ENGINEERING SCIENCES 120
INTRODUCTION TO THE MECHANICS OF SOLIDS

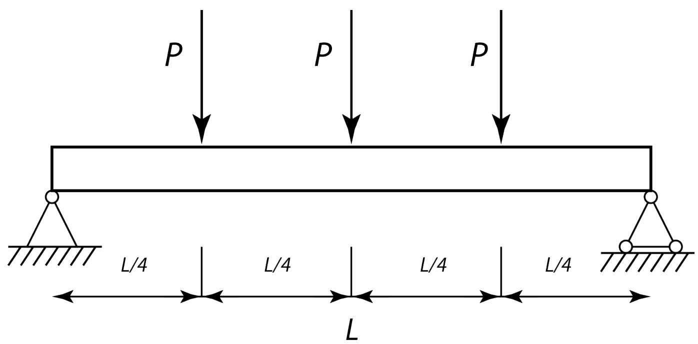
Quiz 2

April 11, 2018

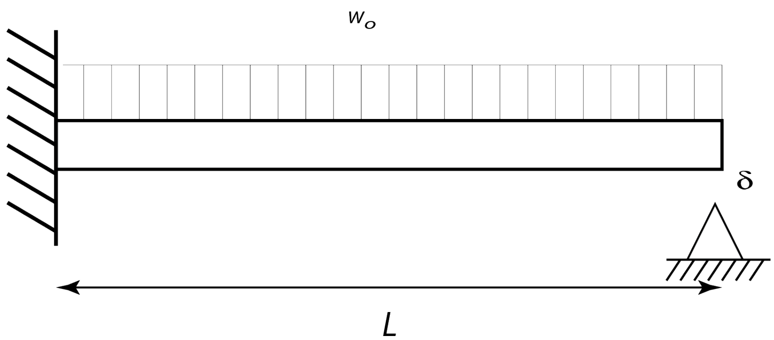
Length: 53 minutes

You are allowed to use both the equation sheet and a calculator when solving the problems. Please make sure your answer is clear and legible. No credit will be given if we cannot read the answer or figure out how you derived it! All questions are weighted equally.

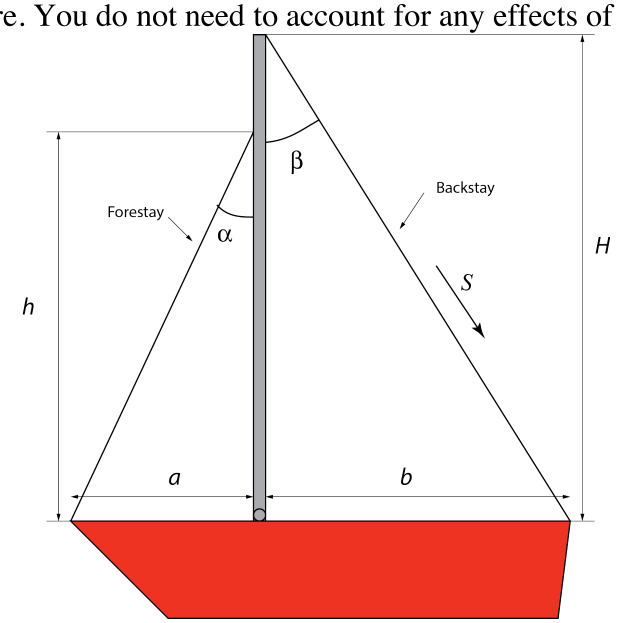
1. Consider a prismatic beam of length L, moment of inertia I, and Young's modulus E, simply supported on either end. The beam supports several discrete loads P (all equal) as shown in the picture. Use singularity functions to derive an expression for the deflection of the beam.



2. Consider a prismatic beam of length L, moment of inertia I, and Young's modulus E, clamped on the left. The beam is also supposed to rest on a support on the right side, but because of faulty construction there is a small gap δ. The beam is loaded by a uniform distributed load w_o (pointing down). (1) Determine the load w_o at which the tip of the beam touches the support. (2) Calculate the force exerted on the support once the beam touches.

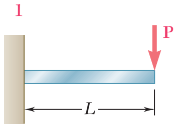
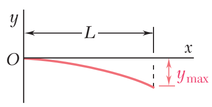
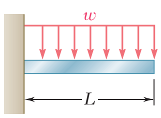
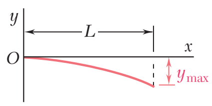
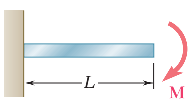
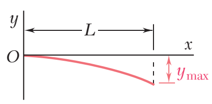
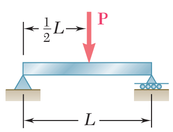
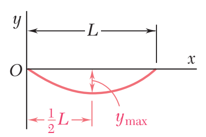
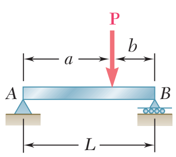
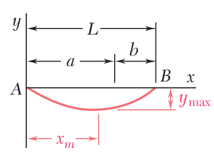
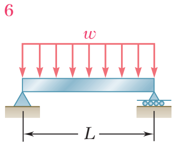
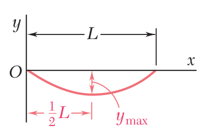
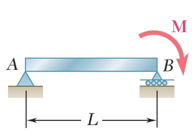
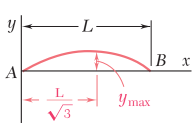


3. In a sailboat with a fractional rig, the tension in the backstay (i.e., the steel cable that runs from the masthead to the stern) is often used to bend the mast and thus shape the sails for optimum sailing performance in a race (bending the mast tends to depower the sails). As the wind increases, sailors increase the tension in the backstay using a system of pulleys or sometimes hydraulics. The figure below shows the geometry. Assume the mast can be modeled as a vertical beam that is simply supported at the deck level and at the point where the forestay (i.e., cable running from bow to mast) is connected to the mast. Assume further that the mast is initially straight. The bending stiffness of the mast is EI. The height of the mast is H; the height of the point where the forestay is connected is h. The distance from the bow to the mast is a, and from the mast to the stern is b. Determine the shape of the mast as a function of the tension S in the backstay. No need to consider buckling here. You do not need to account for any effects of wind in this problem.



ES 120 -- Introduction to the Mechanics of Solids

APPENDIX D Beam Deflections and Slopes

Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y = \frac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\frac{wL^4}{8EI}$	$-\frac{wL^3}{6EI}$	$y = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y = -\frac{M}{2EI}x^2$
4 		$-\frac{PL^3}{48EI}$	$\pm\frac{PL^2}{16EI}$	For $x \le \frac{1}{2}L$: $y = \frac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\frac{L^2 - b^2}{3}}$	$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\frac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\frac{Pa^2b^2}{3EIL}$
6 		$-\frac{5wL^4}{384EI}$	$\pm\frac{wL^3}{24EI}$	$y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$	$y = -\frac{M}{6EIL}(x^3 - L^2x)$