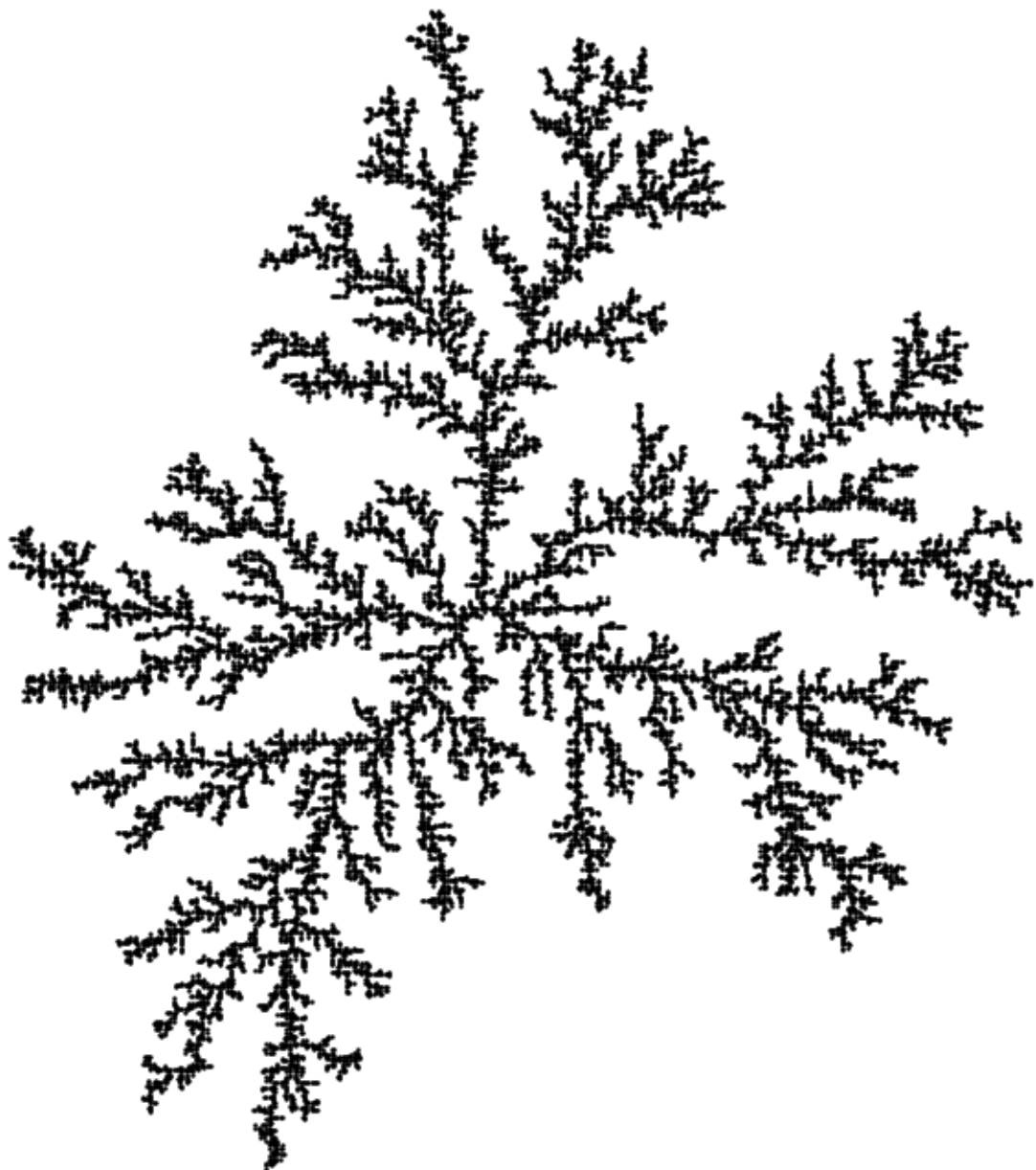


DIFFUSION LIMITED AGGREGATION



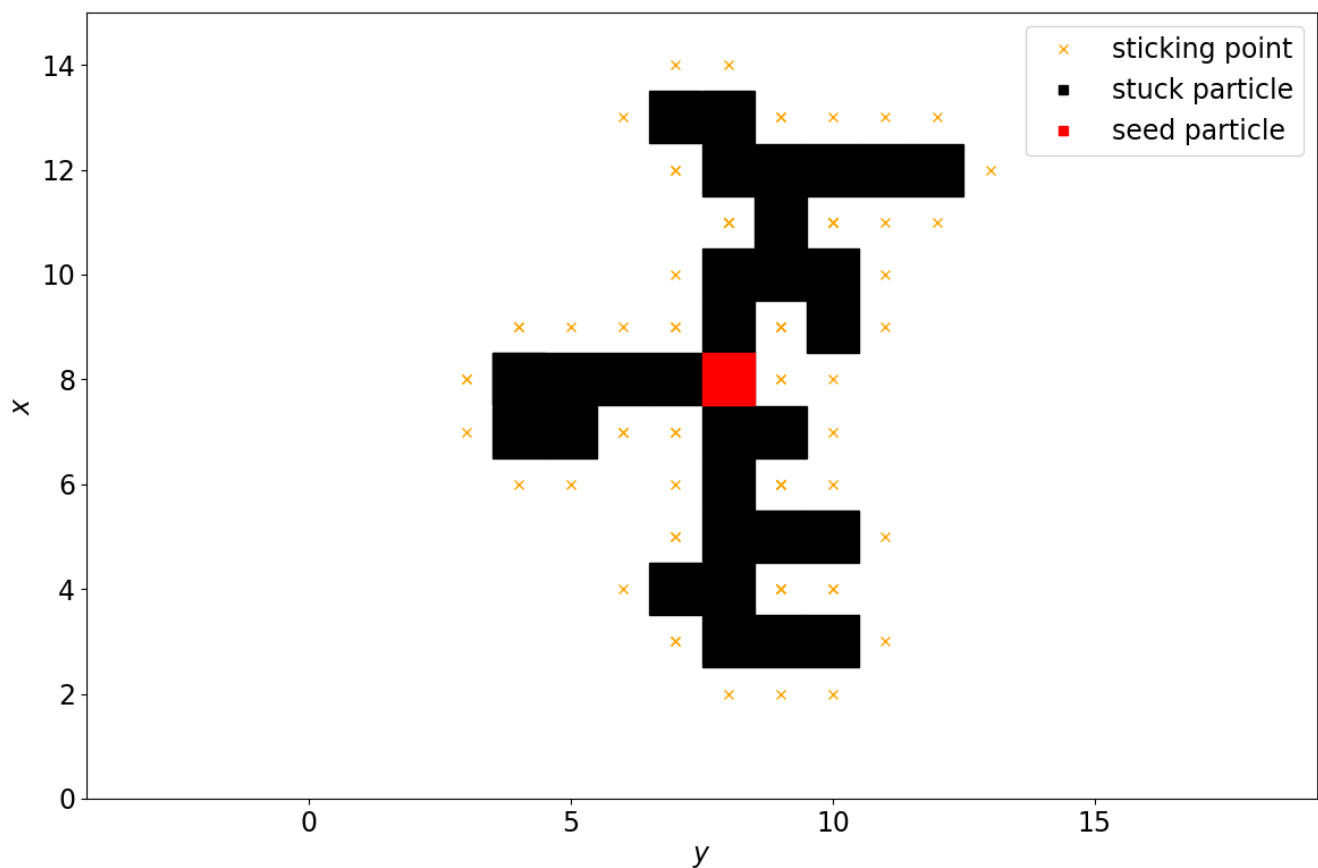


Figure 1: This figure shows a cluster made from the diffusion of particles onto a seed particle placed at the centre of a periodic 16x16 lattice. 30 particles were injected from the edge of the lattice and performed a random walk until hitting a sticking point, this being adjacent the seed or any particle in the cluster. The seed particle is denoted as a red square, stuck particles as black squares and orange crosses as sticking points. Some sticking points are inaccessible as particles would stick before reaching them.

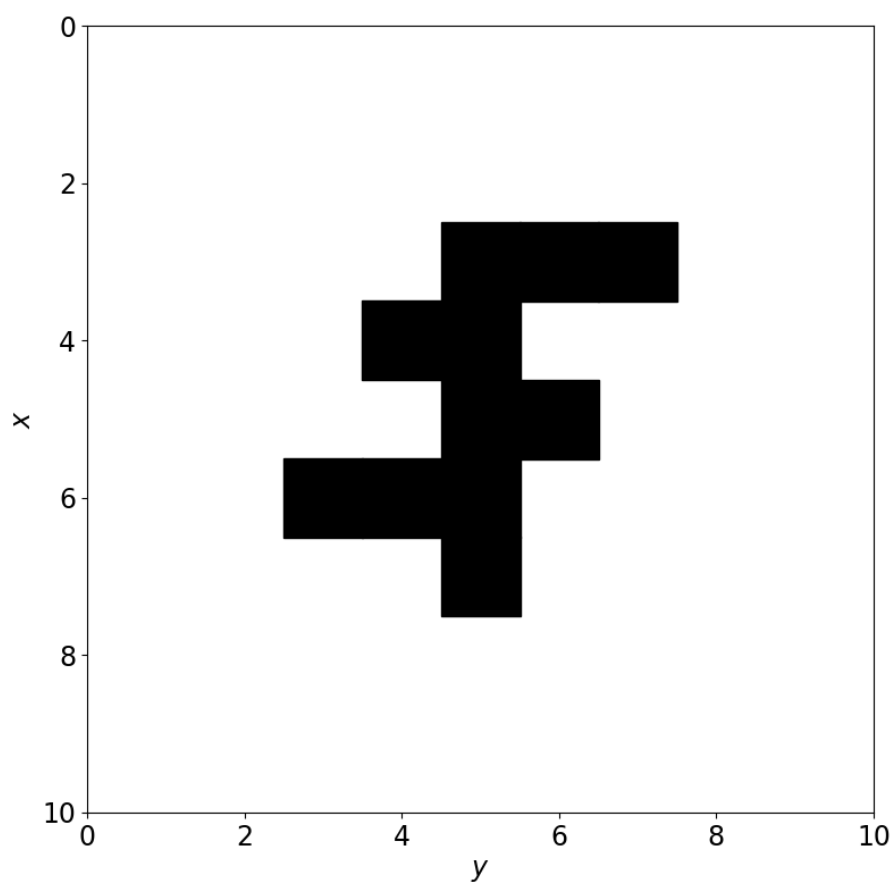


Figure 2: Reproduction of the test case. Particles were injected at given points in a periodic lattice and performed a random walk, from a given seed, to form the cluster in the figure.

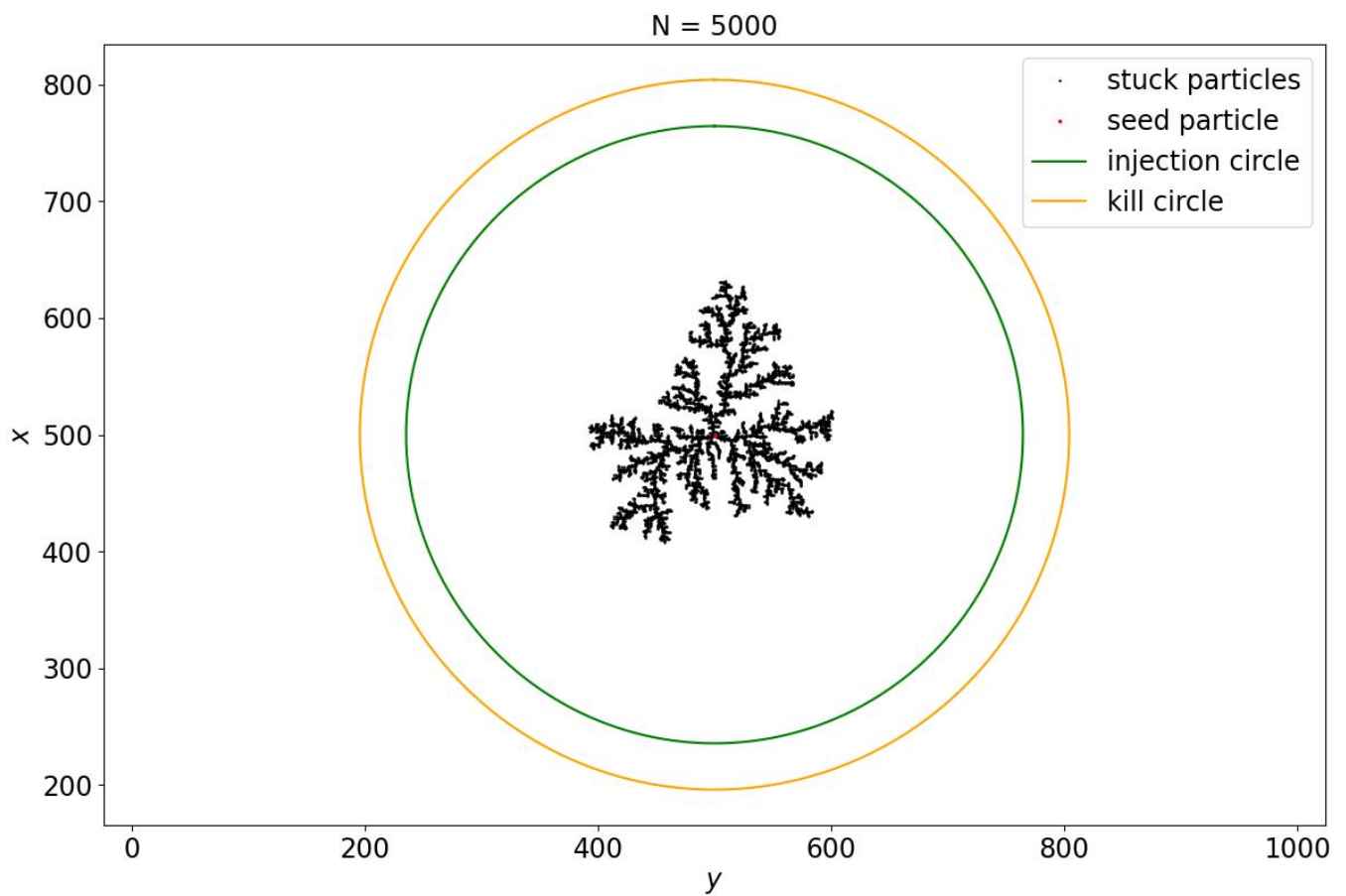


Figure 3: Large scale representation of the diffusion of particles onto a seed particle, shown in red. The cluster shows 5000 stuck particles (excluding the seed) that were injected from the injection circle (green) and performed a random walk until hitting a sticking point. The kill circle (orange) removes the particle from the lattice, to prevent it from moving too far away, then reinjects it back into the injection circle. The radii of these circles increase relatively with the radius of the cluster to simulate the particle being injected from infinity (injection circle) or moving away to infinity (kill circle).

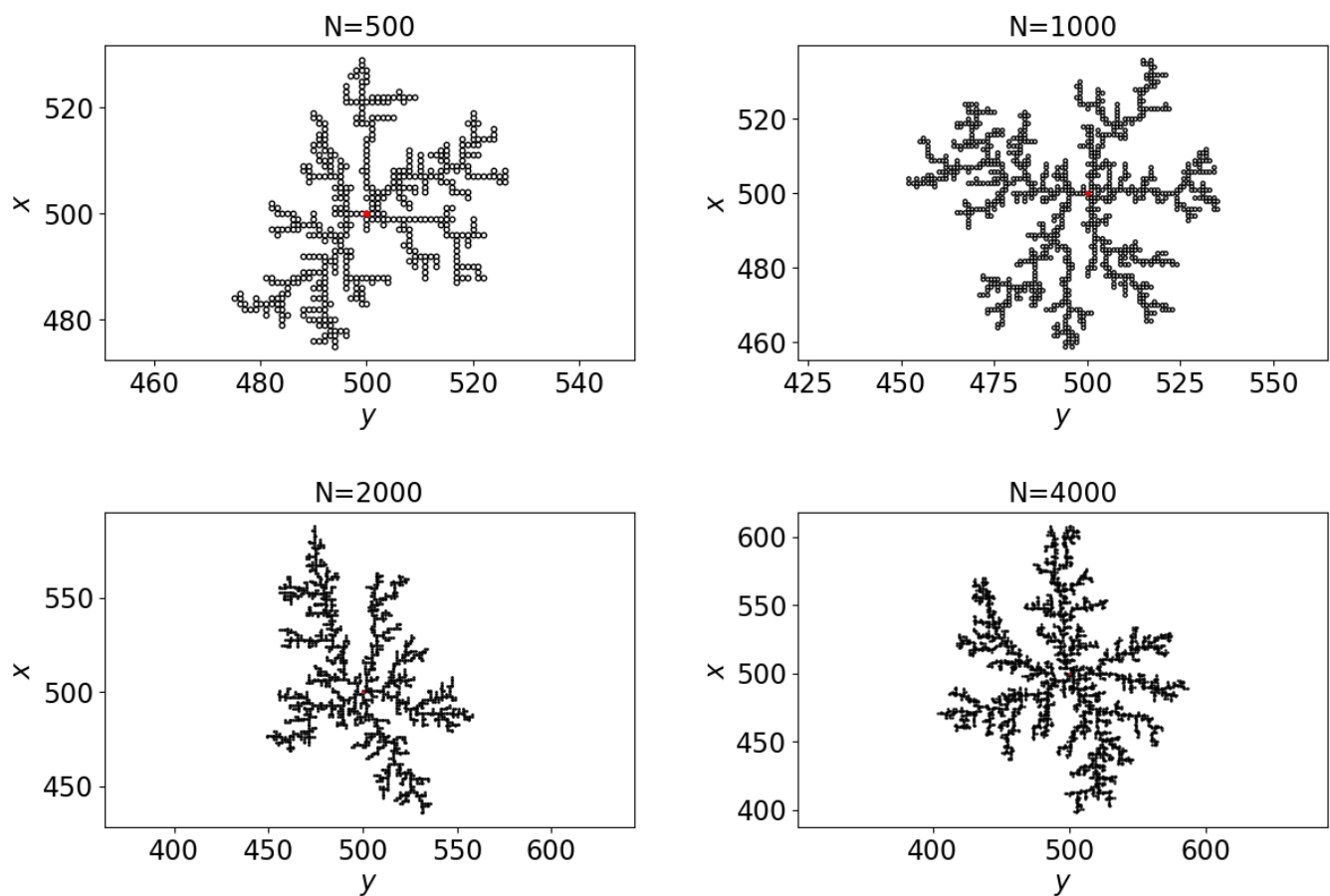


Figure 4: Subplots to show the evolution of the size and appearance of clusters for 500, 1000, 2000 and 4000 particles. This was used to estimate the number of particles sufficient for finding accurate data. 2000 particles should be sufficient as the particles are not visible in contrast to the cluster, also branches of the cluster are more prominent.

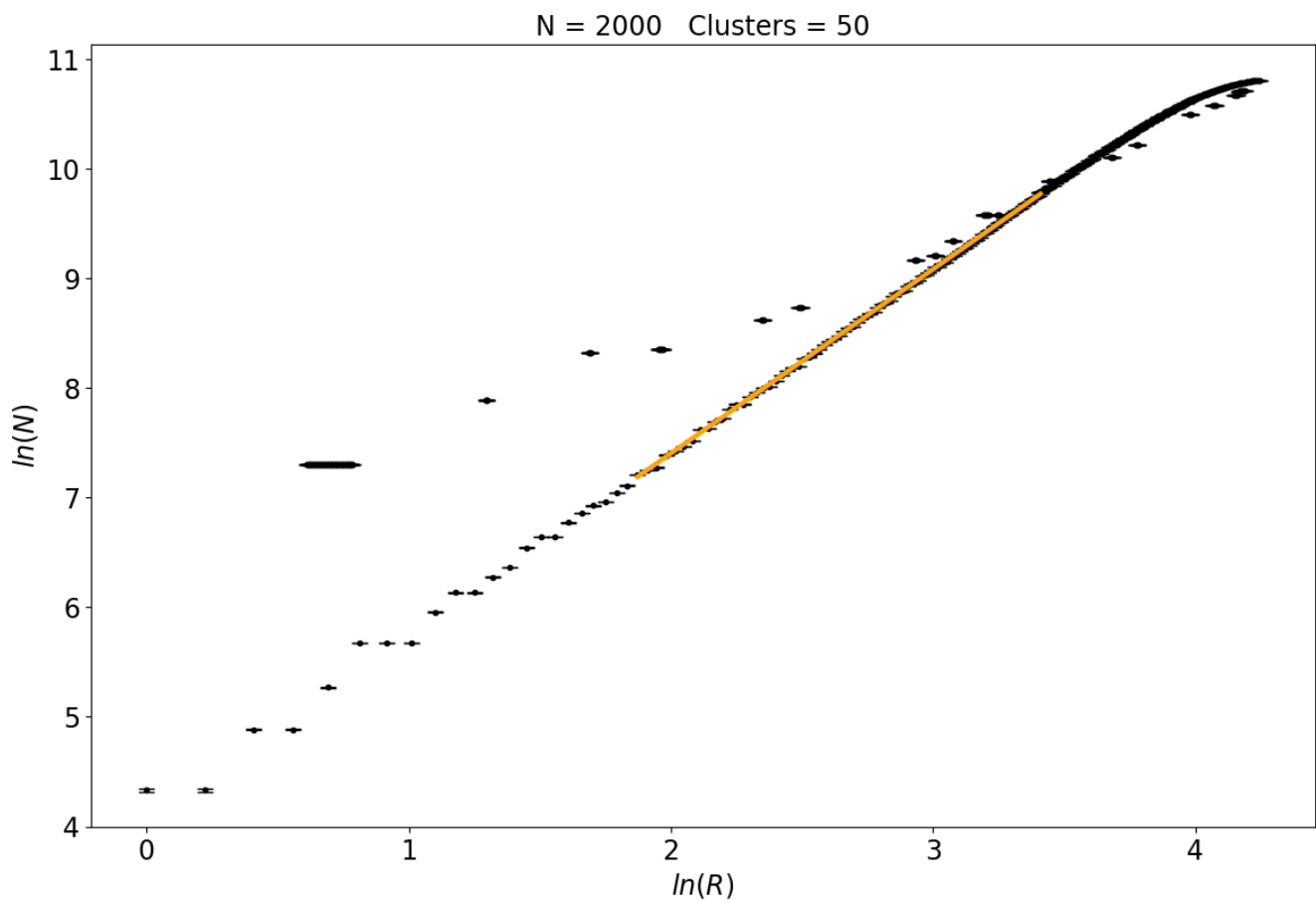


Figure 5: Plot of $\ln(N)$ against $\ln(R)$ to find the fractal dimension, which describes how the number of particles, within a circle, varies with the radius, taking the seed to be the centre. Values of $N \pm 1.5$ were averaged over 50 clusters, to reduce uncertainty, for 2000 particles each. Using the most linear part of the graph, the fractal dimension found was $d_f = 1.68$. This follows the expected result from the reference paper, 'Tenti JM, Hernández Guiance SN, Irurzun IM. Fractal dimension of diffusion-limited aggregation clusters grown on spherical surfaces. Phys Rev E. 2021 Jan;103(1-1):012138. doi: 10.1103/PhysRevE.103.012138. PMID: 33601584'.

The outliers on the plot are a result of some clusters having larger radii than others, meaning they are not averaged correctly. These were not included when calculating the gradient.

$N = 5000$

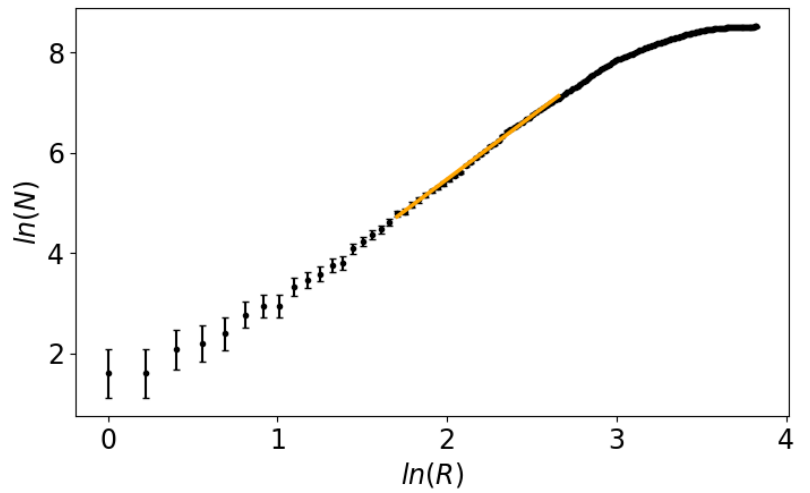
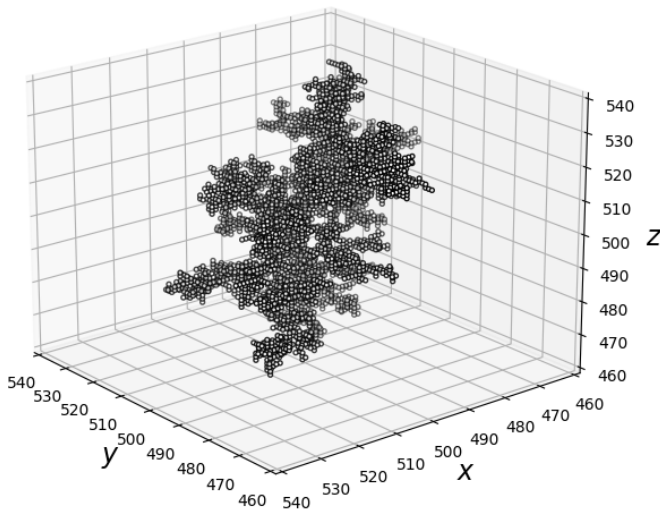


Figure 6: Plot of a 3D cluster (left) to investigate how the fractal dimension varies for an extra dimension. An injection and kill sphere were used, which function similarly to the injection and kill circle. 5000 particles were injected for this model and an uncertainty of $N \pm 1.5$ was used. The fractal dimension from the $\ln(N)$ against $\ln(R)$ plot (right) was found to be $d_f = 2.52$. This is as expected as there will be more available sticking points than clusters with less dimensions, giving a denser cluster.

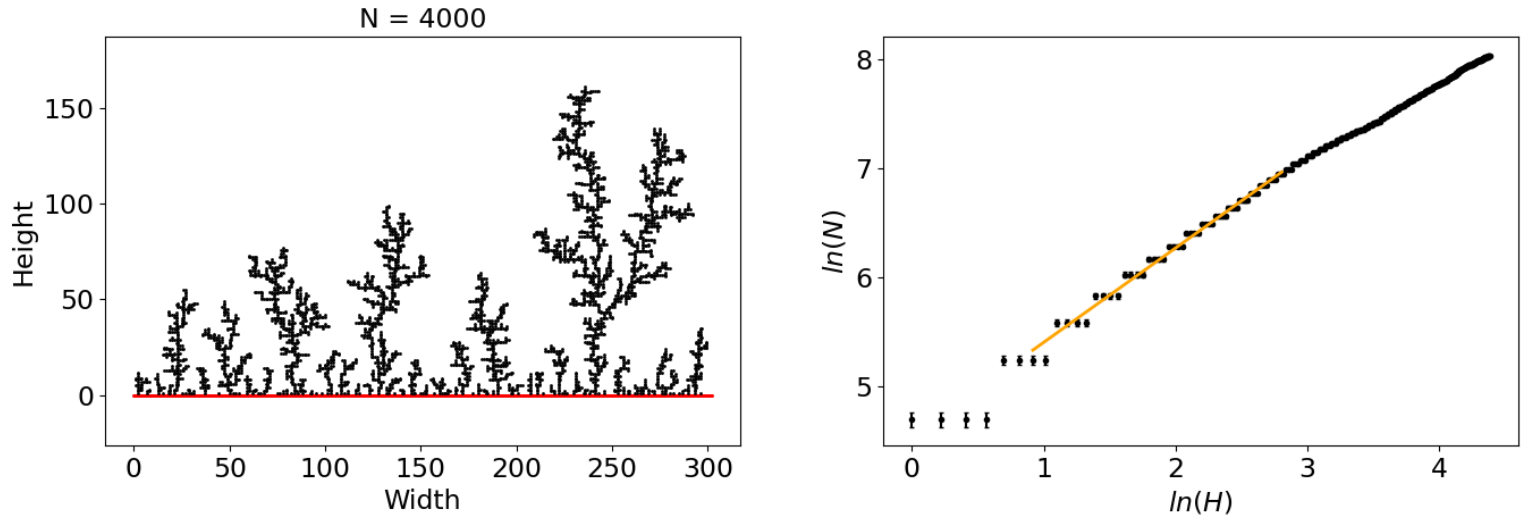


Figure 7: Plot of a line seed (left) to investigate the effect the initial seed has on the fractal dimension. Particles were injected at a height above the seed and performed a random walk until hitting a sticking point. The line seed was modelled for 4000 particles in a periodic lattice. An uncertainty of $N \pm 1.5$ was used and the fractal dimension from the $\ln(N)$ against $\ln(H)$ (right) plot was found to be $d_f = 0.87$. This can be seen in the line seed plot where the larger trees start to dominate as they collect more particles due to more sticking points, decreasing the number of particles stuck per unit height.

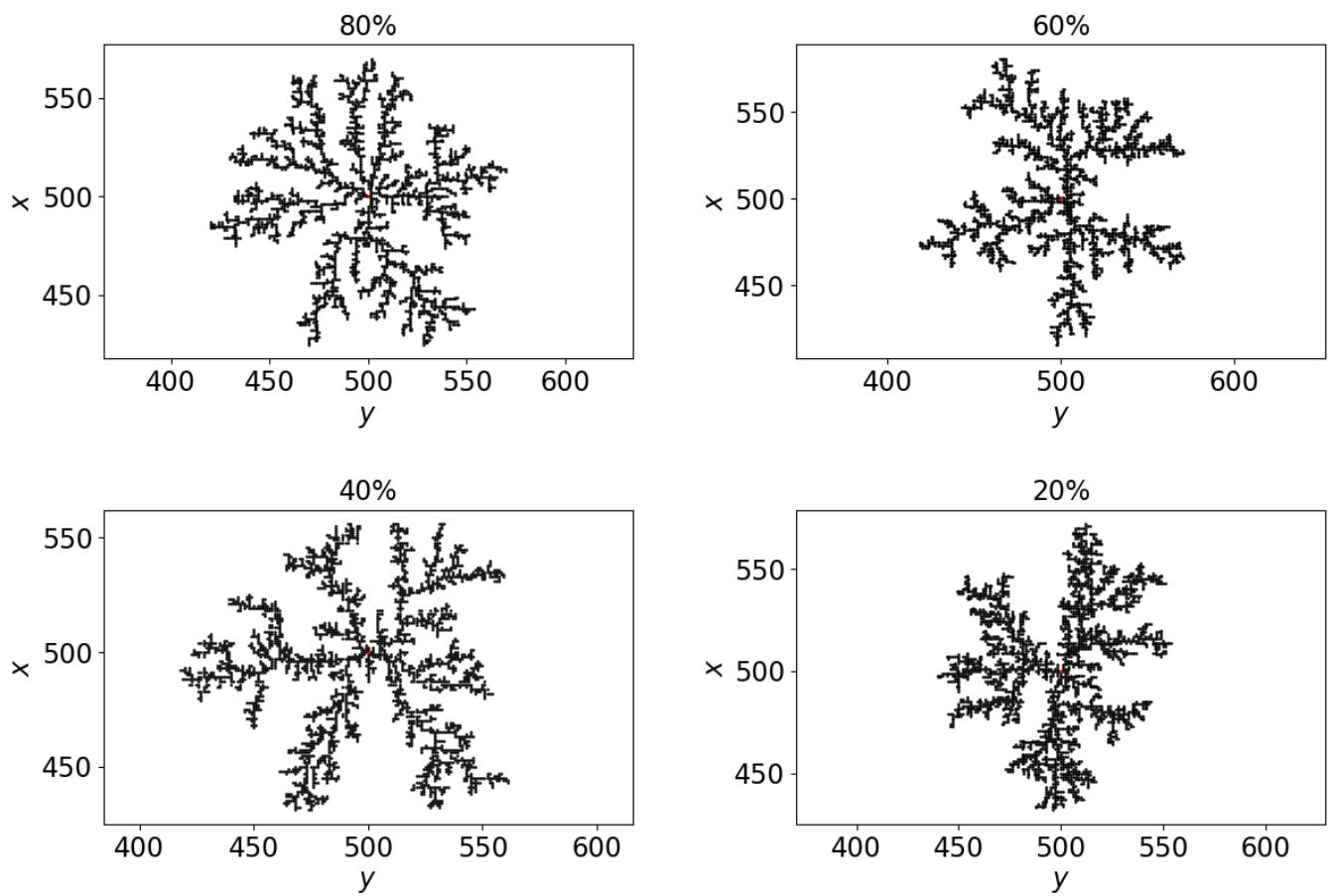


Figure 8: Sticking probability subplots to investigate a more realistic model for cluster growth, where particles do not always stick when hitting a sticking point. Sticking probabilities of 80%, 60%, 40% and 20% were plotted for clusters of 3000 particles. Analyzing this figure visually, it becomes clear that the fractal dimension would increase for a decrease in sticking probability, due to particles clumping closer to the centre. The figures for 80%, 60% and 40% sticking probability do not show much difference, however, the 20% figure shows prominent clumping and less obvious branches.