ES120 Spring 2018 – Section 7 Notes

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Problem 1:

For the beam and loading shown, (a) determine the equations of the shear and bending-moment curves, (b) draw the shear and bending-moment diagrams.

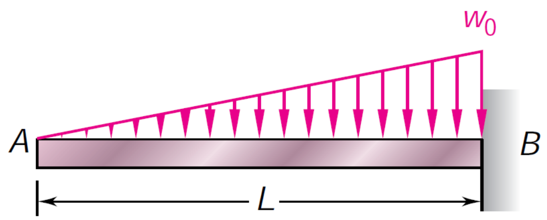

Figure 1

Solution 1

Part (a)

To do this problem we must begin by draying the free body diagram for an arbitrary distance x from edge A to point J.

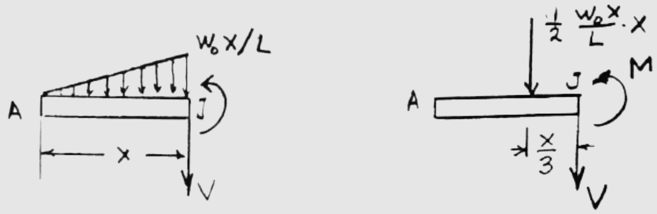

Figure 2

Using this figure, we can compute the force balance in the y direction such that

$$-\frac{1}{2}\frac{w_o x}{L}x - V = 0 \tag{1}$$

$$V = -\frac{w_o x^2}{2L} \tag{2}$$

Now to obtain the equation for the bending moment, we simply integrate $\boldsymbol{V}$ with respect to x, namely:

$$\boldsymbol{M = -\frac{w_o x^3}{6L}} \tag{3}$$

Now, to obtain the maximum value of both, we evaluate the above equations at point B, where $\boldsymbol{x = L}$

$$\boldsymbol{V(L) = -\frac{w_o L}{2}} \tag{4}$$

$$\boldsymbol{M(L) = -\frac{w_o L^2}{6}} \tag{5}$$

Part (b)

Using this information, we can draw the shear and bending moment diagrams as below. Note, that we know the way the function should look like based on the power of x, i.e. $\boldsymbol{V(x) \sim -x^2}$ amd $\boldsymbol{M(x) \sim -x^3}$

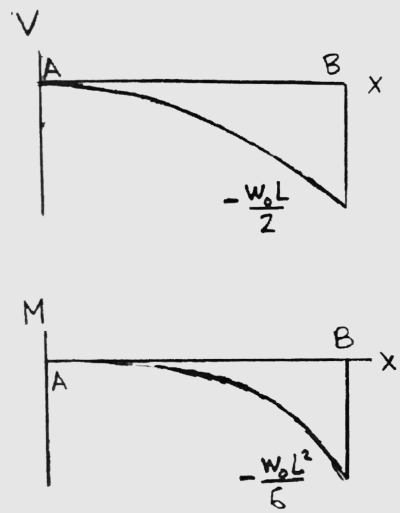

Figure 3

Problem 2:

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

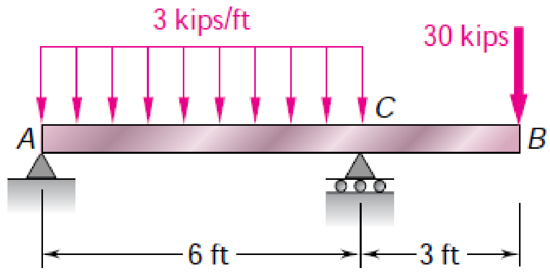

Figure 4

Solution 2

This problem is a bit more complex than the previous one. First, we need to begin finding the unknown reaction forces along the bottom of the beam. To do this, let's use sum of moments about points C and A, respectively (we know that the distributed load can be approximated as a point load at the center between A and C and of the total amount $3 * 6 = 18$ kips):

$$\Sigma M_C = 0 : -6R_A + (3)(18) - (3)(30) = 0 \Rightarrow R_A = -6 \text{ kips (downward)} \tag{6}$$

$$\Sigma M_A = 0 : 6R_C - (3)(18) - (9)(30) = 0 \Rightarrow R_C = 54 \text{ kips (upward)} \tag{7}$$

Using this information, we can separate this problem into two parts, namely A to C and C to B. So let's begin by solely analyzing A to C. This is valid for $- < x < 6$

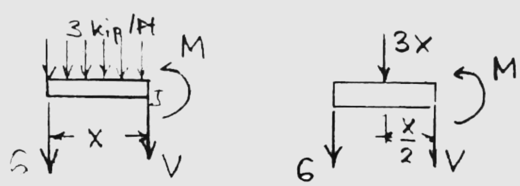

Figure 5

If we do sum of forces in y

$$\Sigma F_y = 0 : -6 - 3x - V = 0 \Rightarrow V = -6 - 3x \text{ kips} \tag{8}$$

So to obtain the bending moment we can take the integral such that we obtain

$$M(x) = -6x - 1.5x^2 \tag{9}$$

Which if we evaluate at pint C which is $x = 6$ we obtain

$$M(6) = -6(6) - 1.5(6)^2 = -90 \tag{10}$$

Now lets analyze C to B

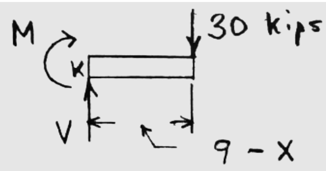

Figure 6

Similar to before, lets do a sum of forces in the y direction

$$\Sigma F_y = 0 : V - 30 = 0 \Rightarrow V = 30 \text{ kips} \tag{11}$$

We can do this integral one of two ways: 1. matching the solution to solve for the integration constant of an indefinite integral or 2. find the bounds we need to use in the integral.
Method 1: Now here we have to be careful when taking the integral as the integration constant does matter, namely we will have

$$M = 30x + k \tag{12}$$

We know that at $x = 6$, M has to match the solution from the previous part (from A to C.) Since we can evaluate to be $M(6) = -90$ then we can solve for the constant k at point C, namely,

$$M = 30(6) + k = -90 \Rightarrow k = -270 \text{ kips} \tag{13}$$

Method 2: We perform the following definite integral

$$M(x) = \int_9^x 30(x)dx \Rightarrow 30x - 30(9) = 30x - 270 \tag{14}$$

So our final expression therefore becomes

$$M(x) = 30x - 270 \tag{15}$$

Using this information we are ready to draw our shear and bending moment diagrams.

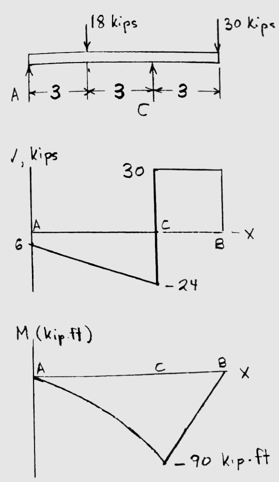

Figure 7

From these graphs, we can decipher that out maximum absolute values for the shear and bending moments are

$$\boxed{|V|_{\text{max}} = 30 \text{ kips}} \tag{16}$$

$$\boxed{|M|_{\text{max}} = 90 \text{ kips} \cdot \text{ft}} \tag{17}$$

Problem 3:

(a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

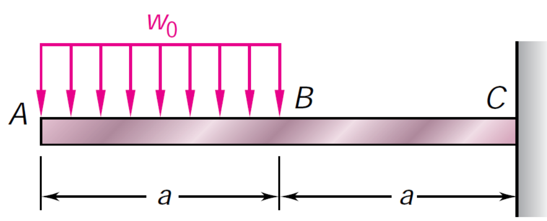

Figure 8

Solution 3

Singularity functions are your friends and are by no means as scary as they sound. They are simply a way to help you write the governing equations in a much simpler way. They simply indicate where a change in the stresses or boundary conditions occur.

Just to formalize the singularity function, let's define what it is:

Singularity Function Definition

$$< x - a >^n = \begin{cases} (x-a)^n & \text{for } x \geq a \\ 0 & \text{for } x < a \end{cases} \tag{18}$$

Let's first begin by drawing the free body diagram of the cantilever beam

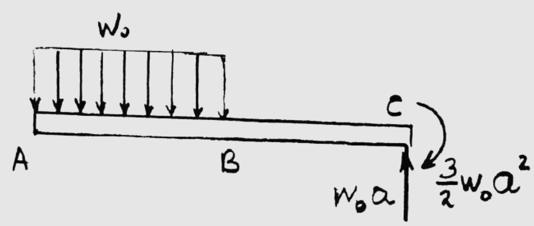

Figure 9

Part (a)

Simply based on the diagram we can write the following for $w(x)$

$$w(x) = w_o - w_o < x - a >^0 \tag{19}$$

Note that $w(x) = -\frac{dV}{dx}$ so we can obtain $V(x)$ as

$$\boxed{V(x) = -w_o x + w_o < x - a >^1} \tag{20}$$

Again note that $V(x) = \frac{dM}{dx}$ so we can obtain $M(x)$ as

$$\boxed{M(x) = -\frac{1}{2}w_o x^2 + \frac{1}{2}w_o < x - a >^2} \tag{21}$$

Part (b)

This asks us to obtain the bending moment at point C. At point C we will have a value of $x = 2a$. So we can evaluate our function $M(x)$ at point $2a$ such that we obtain

$$M(2a) = -\frac{1}{2}w_o(2a)^2 + \frac{1}{2}w_o < (2a) - a >^2 = -\frac{1}{2}w_o(2a)^2 + \frac{1}{2}w_o(a)^2 = \boxed{-\frac{3}{2}w_o a^2} \tag{22}$$

Extra Check

To verify that we did this correctly we can do a sum of moments about point C such that,

$$\Sigma M_c = 0 : \quad \left(\frac{3a}{2}\right)(w_o a) + M_c = 0 \Rightarrow \boxed{M_c = -\frac{3}{2}w_o a^2} \quad \checkmark \tag{23}$$