Module 5: Observational Studies

Fall 2021

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Gov 2003 (Harvard)

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- Up to now: experiments where design makes everything easier.
- Now: what happens when do observational studies?
    - Start with identification, selection on observables, and DAGs.
    - Rest of the course will cover different designs for observational studies.

1/ Identification in observational studies

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   - Sometimes written as $D_i \perp\!\!\!\perp (\mathbf{Y}(1), \mathbf{Y}(0))$

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- **Selection bias**: how different the treated and control groups are in terms of their potential outcome under control.

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- With an unbounded Y_i, we can't even bound the ATT because, in principle, SB could be anywhere from $-\infty$ to ∞.
- We say ATT (and ATE) are **unidentified** without further assumptions.

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    - Or you will have justify them through argument.

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  - Keep them separate: estimator shouldn't drive identification.

What is confounding?

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2/ Selection on observables

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- These are assumptions that **can be wrong**!!

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- Key part of the above proof:

$$\underbrace{\mu_1(\mathbf{x})}_{\text{counterfactual}} = \underbrace{\mathbb{E}[Y_i \mid D_i = 1, \mathbf{X}_i = \mathbf{x}]}_{\text{observational}}, \qquad \mu_0(\mathbf{x}) = \mathbb{E}[Y_i \mid D_i = 0, \mathbf{X}_i = \mathbf{x}]$$

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- These make two very different assumptions about the CEFs!

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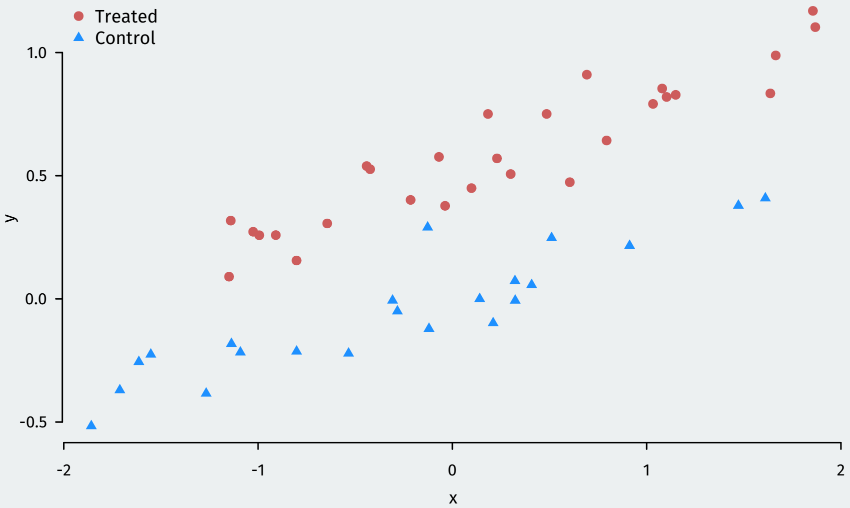
Imputation estimator visualization



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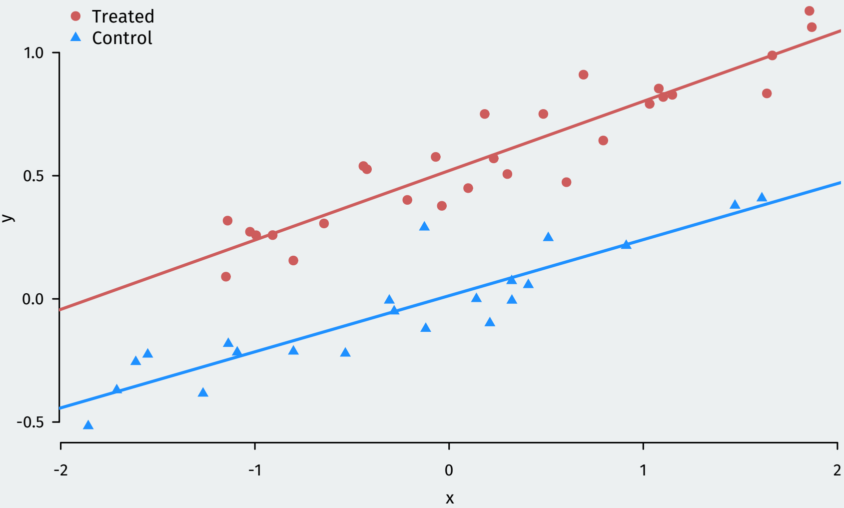

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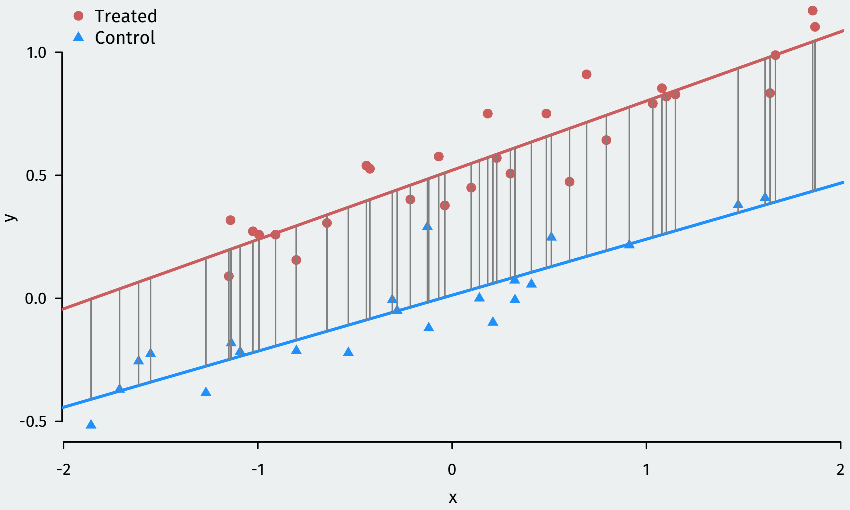

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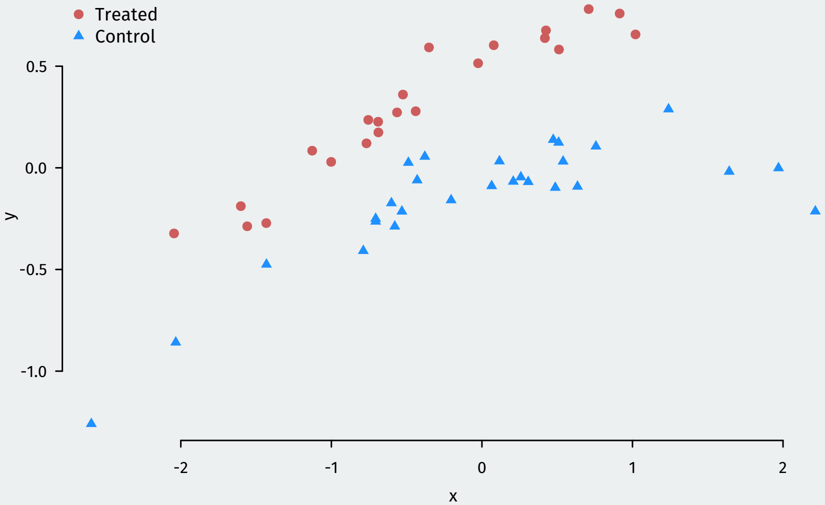

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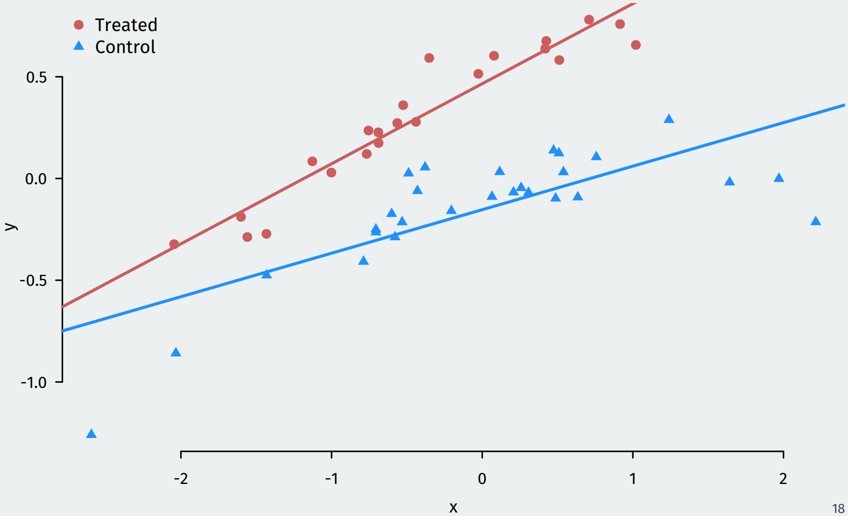

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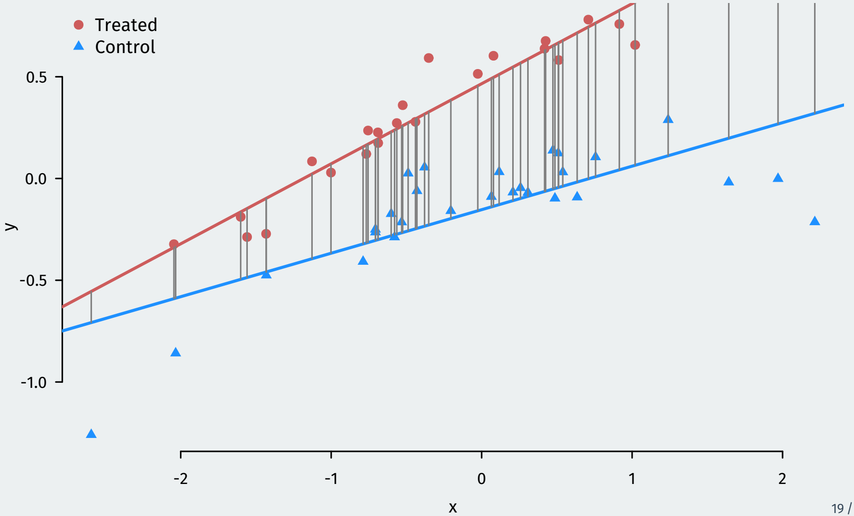

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- Here, CEFs are nonlinear, but we don't know their form.
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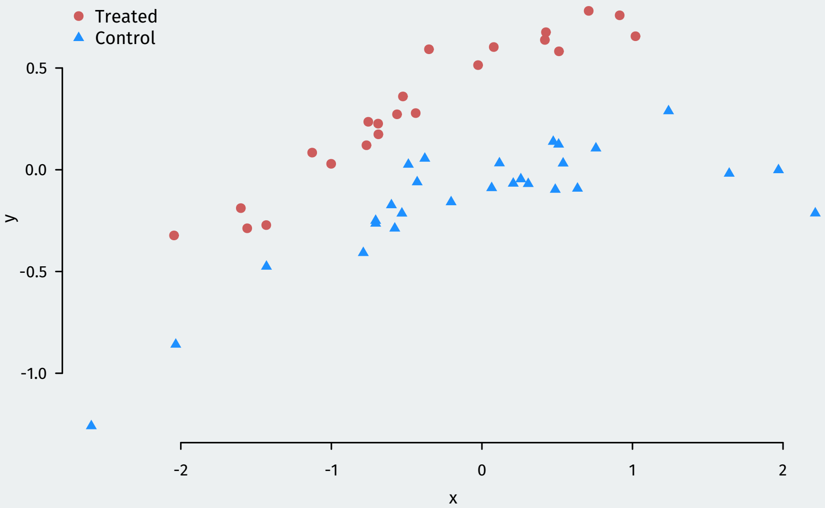
Using semiparametric regression

- Here, CEFs are nonlinear, but we don't know their form.
- We can use GAMs from the `mgcv` package to for flexible estimate:

```
library(mgcv)
mod0 <- gam(y~s(x), subset = d==0)
summary(mod0)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ s(x)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -0.154      0.019    -8.1  5.1e-08 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##       edf Ref.df    F p-value
## s(x) 5.17   6.29 36.9  <2e-16 ***
## ---
```

Using GAMs



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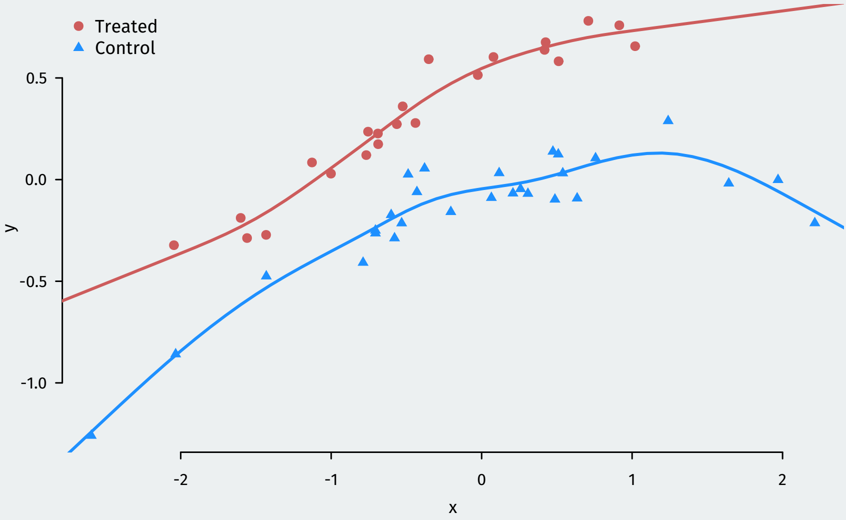

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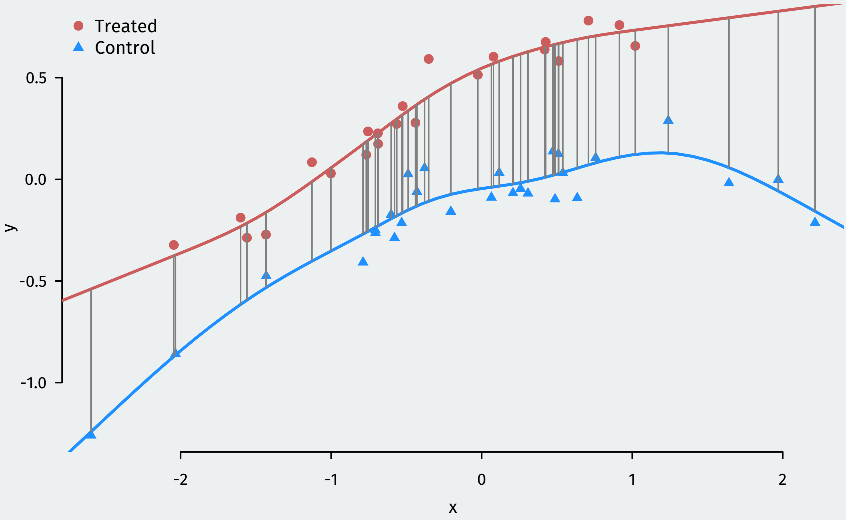

3/ DAGs

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- Another way: use DAGs and look at back-door paths.

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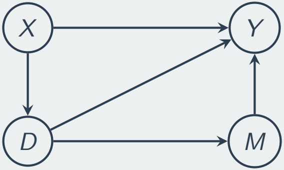
- **Acyclic**: no cycles: a variable cannot cause itself

DAG terminology



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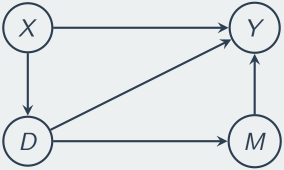

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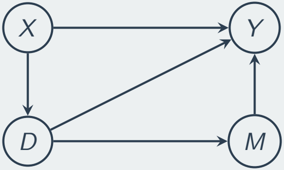

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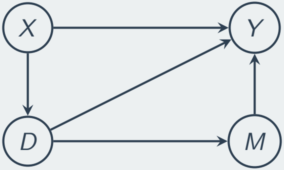

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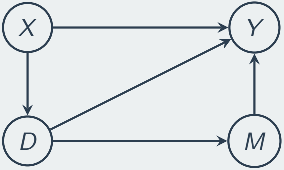

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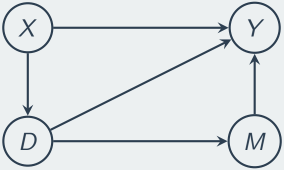

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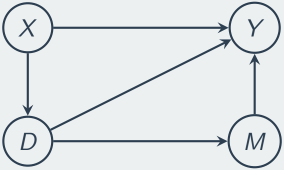

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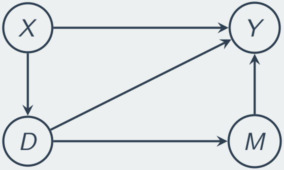

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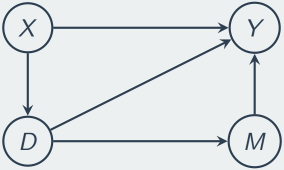

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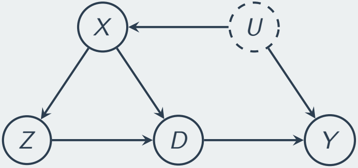
DAGs to distributions



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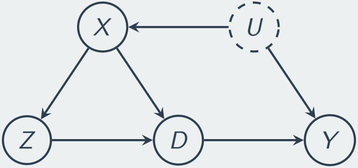

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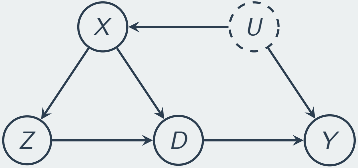

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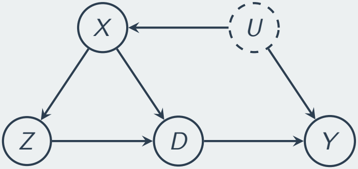

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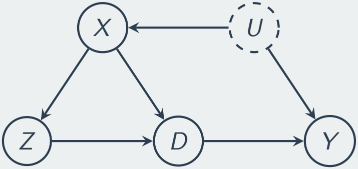

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- Causal DAGs imply the following factorization (some conditions apply):

$$\mathbb{P}(X_1, X_2, \dots, X_J) = \prod_{j=1}^{J} \mathbb{P}(X_j \mid \text{pa}(X_j)) \quad \text{where} \quad \text{pa}(X_j) = \text{parents of } X_j$$

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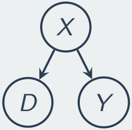
  - If not, then d-connected and A and B dependence conditional on C is compatible with the DAG.

Common structures

Confounder



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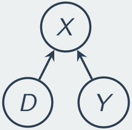
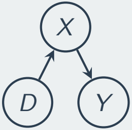
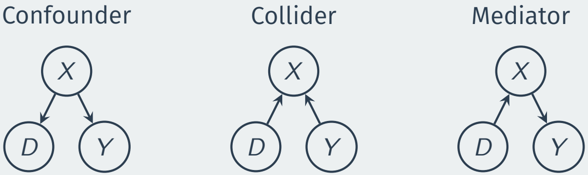


Mediator



Common structures



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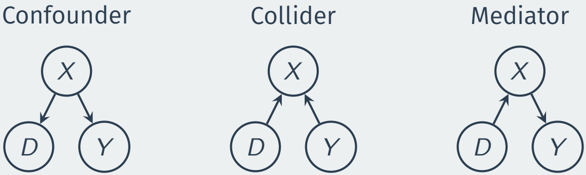

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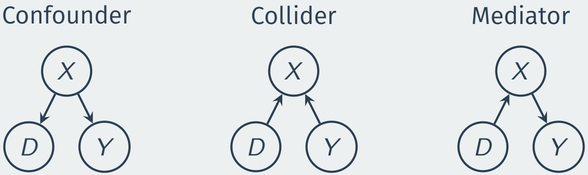

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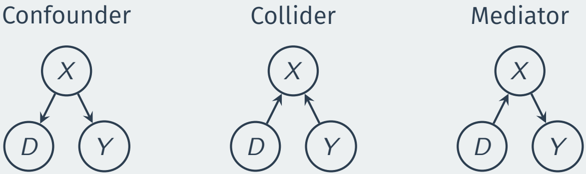

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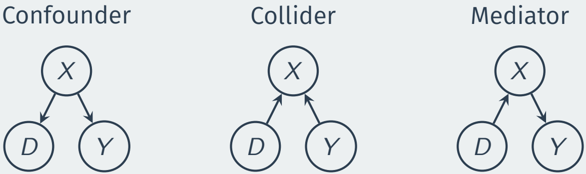

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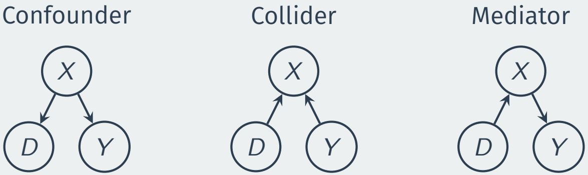

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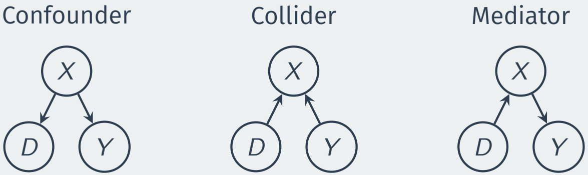

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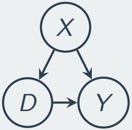

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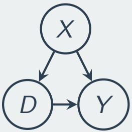
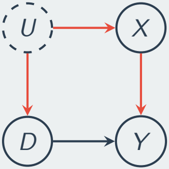


- Here: backdoor path $D \leftarrow X \rightarrow Y$

Other types of confounding



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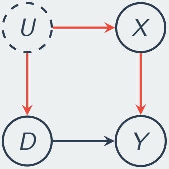

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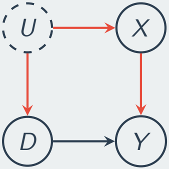

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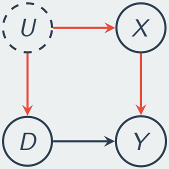

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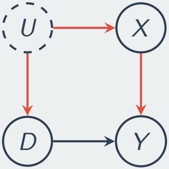

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- The backdoor criterion is fairly powerful. Tells us:
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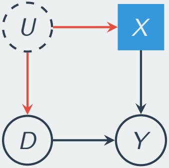
  - what variables to condition on to eliminate the confounding.

Other types of confounding



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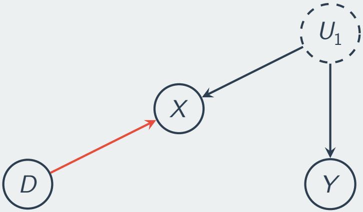

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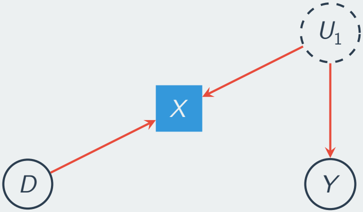

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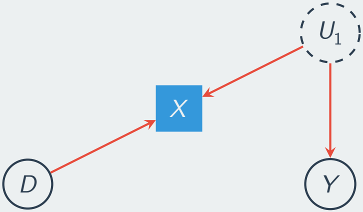

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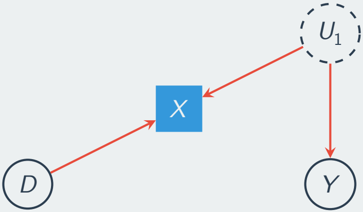

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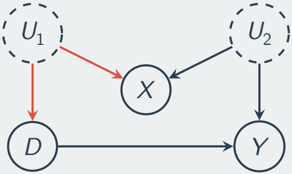
  - But still no causal relationship $\rightsquigarrow$ selection bias.

M-bias



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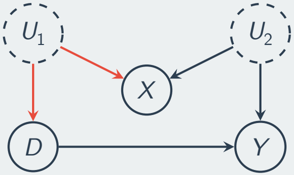

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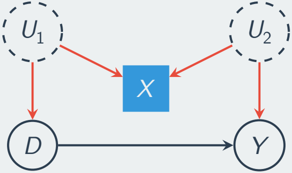

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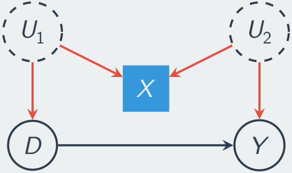

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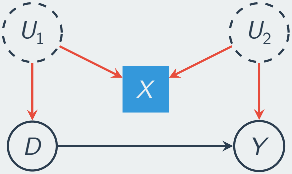

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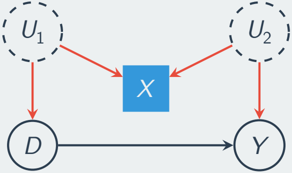

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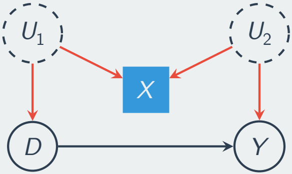

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    - Pearl and others think M-bias is a real threat.