ES120 Spring 2018 – Section 7 Notes

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Problem 1:

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.

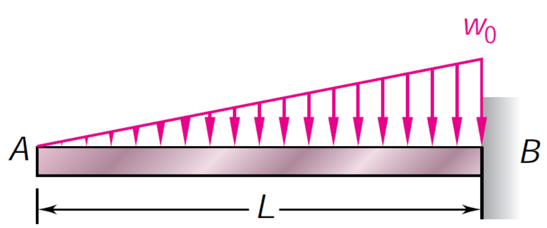

Figure 1

Solution 1

To do this problem we must begin by draying the free body diagram for an arbitrary distance x from edge A to point J.

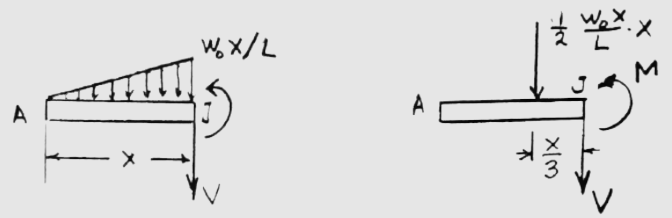

Figure 2

Using this figure, we can compute the force balance in the y direction such that

$$-\frac{1}{2}\frac{w_o x}{L}x - V = 0 \tag{1}$$

$$V = -\frac{w_o x^2}{2L} \tag{2}$$

Now to obtain the equation for the bending moment, we simply integrate V with respect to x, namely:

$$M = -\frac{w_o x^3}{6L} \tag{3}$$

Now, to obtain the maximum value of both, we evaluate the above equations at point B, where $x = L$

$$V(L) = -\frac{w_o L}{2} \tag{4}$$

$$M(L) = -\frac{w_o L^2}{6} \tag{5}$$

Using this, we can draw the shear and bending moment diagrams as

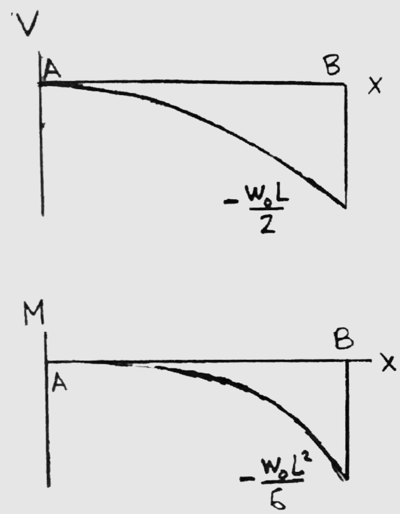

Figure 3

Problem 2:

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

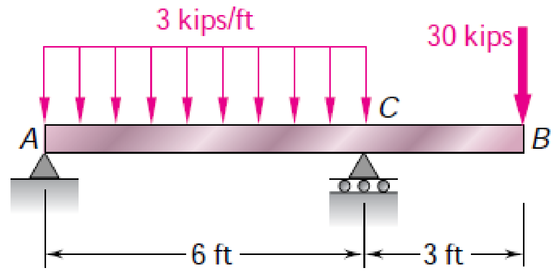

Figure 4

Solution 2

This problem is a bit more complex than the previous one. First, we need to begin finding the unknown reaction forces along the bottom of the beam. To do this, let's use sum of moments about points C and A, respectively (we know that the distributed load can be approximated as a point load at the center between A and C and of the total amount $3 * 6 = 18$ kips):

$$\Sigma M_C = 0 : -6R_A + (3)(18) - (3)(30) = 0 \Rightarrow R_A = -6 \text{ kips} \tag{6}$$
$$\Sigma M_A = 0 : 6R_C - (3)(18) - (9)(30) = 0 \Rightarrow R_C = -54 \text{ kips} \tag{7}$$

Using this information, we can separate this problem into two parts, namely A to C and C to B. So let's begin by solely analyzing A to C. This is valid for $- < x < 6$

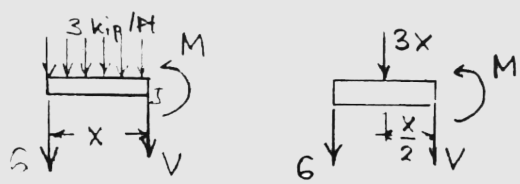

Figure 5

If we do sum of forces in y

$$\Sigma F_y = 0 : -6 - 3x - V = 0 \Rightarrow V = -6 - 3x \text{ kips} \tag{8}$$

So to obtain the bending moment we can take the integral such that we obtain

$$M = -6x - 1.5x^2 \tag{9}$$

Which if we evaluate at pint C which is $x = 6$ we obtain

$$M(6) = -6(6) - 1.5(6)^2 = -90 \tag{10}$$

Now lets analyze C to B

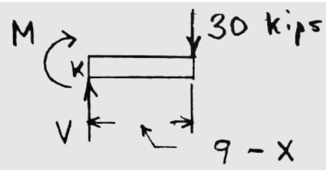

Figure 6

Similar to before, lets do a sum of forces in the y direction

$$\Sigma F_y = 0 : V - 30 = 0 \Rightarrow V = 30 \text{ kips} \tag{11}$$

Now here we have to be careful when taking the integral as the integration constant does matter, namely we will have

$$M = 30x + k \tag{12}$$

We know that at $x = 6$, M has to match the solution from the previous part from A to C. So, since we know that $M(6) = -90$ then we can set it to that and solve for the constant k at point C, namely,

$$M = 30(6) + k = -90 \Rightarrow k = -270 \text{ kips} \tag{13}$$

So our final expression therefore becomes

$$M = 30x - 270 \tag{14}$$

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Using this information we are ready to draw our shear and bending moment diagrams.

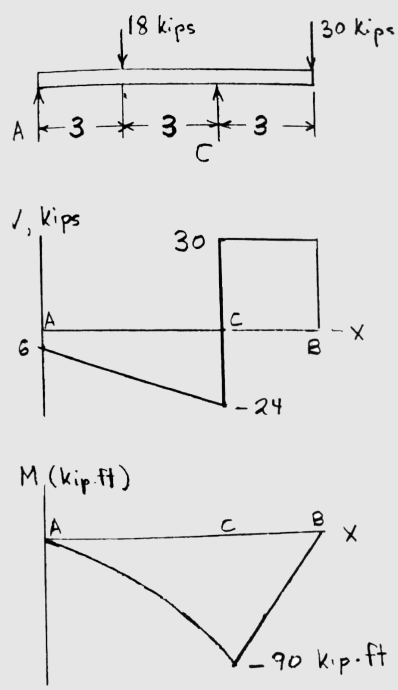

Figure 7

From these graphs, we can decipher that out maximum absolute values for the shear and bending moments are

$$\boxed{|V|_{\text{max=30 kips}}} \tag{15}$$

$$\boxed{|M|_{\text{max=90 kips}}} \tag{16}$$

Problem 3:

(a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

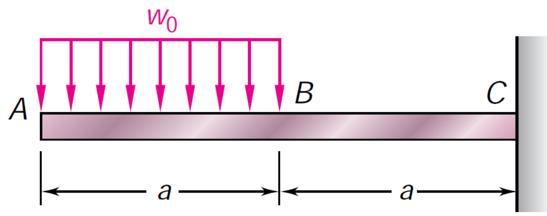

Figure 8

Solution 3

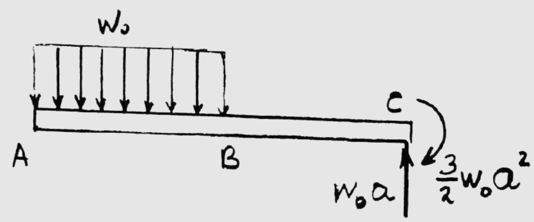

Figure 9