ENGINEERING SCIENCES 120

INTRODUCTION TO THE MECHANICS OF SOLIDS

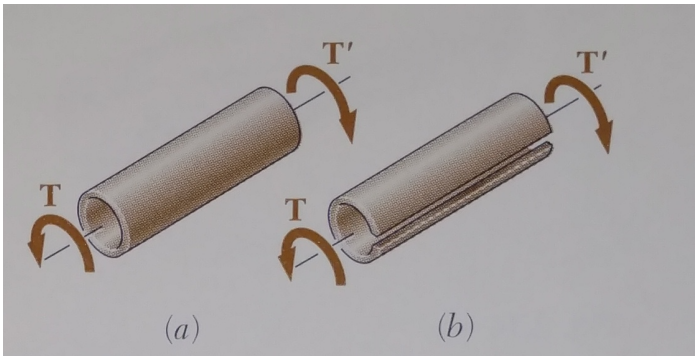
Quiz 1

March 11, 2016

Length: 53 minutes

You are allowed to use a calculator when solving the problems, as well as the equation sheet posted on the web site. Please make sure your answers are clear and legible. No credit will be given if we cannot read an answer or figure out how you derived it! All questions are weighted equally.

1. Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (a) the ratio τ_b/τ_a of the maximum shear stresses in the tubes, (b) the ratio φ_b/φ_a of the angles of twist of the shafts, (c) the radii of solid cylindrical shafts of the same material with the same stiffness as the two thin-walled tubes.



2. Consider the figure below and imagine that *A* represents a diesel engine. Gears *B* and *C* represent a gearbox, and *D* represents the load, in this case a pump. The maximum output of the engine is 40 kW. The characteristics of the engine (*as measured at point D for a fixed fuel supply*) and of several pumps are shown in the graph below.

(a) Given that you want to maximize power to the pump, which pump would you select?

(b) Having selected the pump, determine the corresponding angular velocities at points A and D. What are the internal torques in sections AB and CD of the shaft. What is the total elastic twist angle φ in the system as a result of these torques?

(c) Knowing that the maximum allowable shear stress of the shaft material (mild steel) is 105 MPa, determine the required diameter of shaft *AB* and shaft *CD*. Use metric units in your answer.

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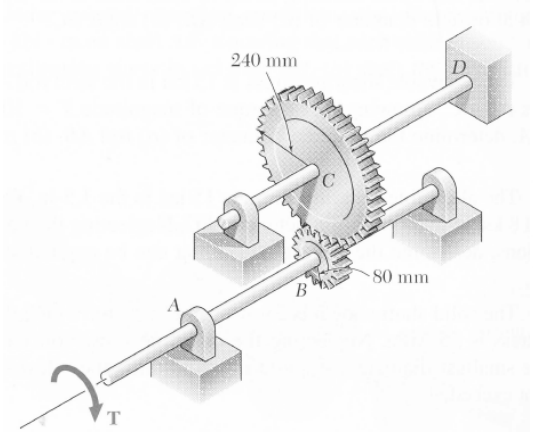
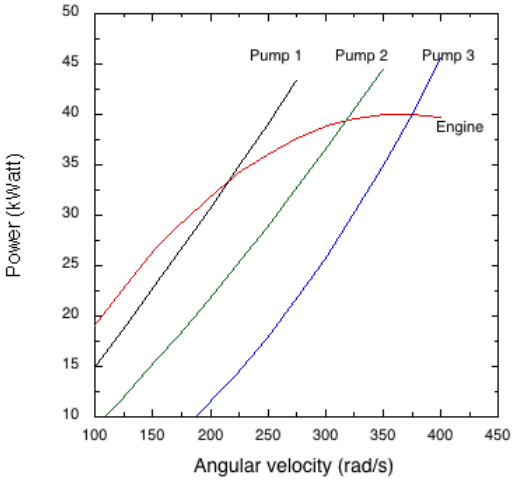




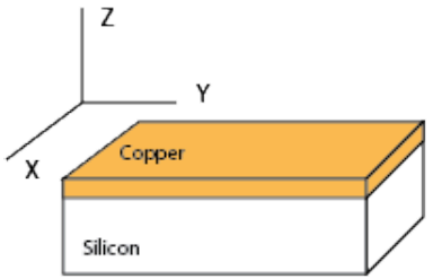
3. At the heart of your iPhone or laptop, there is a small microprocessor. This microprocessor consists essentially of a rectangular piece of silicon coated with many layers of other materials such as copper (to carry the signals) and silicon dioxide (serves as dielectric). Residual stresses in these coatings are a major reliability concern of manufacturers such as Intel or AMD. Let's try to estimate the residual stresses that develop in a thin layer of copper on a silicon substrate.

a. Assume that we have a thick silicon substrate and a very thin layer of copper as indicated in the figure. Both silicon and copper are stress-free at room temperature. When you turn on your iPhone or laptop, the temperature of the microprocessor increases from room temperature to T_o. If the thermal expansion coefficient of silicon is α_{Si} and that of Cu is α_{Cu}, where $\alpha_{Cu} \gg \alpha_{Si}$, do you develop compressive or tensile normal stresses in the copper? Why?

b. Estimate the stress in the copper coating at T_o using the data in the table below. Assume that the copper is isotropic and elastic, and that the same thermal strain develops in all directions in the plane of the coating. Further assume that the stress perpendicular to the coating is zero and that the elastic deformation of the substrate is negligible.

c. What happens if the temperature, T_o, is so large that the stress in the copper in absolute value exceeds the yield stress of the copper? In that case, what is the stress in the copper at T_o? What happens when you turn off your iPhone or laptop and the chip cools down to room temperature? Make a sketch of stress in the copper as a function of temperature.

α_{Cu}	16 x 10^{-6}/K
E_{Cu}	120 GPa
ν_{Cu}	0.35
σ_y	200 MPa
α_{Si}	3 x 10^{-6}/K
Room temperature	20°C
T_o	100°C



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