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## Logic for Computer Science

Final exam (bachelor part)

February 3, 2010

Task 1 (1 point). Write in the empty fields of the table below the cardinalities of respective sets.

$\mathbb{N}\times\{0,1,2\}$	$\{a,b\} \times \mathbb{Q}$	$\mathcal{P}(\mathbb{Q}\setminus[0,1])$	$\mathcal{P}(\mathbb{N}\times\{0,1\})$	$\{2010, \mathbb{Q}, \mathbb{R}\}$	$(\mathbb{Q}\setminus\mathbb{N})^{\mathbb{Q}}$	$(\mathbb{R}\setminus\mathbb{Q})^{\{a,b,c\}}$	$\mathcal{P}(\{0,1\})$
$\aleph_0$	$\aleph_0$	¢	¢	3	c	¢	4

**Task 2 (1 point).** If there exists a bijection  $f : \mathcal{P}(\mathbb{N}) \to \{a, b\}^{\mathbb{N} \times \{0, 1\}}$ , then in the box below write an expression defining any such bijection. Otherwise write the word "NO".

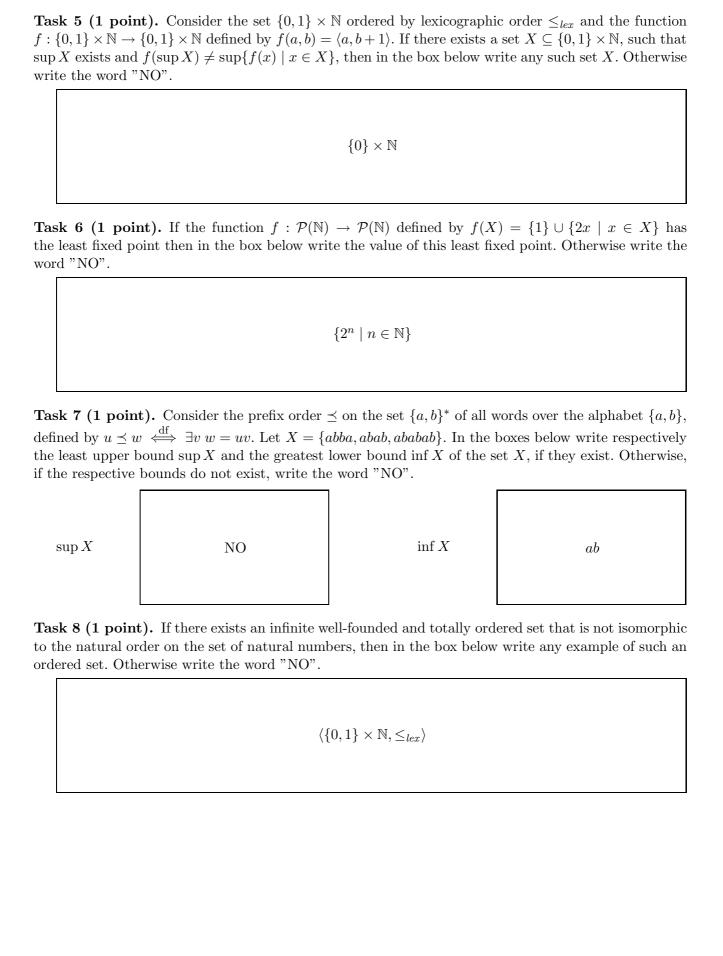
$$f(X)(n,x) = \begin{cases} a, & \text{if } 2n + x \in X \\ b, & \text{otherwise} \end{cases}$$

Task 3 (1 point). If the ordered sets  $\langle \{0,1,2\} \times \{0,1,2\}, \leq_{lex} \rangle$  and  $\langle \{0,1,2,3,4,5,6,7,8\}, \leq \rangle$ , where  $\leq_{lex}$  is the lexicographic extension of the natural order, are isomorphic, then in the box below write any isomorphism of these orders. Otherwise write a justification why such an order does not exist.

$$f(x,y) = 3x + y$$

**Task 4 (1 point).** If the ordered sets  $\langle \mathbb{Q}, \leq \rangle$  and  $\langle \mathbb{Q} \times \mathbb{N}, \leq_{lex} \rangle$  are isomorphic, then in the box below write any isomorphism of these orders. Otherwise write a justification why such an order does not exist.

 $\mathbb{Q}$  is a dense order but there are no elements between (0,0) and (0,1) in  $\mathbb{Q} \times \mathbb{N}$ 



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Task 9 (1 point). If the terms f(h(z), x) and f(y, g(y)) are unifiable, then in the box below write any unifier of these terms. Otherwise write the word "NO".

$$\{x/g(h(z)), y/h(z)\}$$

Task 10 (1 point). If the terms f(h(x), x) and f(y, g(y)) are unifiable, then in the box below write any unifier of these terms. Otherwise write the word "NO".

NO

**Task 11 (1 point).** If the set of clauses  $\{\neg s \lor r, \neg q \lor r, s \lor q, \neg r \lor \neg p, \neg r \lor p\}$  is inconsistent then in the box below write a resolution proof of inconsistency of this set. Otherwise write a valuation satisfying this set.

$$\frac{\neg r \vee \neg p \quad \neg r \vee p}{\neg r} \qquad \frac{s \vee q \quad \neg q \vee r}{r \vee s} \quad \neg s \vee r$$

