ES120 Spring 2018 – Midterm 1 Review

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Document Disclaimer

The list provided below is by no means comprehensive and if you find anything missing that you would like to add please let me know. This review session has been created without prior knowledge of the problems in the exam and should not be treated in any way as hints to problems that will be asked in the exam. We will do our best to go over the topics of the course in detail however please do your own reading of chapters 1,2 and 3 as well as other topics not included in the book. If you find any typos please let me know and I will update and push a new version to Github ASAP.

You may also find my notes from a previous year helpful: `http://fer.me/es120notes`

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Topics Covered Summary

1. Introduction – Concepts of Stress
   - Normal Stress – $\sigma = \frac{P}{A}$, where A is perpendicular to direction of force
   - Shearing Stress – $\tau_{ave} = \frac{P}{A}$, where A is parallel to the direction of force
   - Stresses under general loading conditions - Determining the different components of stress from FBD such as σ_{xx}, σ_{yy} and τ_{xy}.
   - Ultimate stress – $\sigma_U = \frac{P_u}{A}$
   - Factor of Safety – F.S.= $\frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$
   - Truss Systems – How to efficiently solve a truss system using method of sections

2. Stress and Strain – Axial Loading
   - Strain – $\epsilon = \frac{\delta}{L}$
   - Elastic Stress-Strain Diagram – Linear Relationship
   - Plastic Stress-Strain Diagram – Ideal plasticity with yield stress σ_Y
   - True Stress and True Strain – Difference between True and Engineering is the cross-sectional area. True stress uses A of deformed specimen.
   - Hooke's Law – $\sigma = E\epsilon$
   - General Stress State – Symmetric positive definite matrix of σ_{ij}
   - Modulus of Elasticity – E
   - Elastic vs. Plastic Behavior of Material – Necking, yield stress, rupture etc.
   - Fatigue – In cases of cyclic loading, rupture will occur at a stress much lower than the static breaking strength; this phenomenon is called fatigue.
   - Deformations of Members Under Axial Loading – $\delta = \frac{PL}{AE}$
   - Statically Indeterminate Problems – Problems that cannot be determined using statics, but where we need to formulate a compatibility constraint. This occurs when we have more reaction forces to solve for than we have equations.
   - Problems involving temperature changes – Thermal strain $\epsilon_T = \alpha \Delta T$. This does not create a stress until it is statically constrained and as per superposition the thermal strain becomes mechanical strain.

- Superposition Method – In statically indeterminate problems we remove redundant loads and apply superposition to solve for the different unknown loads.
- Poisson's Ratio – Relates lateral and axial strains through $\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$
- Multiaxial loading – Loading through multiple axis and the relationship to strain
- Generalized Hooke's Law – Generalized relationship between all stresses, strains and material parameters through equations of form $\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$, $\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_z)$, $\epsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_y - \nu\sigma_x)$
- Plane Strain – $\epsilon_z = 0$
- Plane Stress – $\sigma_z = 0$
- Bulk Modulus – Change of volume per unit volume described by $k = \frac{E}{3(1-2\nu)}$
- Shearing Strain – Nondimensional deformation due to shearing stress γ_{xy}
- Hooke's law for shearing stress and strain – $\tau_{xy} = G\gamma_{xy}$
- Modulus of Rigidity – Empirical value to relates shear stress to shear strain G. Analogous to modulus of elasticity E.
- Relation among E, ν, G – $\frac{E}{2G} = 1 + \nu$
- Stress Concentrations – $K = \frac{\sigma_{max}}{\sigma_{avg}}$
- Plastic Deformation – Elastoplastic material stress strain curve. Gain intuition from this curve.
- Residual Stresses – Stresses left in a part post plastic deformation

3. Torsion
   - Deformation in a Circular shaft – $\gamma = \frac{\rho\phi}{L}$
   - Average shearing strain – $\gamma = \frac{\rho}{c}\gamma_{max}$
   - Torsion Stresses – Shear stresses due to torsion $\tau = \frac{T\rho}{J}$ where J is the polar moment of inertia: `http://fer.me/git/es120notes/blob/master/Section4/J-list.pdf`
   - Torsion Stresses in the Elastic Range – As long as the yield strength is not exceeded in any part of circualr shaft, the shearing stress in that shaft varies linearly with distance ρ from the axis of the shaft such that $\tau = \frac{\rho}{c}\tau_{max}$
   - Angle of Twist in the Elastic Range – $\phi = \frac{TL}{JG}$
   - Statically Indeterminate Shafts – This is analogous to non-torsional statically indeterminate problems, where we need to find a compatibility equation to constrain the different reaction forces we cannot solve for using statics.
   - Design of Transmission Shafts – This simply builds the relationship of what we have learned to power, frequency and torque, namely, $T = \frac{P}{2\pi f}$
   - Stress concentration in circular shafts – $\tau_{max} = K\frac{Tc}{J}$, where $\frac{Tc}{J}$ is the stress computed for the smaller-diameter shaft and K is a tabulated stress-concentration factor obtained from an empirical curve.
   - Plastic Deformation in Circular Shafts – $R_t = \frac{T_u c}{J}$, where R_t is the modulus of rupture, and T_u is the ultimate torque of the shaft
   - Circular shafts made of Elastoplastic Material – $T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\phi_Y^3}{\phi^3}\right)$ is the relationship between the torque and the angle of twist of a elastoplastic shaft. There are also relationships derived in section 3.10 that discuss it in the form of the radius ρ. It is worth reviewing the derivation of this section in detail.
   - Torsion of non-circular members – Using tabulated values we can find an expression for maximum shear $\tau_{max} = \frac{T}{c_1 ab^2}$ and angle of twist $\phi = \frac{TL}{c_2 ab^3 G}$ where a and b are the lengths of the sides and c_1 and c_2 are coefficients determined empirically. Note that for large values of a/b the coefficients become $c_1 = c_2 \approx \frac{1}{3}$
   - Thin-walled hollow shafts – Integrate through an idea of shear flow to obtain a relationship between an equivalent area and the torque such that we can concentrate the shear only on the thin amount of material to obtain $\tau = \frac{T}{2t\mathcal{A}}$, where $\mathcal{A}$ is the area of the region from the centerline of the thin-wall to the center of the shaft
   - Dynamics Torsion Problems – What is the frequency of a rods vibration if twisted and allowed to freely vibrate torsionally. This can be solved to be a wave equation of form $\frac{\partial^2 \rho}{\partial t^2} = c^2 \frac{\partial^2 \rho}{\partial x^2}$. My notes on this matter can be found here: `http://fer.me/l/Nh1grt`