ES120 Spring 2018 – Section 6 Notes

Matheus Fernandes

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Problem 1:

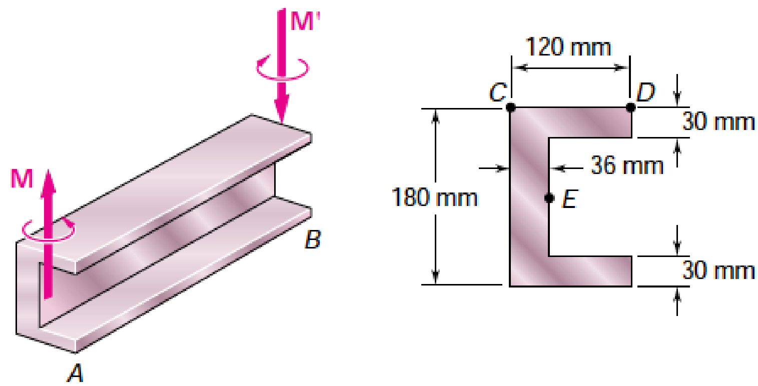

Figure 1

Two equal and opposite couples of magnitude $M = 25$ [kN·m] are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (a) point C, (b) point D, (c) point E.

Solution 1

To do this problem, we need to identify what is the axis of the moment M applied to the beam. Since the axis of both moment arms are in the positive and negative y_o direction (according to the right hand rule), we need to first find what is the centroid of the structure within the plane of the cross section. To do so, we need to divide the cross section into three separate rectangles as shown fig. 2. Using the dimensions provided by the problem statement we can obtain measures for the area A of the different rectangles, and the distance $\hat{x}_o$ in the direction x_o of the center of the rectangle. For these distances, we choose our reference axis to be y_o.

Now we can easily repeat this approach for all of the 3 rectangles in the cross section and formulate the first two columns of the table below. The last column is simply computed by multiplying the first two columns together.

	A, [mm^2]	$\bar{x}_o$, [mm]	$A\bar{x}_o$, [mm^3]
#1	3600	60	216×10^3
#2	4320	18	77.76×10^3
#3	3600	60	216×10^3
Σ	$11,520$		509.76×10^3

Using this table we have all of the information we need to compute the centroid distance $\bar{x}$ from the axis $\bar{y}_o$ using the equation:

$$\begin{aligned} \bar{x}\Sigma A &= \Sigma A\bar{x}_o \\ \bar{x}(11520) &= 509.76 \times 10^3 \\ \bar{x} &= 44.25 \text{ [mm]} \end{aligned} \tag{1}$$

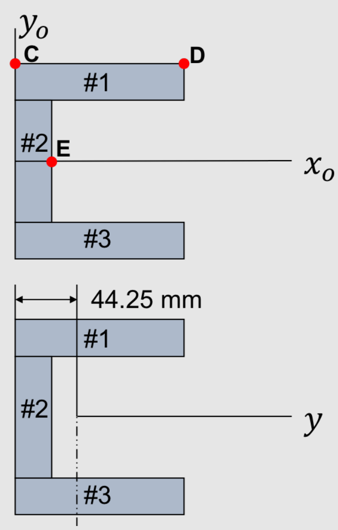

Figure 2: Diagram depicting different sections and centroid of cross-section.

Using the structure's centroid information, we can now compute the distance d from a rectangles center to the centroid of the structure. We will get a different d for each of the rectangles

$$\begin{aligned} d_1 &= 60 - 44.25 = 15.75 \text{ [mm]} \\ d_2 &= 44.25 - 1.8 = 26.26 \text{ [mm]} \\ d_3 &= 60 - 44.25 = 15.75 \text{ [mm]} \end{aligned} \tag{2}$$

We're ready to compute the second moment of inertia for the three different rectangles using the parallel axis theorem. **The parallel axis theorem states** that if the body is made to rotate instead about a new axis which is parallel to the first axis and displaced from it by a distance d, then the moment of inertia I with respect to the new axis is related to I_{cm} (center of mass) by

$$I = I_{cm} + Ad^2 \tag{3}$$

In other words, we must simply compute the moment of inertia of a given rectangle and offset the centroid by the distance between the two axis squared multiplied by the area. Always remember to define b and h according to the axis of rotation, which for this case is y_o. Let's use this information to compute the moment of inertias for this problem

$$I_1 = I_3 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ [mm}^4\text{]} \tag{4}$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(120)(36)^3 + (4320)(26.25)^3 = 3.4433 \times 10^6 \text{ [mm}^4\text{]} \tag{5}$$

All of the work done so far gets us to the final goal of computing the moment of inertia of this slightly complex cross section. So the final step of this journey is to add these offset moment of inertias with respect to the centroid of the cross section together, namely

$$I = I_1 + I_2 + I_3 = 2I_1 + I_2 = 13.8694 \times 10^{-6} \text{ [m}^4\text{]} \tag{6}$$

Now for the different parts of the problem we must compute the distance of the points to our centroid axis y, namely:

$$\begin{aligned} y_1 &= -44.25 \text{ [mm]} \\ y_2 &= 120 - 44.25 = 75.75 \text{ [mm]} \\ y_3 &= 36 - 44.25 = -8.25 \text{ [mm]} \end{aligned} \tag{7}$$

At this point we have all of the information we need to compute the stresses at the different points requested for the different parts of the problem. So let's do this!

0.0.1 Part (a)

For Point C

$$\boxed{\sigma_C = -\frac{My_C}{I} = -\frac{(25 \times 10^3)(-0.04425)}{13.8694 \times 10^{-6}} = 79.8 \times 10^6 \text{ [Pa]}} \tag{8}$$

0.0.2 Part (b)

For Point D

$$\boxed{\sigma_D = -\frac{My_D}{I} = -\frac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6 \text{ [Pa]}} \tag{9}$$

0.0.3 Part (c)

For Point E

$$\boxed{\sigma_E = -\frac{My_E}{I} = -\frac{(25 \times 10^3)(-0.00825)}{13.8694 \times 10^{-6}} = -14.87 \times 10^6 \text{ [Pa]}} \tag{10}$$

Problem 2:

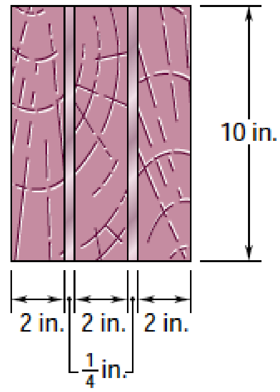

Figure 3

Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	**Steel**
Modulus of elasticity	2×10^6 psi	30×10^6psi
Allowable stress	2000 psi	$22,000$ psi

Solution 2