



Matthias Moulin



# Rendering Algorithms

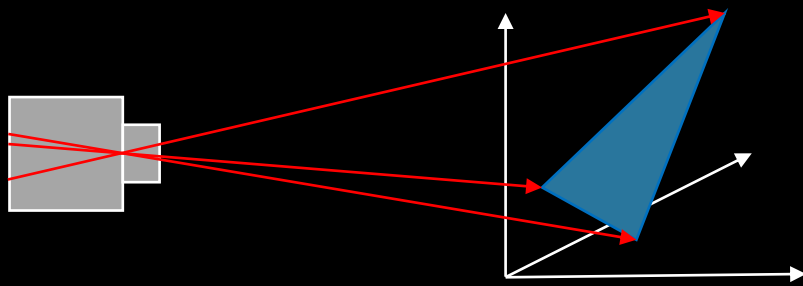
## Ray Tracing

For each pixel do:

construct ray

For each geometric primitive do:

test hit(ray, primitive)



*project image samples into the scene*

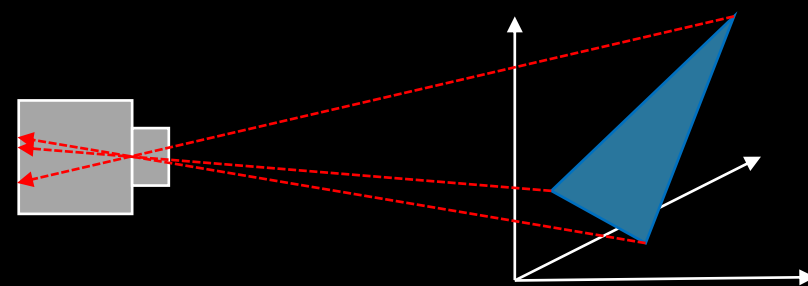
## Rasterization

For each geometric primitive do:

project primitive

For each pixel do:

test hit(pixel, primitive)

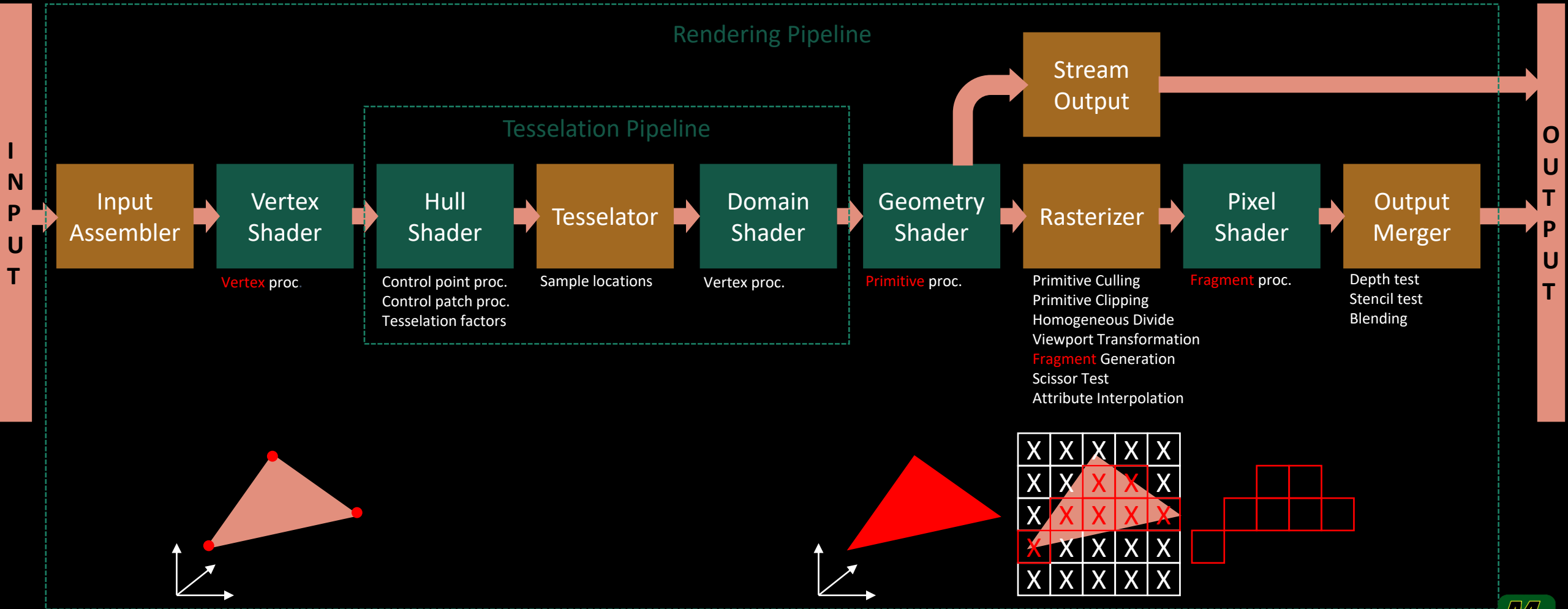


*project geometry onto the image plane*

# Rendering Pipeline

Programmable Stage

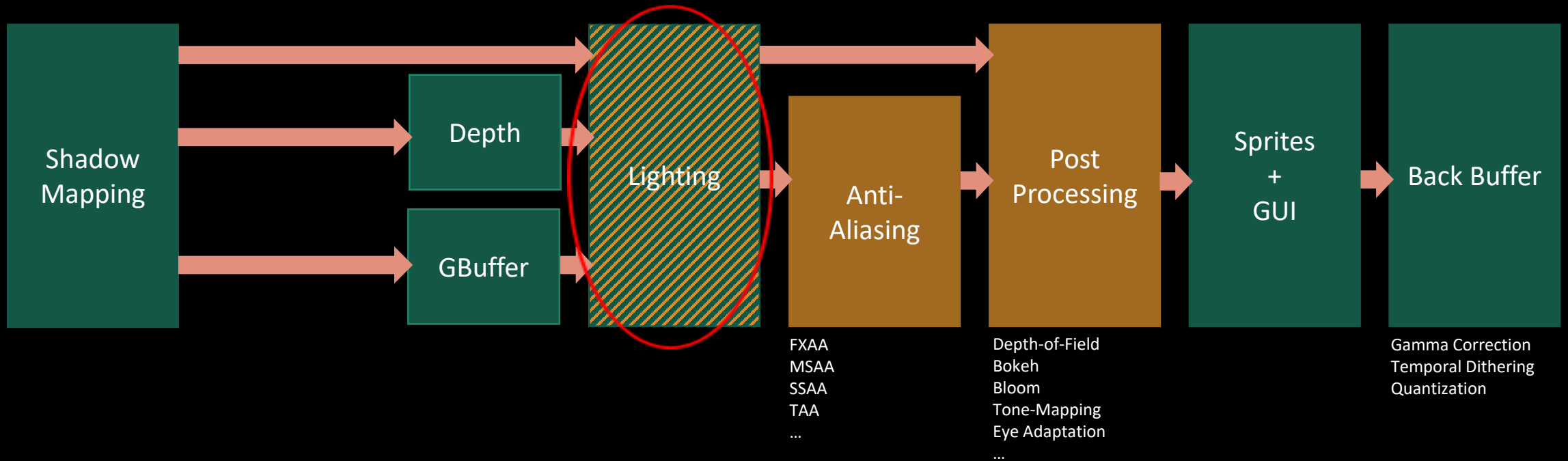
Configurable Stage



# Rendering Engine Pipeline

Rendering Pipeline

Compute Pipeline

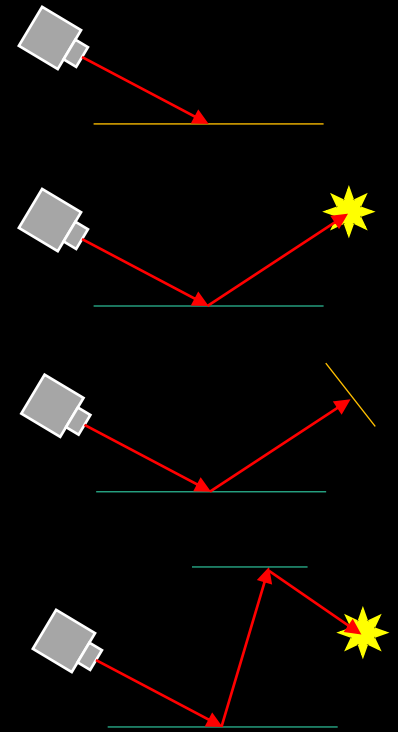


# Lighting

no pre-computation

- Self emission associated with the emissive surfaces
  - 0 bounces/surface interactions
- Direct illumination associated with the point lights
  - 1 bounce/surface interaction
- Direct illumination associated with the emissive surfaces
  - 1 bounce/surface interaction
- Indirect illumination associated with the point lights
  - 2 bounces/surface interactions
- ...

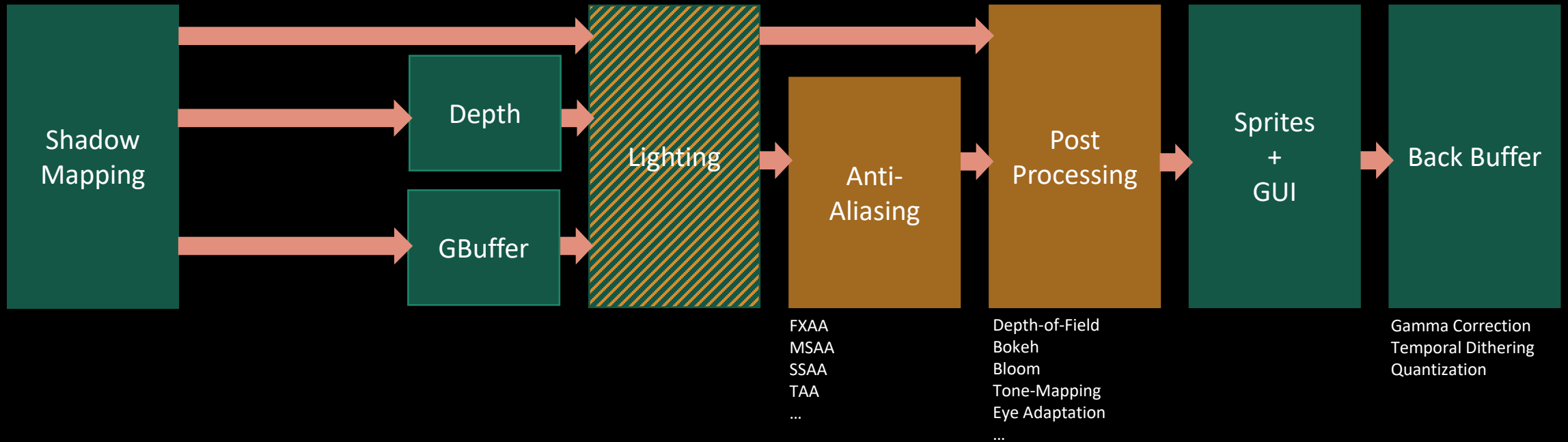
pre-computation



# Rendering Engine Pipeline

Rendering Pipeline

Compute Pipeline

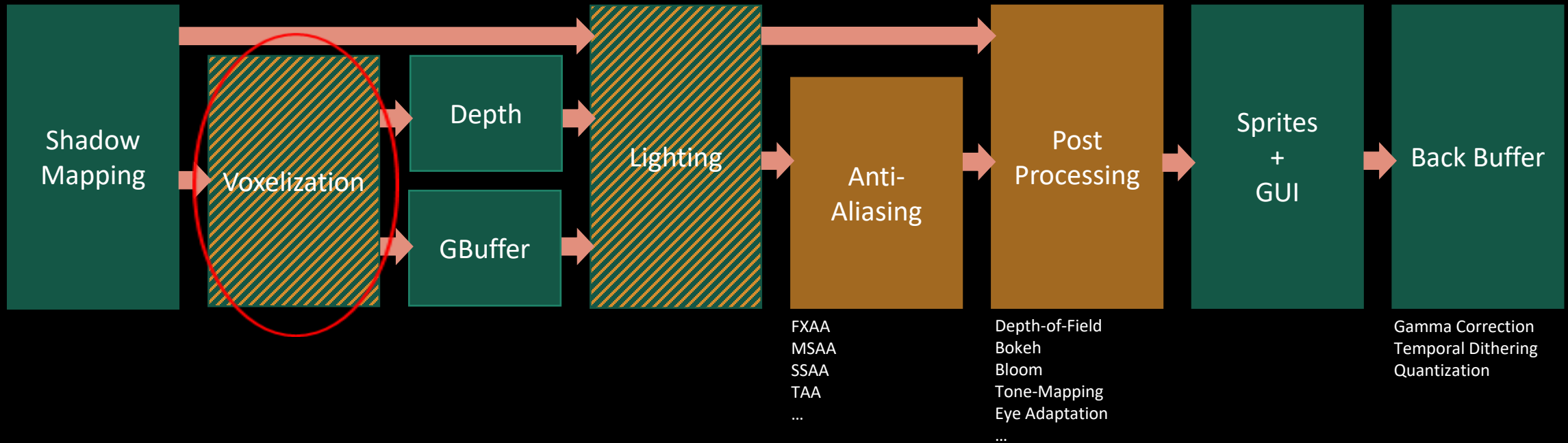




# Rendering Engine Pipeline

Rendering Pipeline

Compute Pipeline



# Voxelization



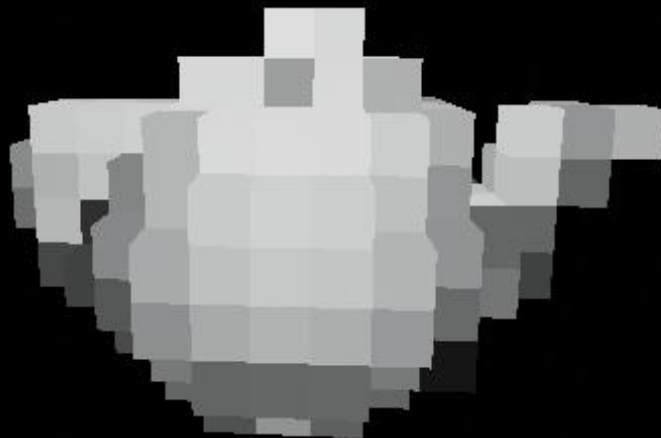
Grid =  $256^3$ , Voxel =  $0.01^3$



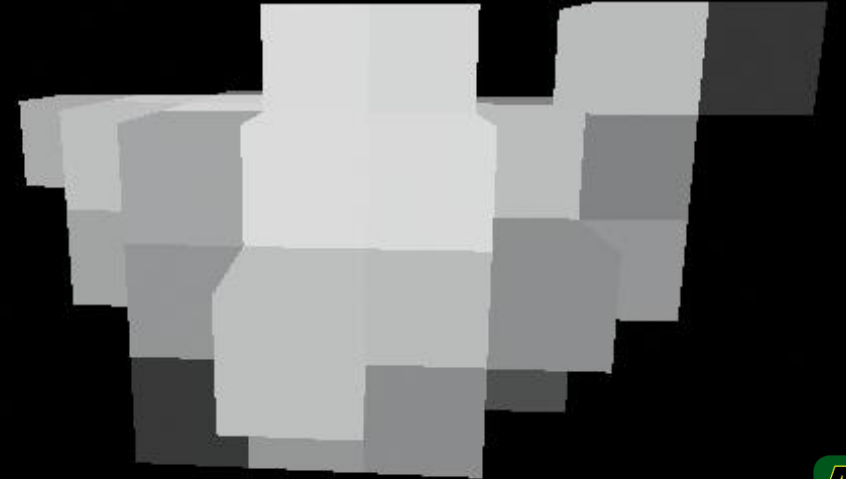
Grid =  $128^3$ , Voxel =  $0.02^3$



Grid =  $64^3$ , Voxel =  $0.04^3$



Grid =  $32^3$ , Voxel =  $0.08^3$

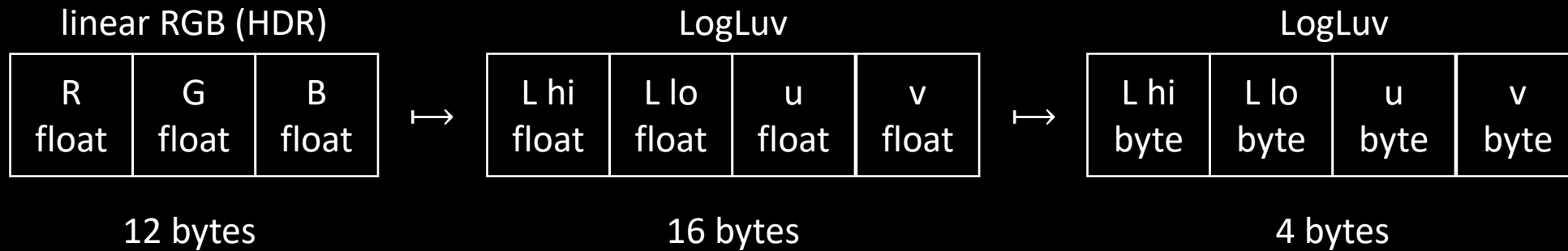


Grid =  $16^3$ , Voxel =  $0.16^3$

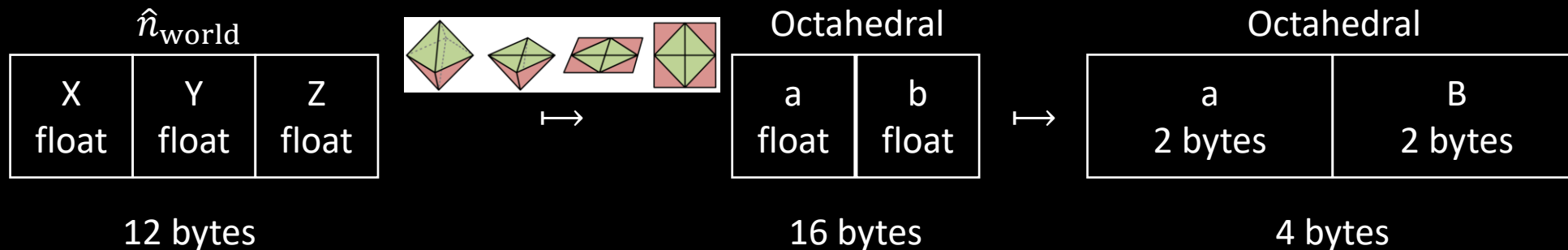


# Voxelization: Voxel Data

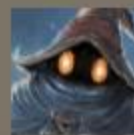
- Radiance *distribution*



- Normal *distribution* (to support more bounces)



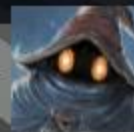


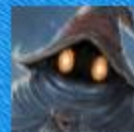
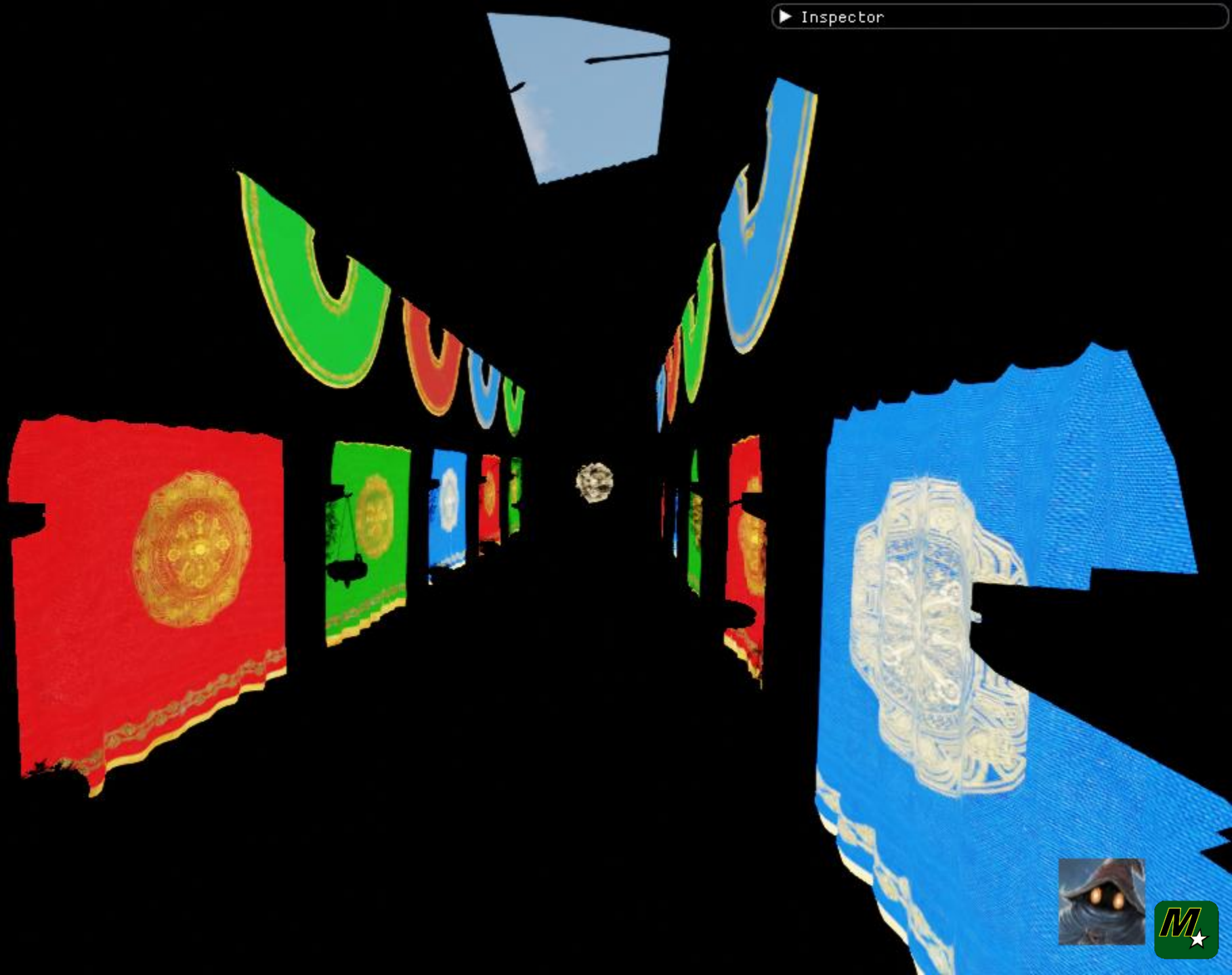


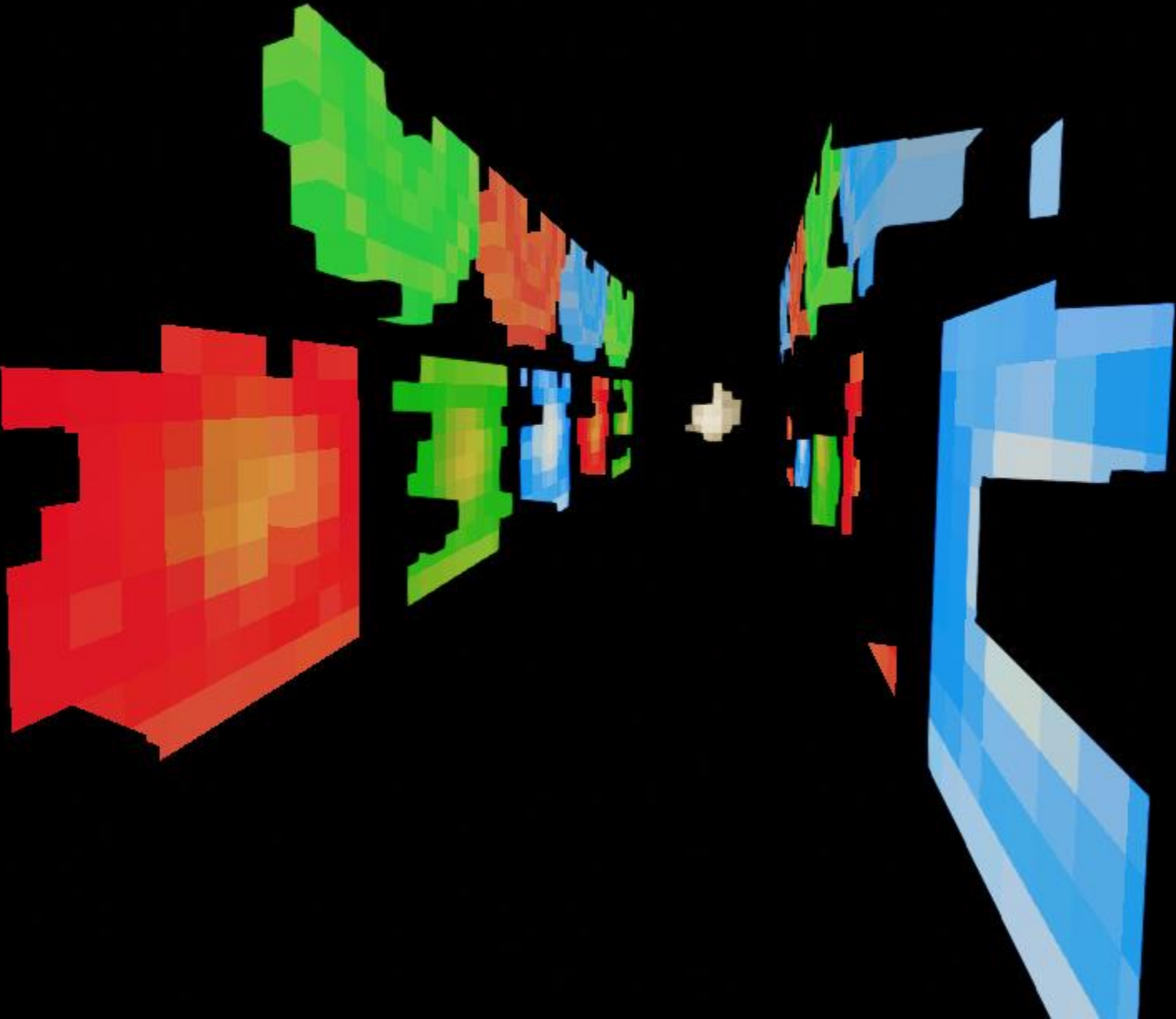










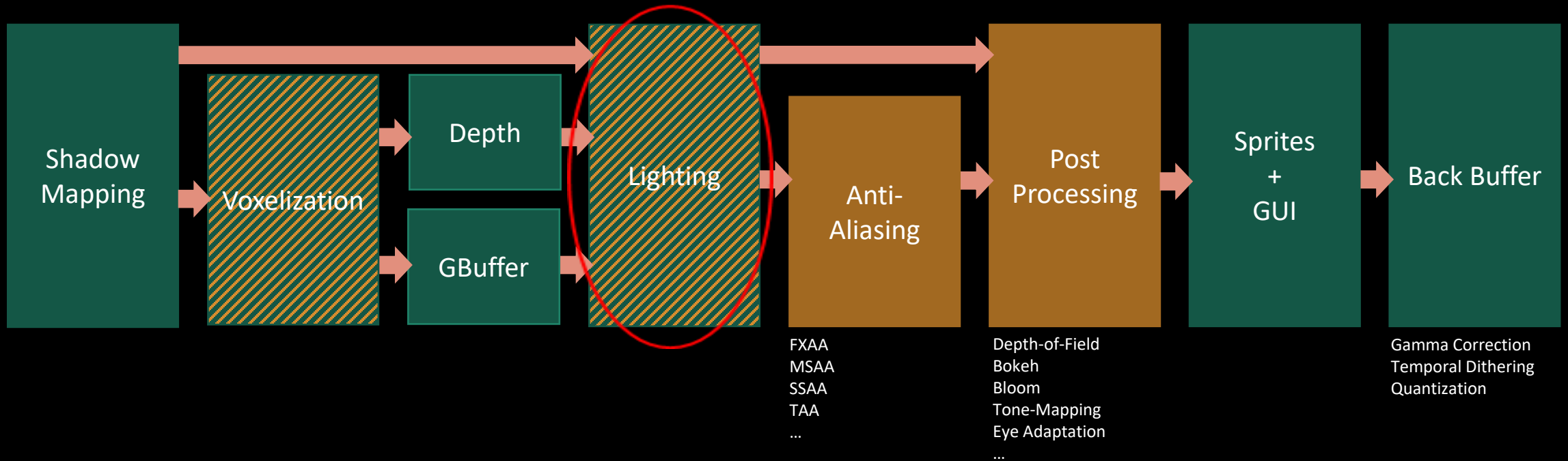




# Rendering Engine Pipeline

Rendering Pipeline

Compute Pipeline



# Voxel Cone Tracing: Ambient Occlusion

$$AO(\vec{p}, \hat{\omega}_o) = \frac{1}{\pi} \int_{\Omega} AO_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$AO(\vec{p}, \hat{\omega}_o) = \frac{1}{\pi} \sum_{j=1}^N \int_{\Omega_j} AO_v(\vec{p}, \hat{\omega}_{j,i}) (\hat{n} \cdot \hat{\omega}_{j,i}) d\hat{\omega}_{j,i}$$

$$AO(\vec{p}, \hat{\omega}_o) \approx \frac{1}{\pi} \sum_{j=1}^N AO_v(\vec{p}, \hat{\omega}_j, \alpha_j) \int_{\Omega_j} (\hat{n} \cdot \hat{\omega}_{j,i}) d\hat{\omega}_{j,i}$$

rays to cones

$$\hat{w}_j = \frac{1}{\pi} \int_{\Omega_j} (\vec{n} \cdot \hat{\omega}_{j,i}) d\hat{\omega}_{j,i}$$

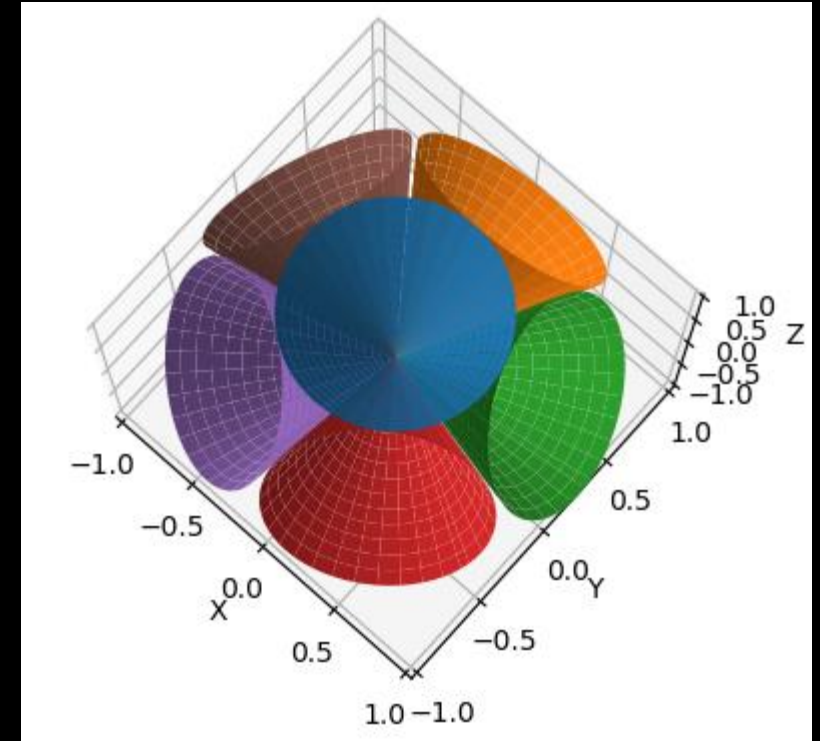
$$AO(\vec{p}, \hat{\omega}_o) \approx \sum_{j=1}^N \hat{w}_j AO_v(\vec{p}, \hat{\omega}_j, \alpha_j)$$

# Voxel Cone Tracing: Ambient Occlusion

$$AO(\vec{p}, \hat{\omega}_o) \approx \sum_{j=1}^N \hat{w}_j AO_v(\vec{p}, \hat{\omega}_j, \alpha_j)$$

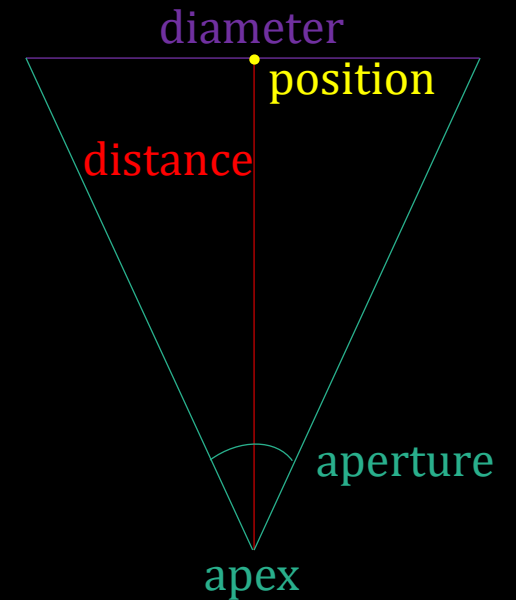
$$\hat{w}_0 = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/6} \cos \theta \sin \theta \, d\theta d\varphi = \frac{1}{4}$$

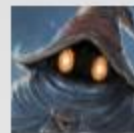
$$\hat{w}_1 = \hat{w}_2 = \hat{w}_3 = \hat{w}_4 = \hat{w}_5 \approx \frac{1 - \hat{w}_0}{5} = \frac{3}{20}$$

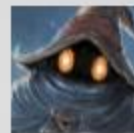
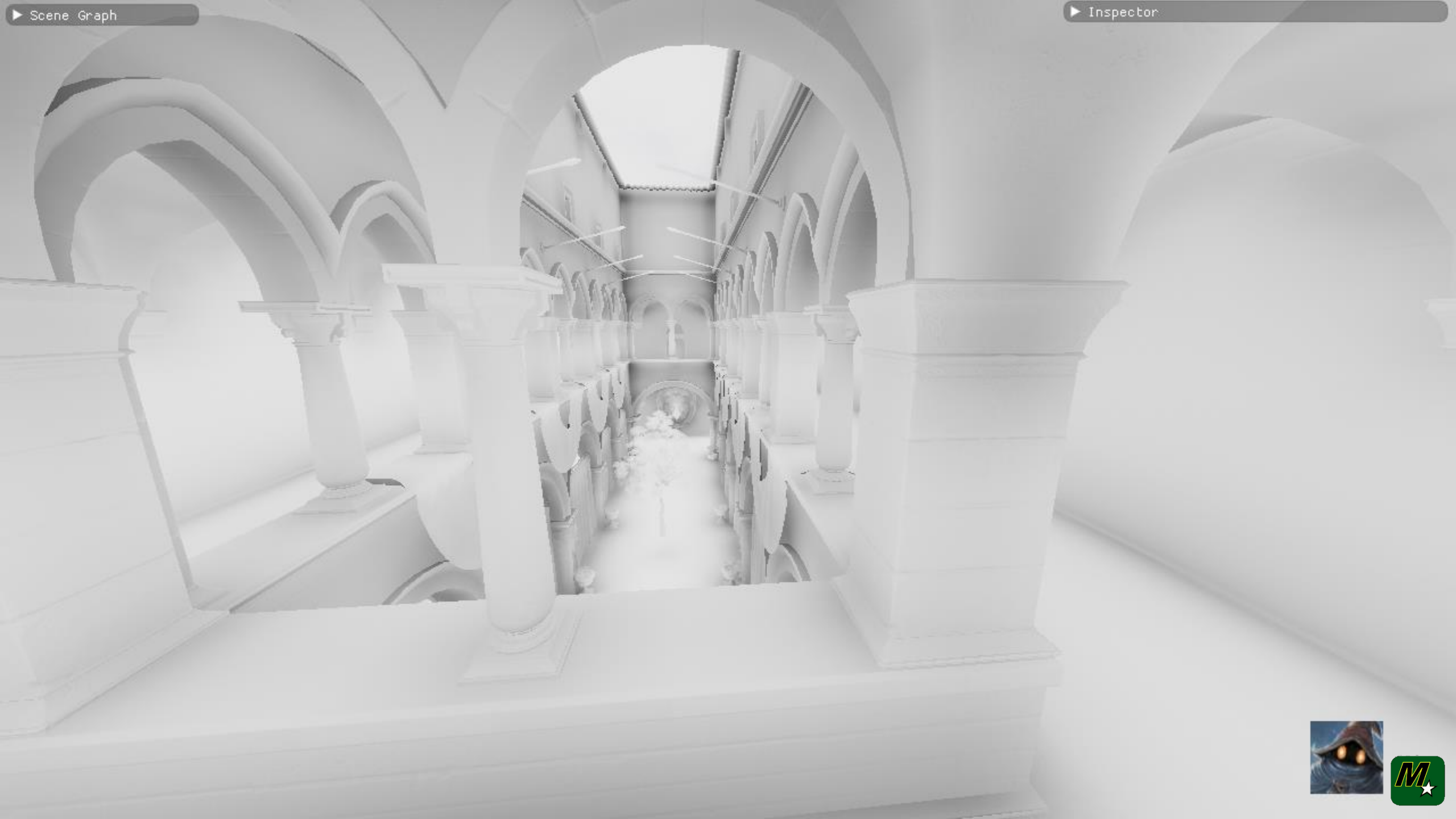


# Voxel Cone Tracing: Ambient Occlusion

```
AOv ← 0
distance ← voxeloffset
while AOv < 1 and distance < distanceAO :
    compute diameter
    compute mip_level
    compute position at distance
    if mip_level ≥ max_mip_level or position ∉ [0,1]3 :
        break
    sample AOstep(position, mip_level)
    AOv ← AOv + (1 - AOv) conestep AOstep
    distance ← distance + conestep diameter
return AOv
```







# Voxel Cone Tracing: Diffuse Illumination

$$L_o(\vec{p}, \hat{\omega}_o) = \int_{\Omega} f_{r,d}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$L_o(\vec{p}, \hat{\omega}_o) = (1 - m) \frac{k_d}{\pi} \sum_{j=1}^N \int_{\Omega_j} L_v(\vec{p}, \hat{\omega}_{j,i}) (\hat{n} \cdot \hat{\omega}_{j,i}) d\hat{\omega}_{j,i}$$

$$L_o(\vec{p}, \hat{\omega}_o) \approx (1 - m) \frac{k_d}{\pi} \sum_{j=1}^N L_v(\vec{p}, \hat{\omega}_j, \alpha_j) \int_{\Omega_j} (\hat{n} \cdot \hat{\omega}_{j,i}) d\hat{\omega}_{j,i}$$

rays to cones

$$\hat{\omega}_j = \frac{1}{\pi} \int_{\Omega_j} (\vec{n} \cdot \hat{\omega}_{j,i}) d\hat{\omega}_{j,i}$$

$$L_o(\vec{p}, \hat{\omega}_o) \approx (1 - m) k_d \sum_{j=1}^N \hat{\omega}_j L_v(\vec{p}, \hat{\omega}_j, \alpha_j)$$

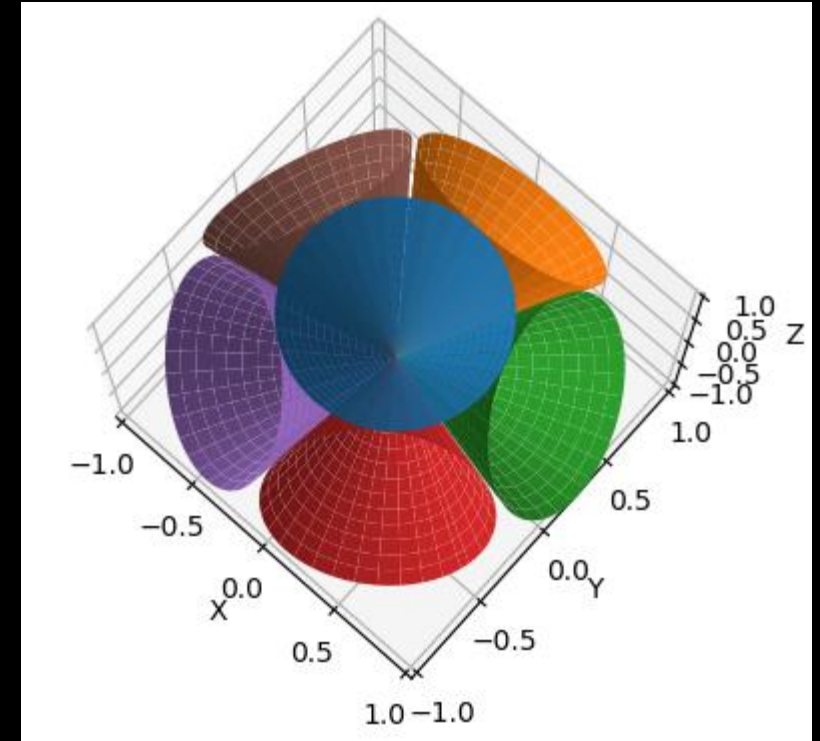


# Voxel Cone Tracing: Diffuse Illumination

$$L_o(\vec{p}, \hat{\omega}_o) \approx (1 - m)k_d \sum_{j=1}^N \hat{w}_j L_v(\vec{p}, \hat{\omega}_j, \alpha_j)$$

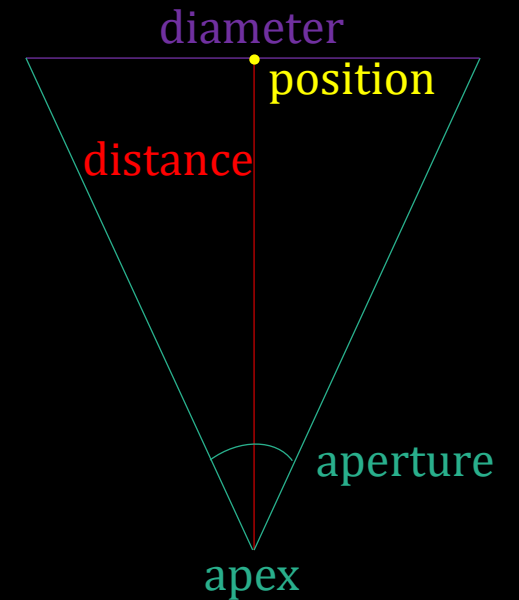
$$\hat{w}_0 = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/6} \cos \theta \sin \theta \, d\theta d\varphi = \frac{1}{4}$$

$$\hat{w}_1 = \hat{w}_2 = \hat{w}_3 = \hat{w}_4 = \hat{w}_5 \approx \frac{1 - \hat{w}_0}{5} = \frac{3}{20}$$

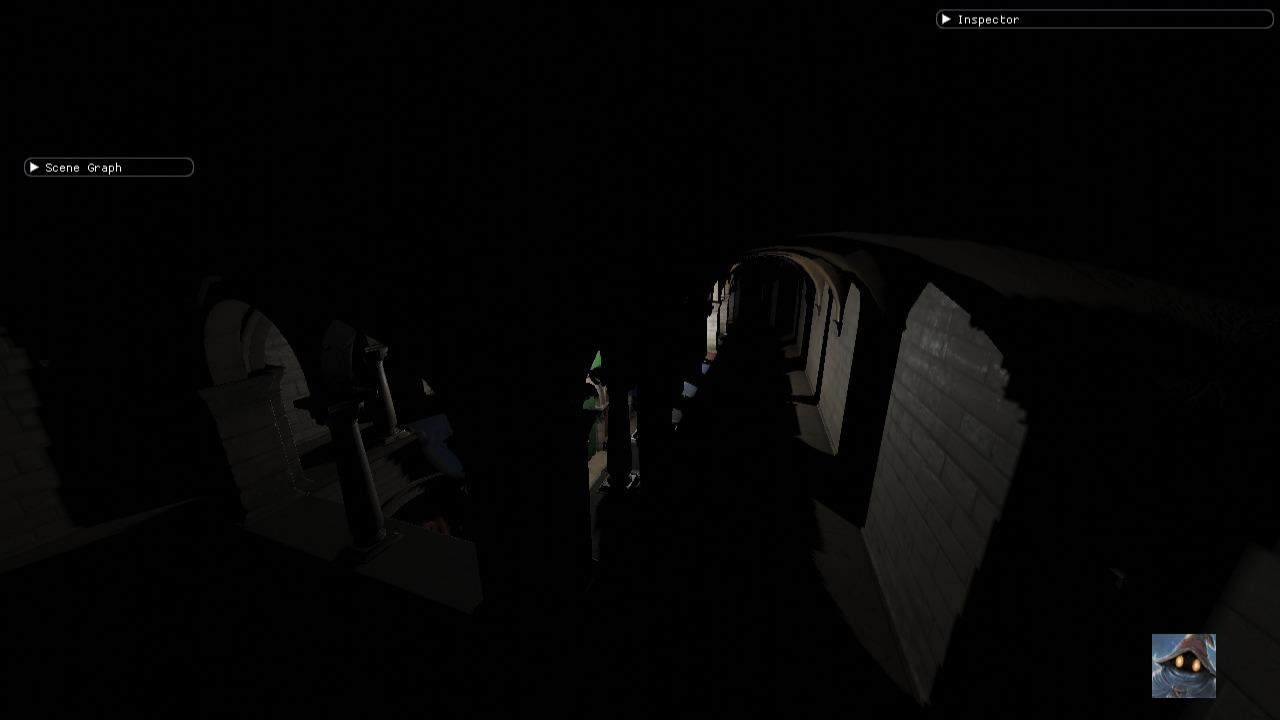
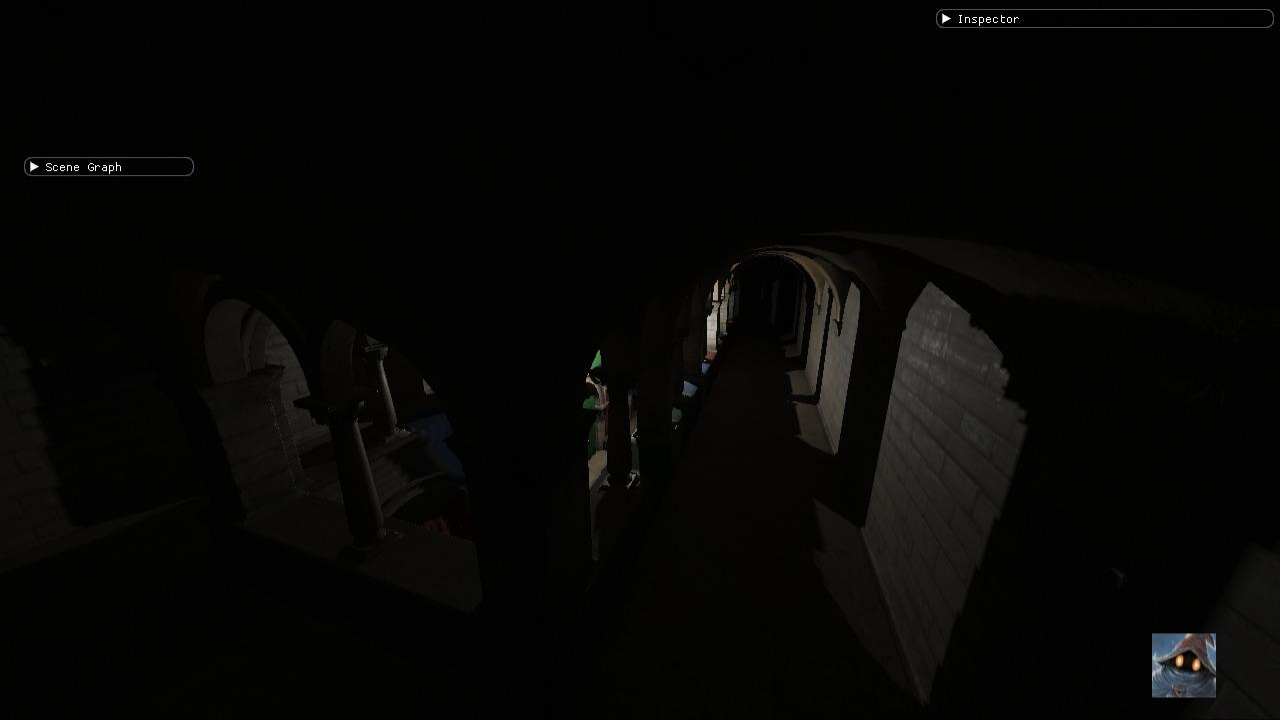


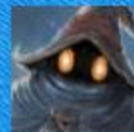
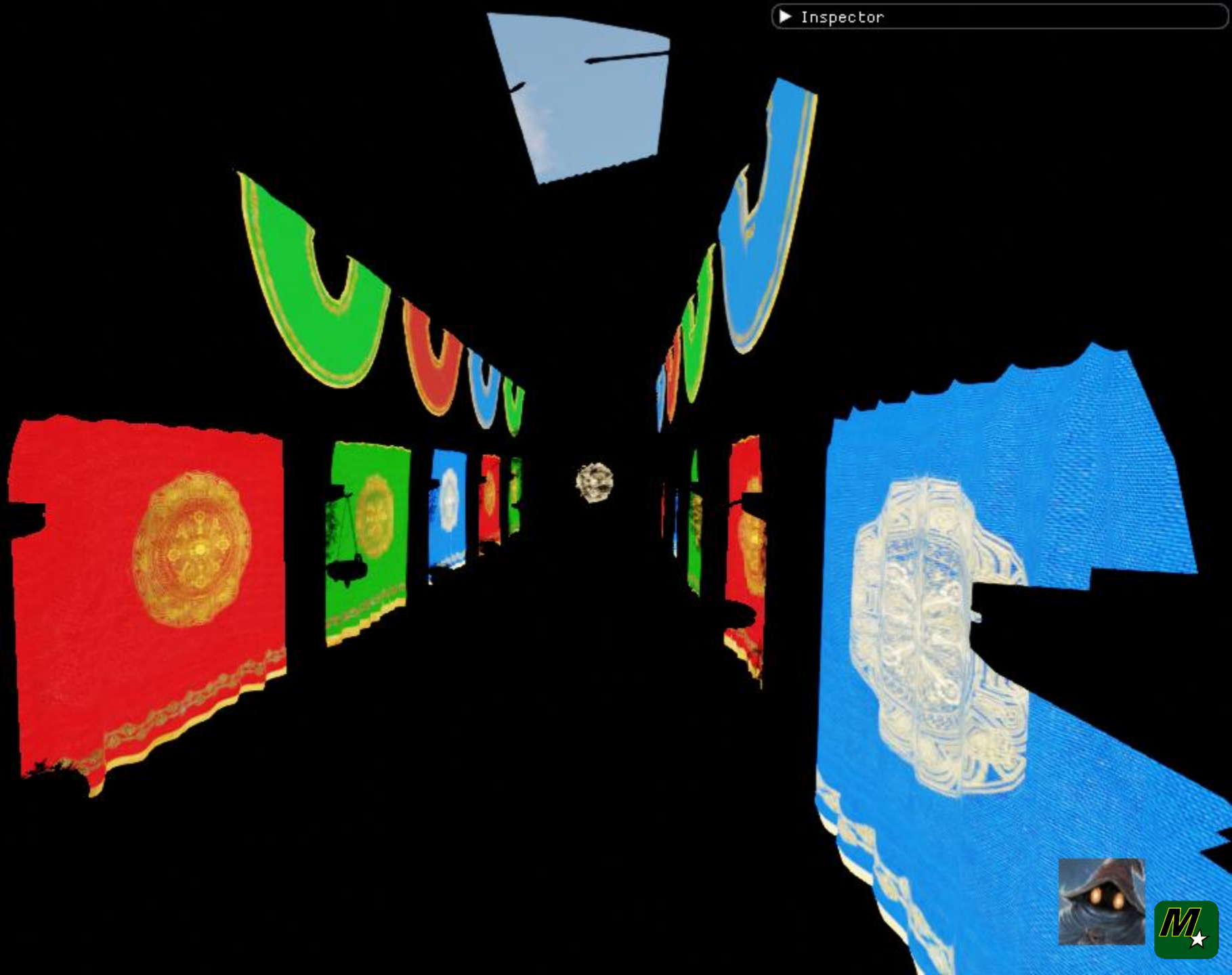
# Voxel Cone Tracing: Marching in UVW Space

```
 $L_v \leftarrow [0, 0, 0, 0]$   
distance  $\leftarrow$  voxel_offset  
while  $L_{v,a} < 1$  :  
    compute diameter  
    compute mip_level  
    compute position at distance  
    if mip_level  $\geq$  max_mip_level or position  $\notin [0,1]^3$  :  
        break  
    sample  $L_{\text{step}}$ (position, mip_level)  
     $L_{v,\text{rgb}} \leftarrow L_{v,\text{rgb}} + (1 - L_{v,a}) \text{cone}_{\text{step}} L_{\text{step},a} L_{\text{step}}$   
     $L_{v,a} \leftarrow L_{v,a} + (1 - L_{v,a}) \text{cone}_{\text{step}} L_{\text{step},a}$   
    distance  $\leftarrow$  distance + cone_step diameter  
return  $L_v$ 
```













► Inspector

► Scene Graph





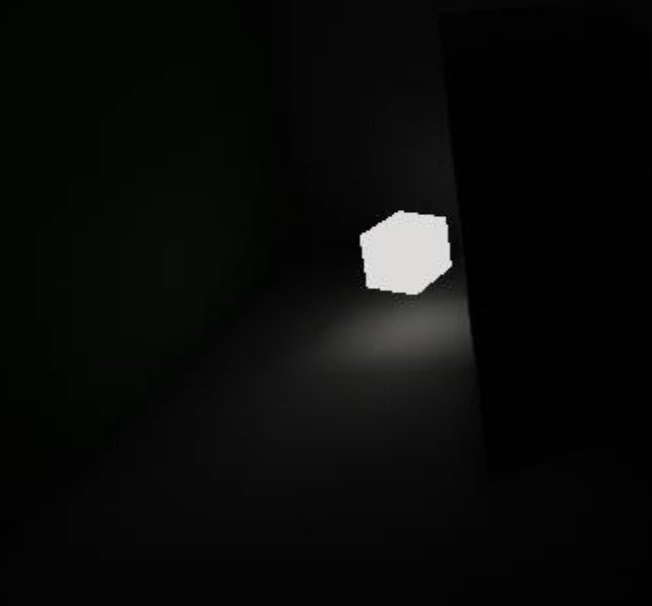
FPS: 153  
SPF: 6.53ms  
CPU: 5.6%  
RAM: 130MB  
DCs: 20

▼ Scene Graph

Player

▼ Cornell Box

- Left Plane
- Right Plane
- Bottom Plane
- Top Plane
- Near Plane
- Far Plane
- Short Box
- Tall Box
- Omni Light



▼ Inspector

Player ☒ Active

- ▶ Transform
- ▶ Perspective Camera
- ▶ Sprite Image
- ▶ Sprite Text



D3D11. F12, PrtScrn to capture. Frame: 148659. 11.87 ms (10.76 .. 13.03) (84. FPS)  
0 Captures saved.

► Inspector

► Scene Graph



# Voxel Cone Tracing: Specular Illumination

$$f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \mathbf{N} \hat{f}_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

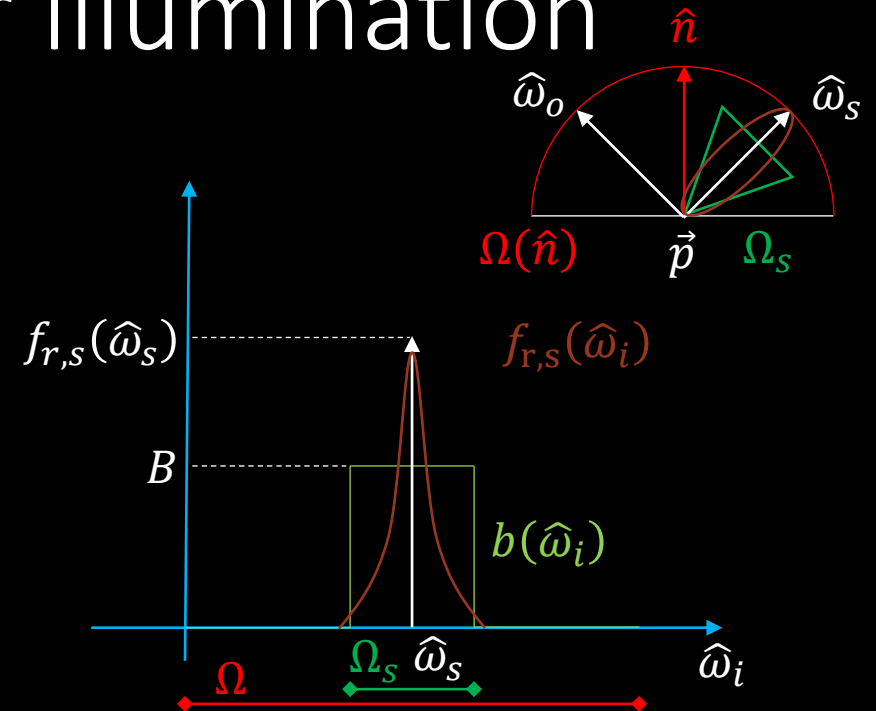
$$\int_{\Omega} f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i = \mathbf{N}$$

$$\int_{\Omega} \hat{f}_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i = 1$$

$$b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \mathbf{N} \hat{b}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

$$\int_{\Omega} b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i = B \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i = \mathbf{N}$$

$$\int_{\Omega} \hat{b}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i = \hat{B} \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i = 1$$



# Voxel Cone Tracing: Specular Illumination

$$\left\{ \begin{array}{l} \int_{\Omega} \hat{f}_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i) \, d\hat{\omega}_i = \hat{B} \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i) \, d\hat{\omega}_i = 1 \\ \int_{\Omega} \hat{f}_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i)^2 \, d\hat{\omega}_i = \hat{B} \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i)^2 \, d\hat{\omega}_i \quad 1^{\text{st}} \text{ moment} \\ \dots \\ M_k = \int_{\Omega} \hat{f}_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i)^k \, d\hat{\omega}_i = \hat{B} \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i)^k \, d\hat{\omega}_i \end{array} \right.$$

$$\Rightarrow \hat{B}$$

$$\Rightarrow \Omega_s$$

$$\Rightarrow (\hat{\omega}_s)$$

# Voxel Cone Tracing: Specular Illumination

$$M_k = \hat{B} \int_{\Omega_s(\hat{n})} (\hat{n} \cdot \hat{\omega}_i)^k d\hat{\omega}_i$$

$$M_k = \hat{B} \int_{\Omega_s(\hat{\omega}_s)} (\hat{n} \cdot \hat{\omega}_i)^k d\hat{\omega}_i$$

$\Omega_s(\hat{n}) \approx \Omega_s(\hat{\omega}_s)$

$$M_k \approx \hat{B} \int_0^{2\pi} \int_0^{\alpha/2} \cos^k \theta \sin \theta d\theta d\varphi$$

$$M_k \approx 2\pi \hat{B} \int_0^{\alpha/2} \cos^k \theta \sin \theta d\theta$$

$$\begin{bmatrix} M_0 \\ M_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \approx \frac{2\pi \hat{B} (1 - \cos^3(\alpha/2))}{3} \end{bmatrix} \approx \frac{\pi \hat{B} (1 - \cos^2(\alpha/2))}{1 - \cos^2(\alpha/2)} = \pi \hat{B} (1 - \cos^2(\alpha/2))$$
$$\begin{bmatrix} M_0 \\ M_1 \end{bmatrix} \approx \frac{2\pi \hat{B} (1 - \cos^3(\alpha/2))}{3} = \frac{2(1 - \cos^3(\alpha/2))}{3(1 - \cos^2(\alpha/2))}$$

# Voxel Cone Tracing: Specular Illumination

$$L_o(\vec{p}, \hat{\omega}_o) = \int_{\Omega} f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$L_o(\vec{p}, \hat{\omega}_o) = \int_{\Omega} b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$L_o(\vec{p}, \hat{\omega}_o) \approx B \int_{\Omega_s} L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

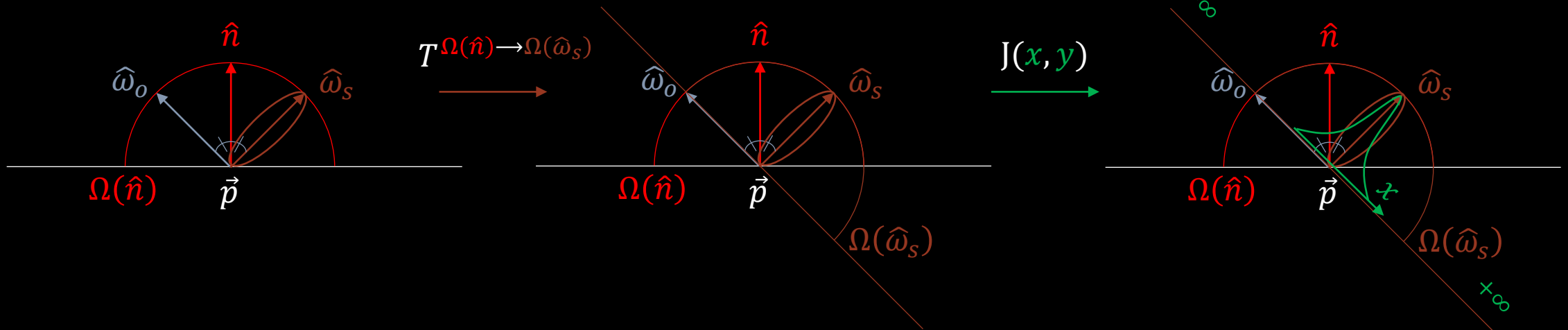
$$L_o(\vec{p}, \hat{\omega}_o) \approx B L_v(\vec{p}, \hat{\omega}_s, \alpha) \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

↙ rays to cone

$$L_o(\vec{p}, \hat{\omega}_o) \approx N L_v(\vec{p}, \hat{\omega}_s, \alpha)$$

# Voxel Cone Tracing: Specular Illumination

$$\hat{\omega}_s = 2(\hat{\omega}_o \cdot \hat{n})\hat{n} - \hat{\omega}_o$$



$$\begin{aligned} x &= \cos \varphi \sin \theta \\ y &= \sin \varphi \sin \theta \end{aligned} \quad \begin{aligned} \theta &= \sin^{-1} \sqrt{x^2 + y^2} \\ \varphi &= \cot^{-1} \frac{x}{y} = \tan^{-1} \frac{y}{x} \end{aligned}$$

$$J(x, y) = \begin{vmatrix} \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \\ \frac{\partial \varphi}{\partial x} & \frac{\partial \varphi}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{x}{\sqrt{1-x^2-y^2}\sqrt{x^2+y^2}} & \frac{y}{\sqrt{1-x^2-y^2}\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{vmatrix} = \frac{1}{\sqrt{1-x^2-y^2}\sqrt{x^2+y^2}}$$

$$J(x, y) = \frac{1}{J(\theta, \varphi)} = \frac{1}{\cos \theta \sin \theta}$$



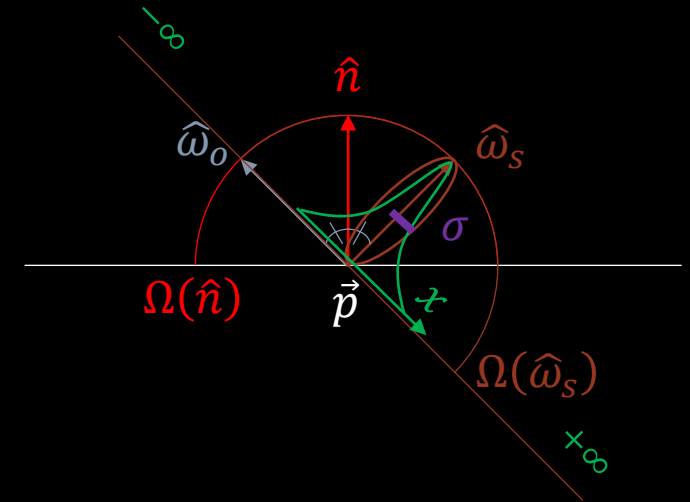
# Voxel Cone Tracing: Specular Illumination

$$G(x, y) = c e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2 \log(G(x, y))}{\partial^2 x} = \frac{\partial^2 \log(G(x, y))}{\partial^2 y} = \frac{-1}{\sigma^2}$$

$$\sin \frac{\alpha}{2} = \sigma \Rightarrow \tan \frac{\alpha}{2} = \sqrt{\frac{\sigma^2}{1-\sigma^2}}$$

$$\left. \frac{\partial^2 \log(f_{r,s}(x, y))}{\partial^2 x} \right|_{(x,y)=(0,0)} = \left. \frac{-1}{\sigma(x, y)^2} \right|_{(x,y)=(0,0)}$$



# Blinn-Phong BRDF

$$f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \frac{F_s}{\mathbf{N}} \left( \cos \frac{\theta}{2} \right)^{N_s}$$
$$f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \frac{F_s}{\mathbf{N}'} \left( \cos \frac{\theta}{2} \right)^{\frac{2}{r^4} - 2}$$

# I. BRDF Normalization

$$\forall \hat{\omega}_i \in \Omega: F_{r,s}(\vec{p}, \hat{\omega}_i) = \int_{\Omega} f_{r,s}(\vec{p}, \hat{\omega}_i, \hat{\omega}_o)(\hat{n} \cdot \hat{\omega}_o) d\hat{\omega}_o \leq 1$$

$$\forall \hat{\omega}_i \in \Omega: F_{r,s}(\vec{p}, \hat{\omega}_i) = \frac{F_S}{N} \int_{\Omega} (\hat{n} \cdot \hat{h})^{N_S} (\hat{n} \cdot \hat{\omega}_o) d\hat{\omega}_o \leq 1$$

$$\text{maximum: } \hat{n} = \hat{\omega}_i \Rightarrow \hat{n} \cdot \hat{h} = \cos \frac{\theta}{2}$$

$$\frac{2\pi}{N} \int_0^{\pi/2} \left( \cos \frac{\theta}{2} \right)^{N_S} \cos \theta \sin \theta d\theta = 1$$

$$\frac{N_S+6}{8\pi} < N = \frac{(N_S+2)(N_S+4)}{8\pi \binom{N_S+2}{2}} < \frac{N_S+8}{8\pi}$$

## II. BRDF/Helmholtz Reciprocity

$$\forall \hat{\omega}_i \in \Omega: F_{r,s}(\vec{p}, \hat{\omega}_i) = \int_{\Omega} f_{r,s}(\vec{p}, \hat{\omega}_i, \hat{\omega}_o)(\hat{n} \cdot \hat{\omega}_o) d\hat{\omega}_o \leq 1$$
$$f_{r,s}(\vec{p}, \hat{\omega}_i, \hat{\omega}_o) \equiv f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

$$\forall \hat{\omega}_o \in \Omega: F_{r,s}(\vec{p}, \hat{\omega}_o) = \int_{\Omega} f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i \leq 1$$

exclude  $F_S = c$  from  $f_{r,s}$

$$f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \hat{f}_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \text{pdf}_{\Omega}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

$$b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \hat{b}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \text{pdf}_{\Omega_s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

### III. Voxel Cone Tracing

$$L_o(\vec{p}, \hat{\omega}_o) = F_S \int_{\Omega} f_{r,s}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$L_o(\vec{p}, \hat{\omega}_o) = F_S \int_{\Omega} b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$L_o(\vec{p}, \hat{\omega}_o) \approx F_S B \int_{\Omega_S} L_v(\vec{p}, \hat{\omega}_i) (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

$$L_o(\vec{p}, \hat{\omega}_o) \approx F_S B L_v(\vec{p}, \hat{\omega}_s, \alpha) \int_{\Omega_S} (\hat{n} \cdot \hat{\omega}_i) d\hat{\omega}_i$$

rays to cone

$$L_o(\vec{p}, \hat{\omega}_o) \approx F_S L_v(\vec{p}, \hat{\omega}_s, \alpha)$$

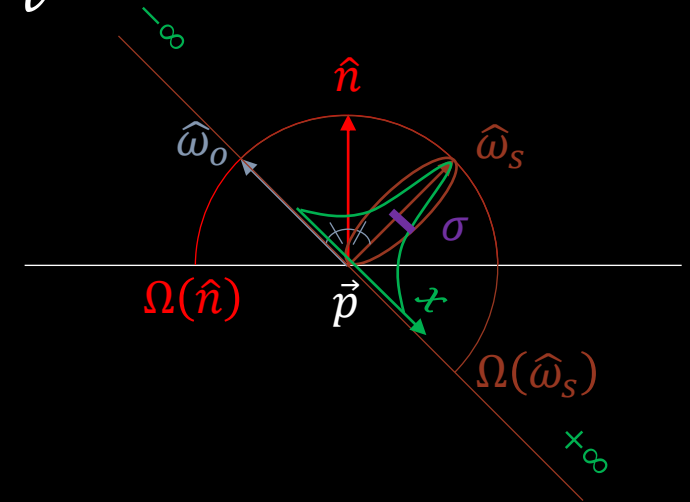
# III. Voxel Cone Tracing

$$a = 1 + \hat{v}_z \quad b = \hat{n}_z + \hat{n} \cdot \hat{v}$$

$$\sigma^2 \approx \frac{r^4}{1-r^4} \left( \frac{a^2 b}{2a\hat{n}_x\hat{v}_x + 2a^2\hat{n}_z - ab\hat{v}_z - \frac{3}{2}b\hat{v}_x^2} \right)$$

$$\sigma^2 \approx \frac{r^4}{1-r^4} \left( \frac{a^2 b}{2a\hat{n}_y\hat{v}_y + 2a^2\hat{n}_z - ab\hat{v}_z - \frac{3}{2}b\hat{v}_y^2} \right)$$

$$\sin \frac{\alpha}{2} = \sigma \Rightarrow \tan \frac{\alpha}{2} = \sqrt{\frac{\sigma^2}{1-\sigma^2}}$$



# III. Voxel Cone Tracing

$$\sigma^2 \approx \frac{r^4}{1-r^4} c_x$$

$$\sigma^2 \approx \frac{r^4}{1-r^4} c_y$$

$$\sin \frac{\alpha}{2} = \sigma \Rightarrow \tan \frac{\alpha}{2} = \sqrt{\frac{\sigma^2}{1-\sigma^2}}$$

$r$	$\sigma^\mp$	$\frac{\alpha}{2}$	$\tan \frac{\alpha}{2}$	1 <sup>st</sup> MIP
0	0	0	0	1
1	1	$\frac{\pi}{2}$	$+\infty$	$+\infty$



