`regsensitivity`: A Stata Package for Regression Sensitivity Analysis

Paul Diegert* Matthew A. Masten† Alexandre Poirier‡

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Introduction

Omitted variables are one of the most important threats to the identification of causal effects. In linear models, the well known omitted variable bias formula shows how an omitted variable can bias the regression coefficient on the covariate of interest when that covariate is correlated with the omitted variable. Since is often implausible to assume that data has been collected on every relevant variable, applied research is often vulnerable to this bias. Nonetheless, omitted variable bias can be quantified under various alternative assumptions about the relationship between the omitted variable and the covariate of interest. Using these techniques, researchers can analyze how sensitive their results are to omitted variable bias.

Several methods of sensitivity analysis for linear models have been proposed in the literature. The `regsensitivity` package implements the methods proposed in Diegert, Masten, and Poirier (2022) and Oster (2019). In each of these papers, the authors define a set of sensitivity parameters which index relaxations of the assumption that the covariate of interest is uncorrelated with any unobserved variables. The parameter of interest in both cases is β_{long}, the coefficient on that covariate of interest in the infeasible regression that includes the unobserved variables. Using this framework, we can ask two questions:

1. What is the set of parameter estimates for β_{long} which are consistent with the relaxed assumptions? That is, what are bounds on the value of β_{long} under the alternate assumptions?
2. How much can we relax the exogeneity assumption before a hypothesis about β_{long} is overturned? This is called the *breakdown point*: the maximum relaxation of the baseline assumption before the hypothesis is overturned.

`regsensitivity` can be used to perform both of these sensitivity analyses using the sensitivity parameters defined in Diegert, Masten, and Poirier (2022) and Oster (2019).

Getting Started

We will illustrate how to use `regsensitivity` with data from Bazzi, Fiszbein, and Gebresilasse (2020), which is used in the empirical application in Diegert, Masten, and Poirier (2022). One of the datasets used in Bazzi, Fiszbein, and Gebresilasse (2020) is included with the package, and can be loaded using the `sysuse` command:

```
. sysuse bfg2020, clear
```

*Department of Economics, Duke University, paul.diegert@duke.edu
†Department of Economics, Duke University, matt.masten@duke.edu
‡Department of Economics, Georgetown University, alexandre.poirier@georgetown.edu

The specification in column (7) of Table III in Bazzi, Fiszbein, and Gebresilasse (2020) and replicated in Diegert, Masten, and Poirier (2022) is as follows,

```
. local y avgrep2000to2016

. local x tye_tfe890_500kNI_100_l6

. local w1 log_area_2010 lat lon temp_mean rain_mean elev_mean d_coa d_riv d_lak ave_gyi

. local w0 i.statea

. local w `w1´ `w0´

. local SE cluster(km_grid_cel_code)

. reg `y´ `x´ `w´, `SE´

Linear regression                               Number of obs     =      2,036
                                                F(39, 379)        =          .
                                                Prob > F          =          .
                                                R-squared         =     0.3321
                                                Root MSE          =     9.6368

                          (Std. err. adjusted for 380 clusters in km_grid_cel_code)
------------------------------------------------------------------------------------
                         |               Robust
        avgrep2000to2016 | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------------------+----------------------------------------------------------------
tye_tfe890_500kNI_100_l6 |   2.054759   .3491648     5.88   0.000     1.368217    2.741302
           log_area_2010 |   .2758775    .979906     0.28   0.778    -1.650856    2.202611
                     lat |    2.26515   1.101151     2.06   0.040     .1000189    4.430281
                     lon |   .0108189   .2913783     0.04   0.970    -.5621017    .5837395
               temp_mean |    1.62737   1.068132     1.52   0.128    -.4728361    3.727577
               rain_mean |   .0164826   .0046086     3.58   0.000      .007421    .0255442
               elev_mean |   .0154764   .0037786     4.10   0.000     .0080468     .022906
                   d_coa |   9.83e-06   3.76e-06     2.62   0.009     2.45e-06    .0000172
                   d_riv |   .0000307   9.91e-06     3.10   0.002     .0000112    .0000502
                   d_lak |   3.05e-07   4.45e-06     0.07   0.945    -8.44e-06    9.05e-06
                 ave_gyi |  -3.779807   10.81002    -0.35   0.727    -25.03493    17.47532
                         |
                  statea |
                      5  |  -4.213545   3.386398    -1.24   0.214    -10.87203    2.444936
                      8  |  -27.31682   6.246914    -4.37   0.000    -39.59977   -15.03387
                     12  |   4.627587   3.354655     1.38   0.169    -1.968479    11.22365
                     13  |   .5398875   2.643504     0.20   0.838    -4.657883    5.737658
                     17  |  -10.25822   3.787414    -2.71   0.007     -17.7052   -2.811248
                     18  |  -5.924452   3.497393    -1.69   0.091    -12.80118    .9522727
                     19  |  -18.02705   4.514016    -3.99   0.000     -26.9027   -9.151398
                     20  |   1.598741   4.633634     0.35   0.730    -7.512109    10.70959
                     21  |   -.504168   3.185907    -0.16   0.874    -6.768436      5.7601
                     22  |   .8939823   3.276872     0.27   0.785    -5.549145    7.337109
                     26  |  -14.06314    4.40552    -3.19   0.002    -22.72546   -5.400816
                     27  |  -18.10308   4.821495    -3.75   0.000    -27.58331   -8.622851
                     28  |  -6.930918   3.675983    -1.89   0.060    -14.15879    .2969573
                     29  |  -4.170334   3.902039    -1.07   0.286    -11.84269    3.502024
                     31  |  -1.342615    4.73751    -0.28   0.777    -10.65771     7.97248
                     35  |  -40.78007   9.264248    -4.40   0.000    -58.99583   -22.56431
                     36  |  -9.821649    4.68884    -2.09   0.037    -19.04105   -.6022507
                     37  |  -13.53756   4.241671    -3.19   0.002    -21.87772   -5.197404
                     38  |  -11.98193   5.512474    -2.17   0.030    -22.82079   -1.143061
                     39  |  -6.190808   3.655508    -1.69   0.091    -13.37843    .9968088
                     40  |   13.60029   4.880539     2.79   0.006     4.003963    23.19661
                     42  |   -3.14623   4.426406    -0.71   0.478    -11.84962    5.557161
                     46  |  -11.84706   5.101547    -2.32   0.021    -21.87794   -1.816175
                     47  |  -3.541445   2.794141    -1.27   0.206    -9.035406    1.952515
                     48  |   12.82591   4.174157     3.07   0.002     4.618502    21.03331
                     51  |  -.8116892   4.047756    -0.20   0.841    -8.770561    7.147182
                     54  |  -3.243583   3.570236    -0.91   0.364    -10.26353    3.776369
                     55  |  -18.92918   4.503985    -4.20   0.000    -27.78511   -10.07325
                     56  |  -19.02288   9.729311    -1.96   0.051    -38.15307    .1073112
```

```
                    _cons |  -73.53523   57.84708    -1.27   0.204    -187.2766    40.20618
```

Without a subcommand **regsensitivity** performs two sensitivity analyses, one from Diegert, Masten, and Poirier (2022), and one from Oster (2019):

```
. regsensitivity `y´ `x´ `w´, compare(`w1´)

Regression Sensitivity Analysis, Bounds

Analysis         : DMP (2022)                 Number of obs     =      2,036
                                              Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6   Beta(medium)      =      2.055
Outcome          : avgrep2000to2016           R2(short)         =      0.033
                                              R2(medium)        =      0.105
                                              Var(Y)            =    101.739
                                              Var(X)            =      0.901
                                              Var(X_Residual)   =      0.882

Hypothesis       : Beta > 0                   Breakdown Point   =       80.4%
Other Params     : rybar = +inf, cbar = 1
--------------------------------------------------------------------------------
 rxbar                               Beta
--------------------------------------------------------------------------------
 0.000                              [ 2.0548,  2.0548 ]
 0.099                              [ 1.9064,  2.2031 ]
 0.198                              [ 1.7535,  2.3560 ]
 0.297                              [ 1.5906,  2.5189 ]
 0.396                              [ 1.4106,  2.6989 ]
 0.495                              [ 1.2026,  2.9069 ]
 0.594                              [ 0.9478,  3.1617 ]
 0.693                              [ 0.6080,  3.5015 ]
 0.792                              [ 0.0868,  4.0227 ]
 0.890                              [-0.9927,  5.1022 ]
 0.989                              [   -inf,    +inf ]
--------------------------------------------------------------------------------

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)
Treatment        : tye_tfe890_500kNI_100_l6
Outcome          : avgrep2000to2016
Hypothesis       : Beta != 0
--------------------------------------------------------------------------------
 R-squared(long)                      Delta(Breakdown)
--------------------------------------------------------------------------------
 0.137                                2328.8%
 0.237                                943.9%
 0.337                                591.9%
 0.437                                431.1%
 0.537                                339.0%
 0.637                                279.4%
 0.737                                237.5%
 0.837                                206.6%
 0.937                                182.8%
 1.000                                170.4%
--------------------------------------------------------------------------------
```

To explore the output, we will consider each sensitivity analysis separately.

Diegert, Masten, and Poirier (2022)

Diegert, Masten, and Poirier (2022) consider the model:

$$Y = \beta_{\text{long}} X + \gamma_0' W_0 + \gamma_1' W_1 + \gamma_2 W_2 + Y^{\perp X,W},$$

where (Y, X, W_0, W_1) are observed and W_2 is an omitted variable that is potentially correlated with (X, W_0, W_1).[1] Restrictions on the joint distribution of (X, W_0, W_1, W_2) are governed by three scalar sensitivity parameters, $(\bar{r}_X, \bar{r}_Y, \bar{c})$. Given the joint distribution of the observed variables, (Y, X, W_0, W_1), and the values of the sensitivity parameters, Diegert, Masten, and Poirier (2022) show how to compute the upper and lower bounds on the identified set for β_{long}, denoted by $\mathcal{B}_I(\bar{r}_X, \bar{r}_Y, \bar{c})$. The identified set is the set of values of β_{long} which are consistent with the distribution of observed data and the maintained assumptions. When $\bar{r}_X > 0$ and $\bar{r}_Y > 0$, β_{long} is not point identified, so we instead estimate these bounds. For more details about the definitions and interpretation of the sensitivity parameters, see Diegert, Masten, and Poirier (2022).

`regsensitivity bounds` can be used with the option `dmp` to calculate the upper and lower bounds of $\mathcal{B}_I(\bar{r}_X, \bar{r}_Y, \bar{c})$. The basic syntax for `regsensitivity` is similar to the `regress` command and its variants:

> `regsensitivity bounds` *depvar indepvar controls*, *options...*,

where *depvar* is the dependant variable, Y, and *indepvar controls* are the independent variables, (X, W_0, W_1). Unlike `regress`, the order of the independent variables matter in the call to `regsensitivity`. The first variable, *indepvar*, is X, the variable of interest for which the sensitivity analysis is conducted while *controls* are additional variables included in the model which are not of interest.

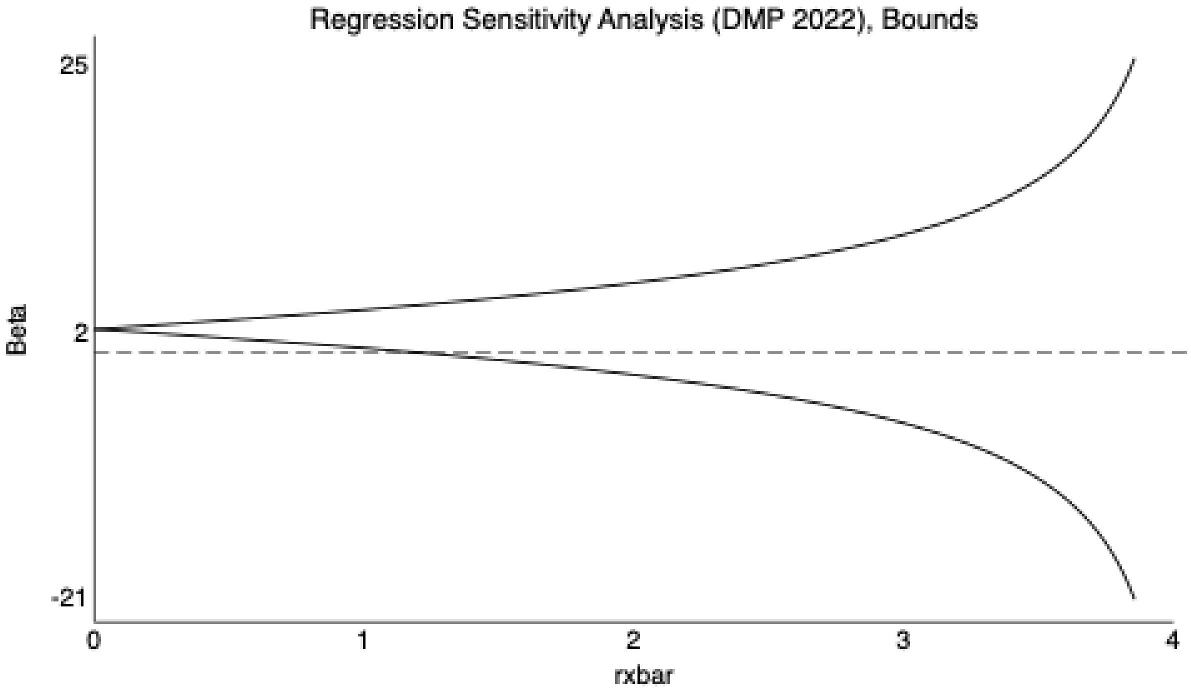
By default, `regsensitivity bounds` calculates the bounds for a range of values of $\bar{r}_X$ holding $\bar{c}$ and $\bar{r}_Y$ fixed. The defaults are to set $\bar{c} = 1$ and $\bar{r}_Y = +\infty$. To specify a different value of $\bar{c}$, use the `cbar` option. For example,

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(.1)

Regression Sensitivity Analysis, Bounds

Analysis         : DMP (2022)                Number of obs      =      2,036
                                             Beta(short)        =      1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =      2.055
Outcome          : avgrep2000to2016          R2(short)          =      0.033
                                             R2(medium)         =      0.105
                                             Var(Y)             =    101.739
                                             Var(X)             =      0.901
                                             Var(X_Residual)    =      0.882

Hypothesis       : Beta > 0                  Breakdown Point    =       119%
Other Params     : rybar = +inf, cbar = .1
-----------------------------------------------------------------------------
 rxbar                               Beta
-----------------------------------------------------------------------------
 0.000                              [  2.05,   2.05 ]
 0.408                              [  1.42,   2.69 ]
 0.816                              [  0.72,   3.39 ]
 1.224                              [ -0.06,   4.17 ]
 1.632                              [ -0.97,   5.08 ]
 2.040                              [ -2.05,   6.16 ]
 2.448                              [ -3.41,   7.52 ]
 2.856                              [ -5.26,   9.37 ]
 3.265                              [ -8.14,  12.25 ]
 3.673                              [-14.21,  18.32 ]
 4.081                              [  -inf,   +inf ]
```

[1]We denote the coefficient on X by β_{long} because it is the regression coefficient in the infeasible "long" regression of Y on $(1, X, W_0, W_1, W_2)$. This helps distinguish it from β_{medium}, the coefficient on X in the regression of Y on $(1, X, W_0, W_1)$, and from β_{short}, the coefficient on X in the regression of Y on $(1, X, W_0)$.

To plot the results, use the `plot` subcommand,

```
. regsensitivity plot
```



Notice that in the call to `regsensitivity bounds`, we also included an option `compare`(*varlist*). This specifies which of the variables in the *controls* are included in W_1 rather than W_0. These are referred to as the *comparison controls* because they are the variables used to calibrate the sensitivity parameters, $(\bar{r}_X, \bar{r}_Y, \bar{c})$. For more details, see section 3.3 in Diegert, Masten, and Poirier (2022).

By including more variables in the comparison controls, the identified set will tend to be larger for a given value of the sensitivity parameters. For example, if the `compare` option is omitted, then all the control variables are included in W_1,

```
. regsensitivity bounds `y´ `x´ `w´, cbar(.1)
Regression Sensitivity Analysis, Bounds
Analysis         : DMP (2022)               Number of obs     =      2,036
                                            Beta(short)       =      1.708
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)      =      2.055
Outcome          : avgrep2000to2016         R2(short)         =      0.027
                                            R2(medium)        =      0.332
                                            Var(Y)            =    136.320
                                            Var(X)            =      1.257
                                            Var(X_Residual)   =      0.882

Hypothesis       : Beta > 0                 Breakdown Point   =       29.7%
Other Params     : rybar = +inf, cbar = .1
--------------------------------------------------------------------------
 rxbar                            Beta
--------------------------------------------------------------------------
 0.000                           [     2,       2 ]
```

```
0.171                                  [      1,       3 ]
0.343                                  [     -0,       4 ]
0.514                                  [     -2,       6 ]
0.685                                  [     -3,       8 ]
0.856                                  [     -6,      10 ]
1.028                                  [     -9,      13 ]
1.199                                  [    -17,      21 ]
1.370                                  [-9,540,    9,544 ]
1.541                                  [-9,540,    9,544 ]
1.713                                  [   -inf,    +inf ]
```

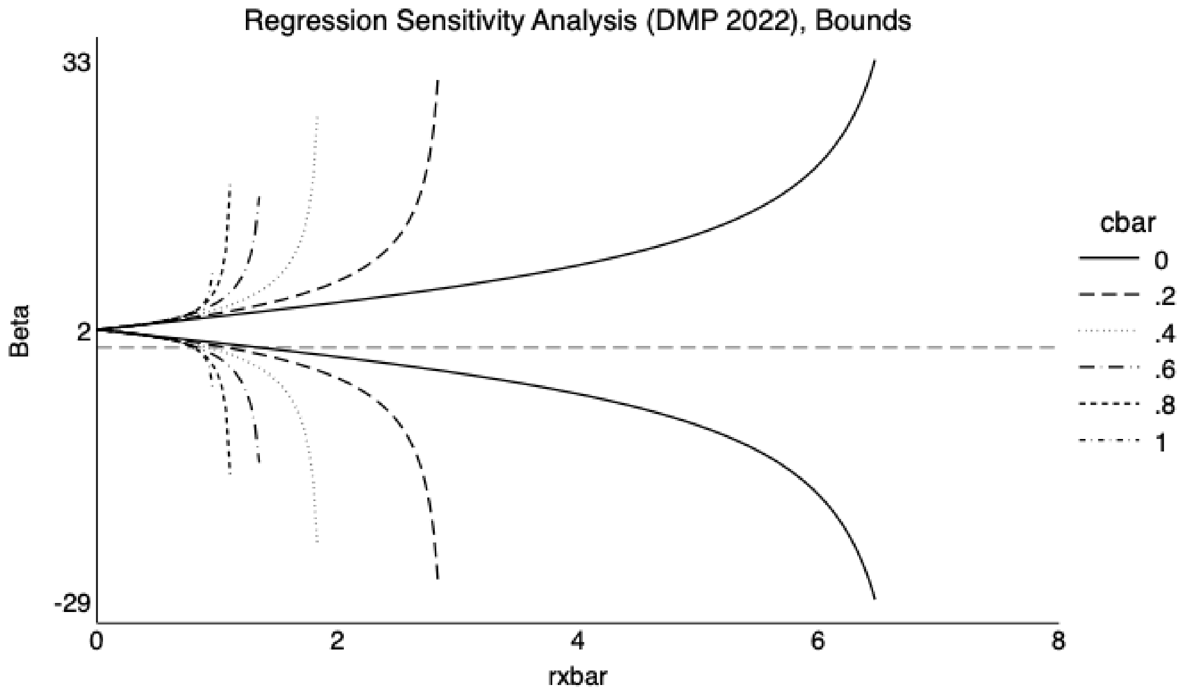
With all the controls included in W_1, the identified set becomes $\mathbb{R}$ at $\bar{r}_X = 1.336$, compared to $\bar{r}_X = 4.063$ when W_1 excludes the state fixed effects (*statea*). To directly compare the bounds under the two choices of W_1, we can manually set the values of `rxbar` to be the same in each case. The output table from the last call to `regsensitivity bounds` are stored in `e(idset_table)`. We can extract the values of `rxbar` from these and rerun the analysis with W_1 excluding state fixed effects as follows,

```
. forvalues i=1/12{
  2.     local rxbar `rxbar´ `=e(idset_table)[`i´, 1]´
  3. }

. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(.1) rxbar(`rxbar´)

Regression Sensitivity Analysis, Bounds

Analysis          : DMP (2022)                Number of obs       =       2,036
                                              Beta(short)         =       1.925
Treatment         : tye_tfe890_500kNI_100_l6  Beta(medium)        =       2.055
Outcome           : avgrep2000to2016          R2(short)           =       0.033
                                              R2(medium)          =       0.105
                                              Var(Y)              =     101.739
                                              Var(X)              =       0.901
                                              Var(X_Residual)     =       0.882

Hypothesis        : Beta > 0                  Breakdown Point     =         119%
Other Params      : rybar = +inf, cbar = .1
--------------------------------------------------------------------------------
 rxbar                                 Beta
--------------------------------------------------------------------------------
 0.000                                 [ 2.0548,  2.0548 ]
 0.171                                 [ 1.7960,  2.3135 ]
 0.343                                 [ 1.5276,  2.5819 ]
 0.514                                 [ 1.2483,  2.8612 ]
 0.685                                 [ 0.9568,  3.1528 ]
 0.856                                 [ 0.6514,  3.4581 ]
 1.028                                 [ 0.3305,  3.7790 ]
 1.199                                 [-0.0084,  4.1179 ]
 1.370                                 [-0.3679,  4.4775 ]
 1.541                                 [-0.7514,  4.8609 ]
 1.713                                 [-1.1630,  5.2725 ]
 .                                     [   -inf,    +inf ]
--------------------------------------------------------------------------------
```

Comparing each line of the table to the previous call where all the *controls* were included in W_1, we can see that the bounds are much tighter for each value of $\bar{r}_X$.

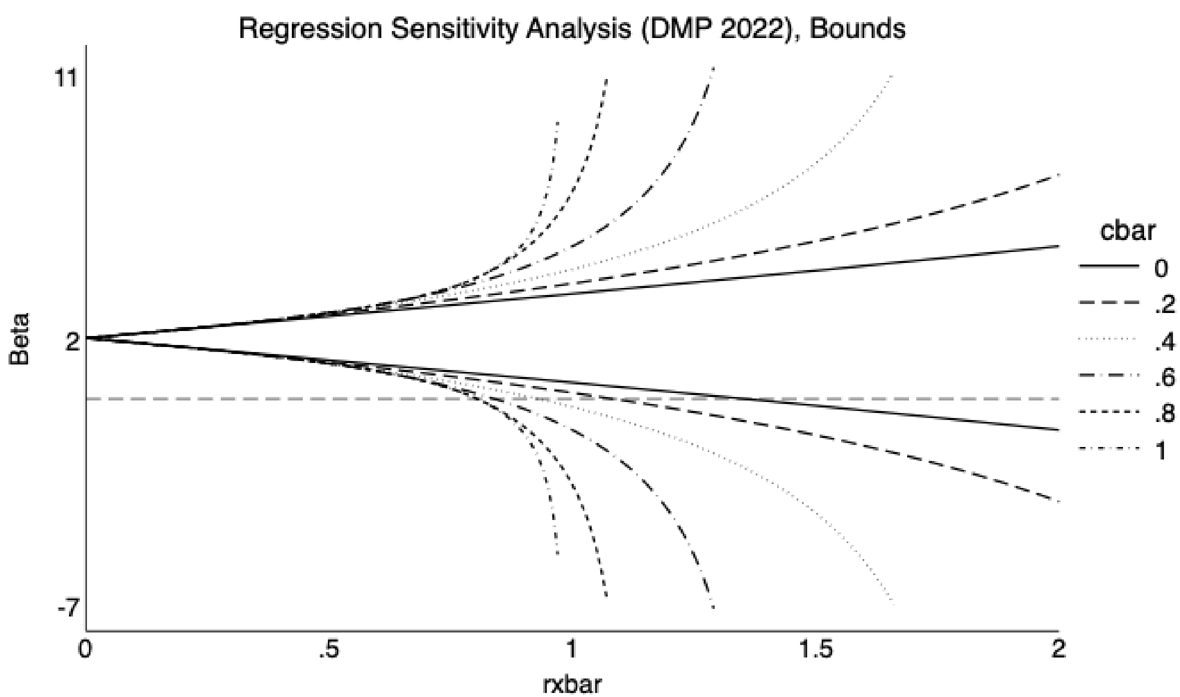
To compare multiple values of $\bar{c}$, a `numlist` can be given in the `cbar` option. With multiple values of $\bar{c}$, the command will show a plot rather than displaying the results in the console.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(0(.2)1)
```



By default, the plot will try to show where the identified set becomes $\mathbb{R}$ for each value of $\bar{c}$. For this example, the plot is dominated visually by the identified set for $\bar{c} = 0$. To see better where the identified set intersects with 0, we can rerun the analysis restricting the range of $\bar{r}_X$.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) cbar(0(.2)1) rxbar(0 2) plot
```



Breakdown Frontier

The output of `regsensitivity bounds` shows a *breakdown point* for a given hypothesis about the parameter β_{long}. For a hypothesis $\beta_{\text{long}} \in B \subseteq \mathbb{R}$, the breakdown point is the smallest value of the sensitivity parameter $\bar{r}_X$ for which the hypothesis does not hold for every β in the identified set. Formally,

$$\bar{r}_X^{bp}(\bar{r}_Y, \bar{c}; B) = \inf\{\bar{r}_X \geq 0 : b \in \mathcal{B}_I(\bar{r}_X, \bar{r}_Y, \bar{c}) \text{ for some } b \in \mathbb{R} \setminus B\}.$$

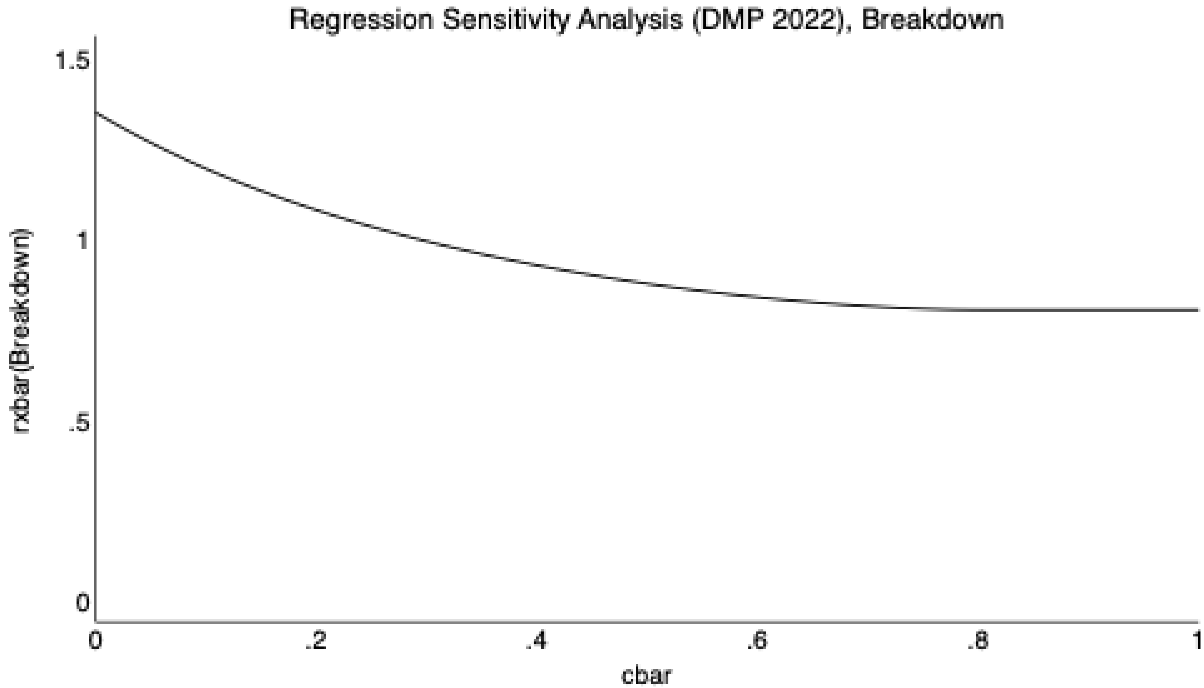
`regsensitivity` can handle hypotheses of the form, $\beta_{\text{long}} \gtrless b$ for any value of b. The default hypothesis is that $\text{sign}(\beta_{\text{long}}) = \text{sign}(\beta_{\text{medium}})$, where β_{medium} is the coefficient on X in a regression of Y on $(1, X, W_0, W_1)$. In this case $\beta_{\text{medium}} > 0$, so the default is to test the hypothesis that $\beta_{\text{long}} > 0$.

The output to `regsensitivity bounds` showed that with W_1 excluding state fixed effects, $\bar{r}_X^{bp}(.1, +\infty; (-\infty, 0]) = 1.195$. To see how this breakdown point varies with the choice of the sensitivity parameter $\bar{c}$, use the `breakdown` subcommand,

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) cbar(0(.1)1)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)                Number of obs      =      2,036
                                             Beta(short)        =      1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =      2.055
Outcome          : avgrep2000to2016          R2(short)          =      0.033
                                             R2(medium)         =      0.105
                                             Var(Y)             =    101.739
                                             Var(X)             =      0.901
Hypothesis       : Beta > 0                  Var(X_Residual)    =      0.882
--------------------------------------------------------------------------------
 cbar                                 rxbar(Breakdown)
--------------------------------------------------------------------------------
 0.000                                135.0%
 0.100                                119.5%
 0.200                                108.0%
 0.300                                99.3 %
 0.400                                92.7 %
 0.500                                87.6 %
 0.600                                83.9 %
 0.700                                81.4 %
 0.800                                80.4 %
 0.900                                80.4 %
 1.000                                80.4 %
--------------------------------------------------------------------------------
```

These results can also be plotted using the `plot` subcommand,

```
. regsensitivity plot
```



To test the hypothesis that $\beta_{\text{long}} > b$ for some other value, b, specify `beta(b lb)` (lb for "lower bound"). The `beta` option can also accept a `numlist` to test a range of hypotheses. For example, the following tests the hypotheses that $\beta_{\text{long}} > b$ for a range of values of b,

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) beta(-1(.2)1 lb)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)                Number of obs      =      2,036
                                             Beta(short)        =      1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)       =      2.055
Outcome          : avgrep2000to2016          R2(short)          =      0.033
                                             R2(medium)         =      0.105
                                             Var(Y)             =    101.739
                                             Var(X)             =      0.901
Hypothesis       : Beta > Beta(Hypothesis)   Var(X_Residual)    =      0.882
Other Params     : cbar = 1,
---------------------------------------------------------------------------
 Beta(Hypothesis)                   rxbar(Breakdown)
---------------------------------------------------------------------------
 -1.000                             89.1 %
 -0.800                             87.9 %
 -0.600                             86.5 %
 -0.400                             84.8 %
 -0.200                             82.8 %
 0.000                              80.4 %
 0.200                              77.4 %
 0.400                              73.8 %
 0.600                              69.5 %
 0.800                              64.1 %
 1.000                              57.5 %
---------------------------------------------------------------------------
```

We can also test a hypothesis of the form $\beta < b$, by specifying `beta(b ub)` (ub for "upper bound"). For example the following checks the hypothesis that $\beta < 4$,

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) cbar(0(.1)1) beta(4 ub)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)               Number of obs     =      2,036
                                            Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6 Beta(medium)      =      2.055
Outcome          : avgrep2000to2016         R2(short)         =      0.033
                                            R2(medium)        =      0.105
                                            Var(Y)            =    101.739
                                            Var(X)            =      0.901
Hypothesis       : Beta < 4                 Var(X_Residual)   =      0.882
--------------------------------------------------------------------------------
 cbar                          rxbar(Breakdown)
--------------------------------------------------------------------------------
 0.000                         128.1%
 0.100                         114.0%
 0.200                         103.6%
 0.300                         95.7 %
 0.400                         89.6 %
 0.500                         85.0 %
 0.600                         81.7 %
 0.700                         79.5 %
 0.800                         78.8 %
 0.900                         78.8 %
 1.000                         78.8 %
--------------------------------------------------------------------------------
```

To see visually where the identified set intersects with 4 we can specify this alternative hypothesis in the `regsensitivity bounds` command. By including the option `beta(4 ub)` the resulting plot will include a horizontal line at 4,

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) rxbar(0 2) cbar(0(.1)1) beta(4 ub)

. regsensitivity plot, nolegend yrange(0 6)
```

Bounds and breakdown frontier with $\bar{r}_Y < \infty$

Starting with version 1.2, it is also possible to estimate the identified set with $\bar{r}_Y < \infty$. For example to restrict $\bar{r}_Y = 2$

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) rybar(2)

Regression Sensitivity Analysis, Bounds

Analysis         : DMP (2022)               Number of obs     =      2,036
                                            Beta(short)       =      1.925
Treatment        : tye_tfe890_500kNI_100_l6 Beta(medium)      =      2.055
Outcome          : avgrep2000to2016         R2(short)         =      0.033
                                            R2(medium)        =      0.105
                                            Var(Y)            =    101.739
                                            Var(X)            =      0.901
                                            Var(X_Residual)   =      0.882

Hypothesis       : Beta > 0                 Breakdown Point   =       80.4%
Other Params     : rybar = 2, cbar = 1
--------------------------------------------------------------------------------
 rxbar                        Beta
--------------------------------------------------------------------------------
 0.000                      [ 2.0548,  2.0548 ]
 0.099                      [ 1.9107,  2.2006 ]
 0.198                      [ 1.7612,  2.3539 ]
 0.297                      [ 1.5983,  2.5185 ]
 0.396                      [ 1.4149,  2.6989 ]
 0.495                      [ 1.2027,  2.9069 ]
 0.594                      [ 0.9478,  3.1617 ]
```

```
0.693                            [ 0.6080,  3.5015 ]
0.792                            [ 0.0868,  4.0227 ]
0.890                            [-0.9927,  5.1021 ]
0.989                            [   -inf,    +inf ]
```

Note, that when $\bar{r}_Y < \infty$ and $\bar{c} > 0$, the bounds are calculated by numerically solving a constrained, nonconvex optimization problem, so this is more computationally demanding. Therefore, by default, only the grid of 10 points is calculated and the resulting plot will be more coarse,

```
. regsensitivity plot
```

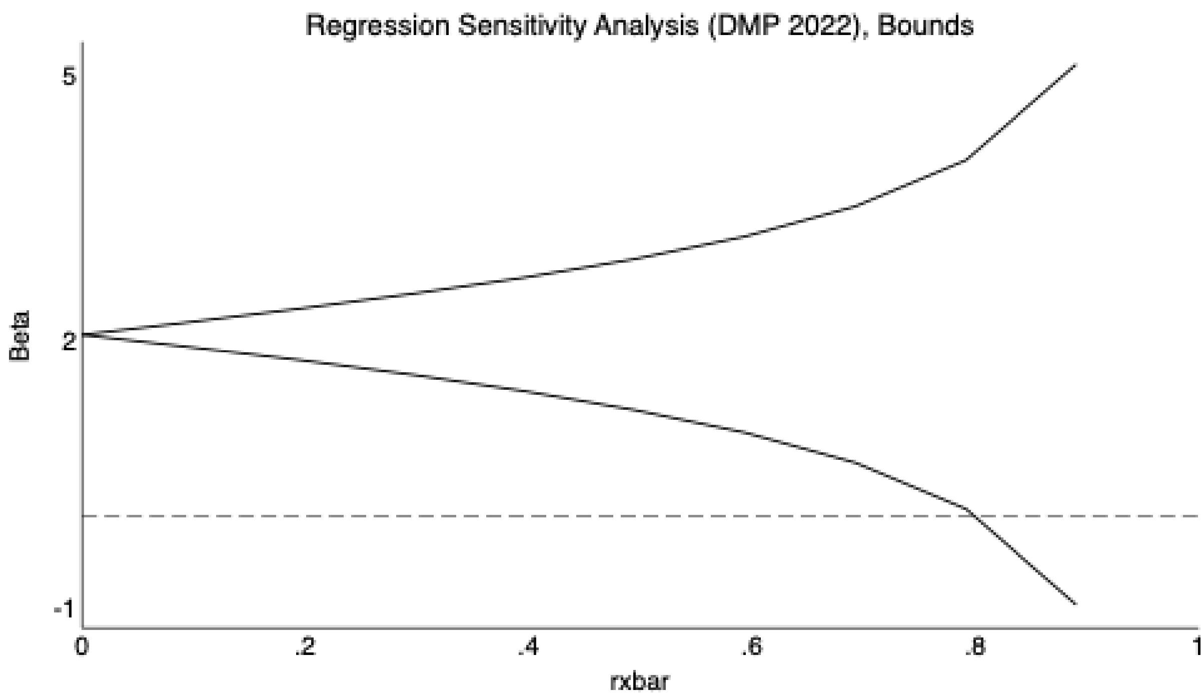


Figure 1: DMP Breakdown 2

It is also possible to calculate a range of values where one of the sensitivity parameters is a function of the others. This is useful in particular for calculate the identified set for a range of values of $\bar{r} := \bar{r}_X = \bar{r}_Y$,

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) rybar(=rxbar)

Regression Sensitivity Analysis, Bounds

Analysis         : DMP (2022)                 Number of obs      =      2,036
                                              Beta(short)        =      1.925
Treatment        : tye_tfe890_500kNI_100_l6   Beta(medium)       =      2.055
Outcome          : avgrep2000to2016           R2(short)          =      0.033
                                              R2(medium)         =      0.105
                                              Var(Y)             =    101.739
                                              Var(X)             =      0.901
                                              Var(X_Residual)    =      0.882

Other Params     : cbar = 1

 rxbar   rybar                      Beta

 0.000   0.000                   [ 2.0548,  2.0548 ]
 0.099   0.099                   [ 2.0506,  2.0589 ]
```

```
 0.198   0.198                   [ 2.0379,  2.0718 ]
 0.297   0.297                   [ 2.0161,  2.0946 ]
 0.396   0.396                   [ 1.9841,  2.1295 ]
 0.495   0.495                   [ 1.9394,  2.1811 ]
 0.594   0.594                   [ 1.8774,  2.2589 ]
 0.693   0.693                   [ 1.7867,  2.3848 ]
 0.792   0.792                   [ 1.6341,  2.6247 ]
 0.890   0.890                   [ 1.2614,  3.2995 ]
 0.989   0.989                   [   -inf,    +inf ]
```

Notice that the output of this command includes a breakdown point. This is the breakdown point for $\bar{r}$ when $\bar{r}_X$ and $\bar{r}_Y$ are constrained to be equal to each other, fixing $\bar{c} = 1$.

a breakdown frontier can be calculated directly with

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) rybar(=rxbar) cbar(0 .5 1)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : DMP (2022)                Number of obs     =       2,036
                                             Beta(short)       =       1.925
Treatment        : tye_tfe890_500kNI_100_l6  Beta(medium)      =       2.055
Outcome          : avgrep2000to2016          R2(short)         =       0.033
                                             R2(medium)        =       0.105
                                             Var(Y)            =     101.739
                                             Var(X)            =       0.901
Hypothesis       : Beta > 0                  Var(X_Residual)   =       0.882
Other Params     : rybar = rxbar
--------------------------------------------------------------------------------
 cbar                              rxbar(Breakdown)
--------------------------------------------------------------------------------
 0.000                             216.0%
 0.500                             124.6%
 1.000                             95.8 %
--------------------------------------------------------------------------------
```

Note that in the current implementation, there are some cases that can't be safely solved where $\bar{r}_Y$ is small. In those cases, `regsensitivity` will simply raise an error, e.g.,

```
. capture noisily regsensitivity bounds `y´ `x´ `w´, compare(`w1´) rybar(.1)
Not implemented Error: not implemented to calculate breakdown point when rybar < rmax(c) (see documentation)
```

Summary statistics

The output of `regsensitivity bounds` and `regsensitivity breakdown` both include a table of summary statistics. These are as follows,

- `Number of observations`
- `Beta(short)`: The coefficient on X in the regression of Y on $(1, X)$
- `Beta(medium)`: The coefficient on X in the regression of Y on $(1, X, W_1)$
- `R2(short)`: The R-squared from the regression of Y on $(1, X)$
- `R2(medium)`: The R-squared from the regression of Y on $(1, X, W_1)$
- `Var(Y)`: Variance of Y
- `Var(X)`: Variance of X
- `Var(X_Residual)`: Variance of $X^{\perp W_1}$, the residual from the regression of X on $(1, W_1)$.

Note: For all the summary statistics reported in this table, (Y, X, W_1) are shorthand for $(Y^{\perp W_0}, X^{\perp W_0}, W_1^{\perp W_0})$ where $Y^{\perp W_0}$ is the residual from the regression of Y on $(1, W_0)$ and likewise for $X^{\perp W_0}$ and $W_1^{\perp W_0}$.

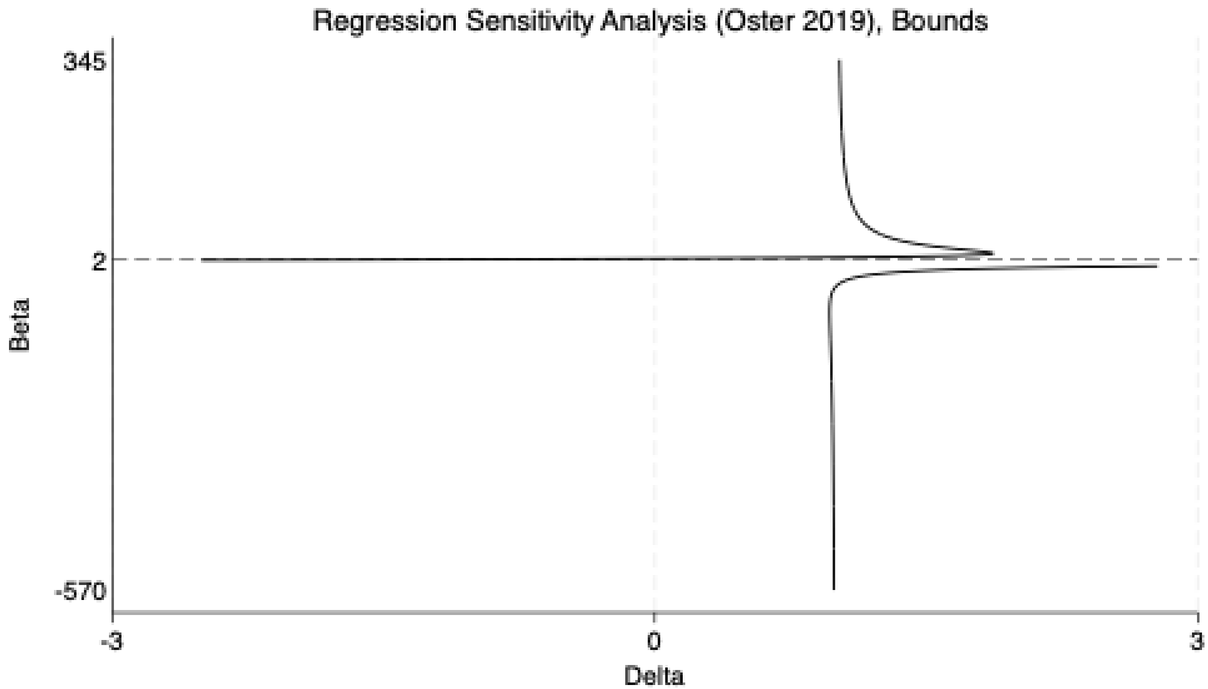
Oster 2019

This paper uses a different set of sensitivity parameters than Diegert, Masten, and Poirier (2022). These parameters are denoted by δ and R^2_{long}. Proposition 2 in Oster (2019) gives the identified set for β as a function of these two sensitivity parameters. This set can be calculated for a range of sensitivity parameter values.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster
ndeltapoints = 21, ngrid = 200

Regression Sensitivity Analysis, Bounds

Analysis        : Oster (2019)               Number of obs     =      2,036
                                             Beta(short)       =      1.925
Treatment       : tye_tfe890_500kNI_100_l6   Beta(medium)      =      2.055
Outcome         : avgrep2000to2016           R2(short)         =      0.033
                                             R2(medium)        =      0.105
                                             Var(Y)            =    101.739
                                             Var(X)            =      0.901
                                             Var(X_Residual)   =      0.882

Hypothesis      : Beta != 0                  Breakdown Point   =       170%
Other Params    : R-squared(long) = 1
--------------------------------------------------------------------------------
 delta                              Beta
--------------------------------------------------------------------------------
 -1.000                           {  0.70,      .,      . }
 -0.900                           {  0.81,      .,      . }
 -0.800                           {  0.93,      .,      . }
 -0.700                           {  1.05,      .,      . }
 -0.600                           {  1.18,      .,      . }
 -0.500                           {  1.31,      .,      . }
 -0.400                           {  1.45,      .,      . }
 -0.300                           {  1.59,      .,      . }
 -0.200                           {  1.74,      .,      . }
 -0.100                           {  1.89,      .,      . }
 0.000                            {  2.05,      .,      . }
 0.100                            {  2.22,      .,      . }
 0.200                            {  2.40,      .,      . }
 0.300                            {  2.58,      .,      . }
 0.400                            {  2.77,      .,      . }
 0.500                            {  2.98,      .,      . }
 0.600                            {  3.19,      .,      . }
 0.700                            {  3.42,      .,      . }
 0.800                            {  3.66,      .,      . }
 0.900                            {  3.92,      .,      . }
 1.000                            {-46.17,   4.19,      . }
--------------------------------------------------------------------------------
```

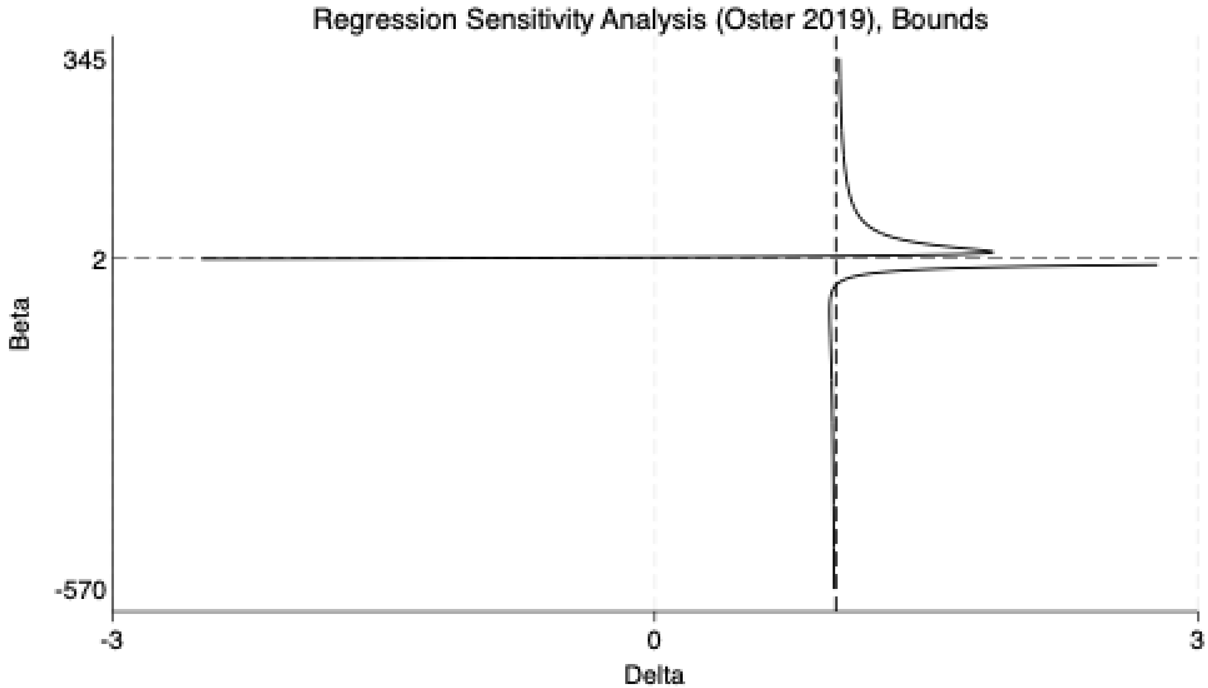
The output also shows the exact zero breakdown point, which is outside the default (-1, 1) range, so we'll expand the range of δ to include the breakdown point, and we'll generate a plot by including the `plot` option.

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(-3 3 eq) plot
ndeltapoints = 2, ngrid = 200
```



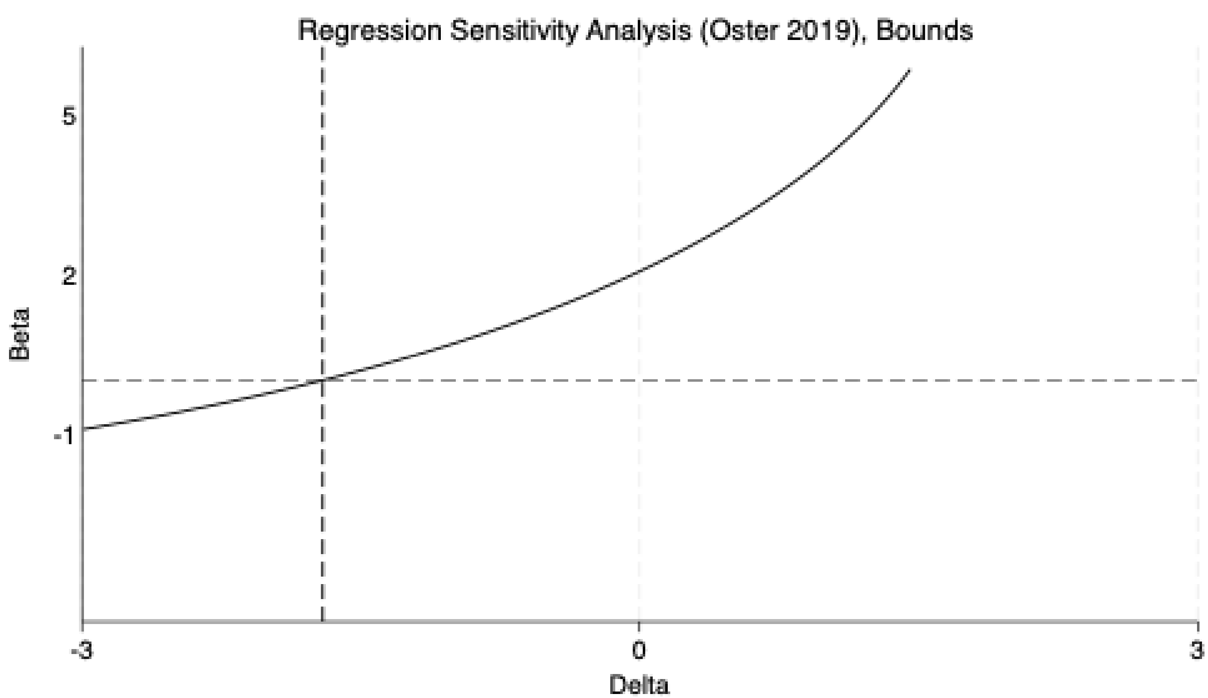
It is apparent in this plot that around $\delta = 1$, the identified set includes arbitrarily small and large values of β. In fact, there is an asymptote at exactly $\delta = 1$. The `regsensitivity plot` command takes additional *twoway options*, so we can add a line at $\delta = 1$ to see this.

```
. regsensitivity plot, xline(1)
```



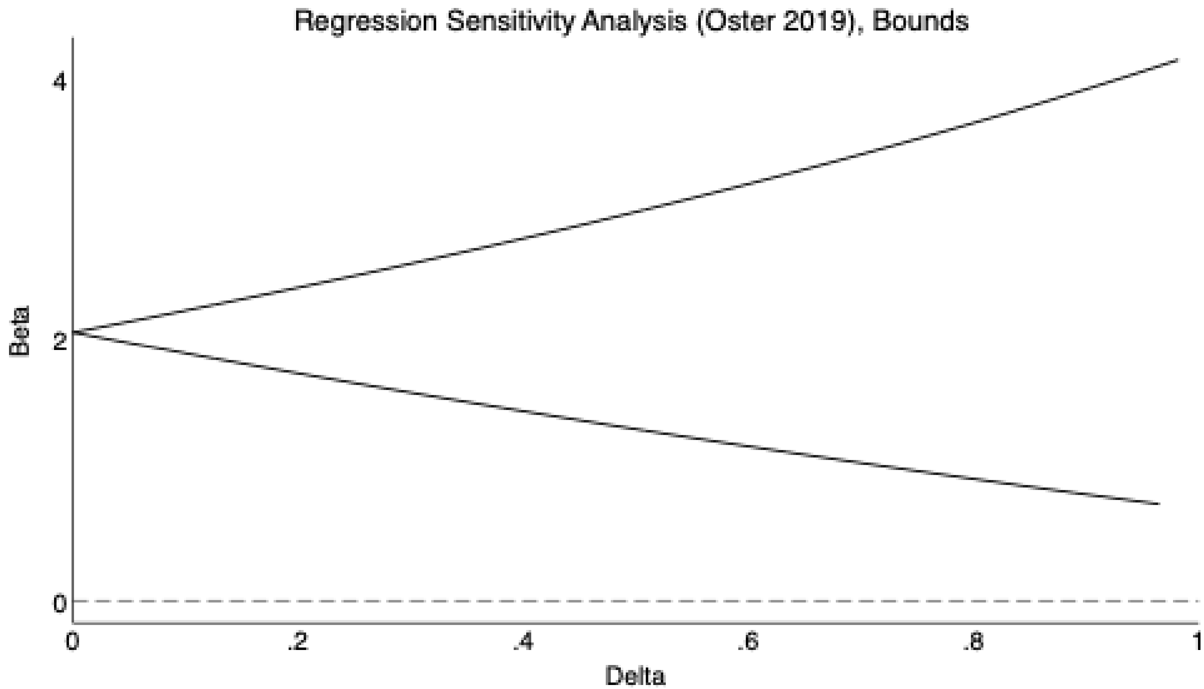
On the other hand, to see that that the exact zero breakdown point is in fact at -1.704, we can restrict the range of the y-axis

```
. regsensitivity plot, ywidth(4) xline(-1.704)
```



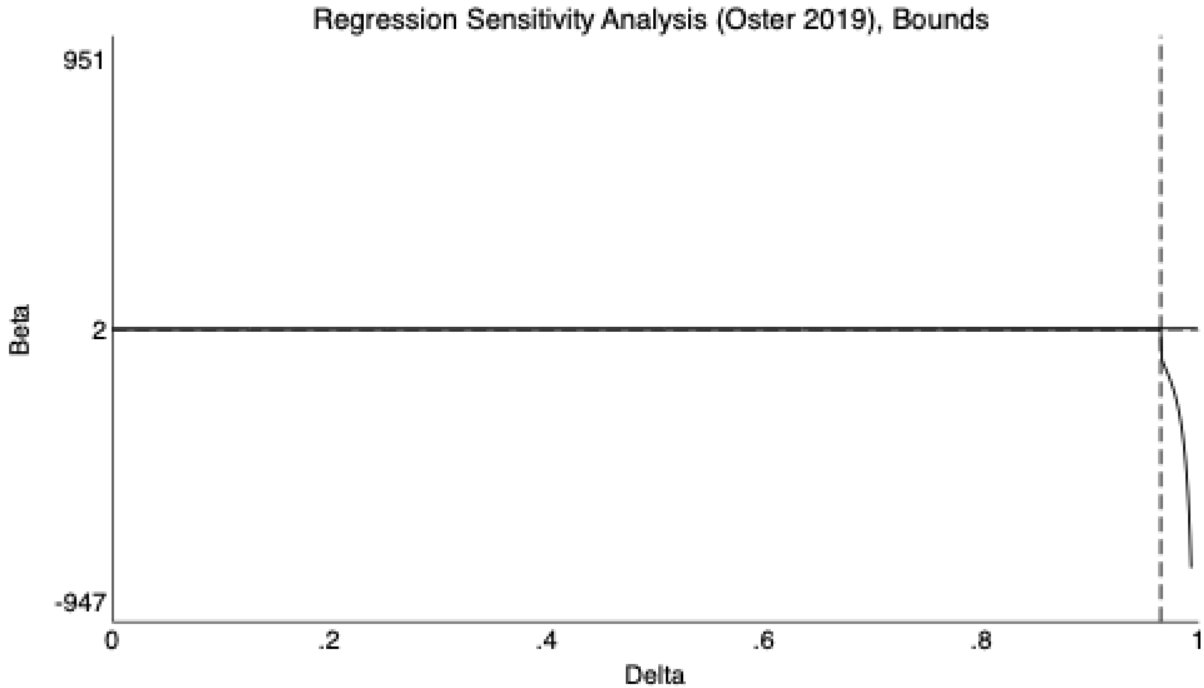
Researchers may be interested in the hypothesis that $\beta > 0$ rather than the hypothesis that $\beta \neq 0$. As in the analysis in Diegert, Masten, and Poirier (2022), we can also calculate the identified set for bounds on the absolute value of δ. That is, for $\bar{\delta}$, we can calculate the identified set of β such that $|\delta| < \bar{\delta}$. To do this, use `bound` in the `delta` option,

```
. regsensitivity bounds `y´ `x´ `w´, compare(`w1´) oster delta(0(.001).999 bound) plot
```



By default, the plot sets the range of the y-axis to avoid being visually dominated by outliers. In this case, we don't see where the sign change breakdown point is or where the set goes to $\mathbb{R}$. To override the default, set the `ywidth` option. We will also point a horizontal line at the breakdown point,

```
. regsensitivity plot, ywidth(1000) xline(.965)
```



Plotted with this wider range for β, we can see that there is a discrete jump at the sign change breakdown point $\bar{\delta}^{\text{bp}} = 0.965$ and then the lower bound tends toward $-\infty$ as $\delta \to 1$.

As in Diegert, Masten, and Poirier (2022) we can also consider how breakdown points change when we change the other sensitivity parameter. The second sensitivity parameter in Oster (2019) is R^2_{long}. To see how this affects the breakdown point use the `breakdown` subcommand with the `oster` option and pass a range of values for `rmax`. To report the exact zero breakdown point relative to $\delta \neq 0$ set the `beta(0 eq)`

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) oster rmax(0(.1)1) beta(0 eq)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)              Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_16  Beta(medium)       =       2.055
Outcome          : avgrep2000to2016          R2(short)          =       0.033
                                             R2(medium)         =       0.105
                                             Var(Y)             =     101.739
                                             Var(X)             =       0.901
Hypothesis       : Beta != 0                 Var(X_Residual)    =       0.882
Other Params     : R-squared(long) = 1
------------------------------------------------------------------------------
 Beta(Hypothesis)                 Delta(Breakdown)
------------------------------------------------------------------------------
 0.000                            170.4%
------------------------------------------------------------------------------
```

To instead consider the sign change breakdown point, specify either `beta(0 lb)` (for *lower bound*) or `beta(sign)`. This is the minimum value of $|\delta|$ such that β has the same sign as β_{med} for all $d > |\delta|$.

```
. regsensitivity breakdown `y´ `x´ `w´, compare(`w1´) oster rmax(0(.1)1) beta(sign)

Regression Sensitivity Analysis, Breakdown Frontier

Analysis         : Oster (2019)              Number of obs      =       2,036
                                             Beta(short)        =       1.925
Treatment        : tye_tfe890_500kNI_100_16  Beta(medium)       =       2.055
```

```
Outcome          : avgrep2000to2016          R2(short)         =       0.033
                                             R2(medium)        =       0.105
                                             Var(Y)            =     101.739
                                             Var(X)            =       0.901
Hypothesis       : Beta > 0                  Var(X_Residual)   =       0.882
Other Params     : R-squared(long) = 1
```

Beta(Hypothesis)	Delta(Breakdown)
0.000	96.5 %

References

Bazzi, Samuel, Martin Fiszbein, and Mesay Gebresilasse. 2020. "Frontier Culture: The Roots and Persistence of Rugged Individualism in the United States." *Econometrica* 88 (6): 2329–68.

Diegert, Paul, Matthew A. Masten, and Alexandre Poirier. 2022. "Assessing Omitted Variable Bias When the Controls Are Endogenous." *arXiv Preprint.*

Oster, Emily. 2019. "Unobservable Selection and Coefficient Stability: Theory and Evidence." *Journal of Business & Economic Statistics* 37 (2): 187–204.