`regsensitivity`: A Stata Package for Regression Sensitivity Analysis

Paul Diegert* Matthew A. Masten† Alexandre Poirier‡

August 2, 2022

Introduction

Omitted variables are one of the most important threats to the identification of causal effects. In linear models, the well known omitted variable bias formula shows how an omitted variable can bias the regression coefficient on the covariate of interest when that covariate is correlated with the omitted variable. Since is often implausible to assume that data has been collected on every relevant variable, applied research is often vulnerable to this bias. Nonetheless, omitted variable bias can be quantified under various alternative assumptions about the relationship between the omitted variable and the covariate of interest. Using these techniques, researchers can analyze how sensitive their results are to omitted variable bias.

Several methods of sensitivity analysis for linear models have been proposed in the literature. The `regsensitivity` package implements the methods proposed in Diegert, Masten, and Poirier (2022), Oster (2019), and Masten and Poirier (2022). In each of these papers, the authors define a set of sensitivity parameters which index relaxations of the assumption that the covariate of interest is uncorrelated with any unobserved variables. The parameter of interest is β_{long}, the coefficient on that covariate of interest in the infeasible regression that includes the unobserved variables. Using this framework, we can ask two questions:

1. What is the set of parameter estimates for β_{long} which are consistent with the relaxed assumptions? That is, what are bounds on the value of β_{long} under the alternate assumptions?
2. How much can we relax the exogeneity assumption before a hypothesis about β_{long} is overturned? This is called the *breakdown point*: the maximum relaxation of the baseline assumption before the hypothesis is overturned.

`regsensitivity` can be used to perform both of these sensitivity analyses using the sensitivity parameters defined in Diegert, Masten, and Poirier (2022), Oster (2019), and Masten and Poirier (2022).

Getting Started

We will illustrate how to use `regsensitivity` with data from Bazzi, Fiszbein, and Gebresilasse (2020), which is used in the empirical application in Diegert, Masten, and Poirier (2022). One of the datasets used in Bazzi, Fiszbein, and Gebresilasse (2020) is included with the package, and can be loaded using the `sysuse` command:

```
. sysuse bfg2020, clear
```

*Department of Economics, Duke University, paul.diegert@duke.edu
†Department of Economics, Duke University, matt.masten@duke.edu
‡Department of Economics, Georgetown Univervsity, alexandre.poirier@georgetown.edu

The specification in column (7) of Table III in Bazzi, Fiszbein, and Gebresilasse (2020) and replicated in Diegert, Masten, and Poirier (2022) is as follows:

```
. local y avgrep2000to2016

. local x tye_tfe890_500kNI_100_l6

. local w1 log_area_2010 lat lon temp_mean rain_mean elev_mean d_coa d_riv d_lak ave_gyi

. local w0 i.statea

. local w `w1´ `w0´

. local SE cluster(km_grid_cel_code)

. reg `y´ `x´ `w´, `SE´

Linear regression                               Number of obs     =      2,036
                                                F(39, 379)        =          .
                                                Prob > F          =          .
                                                R-squared         =     0.3321
                                                Root MSE          =     9.6368

                                    (Std. err. adjusted for 380 clusters in km_grid_cel_code)
```

avgrep2000to2016	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
tye_tfe890_500kNI_100_l6	2.054759	.3491648	5.88	0.000	1.368217	2.741302
log_area_2010	.2758775	.979906	0.28	0.778	-1.650856	2.202611
lat	2.26515	1.101151	2.06	0.040	.1000189	4.430281
lon	.0108189	.2913783	0.04	0.970	-.5621017	.5837395
temp_mean	1.62737	1.068132	1.52	0.128	-.4728361	3.727577
rain_mean	.0164826	.0046086	3.58	0.000	.007421	.0255442
elev_mean	.0154764	.0037786	4.10	0.000	.0080468	.022906
d_coa	9.83e-06	3.76e-06	2.62	0.009	2.45e-06	.0000172
d_riv	.0000307	9.91e-06	3.10	0.002	.0000112	.0000502
d_lak	3.05e-07	4.45e-06	0.07	0.945	-8.44e-06	9.05e-06
ave_gyi	-3.779807	10.81002	-0.35	0.727	-25.03493	17.47532
statea						
5	-4.213545	3.386398	-1.24	0.214	-10.87203	2.444936
8	-27.31682	6.246914	-4.37	0.000	-39.59977	-15.03387
12	4.627587	3.354655	1.38	0.169	-1.968479	11.22365
13	.5398875	2.643504	0.20	0.838	-4.657883	5.737658
17	-10.25822	3.787414	-2.71	0.007	-17.7052	-2.811248
18	-5.924452	3.497393	-1.69	0.091	-12.80118	.9522727
19	-18.02705	4.514016	-3.99	0.000	-26.9027	-9.151398
20	1.598741	4.633634	0.35	0.730	-7.512109	10.70959
21	-.504168	3.185907	-0.16	0.874	-6.768436	5.7601
22	.8939823	3.276872	0.27	0.785	-5.549145	7.337109
26	-14.06314	4.40552	-3.19	0.002	-22.72546	-5.400816
27	-18.10308	4.821495	-3.75	0.000	-27.58331	-8.622851
28	-6.930918	3.675983	-1.89	0.060	-14.15879	.2969573
29	-4.170334	3.902039	-1.07	0.286	-11.84269	3.502024
31	-1.342615	4.73751	-0.28	0.777	-10.65771	7.97248
35	-40.78007	9.264248	-4.40	0.000	-58.99583	-22.56431
36	-9.821649	4.68884	-2.09	0.037	-19.04105	-.6022507
37	-13.53756	4.241671	-3.19	0.002	-21.87772	-5.197404
38	-11.98193	5.512474	-2.17	0.030	-22.82079	-1.143061
39	-6.190808	3.655508	-1.69	0.091	-13.37843	.9968088
40	13.60029	4.880539	2.79	0.006	4.003963	23.19661
42	-3.14623	4.426406	-0.71	0.478	-11.84962	5.557161
46	-11.84706	5.101547	-2.32	0.021	-21.87794	-1.816175
47	-3.541445	2.794141	-1.27	0.206	-9.035406	1.952515
48	12.82591	4.174157	3.07	0.002	4.618502	21.03331
51	-.8116892	4.047756	-0.20	0.841	-8.770561	7.147182
54	-3.243583	3.570236	-0.91	0.364	-10.26353	3.776369
55	-18.92918	4.503985	-4.20	0.000	-27.78511	-10.07325
56	-19.02288	9.729311	-1.96	0.051	-38.15307	.1073112

```
                   _cons |  -73.53523   57.84708    -1.27   0.204    -187.2766    40.20618
```

Without a subcommand **regsensitivity** performs two sensitivity analyses, one from Diegert, Masten, and Poirier (2022), and one from Oster (2019):

```
. regsensitivity `y´ `x´ `w´, compare(`w1´)

Regression Sensitivity Analysis, Bounds

Analysis         : DMP (2022)                 Number of obs     =      2,036
                                              Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6   Beta(medium)      =      2.055
Outcome          : avgrep2000to2016           R2(short)         =      0.033
                                              R2(medium)        =      0.105
                                              Var(Y)            =    101.739
                                              Var(X)            =      0.901
                                              Var(X_Residual)   =      0.882

Hypothesis       : Beta > 0                   Breakdown point   =       80.4%
Other Params     : cbar = 1, rybar = +inf
--------------------------------------------------------------------------------
 rxbar                              Beta
--------------------------------------------------------------------------------
 0.000                             [ 2.0548,  2.0548 ]
 0.100                             [ 1.9042,  2.2053 ]
 0.201                             [ 1.7488,  2.3607 ]
 0.301                             [ 1.5830,  2.5266 ]
 0.402                             [ 1.3991,  2.7104 ]
 0.502                             [ 1.1855,  2.9240 ]
 0.602                             [ 0.9216,  3.1879 ]
 0.703                             [ 0.5647,  3.5448 ]
 0.803                             [ 0.0018,  4.1077 ]
 0.904                             [-1.2579,  5.3675 ]
 0.989                             [   -inf,    +inf ]
--------------------------------------------------------------------------------

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)
Treatment        : tye_tfe890_500kNI_100_l6
Outcome          : avgrep2000to2016
Hypothesis       : Beta != 0
--------------------------------------------------------------------------------
 R-squared(long)                    Delta(Breakdown)
--------------------------------------------------------------------------------
 0.137                              2328.8%
 1.000                              170.4%
--------------------------------------------------------------------------------
```

To explore the output, we will consider each sensitivity analysis separately.

Diegert, Masten, and Poirier (2022)

Diegert, Masten, and Poirier (2022) consider the model:

$$Y = \beta_{\text{long}} X + \gamma_0' W_0 + \gamma_1' W_1 + \gamma_2 W_2 + Y^{\perp X,W},$$

where (Y, X, W_0, W_1) are observed and W_2 is an omitted variable that is potentially correlated with (X, W_0, W_1).[1] Restrictions on the joint distribution of (Y, X, W_0, W_1, W_2) are governed by three scalar

[1] We denote the coefficient on X by β_{long} because it is the regression coefficient in the infeasible "long" regression of Y on $(1, X, W_0, W_1, W_2)$. This helps distinguish it from β_{med}, the coefficient on X in the regression of Y on $(1, X, W_0, W_1)$, and from β_{short}, the coefficient on X in the regression of Y on $(1, X, W_0)$.

sensitivity parameters, $(\bar{r}_X, \bar{r}_Y, \bar{c})$. Given the joint distribution of the observed variables, (Y, X, W_0, W_1), and the values of the sensitivity parameters, Diegert, Masten, and Poirier (2022) show how to compute the upper and lower bounds on the identified set for β_{long}, denoted by $\mathcal{B}_I(\bar{r}_X, \bar{r}_Y, \bar{c})$. The identified set is the set of values of β_{long} which are consistent with the distribution of observed data and the maintained assumptions. When $\bar{r}_X > 0$ and $\bar{r}_Y > 0$, β_{long} is not point identified, so we instead estimate these bounds. For more details about the definitions and interpretation of the sensitivity parameters, see Diegert, Masten, and Poirier (2022).

`regsensitivity bounds` can be used with the option `dmp` to calculate the upper and lower bounds of $\mathcal{B}_I(\bar{r}_X, \bar{r}_Y, \bar{c})$. The basic syntax for `regsensitivity` is similar to the `regress` command and its variants:

`regsensitivity bounds` *depvar indepvar controls*, *options...*,

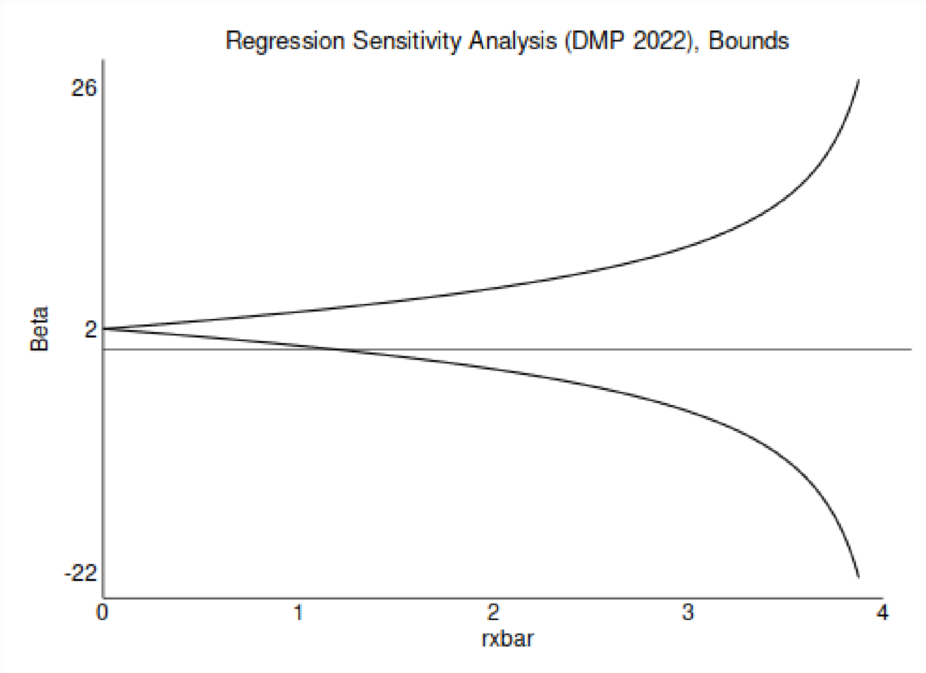
where *depvar* is the dependent variable, Y, and *indepvar controls* are the independent variables, (X, W_0, W_1). Unlike `regress`, the order of the independent variables matter in the call to `regsensitivity`. The first variable, *indepvar*, is X, the variable of interest for which the sensitivity analysis is conducted while *controls* are additional variables included in the model which are not of interest.

By default, `regsensitivity bounds` calculates the bounds for a range of values of $\bar{r}_X$ holding $\bar{c}$ and $\bar{r}_Y$ fixed. The defaults are to set $\bar{c} = 1$ and $\bar{r}_Y = +\infty$. To specify a different value of $\bar{c}$, use the `cbar` option. For example,

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(.1)

Regression Sensitivity Analysis, Bounds

Analysis        : DMP (2022)               Number of obs     =      2,036
                                           Beta(short)       =      1.925
Treatment       : tye_tfe890_500kNI_100_l6  Beta(medium)      =      2.055
Outcome         : avgrep2000to2016         R2(short)         =      0.033
                                           R2(medium)        =      0.105
                                           Var(Y)            =    101.739
                                           Var(X)            =      0.901
                                           Var(X_Residual)   =      0.882

Hypothesis      : Beta > 0                 Breakdown point   =       119%
Other Params    : cbar = .1, rybar = +inf
------------------------------------------------------------------------------
 rxbar                             Beta
------------------------------------------------------------------------------
 0.000                            [  2.05,   2.05 ]
 0.415                            [  1.41,   2.70 ]
 0.829                            [  0.70,   3.41 ]
 1.244                            [ -0.10,   4.21 ]
 1.658                            [ -1.03,   5.14 ]
 2.073                            [ -2.15,   6.26 ]
 2.488                            [ -3.56,   7.67 ]
 2.902                            [ -5.51,   9.62 ]
 3.317                            [ -8.64,  12.75 ]
 3.731                            [-15.85,  19.96 ]
 4.063                            [  -inf,   +inf ]
------------------------------------------------------------------------------
```

To plot the results, use the `plot` subcommand,

```
. regsensitivity plot
```



Notice that in the call to **regsensitivity bounds**, we also included an option **compare**(*varlist*). This specifies which of the variables in the *controls* are included in W_1 rather than W_0. These are referred to as the *comparison controls* because they are the variables used to calibrate the sensitivity parameters, $(\bar{r}_X, \bar{r}_Y, \bar{c})$. For more details, see section 3.3 in Diegert, Masten, and Poirier (2022).

By including more variables in the comparison controls, the identified set will tend to be larger for a given value of the sensitivity parameters. For example, if the compare option is omitted, then all the control variables are included in W_1:

```
. regsensitivity bounds `y´ `x´ `w´, cbar(.1)

Regression Sensitivity Analysis, Bounds

Analysis          : DMP (2022)                  Number of obs      =      2,036
                                                Beta(short)        =      1.708
Treatment         : tye_tfe890_500kNI_100_l6    Beta(medium)       =      2.055
Outcome           : avgrep2000to2016            R2(short)          =      0.027
                                                R2(medium)         =      0.332
                                                Var(Y)             =    136.320
                                                Var(X)             =      1.257
                                                Var(X_Residual)    =      0.882

Hypothesis        : Beta > 0                    Breakdown point    =       29.7%
Other Params      : cbar = .1, rybar = +inf
--------------------------------------------------------------------------------
 rxbar                              Beta
--------------------------------------------------------------------------------
 0.000                             [  2.05,    2.05 ]
 0.134                             [  1.15,    2.96 ]
 0.269                             [  0.20,    3.90 ]
 0.403                             [ -0.82,    4.93 ]
 0.538                             [ -1.97,    6.08 ]
 0.672                             [ -3.32,    7.42 ]
 0.806                             [ -4.97,    9.08 ]
 0.941                             [ -7.21,   11.32 ]
 1.075                             [-10.67,   14.78 ]
 1.210                             [-18.07,   22.17 ]
```

```
1.336                           [  -inf,   +inf ]
```

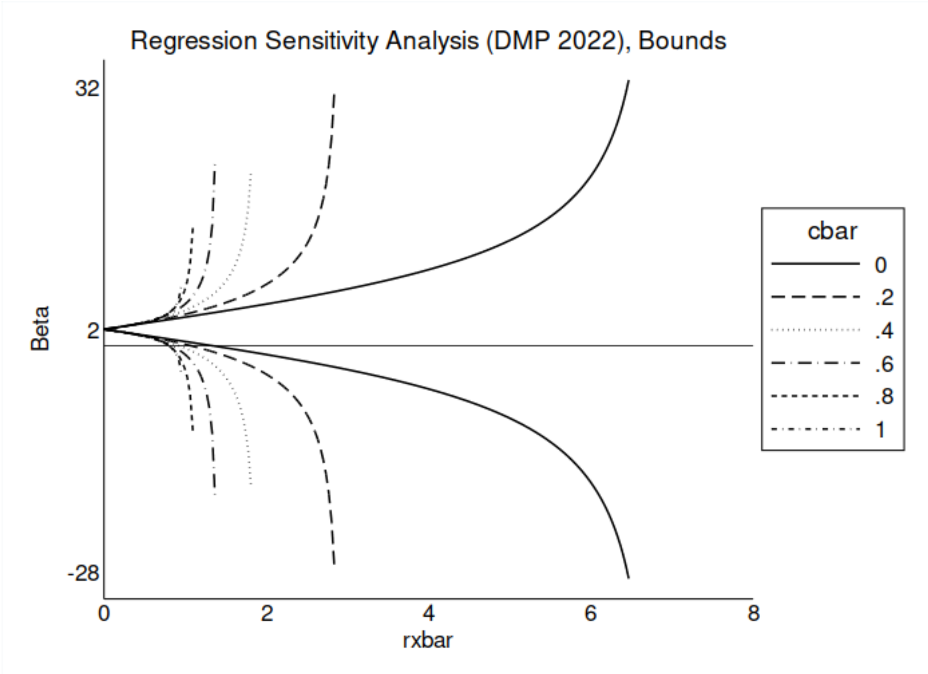
With all the controls included in W_1, the identified set becomes $\mathbb{R}$ at $\bar{r}_X = 1.336$, compared to $\bar{r}_X = 4.063$ when W_1 excludes the state fixed effects (*statea*). To directly compare the bounds under the two choices of W_1, we can manually set the values of `rxbar` to be the same in each case. The output table from the last call to `regsensitivity bounds` are stored in `e(idset_table)`. We can extract the values of `rxbar` from these and rerun the analysis with W_1 excluding state fixed effects as follows:

```
. forvalues i=1/11{
  2.     local rxbar `rxbar´ `=e(idset_table)[`i´, 1]´
  3. }

. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(.1) rxbar(`rxbar´)

Regression Sensitivity Analysis, Bounds

Analysis         : DMP (2022)                Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =       2.055
Outcome          : avgrep2000to2016          R2(short)          =       0.033
                                             R2(medium)         =       0.105
                                             Var(Y)             =     101.739
                                             Var(X)             =       0.901
                                             Var(X_Residual)    =       0.882

Hypothesis       : Beta > 0                  Breakdown point    =        119%
Other Params     : cbar = .1, rybar = +inf
--------------------------------------------------------------------------------
 rxbar                              Beta
--------------------------------------------------------------------------------
 0.000                           [ 2.0548,  2.0548 ]
 0.134                           [ 1.8525,  2.2570 ]
 0.269                           [ 1.6444,  2.4651 ]
 0.403                           [ 1.4299,  2.6796 ]
 0.538                           [ 1.2085,  2.9010 ]
 0.672                           [ 0.9794,  3.1301 ]
 0.806                           [ 0.7420,  3.3676 ]
 0.941                           [ 0.4952,  3.6143 ]
 1.075                           [ 0.2381,  3.8714 ]
 1.210                           [-0.0304,  4.1399 ]
 1.336                           [-0.2937,  4.4032 ]
--------------------------------------------------------------------------------
```

Comparing each line of the table to the previous call where all the *controls* were included in W_1, we can see that the bounds are much tigher for each value of $\bar{r}_X$.

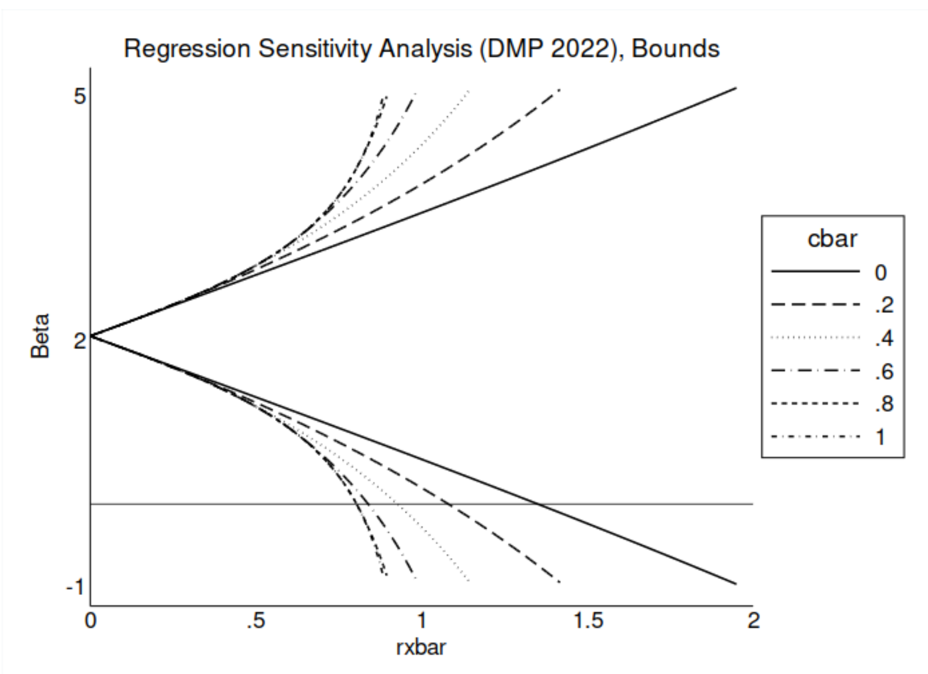
To compare multiple values of $\bar{c}$, a `numlist` can be given in the `cbar` option. With multiple values of $\bar{c}$, the command will show a plot rather than displaying the results in the console.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(0(.2)1)
```



By default, the plot will try to show where the identified set becomes $\mathbb{R}$ for each value of $\bar{c}$. For this example, the plot is dominated visually by the identified set for $\bar{c} = 0$. To see better where the identified set intersects with 0, we can rerun the analysis restricting the range of $\bar{r}_X$.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(0(.2)1) rxbar(0(.2)2) plot
```



Breakdown Frontier

The output of `regsensitivity bounds` shows a *breakdown point* for a given hypothesis about the parameter β_{long}. For a hypothesis $\beta_{\text{long}} \in B \subseteq \mathbb{R}$, the breakdown point is the smallest value of the sensitivity parameter $\bar{r}_X$ for which the hypothesis does not hold for every β value in the identified set. Formally,

$$\bar{r}_X^{\text{bp}}(\bar{r}_Y, \bar{c}; B) = \inf\{\bar{r}_X \geq 0 : b \in \mathcal{B}_I(\bar{r}_X, \bar{r}_Y, \bar{c}) \text{ for some } b \in \mathbb{R} \setminus B\}.$$

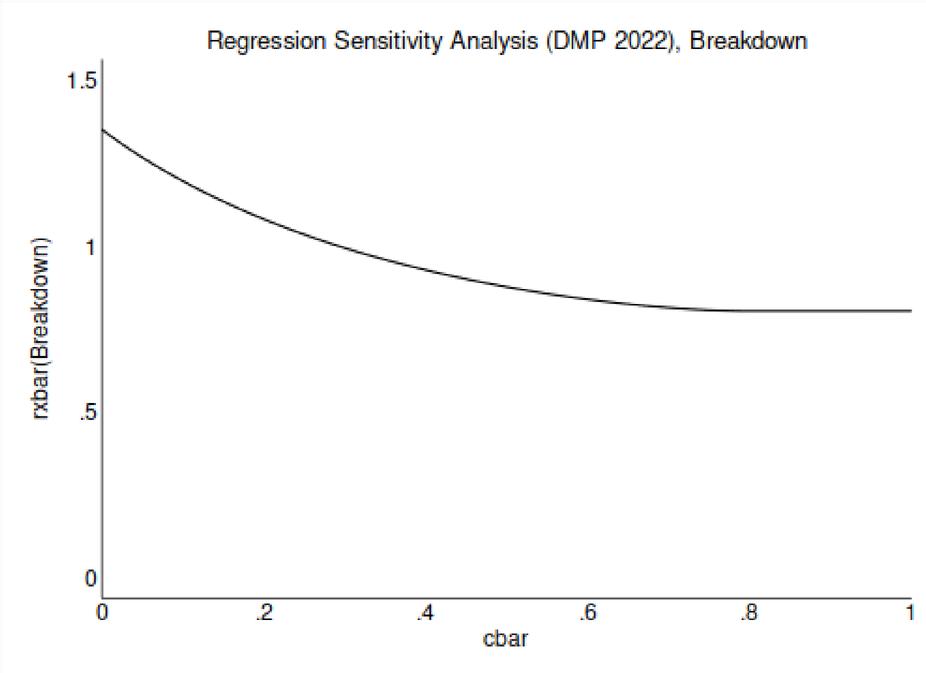
`regsensitivity` can handle hypotheses of the form, $\beta_{\text{long}} \gtrless b$ for any value of b. The default hypothesis is that $\text{sign}(\beta_{\text{long}}) = \text{sign}(\beta_{\text{med}})$, where β_{med} is the coefficient on X in a regression of Y on $(1, X, W_0, W_1)$. In this case $\beta_{\text{med}} > 0$, so the default is to test the hypothesis that $\beta_{\text{long}} > 0$.

The output to `regsensitivity bounds` showed that with W_1 excluding state fixed effects, $\bar{r}_X^{\text{bp}}(.1, +\infty; (-\infty, 0]) = 1.195$. To see how this breakdown point varies with the choice of the sensitivity parameter $\bar{c}$, use the `breakdown` subcommand,

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) cbar(0(.1)1)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)                Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =       2.055
Outcome          : avgrep2000to2016          R2(short)          =       0.033
                                             R2(medium)         =       0.105
                                             Var(Y)             =     101.739
                                             Var(X)             =       0.901
Hypothesis       : Beta > 0                  Var(X_Residual)    =       0.882
Other Params     : rybar = +inf
--------------------------------------------------------------------------------
 cbar                              rxbar(Breakdown)
--------------------------------------------------------------------------------
 0.000                             135.0%
 0.100                             119.5%
 0.200                             108.0%
 0.300                             99.3 %
 0.400                             92.7 %
 0.500                             87.6 %
 0.600                             83.9 %
 0.700                             81.4 %
 0.800                             80.4 %
 0.900                             80.4 %
 1.000                             80.4 %
--------------------------------------------------------------------------------
```

These results can also be plotted using the `plot` subcommand:

```
. regsensitivity plot
```



To test the hypothesis that $\beta_{\text{long}} > b$ for some other value, b, specify **beta(b lb)** (lb for "lower bound"). The **beta** option can also accept a **numlist** to test a range of hypotheses. For example, the following tests the hypotheses that $\beta_{\text{long}} > b$ for a range of values of b:

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) beta(-1(.2)1 lb)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)                Number of obs     =      2,036
                                             Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_16  Beta(medium)      =      2.055
Outcome          : avgrep2000to2016          R2(short)         =      0.033
                                             R2(medium)        =      0.105
                                             Var(Y)            =    101.739
                                             Var(X)            =      0.901
Hypothesis       : Beta > Beta(Hypothesis)   Var(X_Residual)   =      0.882
Other Params     : cbar = 1, rybar = +inf
---------------------------------------------------------------------------
 Beta(Hypothesis)                   rxbar(Breakdown)
---------------------------------------------------------------------------
 -1.000                             89.1 %
 -0.800                             87.9 %
 -0.600                             86.5 %
 -0.400                             84.8 %
 -0.200                             82.8 %
 0.000                              80.4 %
 0.200                              77.4 %
 0.400                              73.8 %
 0.600                              69.5 %
 0.800                              64.1 %
 1.000                              57.5 %
---------------------------------------------------------------------------
```

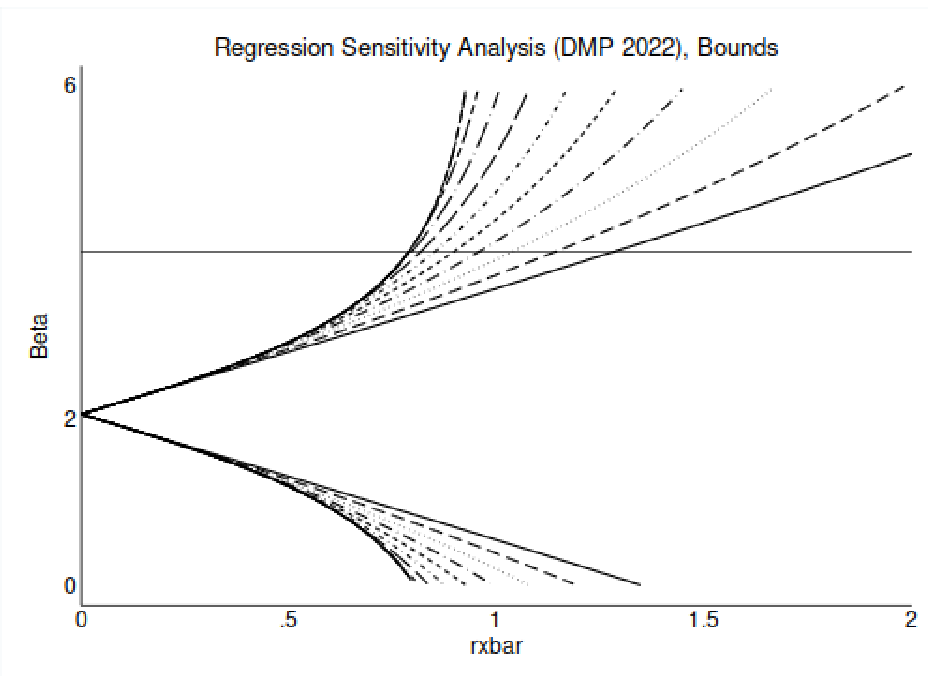
We can also test a hypothesis of the form $\beta_{\text{long}} < b$, by specifying **beta(b ub)** (ub for "upper bound"). For example the following checks the hypothesis that $\beta_{\text{long}} < 4$:

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) cbar(0(.1)1) beta(4 ub)
```

```
Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)                  Number of obs      =      2,036
                                               Beta(short)        =      1.925
Treatment        : tye_tfe890_500kNI_100_l6    Beta(medium)       =      2.055
Outcome          : avgrep2000to2016            R2(short)          =      0.033
                                               R2(medium)         =      0.105
                                               Var(Y)             =    101.739
                                               Var(X)             =      0.901
Hypothesis       : Beta < 4                    Var(X_Residual)    =      0.882
Other Params     : rybar = +inf
-------------------------------------------------------------------------------
 cbar                                 rxbar(Breakdown)
-------------------------------------------------------------------------------
 0.000                                128.1%
 0.100                                114.0%
 0.200                                103.6%
 0.300                                95.7 %
 0.400                                89.6 %
 0.500                                85.0 %
 0.600                                81.7 %
 0.700                                79.5 %
 0.800                                78.8 %
 0.900                                78.8 %
 1.000                                78.8 %
-------------------------------------------------------------------------------
```

To see visually where the identified set intersects with 4 we can specify this alternative hypothesis in the **regsensitivity bounds** command. By including the option **beta(4 ub)** the resulting plot will include a horizontal line at 4:

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) rxbar(0(.2)2) cbar(0(.1)1) beta(4, ub)

. regsensitivity plot, nolegend yrange(0 6)
```



Summary statistics

The output of `regsensitivity bounds` and `regsensitivity breakdown` both include a table of summary statistics. These are as follows,

- `Number of observations`
- `Beta(short)`: The coefficient on X in the regression of Y on $(1, X)$
- `Beta(medium)`: The coefficient on X in the regression of Y on $(1, X, W_1)$
- `R2(short)`: The R-squared from the regression of Y on $(1, X)$
- `R2(medium)`: The R-squared from the regression of Y on $(1, X, W_1)$
- `Var(Y)`: Variance of Y
- `Var(X)`: Variance of X
- `Var(X_Residual)`: Variance of $X^{\perp W_1}$, the residual from the regression of X on $(1, W_1)$.

Note: For all the summary statistics reported in this table, (Y, X, W_1) are shorthand for $(Y^{\perp W_0}, X^{\perp W_0}, W_1^{\perp W_0})$ where $Y^{\perp W_0}$ is the residual from the regression of Y on $(1, W_0)$ and likewise for $X^{\perp W_0}$ and $W_1^{\perp W_0}$.

Oster 2019

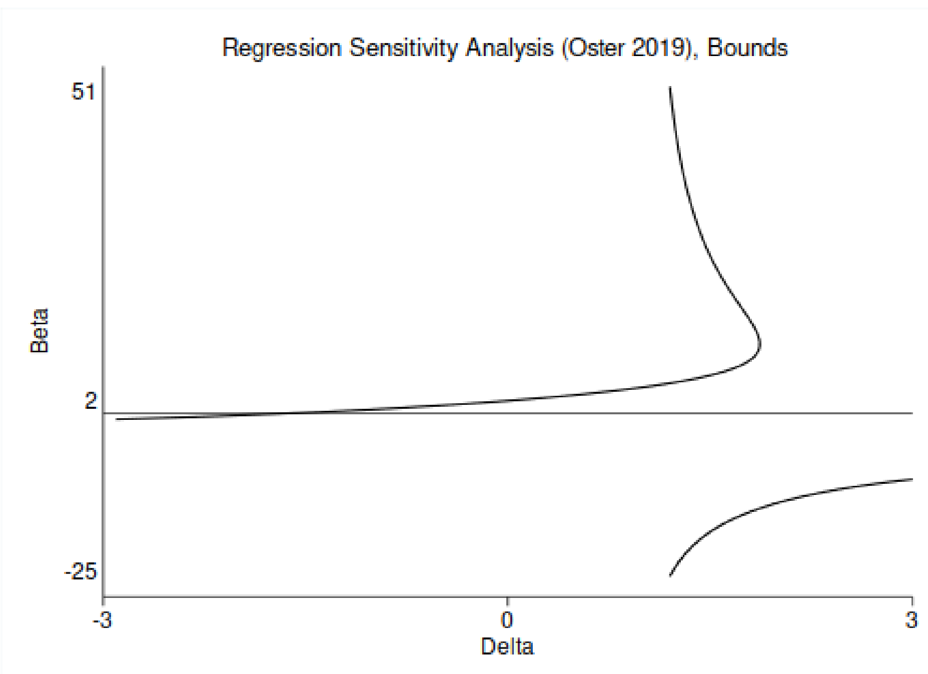
This paper uses a different set of sensitivity parameters than Diegert, Masten, and Poirier (2022). These parameters are denoted by δ and R^2_{long}. Proposition 2 in Oster (2019) gives the identified set for β_{long} as a function of these two sensitivity parameters; also see Theorem 2 in Masten and Poirier (2022). This set can be calculated for a range of sensitivity parameter values.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster

Regression Sensitivity Analysis, Bounds

Analysis         : Oster (2019)              Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =       2.055
Outcome          : avgrep2000to2016          R2(short)          =       0.033
                                             R2(medium)         =       0.105
                                             Var(Y)             =     101.739
                                             Var(X)             =       0.901
                                             Var(X_Residual)    =       0.882

Hypothesis       : Beta != 0                 Breakdown point    =        170%
Other Params     : R-squared(long) = 1
--------------------------------------------------------------------------------
 Delta                            Beta
--------------------------------------------------------------------------------
 -0.990                         {   0.71,        .,        . }
 -0.800                         {   0.93,        .,        . }
 -0.600                         {   1.18,        .,        . }
 -0.400                         {   1.45,        .,        . }
 -0.200                         {   1.74,        .,        . }
 0.000                          {   2.05,        .,        . }
 0.200                          {   2.40,        .,        . }
 0.400                          {   2.77,        .,        . }
 0.600                          {   3.19,        .,        . }
 0.800                          {   3.66,        .,        . }
 0.990                          {-570.85,   -50.04,     4.17 }
--------------------------------------------------------------------------------
```

The output also shows the explain away breakdown point, which is outside the default (-1, 1) range, so we'll expand the range of δ to include the breakdown point, and we'll generate a plot by including the `plot` option.

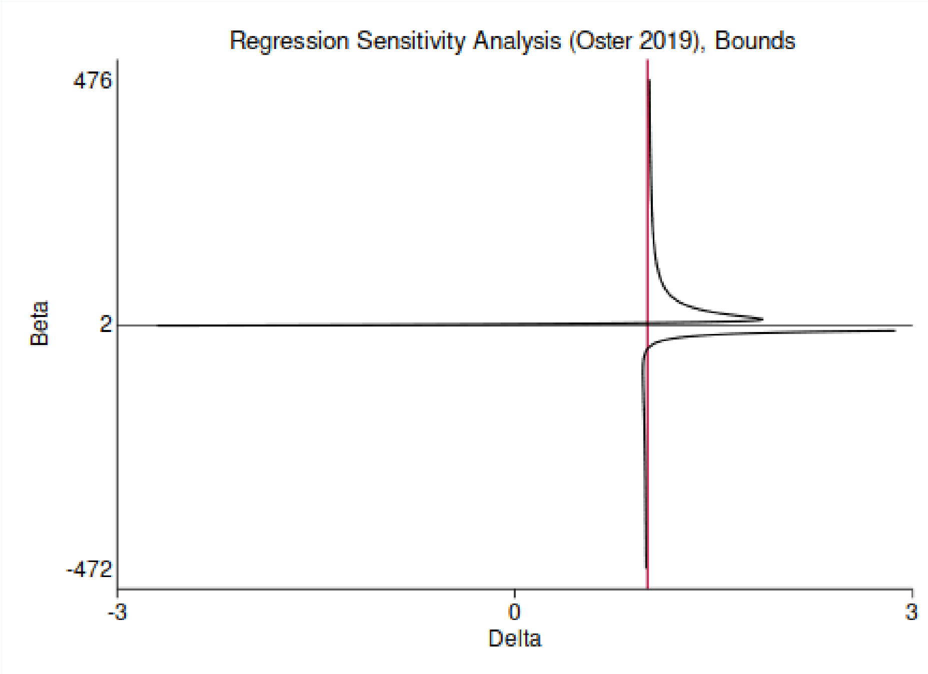
```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(-3(.3)3, eq) plot

Regression Sensitivity Analysis, Bounds
```

```
Analysis         : Oster (2019)              Number of obs     =      2,036
                                             Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_16  Beta(medium)      =      2.055
Outcome          : avgrep2000to2016          R2(short)         =      0.033
                                             R2(medium)        =      0.105
                                             Var(Y)            =    101.739
                                             Var(X)            =      0.901
                                             Var(X_Residual)   =      0.882

Hypothesis       : Beta != 0                 Breakdown point   =       170%
Other Params     : R-squared(long) = 1
--------------------------------------------------------------------------------
 Delta                              Beta
--------------------------------------------------------------------------------
 -3.000                           { -0.93,       .,       . }
 -2.400                           { -0.54,       .,       . }
 -1.800                           { -0.08,       .,       . }
 -1.200                           {  0.48,       .,       . }
 -0.600                           {  1.18,       .,       . }
 0.000                            {  2.05,       .,       . }
 0.600                            {  3.19,       .,       . }
 1.200                            {-25.71,   4.82,   51.71 }
 1.800                            {-15.46,   8.57,   14.59 }
 2.400                            {-12.15,       .,       . }
 3.000                            {-10.36,       .,       . }
--------------------------------------------------------------------------------
```



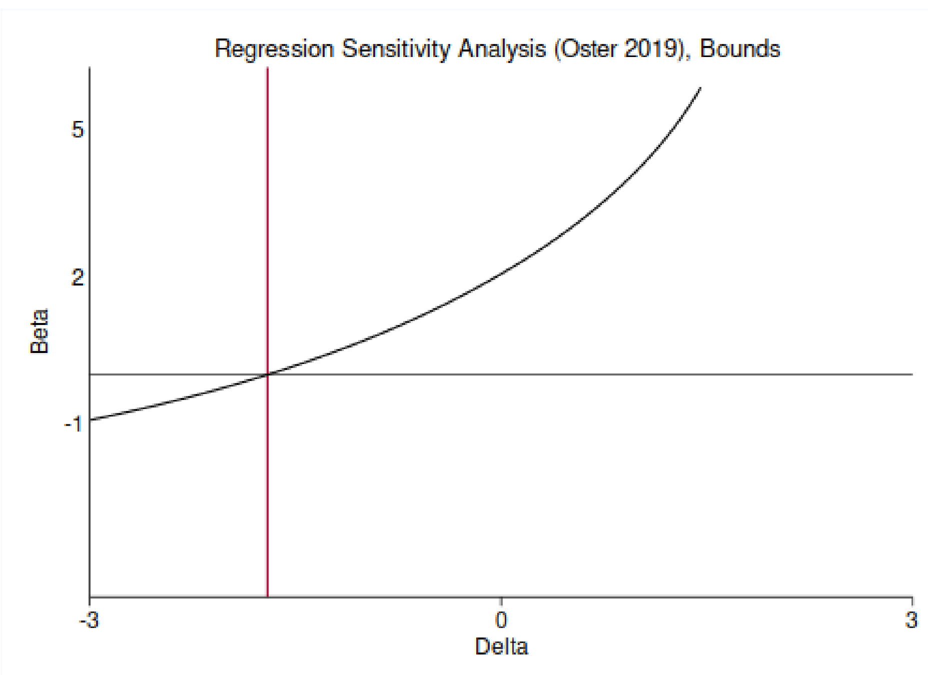
As discussed in Masten and Poirier (2022), the identified set includes arbitrarily small and large values of β_{long} around $\delta = 1$. In fact, there is an asymptote at exactly $\delta = 1$. We can expand the range of the y axis with the `ywidth` option to see this better. The `regsensitivity plot` command accepts additional *twoway options*, so we can also add a line at $\delta = 1$.

```
. regsensitivity plot, ywidth(500) xline(1)
```



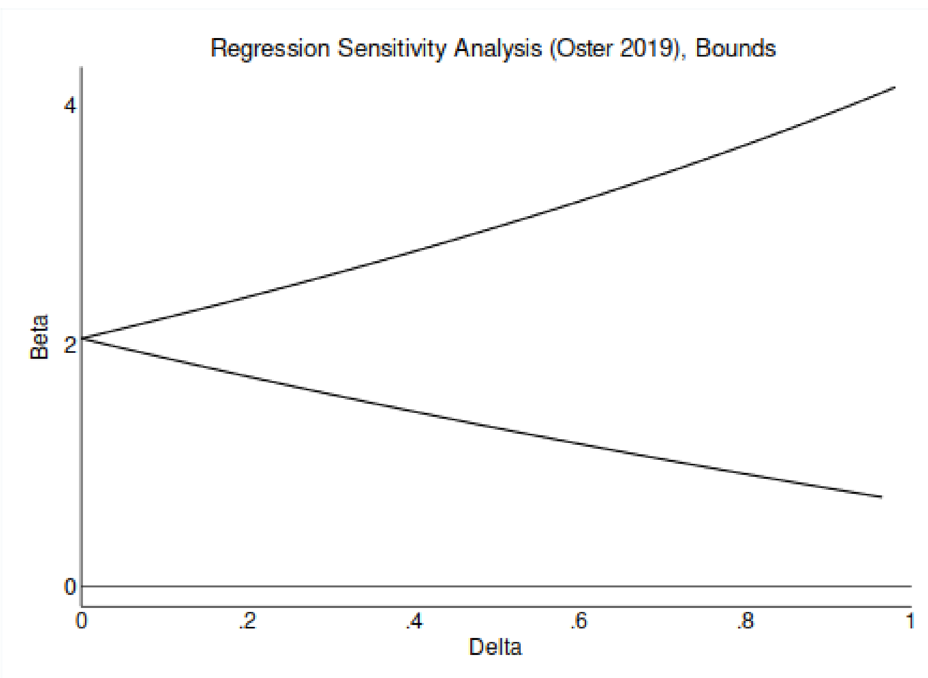
On the other hand, to see that that the explain away breakdown point is in fact at -1.704, we can narrow the range of the y-axis

```
. regsensitivity plot, ywidth(4) xline(-1.704)
```



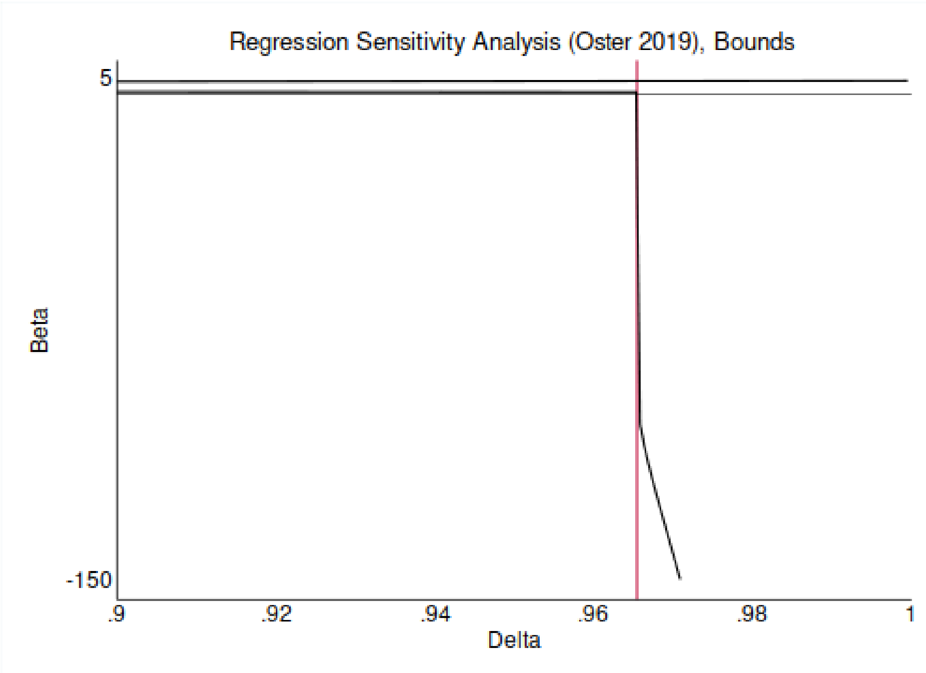
As discussed in Masten and Poirier (2022), researchers may be interested in the hypothesis that $\beta_{\text{long}} > 0$ rather than the hypothesis that $\beta_{\text{long}} \neq 0$. As in the analysis in Diegert, Masten, and Poirier (2022), we can also calculate the identified set for bounds on the absolute value of δ. That is, for a given $\bar{\delta} \geq 0$, we can calculate the identified set of β_{long} such that $|\delta| < \bar{\delta}$. To do this, use `bound` in the `delta` option:

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(0(.1).9 .999 1, bound) plot

Regression Sensitivity Analysis, Bounds

Analysis          : Oster (2019)              Number of obs      =      2,036
                                              Beta(short)        =      1.925
Treatment         : tye_tfe890_500kNI_100_16  Beta(medium)       =      2.055
Outcome           : avgrep2000to2016          R2(short)          =      0.033
                                              R2(medium)         =      0.105
                                              Var(Y)             =    101.739
                                              Var(X)             =      0.901
                                              Var(X_Residual)    =      0.882

Hypothesis        : Beta > 0                  Breakdown point    =      96.5%
Other Params      : R-squared(long) = 1
--------------------------------------------------------------------------------
 Delta                              Beta
--------------------------------------------------------------------------------
 0.000                             [     2,       2 ]
 0.100                             [     2,       2 ]
 0.200                             [     2,       2 ]
 0.300                             [     2,       3 ]
 0.400                             [     1,       3 ]
 0.500                             [     1,       3 ]
 0.600                             [     1,       3 ]
 0.700                             [     1,       3 ]
 0.800                             [     1,       4 ]
 0.900                             [     1,       4 ]
 0.999                             [-6,125,       4 ]
 1.000                             [  -inf,    +inf ]
--------------------------------------------------------------------------------
```



By default, the plot sets the range of the y-axis to avoid being visually dominated by the asymptote at $\delta = 1$. In this case, we don't see where the sign change breakdown point is or where the set goes to $\mathbb{R}$. To see where the breakdown point is, we will plot on a more narrow range for δ and expand the range of β. The previous call to `regsensitivity bounds` stored the exact breakdown point in `e(breakdown)`; we will retrieve that and add a vertical line at the breakdown point as well.

```
. local breakdown = e(breakdown)
. qui regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(0.9(.001).99 .999 1, bound)
. regsensitivity plot, yrange(-150 5) xline(`breakdown´)
```



Plotted with this wider range for β_{long}, we can see that there is a discrete jump at the estimated sign change breakdown point $\widehat{\delta}^{\text{bp,sign}}(R^2_{\text{long}}) = 0.965$ and then the lower bound tends toward $-\infty$ as $\delta \to 1$.

As in Diegert, Masten, and Poirier (2022) we can also consider how breakdown points change when we change the other sensitivity parameter. The second sensitivity parameter in Oster (2019) is R^2_{long}. To see how this affects the breakdown point use the `breakdown` subcommand with the `oster` option and pass a range of values for `r2long`. To report the explain away breakdown point, use the `beta(0 eq)` option:

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) oster r2long(0(.1)1) beta(0, eq)
Regression Sensitivity Analysis, Breakdown Frontier

Analysis          : Oster (2019)               Number of obs     =      2,036
                                               Beta(short)       =      1.925
Treatment         : tye_tfe890_500kNI_100_16   Beta(medium)      =      2.055
Outcome           : avgrep2000to2016           R2(short)         =      0.033
                                               R2(medium)        =      0.105
                                               Var(Y)            =    101.739
                                               Var(X)            =      0.901
Hypothesis        : Beta != 0                  Var(X_Residual)   =      0.882

--------------------------------------------------------------------------------
 R-squared(long)                    Delta(Breakdown)
--------------------------------------------------------------------------------
 0.105                              +inf
 0.200                              1207.1%
 0.300                              685.6%
 0.400                              478.8%
 0.500                              367.8%
 0.600                              298.6%
 0.700                              251.3%
```

```
 0.800                            217.0%
 0.900                            190.9%
 1.000                            170.4%
```

To instead consider the sign change breakdown point, specify either `beta(0 lb)` (for *lower bound*) or `beta(sign)`. This is the minimum value of $|\delta|$ such that β_{long} has the same sign as β_{med} for all $d < |\delta|$.

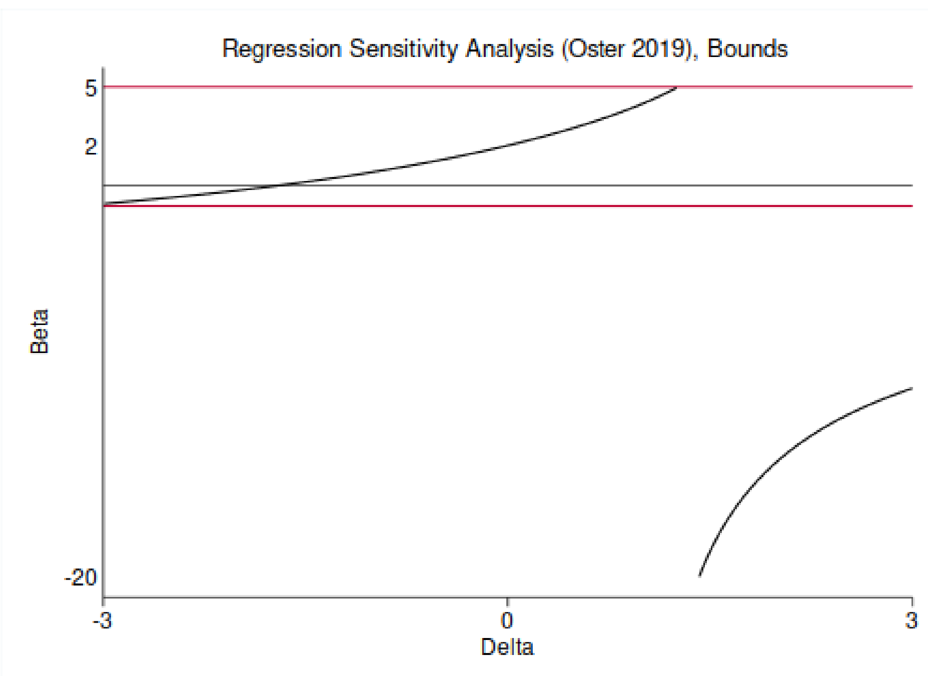
```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) oster r2long(0(.1)1) beta(sign)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)                Number of obs     =      2,036
                                               Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6    Beta(medium)      =      2.055
Outcome          : avgrep2000to2016            R2(short)         =      0.033
                                               R2(medium)        =      0.105
                                               Var(Y)            =    101.739
                                               Var(X)            =      0.901
Hypothesis       : Beta > 0                    Var(X_Residual)   =      0.882
--------------------------------------------------------------------------
 R-squared(long)                  Delta(Breakdown)
--------------------------------------------------------------------------
 0.105                            +inf
 0.200                            97.3 %
 0.300                            97.3 %
 0.400                            97.2 %
 0.500                            97.1 %
 0.600                            97.0 %
 0.700                            96.9 %
 0.800                            96.8 %
 0.900                            96.7 %
 1.000                            96.5 %
--------------------------------------------------------------------------
```

Masten and Poirier (2022) discuss the restriction that $|\beta_{\text{med}} - \beta_{\text{long}}| < M$ for some value of M. To impose this restriction, use the option `maxovb` with `regsensitivity bounds`:

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(0(.1)1, bound) beta(sign) maxovb(3)

Regression Sensitivity Analysis, Bounds

Analysis         : Oster (2019)                Number of obs     =      2,036
                                               Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6    Beta(medium)      =      2.055
Outcome          : avgrep2000to2016            R2(short)         =      0.033
                                               R2(medium)        =      0.105
                                               Var(Y)            =    101.739
                                               Var(X)            =      0.901
                                               Var(X_Residual)   =      0.882

Hypothesis       : Beta > 0                    Breakdown point   =       170%
Other Params     : R-squared(long) = 1, max OVB = 3
--------------------------------------------------------------------------
 Delta                           Beta
--------------------------------------------------------------------------
 0.000                         [ 2.0548,  2.0548 ]
 0.100                         [ 1.8939,  2.2226 ]
 0.200                         [ 1.7396,  2.3981 ]
 0.300                         [ 1.5915,  2.5819 ]
 0.400                         [ 1.4490,  2.7747 ]
 0.500                         [ 1.3120,  2.9776 ]
 0.600                         [ 1.1801,  3.1917 ]
 0.700                         [ 1.0530,  3.4184 ]
 0.800                         [ 0.9305,  3.6595 ]
 0.900                         [ 0.8124,  3.9171 ]
--------------------------------------------------------------------------
```

Although this is not always the case, in this example the `maxovb` restriction is strong enough to imply that the explain away and sign change breakdown points are the same. This is because the `maxovb` restriction rules out all the negative values of β_{long} near $\delta = 1$. To see this, we plot the identified set with horizontal lines to show the magnitude bounds placed on β_{long}:

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(-3(.3)3)

Regression Sensitivity Analysis, Bounds

Analysis          : Oster (2019)              Number of obs     =      2,036
                                              Beta(short)       =      1.925
Treatment         : tye_tfe890_500kNI_100_16  Beta(medium)      =      2.055
Outcome           : avgrep2000to2016          R2(short)         =      0.033
                                              R2(medium)        =      0.105
                                              Var(Y)            =    101.739
                                              Var(X)            =      0.901
                                              Var(X_Residual)   =      0.882

Hypothesis        : Beta != 0                 Breakdown point   =       170%
Other Params      : R-squared(long) = 1
--------------------------------------------------------------------------------
 Delta                              Beta
--------------------------------------------------------------------------------
 -3.000                            { -0.93,      .,       . }
 -2.400                            { -0.54,      .,       . }
 -1.800                            { -0.08,      .,       . }
 -1.200                            {  0.48,      .,       . }
 -0.600                            {  1.18,      .,       . }
 0.000                             {  2.05,      .,       . }
 0.600                             {  3.19,      .,       . }
 1.200                             {-25.71,   4.82,   51.71 }
 1.800                             {-15.46,   8.57,   14.59 }
 2.400                             {-12.15,      .,       . }
 3.000                             {-10.36,      .,       . }
--------------------------------------------------------------------------------

. regsensitivity plot, yline(-1.05) yline(5.05) yrange(-20 5)
```



`regsensitivity breakdown` can also accept the `maxovb` option:

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) oster beta(sign) r2long(0(.1)1) maxovb(22)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)              Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =       2.055
Outcome          : avgrep2000to2016          R2(short)          =       0.033
                                             R2(medium)         =       0.105
                                             Var(Y)             =     101.739
                                             Var(X)             =       0.901
Hypothesis       : Beta > 0                  Var(X_Residual)    =       0.882
Other Params     : Max OVB = 22
-----------------------------------------------------------------------------
 R-squared(long)                  Delta(Breakdown)
-----------------------------------------------------------------------------
 0.105                            +inf
 0.200                            168.5%
 0.300                            164.6%
 0.400                            161.0%
 0.500                            157.5%
 0.600                            154.1%
 0.700                            150.9%
 0.800                            147.8%
 0.900                            144.8%
 1.000                            142.0%
-----------------------------------------------------------------------------
```

Passing a range of values of `maxovb` to `regsensitivity`, these are displayed in the main table,

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) oster beta(sign) maxovb(2(10)100)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)              Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =       2.055
Outcome          : avgrep2000to2016          R2(short)          =       0.033
                                             R2(medium)         =       0.105
                                             Var(Y)             =     101.739
                                             Var(X)             =       0.901
Hypothesis       : Beta > 0                  Var(X_Residual)    =       0.882
Other Params     : R-squared(long) = 1
-----------------------------------------------------------------------------
 OVB(Max)                         Delta(Breakdown)
-----------------------------------------------------------------------------
 2.000                            +inf
 12.000                           170.4%
 22.000                           142.0%
 32.000                           111.7%
 42.000                           102.5%
 52.000                           99.0 %
 62.000                           97.5 %
 72.000                           96.9 %
 82.000                           96.6 %
 92.000                           96.5 %
-----------------------------------------------------------------------------
```

Currently `maxovb` is only implemented for Oster (2019).

Finally, it is sometimes convenient to state the parameters `maxovb` and `r2long` in relative terms. When the `relative` suboption is specified in the `r2long` option, the values are relative to R^2_{med}. For example, the input `r2long(1.3, relative)` is interpreted as $R^2_{\text{long}} = 1.3R^2_{\text{med}}$. Oster (2019) suggests using this as a rule of thumb.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(-3(.3)3, eq) r2long(1.3, relative)

Regression Sensitivity Analysis, Bounds

Analysis         : Oster (2019)             Number of obs     =      2,036
                                            Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6 Beta(medium)      =      2.055
Outcome          : avgrep2000to2016         R2(short)         =      0.033
                                            R2(medium)        =      0.105
                                            Var(Y)            =    101.739
                                            Var(X)            =      0.901
                                            Var(X_Residual)   =      0.882

Hypothesis       : Beta != 0                Breakdown point   =       2329%
Other Params     : R-squared(long) = .137
---------------------------------------------------------------------------
 Delta                            Beta
---------------------------------------------------------------------------
 -3.000                          {  1.88,      .,      . }
 -2.400                          {  1.92,      .,      . }
 -1.800                          {  1.95,      .,      . }
 -1.200                          {  1.99,      .,      . }
 -0.600                          {  2.02,      .,      . }
 0.000                           {  2.05,      .,      . }
 0.600                           {  2.09,      .,      . }
 1.200                           {-31.12,   2.12,  59.83 }
 1.800                           {-19.06,   2.16,  24.61 }
 2.400                           {-15.36,   2.20,  17.57 }
 3.000                           {-13.37,   2.23,  14.22 }
---------------------------------------------------------------------------
```

When the `relative` suboption is specified in the `maxovb` option, the input is interpreted relative to $|\beta_{\text{med}}|$. For example, the input `maxovb(2, relative)` is interpreted as $M = 2|\beta_{\text{med}}|$.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster beta(sign) maxovb(2, relative)

Regression Sensitivity Analysis, Bounds

Analysis         : Oster (2019)             Number of obs     =      2,036
                                            Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6 Beta(medium)      =      2.055
Outcome          : avgrep2000to2016         R2(short)         =      0.033
                                            R2(medium)        =      0.105
                                            Var(Y)            =    101.739
                                            Var(X)            =      0.901
                                            Var(X_Residual)   =      0.882

Hypothesis       : Beta > 0                 Breakdown point   =        170%
Other Params     : R-squared(long) = 1, max OVB = 4.11
---------------------------------------------------------------------------
 Delta                            Beta
---------------------------------------------------------------------------
 0.000                           [ 2.0548,  2.0548 ]
 0.100                           [ 1.8939,  2.2226 ]
 0.200                           [ 1.7396,  2.3981 ]
 0.300                           [ 1.5915,  2.5819 ]
 0.400                           [ 1.4490,  2.7747 ]
 0.500                           [ 1.3120,  2.9776 ]
 0.600                           [ 1.1801,  3.1917 ]
 0.700                           [ 1.0530,  3.4184 ]
 0.800                           [ 0.9305,  3.6595 ]
 0.900                           [ 0.8124,  3.9171 ]
---------------------------------------------------------------------------
```

References

Bazzi, Samuel, Martin Fiszbein, and Mesay Gebresilasse. 2020. "Frontier Culture: The Roots and Persistence of Rugged Individualism in the United States." *Econometrica* 88 (6): 2329–68.

Diegert, Paul, Matthew A. Masten, and Alexandre Poirier. 2022. "Assessing Omitted Variable Bias When the Controls Are Endogenous." *arXiv Preprint*.

Masten, Matthew A., and Alexandre Poirier. 2022. "The Effect of Omitted Variables on the Sign of Regression Coefficients." *arXiv Preprint*.

Oster, Emily. 2019. "Unobservable Selection and Coefficient Stability: Theory and Evidence." *Journal of Business & Economic Statistics* 37 (2): 187–204.