Module 6: Noncompliance and Instrumental Variables

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Gov 2003 (Harvard)

Where are we? Where are we going?

- We've covered randomized experiments (no confounding).
- We've covered selection on observables (no unmeasured confounding).
- What if there is unmeasured confounding? What can we do?
- First approach we'll explore: instrumental variables.
    - First: motivate IV through experiments and noncompliance.
    - Then: how does this relate to classical econometric methods like TSLS?

1/ Randomized experiments with noncompliance

Noncompliance

- GOTV experiment with door-to-door canvassing.
- Households are randomized so treatment assignment is unconfounded.
  - $Z_i = 1$ for assigned to treatment (canvassing attempted),
  - $Z_i = 0$ for assigned to control (no canvassing attempted).
- **Noncompliance**: units don't follow treatment assignment.
  - Units assigned to treatment take control or vice versa.
  - $D_i = 1$ for actually took treatment (heard canvasser message).
  - $D_i = 0$ for actually took control (didn't answer the door).
  - Full compliance means $Z_i = D_i$ for all i

How to handle noncompliance

- Two approaches common seen in applied studies.
- **Intent-to-treat** analysis (ITT): just use randomization.
  - Use Z_i as the treatment and analyze as a typical experiment.
  - Downside: can't learn about the effect of actually being canvassed.
- **As-treated** analysis: just use treatment uptake.
  - Act as if D_i was randomly assigned.
  - Not valid if uptake is **correlated** with the outcome.
  - $\rightsquigarrow$ unmeasured confounding between D_i and Y_i and bias.
- Alternative: leverage latent strata of **compliance types**

Setup

- Treatment assignment, $Z_i \in \{0, 1\}$, treatment uptake $D_i \in \{0, 1\}$
- Treatment uptake now affected by assignment: $D_i(z)$
  - $D_i(1) = 1$ if assigned to canvassing, I **would** open my door.
  - $D_i(1) = 0$ if assigned to canvassing, I **would not** open my door.
  - Noncompliance means $D_i(z) \neq z$ for some i.
- Consistency for the observed treatment as usual:

$$D_i = Z_i D_i(1) + (1 - Z_i) D_i(0)$$

- Canvassing is an example of **one-sided noncompliance**.
  - People might refuse treatment when offered ($D_i(1) = 0$)
  - But no one receives treatment if in control ($D_i(0) = 0, \ \forall i$)
  - **Two-sided noncompliance** is when you can refuse to comply with treatment **or** control.

Potential outcomes

- Outcomes might depend on assignment and uptake: $Y_i(z, d)$.
  - $Y_i(1, 1)$: would I vote if I were assigned to canvassing and received it?
- Can only observe two potential outcomes: $Y_i(1, D_i(1))$ and $Y_i(0, D_i(0))$.
  - $Y_i(1, D_i(1))$: potential outcome when assigned canvassing and whatever uptake occurs for unit *i* when assigned to canvassing.
  - $Y_i(1, 1 - D_i(1))$ not possible to ever observe (cross-world or a prior counterfactual)
- Consistency assumption: $Y_i = Y_i(Z_i, D_i(Z_i))$

Some notation

- Let's use 0/1 subscripts for assignment and t/c subscripts for uptake:

$$n_1 = \sum_{i=1}^{n} Z_i \qquad n_0 = \sum_{i=1}^{n} 1 - Z_i \qquad n_t = \sum_{i=1}^{n} D_i \qquad n_c = \sum_{i=1}^{n} 1 - D_i$$

- Average outcomes and uptake in each assignment group:

$$\overline{Y}_1 = \frac{1}{n_1} \sum_{i=1}^{n} Z_i Y_i \qquad \overline{Y}_0 = \frac{1}{n_0} \sum_{i=1}^{n} (1 - Z_i) Y_i$$
$$\overline{D}_1 = \frac{1}{n_1} \sum_{i=1}^{n} Z_i D_i \qquad \overline{D}_0 = \frac{1}{n_0} \sum_{i=1}^{n} (1 - Z_i) D_i$$

- Assumption 1: **randomization** $[\{Y_i(d,z), \forall d, z\}, D_i(1), D_i(0)] \perp\!\!\!\perp Z_i$
  - For observational uses of IV, might condition on some $\mathbf{X}_i$.

ITT effects

- **Intent-to-treat** (ITT) effects are just the ATEs of Z_i

$$\text{ITT}_D = \frac{1}{n}\sum_{i=1}^{n} D_i(1) - D_i(0) \qquad \text{ITT}_Y = \frac{1}{n}\sum_{i=1}^{n} Y_i(1, D_i(1)) - Y_i(0, D_i(0))$$

  - SATE of assignment on treatment uptake and the outcome.
  - If noncompliance is one-sided, then $\text{ITT}_D \geq 0$
- Standard estimators for these quantities:

$$\widehat{\text{ITT}}_D = \overline{D}_1 - \overline{D}_0 \qquad \widehat{\text{ITT}}_Y = \overline{Y}_1 - \overline{Y}_0$$

- Under randomization of Z_i, everything just like Neyman approach.
  - Variances, tests, CIs all standard.
- Problem: ITT_Y is a combination of true effect of D_i and noncompliance.
  - Effect of D_i is maybe more externally valid than Z_i.

2/ Compliance types

Compliance status

- We can stratify units by their **compliance type**.
  - Compliance type is how they would respond to treatment assignment.
  - Basically it's the value of $(D_i(0), D_i(1))$ for any unit.
- Under one-sided noncompliance, there are two types:
  - **Compliers** with $D_i(1) = 1$ and **noncompliers** with $D_i(1) = 0$.
  - Compliers answer the door when assigned to canvassing
  - Noncompliers don't answer the door when assigned to canvassing
  - Everyone has $D_i(0) = 0$, so no noncompliance there.
- Compliance is a function of potential outcomes so it is **pretreatment**!
  - $\rightsquigarrow$ treatment assignment independent of C_i

ITTs among the compliance groups

- Compliance type indicator $C_i \in \{\text{co}, \text{nc}\}$.
  - Number of compliers: $n_{\text{co}} = \sum_{i=1} \mathbf{1}(C_i = \text{co})$.
  - Proportion of compliers: $\pi_{\text{co}} = n_{\text{co}}/n$
  - Same for noncompliers: n_{nc} and π_{nc}
- ITT on uptake directly related to compliance type:

$$\text{ITT}_{D,\text{co}} = \frac{1}{n_{\text{co}}} \sum_{i:C_i=\text{co}} D_i(1) - D_i(0) = 1$$

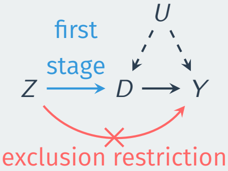
$$\text{ITT}_{D,\text{nc}} = \frac{1}{n_{\text{nc}}} \sum_{i:C_i=\text{nc}} D_i(1) - D_i(0) = 0$$

  - Intuition: no effect of assignment on uptake for noncompliers!
- Implies overall ITT on uptake is equal to the **proportion of compliers**

$$\text{ITT}_D = \pi_{\text{co}}\text{ITT}_{D,\text{co}} + \pi_{\text{nc}}\text{ITT}_{D,\text{nc}} = \pi_{\text{co}}$$

3/ Instrumental variables

Exclusion restriction



- Assumption 2: **first-stage** $\text{ITT}_D = \pi_{\text{co}} \neq 0$
  - At least one person complies with treatment.
- Assumption 3: **exclusion restriction** Z_i only affects Y_i through D_i
  - $Y_i(z, d) = Y_i(z', d)$ for all z, z' and d.
  - Assignment to canvassing only affects turnout through actual canvassing.
  - Not a testable assumption and can't be guaranteed by design.
- Implies that potential outcomes only a function of D_i:

$$Y_i(1) = Y_i(D_i = 1) = Y_i(Z_i = 1, D_i = 1) = Y_i(Z_i = 1, D_i = 1)$$

$$Y_i(0) = Y_i(D_i = 0) = Y_i(Z_i = 1, D_i = 0) = Y_i(Z_i = 1, D_i = 0)$$

Outcome ITTs and compliance types

- We can define the ITTs on the outcome by compliance type as well.
    - $\text{ITT}_{Y,\text{co}}$ effect of assignment on outcome among compliers.
    - $\text{ITT}_{Y,\text{nc}}$ effect of assignment on outcome among noncompliers.
    - Only $\text{ITT}_{Y,\text{co}}$ actually picks up an effect of D_i
- Exclusion restriction has implications for these:
    - Implies that $\text{ITT}_{Y,\text{nc}} = 0$: if D_i doesn't change, Y_i can't change.
    - Implies that $\text{ITT}_{Y,\text{co}}$ is due entirely to treatment uptake.
- Allows us to connect the ITT on the outcome to compliance groups:

$$\text{ITT}_Y = \pi_{\text{co}}\text{ITT}_{Y,\text{co}} + \pi_{\text{nc}}\text{ITT}_{Y,\text{nc}} = \text{ITT}_D\text{ITT}_{Y,\text{co}}$$

LATE

- Under the exclusion restriction, $\text{ITT}_{Y,\text{co}}$ is the effect of treatment receipt:

$$\begin{aligned}\text{ITT}_{Y,\text{co}} &= \frac{1}{n_{\text{co}}} \sum_{i:C_i=\text{co}} Y_i(1, D_i(1)) - Y_i(0, D_i(0)) \\ &= \frac{1}{n_{\text{co}}} \sum_{i:C_i=\text{co}} Y_i(D_i = 1) - Y_i(D_i = 0) = \tau_{\text{LATE}}\end{aligned}$$

- This quantity is the **local ATE** (LATE), local to compliers.
    - It's a conditional ATE, where we condition on being a complier.
    - Also called the **complier average causal effect** (CACE).
- **LATE Theorem** under one-sided noncompliance, exclusion restriction, first-stage, and randomization:

$$\tau_{\text{LATE}} = \text{ITT}_{Y,\text{co}} = \frac{\text{ITT}_Y}{\text{ITT}_D}$$

Wald estimator

- **Wald** or **instrumental variables estimator** for the LATE:

$$\widehat{\tau}_{iv} = \frac{\widehat{\text{ITT}}_Y}{\widehat{\text{ITT}}_D}$$

  - Ratio of the two unbiased ITT estimators.
  - Not unbiased, but it is **consistent** for τ_{LATE}.

- Equivalent to the **two-stage least squares** estimator:
  - Regress D_i on Z_i and get fitted values $\widehat{D}_i$
  - Regress Y_i on $\widehat{D}_i$

- We can use the delta method to find the (superpopulation) variance:

$$\mathbb{V}[\widehat{\tau}_{iv}] = \frac{1}{\text{ITT}_D^2}\mathbb{V}\left[\widehat{\text{ITT}}_Y\right] + \frac{\text{ITT}_Y^2}{\text{ITT}_D^4}\mathbb{V}\left[\widehat{\text{ITT}}_D\right] - 2\frac{\text{ITT}_Y}{\text{ITT}_D^3}\text{cov}\left[\widehat{\text{ITT}}_Y, \widehat{\text{ITT}}_D\right]$$

4/ Two-sided noncompliance

Two-sided noncompliance

- Two-sided noncompliance: those in control can select into treatment.
- **Encouragement design**: randomly assign an encouragement of some treatment.
    - Some may refuse encouragement and opt to not take treatment.
    - Some may take treatment even without encouragement.
- Z_i is the encouragement and D_i is the treatment.
- No change in estimation, just different identification assumptions.

Compliance types

- Four compliance types (or **principal strata**) in this setting:
    - Complier $D_i(1) = 1$ and $D_i(0) = 0$
    - Always-taker $D_i(1) = D_i(0) = 1$
    - Never-taker $D_i(1) = D_i(0) = 0$
    - Defier $D_i(1) = 0$ and $D_i(1) = 1$
- Connections between observed data and compliance types:

	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	Never-taker or Complier	Never-taker or Defier
$D_i = 1$	Always-taker or Defier	Always-taker or Complier

- Let π_{co}, π_{at}, π_{nt}, and π_{de} be the proportions of each type.
- ITT effects on D_i are more murky: $\text{ITT}_D = \pi_{\text{co}} - \pi_{\text{de}}$
    - Defiers really make things messy!

Instrumental variables assumptions

- Canonical IV assumptions for Z_i to be a valid instrument:
    1. Randomization of Z_i
    2. Presence of some compliers $\pi_{\text{co}} \neq 0$ (first-stage)
    3. Exclusion restriction $Y_i(z, d) = Y_i(z', d)$
    4. **Monotonicity**: $D_i(1) \geq D_i(0)$ for all i (no defiers)
- Implies ITT effect on treatment equals proportion compliers: $\text{ITT}_D = \pi_{\text{co}}$
- Implies ITT for the outcome has the same interpretation:

$$
\begin{aligned}
\text{ITT}_Y &= \text{ITT}_{Y,\text{co}}\pi_{\text{co}} + \underbrace{\text{ITT}_{Y,\text{at}}}_{=0 \text{ (ER)}}\pi_{\text{at}} + \underbrace{\text{ITT}_{Y,\text{nt}}}_{=0 \text{ (ER)}}\pi_{\text{nt}} + \text{ITT}_{Y,\text{de}}\underbrace{\pi_{\text{de}}}_{=0 \text{ (mono)}} \\
&= \text{ITT}_{\text{co}}\pi_{\text{co}}
\end{aligned}
$$

- $\rightsquigarrow$ same identification result: $\tau_{\text{LATE}} = \text{ITT}_Y / \text{ITT}_D$

Is the LATE useful?

- The LATE is a unknown subset of the data.
  - Treated units are a mix of always takers and compliers.
  - Control units are a mix of never takers and compliers.
- Without further assumptions, $\tau_{\text{LATE}} \neq \tau$.
- Complier group depends on the instrument $\rightsquigarrow$ different IVs will lead to different identified estimands.
- But we cannot do any better in terms of point estimation without more assumptions.
  - Alternative: bound the ATE?