Matthias Moulin

Rendering Algorithms

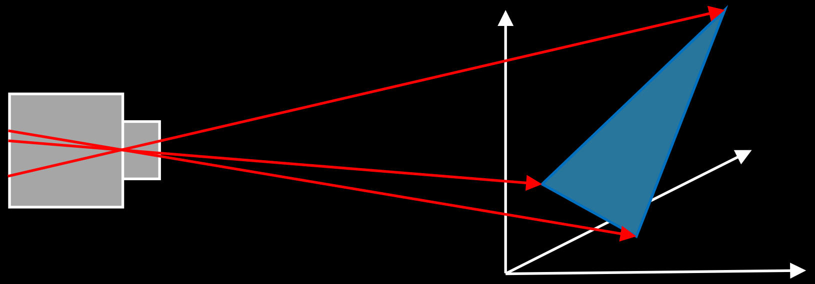
Ray Tracing

For each pixel do:

construct ray

For each geometric primitive do:

test hit(ray, primitive)



project image samples into the scene

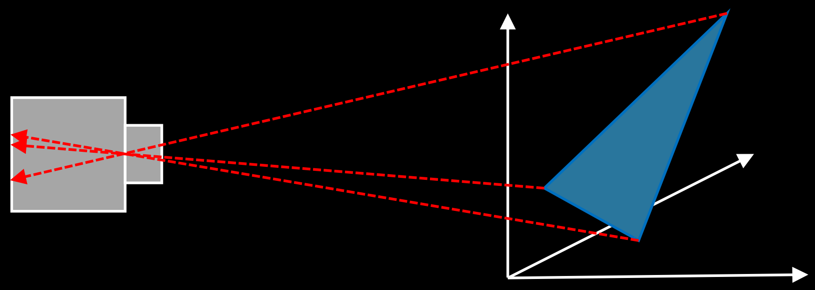
Rasterization

For each geometric primitive do:

project primitive

For each pixel do:

test hit(pixel, primitive)

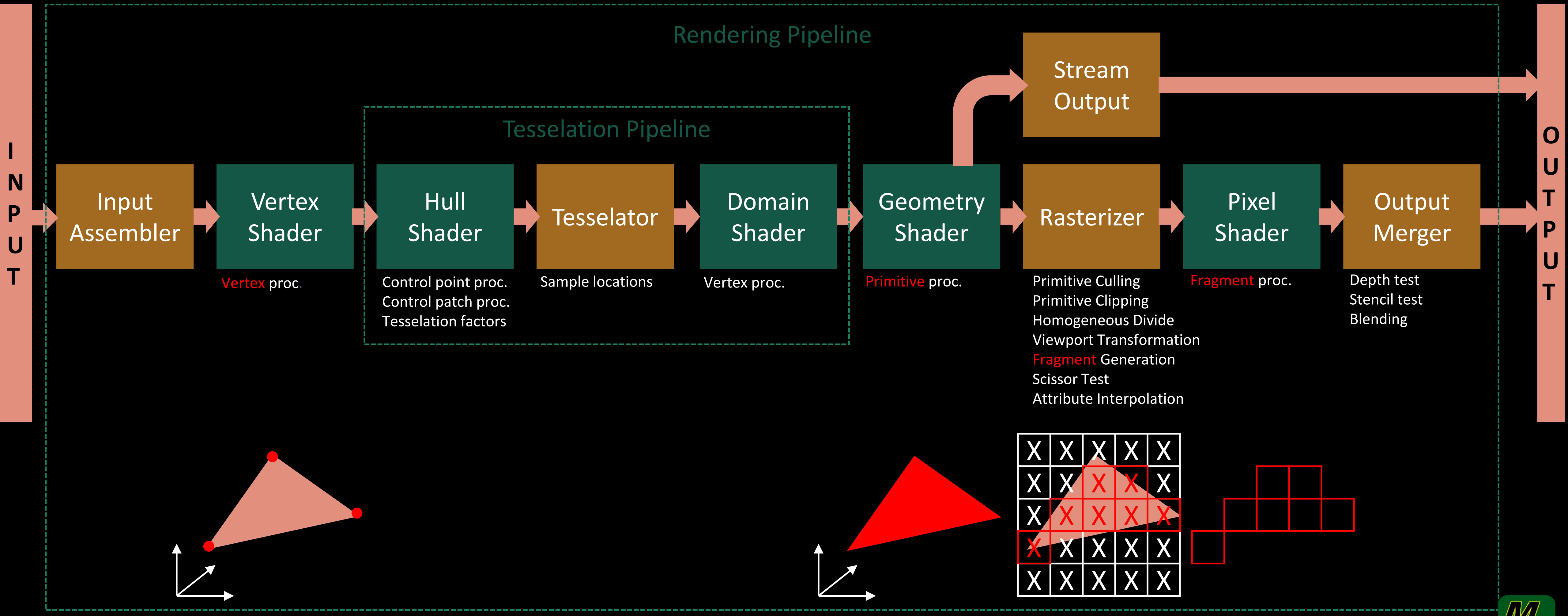


project geometry onto the image plane

Rendering Pipeline

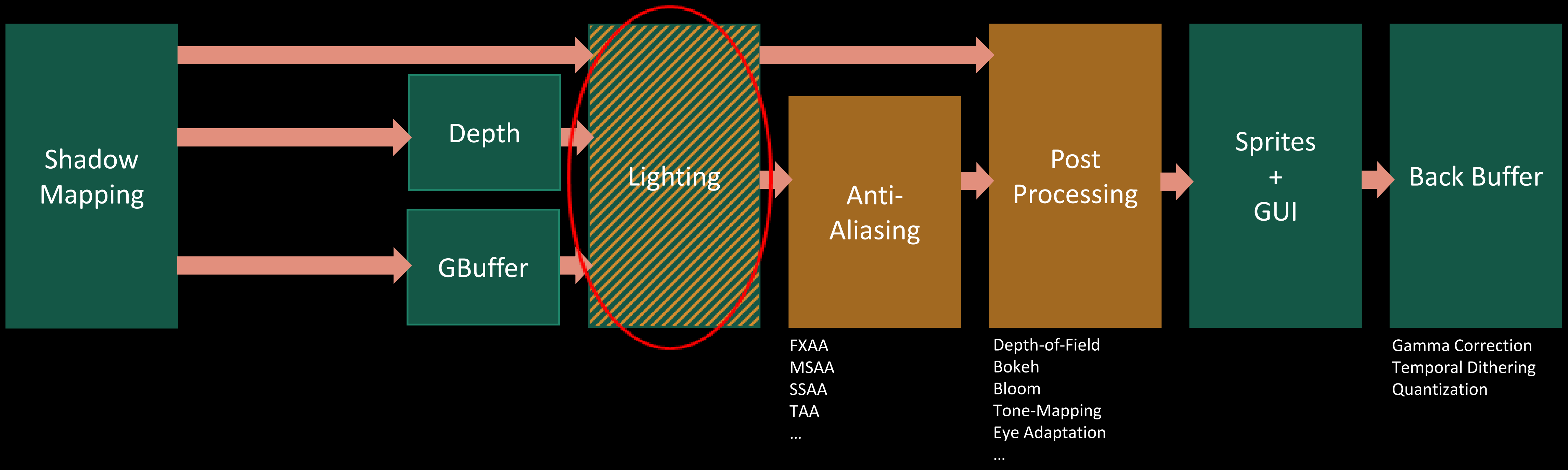
Programmable Stage

Configurable Stage



Rendering Engine Pipeline

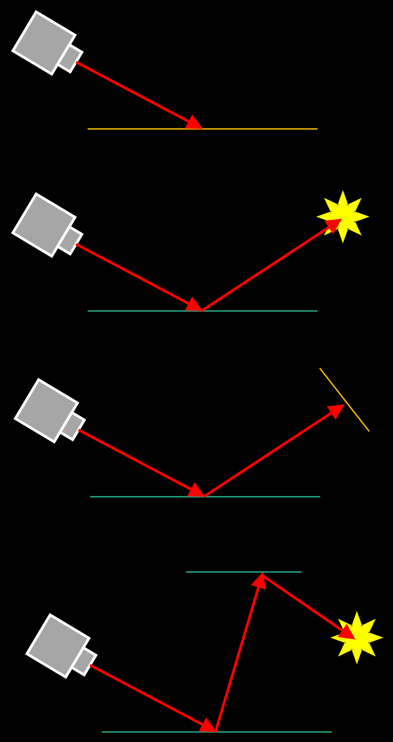
Rendering Pipeline
Compute Pipeline



Lighting

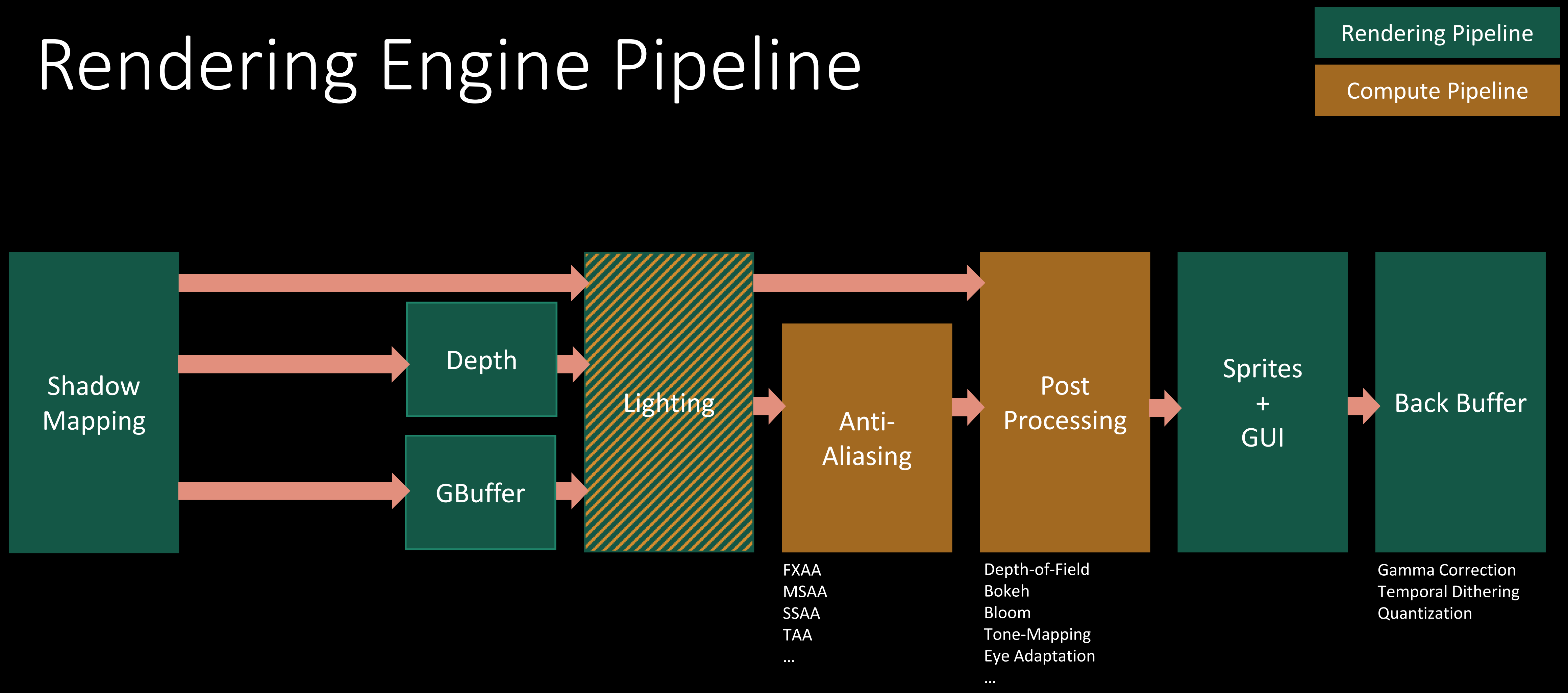
no pre-computation

- Self emission associated with the emissive surfaces
  - 0 bounces/surface interactions
- Direct illumination associated with the point lights
  - 1 bounce/surface interaction
- Direct illumination associated with the emissive surfaces
  - 1 bounce/surface interaction
- Indirect illumination associated with the point lights
  - 2 bounces/surface interactions
- …



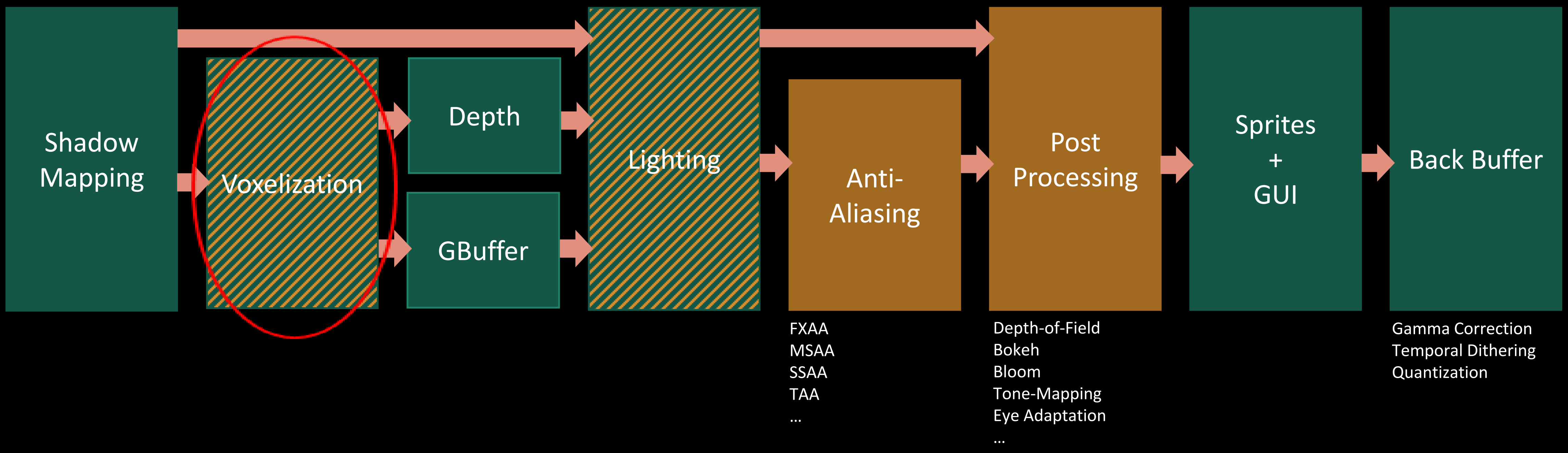
pre-computation

Rendering Engine Pipeline



Rendering Engine Pipeline

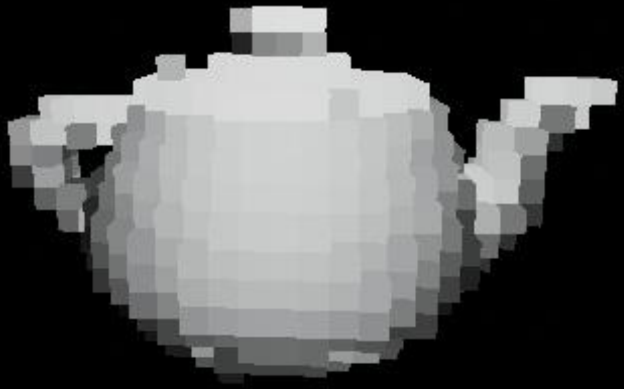
Rendering Pipeline
Compute Pipeline



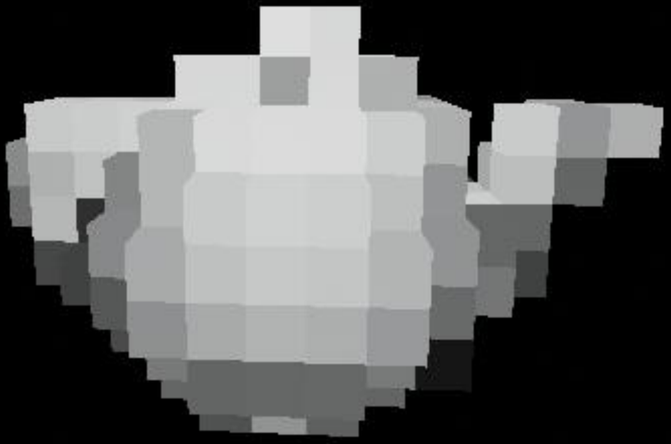
Voxelization

Grid $= 256^3$, Voxel $= 0.01^3$

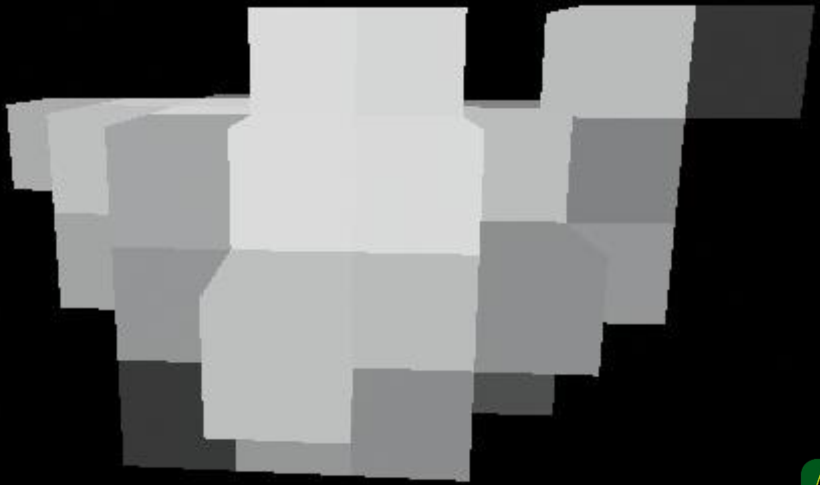
Grid $= 128^3$, Voxel $= 0.02^3$



Grid $= 64^3$, Voxel $= 0.04^3$



Grid $= 32^3$, Voxel $= 0.08^3$



Grid $= 16^3$, Voxel $= 0.16^3$

Voxelization: Voxel Data

- Radiance *distribution*

linear RGB (HDR)

R	G	B
float	float	float

12 bytes

$\longmapsto$

LogLuv

L hi	L lo	u	v
float	float	float	float

16 bytes

$\longmapsto$

LogLuv

L hi	L lo	u	v
byte	byte	byte	byte

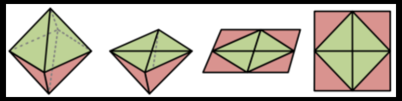
4 bytes

- Normal *distribution* (to support more bounces)

$\hat{n}_{\mathrm{world}}$

X	Y	Z
float	float	float

12 bytes



$\longmapsto$

Octahedral

a	b
float	float

16 bytes

$\longmapsto$

Octahedral

a	B
2 bytes	2 bytes

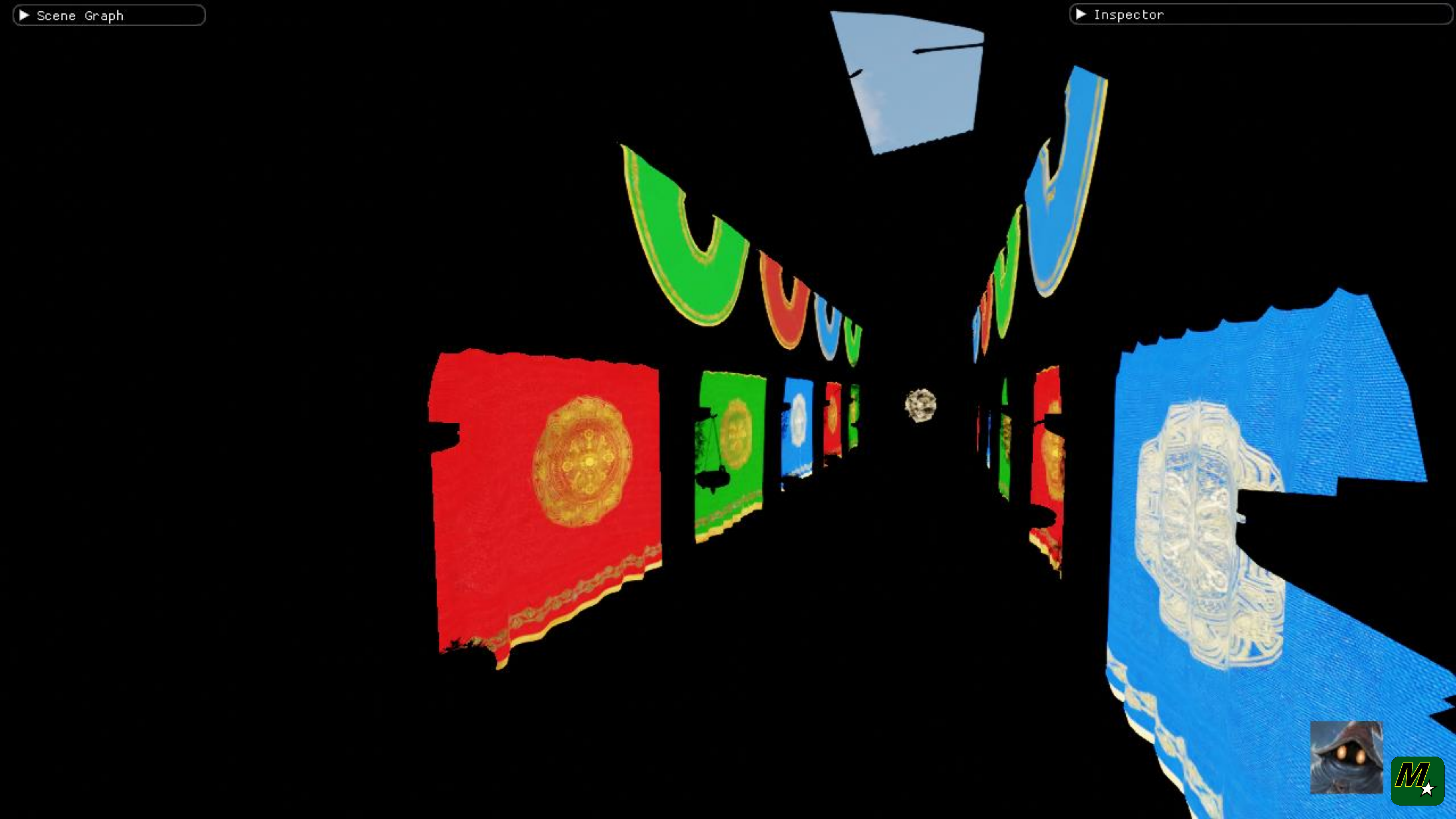
4 bytes

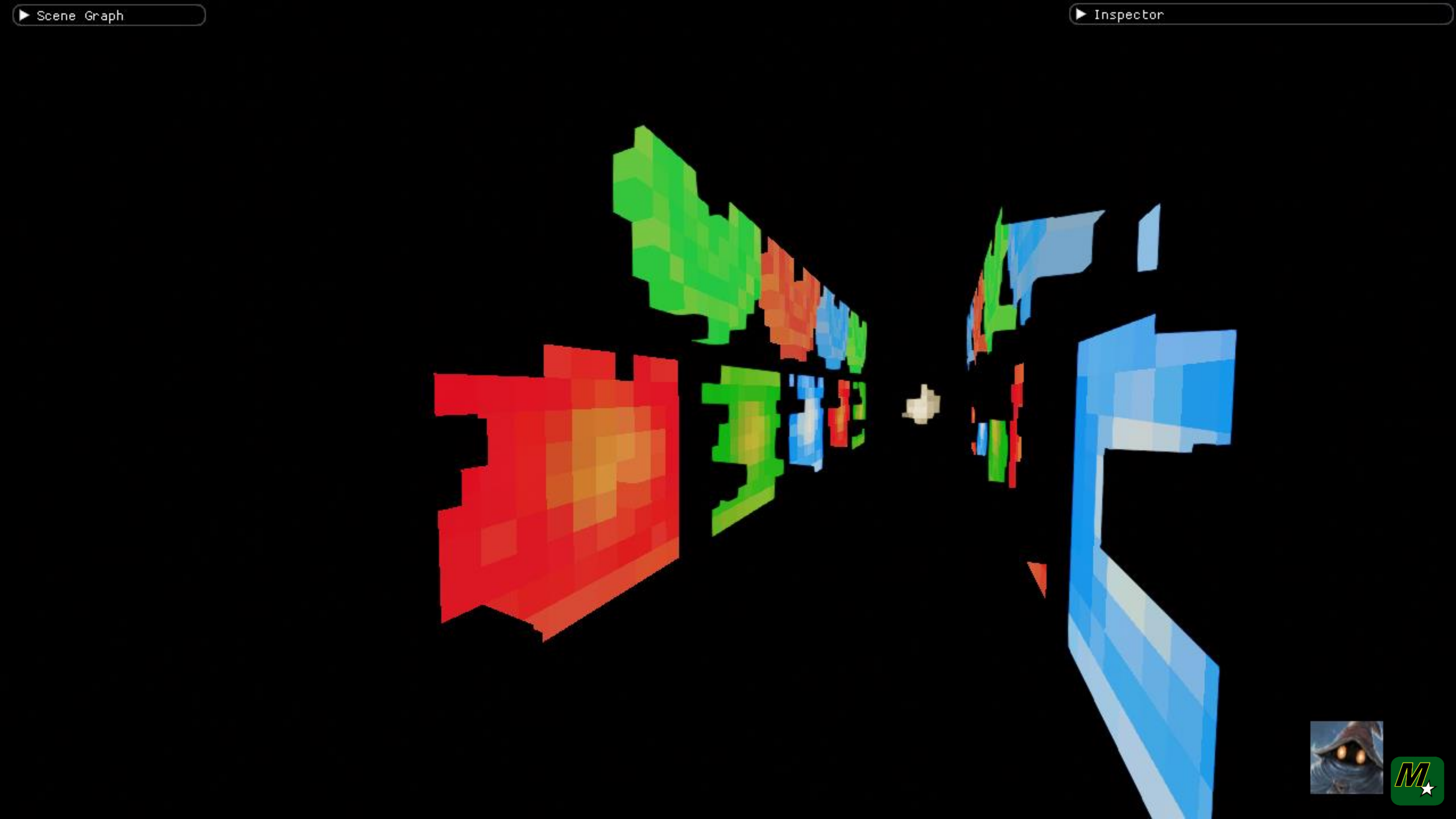

Scene Graph
Inspector


Scene Graph
Inspector


Scene Graph
Inspector

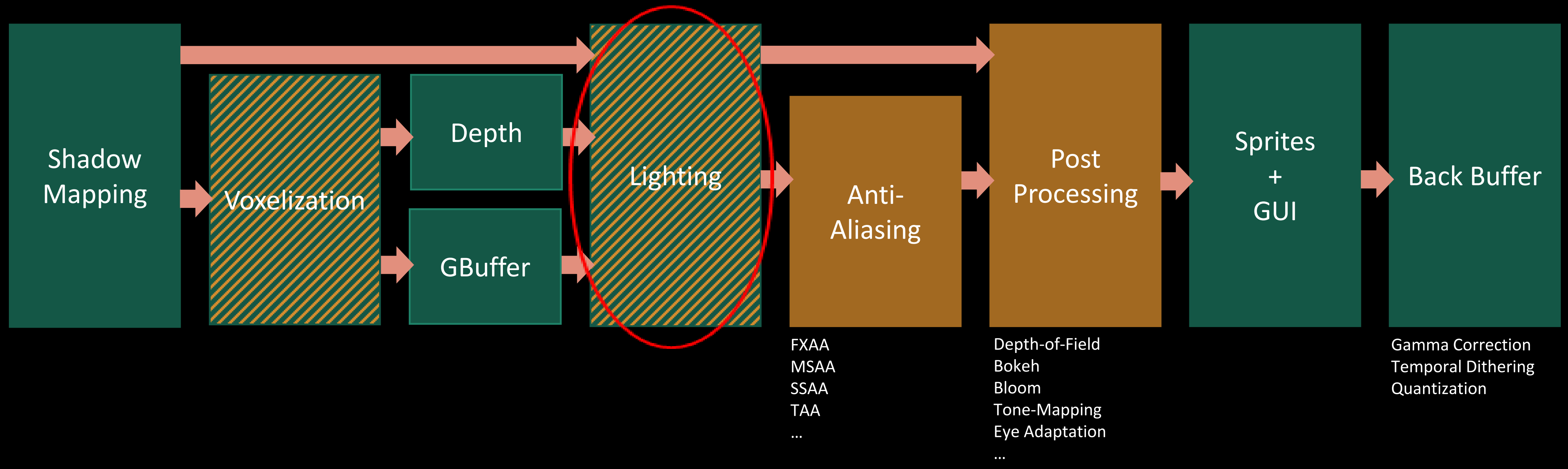

Scene Graph
Inspector


Scene Graph
Inspector


Scene Graph
Inspector

Rendering Engine Pipeline

Rendering Pipeline
Compute Pipeline



Voxel Cone Tracing: Ambient Occlusion

$$\mathrm{AO}(\vec{p},\widehat{\omega}_o) = \frac{1}{\pi}\int_{\Omega} \mathrm{AO_v}(\vec{p},\widehat{\omega}_i)(\hat{n}\cdot\widehat{\omega}_i)\,\mathrm{d}\widehat{\omega}_i$$

$$\mathrm{AO}(\vec{p},\widehat{\omega}_o) = \frac{1}{\pi}\sum_{j=1}^{N}\int_{\Omega_j} \mathrm{AO_v}\left(\vec{p},\widehat{\omega}_{j,i}\right)\left(\hat{n}\cdot\widehat{\omega}_{j,i}\right)\mathrm{d}\widehat{\omega}_{j,i}$$

rays to cones

$$\mathrm{AO}(\vec{p},\widehat{\omega}_o) \approx \frac{1}{\pi}\sum_{j=1}^{N} \mathrm{AO_v}\left(\vec{p},\widehat{\omega}_j,\alpha_j\right)\int_{\Omega_j}\left(\hat{n}\cdot\widehat{\omega}_{j,i}\right)\mathrm{d}\widehat{\omega}_{j,i}$$

$$\widehat{w}_j = \frac{1}{\pi}\int_{\Omega_j}\left(\vec{n}\cdot\widehat{\omega}_{j,i}\right)\mathrm{d}\widehat{\omega}_{j,i}$$

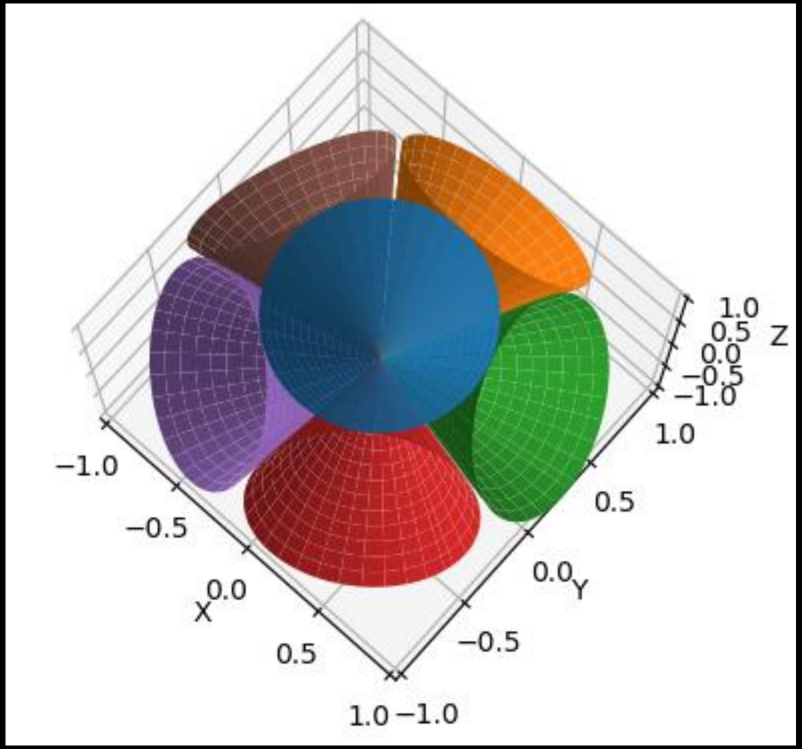
$$\mathrm{AO}(\vec{p},\widehat{\omega}_o) \approx \sum_{j=1}^{N}\widehat{w}_j\,\mathrm{AO_v}\left(\vec{p},\widehat{\omega}_j,\alpha_j\right)$$

Voxel Cone Tracing: Ambient Occlusion

$$\mathrm{AO}(\vec{p}, \hat{\omega}_o) \approx \sum_{j=1}^{N} \hat{w}_j \mathrm{AO_v}\left(\vec{p}, \hat{\omega}_j, \alpha_j\right)$$

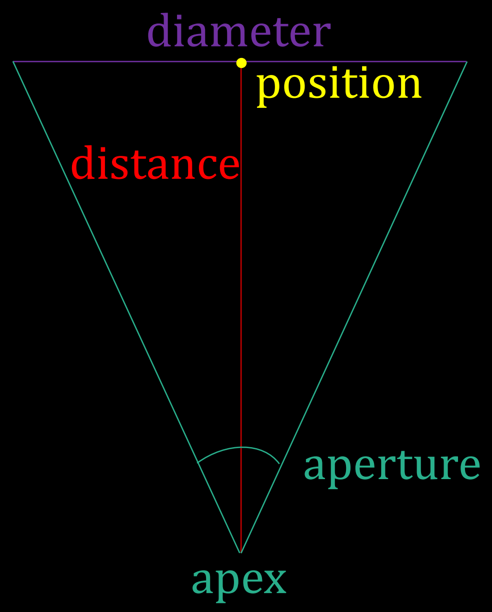
$$\hat{w}_0 = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/6} \cos\theta \sin\theta \, \mathrm{d}\theta \mathrm{d}\varphi = \frac{1}{4}$$

$$\hat{w}_1 = \hat{w}_2 = \hat{w}_3 = \hat{w}_4 = \hat{w}_5 \approx \frac{1 - \hat{w}_0}{5} = \frac{3}{20}$$

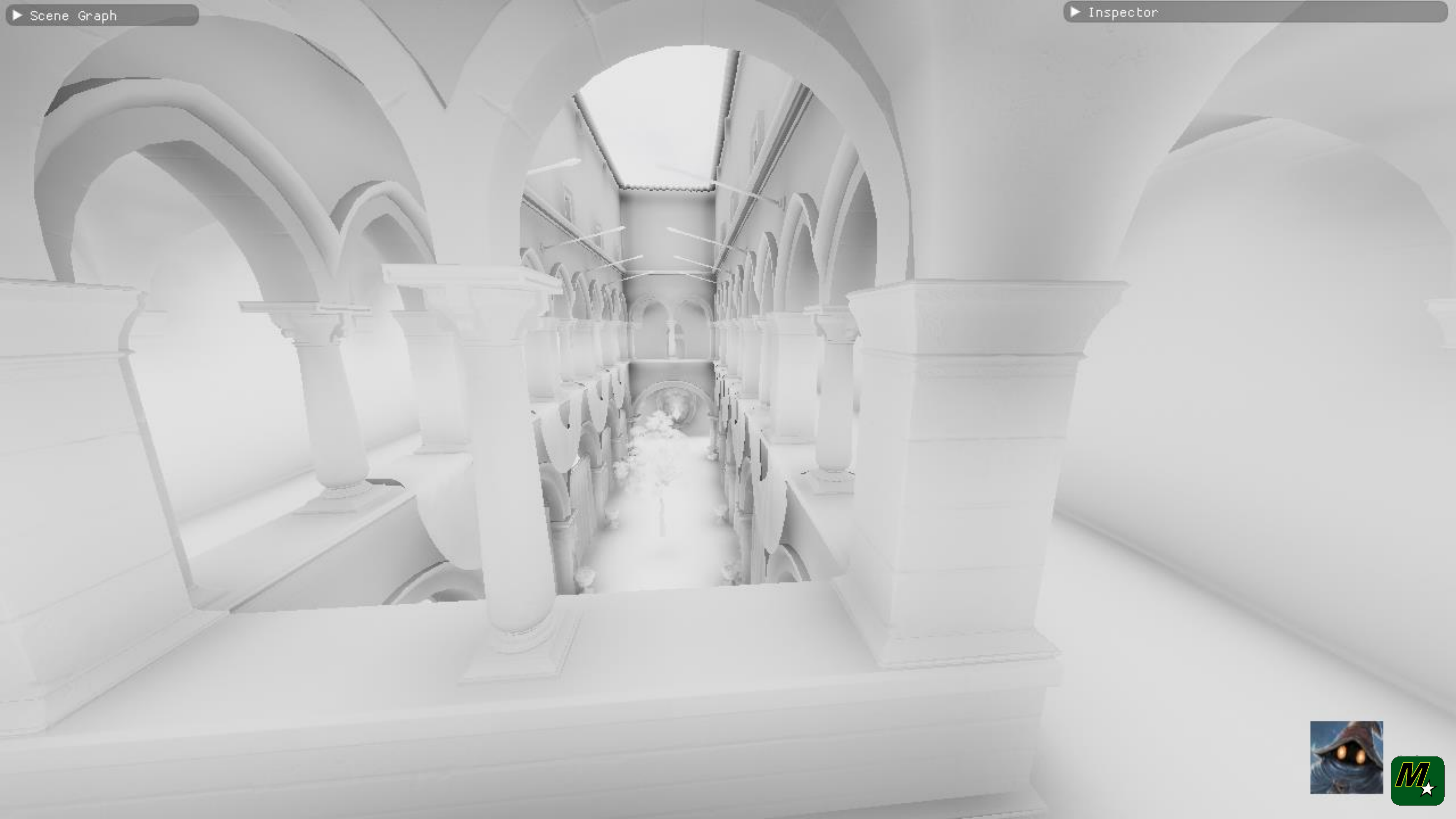


Voxel Cone Tracing: Ambient Occlusion

$\mathrm{AO_v} \leftarrow 0$
distance $\leftarrow \mathrm{voxel_{offset}}$
while $\mathrm{AO_v} < 1$ and distance $< \mathrm{distance_{AO}}$:
    compute diameter
    compute mip_level
    compute position at distance
    if mip_level $\geq$ max_mip_level or position $\notin [0,1]^3$:
        break
    sample $\mathrm{AO_{step}}$(position, mip_level)
    $\mathrm{AO_v} \leftarrow \mathrm{AO_v} + (1 - \mathrm{AO}_v)\, \mathrm{cone_{step}}\, \mathrm{AO_{step}}$
    distance $\leftarrow$ distance $+ \mathrm{cone_{step}}$ diameter
return $\mathrm{AO_v}$




Scene Graph
Inspector


Scene Graph
Inspector

Voxel Cone Tracing: Diffuse Illumination

$$L_o(\vec{p}, \widehat{\omega}_o) = \int_\Omega \mathrm{f}_{\mathrm{r,d}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) L_{\mathrm{v}}(\vec{p}, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)\, \mathrm{d}\widehat{\omega}_i$$

$$L_o(\vec{p}, \widehat{\omega}_o) = (1-m)\frac{k_d}{\pi} \sum_{j=1}^{N} \int_{\Omega_j} L_{\mathrm{v}}\left(\vec{p}, \widehat{\omega}_{j,i}\right)\left(\hat{n} \cdot \widehat{\omega}_{j,i}\right) \mathrm{d}\widehat{\omega}_{j,i}$$

rays to cones

$$L_o(\vec{p}, \widehat{\omega}_o) \approx (1-m)\frac{k_d}{\pi} \sum_{j=1}^{N} L_{\mathrm{v}}\left(\vec{p}, \widehat{\omega}_j, \alpha_j\right) \int_{\Omega_j}\left(\hat{n} \cdot \widehat{\omega}_{j,i}\right) \mathrm{d}\widehat{\omega}_{j,i}$$

$$\widehat{w}_j = \frac{1}{\pi} \int_{\Omega_j}\left(\vec{n} \cdot \widehat{\omega}_{j,i}\right) \mathrm{d}\widehat{\omega}_{j,i}$$

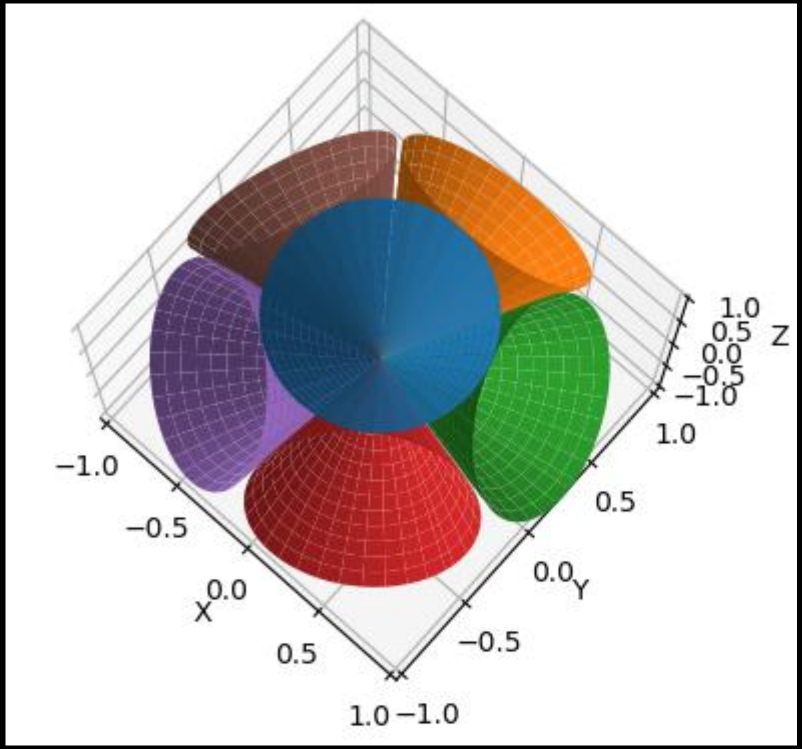
$$L_o(\vec{p}, \widehat{\omega}_o) \approx (1-m) k_d \sum_{j=1}^{N} \widehat{w}_j L_{\mathrm{v}}\left(\vec{p}, \widehat{\omega}_j, \alpha_j\right)$$

Voxel Cone Tracing: Diffuse Illumination

$$L_o(\vec{p}, \widehat{\omega}_o) \approx (1 - m)k_d \sum_{j=1}^{N} \widehat{w}_j L_v(\vec{p}, \widehat{\omega}_j, \alpha_j)$$

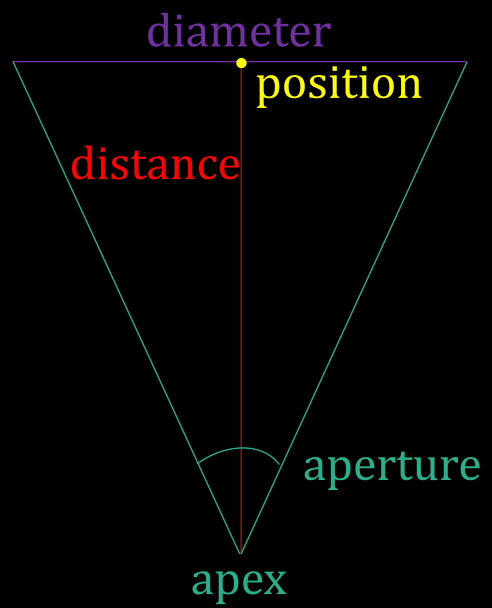
$$\widehat{w}_0 = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/6} \cos\theta \sin\theta \, d\theta d\varphi = \frac{1}{4}$$

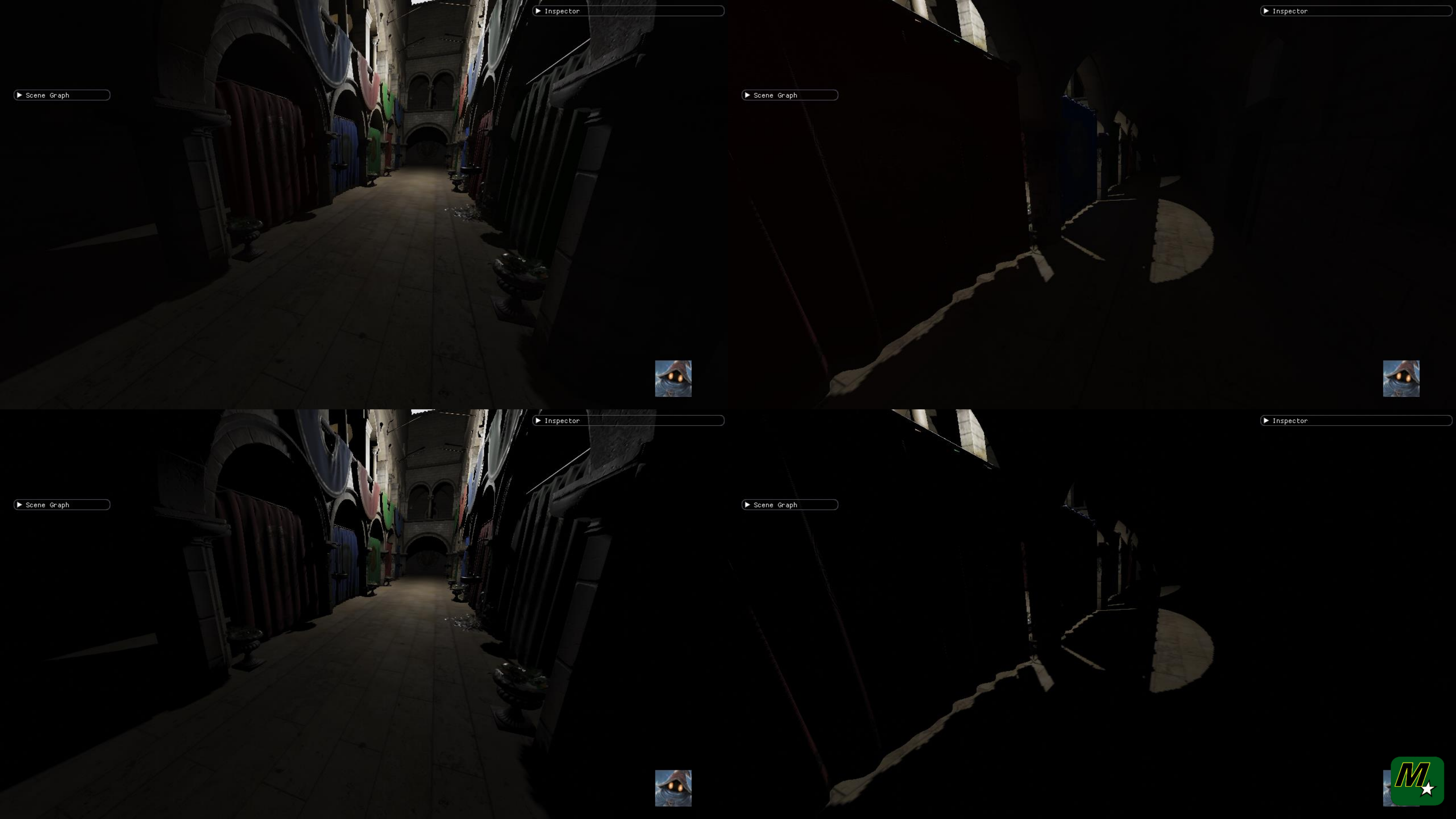
$$\widehat{w}_1 = \widehat{w}_2 = \widehat{w}_3 = \widehat{w}_4 = \widehat{w}_5 \approx \frac{1 - \widehat{w}_0}{5} = \frac{3}{20}$$



Voxel Cone Tracing: Marching in UVW Space

$L_\mathrm{v} \leftarrow [0, 0, 0, 0]$
distance $\leftarrow \mathrm{voxel_{offset}}$
while $L_\mathrm{v,a} < 1$:
  compute diameter
  compute mip_level
  compute position at distance
  if mip_level $\geq$ max_mip_level or position $\notin [0,1]^3$:
    break
  sample L_step(position, mip_level)
  $L_\mathrm{v,rgb} \leftarrow L_\mathrm{v,rgb} + (1 - L_\mathrm{v,a})\, \mathrm{cone_{step}}\, L_\mathrm{step,a}\, L_\mathrm{step}$
  $L_\mathrm{v,a} \leftarrow L_\mathrm{v,a} + (1 - L_\mathrm{v,a})\, \mathrm{cone_{step}}\, L_\mathrm{step,a}$
  distance $\leftarrow$ distance $+\ \mathrm{cone_{step}}$ diameter
return L_v




Inspector
Scene Graph
Inspector
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Scene Graph


Inspector
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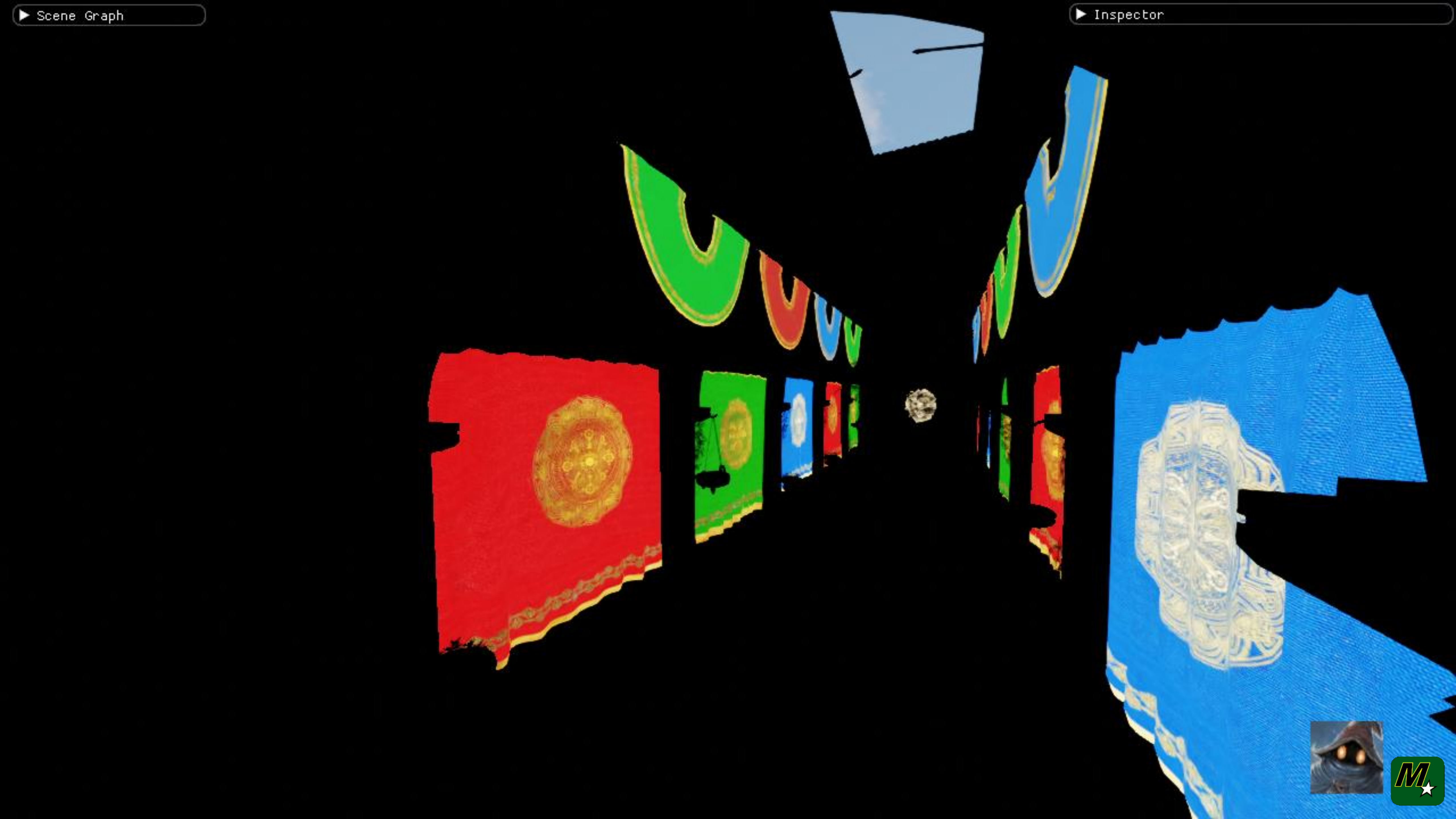
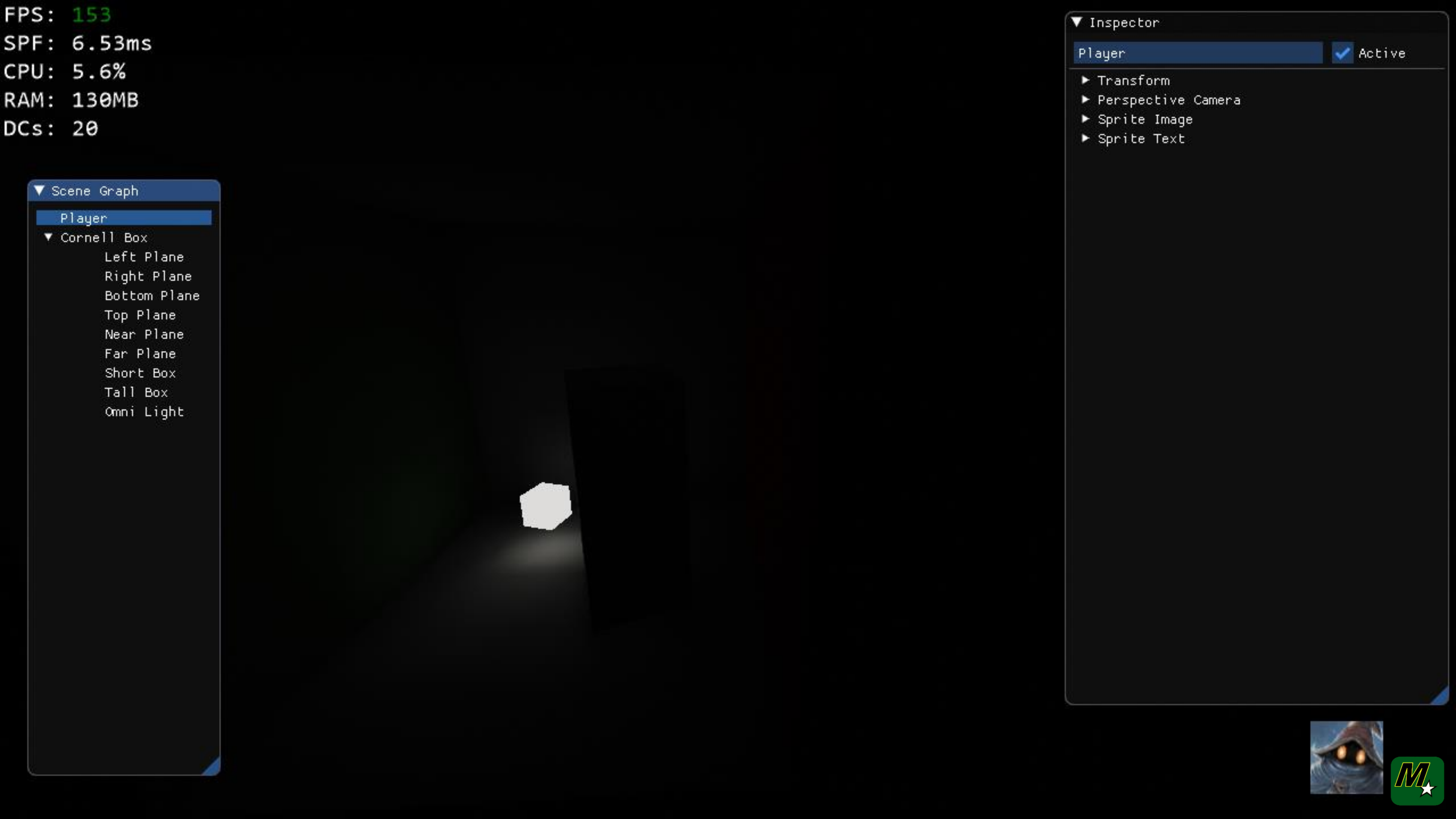
Inspector
Scene Graph

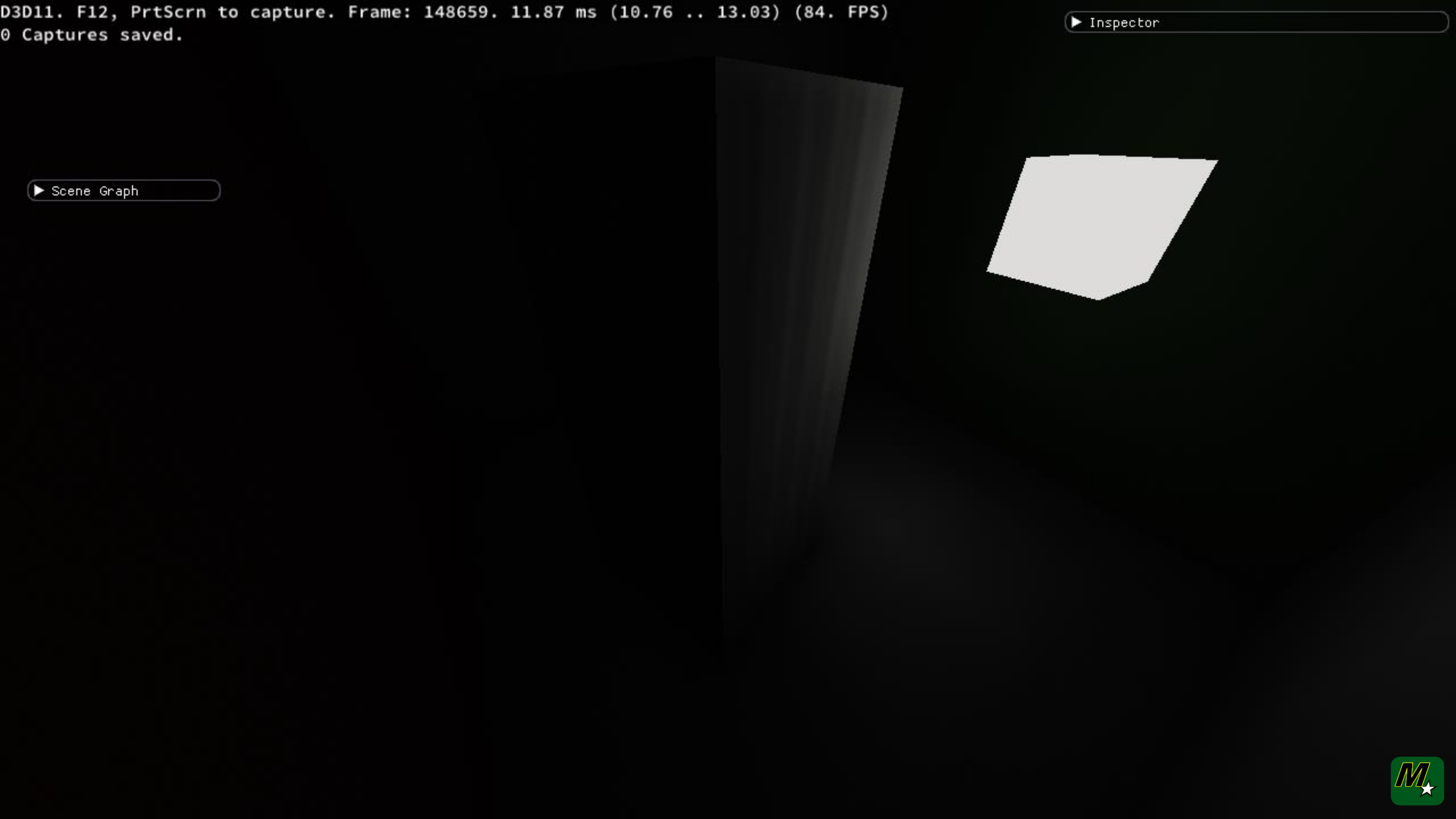


Scene Graph
Inspector

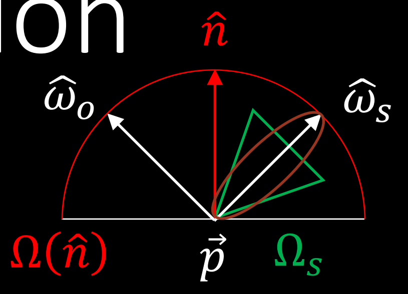

Scene Graph
Inspector


Inspector
Scene Graph


FPS: 153
SPF: 6.53ms
CPU: 5.6%
RAM: 130MB
DCs: 20
Scene Graph
Player
Cornell Box
Left Plane
Right Plane
Bottom Plane
Top Plane
Near Plane
Far Plane
Short Box
Tall Box
Omni Light
Inspector
Player
Active
Transform
Perspective Camera
Sprite Image
Sprite Text


D3D11. F12, PrtScrn to capture. Frame: 148659. 11.87 ms (10.76 .. 13.03) (84. FPS)
0 Captures saved.
Inspector
Scene Graph

Voxel Cone Tracing: Specular Illumination

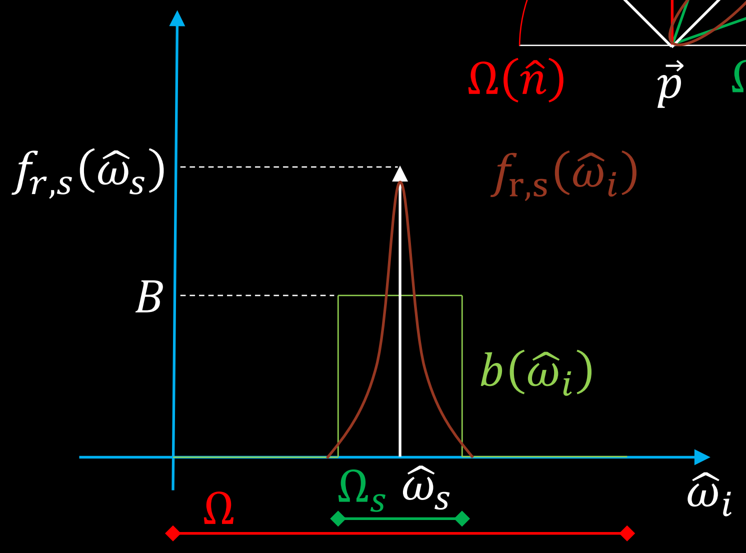


$$f_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \mathrm{N}\, \hat{f}_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

$$\int_{\Omega} f_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i)\, \mathrm{d}\hat{\omega}_i = \mathrm{N}$$

PDF

$$\int_{\Omega} \hat{f}_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i)\, \mathrm{d}\hat{\omega}_i = 1$$



$$b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i) = \mathrm{N}\, \hat{b}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)$$

$$\int_{\Omega} b(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i)\, \mathrm{d}\hat{\omega}_i = \mathrm{B} \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i)\, \mathrm{d}\hat{\omega}_i = \mathrm{N}$$

PDF

$$\int_{\Omega} \hat{b}(\vec{p}, \hat{\omega}_o, \hat{\omega}_i)(\hat{n} \cdot \hat{\omega}_i)\, \mathrm{d}\hat{\omega}_i = \hat{\mathrm{B}} \int_{\Omega_s} (\hat{n} \cdot \hat{\omega}_i)\, \mathrm{d}\hat{\omega}_i = 1$$

Voxel Cone Tracing: Specular Illumination

$$\begin{cases} \int_{\color{red}\Omega} \hat{f}_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i) \; \mathrm{d}\widehat{\omega}_i = \widehat{\mathrm{B}} \int_{\color{green}\Omega_s} (\hat{n} \cdot \widehat{\omega}_i) \; \mathrm{d}\widehat{\omega}_i = 1 \\ \int_{\color{red}\Omega} \hat{f}_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)^2 \; \mathrm{d}\widehat{\omega}_i = \widehat{\mathrm{B}} \int_{\color{green}\Omega_s} (\hat{n} \cdot \widehat{\omega}_i)^2 \; \mathrm{d}\widehat{\omega}_i \quad \text{1}^{\text{st}} \text{ moment} \\ \dots \end{cases}$$

$$M_k = \int_{\color{red}\Omega} \hat{f}_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)^k \; \mathrm{d}\widehat{\omega}_i = \widehat{\mathrm{B}} \int_{\color{green}\Omega_s} (\hat{n} \cdot \widehat{\omega}_i)^k \; \mathrm{d}\widehat{\omega}_i$$

$\Rightarrow \widehat{\mathrm{B}}$

$\Rightarrow {\color{green}\Omega_s}$

$\Rightarrow (\widehat{\omega}_s)$

Voxel Cone Tracing: Specular Illumination

$$M_k = \hat{\mathrm{B}} \int_{\Omega_s(\hat{n})} (\hat{n} \cdot \hat{\omega}_i)^k \, \mathrm{d}\hat{\omega}_i$$

$$M_k = \hat{\mathrm{B}} \int_{\Omega_s(\hat{\omega}_s)} (\hat{n} \cdot \hat{\omega}_i)^k \, \mathrm{d}\hat{\omega}_i$$

$$\Omega_s(\hat{n}) \approx \Omega_s(\hat{\omega}_s)$$

$$M_k \approx \hat{\mathrm{B}} \int_0^{2\pi} \int_0^{\alpha/2} \cos^k\theta \sin\theta \, \mathrm{d}\theta \mathrm{d}\varphi$$

$$M_k \approx 2\pi\hat{\mathrm{B}} \int_0^{\alpha/2} \cos^k\theta \sin\theta \, \mathrm{d}\theta$$

$$\boxed{M_0} = 1 \approx \frac{\pi\hat{\mathrm{B}}(1-\cos\alpha)}{2} = \pi\hat{\mathrm{B}}\left(1 - \cos^2(\alpha/2)\right)$$

$$\boxed{M_1} \approx \frac{2\pi\hat{\mathrm{B}}\left(1-\cos^3(\alpha/2)\right)}{3} = \frac{2\left(1-\cos^3(\alpha/2)\right)}{3\left(1-\cos^2(\alpha/2)\right)}$$

Normalized BRDF

Voxel Cone Tracing: Specular Illumination

$$L_{\mathrm{o}}(\vec{p}, \widehat{\omega}_o) = \int_{\Omega} f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) L_{\mathrm{v}}(\vec{p}, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)\, \mathrm{d}\widehat{\omega}_i$$

$$L_{\mathrm{o}}(\vec{p}, \widehat{\omega}_o) = \int_{\Omega} b(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) L_{\mathrm{v}}(\vec{p}, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)\, \mathrm{d}\widehat{\omega}_i$$

$$L_{\mathrm{o}}(\vec{p}, \widehat{\omega}_o) \approx \mathrm{B} \int_{\Omega_s} L_{\mathrm{v}}(\vec{p}, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)\, \mathrm{d}\widehat{\omega}_i$$

rays to cone

$$L_{\mathrm{o}}(\vec{p}, \widehat{\omega}_o) \approx \mathrm{B}\, L_{\mathrm{v}}(\vec{p}, \widehat{\omega}_s, \alpha) \int_{\Omega_s} (\hat{n} \cdot \widehat{\omega}_i)\, \mathrm{d}\widehat{\omega}_i$$

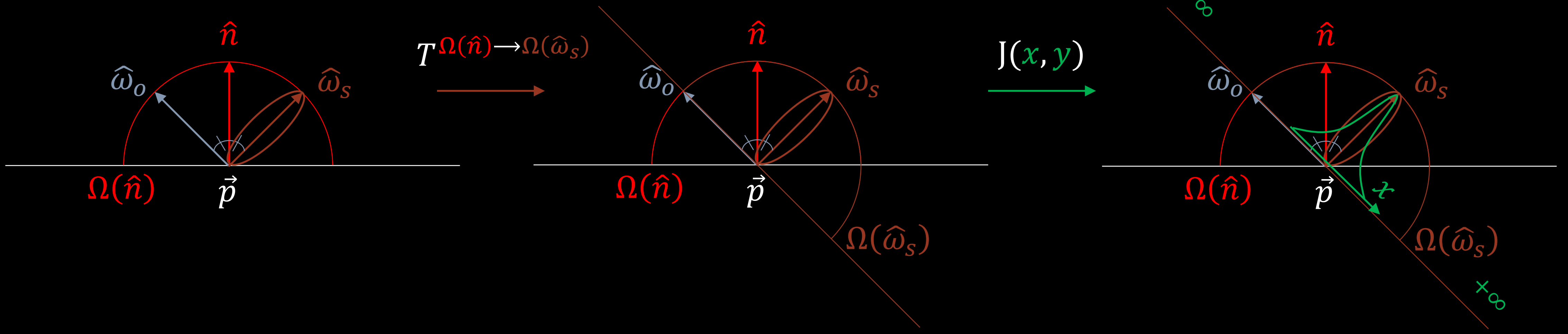
$$L_{\mathrm{o}}(\vec{p}, \widehat{\omega}_o) \approx \mathrm{N}\, L_{\mathrm{v}}(\vec{p}, \widehat{\omega}_s, \alpha)$$

Voxel Cone Tracing: Specular Illumination

$$\hat{\omega}_s = 2(\hat{\omega}_o \cdot \hat{n})\hat{n} - \hat{\omega}_o$$

$$x = \cos\varphi \sin\theta \qquad \theta = \sin^{-1}\sqrt{x^2 + y^2}$$

$$y = \sin\varphi \sin\theta \qquad \varphi = \cot^{-1}\frac{x}{y} = \tan^{-1}\frac{y}{x}$$



$$J(x, y) = \begin{vmatrix} \dfrac{\partial\theta}{\partial x} & \dfrac{\partial\theta}{\partial y} \\ \dfrac{\partial\varphi}{\partial x} & \dfrac{\partial\varphi}{\partial y} \end{vmatrix} = \begin{vmatrix} \dfrac{x}{\sqrt{1 - x^2 - y^2}\sqrt{x^2 + y^2}} & \dfrac{y}{\sqrt{1 - x^2 - y^2}\sqrt{x^2 + y^2}} \\ \dfrac{-y}{x^2 + y^2} & \dfrac{x}{x^2 + y^2} \end{vmatrix} = \frac{1}{\sqrt{1 - x^2 - y^2}\sqrt{x^2 + y^2}}$$

$$J(x, y) = \frac{1}{J(\theta, \varphi)} = \frac{1}{\cos\theta \sin\theta}$$

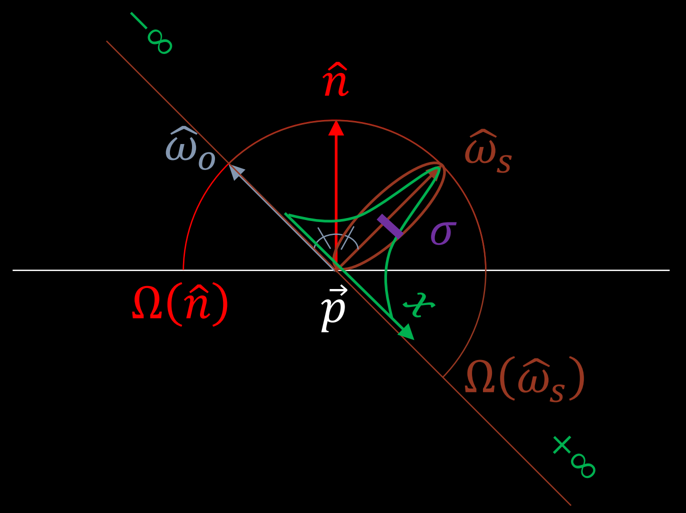
Voxel Cone Tracing: Specular Illumination

$$G(x,y) = \mathrm{c}\, e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2 \log(G(x,y))}{\partial^2 x} = \frac{\partial^2 \log(G(x,y))}{\partial^2 y} = \frac{-1}{\sigma^2}$$

$$\sin\frac{\alpha}{2} = \sigma \Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{\sigma^2}{1-\sigma^2}}$$

$$\left.\frac{\partial^2 \log\left(f_{\mathrm{r,s}}(x,y)\right)}{\partial^2 x}\right|_{(x,y)=(0,0)} = \left.\frac{-1}{\sigma(x,y)^2}\right|_{(x,y)=(0,0)}$$



Blinn-Phong BRDF

$$f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) = \frac{\mathrm{F_S}}{\mathrm{N}}\left(\cos\frac{\theta}{2}\right)^{\mathrm{N_S}}$$

$$f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) = \frac{\mathrm{F_S}}{\mathrm{N}'}\left(\cos\frac{\theta}{2}\right)^{\frac{2}{r^4}-2}$$

I. BRDF Normalization

$$\forall \hat{\omega}_i \in \Omega: F_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_i) = \int_\Omega f_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_i, \hat{\omega}_o)(\hat{n} \cdot \hat{\omega}_o)\mathrm{d}\hat{\omega}_o \leq 1$$

$$\forall \hat{\omega}_i \in \Omega: F_{\mathrm{r,s}}(\vec{p}, \hat{\omega}_i) = \frac{\mathrm{F_S}}{\mathrm{N}} \int_\Omega \left(\hat{n} \cdot \hat{h}\right)^{\mathrm{N_S}} (\hat{n} \cdot \hat{\omega}_o)\mathrm{d}\hat{\omega}_o \leq 1$$

$$\text{maximum: } \hat{n} = \hat{\omega}_i \Rightarrow \hat{n} \cdot \hat{h} = \cos\frac{\theta}{2}$$

$$\frac{2\pi}{\mathrm{N}} \int_0^{\pi/2} \left(\cos\frac{\theta}{2}\right)^{\mathrm{N_S}} \cos\theta \sin\theta \, \mathrm{d}\theta = 1$$

$$\frac{\mathrm{N_S}+6}{8\pi} < \mathrm{N} = \frac{(\mathrm{N_S}+2)(\mathrm{N_S}+4)}{8\pi\left(\mathrm{N_S}+2^{\frac{-\mathrm{N_S}}{2}}\right)} < \frac{\mathrm{N_S}+8}{8\pi}$$

II. BRDF/Helmholtz Reciprocity

$$\forall \widehat{\omega}_i \in \Omega: F_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_i) = \int_\Omega f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_i, \widehat{\omega}_o)(\hat{n} \cdot \widehat{\omega}_o)\mathrm{d}\widehat{\omega}_o \leq 1$$

$$f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_i, \widehat{\omega}_o) \equiv f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)$$

$$\forall \widehat{\omega}_o \in \Omega: F_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o) = \int_\Omega f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)(\hat{n} \cdot \widehat{\omega}_i)\mathrm{d}\widehat{\omega}_i \leq 1$$

exclude $\mathrm{F_S} = \mathrm{c}$ from $f_{\mathrm{r,s}}$

$$f_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) = \hat{f}_{\mathrm{r,s}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) = \mathrm{pdf}_{\Omega}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)$$

$$b(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) = \hat{b}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i) = \mathrm{pdf}_{\Omega_{\mathrm{S}}}(\vec{p}, \widehat{\omega}_o, \widehat{\omega}_i)$$

III. Voxel Cone Tracing

$$L_{\mathrm{o}}(\vec{p},\widehat{\omega}_o) = \mathrm{F_S} \int_{\Omega} f_{\mathrm{r,s}}(\vec{p},\widehat{\omega}_o,\widehat{\omega}_i) L_{\mathrm{v}}(\vec{p},\widehat{\omega}_i)(\hat{n}\cdot\widehat{\omega}_i)\,\mathrm{d}\widehat{\omega}_i$$

$$L_{\mathrm{o}}(\vec{p},\widehat{\omega}_o) = \mathrm{F_S} \int_{\Omega} b(\vec{p},\widehat{\omega}_o,\widehat{\omega}_i) L_{\mathrm{v}}(\vec{p},\widehat{\omega}_i)(\hat{n}\cdot\widehat{\omega}_i)\,\mathrm{d}\widehat{\omega}_i$$

$$L_{\mathrm{o}}(\vec{p},\widehat{\omega}_o) \approx \mathrm{F_S}\,\mathrm{B} \int_{\Omega_s} L_{\mathrm{v}}(\vec{p},\widehat{\omega}_i)(\hat{n}\cdot\widehat{\omega}_i)\,\mathrm{d}\widehat{\omega}_i$$

rays to cone

$$L_{\mathrm{o}}(\vec{p},\widehat{\omega}_o) \approx \mathrm{F_S}\,\mathrm{B}\, L_{\mathrm{v}}(\vec{p},\widehat{\omega}_s,\alpha) \int_{\Omega_s} (\hat{n}\cdot\widehat{\omega}_i)\,\mathrm{d}\widehat{\omega}_i$$

$$L_{\mathrm{o}}(\vec{p},\widehat{\omega}_o) \approx \mathrm{F_S}\, L_{\mathrm{v}}(\vec{p},\widehat{\omega}_s,\alpha)$$

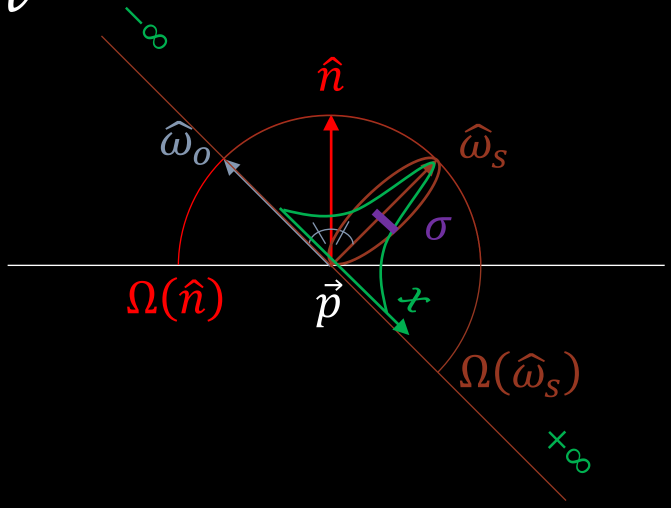
III. Voxel Cone Tracing

$$a = 1 + \hat{v}_z \qquad b = \hat{n}_z + \hat{n}\cdot\hat{v}$$

$$\sigma^2 \approx \frac{r^4}{1-r^4}\left(\frac{a^2 b}{2a\hat{n}_x\hat{v}_x + 2a^2\hat{n}_z - ab\hat{v}_z - \frac{3}{2}b{\hat{v}_x}^2}\right)$$

$$\sigma^2 \approx \frac{r^4}{1-r^4}\left(\frac{a^2 b}{2a\hat{n}_y\hat{v}_y + 2a^2\hat{n}_z - ab\hat{v}_z - \frac{3}{2}b{\hat{v}_y}^2}\right)$$

$$\sin\frac{\alpha}{2} = \sigma \Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{\sigma^2}{1-\sigma^2}}$$



III. Voxel Cone Tracing

$$\sigma^2 \approx \frac{r^4}{1-r^4} c_x$$

$$\sigma^2 \approx \frac{r^4}{1-r^4} c_y$$

$$\sin\frac{\alpha}{2} = \sigma \Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{\sigma^2}{1-\sigma^2}}$$

r	$\sigma^{\mp}$	$\frac{\alpha}{2}$	$\tan\frac{\alpha}{2}$	1st MIP
0	0	0	0	1
1	1	$\frac{\pi}{2}$	$+\infty$	$+\infty$