ES120 Spring 2018 – Final Exam Review w/ Solutions

Matheus Fernandes

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Document Disclaimer

The list provided below is by no means comprehensive and if you find anything missing that you would like to add please let me know. This review session has been created without prior knowledge of the problems in the exam and should not be treated in any way as hints to problems that will be asked in the exam. We will do our best to go over the topics of the course in detail however please do your own reading of chapters 1 through 11 (with the exception of chapter 8) as well as other topics not included in the book. If you find any typos please let me know and I will update and push a new version to Github.

Relevant Reading Material from Textbook

- 6.1-6.4
- 6.6
- 6.7
- 7.1-7.4
- 7.5 (limited to what we covered in class)
- 7.7-7.8 (limited to what we covered in class)
- 9.1-9.8
- 10.3-10.4
- 11.1-11.5
- 11.6 (limited to what we covered in class)
- 11.9-11.14

Topics Covered Summary

Ch. 1 **Introduction – Concepts of Stress**

- **Normal Stress** – $\sigma = \frac{P}{A}$, where A is perpendicular to direction of force
- **Shearing Stress** – $\tau_{ave} = \frac{P}{A}$, where A is parallel to the direction of force
- **Stresses under general loading conditions** - Determining the different components of stress from FBD such as σ_{xx}, σ_{yy} and τ_{xy}.
- **Ultimate stress** – $\sigma_U = \frac{P_u}{A}$
- **Factor of Safety** – F.S.$= \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$
- **Truss Systems** – How to efficiently solve a truss system using method of sections

Ch. 2 **Stress and Strain – Axial Loading**

- **Strain** – $\epsilon = \frac{\delta}{L}$
- **Elastic Stress-Strain Diagram** – Linear Relationship
- **Plastic Stress-Strain Diagram** – Ideal plasticity with yield stress σ_Y
- **True Stress and True Strain** – Difference between True and Engineering is the cross-sectional area. True stress uses A of deformed specimen.
- **Hooke's Law** – $\sigma = E\epsilon$
- **General Stress State** – Symmetric positive definite matrix of σ_{ij}
- **Modulus of Elasticity** – E
- **Elastic vs. Plastic Behavior of Material** – Necking, yield stress, rupture etc.
- **Fatigue** – In cases of cyclic loading, rupture will occur at a stress much lower than the static breaking strength; this phenomenon is called fatigue.
- **Deformations of Members Under Axial Loading** – $\delta = \frac{PL}{AE}$

- **Statically Indeterminate Problems** – Problems that cannot be determined using statics, but where we need to formulate a compatibility constraint. This occurs when we have more reaction forces to solve for than we have equations.
- **Problems involving temperature changes** – Thermal strain $\epsilon_T = \alpha \Delta T$. This does not create a stress until it is statically constrained and as per superposition the thermal strain becomes mechanical strain.
- **Superposition Method** – In statically indeterminate problems we remove redundant loads and apply superposition to solve for the different unknown loads.
- **Thermal Stress in a Film on a Substrate** – This is the problem he worked out in class. My notes can be found here `http://fer.me/l/pbFeFA`
- **Poisson's Ratio** – Relates lateral and axial strains through $\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$
- **Multiaxial loading** – Loading through multiple axis and the relationship to strain
- **Generalized Hooke's Law** – Generalized relationship between all stresses, strains and material parameters through equations of form $\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$, $\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_z)$, $\epsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_y - \nu\sigma_x)$
- **Plane Strain** – $\epsilon_z = 0$
- **Plane Stress** – $\sigma_z = 0$
- **Bulk Modulus** – Change of volume per unit volume described by $k = \frac{E}{3(1-2\nu)}$
- **Shearing Strain** – Nondimensional deformation due to shearing stress γ_{xy}
- **Hooke's law for shearing stress and strain** – $\tau_{xy} = G\gamma_{xy}$
- **Modulus of Rigidity** – Empirical value to relates shear stress to shear strain G. Analogous to modulus of elasticity E.
- **Relation among E, ν, G** – $\frac{E}{2G} = 1 + \nu$
- **Stress Concentrations** – $K = \frac{\sigma_{max}}{\sigma_{avg}}$
- **Plastic Deformation** – Elastoplastic material stress strain curve. Gain intuition from this curve.
- **Residual Stresses** – Stresses left in a part post plastic deformation

Ch. 3 Torsion

- **Deformation in a Circular shaft** – $\gamma = \frac{\rho\phi}{L}$
- **Average shearing strain** – $\gamma = \frac{\rho}{c}\gamma_{max}$
- **Torsion Stresses** – Shear stresses due to torsion $\tau = \frac{T\rho}{J}$ where J is the polar moment of inertia: `http://fer.me/git/es120notes/blob/master/Section4/J-list.pdf`
- **Torsion Stresses in the Elastic Range** – As long as the yield strength is not exceeded in any part of circualr shaft, the shearing stress in that shaft varies linearly with distance ρ from the axis of the shaft such that $\tau = \frac{\rho}{c}\tau_{max}$
- **Angle of Twist in the Elastic Range** – $\phi = \frac{TL}{JG}$
- **Statically Indeterminate Shafts** – This is analogous to non-torsional statically indeterminate problems, where we need to find a compatibility equation to constrain the different reaction forces we cannot solve for using statics.
- **Design of Transmission Shafts** – This simply builds the relationship of what we have learned to power, frequency and torque, namely, $T = \frac{P}{2\pi f}$
- **Stress concentration in circular shafts** – $\tau_{max} = K\frac{Tc}{J}$, where $\frac{Tc}{J}$ is the stress computed for the smaller-diameter shaft and K is a tabulated stress-concentration factor obtained from an empirical curve.
- **Plastic Deformation in Circular Shafts** – $R_t = \frac{T_u c}{J}$, where R_t is the modulus of rupture, and T_u is the ultimate torque of the shaft
- **Circular shafts made of Elastoplastic Material** – $T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\phi_Y^3}{\phi^3}\right)$ is the relationship between the torque and the angle of twist of a elastoplastic shaft. There are also relationships derived in section 3.10 that discuss it in the form of the radius ρ. It is worth reviewing the derivation of this section in detail.
- **Torsion of non-circular members** – Using tabulated values we can find an expression for maximum shear $\tau_{max} = \frac{T}{c_1 ab^2}$ and angle of twist $\phi = \frac{TL}{c_2 ab^3 G}$ where a and b are the lengths of the sides and c_1 and c_2 are coefficients determined empirically. Note that for large values of a/b the coefficients become $c_1 = c_2 \approx \frac{1}{3}$
- **Thin-walled hollow shafts** – Integrate through an idea of shear flow to obtain a relationship between an equivalent area and the torque such that we can concentrate the shear only on the thin amount of material to obtain $\tau = \frac{T}{2t\mathcal{a}}$, where $\mathcal{a}$ is the area of the region from the centerline of the thin-wall to the center of the shaft
- **Dynamics Torsion Problems** – What is the frequency of a rods vibration if twisted and allowed to freely

vibrate torsionally. This can be solved to be a wave equation of form $\frac{\partial^2 \rho}{\partial t^2} = c^2 \frac{\partial^2 \rho}{\partial x^2}$. My notes on this matter can be found here: `http://fer.me/l/Nh1grt`

Ch. 4 **Pure bending**

- **Geometry** – any cross section perpendicular to the axis of the member remains plane and remains perpendicular to the centroid line.
- **Normal strain and normal stress** – normal strain $\epsilon_{xx} = -\frac{y}{\rho}$ and normal stress $\sigma_{xx} = -\frac{Ey}{\rho}$, where ρ is the radius of the beam. We also have $\epsilon_{xx} = -\frac{y}{c}\epsilon_m$ and $\sigma_{xx} = -\frac{y}{c}\sigma_m$, where ϵ_m and σ_m are maximum strain and stress in the beam.
- **Force** & **Position centroid line** – the force in the beam can be calculated by integrating normal stress over the cross section: $F = \int \sigma_{xx}\ \mathrm{d}A$, if the material is within elastic range, $= -\frac{E}{\rho}\int y\ \mathrm{d}A$, where $Q = \int y\ \mathrm{d}A$ is called first moment (see Appendix A.2 in text book). Force in beam should be zero, that means $Q = 0$. Solving $Q = 0$ gives the position of centroid line.
- **Moment** – moment in the beam is calculated by $M = -\int y\sigma_{xx}\ \mathrm{d}A = \frac{E}{\rho}\int y^2\ \mathrm{d}A$, where $I = \int y^2\ \mathrm{d}A$ is called the second moment (see Appendix A.3 for details). Now we have $M = \frac{EI}{\rho}$, and $\frac{1}{\rho} = \frac{M}{EI}$ (curvature - moment relation).
- **Connect stress with moment** – $\sigma_{xx} = -\frac{My}{I}$ and $\sigma_m = \frac{Mc}{I}$. Introducing elastic section modulus $S = I/c$, so that we have $\sigma_m = \frac{M}{S}$.
- **Composite beams** – two materials with young's modulus E_1 and E_2, let $n = E_2/E_1$, the resistance to bending of the bar would remain the same if both portions were made of the first material E_1, provided that the **width** of each element of the lower portion were multiplied by the factor n. To obtain the **stress** σ_2 for material 2, we must multiply by n (see more on textbook page 230).
- **Reinforced concrete beams** – (1) replace the total cross-sectional area of the steel bars A_s by an equivalent area nA_s; (2) only the portion of the cross section in compression should be used in the transformed section (see textbook 233).
- **Eccentric axial loading** – $\sigma_{xx} = (\sigma_{xx})_{centric} + (\sigma_{xx})_{bending} = \frac{P}{A} - \frac{My}{I}$

Ch. 5 **Analysis and Design of Beams for Bending**

- **Shear and Bending moments diagrams** – Drawing shear forces through a uniform beam. Remember which direction is positive and which direction is negative. Note that the bending moment is the integral of the shear diagram, see Figure 5.7 of textbook.
- **Relations among load shear and bending moment** – The overall relationship of all of these are integrals. Specifically they are related through these equations: $-w = \frac{dV}{dx}, V = \frac{dM}{dx}$. In other words, they are the area under the curve of each other.
- **Design of Prismatic Beams for Bending** – How to design beam cross-sections such that you achieve the most economical design possible to efficiently withstand specific load conditions. This is a procedure detailed on page 333 of textbook. The main idea of the procedure is to use bending moment diagrams in conjunction with the fact that the maximum stress occurs at the edge of the section to come up with a cross-section height that is appropriate for the loading conditions.
- **Using Singularity Functions for Shear and Bending Moments** – Singularity functions are defined as $< x - a >^n = \begin{cases} (x-a)^n & \text{for } n \geq a \\ 0 & \text{for } n < a \end{cases}$ They are useful to organize the discontinuities in the distribution of shear forces, bending moments and weights through the beam. It simplifies by not having to account for each part of the beam in separate equations, but instead condenses all information into one equation. This is also useful when performing the integrals from weight to shear forces to bending moments
- **Nonprismatic beams** – Unlike primsmatic beams, we now relax the assumption that the beam cross-section is constant. Now the goal of this is to have a beam of constant strength for a particular loading condition and allow the cross section of the beam to be controlled via the loading conditions. This is very similar to prismatic beam design only now $S = \frac{|M|}{\sigma_{\text{all}}}$. This is covered in more detail in an example problem below.

Ch. 6 **Shearing Stresses in Beams and Thin-Walled Members**

- **Horizontal shear per unit length** – $q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$ This expression is obtained by considering horizontal equilibrium of a material element (with shearing forces ΔH, and net force across the two faces, $\int \sigma_{xx} dA$)

- **Shearing Stresses in a beam** – $\tau_{avg} = \frac{\Delta H}{\Delta A} = \frac{VQ}{It}$, where t is the thickness of the area over which the shear stress is considered. For narrow rectangular beams, the variation of this shear stress across the beam width is small (less than 0.8% for $b \leqslant h/4$).
- **Shearing Stresses in a rectangular beam** – $\tau_{xy} = \frac{3}{2}\frac{V}{A}(1 - \frac{y^2}{c^2})$, where c is the distance from neutral axis to the beam surface. Note the parabolic nature and the location/magnitude of the maximum shear stress.
- **Longitudinal shear on a beam element of arbitrary shape** – A similar procedure is used from the derivations for shear per unit length, q, but applied to arbitrary cross sections. The result is in essence the same as the above. $q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$
- **Plasticity** – Similar treatments to other cases of plasticity in torsion and normal loading. As before, we have a critical strain or load that causes yielding. For a rectangular beam: $M_y = \frac{2}{3}\sigma_y bc^2$. This leads to a critical curvature, $\frac{1}{\rho_y} = \frac{\sigma_y}{EC}$ and an expression for the moment: $M = \frac{3}{2}M_y(1 - \frac{\rho^2}{3{\rho_y}^2})$

Ch. X **Dynamic Beams** (please see handout on Canvas for reading on this topic)

- **The differential equation** – $\frac{\delta^2\phi}{\delta t^2} = c^2\frac{\delta^2\phi}{\delta x^2}, c^2 = \frac{G}{\rho}$
- **Sep of Variables** – $\phi(x,t) = X(x)T(t)$
- **Solutions** – $X(x) = A\cos\frac{\omega}{c}x + B\sin\frac{\omega}{c}x, T(t) = C\cos\omega t + D\sin\omega t$, and relevant boundary conditions.

Ch. 7 **Transformation of Stress and Strain**

- **Transformation of Plane Stress** – Geometry/trigonometry for transformation of coordinate systems by rotating the x-axis by an angle θ. $\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \tau_{x'y'} = \frac{-\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta, \sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$
- **Principal stress / Max Shearing Stress** – Note there are some invariants that appear from the writing of these transformed coordinates (this allows us to note we can construct a circle out of these variables). $\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}, R^2 = \frac{\sigma_x+\sigma_y}{2}^2 + {\tau_{xy}}^2$. We also note that $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$. This is the basis for constructing Mohr's Circle!
- **Mohr's Circle** – Construct as per Prof. Vlassak's method of aligning normal stress directions with "normal stress" axis and inverting shear stress direction. Angles in Mohr's circle are double real rotations.
- **Yield Criteria** – The failure criterion covered in class were von Mises: $\sigma_{VM}^2 = {\sigma_1}^2 + {\sigma_2}^2 + {\sigma_3}^2$; and Mohr's: $\sigma_a = \sigma_{ult}$ (both of which use principle stresses).

Ch. 9 **Deflection of Beams**

- **Equation of the elastic curve** – $\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$.
- **Integration** – $y = \int_0^x dx \int_0^x \frac{M(x)}{EI}dx + C_1x + C_2$, where C_1 and C_2 are determined by boundary conditions.
- **Statically indeterminate beams** – superposition of deflection (see more in textbook on page 560).
- **Beam vibrations** – Know the governing PDE and the general solutions for varying boundary conditions. Read lecture notes on this and also notes posted to Canvas.

Ch. 10 **Columns**

- **Euler's formula: critical load** – $P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$, where L_{eff} depends on the boundary conditions:
  both end pinned – $L_{eff} = L$
  one fixed end one free end – $L_{eff} = 2L$
  one fixed end one pinned – $L_{eff} = 0.7L$
  both ends fixed – $L_{eff} = 0.5L$
- **Critical stress** – $\sigma_{cr} = P_{cr}/A$

Ch. 11 **Energy Methods**

- **Strain-energy density for general stress state** (page 680)
  $u = \frac{1}{2E}\left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \nu(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)\right] + \frac{1}{2G}(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)$
- **Strain-energy for special case** (page 675) – Axial loading: $U = \frac{P^2L}{2AE}$, Bending: $U = \int_0^L \frac{M^2}{2EI}dx$, Torsion: $U = \int_0^L \frac{T^2}{2EI_p}dx$
- **Work of single load** (page 696) – force: $U = \frac{1}{2}P_1x_1$, moment: $U = \frac{1}{2}M_1\theta_1$, torque: $U = \frac{1}{2}T_1\phi_1$.
- **Work of two loads** (page 709) – $U = \frac{1}{2}(\alpha_{11}P_1^2 + 2\alpha_{12}P_1P_2 + \alpha_{22}P_2^2)$

- **Work of n loads** – $U = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}\alpha_{ik}P_iP_k$
- **Castigliano's theorem** (page 711) – displacement (deflection): $x_j = \frac{\partial U}{\partial P_j}$, angle (slope): $\theta_j = \frac{\partial U}{\partial M_j}$.
- **Deflections by Castigliano's theorem** (page 712) – $x_j = \frac{\partial U}{\partial P_j} = \int_0^L \frac{M}{2EI}\frac{\partial M}{\partial P_j}dx$.

Review Problems

Problem 1:

Knowing that the torsional spring at B is of constant K and that the bar AB is rigid, determine the critical load P_{cr}.

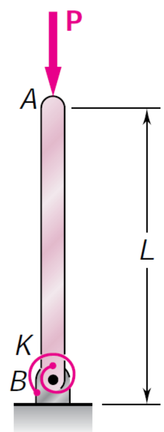
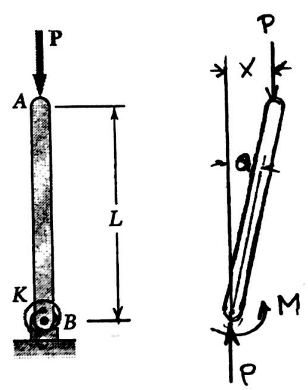


Figure 1

Solution 1



Let θ be the angle change of bar AB.

$M = K\theta$, $x = L\sin\theta \approx L\theta$

$\circlearrowleft M_B = 0$: $M - Px = 0$ $K\theta - PL\theta = 0$

$(K - PL)\theta = 0$ $P_{cr} = K/L$

Problem 2:

Two rigid bars AC and BC are connected as shown to a spring of constant k. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.

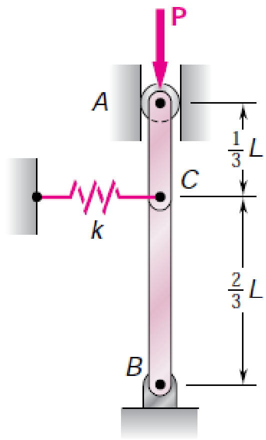
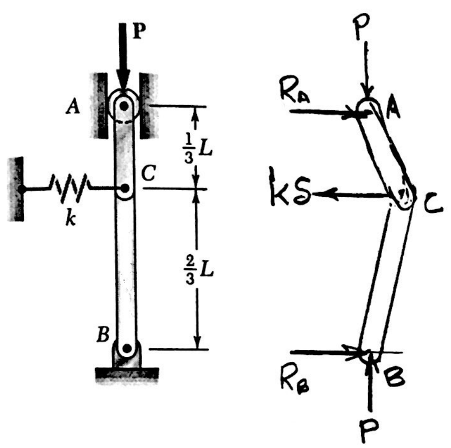


Figure 2

Solution 2



Let δ be the deflection of point C.

Using free body AC and $+\circlearrowleft \Sigma M_C = 0$:

$$-\tfrac{1}{3}L R_A + P\delta = 0 \qquad R_A = \frac{3P\delta}{L}$$

Using free body BC and $+\circlearrowleft \Sigma M_C = 0$:

$$\tfrac{2}{3}L R_B - P\delta = 0 \qquad R_B = \frac{3P\delta}{2L}$$

Using both free bodies together,

$$\xrightarrow{+} \Sigma F_x = 0: \quad R_A + R_B - k\delta = 0$$

$$\frac{3P\delta}{L} + \frac{3P\delta}{2L} - k\delta = 0$$

$$\left(\frac{9}{2}\frac{P}{L} - k\right)\delta = 0$$

$$P_{cr} = \frac{2kL}{9}$$

Problem 3:

A frame consists of four L-shaped members connected by four torsional springs, each of constant K. Knowing that equal loads P are applied at points A and D as shown, determine the critical value P_{cr} of the loads applied to the frame.

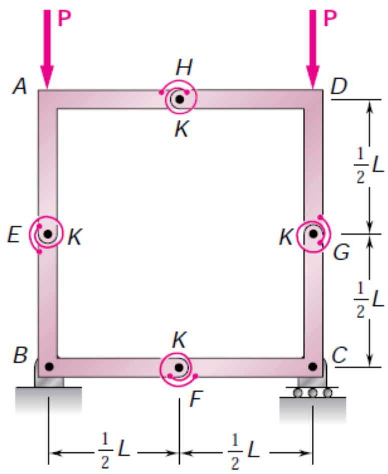
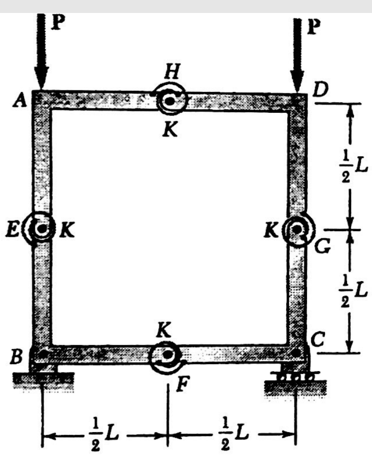
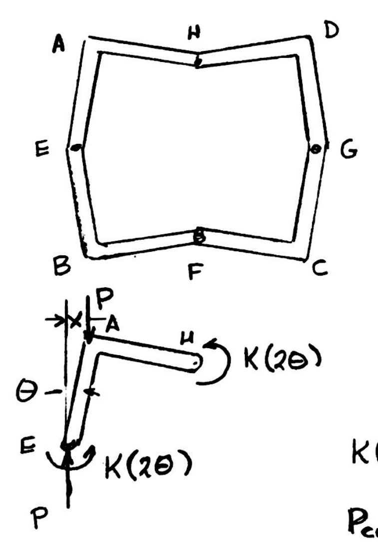


Figure 3

Solution 3





Let θ be the rotation of each L-shaped member.

Angle change across each torsional spring is 2θ.

$$x = \tfrac{1}{2}L\sin\theta \approx \tfrac{1}{2}L\theta$$

$\Sigma M_E = 0$:

$$K(2\theta) + K(2\theta) - Px = 0$$

$$P_{cr} = \frac{4K\theta}{x} \qquad P_{cr} = \frac{8K}{L}$$

Problem 4:

Determine (a) the critical load for the steel strut, (b) the dimension d for which the aluminum strut will have the same critical load. (c) Express the weight of the aluminum strut as a percent of the weight of the steel strut.

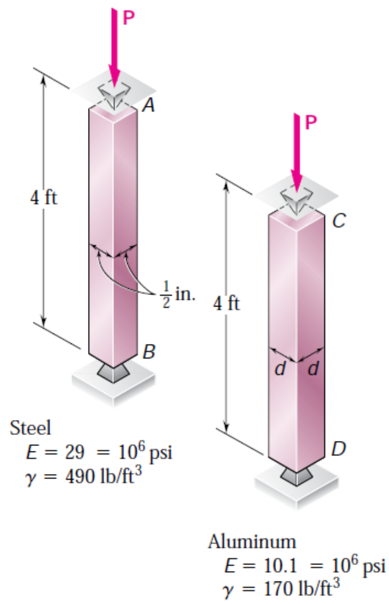
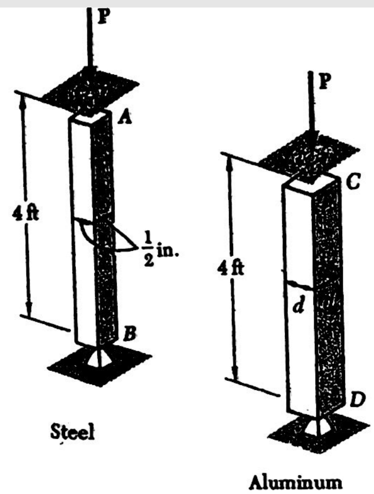


Figure 4

Solution 4



Steel: $E = 29 \times 10^6$ psi

$\gamma = 490$ lb/ft^3 $= 0.28356$ lb/in^3

Aluminum: $E = 10.1 \times 10^6$ psi

$\gamma = 170$ lb/ft^3 $= 0.09838$ lb/in^3

Length: $L = 4$ ft $= 48$ in.

(a) Steel strut:

$$I = \frac{1}{12} d_s^4 = \frac{1}{12}\left(\frac{1}{2}\right)^4 = 5.2083 \times 10^{-3} \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29 \times 10^6)(5.2083 \times 10^{-3})}{(48)^2} \qquad P_{cr} = 647 \text{ lb}$$

Weight: $W_s = \gamma_s L d_s^2 = (0.28356)(48)\left(\frac{1}{2}\right)^2 = 3.4028$ lb

(b) Aluminum strut: $P_{cr} = \frac{\pi^2 EI}{L^2}$

$$I = \frac{P_{cr} L^2}{\pi^2 E} = \frac{(647)(48)^2}{\pi^2 (10.1 \times 10^6)} = 14.9546 \times 10^{-3} \text{ in}^4$$

$$I = \frac{1}{12} d^4 \qquad d = \sqrt[4]{12 I} = \sqrt[4]{(12)(14.9546 \times 10^{-3})} \qquad d = 0.651 \text{ in.}$$

Weight: $W_a = \gamma_a L d^2 = (0.09838)(48)(0.651)^2 = 2.0004$ lb

(c) Weight ratio as a percent: $\frac{W_a}{W_s} \times 100\%$

$$\frac{2.0004}{3.4028} \times 100\% \qquad 58.8\%$$

Problem 5:

Knowing that a factor of safety of 2.6 is required, determine the largest load P that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

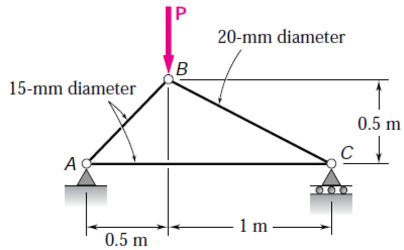
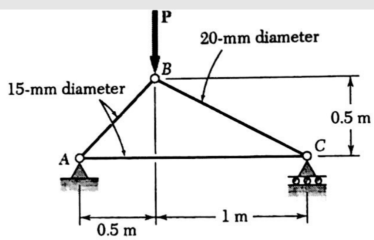


Figure 5

Solution 5



BC:

$L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180$ m

$I = \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64}(20)^4 = 7.854 \times 10^3$ mm^4 $= 7.854 \times 10^{-9}$ m^4

$P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 (200 \times 10^9)(7.854 \times 10^{-9})}{(1.1180)^2}$

$= 12.403 \times 10^3$ N $= 12.403$ kN

$F_{BC,all} = \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770$ kN

AB:

$L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711$ m

$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(15)^4 = 2.485 \times 10^3$ mm^4 $= 2.485 \times 10^{-9}$ m^4

$P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 (200 \times 10^9)(2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3$ N $= 9.8106$ kN

$F_{AB,all} = \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773$ kN

Joint B

P

F_{AB} F_{BC}

$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$

$F_{BC} = 0.79057 F_{AB}$

$+\uparrow \Sigma F_y = 0: \quad \frac{.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} + P = 0$

$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$

$P = 1.06066 F_{AB}$

$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC}$

Allowable value for P.

$P < 1.06066 F_{AB,all} = (1.06066)(3.773) = 4.00$ kN

$P < 1.3416 F_{BC,all} = (1.3416)(4.770) = 6.40$ kN

$P_{all} = 4.00$ kN

Problem 6:

A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support A and to a rigid cap B. The steel rod EF, of 0.75-in. diameter, is welded to cap B. Knowing that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (a) the total strain energy of the system when $P = 10$ kips, (b) the corresponding strain-energy density of the pipe CD and in the rod EF.

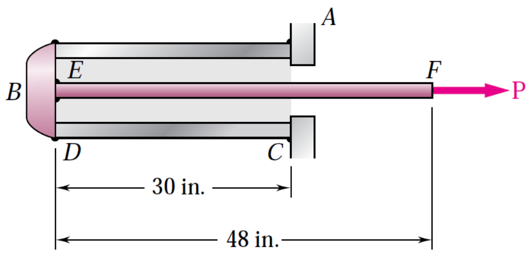
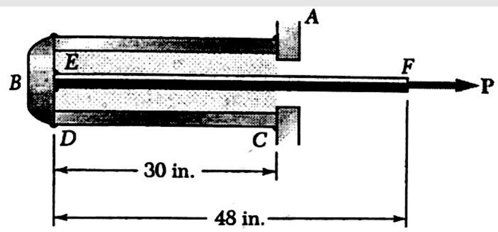


Figure 6

Solution 6



For EF: $A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$

CD: $U_{CD} = \dfrac{P^2 L}{2EA} = \dfrac{(10\times10^3)^2(30)}{(2)(10.6\times10^6)(1.85)} = 76.49$ in.lb

EF: $U_{EF} = \dfrac{P^2 L}{2EA} = \dfrac{(10\times10^3)^2(48)}{(2)(29\times10^6)(0.4418)} = 187.33$ in.lb.

(a) Total: $U = U_{CD} + U_{EF} =$ $U = 264$ in.lb

(b) CD: $\sigma = -\dfrac{10000}{1.85} = -5405$ psi, $u = \dfrac{\sigma^2}{2E} = \dfrac{(-5405)^2}{(2)(10.6\times10^6)} = 1.378$ in-lb/in^3

EF: $\sigma = \dfrac{10000}{0.4418} = 22635$ psi, $u = \dfrac{\sigma^2}{2E} = \dfrac{22635^2}{(2)(29\times10^6)} = 8.83$ in-lb/in^3

Problem 7:

In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.

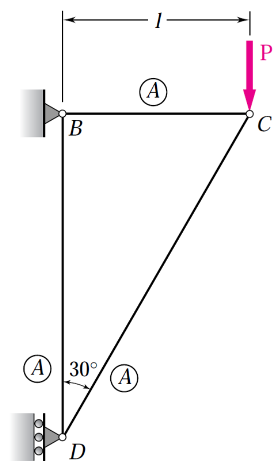
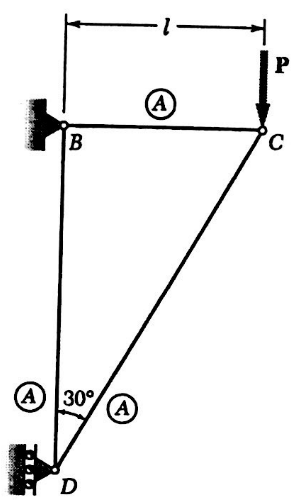
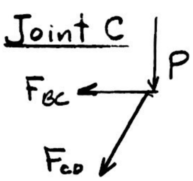


Figure 7

Solution 7



Joint C



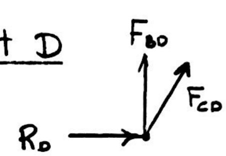
$$+\uparrow \Sigma F_y = 0: \quad -\frac{\sqrt{3}}{2} F_{CD} - P = 0$$

$$F_{CD} = -\frac{2}{\sqrt{3}} P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{BC} - \frac{1}{2} F_{CD} = 0$$

$$F_{BC} = \frac{1}{\sqrt{3}} P$$

Joint D



$$+\uparrow \Sigma F_y = 0:$$

$$F_{BD} + \frac{\sqrt{3}}{2} F_{CD} = 0$$

$$F_{BD} = P$$

Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2 l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2 l/A$
BD	P	$\sqrt{3}l$	A	$\sqrt{3}P^2 l/A$
Σ				$4.732\, P^2 l/A$

$$U = \Sigma \frac{1}{2} \frac{F^2 L}{EA}$$

$$= \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

$$= \frac{1}{2E}\left(4.732 \frac{P^2 l}{A}\right)$$

$$U = 2.37 \frac{P^2 l}{EA}$$

Problem 8:

The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. The steel drill pipe has an outer diameter of 8 in. and a uniform wall thickness of 0.5 in. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at B starts to operate and using $G = 11.2 \times 10^6$ psi, determine the maximum strain energy acquired by the drill pipe.

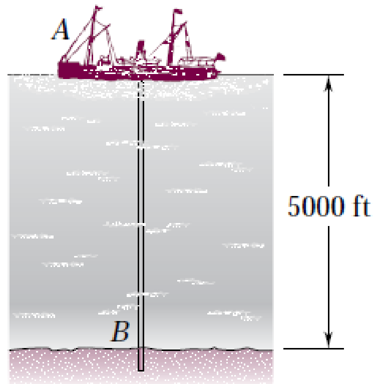
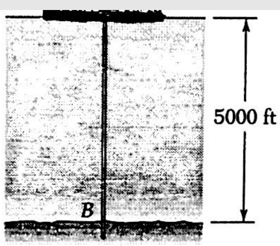


Figure 8

Solution 8



$\varphi = (2)(2\pi) = 4\pi$ rad

$L = 5000$ ft $= 60 \times 10^3$ in

$c_o = \frac{d_o}{2} = 4$ in $\quad c_i = c_o - t = 3.5$ in

$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 166.406$ in^4

$\varphi = \frac{TL}{GJ} \quad T = \frac{GJ\varphi}{L} \quad U = \frac{T^2 L}{2GJ} = \left(\frac{GJ\varphi}{L}\right)^2 \frac{L}{2GJ} = \frac{GJ\varphi^2}{2L}$

$U = \frac{(11.2\times10^6)(166.406)(4\pi)^2}{(2)(60\times10^3)}$

$U = 2.45 \times 10^6$ in-lb

Problem 9:

The thin-walled hollow cylindrical member AB has a noncircular cross section of nonuniform thickness. Using the expression given in book Eq. (3.53) $\tau = \frac{T}{2ta}$, and the expression for the strain-energy density given in book Eq. (11.19) $u = \frac{\tau^2}{2G}$, show that the angle of twist of member AB is

$$\mathbf{f} = \frac{TL}{4a^2G}\oint \frac{ds}{t}$$

where ds is an element of the center line of the wall cross section and A is the area enclosed by that center line.

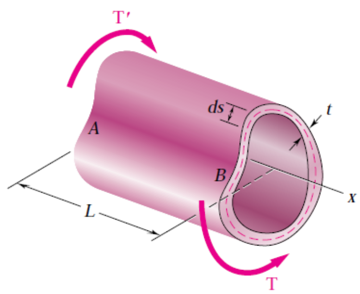
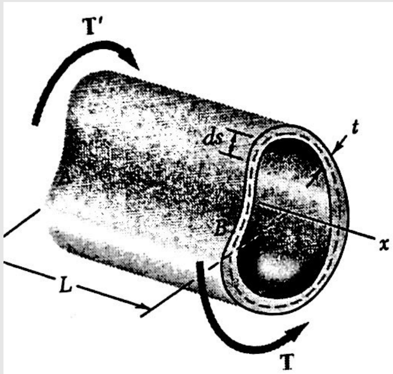


Figure 9

Solution 9



From equation (3.53), $\tau = \frac{T}{2ta}$

Strain energy density:

$$u = \frac{\tau^2}{2G} = \frac{T^2}{8Gt^2a^2}$$

$$U = \int_0^L \oint u\,t\,ds\,dx$$

$$= \int_0^L \frac{T^2}{8Ga^2}\oint \frac{ds}{t}\,dx = \frac{T^2L}{8Ga^2}\oint \frac{ds}{t}$$

Work of torque $= \frac{1}{2}T\varphi = \frac{T^2L}{8Ga^2}\oint \frac{ds}{t}$ $\qquad \varphi = \frac{TL}{4Ga^2}\oint \frac{ds}{t}$

Problem 10:

For the prismatic beam shown, determine the slope at point B.

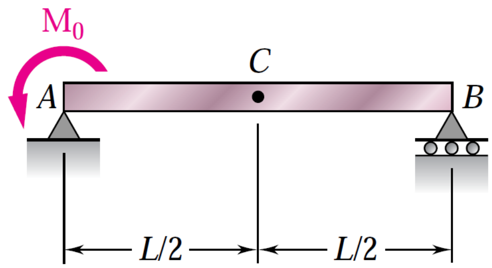
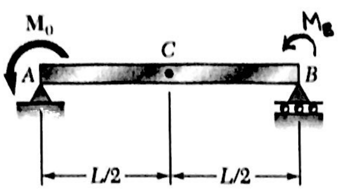


Figure 10

Solution 10



Add couple M_B at point B as shown.

Reactions. $R_A = \frac{1}{L}(M_0 + M_B) \uparrow$

Strain energy. $U = \int_0^L \frac{M^2}{2EI}\,dx$

Slope at point B. $\theta_B = \frac{\partial U}{\partial M_B}$

$$M = R_A x - M_0 = (M_0 + M_B)\frac{x}{L} - M_0$$

$$\frac{\partial M}{\partial M_B} = \frac{x}{L} \qquad \text{With } M_B = 0 \quad M = M_0\left(\frac{x}{L} - 1\right)$$

$$\frac{\partial U}{\partial M_B} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx = \frac{M_0}{EI}\int_0^L\left(\frac{x}{L} - 1\right)\frac{x}{L}\,dx = \frac{M_0}{EIL^2}\int_0^L (x - L)x\,dx$$

$$= \frac{M_0}{EIL^2}\int_0^L (x^2 - Lx)\,dx = \frac{M_0}{EI}\left(\frac{x^3}{3} - \frac{Lx^2}{2}\right)\Big|_0^L = -\frac{M_0 L}{6EI}$$

$$\theta_B = -\frac{M_0 L}{6EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

Problem 11:

Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.

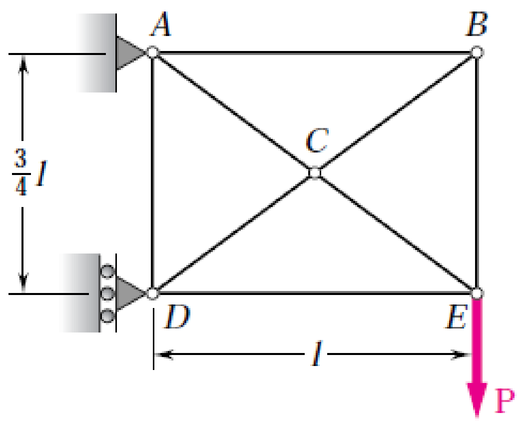
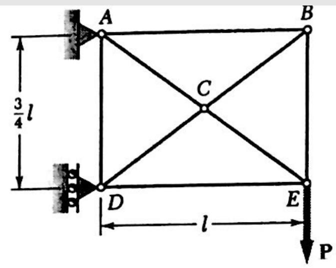


Figure 11

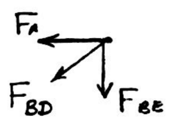
Solution 11

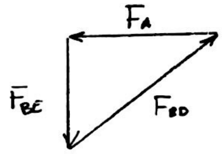


Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$\delta_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A}\sum \frac{F^2 L}{2EA} = \frac{1}{EA}\sum F \frac{\partial F}{\partial F_A} L = 0$$

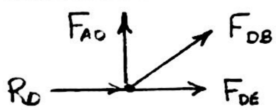
Joint B





$F_{BD} = -\frac{5}{4}F_A \qquad F_{BE} = \frac{3}{4}F_A$

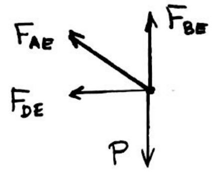
Joint D



$+\uparrow \sum F_y = 0: \quad F_{AD} + \frac{3}{5}F_{DB} = 0$

$F_{AD} = -\frac{3}{5}F_{DB} = -\frac{3}{4}F_A$

Joint E



$+\uparrow \sum F_y = 0:$

$F_{BE} - P + \frac{3}{5}F_{AE}$

$F_{AE} = \frac{5}{3}P - \frac{5}{3}F_{BE}$

$= \frac{5}{3}P - \frac{5}{4}F_A$

$+\rightarrow \sum F_x = 0: \quad -\frac{4}{5}F_{AE} - F_{DE} = 0$

$F_{DE} = -\frac{4}{5}F_{AE} = -\frac{4}{3}P + F_A$

Member	F	$\partial F/\partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4}F_A$	$-\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
AE	$\frac{5}{3}P - \frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48}Pl + \frac{125}{64}F_A l$
BD	$-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64}F_A l$
BE	$\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
DE	$-\frac{4}{3}P + F_A$	1	l	$-\frac{4}{3}Pl + F_A l$
Σ				$-\frac{63}{16}Pl + \frac{27}{4}F_A l$

$$\delta_A = \frac{1}{EA}\left(-\frac{63}{16}Pl + \frac{27}{4}F_A l\right) = 0 \qquad F_A = \frac{7}{12}P$$

$F_{AB} = F_A = \qquad F_{AB} = \frac{7}{12}P$