ES120 Spring 2018 – Section 5 Notes

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Problem 1:

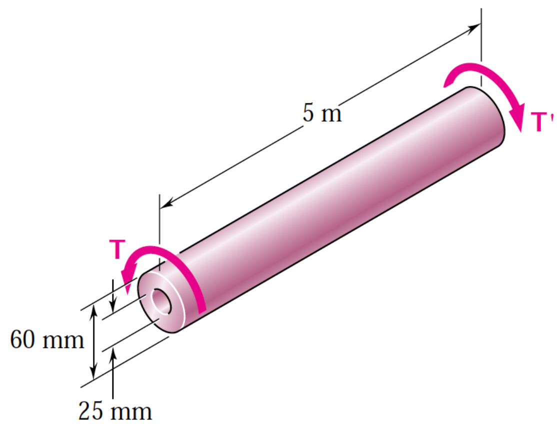

Figure 1

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed.

Solution 1

For this problem we know that the inner and outer radii are given by:

$$c_1 = 12.5 \text{ mm} \qquad c_2 = 30 \text{ mm} , \tag{1}$$

respectively. We know that when the plastic zone reaches the inner surface the stress must be equal to τ_Y given that the rest of the cross section still has resistance to the shear. Therefore, that resistance will not allow the entire corss-section to plastically deform throughout the cross-section. Using this and the fact that the incremental change in torque is a result of the increment of change of the shear and the area we can compute the corresponding torque by integration, namely,

$$dT = \rho\tau dA = \rho\tau_Y(2\pi\rho d\rho) = 2\pi\tau_Y\rho^2 d\rho \tag{2}$$

where ρ is the radius and the equation relates the incremental change in torque as a function of the incremental change in radius. Integrating this equation in ρ we get:

$$
\begin{aligned}
T &= 2\pi\tau_Y \int_{c_1}^{c_2} \rho^2 d\rho = \frac{2\pi}{3}\tau_Y (c_2^3 - c_1^3) \\
&= \frac{2\pi}{3}(145 \times 10^6)\left[(30 \times 10^{-3})^3 - (12.5 \times 10^{-3})^3\right] = \boxed{7.61 \times 10^3 \ \mathrm{N \cdot m}}
\end{aligned} \tag{3}
$$

Problem 2:

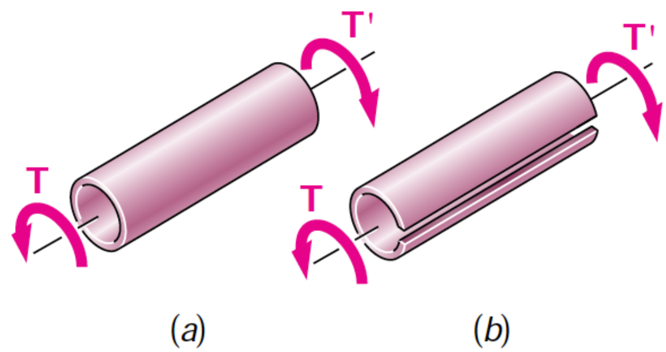

Figure 2

Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (a) the ratio τ_b/τ_a of the maximum shearing stresses in the tubes, (b) the ratio ϕ_b/ϕ_a of the angles of twist of the shafts.

Solution 2

Part (a)

Part (b)