Empirical Methods for Policy Evaluation

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Shift-Share Instrumental Variables (3 Classes)

1. Shift-Share Designs
   - Instrumental Variables and SSIV
   - Identification from many exogenous shifts
   - Identification from exogenous shares
2. Application

   ⇒ Rural migration and urban informality (Imbert and Ulyssea, ECMA forth.)

Instrumental Variables and SSIV

Instrumental Variables (IV)

- Recall from the intro slides

$$
\begin{aligned}
\text{ATE} &= \mathbb{E}(Y_i^1 - Y_i^0) \\
&= \mathbb{E}(Y_i^1 \mid D_i = 1) - \mathbb{E}(Y_i^0 \mid D_i = 0) \\
&= \underbrace{\mathbb{E}(Y_i^1 - Y_i^0 \mid D_i = 1)}_{\text{ATT}} + \underbrace{\mathbb{E}(Y_i^0 \mid D_i = 1) - \mathbb{E}(Y_i^0 \mid D_i = 0)}_{\text{Selection bias}}
\end{aligned}
$$

$\Rightarrow$ A standard solution is to find Z_i such that

$$\mathbb{E}(Y_i^0 \mid Z_i, D_i = 1) = \mathbb{E}(Y_i^0 \mid Z_i, D_i = 0)$$

- ATT can be consistently estimated by 2SLS (assuming $Y_i^1 - Y_i^0 = \Delta$, $\forall i$)

Shift-Share Instrumental Variables

- General structure of shift-share IV (SSIV)

$$Z_i = \sum_{k=1}^{K} S_{ik} g_k$$

⇒ $g_1, ..., g_K$ are shifts not specific to i (e.g. industry-level employment growth)

⇒ S_{ik} are exposure shares (e.g. initial share of k in i's employment)

- When $\sum_k S_{ik} = 1\ \forall i$, Z_i is a (share-)weighted average of the shifts

Shift-Share IV: Decomposition

- $D_i = \frac{X_{i1} - X_{i0}}{X_{i0}}$
- This can be decomposed as $X_{it} = \sum_k X_{ikt}$, hence

$$\frac{X_{i1} - X_{i0}}{X_{i0}} = \sum_k \underbrace{\frac{X_{ik0}}{X_{i0}}}_{\text{Share}} \underbrace{\frac{X_{ik1} - X_{ik0}}{X_{ik0}}}_{\text{Local shift}}$$

$\Rightarrow$ Use a set of common shifts $g_k = \frac{X_{k1} - X_{k0}}{X_{k0}}$ to replace the local shifts

- Instruments constructed in this way tend to be highly correlated with D_i

Shift-Share IV: Example 1 (Bartik, 1991)

- Consider the (inverse) regional supply equation:

$$\Delta \log w_i = \tau \underbrace{\Delta \log L_i}_{= \sum_k S_{ik} \Delta \log L_{ik}} + \underbrace{\epsilon_i}_{\text{labor supply shocks}}$$

- Need a regional labor demand shock as IV

- $Z_i = \sum_k S_{ik} g_k$ by replacing $\Delta \log L_{ik}$ with national industry shift g_k

  $\Rightarrow$ S_{ik} = share of employment in industry k

  $\Rightarrow$ g_k = observed growth rate of industry employment

Shift-Share IV: Example 2 (Card, 2009)

- Consider the (inverse) elasticity of substitution btw. native and immigrants

$$\Delta \text{log} \frac{w_i^{\text{Immigrant}}}{w_i^{\text{Native}}} = \tau \Delta \text{log} \frac{L_i^{\text{Immigrant}}}{L_i^{\text{Native}}} + \underbrace{\epsilon_i}_{\text{change in relative labor demand in region } i}$$

- Need a relative labor supply shock as IV

- $Z_i = \sum_k S_{ik} g_k$: migration prediction from historic enclaves & "push shocks"
  - $\Rightarrow$ S_{ik} = initial share of immigrants of origin k in i's population
  - $\Rightarrow$ g_k = observed national migration rate from k

The SSIV Exogeneity Challenge

- How should we think about $E[Z_i \epsilon_i] = 0$?
  - $\Rightarrow$ Which properties of the shifts and/or shares can make Z_i exogenous ?
  - $\Rightarrow$ Do we need exogeneity of both shifts and shares, or just one ?
- Two approaches + sets of sufficient conditions
  1. Many exogenous shifts: leveraging a shift-level natural experiment
  2. Exogenous shares: pooling diff-in-diffs based on heterog. exposure shares

Identification from Many Exogenous Shifts

Intuition

- Suppose shifts are as-good-as-random: $\mathbb{E}[g_k \mid \epsilon, S] = \alpha$
- Then $\mathbb{E}(Z_i) = \mathbb{E}[\sum_k S_{ik} g_k \mid \epsilon, S] = \alpha \sum_k S_{ik}$
  - $\Rightarrow$ If shares add up to one, then $\mu_i =$ constant
- A weighted average of as-good-random shifts is as-good-as random
  - $\Rightarrow$ If industries with high vs. low g_k are comparable, regions specializing in those industries are comparable
  - $\Rightarrow$ True whatever the initial shares are, we don't need Cov $[S_{ik}, \epsilon_i] = 0$
- Translate random variation across k into random variation across i

Identification

- $\frac{1}{N}\sum_i Z_i\epsilon_i \to 0$ if
  1. g_k are uncorrelated with $\bar{\epsilon}_k = \sum_i S_{ik}\epsilon_i / \sum_i S_{ik}$
  2. The effective number of shifts is large (otw corr $(g_k, \bar{\epsilon}_k) \neq 0$ by chance)
  3. Shares add up to one for each obs. i (otw g_k vary across i through $\sum_i S_{ik}$)

$\Rightarrow$ Holds because

$$\frac{1}{N}\sum_i Z_i\epsilon_i = \frac{1}{N}\sum_{i,k} S_{ik}g_k\epsilon_i = \sum_k s_k g_k \bar{\epsilon}_k,$$

- Where $s_k = \frac{1}{N}\sum_i S_{ik}$

Incomplete Shares

- What if $\sum_k S_{ik} \neq 1$, so Z_i is a weighted sum, not a weighted average?
- E.g. lottery of randomly assigns subsidies g_k to manufacturing industries k
- Regions with more manufacturing will mechanically get higher Z_i
- But recall that the expectation of the instrument is $\mu_i = \alpha \sum_k S_{ik}$

$\Rightarrow$ Thus, controlling for $\sum_k S_{ik}$ avoids bias

Shift-level Controls

- What if shifts are as-good-as-random only after controlling for some q_k?
- E.g. industry subsidies are random only controlling for dummy of hi-tech
- Expected instrument is $\mu_i = \alpha \sum_k S_{ik}(\alpha_0 + \alpha_1 q_k)$
- To avoid bias, it's sufficient to control for $\sum_k S_{ik}$ and $\sum_k S_{ik} q_k$
- E.g. control for the local share of hi-tech industries
  - $\Rightarrow$ Leverage variation btw regions with similar composition of high-tech
  - $\Rightarrow$ But different exposure to subsidies within each group

Inference

- Obs. with similar shares are exposed to same shocks: g_k and (unobserved) ν_k
- Conventional Liang-Zeger clustering (e.g. by state) wouldn't capture that
  - $\Rightarrow$ Placebo shifts $g_k^\star$ generate significant reduced-form in $\sim$50% simulations
- If each region was exposed to just one industry, we'd cluster by industry

$\Rightarrow$ But how to do it in a "fuzzy" case?

Shift-level ("Exposure-robust") Standard Errors

- Residualize outcomes and treatment on i-level controls: $V_i^{\perp}$
- Average across observations with weights S_{ik}: $\bar{V}_k^{\perp} = \sum_i S_{ik} V_i^{\perp} / \sum_i S_{ik}$

$\Rightarrow$ SSIV estimate is equal to the estimate from a weighted 2SLS with g_k as IV

$$\bar{Y}_k^{\perp} = \tau \bar{D}_k^{\perp} + \gamma' q_k + \bar{\epsilon}_k$$

- Where the weights are $s_k = \frac{1}{N} \sum_i S_{ik}$
- Then use robust or clustered SE as in any other regression

Heterogenous Treatment Effects

- SSIV identifies convex averages of treatment effects under monotonicity

- Averages across units i and shifts k, with certain weights $\pi_{ik} \geq 0$

$$D_{ik} = \pi_{ik} g_k + u_{ik}$$

$$Y_i = \sum_k \beta_{ik} S_{ik} D_{ik} + \epsilon_i$$

$\Rightarrow$ β_{ik}: Local labor supply elasticity may depend on the region i or industry k

- When shifts g_k have mean zero, variance σ_k^2 and no mutual corr., then SSIV:

$$\beta = \frac{\sum_i \sum_k \pi_{ik} S_{ik}^2 \sigma_k^2 \cdot \beta_{ik}}{\sum_i \sum_k \pi_{ik} S_{ik}^2 \sigma_k^2}$$

Heterogenous Treatment Effects (continued)

- Suppose $\pi_{ik} = \pi$ and $\sigma_k^2 = \sigma^2$

$\Rightarrow$ Then, $\beta_{ik} = \beta_i$ and $\pi = \sum_k S_{ik}^2$ (Herfindahl of local industry concentration)

- If instead $\beta_{ik} = \beta_k$, then $\pi = \sum_i S_{ik}^2$ (not Herfindahl since $\sum_i S_{ik}^2 \neq 1$)

$\Rightarrow$ This is higher for larger industries/when local k shares are unequal across i

- E.g. tradable industries play a larger role than nontradable of a similar size

$\Rightarrow$ Since typical tradable industries concentrate in a small number of regions

A Checklist for the Shift-Based Approach

1. Motivate SSD with a shift-level idealized experiment
2. Bridge the gap between the observed and ideal shifts
3. Include the "incomplete share" control
4. Lag shares to the beginning of the natural experiment
5. Report descriptives for shifts in addition to observations
6. Implement balance tests for shifts in addition to the instrument
7. Produce main estimates with correct SE and check sensitivity

Software Implementation

- Package `ssaggregate` automate the transformation for shift-level regressions

  ⇒ Various types of dependence in the shifts (e.g. spatial clusters, serial corr.)

  ⇒ Exposure-robust first-stage F-stat for IV strength

Example: The China Shock (Autor, Dorn, Hanson, 2013)

- ADH study the effects of import competition on local labor market outcomes
- Region $i =$ commuting zone (N=722), industry $k =$ SIC4 manuf. (K=397)
- Two periods t: 1991-2000 and 2000-2007
- $Y_{it} =$ local change in manufacturing employment rate
- $D_{it} =$ local growth of exposure to Chinese imports

$$D_{it} = \sum_k S^L_{ikt_0} \frac{\Delta \text{US imports from China}_{kt}}{L_{kt_0}}$$

- Control for period FE and some initial regional characteristics

The China Shock: Picking Controls

- Hypothesize that shifts g_k are as-good-as-random controlling for period FEs

- Regional regression should thus control for

$$\sum_k S_{ikt} q_t = \left(\sum_k S_{ikt} \right) \times q_t$$

$\Rightarrow$ Lagged total share of manufacturing interacted with period dummies

- Other industry controls, such as initial skill intensity of the industry

The China Shock: Balance Tests

- Shifts are uncorrelated with industry Xs, controlling for period FE
- Uncorrelated with regional Xs, controlling for period FExlagged manuf. share

Balance variable	Coef.	SE
Panel A: Industry-level balance		
Production workers' share of employment, 1991	−0.011	(0.012)
Ratio of capital to value-added, 1991	−0.007	(0.019)
Log real wage (2007 USD), 1991	−0.005	(0.022)
Computer investment as share of total, 1990	0.750	(0.465)
High-tech equipment as share of total investment, 1990	0.532	(0.296)
No. of industry-periods	794	
Panel B: Regional balance		
Start-of-period % of college-educated population	0.915	(1.196)
Start-of-period % of foreign-born population	2.920	(0.952)
Start-of-period % of employment among women	−0.159	(0.521)
Start-of-period % of employment in routine occupations	−0.302	(0.272)
Start-of-period average offshorability index of occupations	0.087	(0.075)
Manufacturing employment growth, 1970s	0.543	(0.227)
Manufacturing employment growth, 1980s	0.055	(0.187)
No. of region-periods	1,444	

The China Shock: Estimates/SE from Industry-year Reg.

- China shock is larger in the 2000s (post WTO entry)
- This is when overall manuf. decline is stronger for other reasons

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Coefficient	−0.596 (0.114)	−0.489 (0.100)	−0.267 (0.099)	−0.314 (0.107)	−0.310 (0.134)	−0.290 (0.129)	−0.432 (0.205)
Regional controls							
Autor *et al.* (2013) controls	✓	✓	✓		✓	✓	✓
Start-of-period mfg. share	✓						
Lagged mfg. share		✓	✓	✓	✓	✓	✓
Period-specific lagged mfg. share			✓	✓	✓	✓	✓
Lagged 10-sector shares					✓		✓
Local Acemoglu *et al.* (2016) controls						✓	
Lagged industry shares							✓
SSIV first stage F-stat.	185.6	166.7	123.6	272.4	64.6	63.3	27.6
No. of region-periods	1,444	1,444	1,444	1,444	1,444	1,444	1,444
No. of industry-periods	796	794	794	794	794	794	794

Identification from Exogenous Shares

Special Case 1: One Time Period, Two Industries

- $S_{i2} = 1 - S_{i1}$
- $Z_i = S_{i1}g_1 + S_{i2}g_2 = S_{i1}g_1 + (1 - S_{i1})g_2 = g_2 + (g_1 - g_2)S_{i1}$
- First Stage

$$D_i = \gamma_0 + \gamma Z_i + \eta_i$$
$$D_i = \underbrace{\gamma_0 + \gamma g_2}_{\text{constant}} + \underbrace{\gamma(g_1 - g_2)}_{\text{coefficient}} S_{i1} + \eta_i$$

$\Rightarrow$ The instrument is S_{i1}, while g_k affects relevance

Special Case 2: T Periods, Two Industries

- $Z_{it} = S_{i10}g_{1t} + S_{i20}g_{2t} = g_{2t} + (\Delta_{gt})S_{i10}$
- First Stage

$$D_i = \tau_i + \tau_t + \gamma Z_{it} + \eta_{it}$$
$$D_i = \tau_i + \underbrace{\tau_t + \gamma g_{2t}}_{\tilde{\tau}_t} + \underbrace{\gamma\Delta_{gt}}_{\tilde{\gamma}_t} S_{i10} + \eta_{it}$$

$\Rightarrow$ Industry shares $\times$ time period is the instrument: $Z_{it} = \underbrace{\Delta_{gt}}_{\text{size}} \underbrace{S_{i10}}_{\text{exposure}}$

$\Rightarrow$ Analogy to continuous diff-in-diff: use "pre-period" to test // pre-trends

General Case; Pooling Diff-in-Diffs

- Assume exogenous shares: Cov $[\epsilon_i, S_{ik}] = 0$ for every k
- With Y_i measured in differences, this is K parallel trend assumptions
- Shares are measured in the pre-period
- Rules out any unobserved shocks ν_k that affect Y_i via S_{ik} (or similar shares)
  - $\Rightarrow$ Even if shares are drawn at random, these shocks would violate // trends
- Then we have K valid IVs: $S_{i1}, ..., S_{iK}$
  - $\Rightarrow$ SSIV with $Z_i = \sum_k S_{ik} g_k$ is one reasonable way to combine them
  - $\Rightarrow$ 2SLS (for small K, or even one k) is another alternative

Plausible Vs. Implausible Exogenous Shares

- Construct the instrument with shares that are tailored to the treatment
- Mediating only the shocks to D_{it} and not other shocks that might affect Y_{it}
- E.g. Immigration shares: local migration from various origins in the past
- E.g. industry employment shares are generic: reflect exposure to other shocks

$\Rightarrow$ Same instrument for different treatments $\rightarrow$ problematic

Rotemberg Weights

- $\hat{\tau} = \sum_k \hat{\alpha}_k \hat{\tau}_k$ for $\hat{\tau}_k$ that uses S_{ik} as IV one at a time
- $\hat{\alpha}_k$ are higher for k with more extreme shifts and larger first stages
- $\hat{\alpha}_k$ add up to one but can be negative (potential issue w/heterog. effects)
- Scrutinize validity of the share IVs with highest (Rotemberg) weights

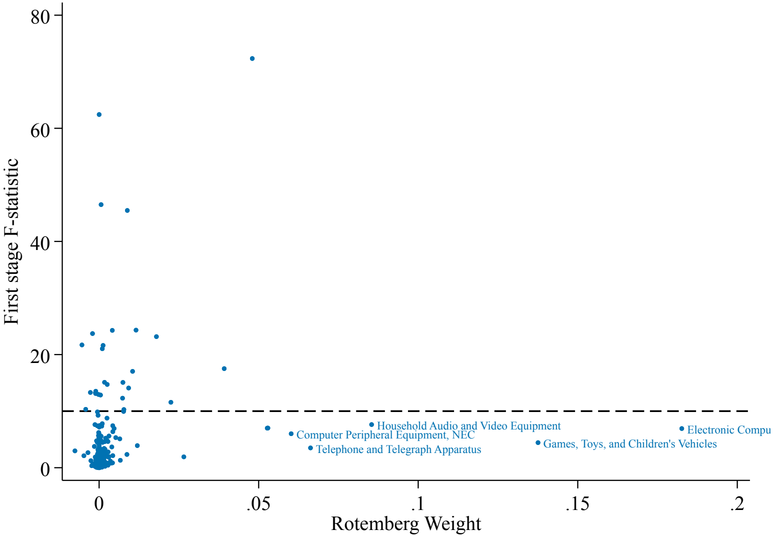
$\Rightarrow$ Sensitivity of the SSIV estimate to violations of exogeneity by each share IV

Software Implementation

- `bartik_weight` command in Stata and R computes the Rotemberg weights

$\Rightarrow$ Each weight returned in `cmd:r(alpha)` corresponds to the misspecification elasticity for each individual instrument, when using the Bartik instrument defined by the weights in opt `weightstub()`

China Shock Example (cont'ed): Rotemberg Weights



A Checklist for the Share-Based Approach

1. Determine whether the exposure shares are potentially suitable instruments
2. Choose the necessary unit-level controls
3. Characterize which shares matter the most for the estimate
4. Implement balance tests for individual shares in addition to the instrument
5. Check sensitivity to how share instruments are combined

Summary of Main Practical Takeaways

	Approach	
	Many exogenous shifts (1)	**Exogenous shares** (2)
Identification argument	Shifts are as-good-as-randomly assigned and only affect the outcome through the treatment	Each share satisfies parallel trends: the outcomes of units with high vs. low shares would have trended the same if not for the treatment
Estimation	Control for the sum of shares (if not one) and shift-share aggregates of any shift-level controls	Check robustness to using share instruments directly: e.g., one share at a time or pooled via two-stage least squares or limited information maximum likelihood
Statistical inference	Get exposure-robust standard errors from the equivalent shift-level instrumental variable regression	Use conventional heteroskedasticity- or cluster-robust standard errors
Balance tests	For both the shift-share instrument and the shifts	For both the shift-share instrument and the shares with high Rotemberg weights
Do not use when...	You would not want to use the shifts directly as an instrument in a shift-level regression, e.g. because they are too few or endogenous	You would not want to use shares directly as instruments, e.g. because they are "generic" (capturing the unit's exposure to many types of shocks)

Rural migration and Urban Informality: Evidence from Brazil

Imbert and Ulyssea (2025)

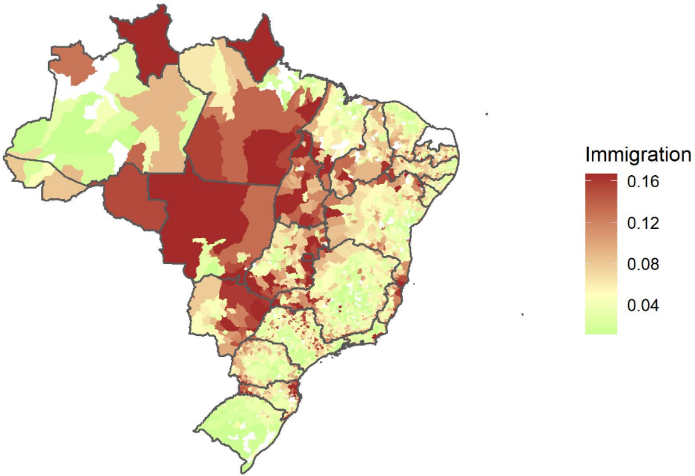
- Study the labor market effects of rural-urban migration on Brazilian cities
- SSIV to identify the causal effects of immigration at destination
  - $\Rightarrow$ $\uparrow$ formal employees, $\downarrow$ informal employees and $\downarrow$ wages
  - $\Rightarrow$ $\uparrow$ formal firms, $\uparrow$ jobs, $\uparrow$ entry, and $\uparrow$ exit
- Model of firms dynamics with inf. and heterog. growth profiles across firms

$\Rightarrow$ Eq. effects of migration with more/less enforc., with/without wage rigidity

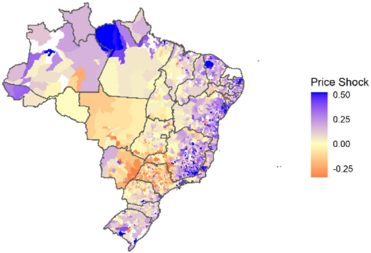
Data

- Migration and Labor Market Outcomes: Population Census, 1991-2010
  - $\Rightarrow$ Migrants $=$ in their current location $\leq$ 10 years
  - $\Rightarrow$ Focus migration to urban areas (88%), and cross-state borders
- Firms' data
  - $\Rightarrow$ Matched employer-employee, universe of formal firms and workers
  - $\Rightarrow$ Survey data on small (up to 5 employees) formal and informal firms
- Push shocks
  - $\Rightarrow$ Int. agri. commodity price shocks $\times$ crop shares at the muni. level (in 1980)
  - $\Rightarrow$ Drought index $\times$ growing season by crop $\times$ crop shares

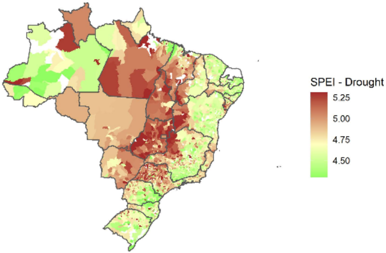
Immigration, 2000-2010



Migration Push Shocks



(a) Price Shocks



(b) Drought Shocks

Shift-Share Design

$$\Delta y_d = \beta_0 + \beta_1 Mig_d + \beta_2' X_d + u_d$$

- $\Delta y_d = y_{d,2010} - y_{d,2000}$ (Census or RAIS)
- $Mig_d = \sum_o \sum_{t=2001}^{2010} \frac{Mig_{o,d,t}}{Pop_{d,2000}}$
- X_d: share of male, young, and high skill (HS completed) in 2000

$\Rightarrow$ Instrument: $Z_d = \sum_o \underbrace{\lambda_{o,d}}_{\text{shares}} \underbrace{s_o^{\text{drought}}}_{\text{shift}}$

Shift-Share Design (cont'd)

- Baseline migration shares may be endogenous to future outcomes

$\Rightarrow$ Push-shocks (shifts) are as good as randomly assigned

$\Rightarrow$ Instrument only affects the dependent variables via the immigration shares

- A few specification/robustness checks
  - Pre-trends: dynamic effects + lagged changes in outcomes as a control
  - Persistence of migration (the shares): control for lagged migration rates
  - Capital Channel: Control for exposure through bank network
  - Demand Channel: Control for local price shocks and shocks to neighboring regions weighted by distance
  - Second push instrument coming from variations in international crop prices

Effects of Immigration on Workers

	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
Panel A: OLS						
Immigration	0.037*	0.105***	−0.068***	0.062	0.032	0.035
	(0.019)	(0.023)	(0.014)	(0.076)	(0.068)	(0.092)
Panel B: IV-Drought						
Immigration	−0.014	0.271***	−0.284***	−0.126	−0.671**	−0.200
	(0.060)	(0.089)	(0.072)	(0.284)	(0.336)	(0.352)
F Statistic (IV)	18.11	18.11	18.11	18.11	18.11	18.11
Baseline Mean	0.332	0.229	0.103	-	-	-
Observations	3,545	3,545	3,545	3,545	3,545	3,545

Notes: Robust standard errors in parenthesis. All regressions control for the share of women, youth (less than 18 years old), and high skill individuals (at least completed high-school) measured in 2000. All regressions are weighted by municipality's population in 2000.

Effects of Immigration on Firms

	Nb firms (1)	Entry (2)	Exit (3)	Nb jobs (4)	Firm wage (5)
Panel A: OLS					
Immigration	1.342***	1.167***	1.588***	1.071***	0.372***
	(0.108)	(0.203)	(0.274)	(0.270)	(0.101)
Panel B: IV-Drought					
Immigration	1.625***	2.555***	2.758**	2.031***	−0.747
	(0.306)	(0.877)	(1.123)	(0.624)	(0.554)
F Statistic (IV)	18.11	18.11	18.11	18.11	18.11
Observations	3,545	3,545	3,545	3,545	3,545

Notes: Robust standard errors in parenthesis. All regressions control for the share of women, youth (less than 18 years old), and high skill individuals (at least completed high-school) measured in 2000. All regressions are weighted by municipality's population in 2000. Columns 1-3 refer to the log of total number of firms, entrants and exiting firms, respectively.

Shift-Share Design: Summary of Results

- 1p.p. $\uparrow$ in $Mig_d(14.5\%SD) \rightarrow \uparrow$ 0.4 p.p. in share of formal $\approx$ 2.1% increase

$\Rightarrow$ Formalization effect driven by workers moving from informal to formal jobs

- 1p.p. $\uparrow$ in $Mig_d \rightarrow \downarrow$ formal wages by 0.7%, no effect on informal wages

- 1p.p. $\uparrow$ in $Mig_d \rightarrow \uparrow$ 2.4% in the number of firms, 2.2% in nb of formal jobs

$\Rightarrow$ There is greater churn, effect on entry is slightly higher

Model: Overview

- Continuum of firms indexed by their individual productivity θ
- Same technology, labor only input: $f(\ell) = \theta q(\ell), q' > 0, q'' < 0$
- Tradeoffs are driven by regulations and enforcement
  - $\Rightarrow$ Informal firms: lower entry costs, and no regulation costs (e.g. taxes), but cost of operation is increasing in firm size
  - $\Rightarrow$ Formal firms: face all regulatory costs, but constant marginal costs; can evade labor regulations by hiring informal workers
- Endogenous exit + exogenous death shock (different across sectors)
- No aggregate shocks, homogenous labor, and labor supply is fixed

Firm Profits

- Informal firms avoid taxes but pay an informality cost

$$\Pi_i(\theta, w) = \max_{\ell}\{\theta q(\ell) - \tau_i(\ell) w\}, \quad \tau_i' > 0, \tau_i'' > 0$$

- Formal firms pay revenue taxes but can evade payroll tax

$$\Pi_f(\theta, w) = \max_{\ell}\{(1-\tau_y)\theta q(l) - C(\ell)\}$$

$$C(l) = \begin{cases} \tau_f(\ell) w & \text{for } \ell \leq \tilde{\ell} \\ \tau_f(\tilde{\ell}) w + (1+\tau_w)(\ell - \tilde{\ell}) & \text{for } \ell > \tilde{\ell}, \end{cases}$$

- Where $\tau_f' > 0, \tau_f'' > 0$ and $\tilde{\ell}$ such that $\tau_f'(\tilde{\ell}) = 1 + \tau_w$

Firm Dynamics

- Dynamics is driven by the evolution of firms' idiosyncratic productivity θ
- Productivity process is given by

$$\begin{aligned} \ln\theta_{j,1} =& \ln\nu_j + \ln\epsilon_{j,1} \\ \ln\theta_{j,t} =& \rho_s \ln\theta_{j,t-1} + (1-\rho_s)\ln\nu_j + \ln\epsilon_{j,t}, \quad t \geq 2 \end{aligned}$$

- Where j indexes firms, $s \in \{i, f\}$ denotes sector, and $\ln\epsilon \sim \ln N(0, \sigma_s^2)$
- Formal firms cannot become informal
- Informal firms can pay $\tilde{c}^e = c_f^e - c_i^e$ and formalize

Firm Dynamics (cont'd)

- The value functions of formal and informal incumbents

$$
\begin{aligned}
V_f(\theta, w) =& \tau_f(\theta, w) + (1-\delta_f)\beta \max\{0, E_\nu[V_f(\theta', w) \mid \theta]\} \\
V_i(\theta, w) =& \tau_i(\theta, w) \\
& + \beta \max\{0, (1-\delta_i)E_\nu[V_i(\theta', w) \mid \theta], (i-\delta_f)E_\nu[V_f(\theta', w) \mid \theta] - \tilde{c}^e\}
\end{aligned}
$$

- where β is the discount factor, δ_s the exogenous exit

- Exit decisions and informal-to-formal transitions follow cutoff rules

$$
\begin{aligned}
E_\nu[V_f(\theta', w) \mid \underline{\theta}_s] = 0, \quad s \in \{i, f\} \\
E_\nu[V_f(\theta', w) - V_i(\theta', w) \mid \overline{\theta}_i] = \tilde{c}^e
\end{aligned}
$$

Firm Entry

- Entrants in both sectors pay a fixed cost of entry, denoted by $c_s^e, s \in \{i, f\}$
- These parameters will be estimated, but theory suggests that $c_f^e > c_i^e$
- Expected value of entry for firm with long-run productivity ν: $E[V_s(\theta, w) \mid \nu]$
- Entry is also characterized by cutoff rules

$$E_\nu[V_i(\theta, w) \mid \underline{\nu}_i] = c_i^e$$
$$E_\nu[V_f(\theta, w) - V_i(\theta, w) \mid \underline{\nu}_f] = c_f^e - c_i^e$$

- Where $\underline{\nu}_f$ characterizes the last firm to enter sector $s = i, f$

Empirical Predictions

- Immigration shock: $\uparrow$ both labor supply and demand for the final good
- Lower real wages and higher labor demand for incumbent firms
- Incumbents are limited by scale, so $\uparrow$ in labor supply $\rightarrow$ greater firm entry
- In eq. not only effect on entry rates but also exit rates and firm size distrib.
- Net effect on informality, or output per worker, is unclear
  - $\Rightarrow$ If migrants are largely absorbed by informal firms and/or low-productivity formal firms: $\uparrow$ informality among firms and workers
  - $\Rightarrow$ if lower wages induce more entry then more firm growth in the formal sector and lower informality

Model Parametrization

- Production function: $y(\theta, \ell) = \theta \ell^{\alpha}, \quad 0 < \alpha < 1$
- Cost functions: $\tau_s(\ell) = \left(1 + \frac{\ell}{\varphi_s}\right)\ell, \quad \varphi_s > 0$

$\Rightarrow$ The larger φ_i the more informal firms can grow, the larger φ_f the easier for formal firms to hire informal workers

- Long-run productivity $\nu \sim H$, observed before entry occurs

$$H(\nu \geq x) = \begin{cases} \left(\frac{\nu_0}{x}\right)^{\xi} & \text{for } x \geq \nu_0 \\ 1 & \text{for } x < \nu_0 \end{cases}$$

$\Rightarrow$ First productivity draw $\theta_1 = \nu \epsilon_1$ has a Pareto-Lognormal distribution

Estimation

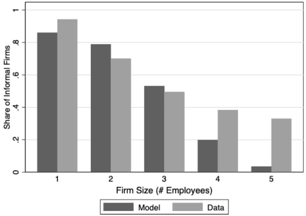
- Two step SMM procedure (set τ_w, tau_y to statutory values)
  - ⇒ First step: system GMM and panel data on productivity process
  - ⇒ Second step: 12 remaining parameter estimated using SMM
- Moments
  - ⇒ Overall share of informal firms and by size brackets
  - ⇒ Average share of informal workers within formal firms
  - ⇒ Formal and informal firm growth at ages 5 and 10 relative to age 1
  - ⇒ Formal/informal firms size distribution by size brackets

Model's parameters

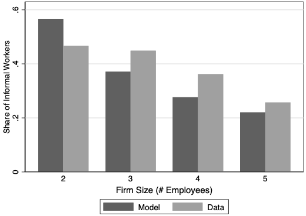
Parameter	Description	Source	Value
First Step			
τ_w	Payroll Tax	Statutory values	0.375
τ_y	Revenue Tax	Statutory values	0.293
ρ	Productivity Process: Persistence Parameter	GMM Estimation	0.92
ν_0	Pareto's Location Parameter	Calibrated	7.3
γ_f	Per-period fixed cost (Formal)	Calibrated	0.7
Second Step: Min. Dist. Calibration			
φ_f	Intensive margin: $\tau_f = \left(1 + \frac{\ell}{\varphi_f}\right)\ell$	–	5.830
φ_i	Extensive margin: $\tau_i = \left(1 + \frac{\ell}{\varphi_i}\right)\ell$	–	5.427
δ_i	Informal death shock	–	0.148
δ_f	Formal death shock	–	0.066
γ_i	Per-period fixed cost (Informal)	–	0.340
ξ	Pareto shape parameter	–	3.801
$c_f^{e\,\dagger}$	Formal sector's entry cost	–	6,205
$c_i^{e\,\dagger}$	Informal sector's entry cost	–	2,800
α	Span-of-control	–	0.643
σ_i	Informal productivity process: SD	–	0.144
σ_f	Formal productivity process: SD	–	0.145
ρ_i	Informal productivity process: persistence	–	0.935

† Estimates and SD expressed in R$ of 2003.

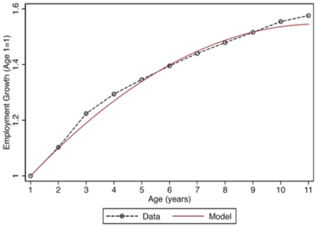
Model Fit: Non-targeted Moments



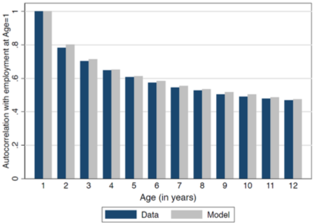
(a) Extensive Mg.



(b) Intensive Mg.



(c) Formal Firms' growth



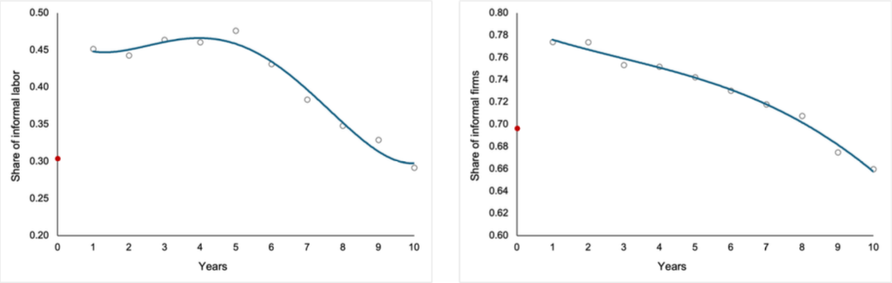
(d) Log-Employment Autocorrelations

Counterfactuals

	IV Estimation	Labor Supply Shock	LS Shock + Enforcement
Share Informal Workers ($\Delta\%$)	-7.3	-4.3	-38.2
Wages ($\Delta\%$)	-5.3	-3.4	-2.1
Number Formal Firms ($\Delta\%$)	20.7	16.3	128
Newly created firms	–	9.9	–
Previously informal firms	–	6.4	–
Share Informal Firms ($\Delta\%$)	–	-5.3	-68.2
Average Firm Productivity ($\Delta\%$)	–	-1.4	2.5
Output ($\Delta\%$)	–	7.1	8.3
Taxes ($\Delta\%$)	–	8.7	30.9

Notes: IV estimation results from a regression contrasting municipalities below and above the median immigration rate (see text). Labor supply shock corresponds to simulating a permanent increase of 10% in population, and effects are measured as percentage change relative to baseline values.

From Short- to Long-Run: Wage Rigidity in the Formal Sector



(a) Labor Informality

(b) Firm Informality

Notes: Red dot indicates the initial steady state of the baseline economy.

Main Takeways from the Estimated Model

- Model revisits empirical evidence on the LMK effect of (internal) immigration
- $\downarrow$ wages, $\uparrow$ entry of formal firms, nb of formal firms, jobs and formal. share

$\Rightarrow$ Gains do not accrue to the most productive firms $\rightarrow$ output per worker falls

$\Rightarrow$ Despite its role as a "stepping-stone" in the short run, the informal sector reduces the aggregate benefits of immigration

$\Rightarrow$ The typical Harris-Todaro-Fields result arises with downward wage rigidity in the formal sector: higher informality + lower output gains

SSIV $\Leftrightarrow$ Economic Model

- Model attempts to rationalize surprising results from SSIV
- The model quantitatively replicates the long-run IV results
- Model sheds light on mechanism for $\uparrow$ in formal firms (40% comes from the formalization of informal firms, and 60% comes from the creation of new formal firms)

$\Rightarrow$ Firm dynamics and the linkages between the informal and formal sectors