Module 9: Panel Data

Fall 2021

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Gov 2003 (Harvard)

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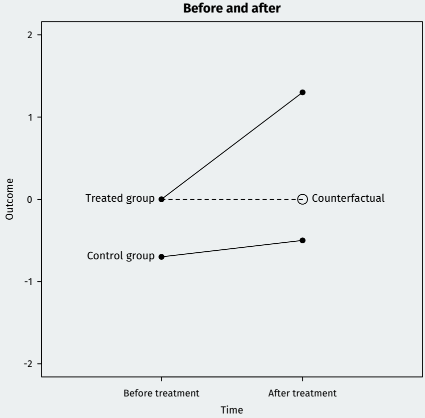
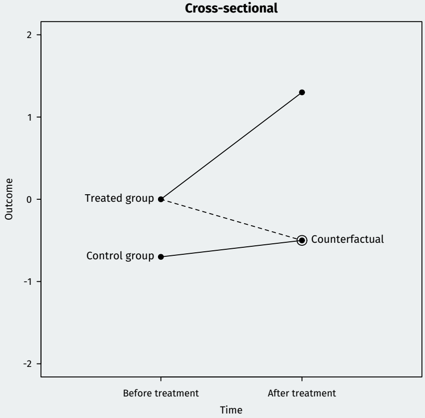
    - Exploit **cross-sectional** variation in treatment.
    - Exploit variation in treatment **within a unit over time** (before/after)

Cross-sectional vs before/after





1/ Difference in differences

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- Look at eastern PA and NJ fast food restaurants.
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    - Most likely to be affected by the change.

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- Part (a) is just a conditional average of observed data $\rightsquigarrow$ identified.
- Part (b) is a counterfactual: what would the average outcome in the treated group have been if it have been in control?

Three control strategies

$$\tau_{ATT} = \mathbb{E}[Y_{i1}(1)|G_i = 1] - \mathbb{E}[Y_{i1}(0)|G_i = 1]$$

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- **Cross-sectional design**
  - Assumption: mean independence of treatment

$$\mathbb{E}[Y_{i1}(0)|G_i = 1] = \mathbb{E}[Y_{i1}(0)|G_i = 0]$$

  - Use post-treatment control group:

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- **Before-and-after design**
    - Assumption: no trends

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- Use pre-period outcome in treated group:

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- **Difference-in-differences**:
    - Assumption: parallel trends

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    - Use pre-period treated outcome plus trend in control group:

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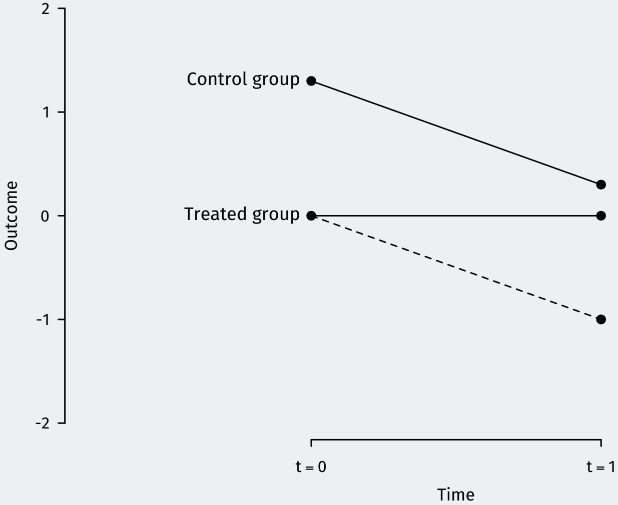
  - Allows for (common) secular trends in the outcome over time (unlike FE).
- Not invariant to nonlinear transformations!
  - Parallel trends for Y_{it} implies non-parallel trends for $\log(Y_{it})$ and vice versa.

Parallel trends in a graph



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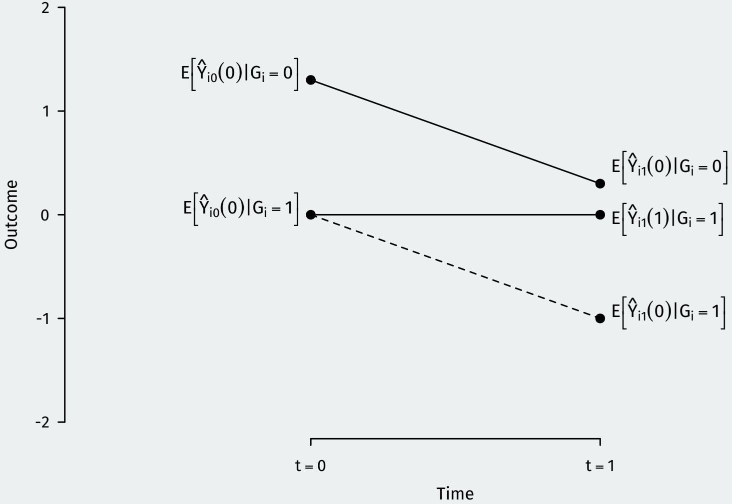

Identification

- Identification result:

$$\tau_{ATT} = (\mathbb{E}[Y_{i1}|G_i = 1] - \mathbb{E}[Y_{i0}|G_i = 1]) \\ - (\mathbb{E}[Y_{i1}|G_i = 0] - \mathbb{E}[Y_{i0}|G_i = 0])$$

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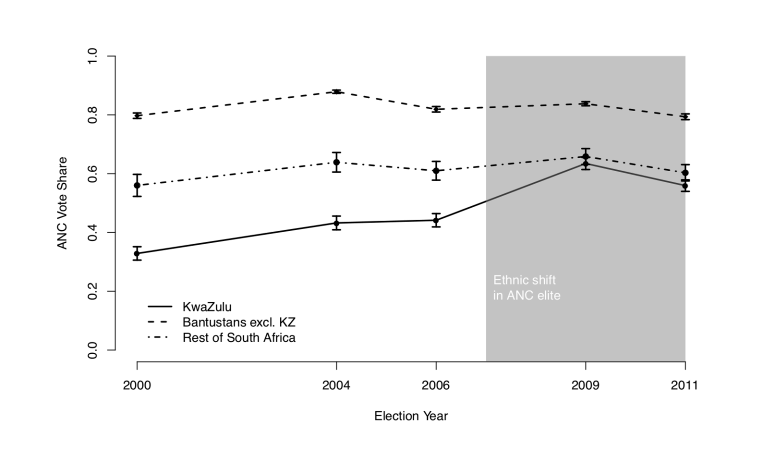
- Threat to identification: non-parallel trends
  - **unmeasured time-varying confounding**
  - **Ashenfelter's dip**: empirical finding that people who enroll in job training programs see their earnings decline prior to that training.
- Falsification test: check pre-treatment parallel trends.
  - Doesn't imply parallel trends hold for the post-period however!

Checking parallel trends (de Kadt/Larreguy, 2018)



Estimation

- Estimation with panel data:

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- Regression implementation:
  - Regress $\Delta Y_i = Y_{i1} - Y_{i0}$ on G_i.

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- Also possible to use DID on repeated cross sections.

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  - Basically, TWFE is an odd weighted average of DID effects with sometimes negative weights.

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  - LDV: previous outcome directly affects treatment assignment.

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- **Matching**: conduct DID analysis on units with similar values of $\mathbf{X}_i$

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    - Possible model misspecification!

2/ Fixed effects

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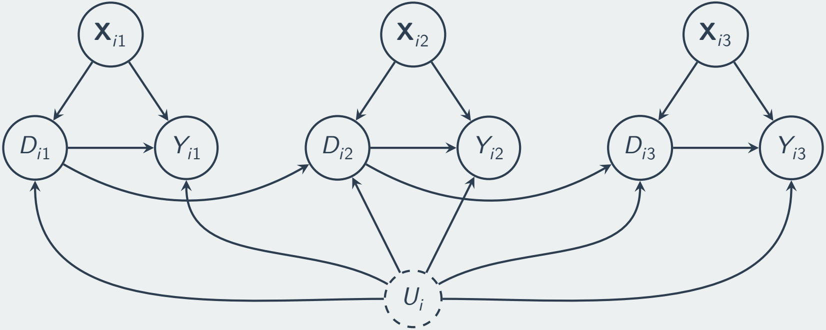
    - $\rightsquigarrow$ lags of treatments become part of time-varying confounders.

Strict exogeneity DAG



Strict exogeneity implied by strict ignorability $Y_{it}(d) \perp\!\!\!\perp \overline{D}_i \mid \overline{X}_i, U_i$

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