Module 9(b): Synthetic Control Methods

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Matthew Blackwell

Gov 2003 (Harvard)

Synthetic controls

- Abadie and Gardeazabal (2003) use a DID approach for "quantitative case studies."
- Application: effect of an intervention in a single country/state at one point in time.
- Basic idea: 1 treated group, many controls.
  - Compare the time-series outcomes in the treated group to the control.
  - But which control group should you use?
  - Many possible choices and they may not be comparable to the treated.
- **Synthetic control**: use a convex combination of the controls to create a synthetic control.
  - Choose the weights that minimize the pretreatment differences between treated and synthetic control.

Intervention study

	Time period						
	1	2	$\cdots$	T_0	T_0+1	$\cdots$	T
Treated unit $(i=1)$	0	0	0	0	1	1	1
Control group $(i=2,\dots,J+1)$	0	0	0	0	0	0	0

- Treatment:
  - All units untreated for T_0 periods.
  - Unit 1 starts treatment at T_0, continues until T.
- Potential outcomes:
  - $Y_{it}(1)$: potential outcome at time t if i had been in the treated group.
  - $Y_{it}(0)$: potential outcome at time t if i had been in the control group.
  - No pre-intervention impacts: $Y_{it}(1) = Y_{it}(0)$ for all $t \leq T_0$.
- $\mathbf{X}_i$ is an $r \times 1$ vector of (pretreatment) covariates.
- Treatment effects: $\tau_{it} = Y_{it}(1) - Y_{it}(0)$
- Goal: estimate $\left(\tau_{1,T_0+1}, \dots, \tau_{1,T}\right)$.

Missing counterfactuals

- By consistency, for $t > T_0$:

$$\tau_{1t} = Y_{1t}(1) - Y_{1t}(0) = Y_{1t} - Y_{1t}(0)$$

- Need to impute missing potential outcomes, $Y_{1t}(0)$.
- **Synthetic control**: Choose weights $(w_2, \ldots, w_{J+1})'$ such that:
    - $w_j \geq 0$ and $\sum_j w_j = 1$.
    - for all $t \leq T_0$ minimize

$$\left| Y_{1t} - \sum_{j=2}^{J+1} w_j Y_{jt} \right|, \qquad \left| \mathbf{Z}_1 - \sum_{j=2}^{J+1} w_j \mathbf{Z}_j \right|$$

    - Can also add a penalty for how dispersed the weights are.
- We hope this implies for $t > T_0$: $\sum_{j=2}^{J+1} w_j Y_{jt} \approx Y_{1t}(0)$

Without synthetic controls

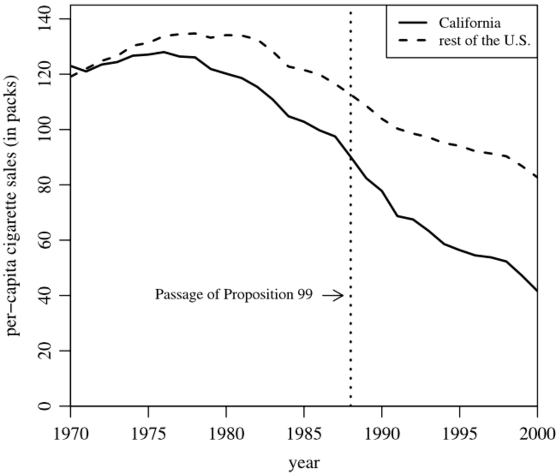

Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

With synthetic controls

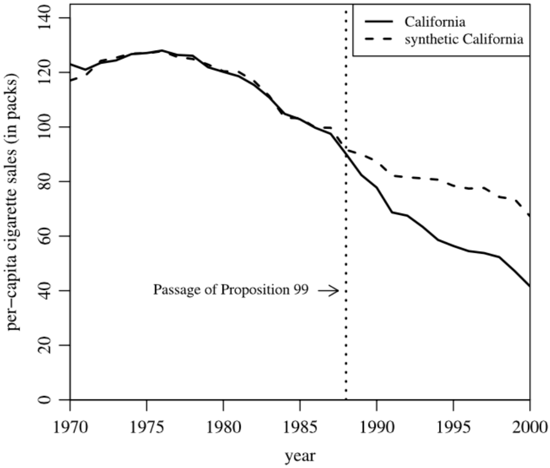

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

Weights

Table 2. State weights in the synthetic California

State	Weight	State	Weight
Alabama	0	Montana	0.199
Alaska	–	Nebraska	0
Arizona	–	Nevada	0.234
Arkansas	0	New Hampshire	0
Colorado	0.164	New Jersey	–
Connecticut	0.069	New Mexico	0
Delaware	0	New York	–
District of Columbia	–	North Carolina	0
Florida	–	North Dakota	0
Georgia	0	Ohio	0
Hawaii	–	Oklahoma	0
Idaho	0	Oregon	–
Illinois	0	Pennsylvania	0
Indiana	0	Rhode Island	0
Iowa	0	South Carolina	0
Kansas	0	South Dakota	0
Kentucky	0	Tennessee	0
Louisiana	0	Texas	0
Maine	0	Utah	0.334
Maryland	–	Vermont	0
Massachusetts	–	Virginia	0
Michigan	–	Washington	–
Minnesota	0	West Virginia	0
Mississippi	0	Wisconsin	0
Missouri	0	Wyoming	0

Inference

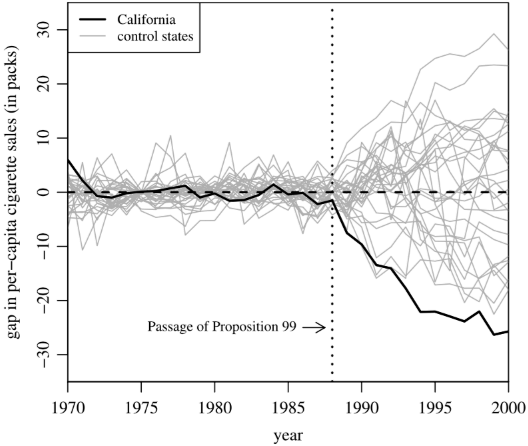

Figure 6. Per-capita cigarette sales gaps in California and placebo gaps in 29 control states (discards states with pre-Proposition 99 MSPE five times higher than California's).

Synthetic control justification

- ADH provide two **model-based** justifications for SC.
- **Model 1**: Interacted factor model

$$Y_{it}(0) = \mathbf{X}_i'\boldsymbol{\beta}_t + \alpha_i + \delta_t + \boldsymbol{\lambda}_t\boldsymbol{\mu}_i + \varepsilon_{it}$$

  - $\boldsymbol{\beta}_t$ are time-varying coefficients on covariates.
  - $\boldsymbol{\lambda}_t$ is a $1 \times F$ vector of common factors
  - $\boldsymbol{\mu}_i$ is a $F \times 1$ vector of factor loadings
  - $\boldsymbol{\lambda}_t\boldsymbol{\mu}_i$ allows time-varying confounding in a structured way.
  - Common time shocks affect each unit in a time-constant way.
- **Model 2**: autoregressive model without fixed effects

$$Y_{i,t+1}(0) = \alpha_t Y_{it}(0) + \boldsymbol{\beta}_{t+1}\mathbf{X}_{i,t+1} + u_{i,t+1}$$
$$\mathbf{X}_{i,t+1} = \gamma_t Y_{it}(0) + \boldsymbol{\Pi}_t\mathbf{X}_{it} + \mathbf{v}_{i,t+1}$$

- Either fixed effects OR lagged dependent variables, not both.

SCM properties

- Suppose perfect balancing weights exist $(w_2^*, \dots, w_{J+1}^*)$ such that:

$$\sum_{j=2}^{J+1} w_j^* Y_{jt} = Y_{1t} \qquad \sum_{j=2}^{J+1} w_j^* \mathbf{X}_j = \mathbf{X}_i$$

- Let $\widehat{Y}_{1t}(0) = \sum_{j=2}^{J+1} w_j^* Y_{jt}$ for post-intervention periods.
- Under Model 1, $\widehat{Y}_{1t}(0) \to Y_{1t}(0)$ as $T_0 \to \infty$
    - As length of pre-intervention period grows, estimates get better.
- Under Model 2, $\mathbb{E}\left[\widehat{Y}_{1t}(0)\right] = \mathbb{E}[Y_{1t}(0)]$
    - Unbiased only based on one pre-treatment periods.
    - But it assumes away unmeasured confounding!
- Outside of those models: ?????

Bias correction

- When pre-treatment fit is imperfect $\rightsquigarrow$ significant bias in SCM
- **Augmented SCM**: use regression models to correct for bias
    - Let $\widehat{m}_{it} = \widehat{m}_{it}(\overline{Y}_{i,t-1})$ be predicted values for a regression of post-treatment outcomes on pre-treatment outcomes.
    - Augment estimator (Ben-Michael, et al, 2021, JASA):

$$\widehat{Y}_{1t}^{\text{aug}}(0) = \sum_{j=2}^{J+1} w_j Y_{jt} + \left(\widehat{m}_{1t} - \sum_{j=2}^{J+1} w_j \widehat{m}_{jt} \right)$$

    - Can add covariates fairly easily.
- Very similar to bias correction in matching.

Generalizing to more treated units

- Two estimation methods to generalize to any number of treated units.
- **Interactive fixed effects**: $Y_{it}(0) = \mathbf{X}'_{it}\beta + \alpha_i + \delta_t + \boldsymbol{\lambda}_t \boldsymbol{\mu}_i$
  - Instead of weights, directly estimate IFE using iterative procedure:
    1. Treat IFE terms as fixed and fit parametric part on untreated units to get new $\hat{\beta}$
    2. Treat covariate coefficients as fixed and use factor analysis to estimate IFE terms.
    3. Repeat until convergence.
- **Matrix completion** methods (Athey et al, 2021)
  - Treat matrix of control POs, $\mathbf{Y}(\mathbf{0})$ as missing data problem.
  - Estimate lower-rank matrix $\mathbf{L}$ as best approximation to observed parts of $\mathbf{Y}(\mathbf{0})$ subject to regularization.