Module 9(b): Synthetic Control Methods

Fall 2021

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Gov 2003 (Harvard)

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    - Choose the weights that minimize the pretreatment differences between treated and synthetic control.

Intervention study

	Time period						
	1	2	$\cdots$	T_0	$T_0 + 1$	$\cdots$	T
Treated unit $(i = 1)$	0	0	0	0	1	1	1
Control group $(i = 2, \dots, J + 1)$	0	0	0	0	0	0	0

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- Treatment effects: $\tau_{it} = Y_{it}(1) - Y_{it}(0)$
- Goal: estimate $\left(\tau_{1,T_0+1}, \ldots, \tau_{1,T}\right)$.

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    - Can also add a penalty for how dispersed the weights are.
- We hope this implies for $t > T_0$: $\sum_{j=2}^{J+1} w_j Y_{jt} \approx Y_{1t}(0)$

Without synthetic controls

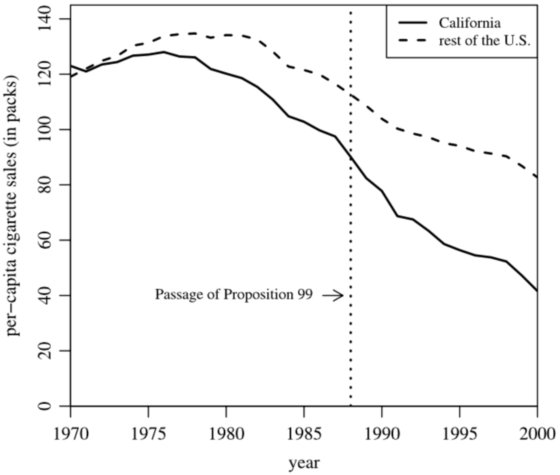

Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

With synthetic controls

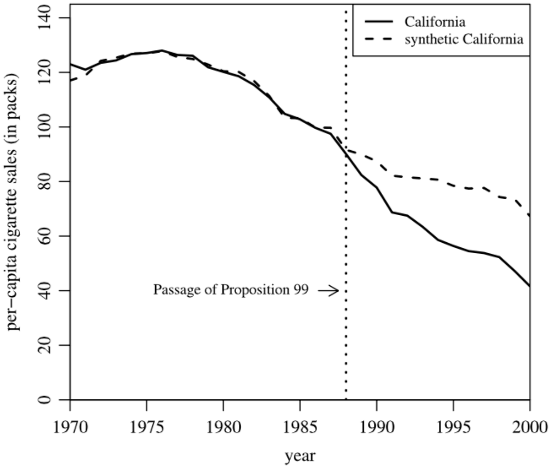

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

Weights

Table 2. State weights in the synthetic California

State	Weight	State	Weight
Alabama	0	Montana	0.199
Alaska	–	Nebraska	0
Arizona	–	Nevada	0.234
Arkansas	0	New Hampshire	0
Colorado	0.164	New Jersey	–
Connecticut	0.069	New Mexico	0
Delaware	0	New York	–
District of Columbia	–	North Carolina	0
Florida	–	North Dakota	0
Georgia	0	Ohio	0
Hawaii	–	Oklahoma	0
Idaho	0	Oregon	–
Illinois	0	Pennsylvania	0
Indiana	0	Rhode Island	0
Iowa	0	South Carolina	0
Kansas	0	South Dakota	0
Kentucky	0	Tennessee	0
Louisiana	0	Texas	0
Maine	0	Utah	0.334
Maryland	–	Vermont	0
Massachusetts	–	Virginia	0
Michigan	–	Washington	–
Minnesota	0	West Virginia	0
Mississippi	0	Wisconsin	0
Missouri	0	Wyoming	0

Inference

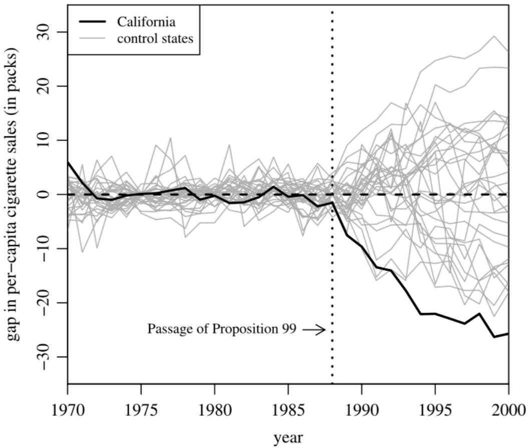

Figure 6. Per-capita cigarette sales gaps in California and placebo gaps in 29 control states (discards states with pre-Proposition 99 MSPE five times higher than California's).

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- Either fixed effects OR lagged dependent variables, not both.

SCM properties

- Suppose perfect balancing weights exist $(w_2^*, \ldots, w_{J+1}^*)$ such that:

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- Outside of those models: ?????

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- Very similar to bias correction in matching.

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