

ES120 Spring 2018 – Section 4 Notes

Matheus Fernandes

February 22, 2018

Problem 1:

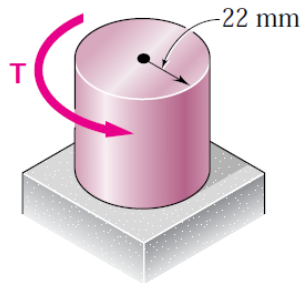


Figure 1

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T=1.5$ kN·m.

Solution 1

In order to determine the maximum shear stress caused by a torque onto the shaft we must convert the physical torque (which is equivalent to a force) to a stress. The relationship between torque and maximum shear stress is given by:

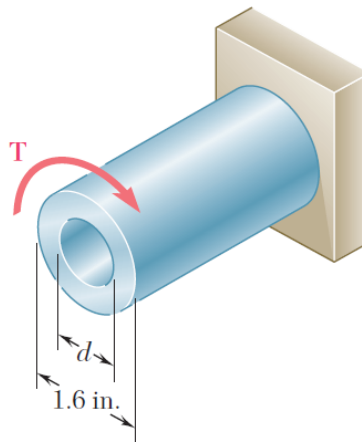
$$\tau_{max} = \frac{Tr}{J}, \quad (1)$$

where J is the polar moment of inertia for a cylinder as described per the document at the end of the notes. We can then obtain the equation for J as:

$$J = \frac{\pi}{2}c^4 \quad (2)$$

such that we can solve for τ_{max} as

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi(0.022)^3} = 89.7\text{MPa} \quad (3)$$

Problem 2:**Figure 2**

Knowing that the internal diameter of the hollow shaft shown is $d=0.9$ in., determine the maximum shearing stress caused by a torque of magnitude $T=9$ kip-in.

Solution 2

Similar to the previous problem we know that the maximum shear stress is dictated by the equation

$$\tau_{max} = \frac{Tr}{J}, \quad (4)$$

where now our radii differ in that for an annulus. So our c 's become

$$c_1 = 0.5 * 0.9 = 0.45 \text{ in} \quad (5)$$

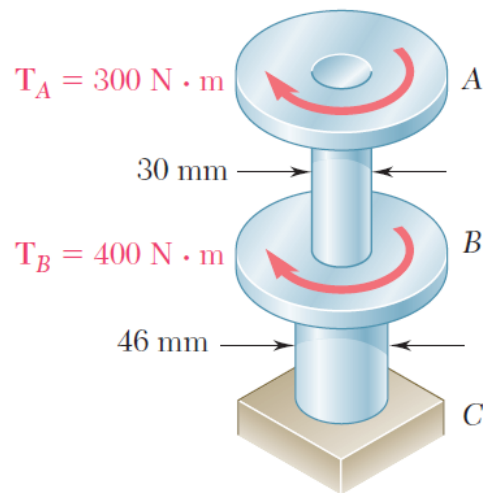
$$c_2 = 0.5 * 1.6 = 0.8 \text{ in} \quad (6)$$

Our second moment of an annulus can be calculated by subtracting the moment of the inner from the outer such that:

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4 \quad (7)$$

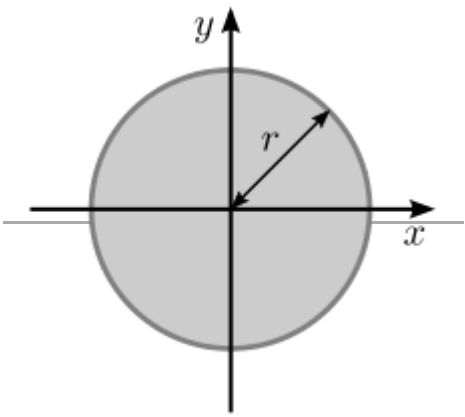
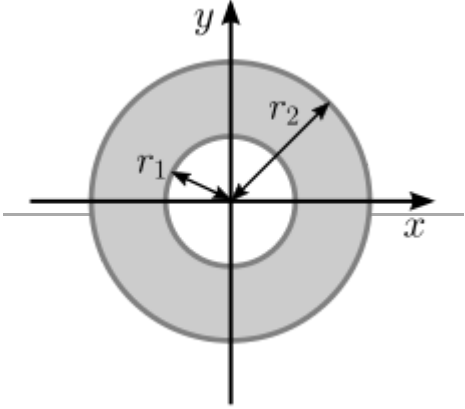
Applying that in to the equation for τ_{max} we get

$$\tau_{max} = \frac{(9)(0.8)}{0.5790} = 12.44 \text{ ksi} \quad (8)$$

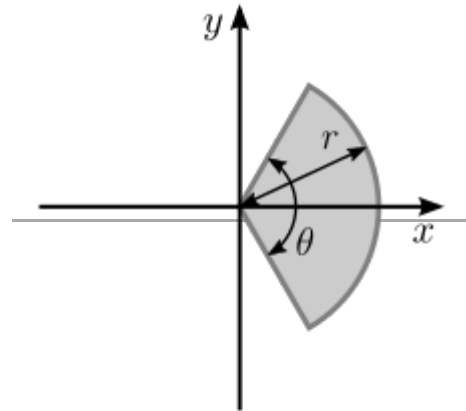
Problem 3:**Figure 3:** 3.9

The torques shown are exerted on pulleys A and B. Knowing that each shaft is solid, determine the maximum shearing stress (a) in shaft AB, (b) in shaft BC.

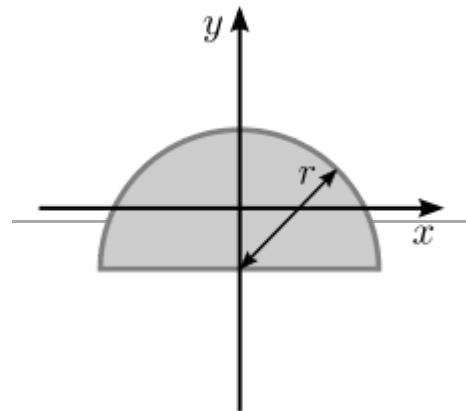
Solution 3**Part A****Part B**

Description	Figure	Area moment of inertia	Comment
A filled circular area of radius r		$I_x = \frac{\pi}{4} r^4$ $I_y = \frac{\pi}{4} r^4$ $I_z = \frac{\pi}{2} r^4 \text{ [1]}$	I_z is the <u>Polar moment of inertia</u> .
An <u>annulus</u> of inner radius r_1 and outer radius r_2		$I_x = \frac{\pi}{4} (r_2^4 - r_1^4)$ $I_y = \frac{\pi}{4} (r_2^4 - r_1^4)$ $I_z = \frac{\pi}{2} (r_2^4 - r_1^4)$	For thin tubes, $r \equiv r_1 \approx r_2$ and $r_2 \equiv r_1 + t$. So, for a thin tube, $I_x = I_y \approx \pi r^3 t$. I_z is the <u>Polar moment of inertia</u> .
A filled circular sector of angle θ in radians and radius r with respect to an axis through the centroid of the sector and the center of the circle		$I_x = (\theta - \sin \theta) \frac{r^4}{8}$	This formula is valid only for $0 \leq \theta \leq \pi$

5/8



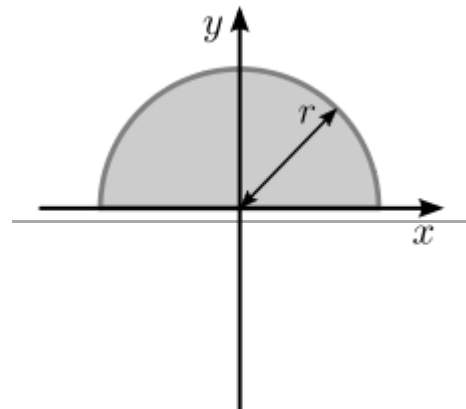
A filled semicircle with radius r with respect to a horizontal line passing through the centroid of the area



$$I_x = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) r^4 \approx 0.1098r^4$$

$$I_y = \frac{\pi r^4}{8} \quad [2]$$

A filled semicircle as above but with respect to an axis collinear with the base



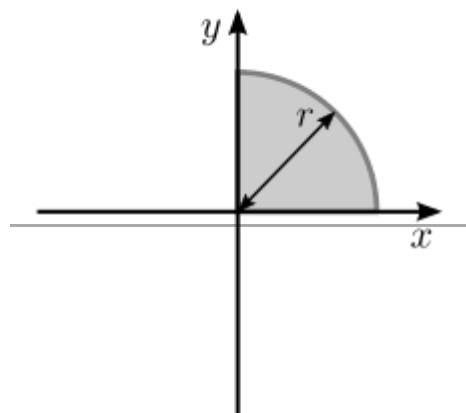
$$I_x = \frac{\pi r^4}{8}$$

$$I_y = \frac{\pi r^4}{8} \quad [2]$$

I_x : This is a consequence of the parallel axis theorem and the fact that the distance between the x axes of the previous one and this one is $\frac{4r}{3\pi}$

A filled quarter circle with radius r with the

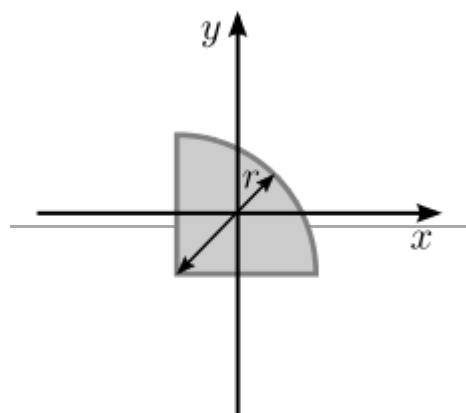
axes passing through the bases



$$I_x = \frac{\pi r^4}{16}$$

$$I_y = \frac{\pi r^4}{16} \quad [3]$$

A filled quarter circle with radius r with the axes passing through the centroid

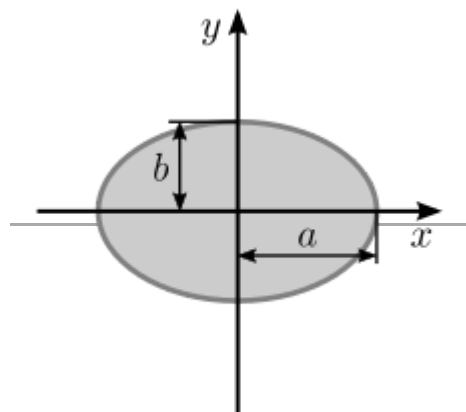


$$I_x = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) r^4 \approx 0.0549r^4$$

$$I_y = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) r^4 \approx 0.0549r^4 \quad [3]$$

This is a consequence of the parallel axis theorem and the fact that the distance between these two axes is $\frac{4r}{3\pi}$

A filled ellipse whose radius along the x-axis is a and whose radius along the y-axis is b

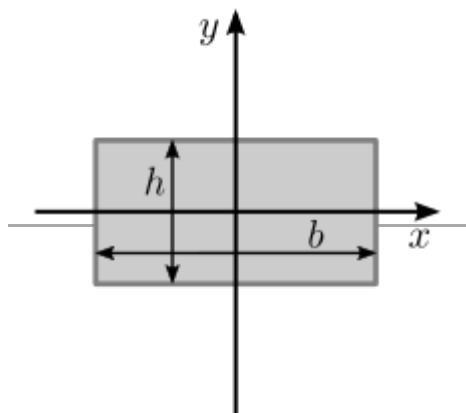


$$I_x = \frac{\pi}{4} ab^3$$

$$I_y = \frac{\pi}{4} a^3 b$$

A filled rectangular area with a base width of

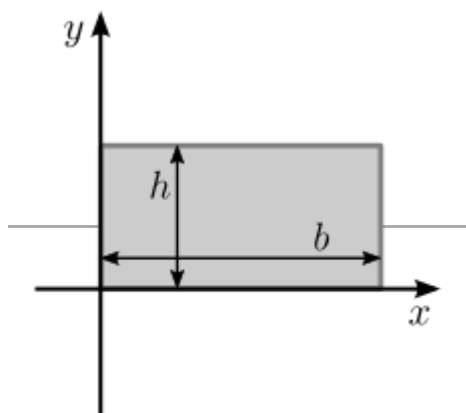
b and height h



$$I_x = \frac{bh^3}{12}$$

$$I_y = \frac{b^3h}{12} \quad [4]$$

A filled rectangular area as above but with respect to an axis collinear with the base

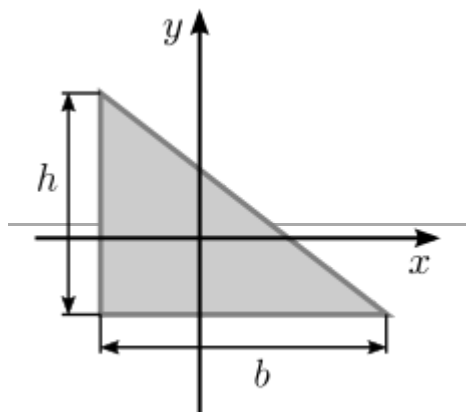


$$I_x = \frac{bh^3}{3}$$

$$I_y = \frac{b^3h}{3} \quad [4]$$

This is a result from the parallel axis theorem

A filled triangular area with a base width of b and height h with respect to an axis through the centroid

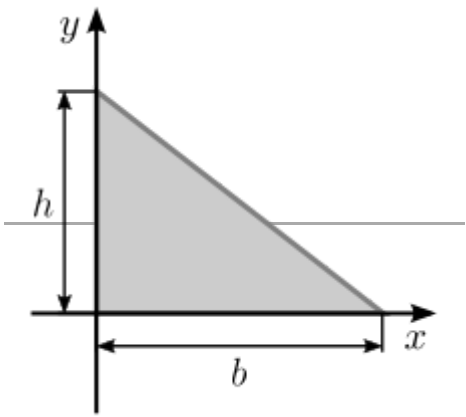
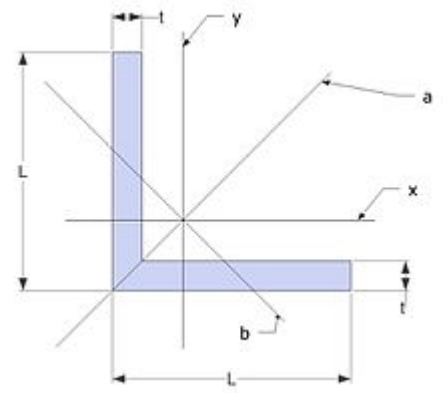
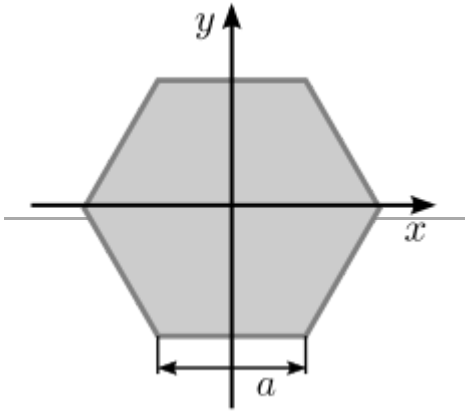


$$I_x = \frac{bh^3}{36}$$

$$I_y = \frac{b^3h}{36} \quad [5]$$

A filled triangular area as above but with

This is a

respect to an axis collinear with the base		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3h}{12} \quad [5]$	consequence of the <u>parallel axis theorem</u>
An equal legged angle, commonly found in engineering applications		$I_x = I_y = \frac{t(5L^2 - 5Lt + t^2)(L^2 - Lt + t^2)}{12(2L - t)}$ $I_{(xy)} = \frac{L^2t(L - t)^2}{4(t - 2L)}$ $I_a = \frac{t(2L - t)(2L^2 - 2Lt + t^2)}{12}$ $I_b = \frac{t(2L^4 - 4L^3t + 8L^2t^2 - 6Lt^3 + t^4)}{12(2L - t)}$	$I_{(xy)}$ is the often unused product of inertia, used to define inertia with a rotated axis
A filled <u>regular hexagon</u> with a side length of a		$I_x = \frac{5\sqrt{3}}{16}a^4$ $I_y = \frac{5\sqrt{3}}{16}a^4$	The result is valid for both a horizontal and a vertical axis through the centroid, and therefore is also valid for an axis with arbitrary direction that passes through the origin.