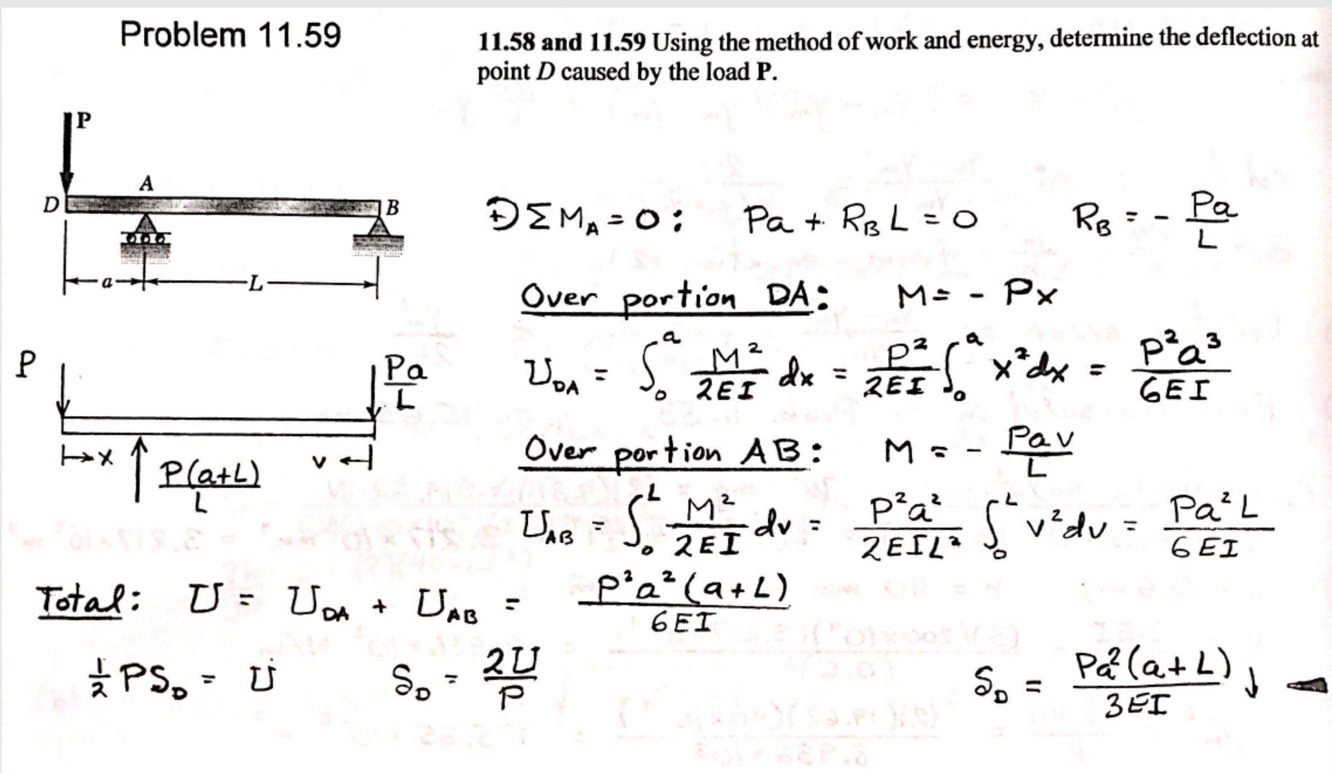


Figure 4

Solution 3



Problem 11.59

11.58 and 11.59 Using the method of work and energy, determine the deflection at point D caused by the load **P**.

$\circlearrowleft \Sigma M_A = 0: \quad Pa + R_B L = 0 \qquad R_B = -\frac{Pa}{L}$

Over portion DA: $\quad M = -Px$

$$U_{DA} = \int_0^a \frac{M^2}{2EI}\,dx = \frac{P^2}{2EI}\int_0^a x^2\,dx = \frac{P^2a^3}{6EI}$$

Over portion AB: $\quad M = -\frac{Pav}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}\,dv = \frac{P^2a^2}{2EIL^2}\int_0^L v^2\,dv = \frac{Pa^2L}{6EI}$$

Total: $\quad U = U_{DA} + U_{AB} = \frac{P^2a^2(a+L)}{6EI}$

$$\frac{1}{2}P\delta_D = U \qquad \delta_D = \frac{2U}{P} \qquad \delta_D = \frac{Pa^2(a+L)}{3EI} \downarrow$$

Figure 5

Problem 4:

For the uniform rod and loading shown and using Castigliano's theorem, determine the deflection of point B.

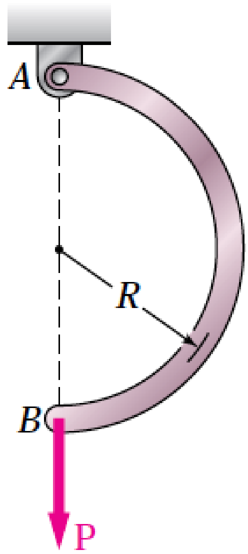
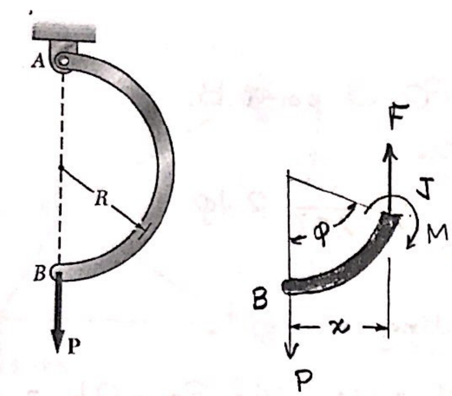


Figure 6

Solution 4



Use polar coordinate φ.
Calculate the bending moment $M(\varphi)$ using free body BJ.

$$+\circlearrowleft \Sigma M_J = 0: \quad Px - M = 0$$

$$M = Px = PR\sin\varphi$$

Strain energy: $U = \int_0^L \frac{M^2}{2EI}\, ds$

$$U = \int_0^{\pi} \frac{(PR\sin\varphi)^2}{2EI}(R\,d\varphi) = \frac{P^2R^3}{2EI}\int_0^{\pi} \sin^2\varphi\, d\varphi$$

$$= \frac{P^2R^3}{2EI}\int_0^{\pi} \frac{1-\cos 2\varphi}{2}\, d\varphi$$

$$= \frac{P^2R^2}{2EI}\left(\frac{1}{2}\varphi\Big|_0^{\pi} - \frac{1}{4}\sin 2\varphi\Big|_0^{\pi}\right) = \frac{\pi P^2 R}{4EI}$$

By Castigliano's theorem, $\delta = \frac{\partial U}{\partial P}$ $\qquad \delta = \frac{\pi P R^3}{2EI} \downarrow$

Figure 7