ES120 Spring 2018 – Section 4 Notes

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Problem 1:

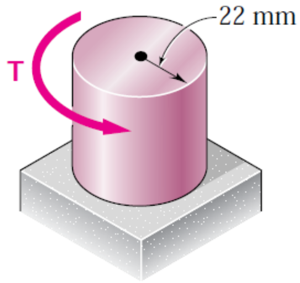


Figure 1

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude T=1.5 kN·m.

Solution 1

In order to determine the maximum shear stress caused by a torque onto the shaft we must convert the physical torque (which is equivalent to a force) to a stress. The relationship between torque and maximum shear stress is given by:

$$\tau_{max} = \frac{Tr}{J}, \tag{1}$$

where J is the polar moment of inertia for a cylinder as described per the document at the end of the notes. We can then obtain the equation for J as:

$$J = \frac{\pi}{2} c^4 \tag{2}$$

such that we can solve for τ_{max} as

$$\boxed{\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi (0.022)^3} = 89.7\text{MPa}} \tag{3}$$

Problem 2:

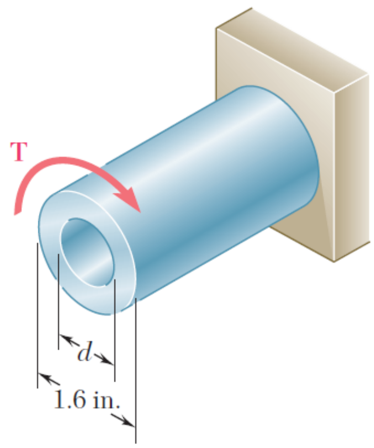


Figure 2

Knowing that the internal diameter of the hollow shaft shown is d=0.9 in., determine the maximum shearing stress caused by a torque of magnitude T=9 kip·in.

Solution 2

Similar to the previous problem we know that the maximum shear stress is dictated by the equation

$$\tau_{max} = \frac{Tr}{J}, \tag{4}$$

where now our radii differ in that for an annulus. So our c's become

$$c_1 = 0.5 * 0.9 = 0.45 \text{ in} \tag{5}$$

$$c_2 = 0.5 * 1.6 = 0.8 \text{ in} \tag{6}$$

Our second moment of an annulus can be calculated by subtracting the moment of the inner from the outer such that:

$$J = \frac{\pi}{2}(c_2^4 - c_2^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4 \tag{7}$$

Applying that in to the equation for τ_{max} we get

$$\boxed{\tau_{max} = \frac{(9)(0.8)}{0.5790} = 12.44 \text{ ksi}} \tag{8}$$

Problem 3:

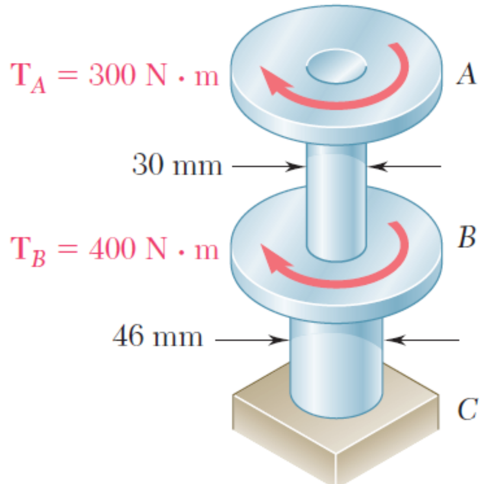


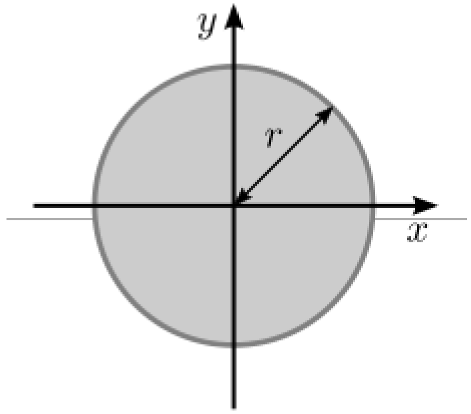
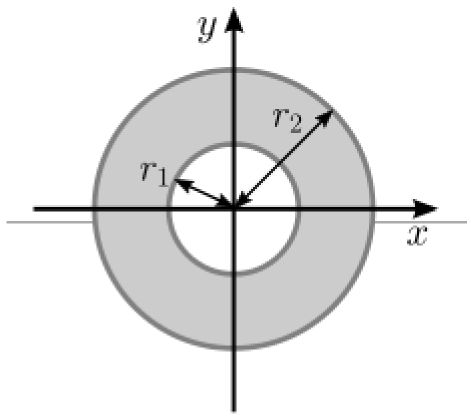
Figure 3

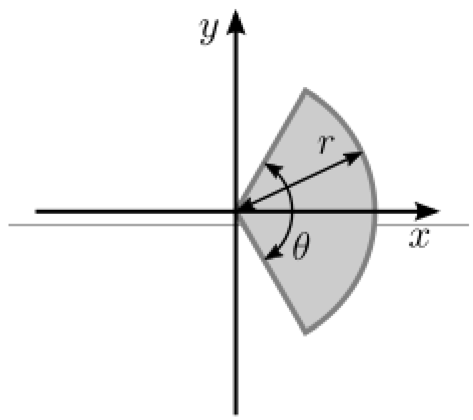
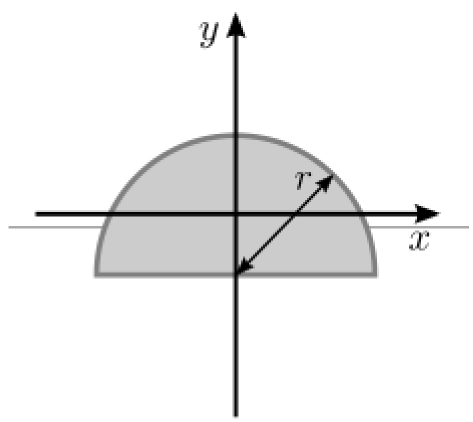
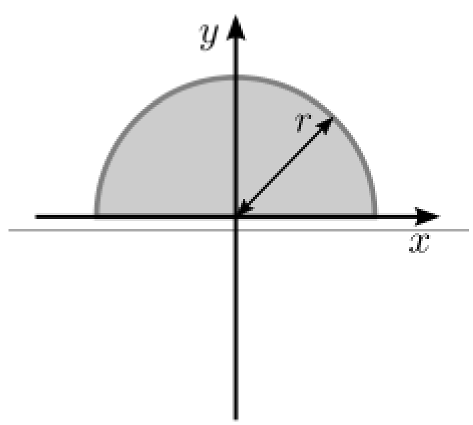
The torques shown are exerted on pulleys A and B. Knowing that each shaft is solid, determine the maximum shearing stress (a) in shaft AB, (b) in shaft BC.

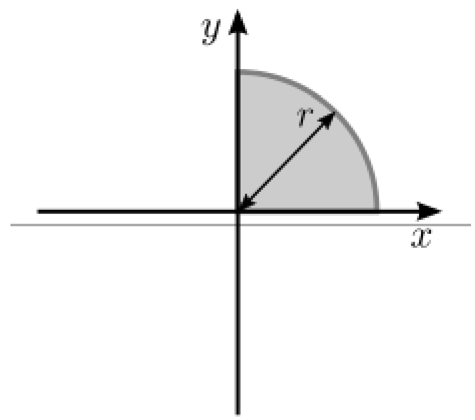
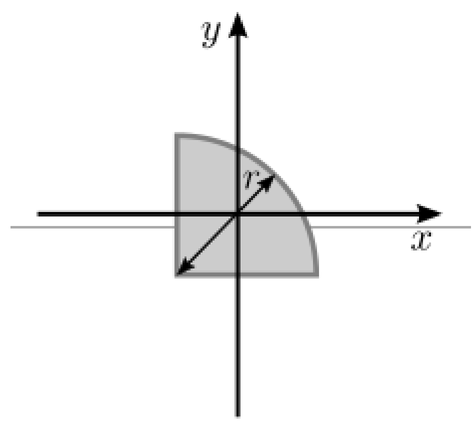
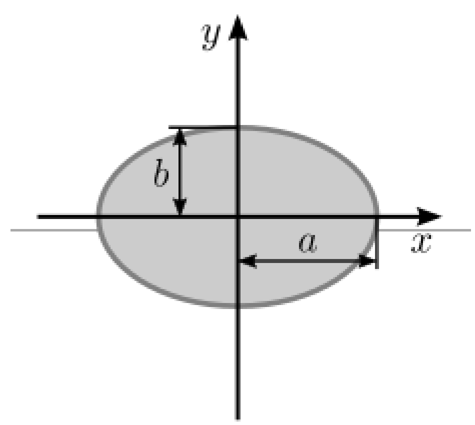
Solution 3

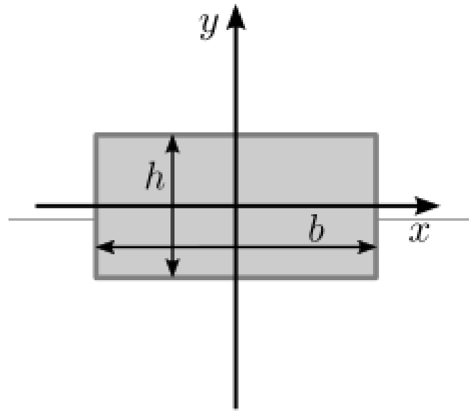
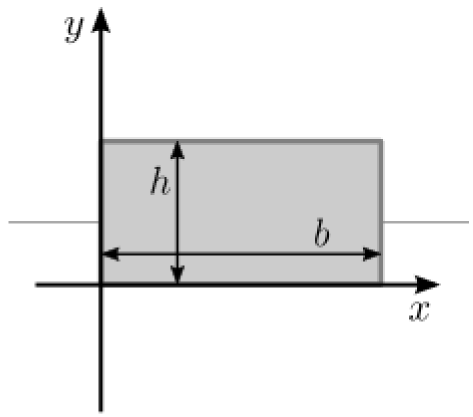
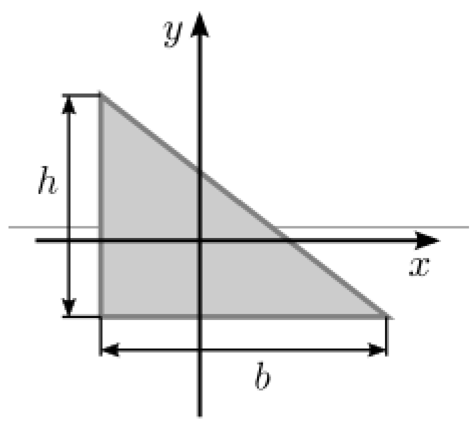
Part A

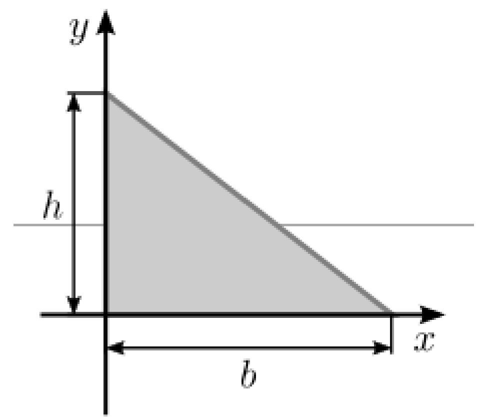
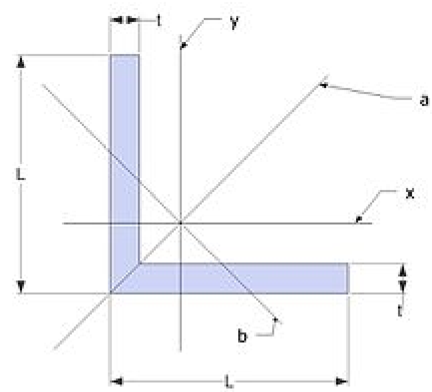
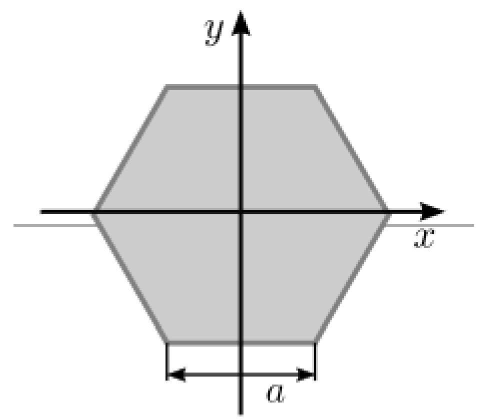
Part B

Description	Figure	Area moment of inertia	Comment
A filled circular area of radius *r*		$I_x = \frac{\pi}{4} r^4$ $I_y = \frac{\pi}{4} r^4$ $I_z = \frac{\pi}{2} r^4$ [1]	I_z is the Polar moment of inertia.
An annulus of inner radius r_1 and outer radius r_2		$I_x = \frac{\pi}{4}\left({r_2}^4 - {r_1}^4\right)$ $I_y = \frac{\pi}{4}\left({r_2}^4 - {r_1}^4\right)$ $I_z = \frac{\pi}{2}\left({r_2}^4 - {r_1}^4\right)$	For thin tubes, $r \equiv r_1 \approx r_2$ and $r_2 \equiv r_1 + t$. So, for a thin tube, $I_x = I_y \approx \pi r^3 t$. I_z is the Polar moment of inertia.
A filled circular sector of angle θ in radians and radius *r* with respect to an axis through the centroid of the sector and the center of the circle		$I_x = (\theta - \sin\theta)\frac{r^4}{8}$	This formula is valid only for $0 \le \theta \le \pi$

			
A filled semicircle with radius *r* with respect to a horizontal line passing through the centroid of the area		$I_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 \approx 0.1098 r^4$ $I_y = \frac{\pi r^4}{8}$ [2]	
A filled semicircle as above but with respect to an axis collinear with the base		$I_x = \frac{\pi r^4}{8}$ $I_y = \frac{\pi r^4}{8}$ [2]	I_x: This is a consequence of the parallel axis theorem and the fact that the distance between the x axes of the previous one and this one is $\frac{4r}{3\pi}$
A filled quarter circle with radius *r* with the			

axes passing through the bases		$I_x = \dfrac{\pi r^4}{16}$ $I_y = \dfrac{\pi r^4}{16}$ [3]	
A filled quarter circle with radius r with the axes passing through the centroid		$I_x = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4 \approx 0.0549 r^4$ $I_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4 \approx 0.0549 r^4$ [3]	This is a consequence of the parallel axis theorem and the fact that the distance between these two axes is $\dfrac{4r}{3\pi}$
A filled ellipse whose radius along the x-axis is a and whose radius along the y-axis is b		$I_x = \dfrac{\pi}{4} a b^3$ $I_y = \dfrac{\pi}{4} a^3 b$	
A filled rectangular area with a base width of			

b and height *h*		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3h}{12}$ [4]	
A filled rectangular area as above but with respect to an axis collinear with the base		$I_x = \frac{bh^3}{3}$ $I_y = \frac{b^3h}{3}$ [4]	This is a result from the parallel axis theorem
A filled triangular area with a base width of *b* and height *h* with respect to an axis through the centroid		$I_x = \frac{bh^3}{36}$ $I_y = \frac{b^3h}{36}$ [5]	
A filled triangular area as above but with			This is a

respect to an axis collinear with the base		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3h}{12}$ [5]	consequence of the parallel axis theorem
An equal legged angle, commonly found in engineering applications		$I_x = I_y = \frac{t(5L^2 - 5Lt + t^2)(L^2 - Lt + t^2)}{12(2L - t)}$ $I_{(xy)} = \frac{L^2t(L-t)^2}{4(t-2L)}$ $I_a = \frac{t(2L-t)(2L^2 - 2Lt + t^2)}{12}$ $I_b = \frac{t(2L^4 - 4L^3t + 8L^2t^2 - 6Lt^3 + t^4)}{12(2L-t)}$	$I_{(xy)}$ is the often unused product of inertia, used to define inertia with a rotated axis
A filled regular hexagon with a side length of *a*		$I_x = \frac{5\sqrt{3}}{16}a^4$ $I_y = \frac{5\sqrt{3}}{16}a^4$	The result is valid for both a horizontal and a vertical axis through the centroid, and therefore is also valid for an axis with arbitrary direction that passes through the origin.