ES 120 Formula sheet

Strain and Stress

$\varepsilon = \frac{L-L_0}{L_0} = \frac{\delta}{L_0}$ $\qquad$ $\varepsilon = \frac{d\delta}{dx}$

$\sigma = \frac{P}{A}$ $\qquad$ $\sigma = E\varepsilon$ $\qquad$ $\tau_{ave} = \frac{F}{A}$ $\qquad$ $\tau = G\gamma$

$G = \frac{E}{2(1+\nu)}$ $\qquad$ $B = \frac{E}{3(1-2\nu)}$

Thermal strain $\varepsilon_T = \alpha\Delta\mathrm{T}$

Factor of safety $= \frac{ultimate\ load}{allowable\ load}$

Stress concentration factor: $K = \frac{\sigma_{max}}{\sigma_{ave}}$

Generalized Hooke's Law

$\epsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz})$ $\qquad$ $\gamma_{xy} = \frac{\sigma_{xy}}{G}$

$\epsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx} - \nu\sigma_{zz})$ $\qquad$ $\gamma_{yz} = \frac{\sigma_{yz}}{G}$

$\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy})$ $\qquad$ $\gamma_{zx} = \frac{\sigma_{zx}}{G}$

Uniaxial tension

$\sigma = \frac{P}{A_0}\cos^2\vartheta$ $\qquad$ $\tau = \frac{P}{A_0}\sin\vartheta\cos\vartheta$

$\delta = \frac{PL}{AE}$ $\qquad$ $\delta = \sum_i \frac{P_i L_i}{A_i E_i}$ $\qquad$ $\delta = \int_0^L \frac{P}{AE}dx$

The 15 equations of small-strain elasticity

Equilibrium equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + b_x = \rho \frac{\partial^2 u}{\partial t^2}.$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + b_y = \rho \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = \rho \frac{\partial^2 w}{\partial t^2}$$

Hooke's law

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{zz} + \sigma_{xx}\right)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\sigma_{xy}, \gamma_{yz} = \frac{2(1+\nu)}{E}\sigma_{yz}, \gamma_{zx} = \frac{2(1+\nu)}{E}\sigma_{zx}$$

Kinematic equations

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

Torsion

$$\gamma = \frac{\varphi\rho}{L}$$

$$\varphi = \frac{TL}{GI_p} \qquad \varphi = \sum_i \frac{T_i L_i}{G_i I_{p_i}} \qquad \varphi = \int_0^L \frac{T}{GI_p} dx$$

$$\gamma = \frac{T\rho}{GI_p} \qquad \tau = \frac{T\rho}{I_p}$$

$$P = T\omega = 2\pi f T$$

$$T = \frac{4}{3} T_y \left(1 - \frac{1}{4}\frac{\varphi_y^3}{\varphi^3}\right)$$

$$\tau = \frac{T}{2t\mathcal{A}} \qquad \varphi = \frac{TL}{4\mathcal{A}^2 G}\oint \frac{ds}{t}$$

$$\tau_{max} = \frac{T}{c_1 ab^2} \qquad \varphi = \frac{TL}{c_2 ab^3 G}$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) \text{ for } \frac{a}{b} \geq 5$$

$$I_p = \int_A \rho^2 \, dA$$

Bending

$$\varepsilon_{xx} = -\frac{y}{\rho} \qquad \sigma_{xx} = -\frac{My}{I} \qquad I = \int_A y^2\, dA$$

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \frac{1}{\rho'} = \frac{\nu}{\rho} \qquad \varepsilon_{yy} = \varepsilon_{zz} = \frac{\nu y}{\rho}$$

$$dF_1 = -\frac{E_1 y}{\rho} dA \qquad dF_2 = -n\frac{E_1 y}{\rho} dA$$

$$\frac{1}{2}bx^2 + nA_s x - nA_s d = 0$$

$$\sigma_M = K\frac{Mc}{I}$$

$$M = -b\int_{-c}^{c} y\sigma_{xx}\, dy$$

$$M_Y = \frac{2}{3}bc^2\sigma_Y$$

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right) = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right)$$

$$\sigma_{xx} = \frac{P}{A} - \frac{My}{I}$$

$$\int_A yz\,dA = 0 \qquad \sigma_{xx} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad \tan\varphi = \frac{I_z}{I_y}\tan\vartheta$$

$$\sigma_{xx} = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Analysis of beams

$$\frac{dV}{dx} = -w \qquad \frac{dM}{dx} = V \qquad I = \int_A y^2 dA$$

$$\gamma_D M_D + \gamma_L M_L \le \phi M_U$$

$$\langle x-a \rangle^n = \begin{cases} (x-a)^n & when \;\; x \ge a \\ 0 & when \;\; x < a \end{cases}$$

Transformation of strains and stresses

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Mohr's circle

Principal stresses are the eigenvalues of the stress matrix

Tresca: σ_{max}-σ_{min} = σ_y

von Mises: $(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2\,\sigma_y^2$

Shearing Stress in beams

$$\Delta H = \frac{VQ}{I}\Delta x$$

$$q = \frac{VQ}{I} \qquad Q: first\ moment\ with\ \ neutral\ axis\, Q = \int_{y_1}^{b} y\,dA$$

$$\tau = \frac{VQ}{It}$$

Deflection of beams

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$EI\frac{d^3y}{dx^3} = V(x)$$

$$EI\frac{d^4y}{dx^4} = -w(x)$$

Singularity functions $< x - a >^n$

Buckling

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$

$L_{eff} = 2L$	one fixed end, one free end
$L_{eff} = L$	both ends pinned
$L_{eff} = 0.7L$	one fixed end, one pinned end
$L_{eff} = 0.5L$	both ends fixed

Energy method

$$U = \int_V u\, dV$$

$$u = \int_0^{\varepsilon_1} \sigma_{xx}\, d\varepsilon_{xx} \quad = \frac{\sigma^2}{2E}$$

$$u = \int_0^{\varepsilon_1} \sigma_{xy}\, d\gamma_{xy} = \frac{\tau^2}{2G}$$

$$u = \frac{1}{2}\left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{yy}\varepsilon_{yy} + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{zx}\gamma_{zx}\right)$$

$$U = \int_0^L \frac{P^2}{2AE} dx = \frac{P^2 L}{2AE}$$

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$U = \int_0^L \frac{T^2}{2GI_p} dx$$

$$x_j = \frac{\partial U}{\partial P_j}$$

$$\theta_j = \frac{\partial U}{\partial M_j}$$

$$\phi_j = \frac{\partial U}{\partial T_j}$$