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Logic for Computer Science

Final exam (bachelor part)

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Task 1 (1 point). Write in the empty fields of the table below the cardinalities of respective sets.

$\mathbb{N} \times \{0,1,2\}$	$\{a,b\} \times \mathbb{Q}$	$\mathcal{P}(\mathbb{Q} \setminus [0,1])$	$\mathcal{P}(\mathbb{N} \times \{0,1\})$	$\{2010, \mathbb{Q}, \mathbb{R}\}$	$(\mathbb{Q} \setminus \mathbb{N})^{\mathbb{Q}}$	$(\mathbb{R} \setminus \mathbb{Q})^{\{a,b,c\}}$	$\mathcal{P}(\{0,1\})$
$\aleph_0$	$\aleph_0$	$\mathfrak{c}$	$\mathfrak{c}$	3	$\mathfrak{c}$	$\mathfrak{c}$	4

Task 2 (1 point). If there exists a bijection $f : \mathcal{P}(\mathbb{N}) \to \{a,b\}^{\mathbb{N}\times\{0,1\}}$, then in the box below write an expression defining any such bijection. Otherwise write the word "NO".

$$f(X)(n,x) = \begin{cases} a, & \text{if } 2n + x \in X \\ b, & \text{otherwise} \end{cases}$$

Task 3 (1 point). If the ordered sets $\langle \{0,1,2\} \times \{0,1,2\}, \leq_{lex} \rangle$ and $\langle \{0,1,2,3,4,5,6,7,8\}, \leq \rangle$, where $\leq_{lex}$ is the lexicographic extension of the natural order, are isomorphic, then in the box below write any isomorphism of these orders. Otherwise write a justification why such an order does not exist.

$$f(x,y) = 3x + y$$

Task 4 (1 point). If the ordered sets $\langle \mathbb{Q}, \leq \rangle$ and $\langle \mathbb{Q} \times \mathbb{N}, \leq_{lex} \rangle$ are isomorphic, then in the box below write any isomorphism of these orders. Otherwise write a justification why such an order does not exist.

$\mathbb{Q}$ is a dense order but there are no elements between $\langle 0,0 \rangle$ and $\langle 0,1 \rangle$ in $\mathbb{Q} \times \mathbb{N}$

Task 5 (1 point). Consider the set $\{0,1\} \times \mathbb{N}$ ordered by lexicographic order $\leq_{lex}$ and the function $f : \{0,1\} \times \mathbb{N} \to \{0,1\} \times \mathbb{N}$ defined by $f(a,b) = \langle a, b+1 \rangle$. If there exists a set $X \subseteq \{0,1\} \times \mathbb{N}$, such that $\sup X$ exists and $f(\sup X) \neq \sup\{f(x) \mid x \in X\}$, then in the box below write any such set X. Otherwise write the word "NO".

$\{0\} \times \mathbb{N}$

Task 6 (1 point). If the function $f : \mathcal{P}(\mathbb{N}) \to \mathcal{P}(\mathbb{N})$ defined by $f(X) = \{1\} \cup \{2x \mid x \in X\}$ has the least fixed point then in the box below write the value of this least fixed point. Otherwise write the word "NO".

$\{2^n \mid n \in \mathbb{N}\}$

Task 7 (1 point). Consider the prefix order $\preceq$ on the set $\{a,b\}^*$ of all words over the alphabet $\{a,b\}$, defined by $u \preceq w \overset{\text{df}}{\iff} \exists v\ w = uv$. Let $X = \{abba, abab, ababab\}$. In the boxes below write respectively the least upper bound $\sup X$ and the greatest lower bound $\inf X$ of the set X, if they exist. Otherwise, if the respective bounds do not exist, write the word "NO".

$\sup X$	NO	$\inf X$	ab

Task 8 (1 point). If there exists an infinite well-founded and totally ordered set that is not isomorphic to the natural order on the set of natural numbers, then in the box below write any example of such an ordered set. Otherwise write the word "NO".

$\langle \{0,1\} \times \mathbb{N}, \leq_{lex} \rangle$

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Task 9 (1 point). If the terms $f(h(z), x)$ and $f(y, g(y))$ are unifiable, then in the box below write any unifier of these terms. Otherwise write the word "NO".

$$\{x/g(h(z)),\ y/h(z)\}$$

Task 10 (1 point). If the terms $f(h(x), x)$ and $f(y, g(y))$ are unifiable, then in the box below write any unifier of these terms. Otherwise write the word "NO".

NO

Task 11 (1 point). If the set of clauses $\{\neg s \vee r,\ \neg q \vee r,\ s \vee q,\ \neg r \vee \neg p,\ \neg r \vee p\}$ is inconsistent then in the box below write a resolution proof of inconsistency of this set. Otherwise write a valuation satisfying this set.

$$\dfrac{\dfrac{\neg r \vee \neg p \quad \neg r \vee p}{\neg r} \qquad \dfrac{\dfrac{s \vee q \quad \neg q \vee r}{r \vee s} \quad \neg s \vee r}{r}}{\perp}$$

Task 12 (1 point). If the set of clauses $\{\neg p \vee \neg q \vee r,\ \neg p \vee q,\ \neg r \vee \neg s,\ \neg r \vee s\}$ is inconsistent then in the box below write a resolution proof of inconsistency of this set. Otherwise write a valuation satisfying this set.

$$\sigma(p) = \mathtt{F},\ \sigma(q) = \mathtt{F},\ \sigma(r) = \mathtt{F},\ \sigma(s) = \mathtt{F}$$