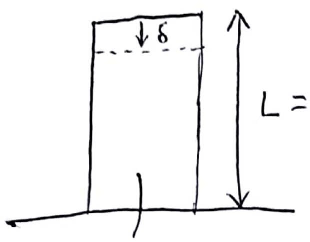
1) Empire State modeled as:




• constant Cross section

a) $\sigma(h)$ due to weight of building, height, $h$

b) $\delta$ due to building weight at top of building

c) Shape of building needed for cst stress w.r.t. $h$?

$E = 30\ GPa$

$\rho = 2400\ kg/m^3$

a) $\sigma(h) = \dfrac{F(h)}{A(h)}$ → force due to gravity (from on top) $= \int g\rho\, dV = \int g\rho\, dz\, dA = g\rho \int dz\, dA$

$= \int_h^L g\rho A\, dz = g\rho A(L-h)$ ($A$ cst w.r.t. height)

$A(h)$ → cst by specification

$= \dfrac{g\rho(L-h)A}{A} \Rightarrow \sigma(h) = g\rho(L-h)$

b) Deflection of one segment in compression, $d\delta = \dfrac{P\,dh}{EA} = \dfrac{\sigma\, dh}{E}$

For building, $L \to h$, $\delta = \int d\delta = \int \dfrac{\sigma(h)}{E} dh = \int_0^L \dfrac{g\rho(L-h)}{E} dh = \dfrac{g\rho}{E}\left[Lh - \tfrac{1}{2}h^2\right]_0^L$

$\delta = \dfrac{g\rho L^2}{2E}$ $\xrightarrow{\#'s}$ $\delta = \dfrac{9.81 \cdot 2400 \cdot (381)^2}{2 \cdot 30 \times 10^9}$ $\left[\dfrac{m/s^2 \cdot kg/m^3 \cdot m^2}{N/m^2}\right] \to [m]$

$= 0.05696\ m$

c) You'd want the cross sectional area decreasing as you go up the building since the force contributed as you go higher must decrease for stresses below it to be constant w.r.t. height. This yields (side view)


For example, consider circular X-section


$r = e^{-h}$ (decreasing radius w/ height)

$\Rightarrow A = \pi e^{-2h}$

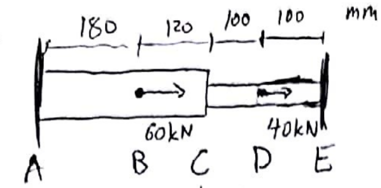
So $F(h) = \int g\rho\, dV = \int_h^L g\rho\pi e^{-2\bar{h}}\, d\bar{h} = \dfrac{g\rho\pi}{2}\left[e^{-2\bar{z}}\right]_L^h$

So $\sigma(h) = \dfrac{\frac{g\rho\pi}{2}\left(e^{-2h} - e^{-2L}\right)}{\pi e^{-2h}} = \dfrac{g\rho}{2}\left(1 - e^{2(h-L)}\right)$

... and observe that as $L \to \infty$, $\sigma(h) \to \dfrac{g\rho}{2}$ (no $h$!)

$e^{2(h-L)} \to 0$

2.




Steel | Brass

$E_S = 210\text{ GPa}$ | $E_B = 120\text{ GPa}$

$\alpha_S = 13\times10^{-6}K^{-1}$ | $\alpha_B = 19\times10^{-6}K^{-1}$

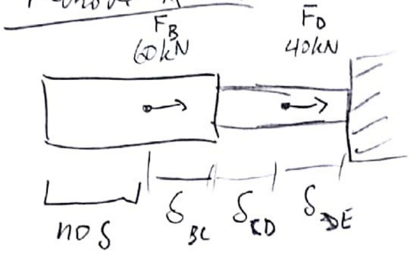
$\varnothing = 40\text{mm}$ | $\varnothing = 30\text{mm}$

a) Reaction @ A, E

b) Deflection @ C

c) $\Delta T = 100K$, what is new $\delta_c$ (assume no plastic flow)

remove A constraint



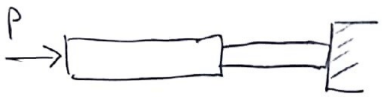

$$\delta_{tot} = \delta_{BC} + \delta_{CD} + \delta_{DE}$$

$L_{CD} = L_{DE}$

$$= \frac{F_B L_{BC}}{E_S A_{AC}} + \frac{F_B L_{CD}}{E_B A_{CE}} + \frac{(F_B+F_D)L_{DE}}{E_B A_{CE}}$$

$$= \frac{(60kN)(120mm)}{(210\text{ GPa})(\pi(20mm)^2)} + \frac{100mm}{(120\text{ GPa})(\pi(15mm)^2)}\cdot(2\cdot60kN + 40kN)$$

$$= 0.2159\text{ mm}$$




$$\delta_P = -\delta_{tot} = -0.2159\text{ mm} = P\left(\frac{L_{AC}}{E_S A_{AC}} + \frac{L_{CE}}{E_B A_{CE}}\right)$$

$$\Rightarrow P = \frac{-0.2159\text{ mm}}{3.495\times10^{-9}\frac{m}{N}} = -61.783\text{ kN} = \text{reaction @ A}$$

$F_A$

$\Rightarrow$ By equilibrium we know (or sectioning between AB)

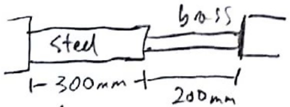
$$\underbrace{F_A}_{P} + \underbrace{F_E}_{unk} + \underbrace{F_B + F_D}_{100kN} = 0$$

$$\Rightarrow F_E = 100kN - 61.783kN$$
$$= 38.216\text{ kN}$$

Deflection @ C $= \delta_c = \delta_{P,AC} + \delta_{F_E,BC} = \frac{-P L_{AC}}{E_S A_{AC}} + \frac{F_B L_{BC}}{E_S A_{AC}} = \frac{1}{E_S A_{AC}}\left(F_B L_{BC} - P L_{AC}\right)$

$$= -0.04295\text{ mm}$$

c) Entire rod system is linearly elastic, so we consider the thermal expansion separately & superpose onto solution from b)




$\phi = 40mm$, $\phi = 30mm$

$E_s = 210\,GPa$, $E_B = 120\,GPa$

Total thermal exp. (free) $\delta_{T,tot} = (L_{AC}\cdot\alpha_S + L_{CE}\cdot\alpha_B)\cdot\Delta T = 0.77mm$

Deflection from constraint $\delta_P = \delta_{T,tot} = \dfrac{-P'L_{AC}}{E_s A_{AC}} - \dfrac{P'L_{CE}}{E_B A_{CE}}$

$= -P'\left(\dfrac{L_{AC}}{E_s A_{AC}} + \dfrac{L_{CE}}{E_B A_{CE}}\right)$

$\dfrac{L_{AC}}{E_s\cdot\pi(20mm)^2}$

$= -P'(2.484\cdot 10^{-9}\ \frac{m}{N})$

$\Rightarrow P' = -309.96\ kN$

So deflection of point c from thermal expansion

$$\delta_{C,th} = \delta_{AC} = \underbrace{\frac{-P'L_{AC}}{E_S A_{AC}}}_{= -0.3523mm} + \underbrace{\Delta T\cdot L_{AC}\,\alpha_S}_{0.39mm}$$

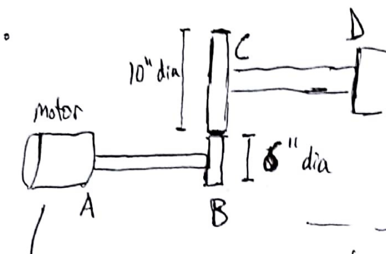
$\Rightarrow \delta_{c,th} = 0.0376\ mm$ (to the right)

$-0.04295\ mm$

Superposing this onto b) yields $\delta_{c,tot} = \delta_{c,mech} + \delta_{c,th}$

$= 0.005353\ mm$ //

3.




operating @ 1260 RPM
15 kW

$\tau_{max,allow} = 50\ MPa$

a) Req. ~~shaft~~ diameter at AB

b) " ——————— " CD

a) We know $P = 2\pi f T$, $f = 1260 \div 60\ s^{-1} = 21\ s^{-1}$

$$\Rightarrow T = \frac{15W \cdot 10^3}{2\pi \cdot 21\ s^{-1}} = 113.68\ Nm$$

For a solid shaft we know $\tau = \frac{T\rho}{I_p}$, where for circ solid shaft $I_p = \frac{1}{4}c^4$

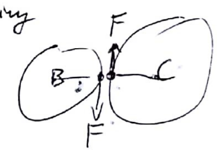
$$\Rightarrow \tau_{max} = \frac{Tc}{\frac{1}{4}c^4} = \frac{4T}{c^3}$$

Since T is cst along shaft, req. diameter = 2c

$$c = \left(\frac{4T}{\tau_{max}}\right)^{1/3} = \left(\frac{4 \cdot 113.68}{50 \cdot 10^6}\ \frac{Nm}{Pa}\right)^{1/3} = 0.02087\ m \text{ or } 20.9\ mm$$

req dia = 41.8 mm

b) Because we have circumferential contact at CB, force balancing gives us $F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \Rightarrow T_{AB} = T_{CD}\frac{r_B}{r_C}$




$T_{AB} = 113.68\ Nm$ from before so $T_{CD} = T_{AB}\frac{r_C}{r_B} = \frac{5}{3} \cdot 113.68$
$= 189.47\ Nm$

If we also assume a solid shaft in CD

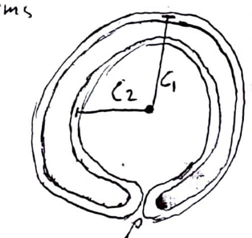
$$\Rightarrow c_{req,CD} = \left(\frac{4T_{CD}}{\tau_{max}}\right)^{1/3} = 0.02475m \text{ or } 24.75\ mm$$

req dia CD = 49.50 mm

# Bonus Q.

Cooling tube of 3mm thick steel sheet. Apply $T = 3\,kNm$ to tube.

forms




negligible opening dimension

$C_1 = 150\,mm$

$C_2 = 100\,mm$

a) Max shear stress in tube.

b) Torque carried by outer circular shell.

a) We look at the area bound by the centerlines of the tube

$$A = \pi (C_1^2 - C_2^2)$$

Shear stress is then given by $\tau = \dfrac{T}{2tA}$

$$= \frac{3\,kNm}{2 \cdot 3\,mm \cdot \pi((150\,mm)^2 - (100\,mm)^2)}$$

$$= \frac{\cancel{6}\, 3\,Nm \cdot 10^3}{6 \cdot \pi \cdot 12500 \cdot 10^{-9}\,m}$$

$$= 12.7\,MPa$$

b) As per analysis from thin walled hollow shafts we know that the shear flow $q = \tau \cdot t_{\text{thickness}}$ is constant along cross section.

We thus know torque will thus be distributed by ratio of the crosssection of inner vs. outer shells. Since thickness is uniform $\Rightarrow T_{outer} \approx \dfrac{2\pi C_1}{2\pi (C_1 + C_2)} T$.

$$\frac{150}{250} \cdot 3\,kNm$$

$$T_{outer} \simeq 1.8\,kNm$$