

# ES120 Spring 2018 – Section 5 Notes

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## Problem 1:

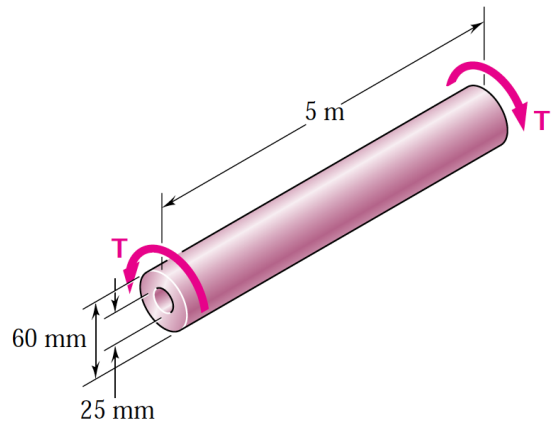


Figure 1

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with  $\tau_Y = 145$  MPa and  $G = 77.2$  GPa. The magnitude  $T$  of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; calculate  $T$ .

### Solution 1

For this problem we know that the inner and outer radii are given by:

$$c_1 = 12.5 \text{ mm} \quad c_2 = 30 \text{ mm} , \quad (1)$$

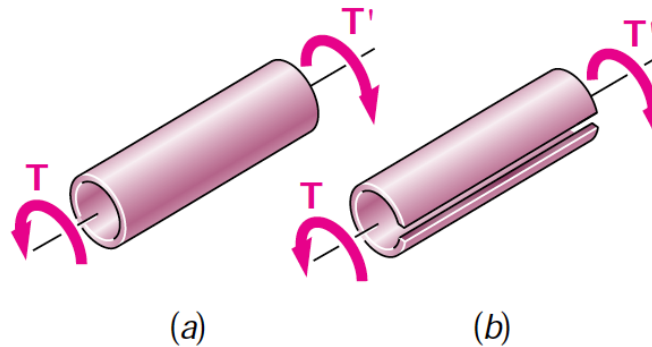
respectively. We know that when the plastic zone reaches the inner surface the stress must be equal to  $\tau_Y$  given that the rest of the cross section still has resistance to the shear. Therefore, that resistance will not allow the entire cross-section to plastically deform throughout the cross-section. Using this and the fact that the incremental change in torque is a result of the increment of change of the shear and the area we can compute the corresponding torque by integration, namely,

$$dT = \rho \tau dA = \rho \tau_Y (2\pi \rho d\rho) = 2\pi \tau_Y \rho^2 d\rho \quad (2)$$

where  $\rho$  is the radius and the equation relates the incremental change in torque as a function of the incre-

mental change in radius. Integrating this equation in  $\rho$  we get:

$$\begin{aligned} T &= 2\pi\tau_Y \int_{c_1}^{c_2} \rho^2 d\rho = \frac{2\pi}{3}\tau_Y(c_2^3 - c_1^3) \\ &= \frac{2\pi}{3}(145 \times 10^6) [(30 \times 10^{-3})^3 - (12.5 \times 10^{-3})^3] = \boxed{7.61 \times 10^3 \text{ N} \cdot \text{m}} \end{aligned} \quad (3)$$

**Problem 2:****Figure 2**

Equal torques are applied to thin-walled tubes of the same length  $L$ , same thickness  $t$ , and same radius  $c$ . One of the tubes has been slit lengthwise as shown. Determine (a) the ratio  $\tau_b/\tau_a$  of the maximum shearing stresses in the tubes, (b) the ratio  $\phi_b/\phi_a$  of the angles of twist of the shafts.

**Solution 2**

For this problem let's separate into separate geometries, namely 'a' for the one without the slit and 'b' for the one with the slit. This is not part a and part b of the problem, but simply a step towards the goals of the problem.

Here we are only calculating the different values for the different geometries. So for the geometry without the slit we can compute the following maximum shear stresses using the fact that we know this to be a thin-walled hollow shaft, which can be read in section 3.13 of the book. We know that the torque applied to a hollow member with constant shear flow is

$$\tau_a = \frac{T}{2t\mathcal{A}} \quad (4)$$

where  $\mathcal{A}$  is the area bounded by the centerline of the wall cross section. Which given that  $t$  is very thin, we can approximate it to be simply  $c$  which gives us  $\mathcal{A} = \pi c^2$ . Therefore,

$$\tau_a = \frac{T}{2\pi c^2 t} \quad (5)$$

$$J \approx 2\pi c^3 t \quad (6)$$

So therefore, we can solve for the angle of twist as

$$\phi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G} \quad (7)$$

Now for the case for the geometry with the slit, we can approximate it as a non-circular member as described in section 3.12 of the book. For this approximation we can compute the length of the wider face  $a$  to be the circumference of the cylinder

$$a = 2\pi c \quad (8)$$

and the length of the lesser wide face to be

$$b = t. \quad (9)$$

We can see that

$$\frac{a}{b} = \frac{2\pi c}{t} \gg 1 \quad (10)$$

which from Table 3.1 of the book indicates that  $c_1 = c_2 = \frac{1}{3}$ . From the equation in section 3.12 we can therefore compute the  $\tau_b$  to be

$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2} \quad (11)$$

Now for the angle of twist we also know from section 3.12 of the book that it is given by:

$$\phi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G} \quad (12)$$

### Part (a)

Now solving for the question on the stress ratio we can plug in our findings to get:

$$\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \boxed{\frac{3c}{t}} \quad (13)$$

### Part (b)

Now solving for the question on the twist ratio we can plug in our findings to get:

$$\frac{\phi_b}{\phi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \boxed{\frac{3c^2}{t^2}} \quad (14)$$

Now let's look at these solutions a little deeper for some intuition. It is cool to see that when we add a slit to the thin shell cylinder we get higher maximum shear stresses than without the slit (assuming  $c > t$ ). Furthermore, not only is our angle of twist also larger for the geometry with the slit but it scales  $\left(\frac{c}{t}\right)^2$  which is much faster than the maximum shear stresses.