Data Structures I: Recursion

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Cocktail of the day: Grasshoper

Disclaimer: Keep alcohol out of the hands of minors.

Cocktail of the day: Grasshoper

- 30 ml Crème de menthe
- 30 ml Crème de cacao
- 30 ml Fresh cream

Formats to transmit data objects use trees

https://www.youtube.com/watch?v=vrPBtQAxw7c

Function call

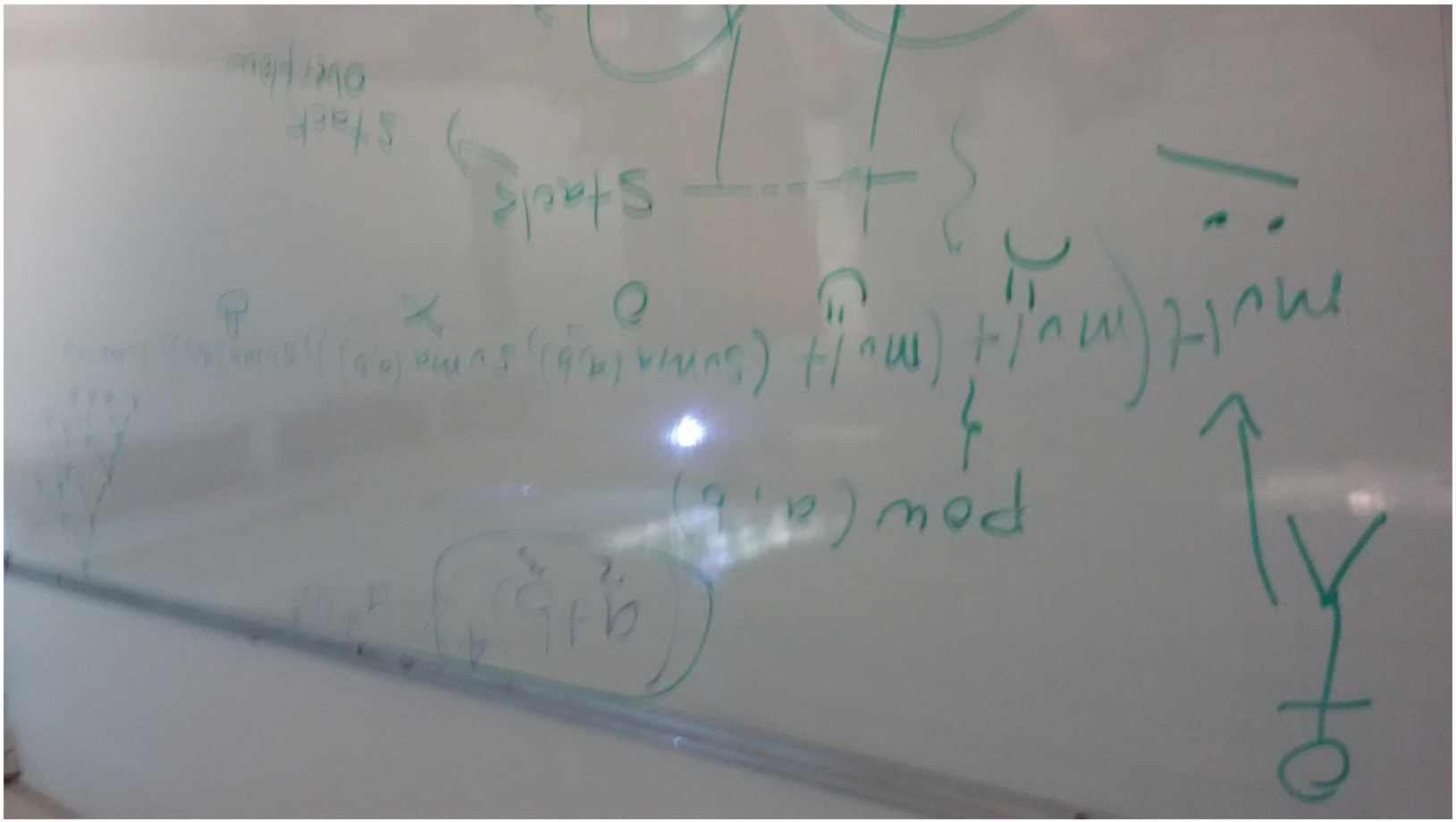


Figure: Function call to perform $(a + b)^4$

Print numbers recursively

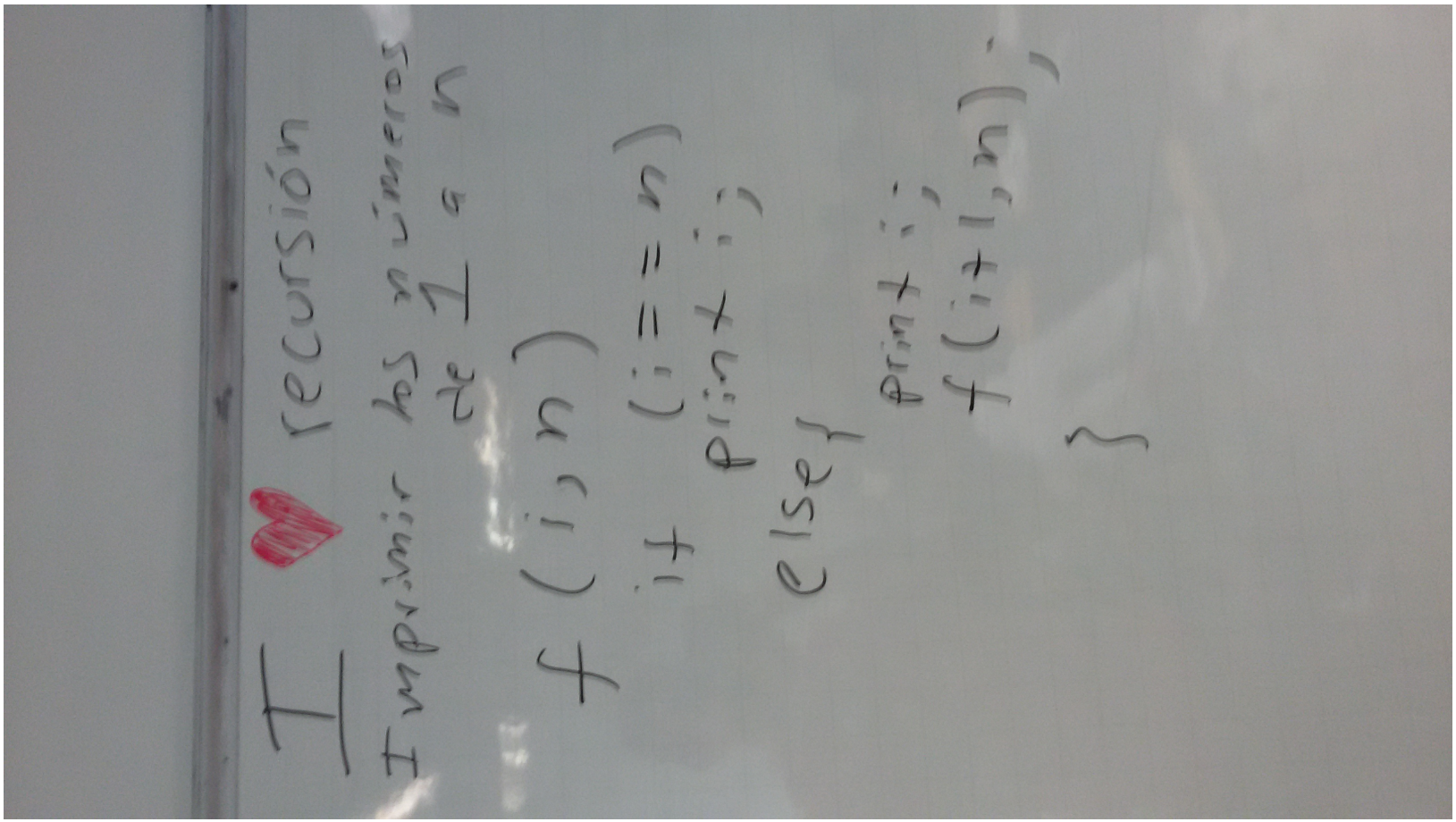


Figure: Print numbers recursively

Sum numbers recursively

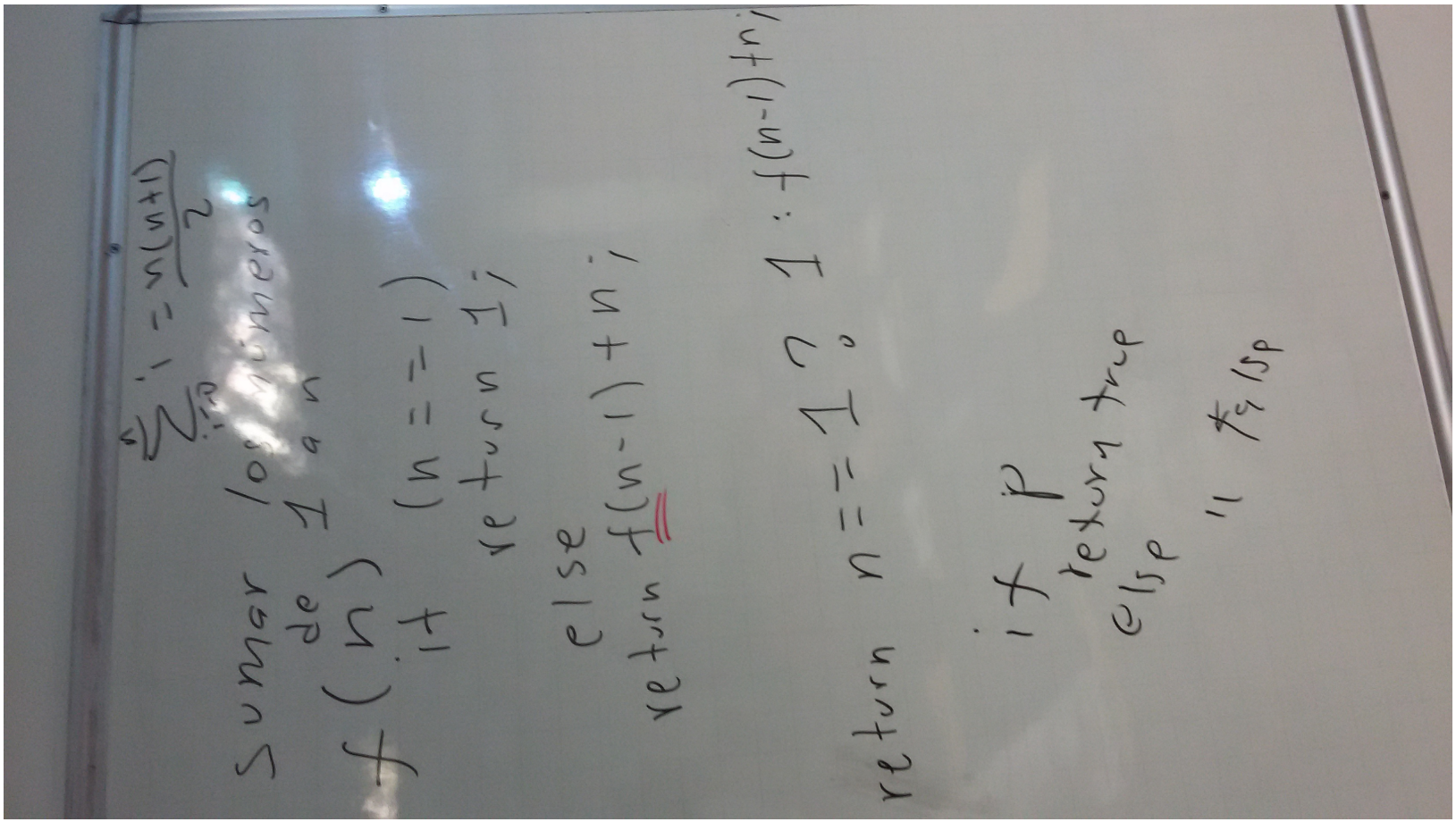
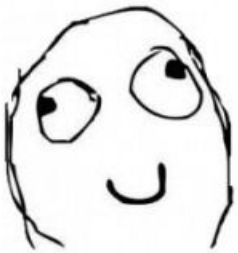


Figure: Sum numbers recursively

Recursive factorial

$$\frac{7!}{4!\,3!}$$

How other people read it:



7 factorial,
divided by 4 factorial,
divided by 3 factorial.

How I read it:

7!!!
4!!!!
3!!!!

Figure: Comic about factorial

Recursive factorial

```
factorial(5)
   factorial(4)
      factorial(3)
         factorial(2)
            factorial(1)
               return 1
            return 2*1 = 2
         return 3*2 = 6
      return 4*6 = 24
   return 5*24 = 120
```

Simulation: http://visualgo.net/recursion.html

Recursive factorial

```
public static int factorial(int N) {
    if (N == 1) return 1;
    return N * factorial(N-1);
}
```

Factorial: Recursion tree

Figure: Recursion tree of factorial function

The rabbit problem

Un conejo está ubicado en la celda $(0, 0)$ de un tablero A de $n \times m$. Si el conejo está en la casilla (i, j), el conejo puede avanzar –únicamente– horizontalmente a la casilla $(i + 1, j)$ o verticalmente a la casilla $(i, j + 1)$. En algunas celdas (ik, jk), hay manzanas que le darán un grado de satisfacción d al conejo y, en otras celdas (ih, jh), hay zanahorias que le darán satisfacción k al conejo. Donde hay manzanas estará el carácter '*' y donde hay zanahoras estará el carácter '#'. Donde no hay ni zanahorias ni manzanas, está el carácter '.' ¿Cuál es la mayor satisfacción que puede tener el conejo, si el conejo recorre el tablero de la manera anteriormente mencionada y tiene que llegar a la posición $(n - 1, m - 1)$ del tablero?

The rabbit problem

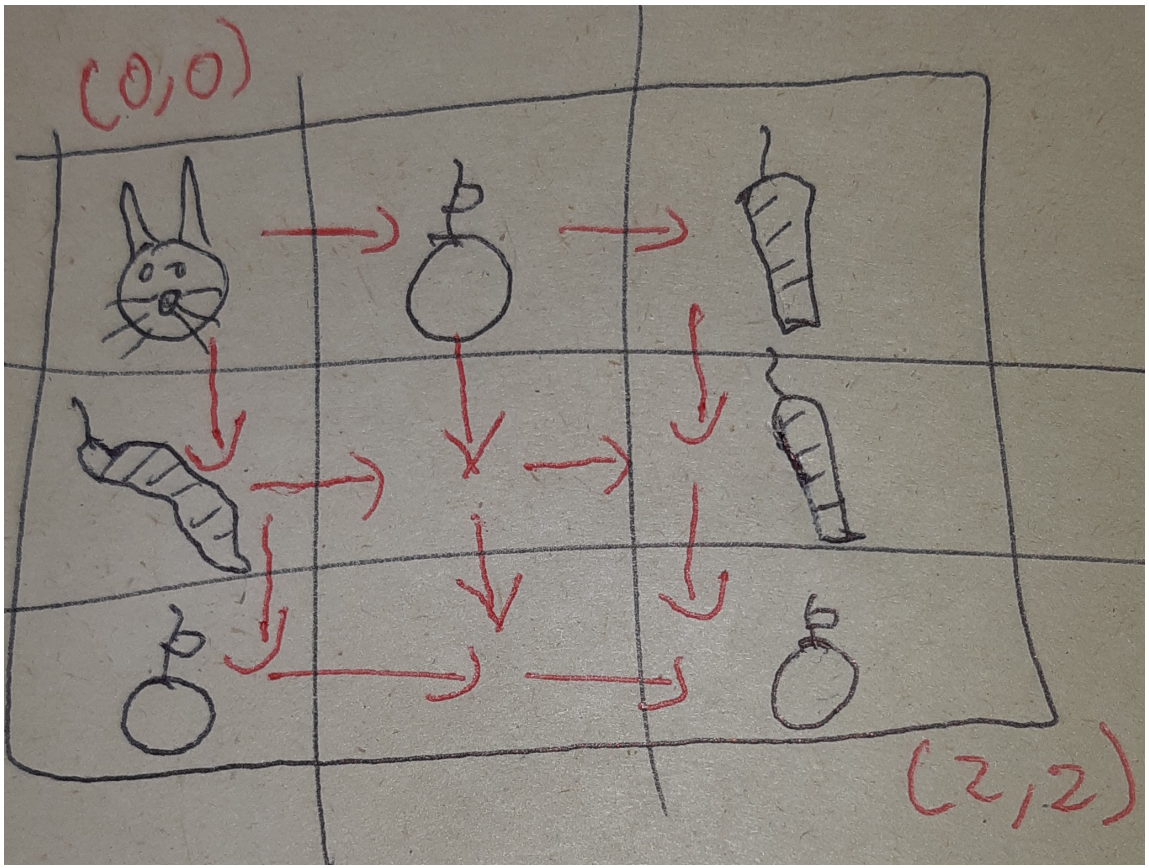


Figure: Example of input and output of the rabbit problem.

Complexity of Recursive factorial

- $T(n) = C + T(n-1), n > 1$
- $T(n)$ is $O(n)$

Euclid's algorithm

Figure: Euclid, the inventor of Euclid's algorithm

Euclid's algorithm

- The greatest common divisor (gcd) of two positive integers is the largest integer that divides evenly into both of them.
- The gcd of 102 and 68 is 34 since both 102 and 68 are multiples of 34.
- We can compute the gcd using the following property, which holds for positive integers p and q:
  - If $p > q$, the gcd of p and q is the same as the gcd of q and $p\%q$.

Taken from
`http://introcs.cs.princeton.edu/java/23recursion/`

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Subset generation

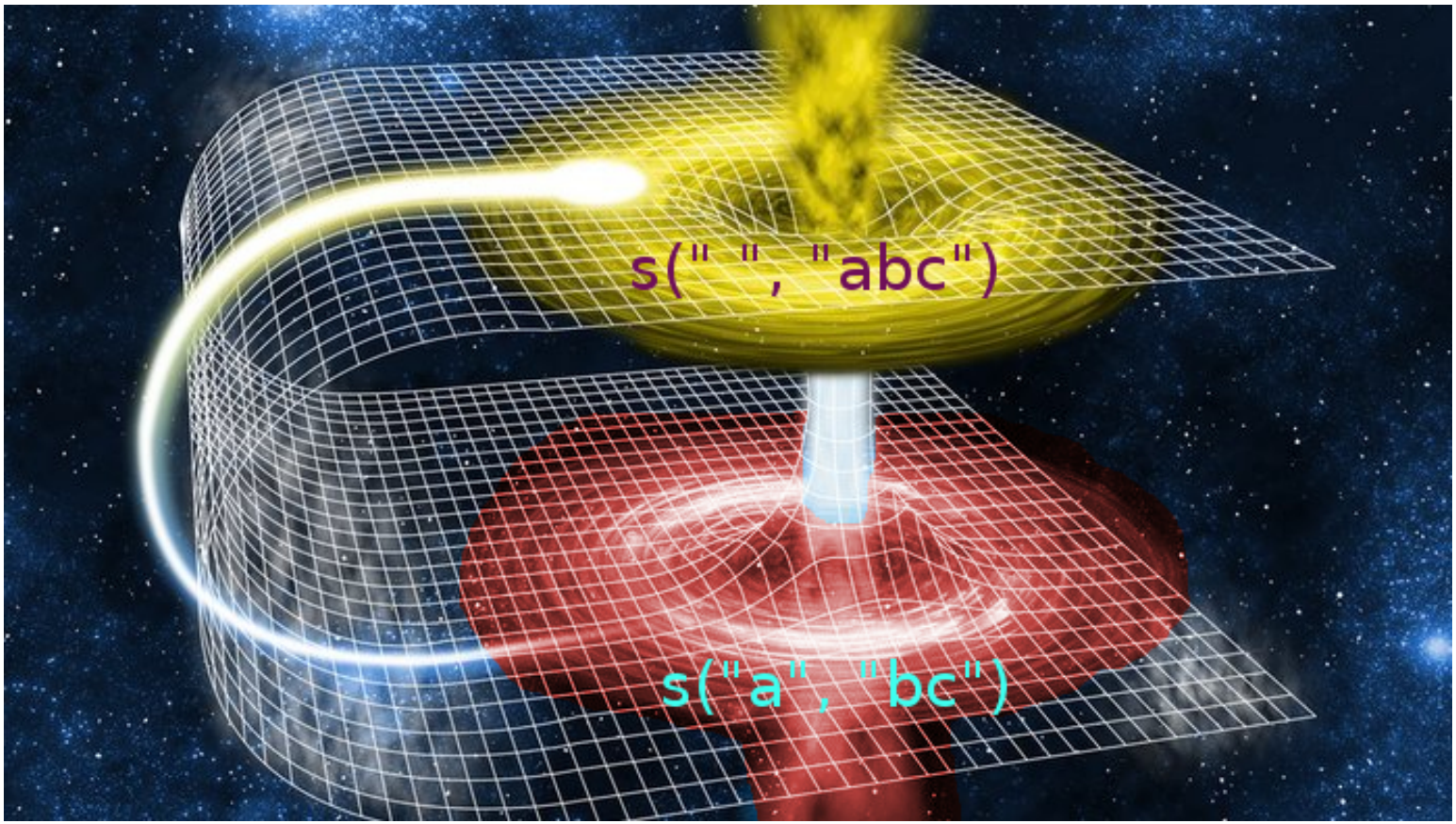


Figure: A universe where “a” belongs to the solution.

Subset generation

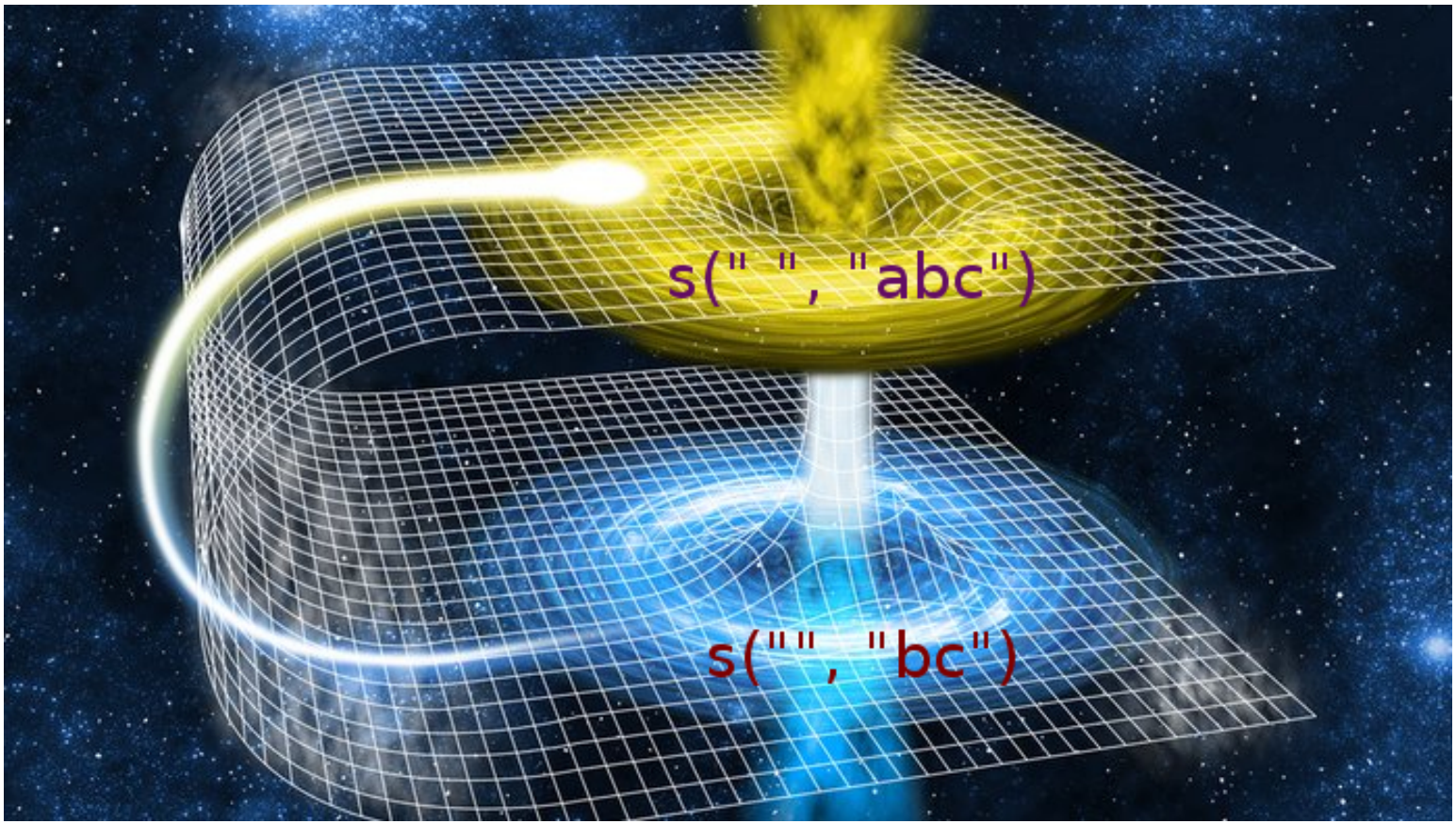


Figure: A universe where “a” does NOT belong to the solution.

Subset generation

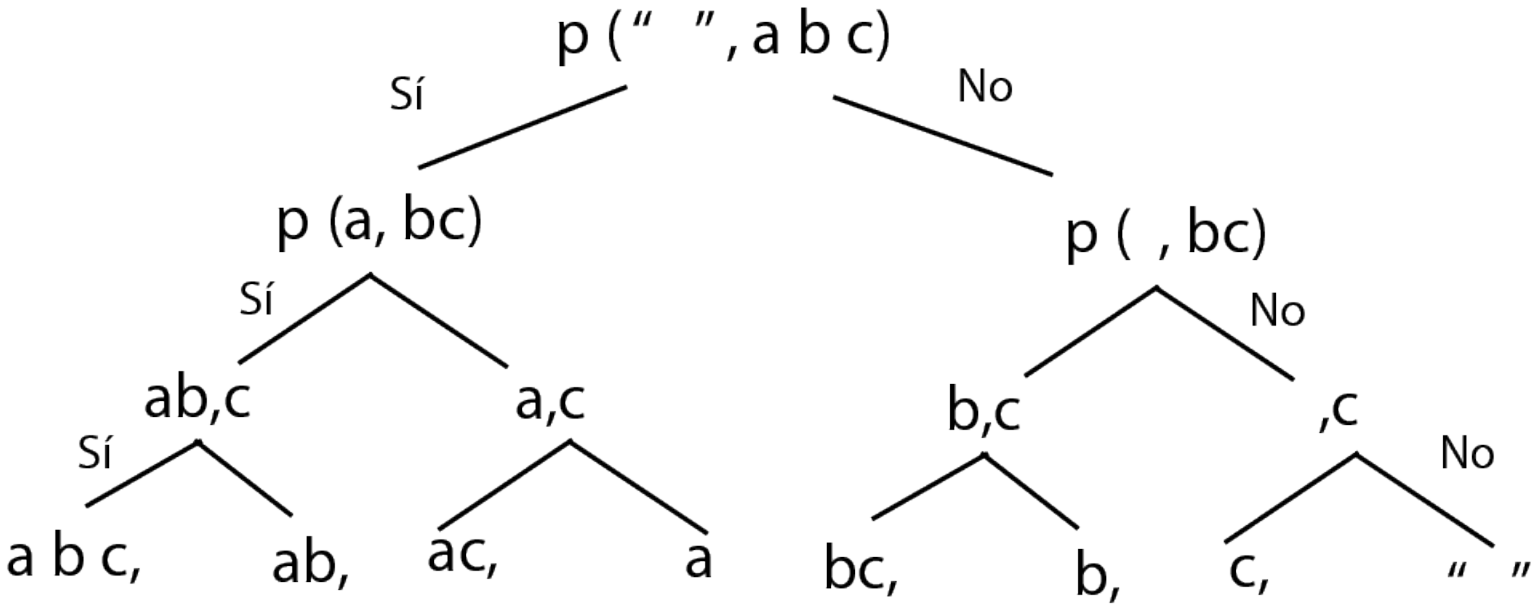


Figure: Recursion tree for subset generation

Euclid's algorithm

```
public static int gcd(int p, int q) {
        if (q == 0) return p;
        else return gcd(q, p % q);
}
```

Simulation: http://visualgo.net/recursion.html

Towers of Hanoi

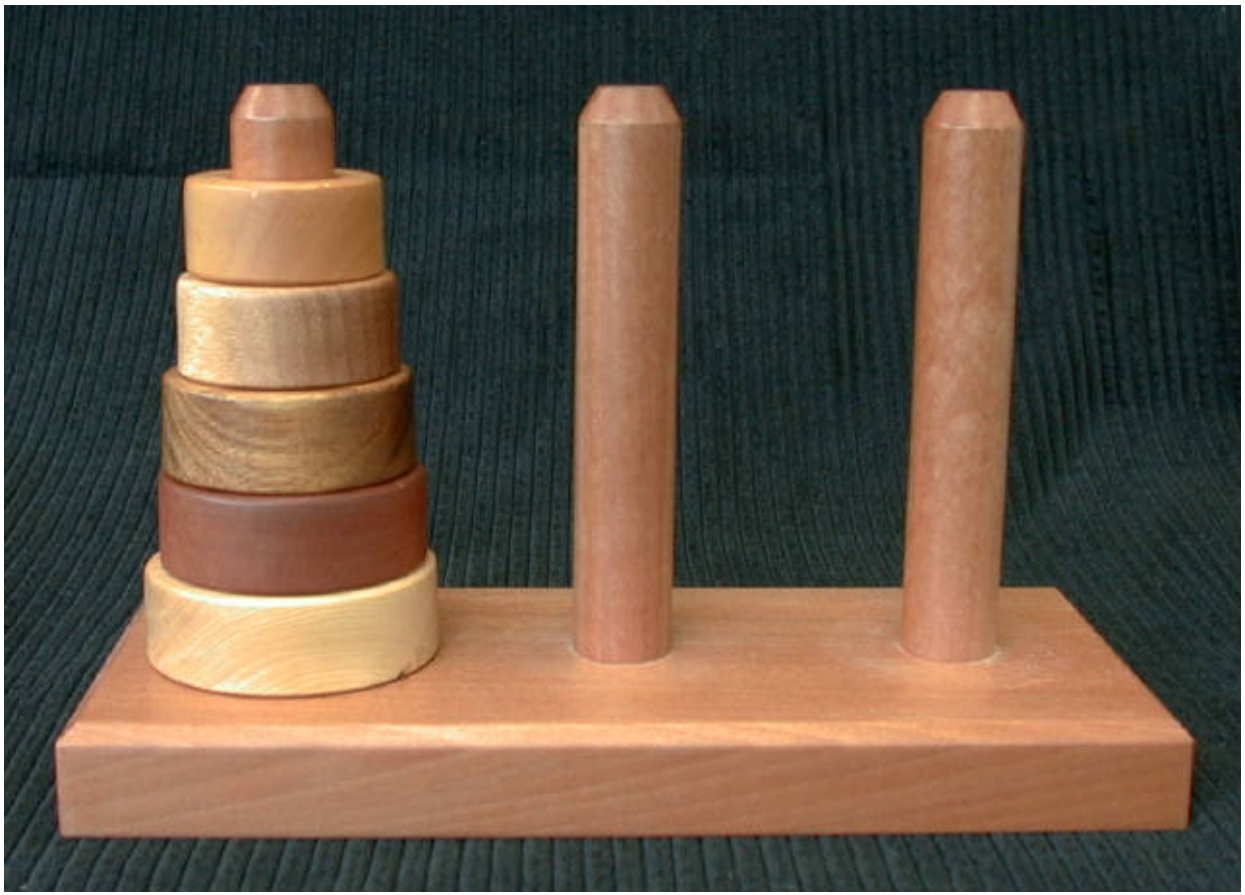


Figure: Towers of Hanoi

Towers of Hanoi

- We have 3 poles and n discs that fit onto the poles.
- The discs differ in size and are initially arranged on one of the poles, in order from largest (disc n) at the bottom to smallest (disc 1) at the top.
- The task is to move the stack of discs to another pole, while obeying the following rules:
  1. Move only one disc at a time.
  2. Never place a disc on a smaller one.

Towers of Hanoi

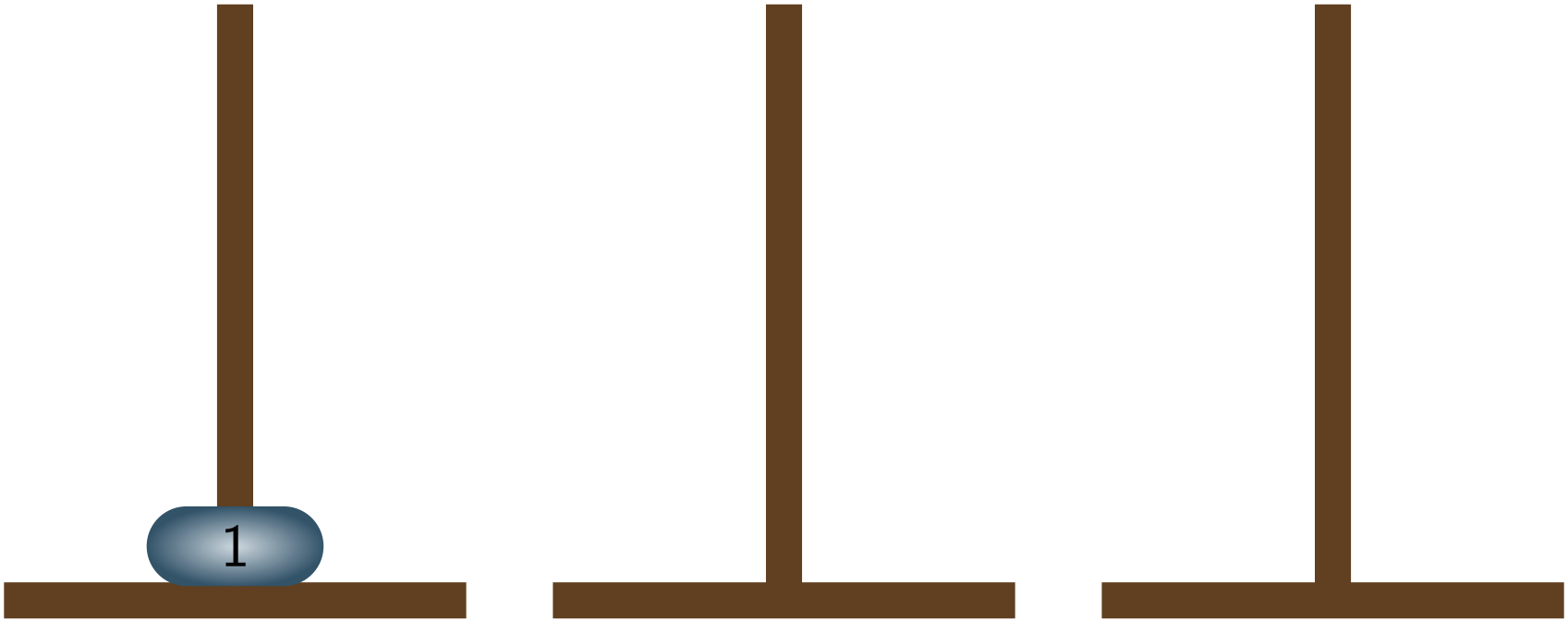
- One legend says that the world will end when a certain group of monks solved the Towers of Hanoi problem in a temple with 64 golden discs on three diamond needles.

Taken from
`http://introcs.cs.princeton.edu/java/23recursion/`

Towers of Hanoi: Algorithm

```
hanoi(topN, A, B, C)
  if (topN == 1)
    Move Disk 1 from A to C
  else
    hanoi(topN - 1, A, C, B)
    Move Disk topN from A to C
    hanoi(topN - 1, B, A, C)
```

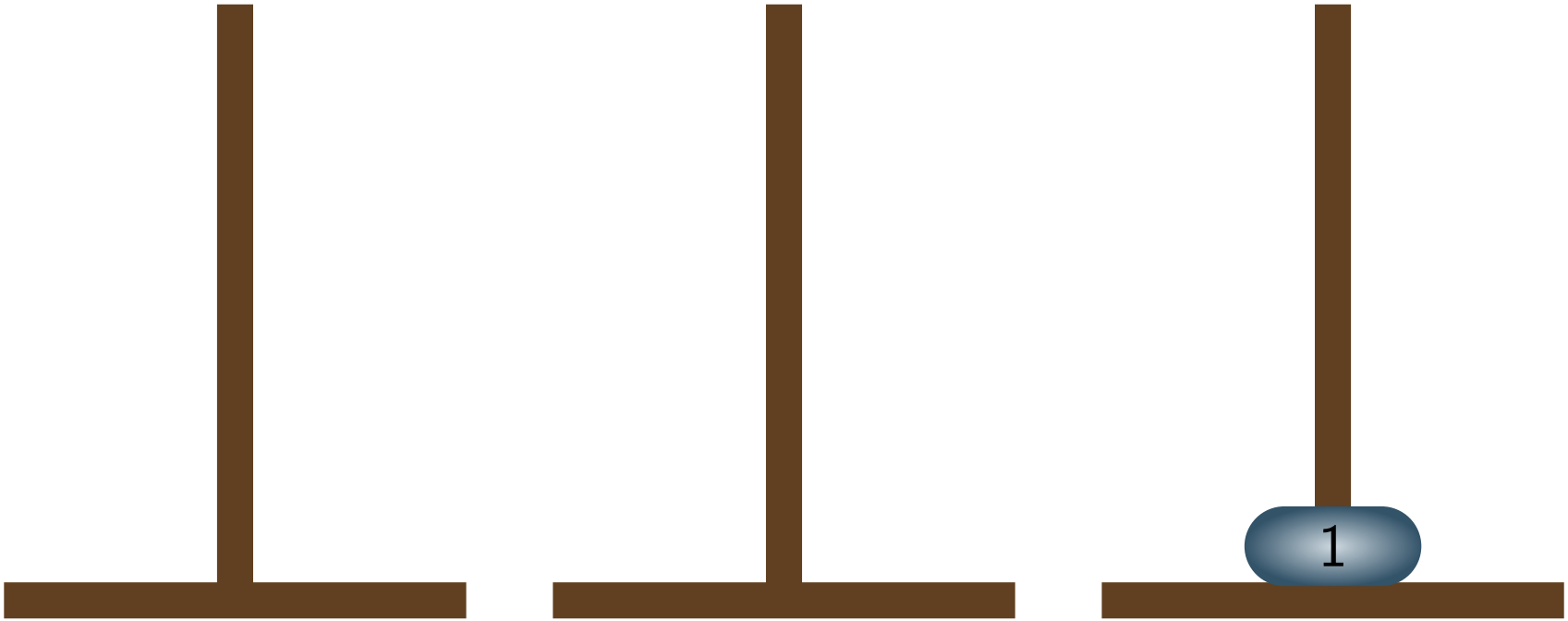
Tower of Hanoi – 1 Disc



Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 1 Disc



Moved disc from pole 1 to pole 3.

Taken from

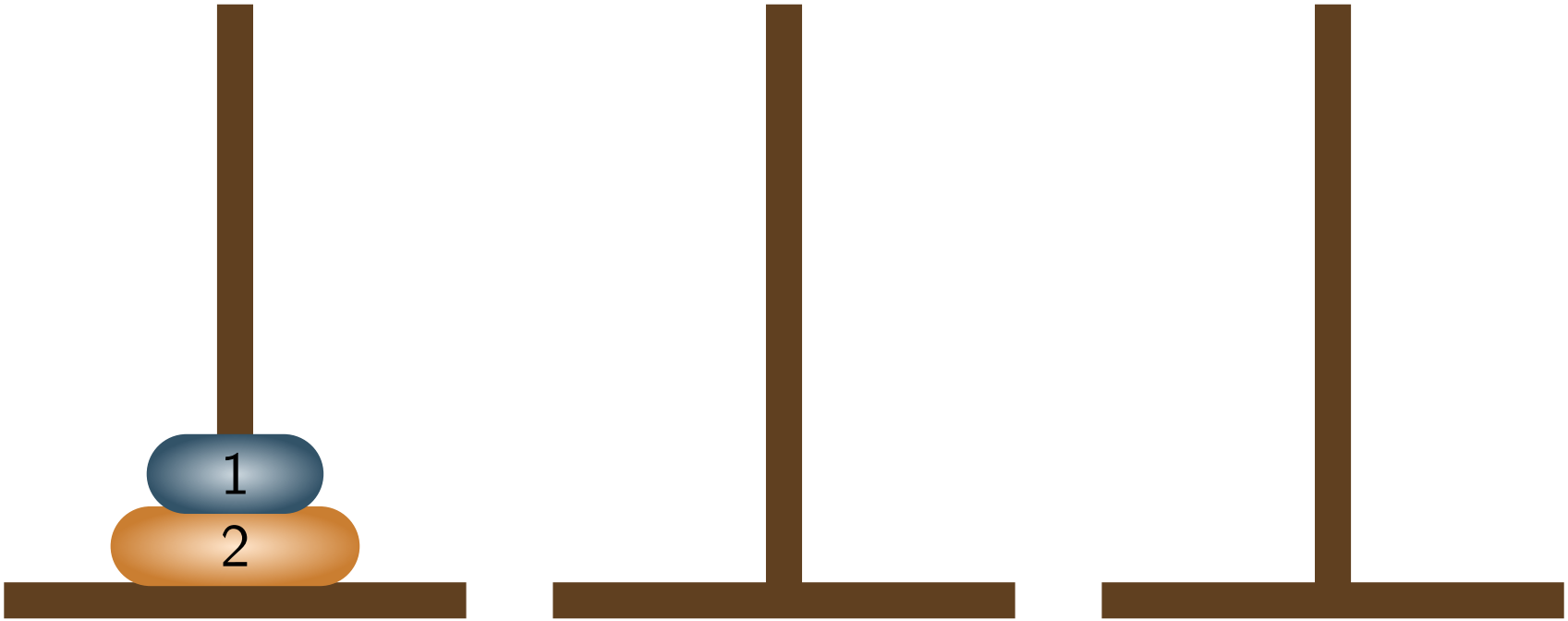
`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 1 Disc

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

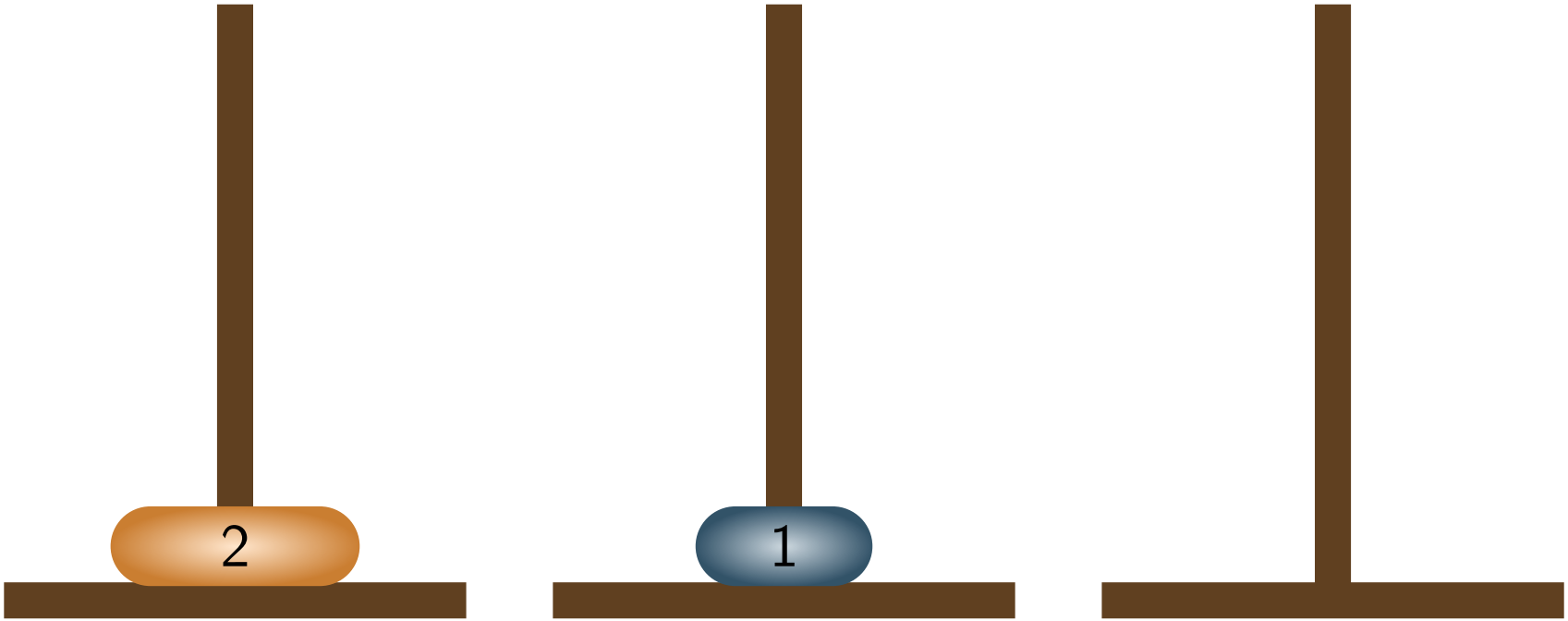
Tower of Hanoi – 2 Discs



Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 2 Discs

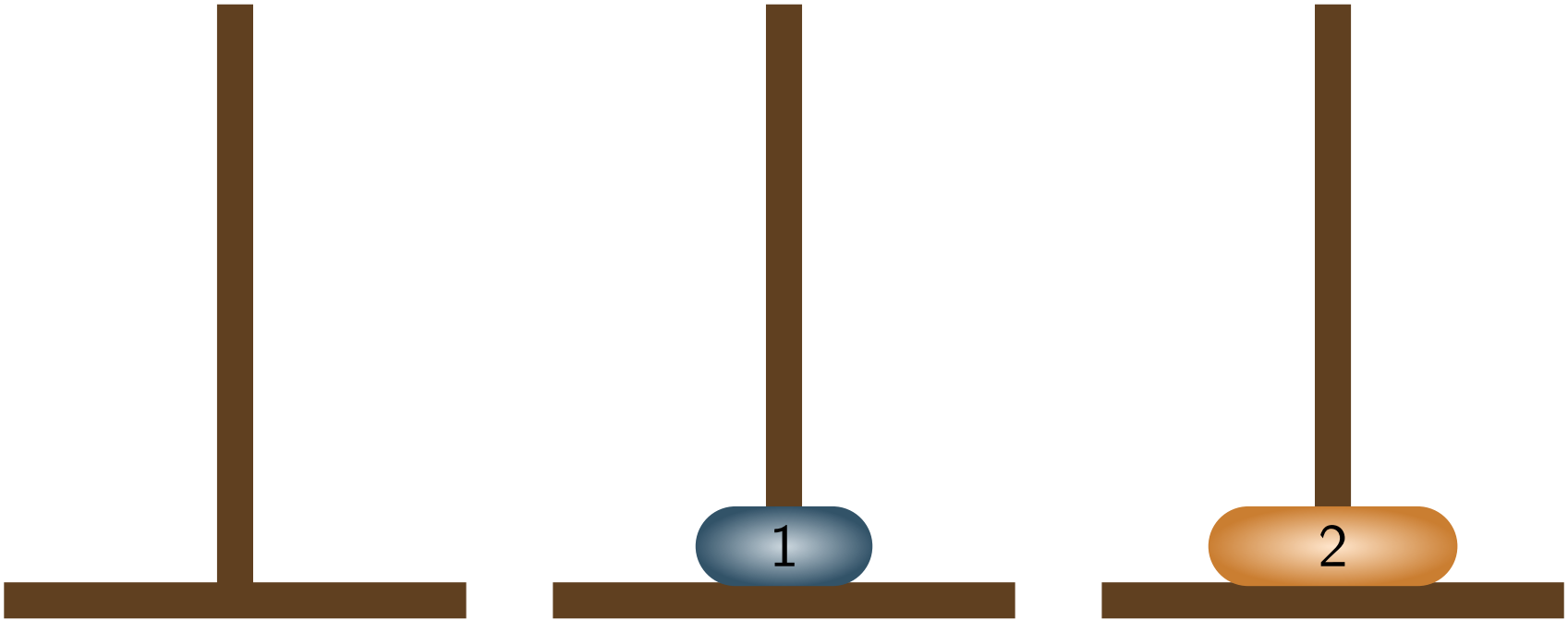


Moved disc from pole 1 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 2 Discs

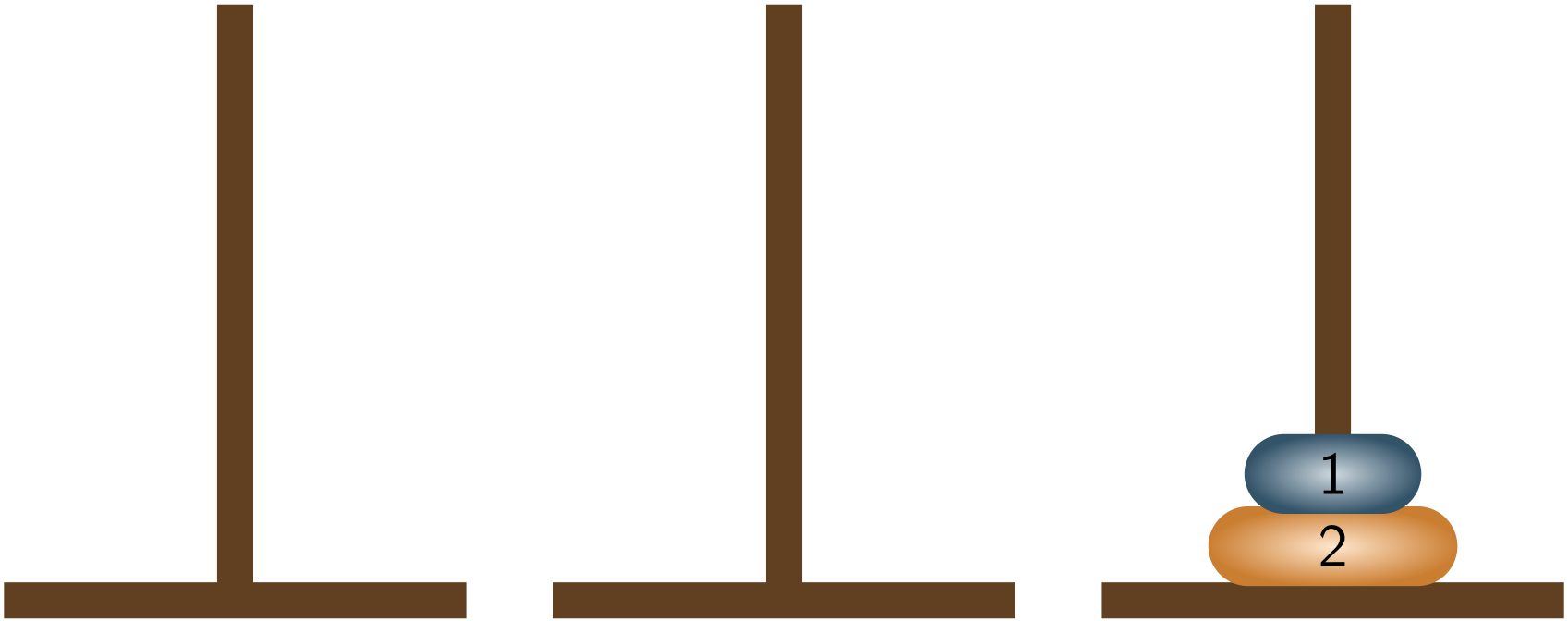


Moved disc from pole 1 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 2 Discs



Moved disc from pole 2 to pole 3.

Taken from

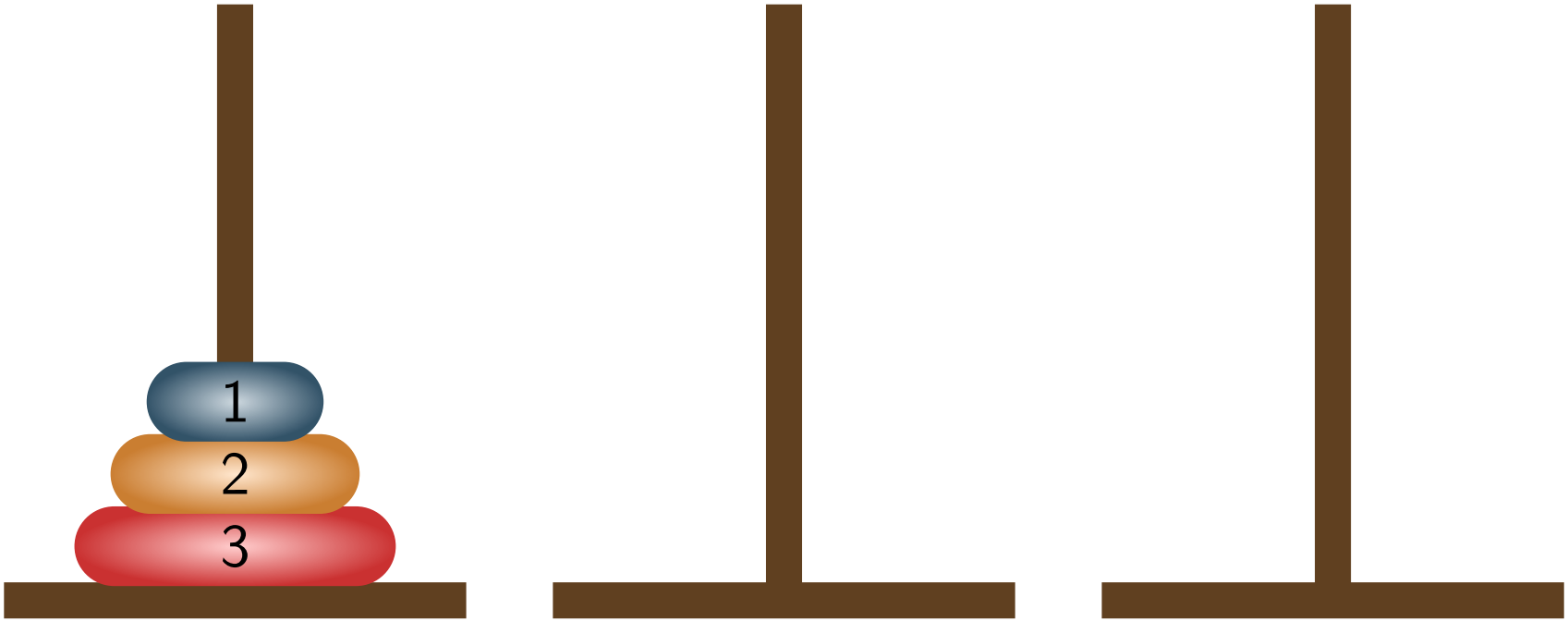
`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 2 Discs

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

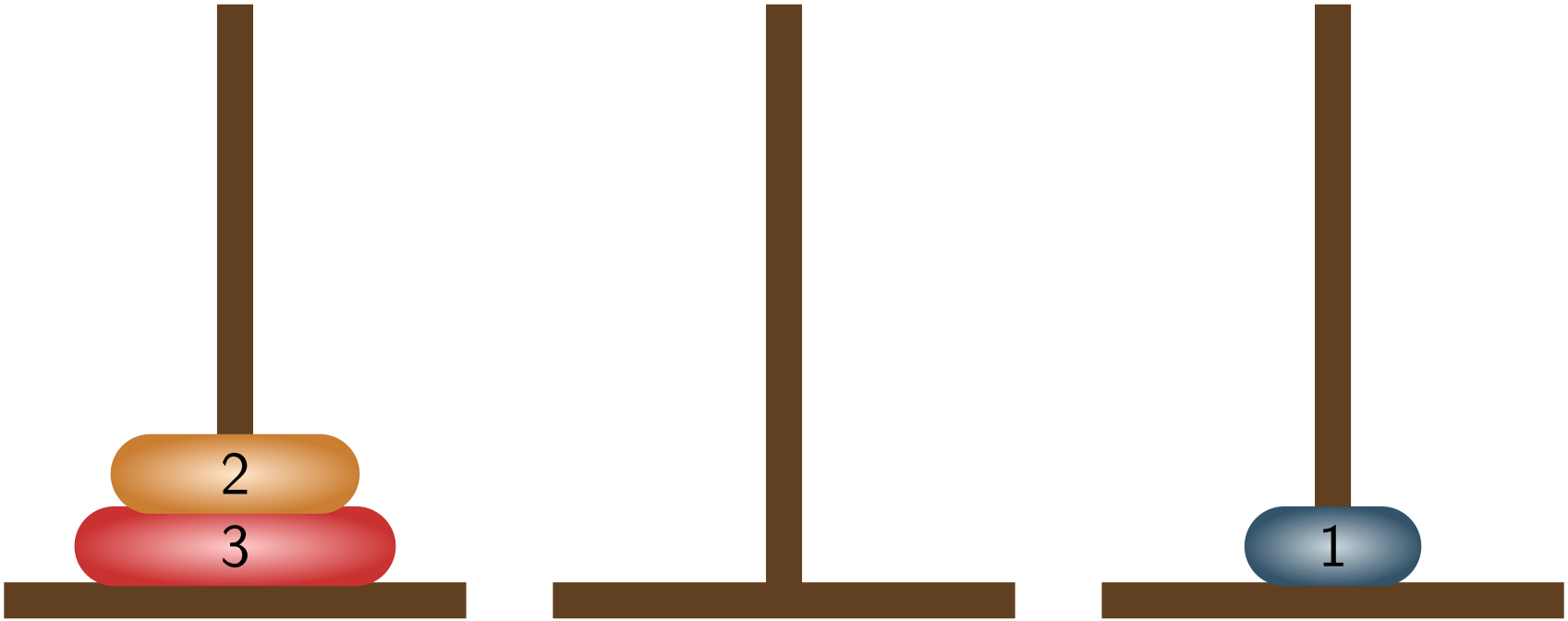
Tower of Hanoi – 3 Discs



Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

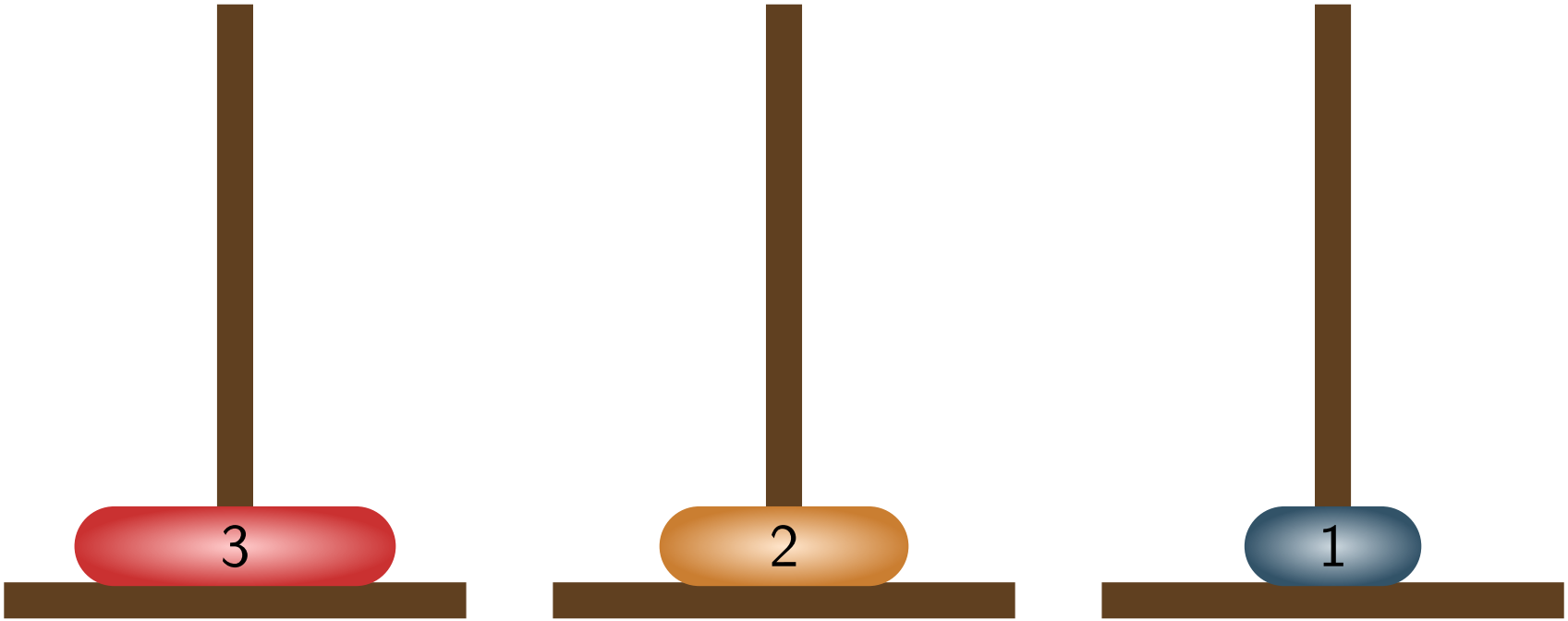


Moved disc from pole 1 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

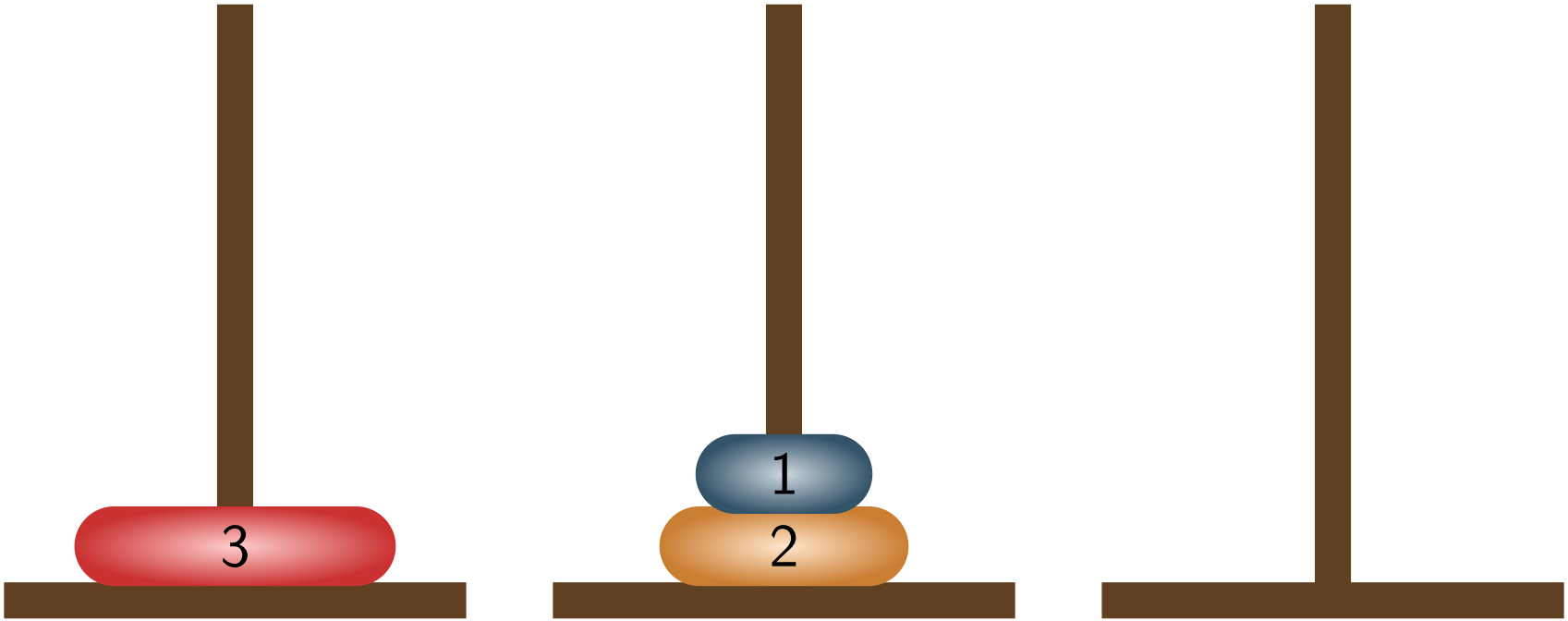


Moved disc from pole 1 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

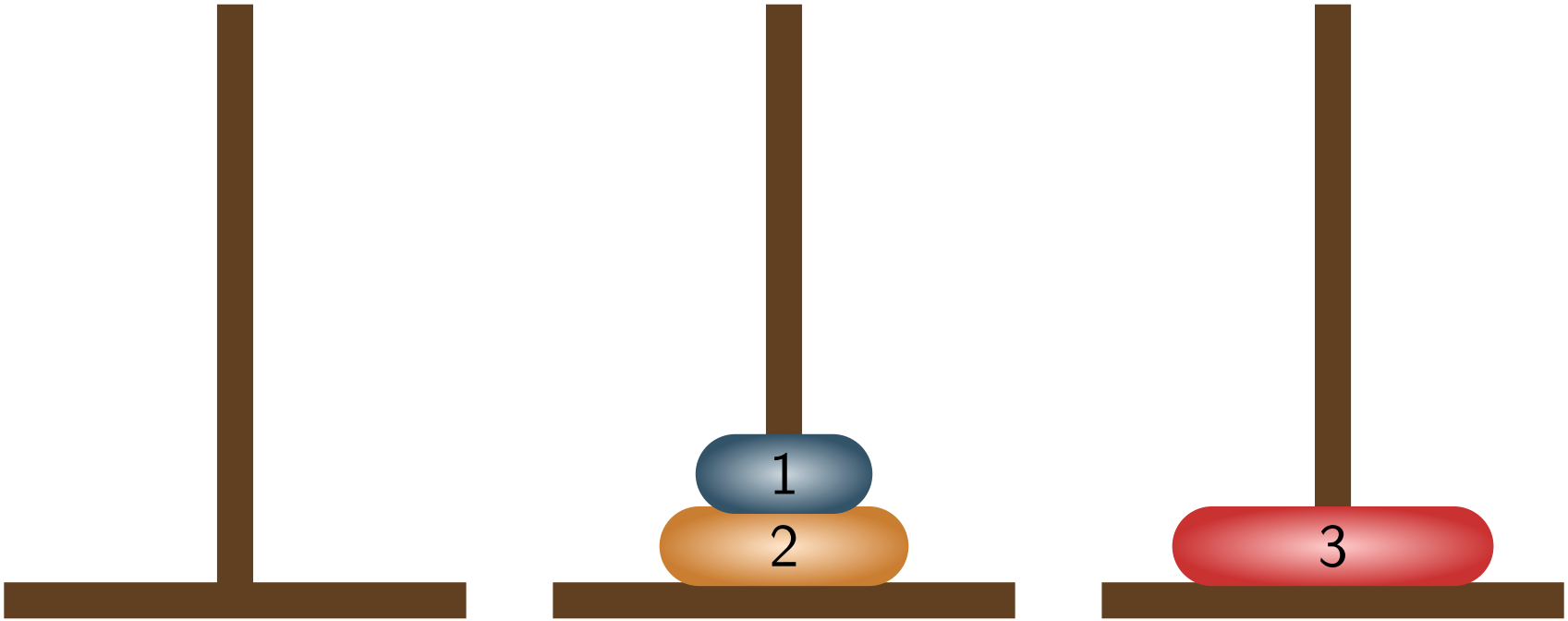


Moved disc from pole 3 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

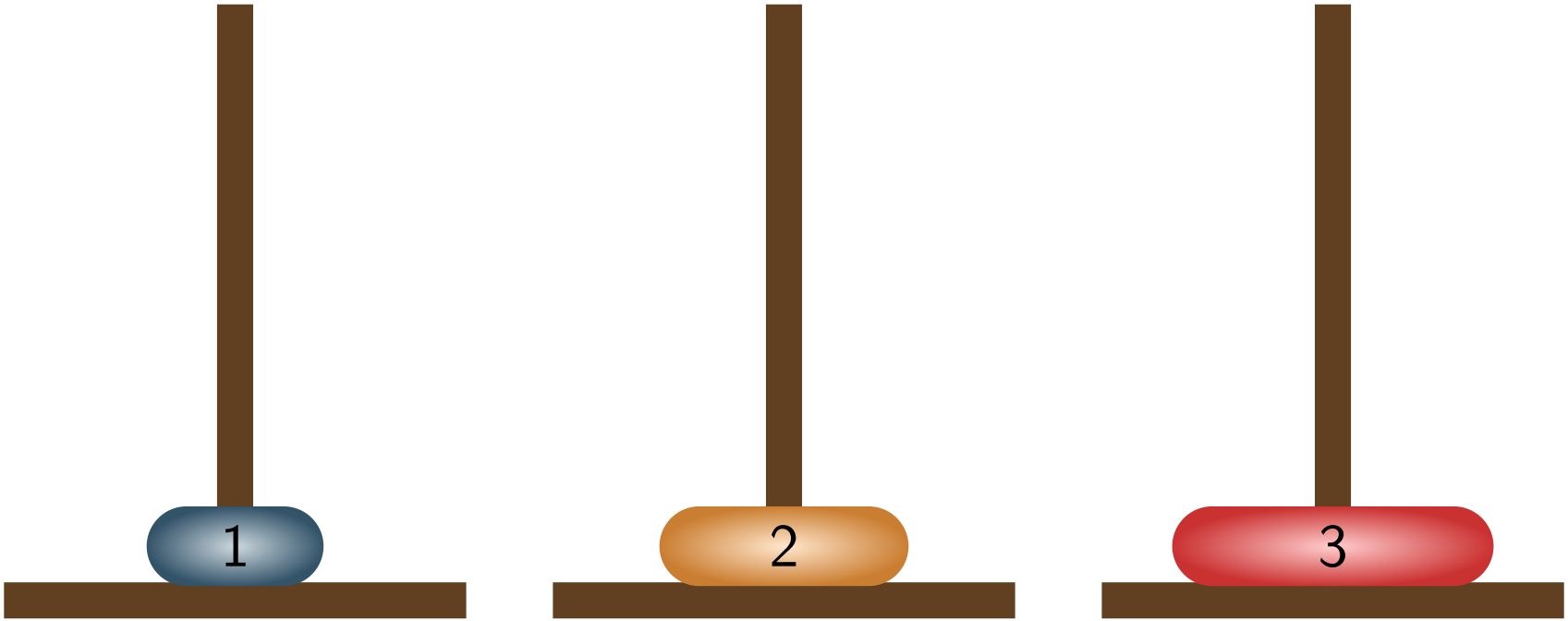


Moved disc from pole 1 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

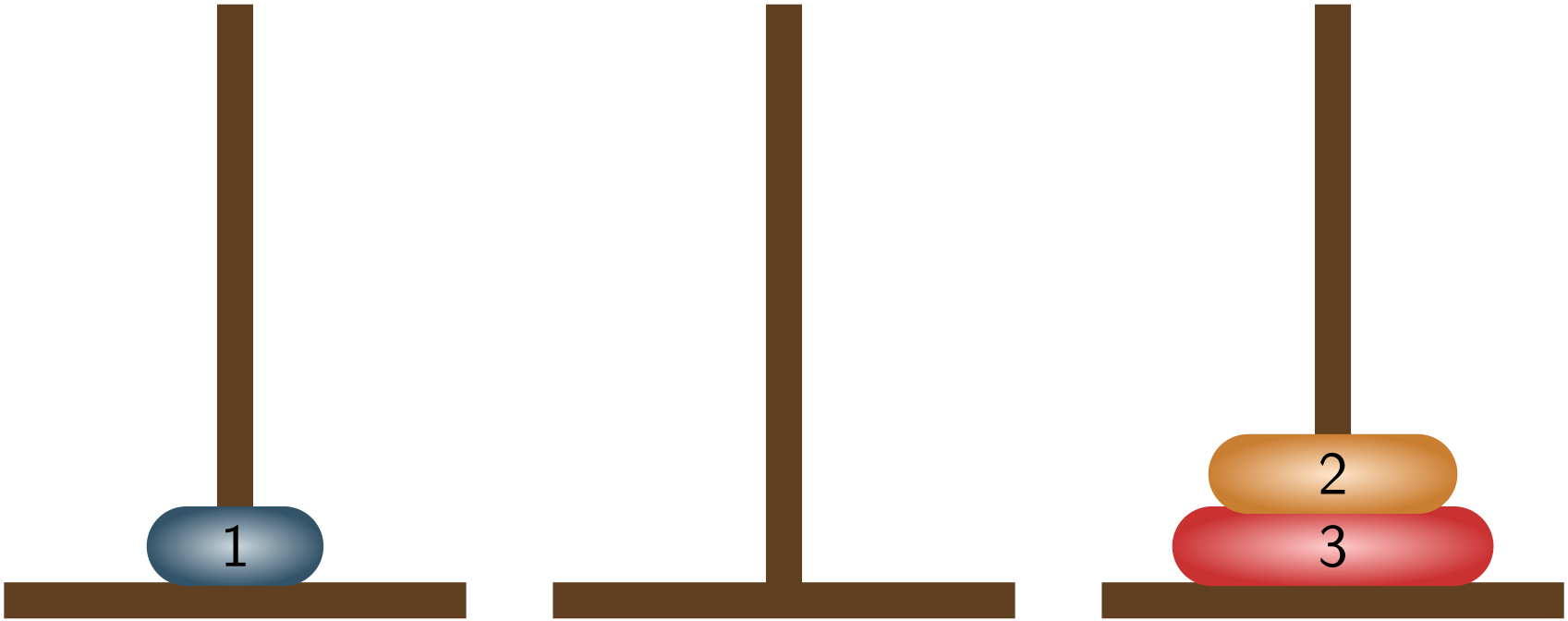


Moved disc from pole 2 to pole 1.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

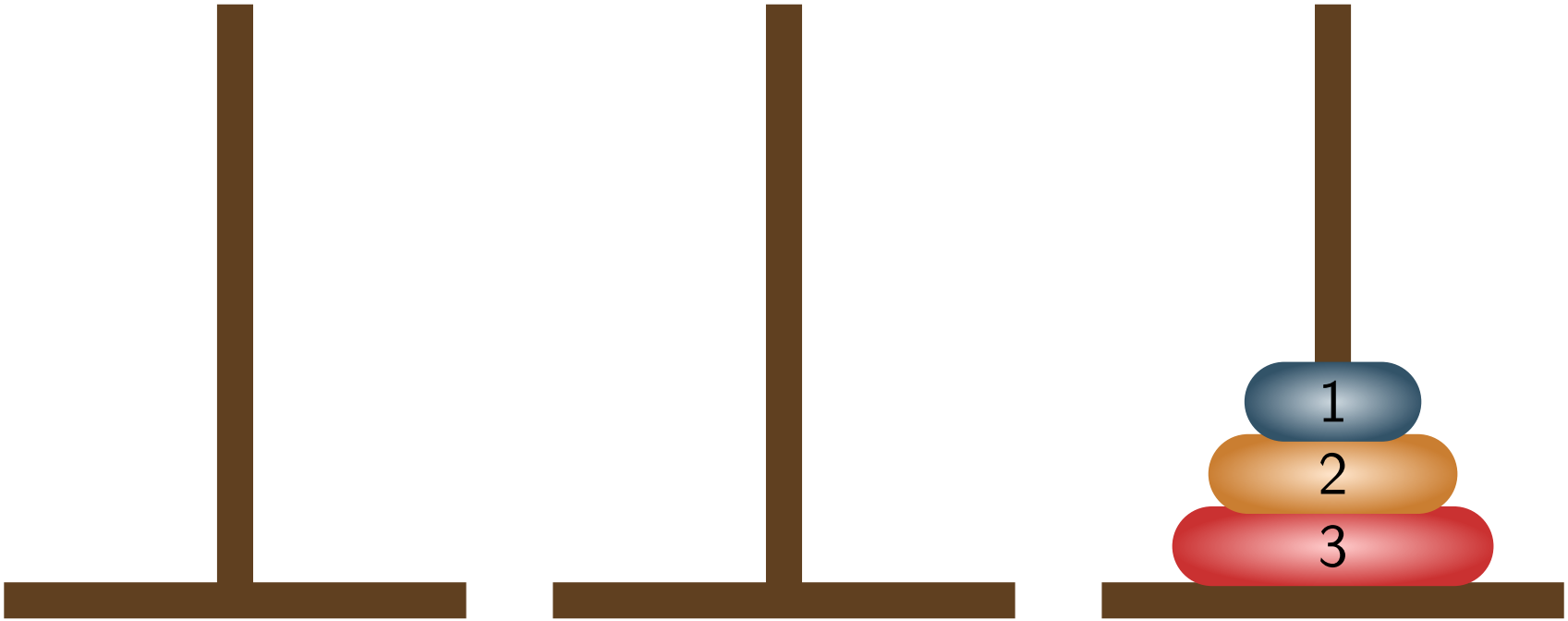


Moved disc from pole 2 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs



Moved disc from pole 1 to pole 3.

Taken from

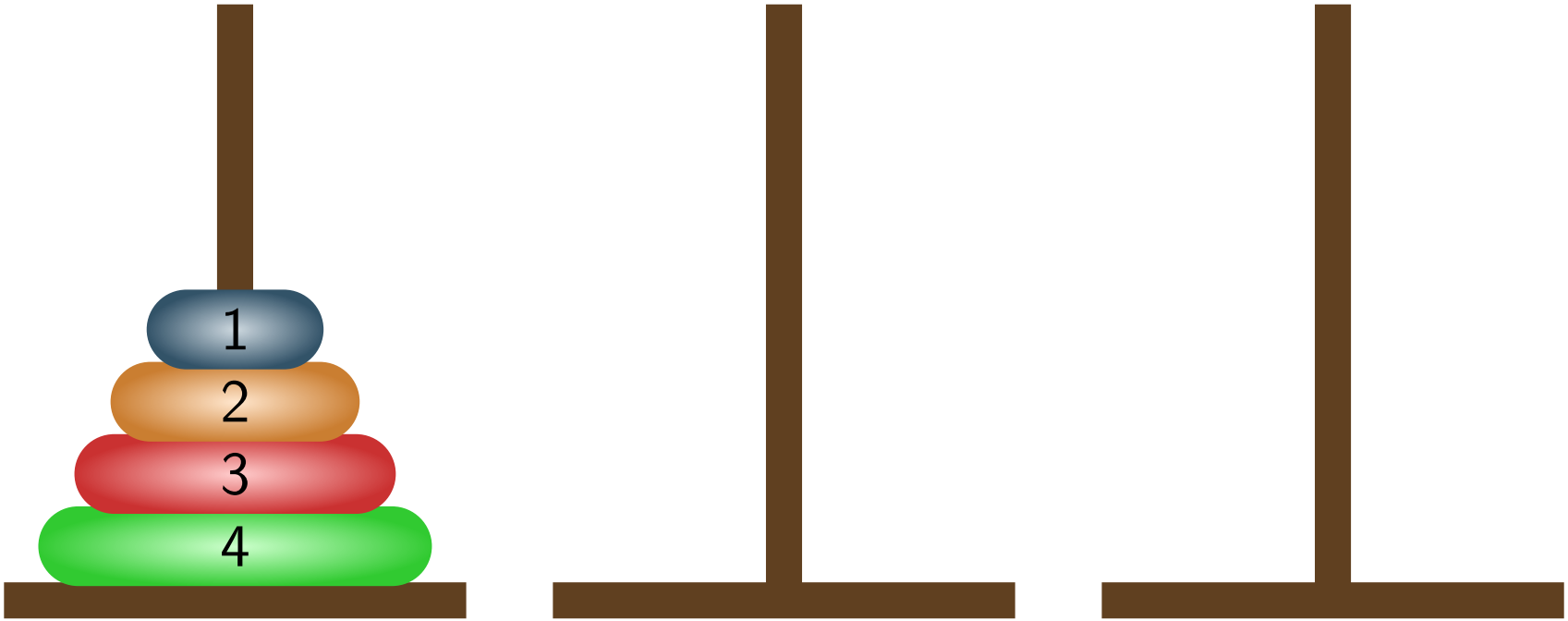
`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 3 Discs

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

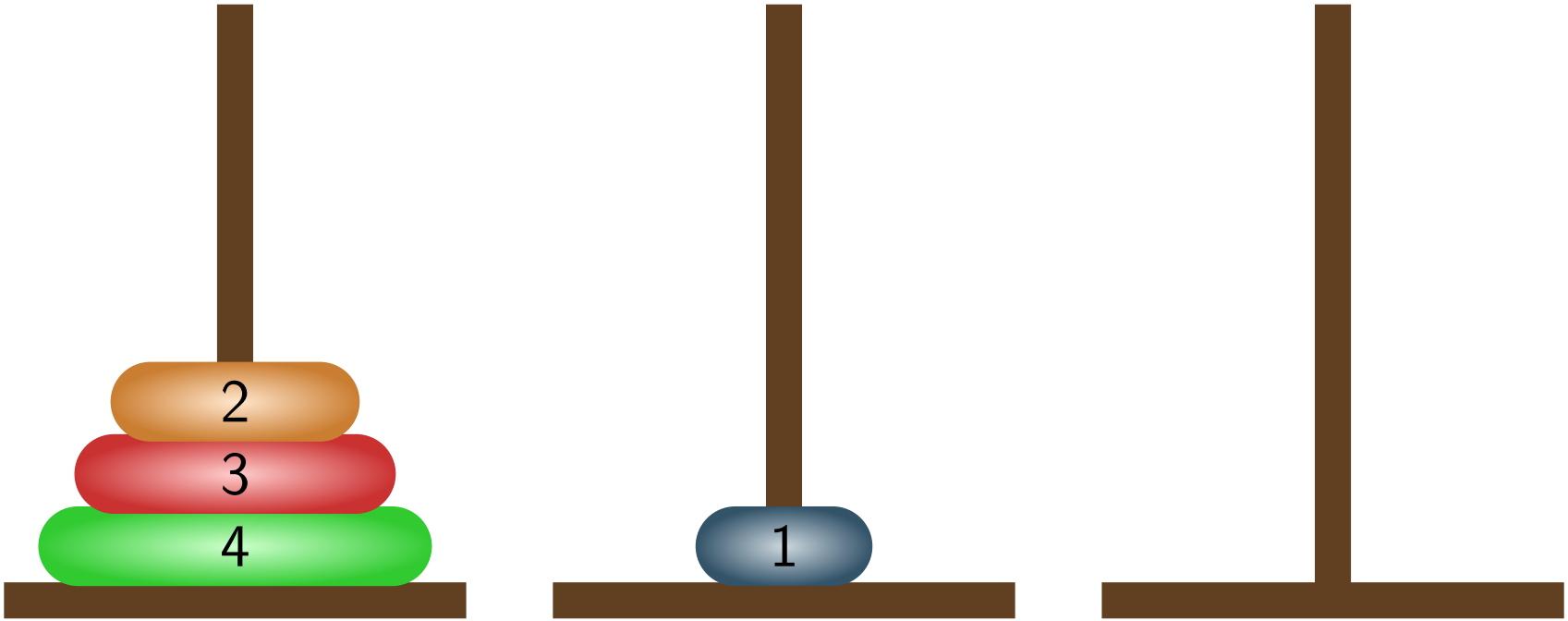
Tower of Hanoi – 4 Discs



Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

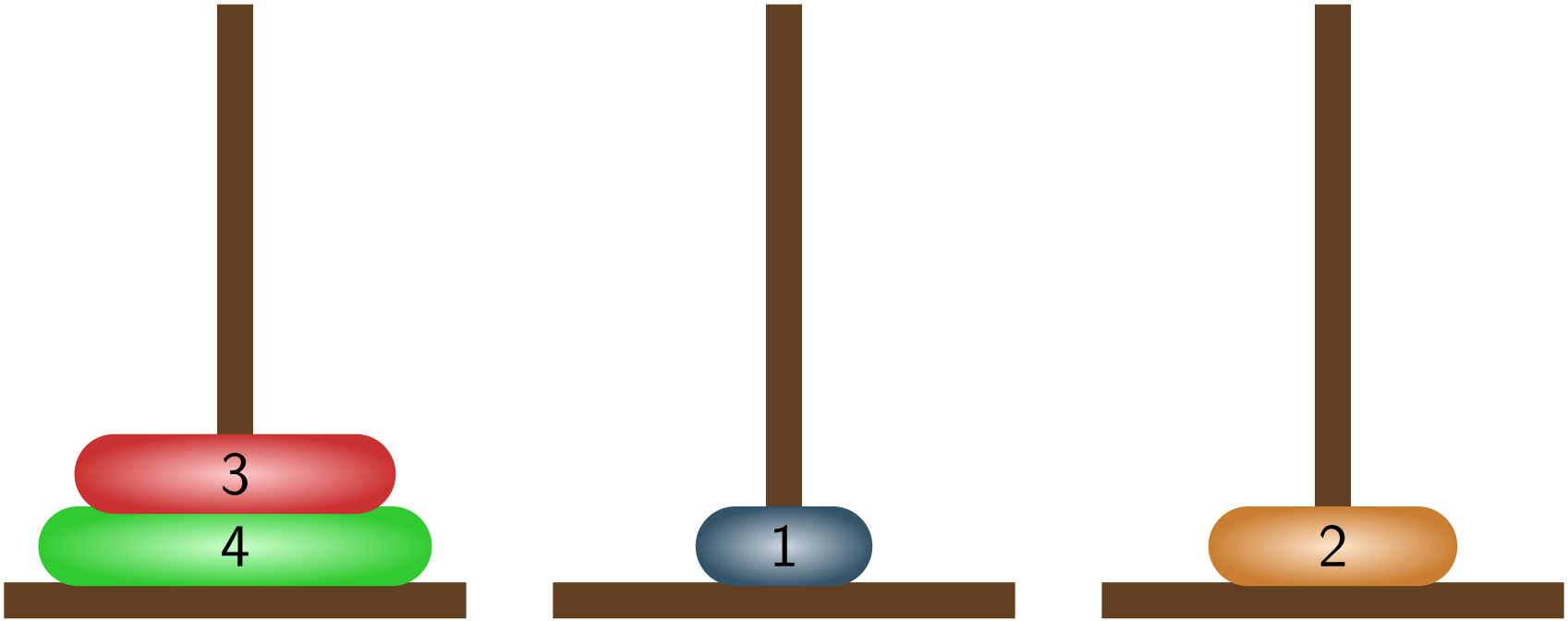


Moved disc from pole 1 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

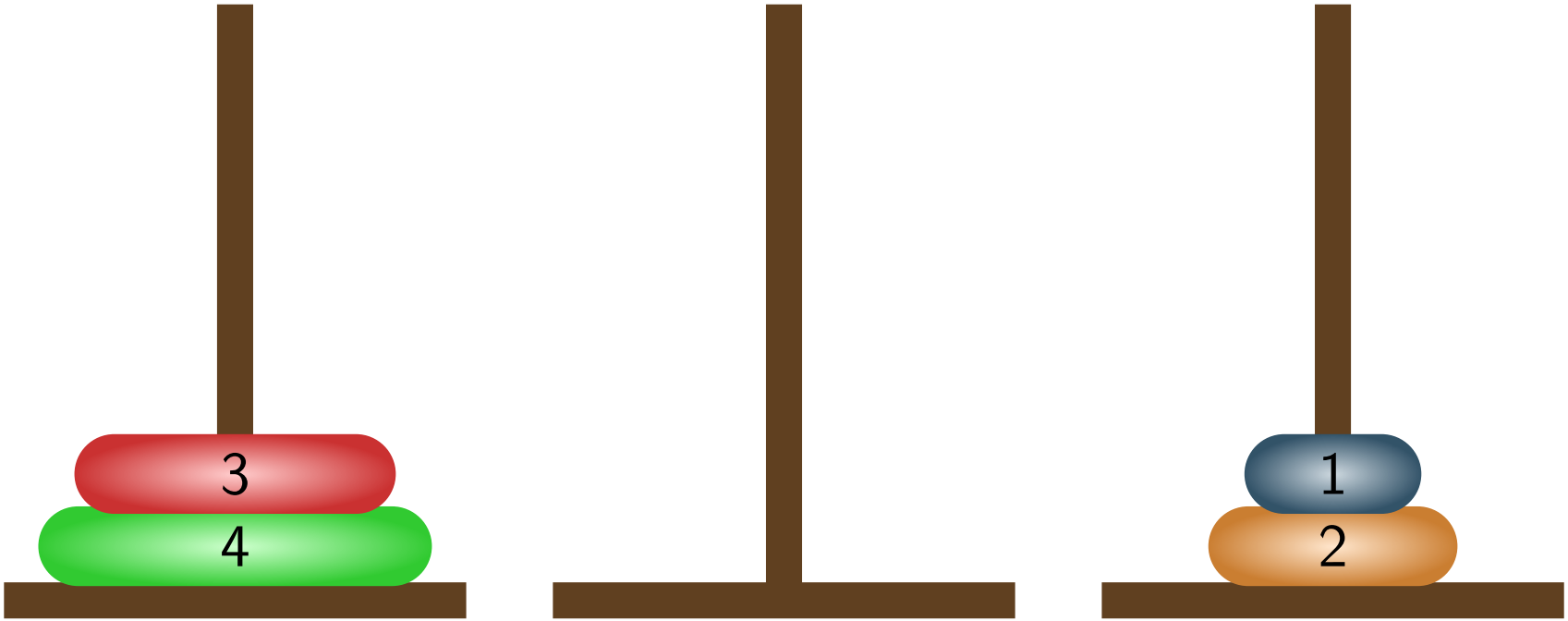


Moved disc from pole 1 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

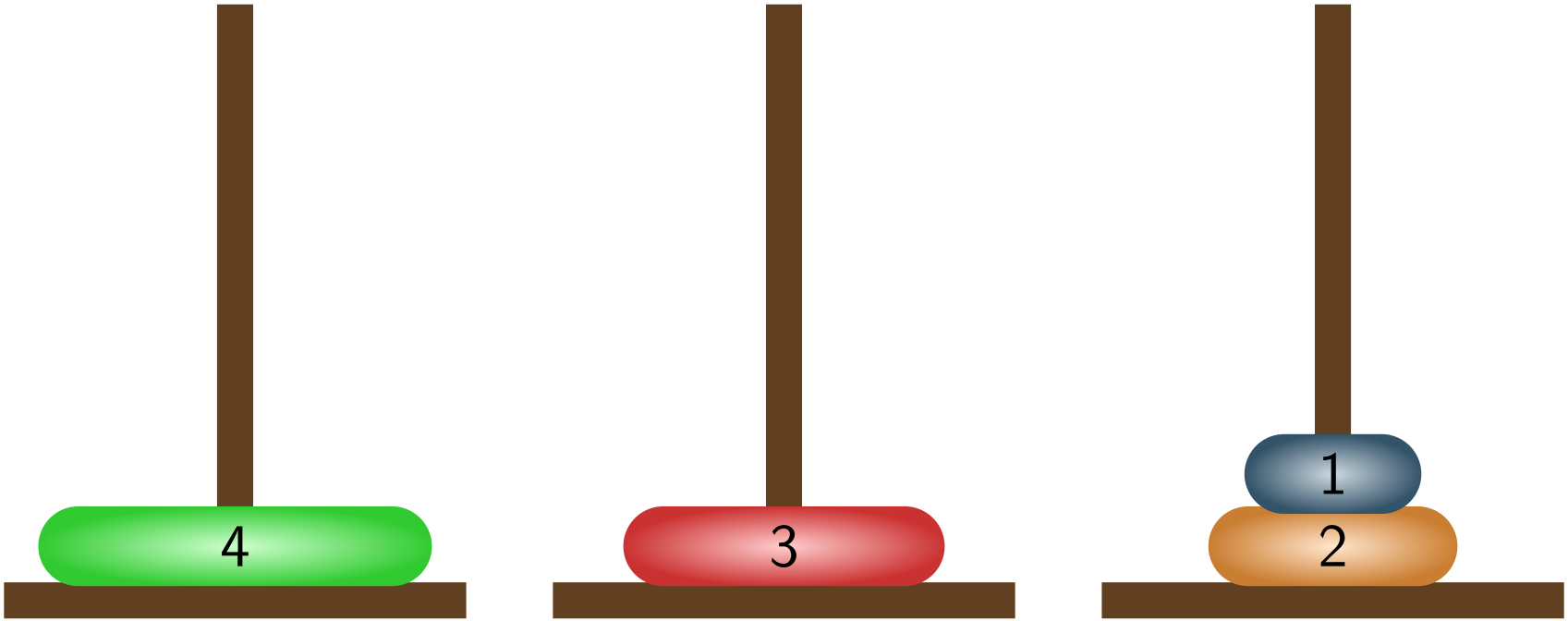


Moved disc from pole 2 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

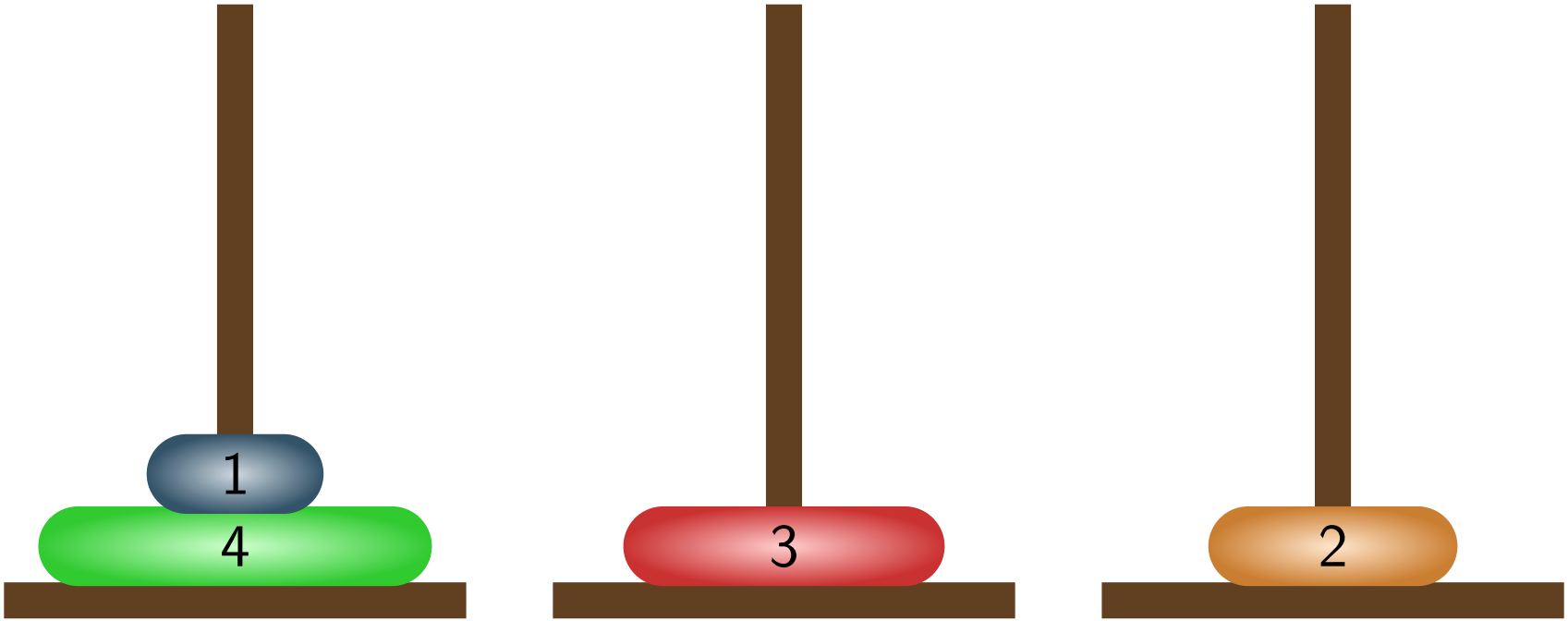


Moved disc from pole 1 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

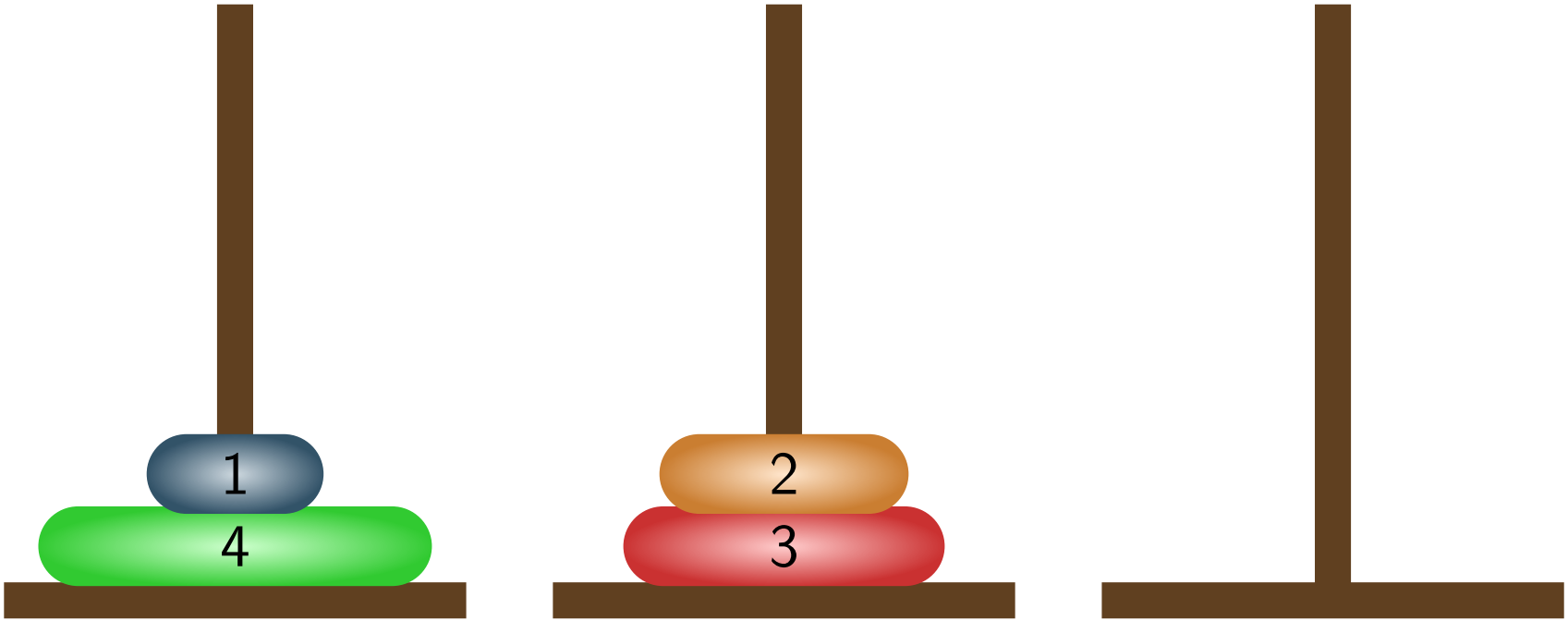


Moved disc from pole 3 to pole 1.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

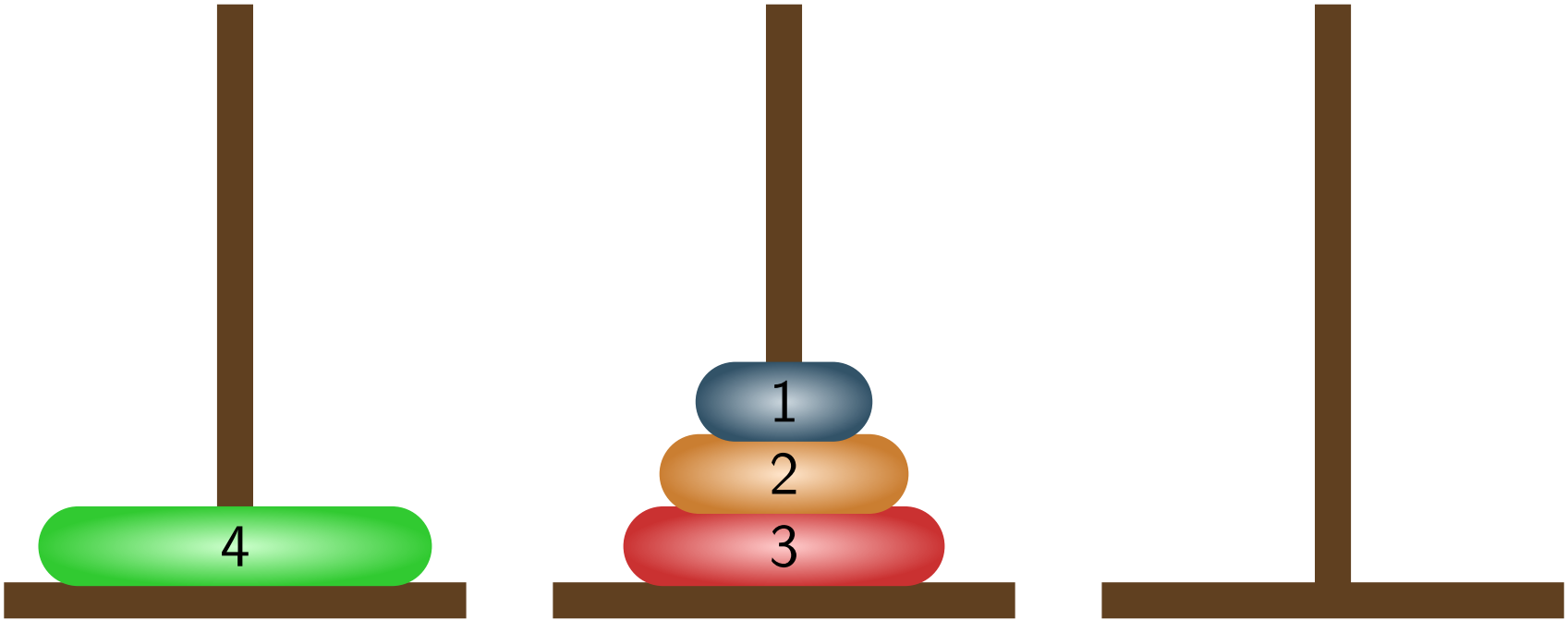


Moved disc from pole 3 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

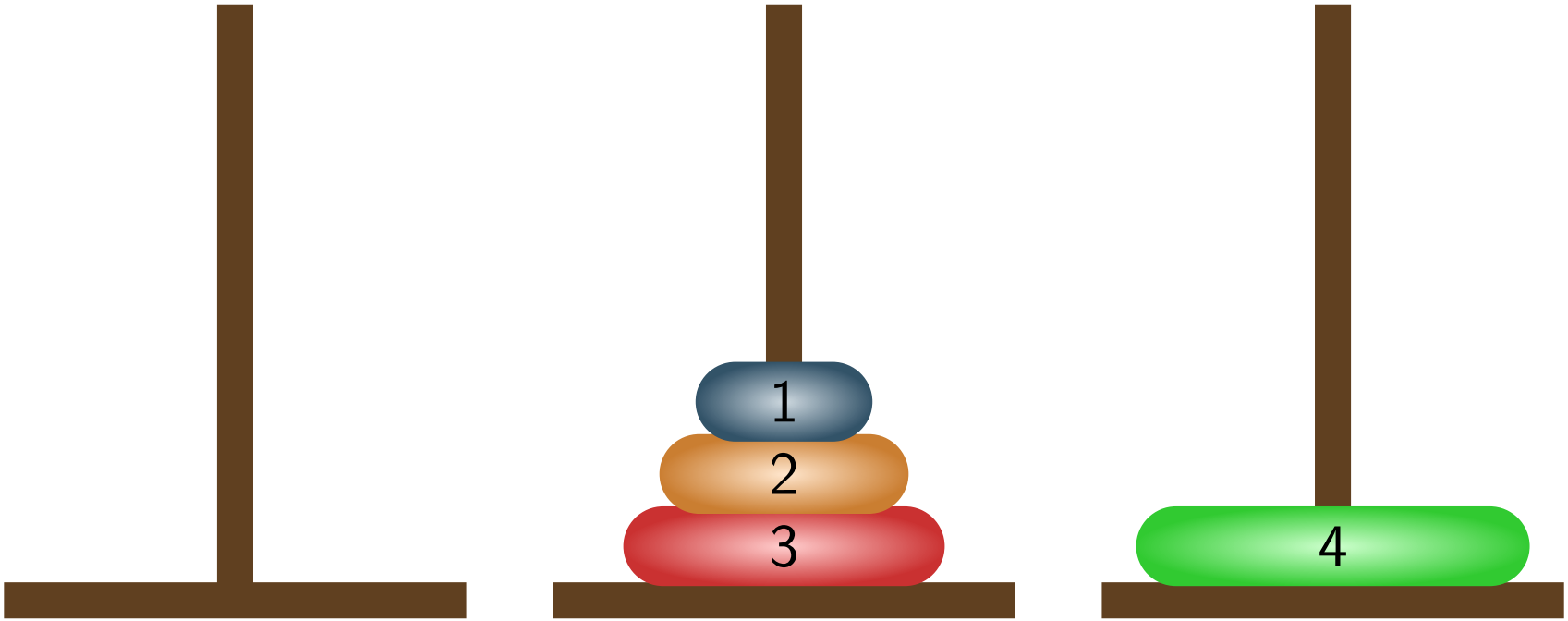


Moved disc from pole 1 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

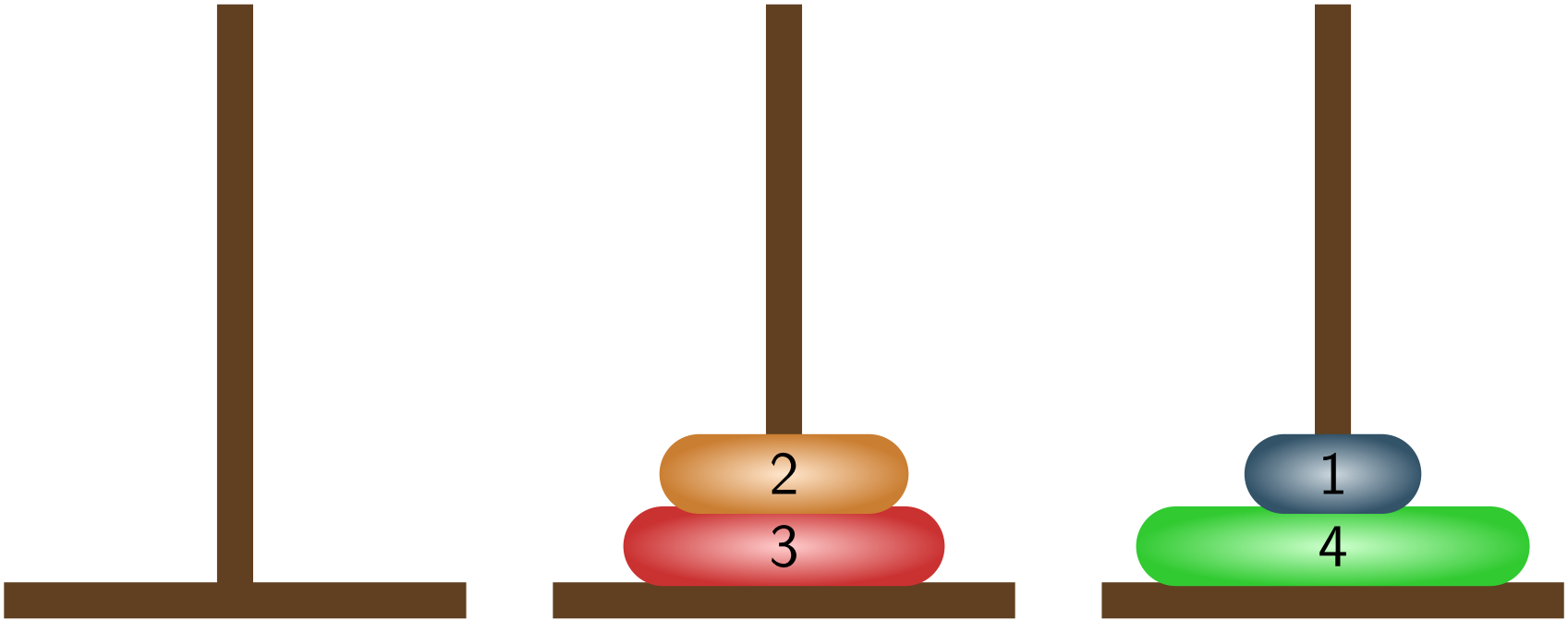


Moved disc from pole 1 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

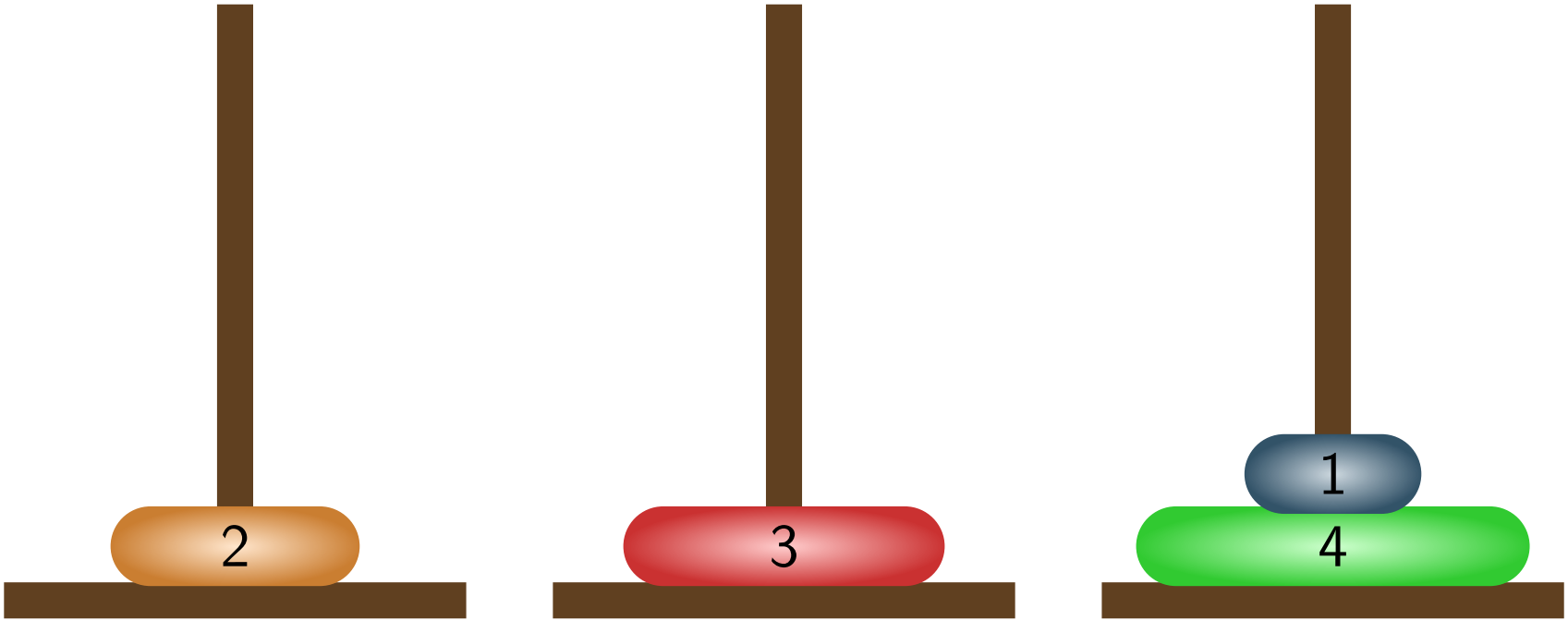


Moved disc from pole 2 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs



Moved disc from pole 2 to pole 1.

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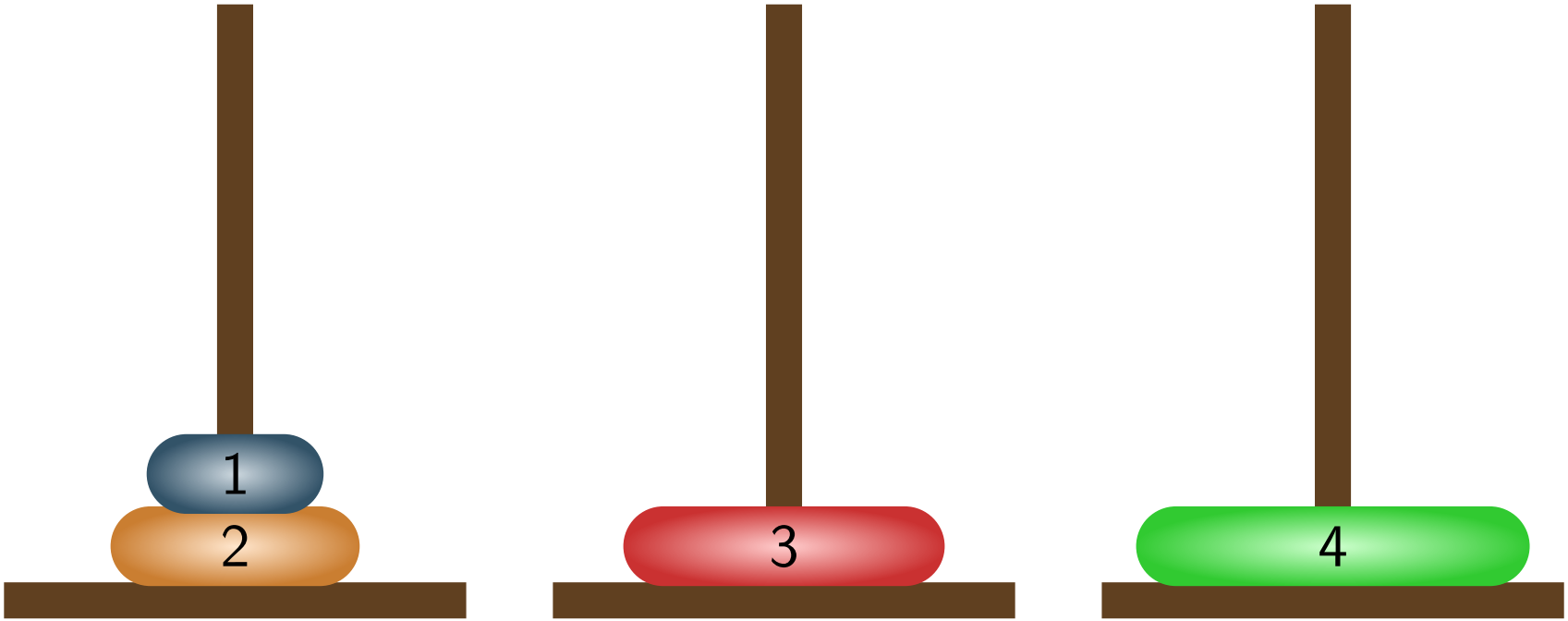
Tower of Hanoi – 4 Discs

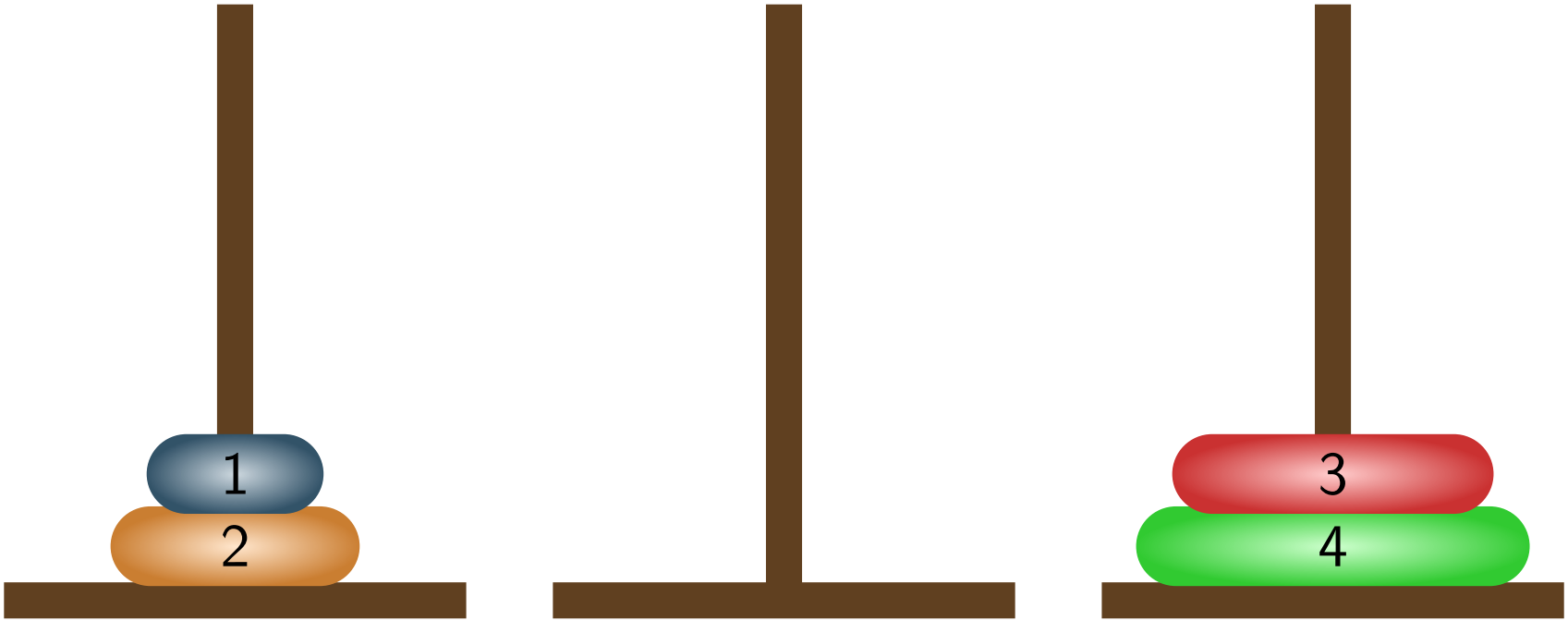


Moved disc from pole 3 to pole 1.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

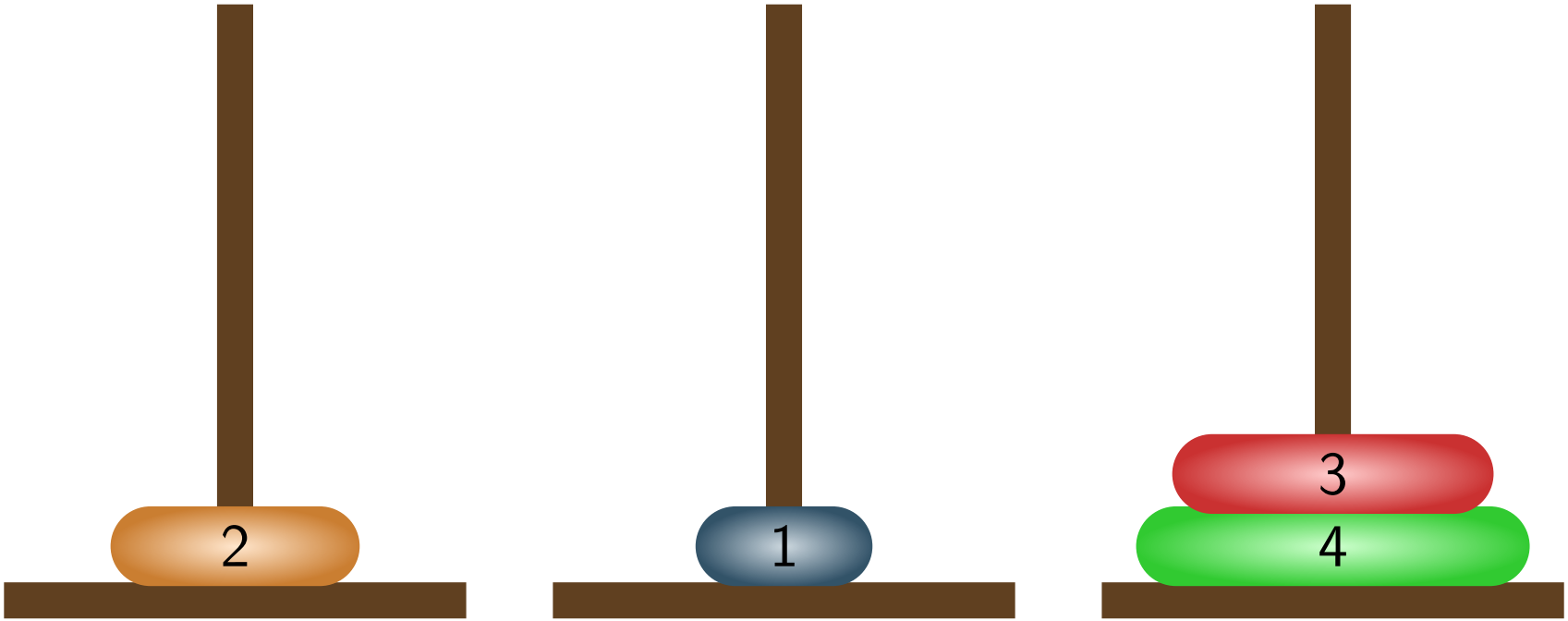


Moved disc from pole 2 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

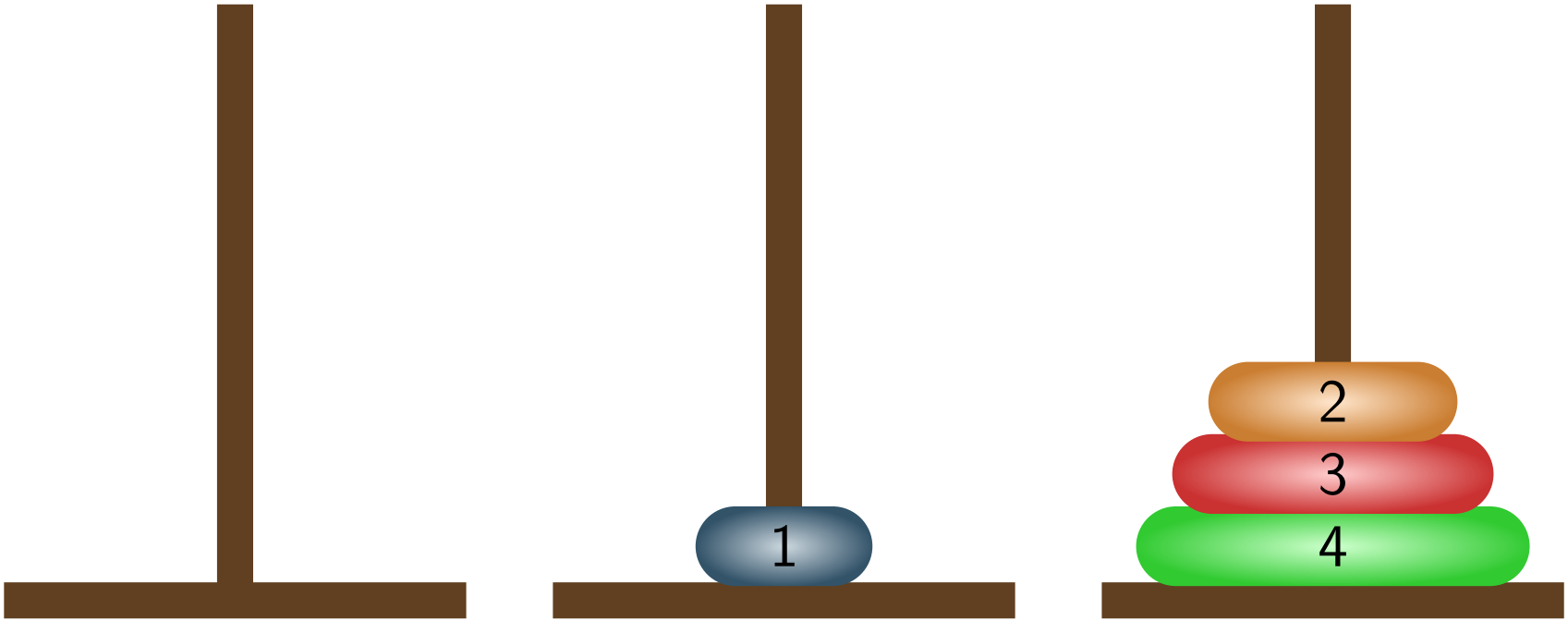


Moved disc from pole 1 to pole 2.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

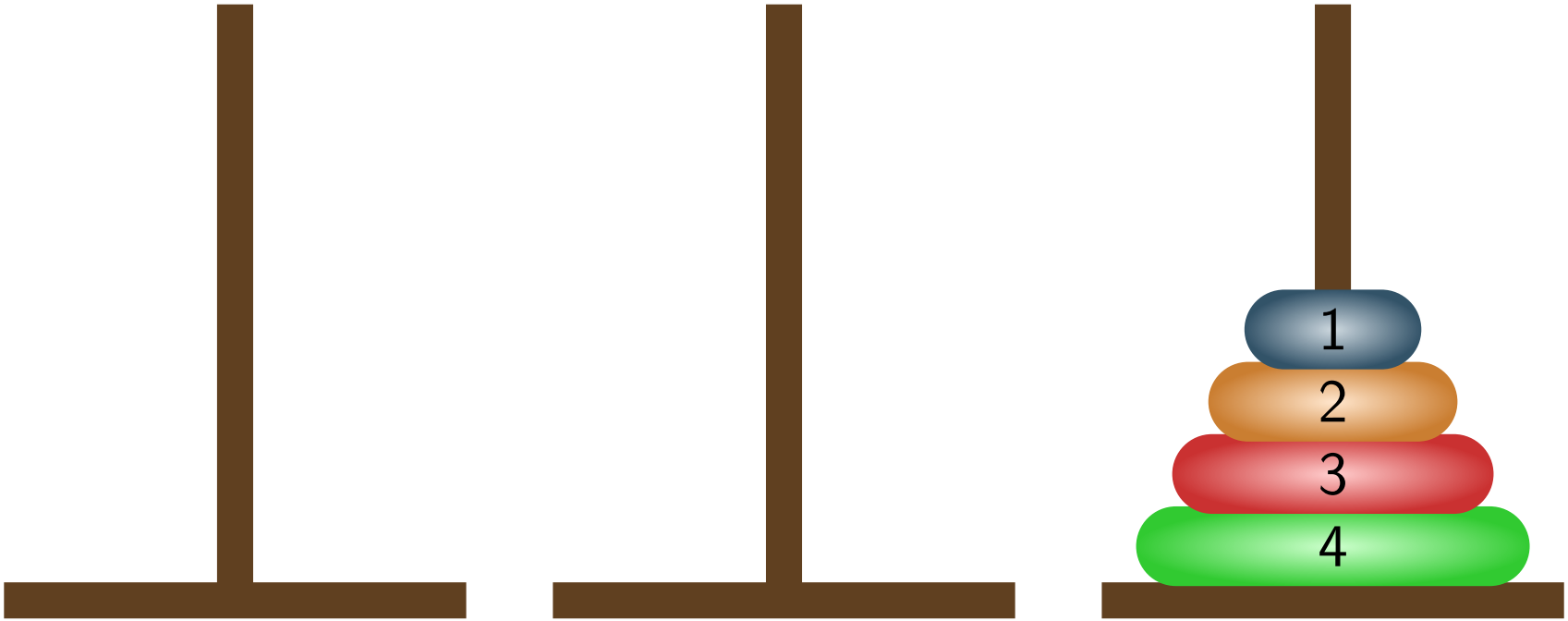


Moved disc from pole 1 to pole 3.

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs



Moved disc from pole 2 to pole 3.

Taken from

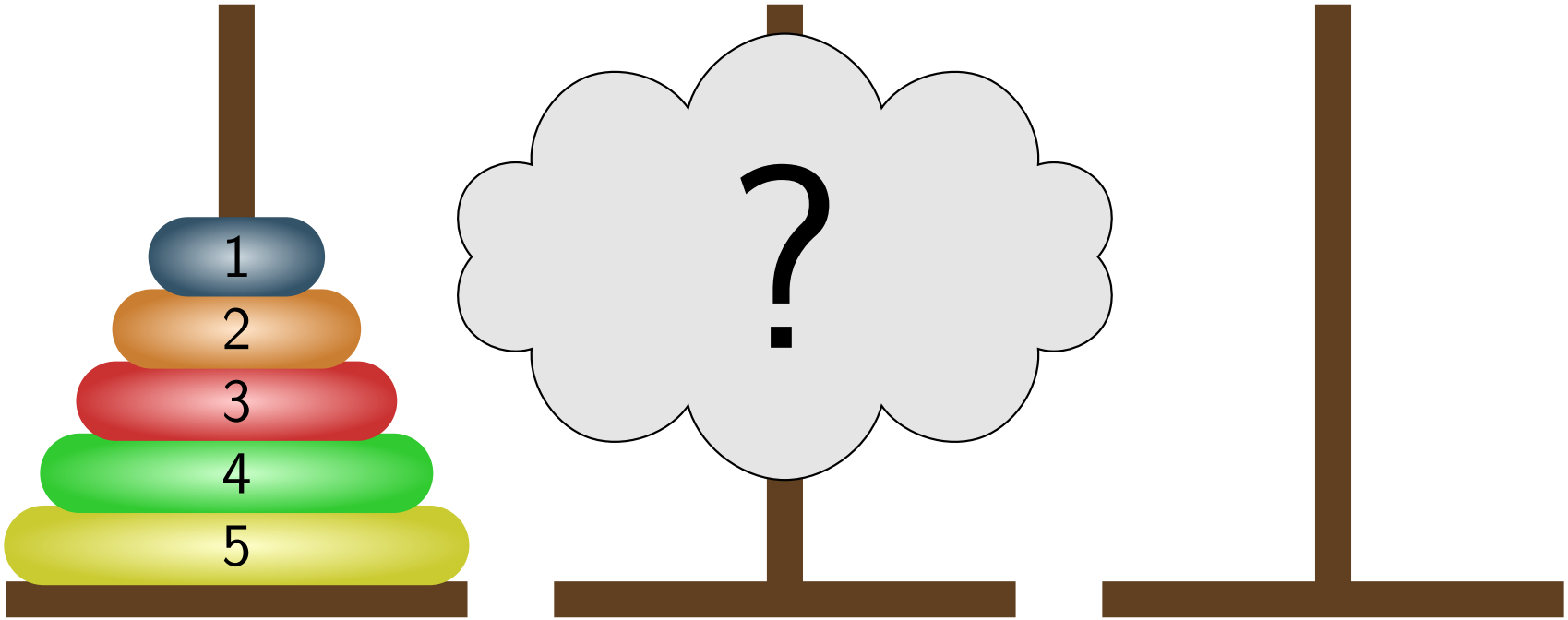
`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 4 Discs

Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Tower of Hanoi – 5 Disc



Taken from

`http://www.texample.net/tikz/examples/towers-of-hanoi/`

Towers of Hanoi

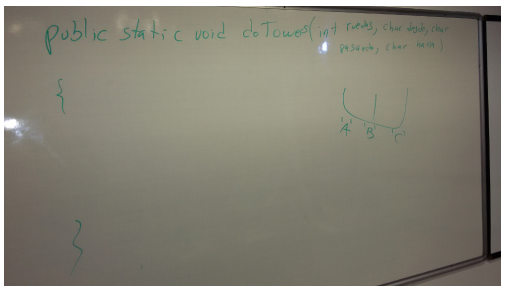


Figure: Algorithm

Towers of Hanoi

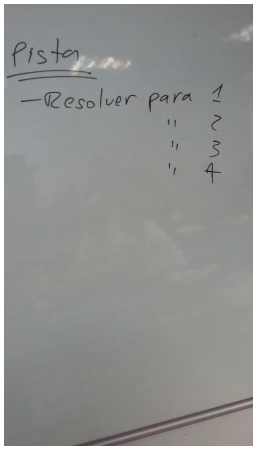


Figure: Algorithm

Towers of Hanoi

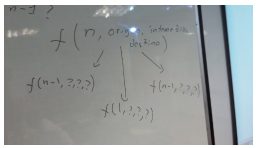


Figure: Algorithm

Towers of Hanoi

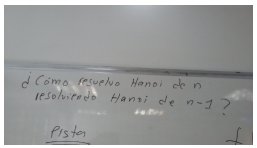


Figure: Algorithm

Towers of Hanoi

```
public static void t(int topN, char from,
                     char inter, char to) {
 if (topN == 1)
  print("Disk␣1␣from␣" + from + "␣to␣" + to);
 else {
  t(topN - 1, from, to, inter);
  print("Disk␣" + topN + "␣from␣" + from +
        "␣to␣" + to);
  t(topN - 1, inter, from, to);
}
```

Complexity of Hanoi Towers

- $T(1) = 1$
- $T(n) = 2T(n-1), n > 1$
- $T(n) = 2^n - 1$
- $T(n)$ is $O(2^n)$

Complexity of Hanoi Towers

- If the monks move discs at the rate of one per second, it would take
  1. more than 31 years to finish a 30-disc problem
  2. more than 348 centuries for them to finish a 40-disc problem
  3. more than 5.8 billion centuries to solve the 64-disc problem

Taken from
`http://introcs.cs.princeton.edu/java/23recursion/`

Recursion over linked lists

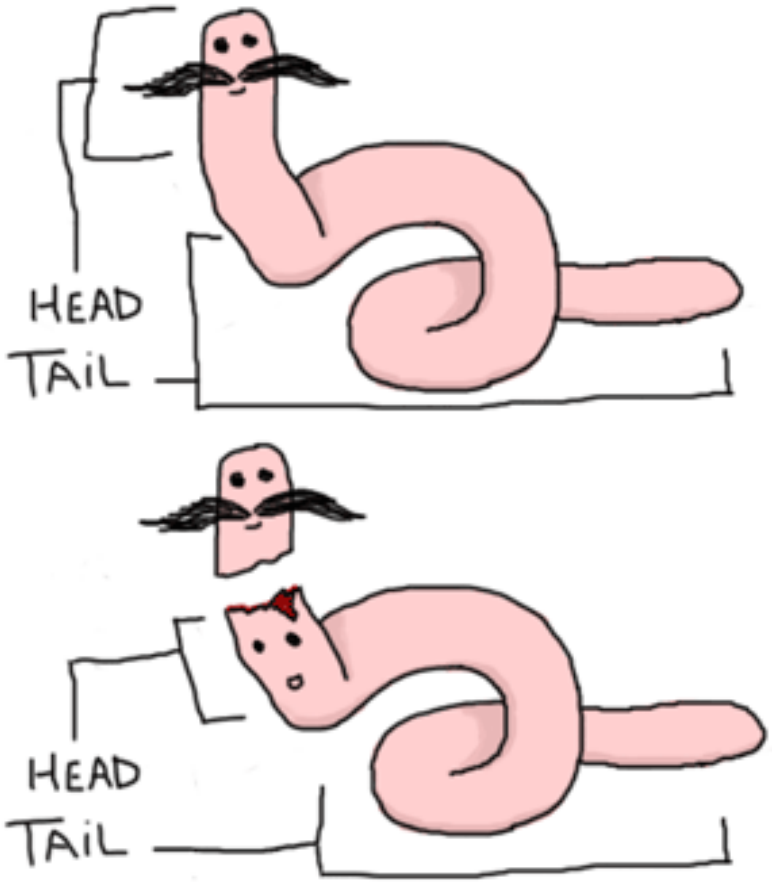


Figure: Tail/Head analogy

Recursion over linked lists

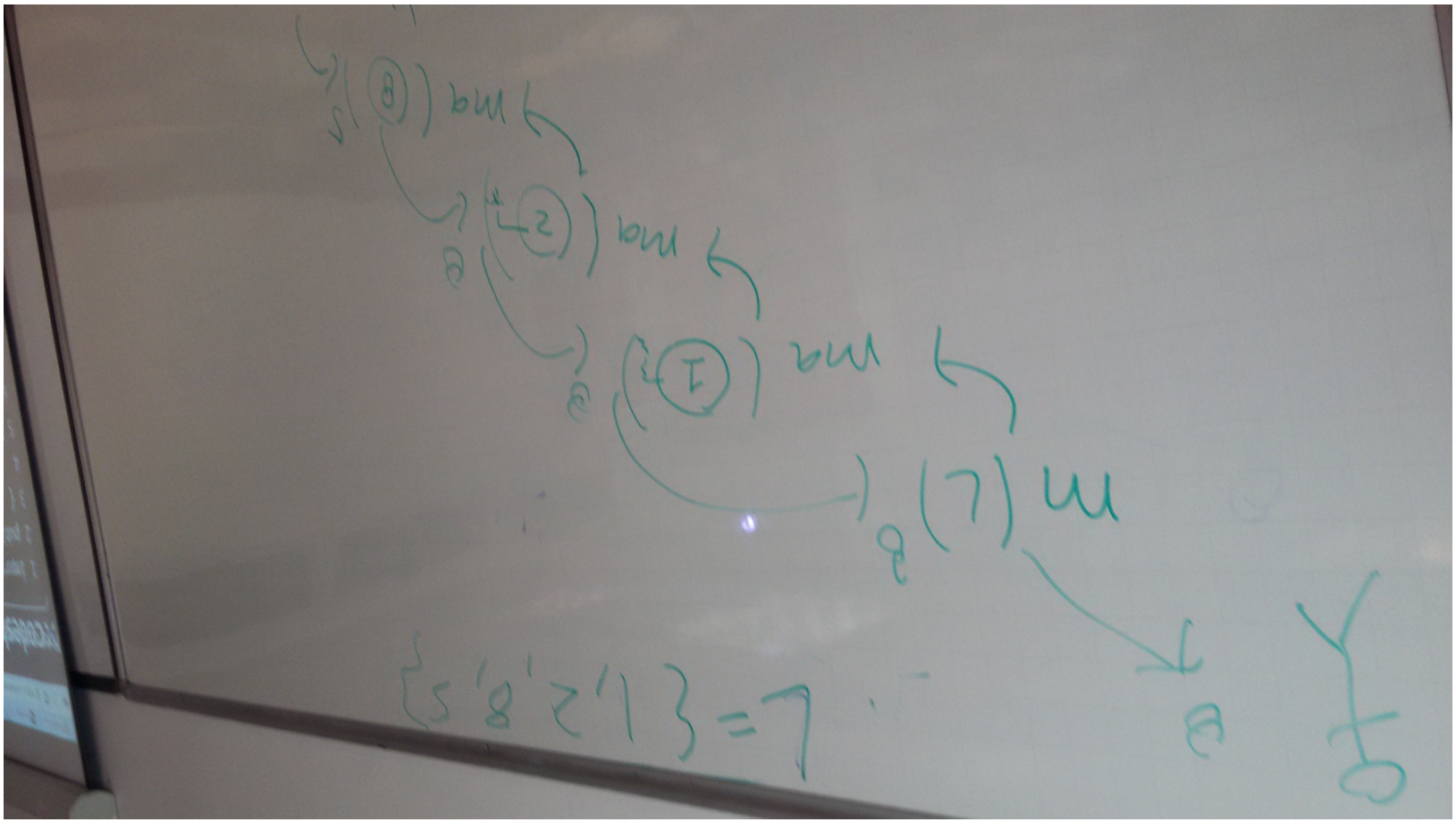


Figure: Example

Quiz questions

- A recursive method is a method that call itself.
- The complexity of recursive factorial is $O(n)$.
- The complexity of solving the towers of Hanoi is $O(2^n)$.

References

- Please learn how to reference images, trademarks, videos and fragments of code.
- Avoid plagiarism

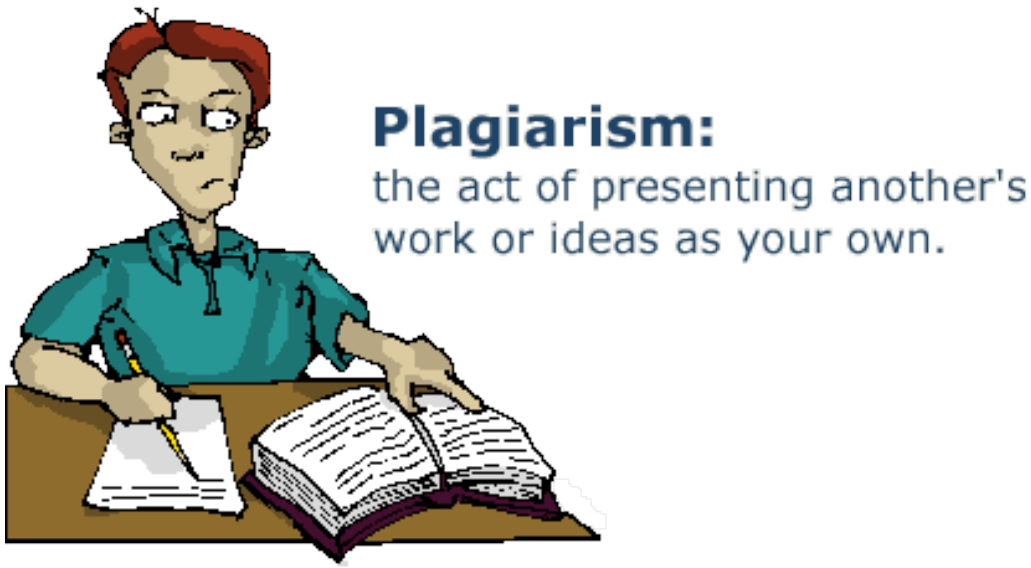


Figure: Figure about plagiarism, University of Malta [Uni09]

References

University of Malta.

Plagarism — The act of presenting another's work or ideas as your own, 2009.

[Online; accessed 29-November-2013].

Further reading

- Recursion
  - http://introcs.cs.princeton.edu/java/23recursion/