Module 7a: Matching Estimators

Fall 2021

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Gov 2003 (Harvard)

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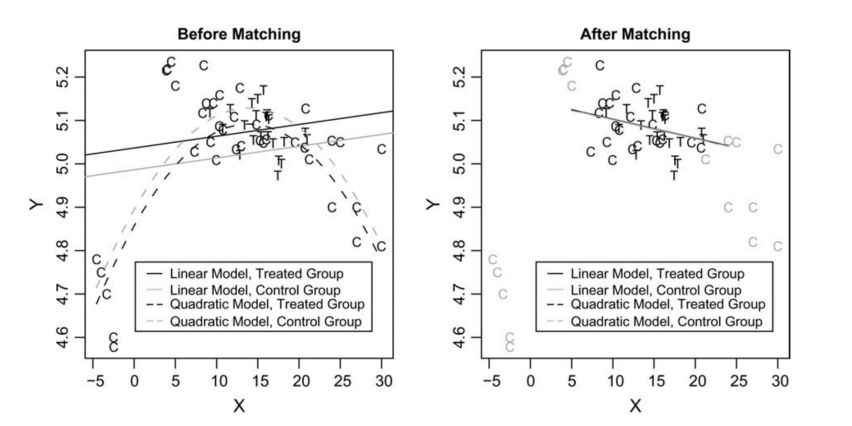
    - But this model might be wrong $\rightsquigarrow$ wrong causal estimates.

Model dependence



What is matching?

- **Matching** is a nonparametric imputation estimator:

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- What matching isn't: a solution for selection on unobservables.
  - Matching is an **estimation** technique, not an identification strategy.

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  - Allows you to control bias/variance tradeoff through coarsening.

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    - Some use the linear predictor: $D(\mathbf{X}_i, \mathbf{X}_j) = |\text{logit}(\widehat{\pi}(\mathbf{X}_i)) - \text{logit}(\widehat{\pi}(\mathbf{X}_j))|$

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    - Somewhat deflates the benefits of PS matching/balancing.

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- In observational data we never know the true $\pi(\mathbf{x}) \rightsquigarrow$ estimate it $\widehat{\pi}(\mathbf{x})$.
- Is balancing on $\widehat{\pi}(\mathbf{x})$ sufficient? **No idea!!**
    - Have to check if $\mathbf{X}_i$ is actually balanced.
    - Somewhat deflates the benefits of PS matching/balancing.
- $\rightsquigarrow$ "propensity score tautology"

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  - Dropping units can lower power ($\uparrow$ p-values) without a change in balance.

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  - Abadie and Imbens (2006) provides matching-based variance estimators.