

ES120 Spring 2018 – Section 9 Notes

Matheus Fernandes

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Problem 1:

Determine the dimension d so that the aluminum and steel struts will have the same weight, and compute the critical load for each strut.

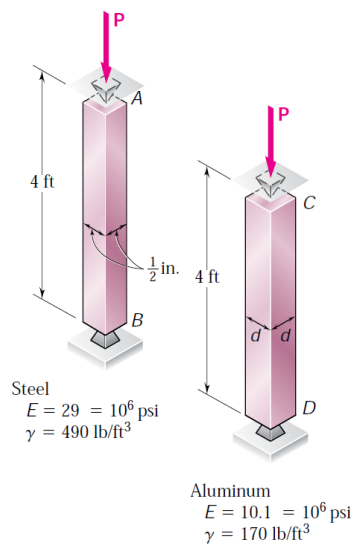


Figure 1

Solution 1

For this problem we must first obtain the total weight of the different beams, namely

$$W = \gamma L d_s^2, \quad (1)$$

for the steel the weight is

$$W_s = (0.2835)(4 * 12)(0.5)^2 = 3.4028 \text{ lb} \quad (2)$$

for the aluminum the weight as a function of the dimension d is

$$W_a = (0.09838)(4 * 12)d^2 = 4.7222d^2 \quad (3)$$

So we want them to be the same weight, thus,

$$W_s = W_a \Rightarrow 3.4028 = 4.7222d^2 \Rightarrow \boxed{d = 0.849 \text{ in}} \quad (4)$$

Now to compute the critical load for each strut we need to solve for the P_{cr} given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (5)$$

For the steel strut, we have that

$$I = \frac{1}{12} d_s^4 = \frac{1}{12} \left(\frac{1}{2} \right)^4 = 5.208 \times 10^{-3} \text{ in}^4 \quad (6)$$

$$P_{cr} = \frac{\pi^2 (29 \times 10^6) (5.2083 \times 10^{-3})}{(4 * 12)^2} = \boxed{647 \text{ lb}} \quad (7)$$

For the aluminum strut, we have that

$$I = \frac{1}{12} d_s^4 = \frac{1}{12} (0.849)^4 = 43.271 \times 10^{-3} \text{ in}^4 \quad (8)$$

$$P_{cr} = \frac{\pi^2 (10.1 \times 10^6) (43.271 \times 10^{-3})}{(4 * 12)^2} = \boxed{1872 \text{ lb}} \quad (9)$$

Problem 2:

Knowing that a factor of safety of 2.6 is required, determine the largest load P that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

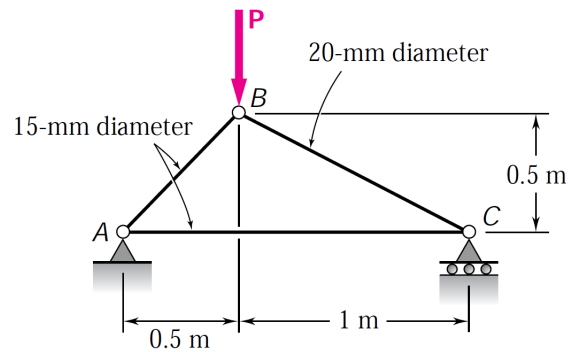


Figure 2

Solution 2

For this problem we must first find the length of the individual members

$$L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180 \text{ m} \quad (10)$$

$$L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711 \text{ m} \quad (11)$$

We can also compute the second moment of inertias as:

$$I_{BC} = \frac{\pi}{64} (20)^4 = 7.854 \times 10^{-9} \text{ m}^4 \quad (12)$$

$$I_{AB} = \frac{\pi}{64} (15)^4 = 2.485 \times 10^{-9} \text{ m}^4 \quad (13)$$

We can now compute what the critical load for both members BC and AB using

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (14)$$

$$P_{cr,BC} = \frac{\pi^2 (200 \times 10^9) (7.854 \times 10^{-9})}{(1.1180)^2} = 12.403 \text{ kN} \quad (15)$$

$$P_{cr,AB} = \frac{\pi^2 (200 \times 10^9) (2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \text{ kN} \quad (16)$$

Given our factor of safety FS we can compute the allowable force recalling that

$$F_{all} = \frac{P_{cr}}{FS} \quad (17)$$

So for the different members that becomes

$$F_{all,BC} = \frac{12.403}{2.6} = 4.770 \text{ kN} \quad (18)$$

$$F_{all,AB} = \frac{9.8106}{2.6} = 3.773 \text{ kN} \quad (19)$$

Now we can obtain the free body diagram for point B as

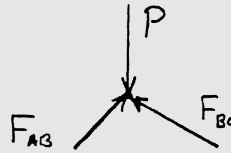


Figure 3

Performing force balance in the horizontal and vertical directions respectively, we obtain

$$\Sigma F_x = 0 : \quad \frac{0.5}{0.70711} F_{AB} - \frac{1}{1.1180} F_{BC} = 0 \Rightarrow F_{BC} = 0.7905 F_{AB} \quad (20)$$

$$\Sigma F_y = 0 : \quad \frac{0.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} - P = 0 \Rightarrow P = 1.06066 F_{AB} \quad (21)$$

Which combining both we can also obtain a relationship between P and F_{BC} , namely

$$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC} \quad (22)$$

Therefore, solving for the allowable value for P yields,

$$P < 1.06066 F_{all, AB} = (1.06066)(3.773) = 4.0 \text{ kN} \quad (23)$$

$$P < 1.3416 F_{all, BC} = (1.3416)(4.770) = 6.4 \text{ kN} \quad (24)$$

The smallest of the the two gives us the largest load namely

$$\boxed{P_{all} = 4.0 \text{ kN}} \quad (25)$$

Problem 3:

Using the method of work and energy, determine the deflection at point D caused by the load P .

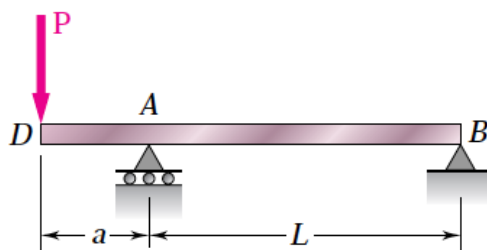
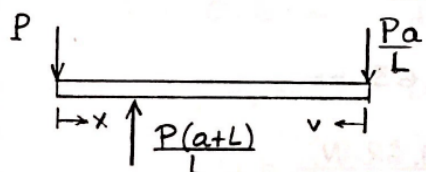
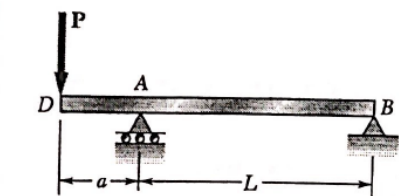


Figure 4

Solution 3**Problem 11.59**

11.58 and 11.59 Using the method of work and energy, determine the deflection at point D caused by the load P .



$$\sum M_A = 0; \quad Pa + R_B L = 0 \quad R_B = -\frac{Pa}{L}$$

Over portion DA: $M = -Px$

$$U_{DA} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over portion AB: $M = -\frac{Pa v}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^L v^2 dv = \frac{Pa^2 L}{6EI}$$

Total: $U = U_{DA} + U_{AB} = \frac{P^2 a^2 (a+L)}{6EI}$

$$\frac{1}{2} P S_D = U$$

$$S_D = \frac{2U}{P}$$

$$S_D = \frac{Pa^2(a+L)}{3EI} \downarrow$$

Figure 5

Problem 4:

For the uniform rod and loading shown and using Castigliano's theorem, determine the deflection of point B .

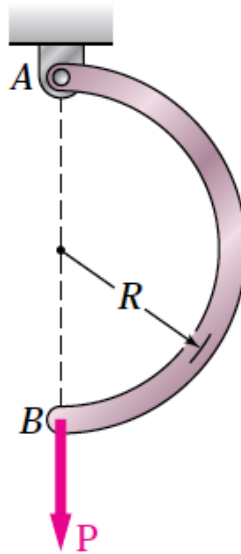
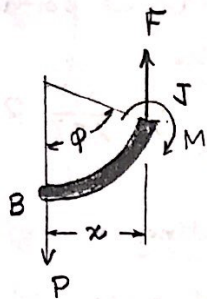
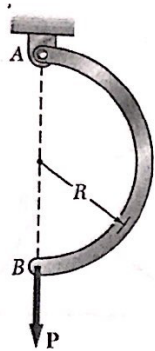


Figure 6

Solution 4

Use polar coordinate φ .

Calculate the bending moment $M(\varphi)$ using free body B.J.

$$+\circlearrowleft \sum M_J = 0: Px - M = 0$$

$$M = Px = PR \sin \varphi$$

Strain energy: $U = \int_0^L \frac{M^2}{2EI} ds$

$$U = \int_0^\pi \frac{(PR \sin \varphi)^2}{2EI} (R d\varphi) = \frac{P^2 R^3}{2EI} \int_0^\pi \sin^2 \varphi d\varphi$$

$$= \frac{P^2 R^3}{2EI} \int_0^\pi \frac{1 - \cos 2\varphi}{2} d\varphi$$

$$= \frac{P^2 R^3}{2EI} \left(\frac{1}{2} \varphi \Big|_0^\pi - \frac{1}{4} \sin 2\varphi \Big|_0^\pi \right) = \frac{\pi P^2 R^3}{4EI}$$

By Castigliano's theorem,

$$\delta = \frac{\partial U}{\partial P}$$

$$\delta = \frac{\pi P R^3}{2EI} \downarrow$$

Figure 7