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HW3
MG6392

Question 7

a) 3.1.1 a - g

a) $27 \in A$
True. 27 is an integer multiple of 3 : 3 * 9 = 27

b) $27 \in B$
False. There is no integer x such that $x^2 = 27$.

c) $100 \in B$
True. $10^2 = 100$.

d) $E \subseteq C$ or $C \subseteq E$
False. $E \subseteq C$ is false as $3 \in E$ but 3 does not belong to C. $C \subseteq E$ is false as $4 \in C$ but does not belong to E

e) $E \subseteq A$
True. 3, 6, and 9 are all integer multiples of 3 (3*1, 3*2, 3*3)

f) $A \subset E$
False. Many counterexamples, including 15.

g) $E \in A$
False. E is a set, and only elements belong to A, not sets

b) 3.1.2 a - e

a) $15 \subset A$
False. 15 is not a set

b) $\{15\} \subset A$
True. {15} belongs to A as 3*5 = 15 and there are other elements in A, such as 12, that do not belong to {15}

c) $\varnothing \subset C$
True. This is always true.

d) $D \subseteq D$
True.

e) $\varnothing \in B$
False. $\varnothing$ is a set while the elements of B are not sets, rather integers

c) 3.1.5 b, d

b) $\{ x \in \mathbb{N} :$ x is an integer multiple of 3$\}$ - Infinite set
d) $\{ x \in \mathbb{N} : x <= 1000$ and x is an integer multiple of 10$\}$ - Cardinality = 101

d) 3.2.1 a - k

a) $2 \in X$
True

b) $\{2\} \subseteq X$
True as $2 \in X$

c) $\{2\} \in X$
False as the set $\{2\}$ is not an element of X

d) $3 \in X$
False as the element 3 does not belong to X, only the set $\{3\}$

e) $\{1, 2\} \in X$
True, the set of $\{1,2\}$ is an element of X

f) $\{1, 2\} \subseteq X$
True, as the subset of elements 1 and 2 is a subset of X

g) $\{2, 4\} \subseteq X$
True, as the subset of elements 2 and 4 is a subset of X

h) $\{2, 4\} \in X$
False, this set is not an element in X

i) $\{2, 3\} \subseteq X$
False, only $\{2, \{3\}\}$ would be a subset of X. The element 3 does not belong to X

j) $\{2, 3\} \in X$
False, this element does not appear in X

k) $|X| = 7$
False. $|X| = 6$

Question 8

3.2.4b

Let A = {1, 2, 3}. What is $\{X \in P(A): 2 \in X\}$?

Define x such that X is an element in the power set P(A) where 2 belongs to X.

P(A) = {empty, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3} }

$\{X \in P(A): 2 \in X\}$ = { {2}, {1,2}, {2,3}, {1,2,3} }

Question 9

a) 3.3.1 c - e

c) $A \cap C$
{-3, 1, 17}

d) $A \cup (B \cap C)$
$A \cup \{-5, 1, 6\} = \{-3, 0, 1, 4, 17\} \cup \{-5, 1, 6\}$
{-5, -3, 0, 1, 4, 17}

e) $A \cap B \cap C$
$A \cap C \cap B$, Commutative. Use $A \cap C$ from part (c)
$\{-3, 1, 17\} \cap \{-12, -5, 1, 4, 6\}$
{1}

b) 3.3.3 a, b, e, f

a) $A_2 \cap A_3 \cap A_4 \cap A_5$
$\{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\}$
{1}

b) $A_2 \cup A_3 \cup A_4 \cup A_5$
$\{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\}$
{1, 2, 3, 4, 5, 9, 16, 25}

e) $\{x \in \mathbf{R} : -1/1 <= x <= 1/1\} \cap$
$\{x \in \mathbf{R} : -1/2 <= x <= 1/2\} \cap$
…
$\cap \{x \in \mathbf{R} : -1/100 <= x <= 1/100\}$
$= \{x \in \mathbf{R} : -1/100 <= x <= 1/100\}$, as this subset is inside of all the sets

f) $\{x \in \mathbf{R} : -1/1 <= x <= 1/1\} \cup$
$\{x \in \mathbf{R} : -1/2 <= x <= 1/2\} \cup$
…
$\cup \{x \in \mathbf{R} : -1/100 <= x <= 1/100\}$
$= \{x \in \mathbf{R} : -1/1 <= x <= 1/1\}$, as this is the "widest" set that encompasses all the other subsets.

c) 3.3.4 b, d

b) $P(A \cup B)$
$P(\{a, b, c\})$
$\{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

d) $P(A) \cup P(B)$
$P(A) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$
$P(B) = \{\varnothing, \{b\}, \{c\}, \{b, c\}\}$
$P(A) \cup P(B) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Question 10

a) 3.5.1 b, c

b) Write an element from the set $B \times A \times C$.

{ foam, tall, non-fat }

c) Write the set $B \times C$ using roster notation.

{ {foam, non-fat}, {foam, whole}, {no-foam, non-fat}, {no-foam, whole} }

b) 3.5.3 b, c, e

b) $\mathbf{Z}^2 \subseteq \mathbf{R}^2$

True. A set consisting of all {Integer, Integer} will be a subset of a set consisting of all {Real number, Real number}

c) $\mathbf{Z}^2 \cap \mathbf{Z}^3 = \varnothing$

True. A set of 2-tuples will have no overlap with a set of 3-tuples.

e) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$.

True. If $(x,y) \in A \times C$, then $x \in A$ and $y \in C$. Since $A \subseteq B$, x is an element of B as well. Therefore $(x, y) \in B \times C$.

c) 3.5.6 d, e

d) $\{xy\text{: where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$

{01, 011, 001, 0011}

e) $\{xy\text{: } x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

{aaa, aaaa, aba, abaa}

d) 3.5.7 c, f, g

c) $(A \times B) \cup (A \times C)$

$\{ab, ac\} \cup \{aa, ab, ad\}$

{aa, ab, ac, ad}

f) $P(A \times B)$

P({ab, ac})

$\{\varnothing, \{ab\}, \{ac\}, \{ab, ac\} \}$

g) $P(A) \times P(B)$

$P(A) = \{\varnothing, \{a\}\}$

$P(B) = \{\varnothing, \{b\}, \{c\}, \{bc\}\}$

$P(A) \times P(B) = \{ (\varnothing, \varnothing), (\varnothing, \{b\}), (\varnothing, \{c\}), (\varnothing, \{bc\}),$
$(\{a\}, \varnothing), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{bc\}) \}$

Question 11

a) 3.6.2 b, c

b) 1. $(B \cup A) \cap (\overline{B} \cup A) = A$

2. $(A \cup B) \cap (\overline{B} \cup A)$, Commutative 1

3. $(A \cup B) \cap (A \cup \overline{B})$, Commutative 2

4. $A \cup (B \cap \overline{B})$, Distributive 3

5. $A \cup \varnothing$, Complement Law 4

6. A, Identity Law

c) 1. $\overline{A \cap \overline{B}} = \overline{A} \cup B$

2. $\overline{A} \cup \overline{\overline{B}}$, DeMorgan's 1

3. $\overline{A} \cup B$, Double Complement 2

b) 3.6.3 b, d

b) $A - (B \cap A) = A$

When $A = \{1, 2, 3\}$ and $B = \{1, 2, 4\}$, $B \cap A = \{1, 2\}$. $\{1, 2, 3\} - \{1, 2\} = \{3\}$ which is not equal to A

d) $(B - A) \cup A = A$

When $B = \{1, 2, 4\}$ and $A = \{1, 2, 5\}$, $(B-A) = \{4\}$. $\{4\} \cup \{1, 2, 5\} = \{1, 2, 4, 5\}$ which is not equal to A ($\{1, 2, 5\}$).

c) 3.6.4 b, c

b) 1. $A \cap (B - A) = \varnothing$

2. $A \cap (B \cap \overline{A})$, Subtraction Law 1

3. $A \cap (\overline{A} \cap B)$, Commutative 2

4. $(A \cap \overline{A}) \cap B$, Associative 3

5. $\varnothing \cap B$, Complement Law 4

6. $\varnothing$, Domination Law 5

c) 1. $A \cup (B - A) = A \cup B$

2. $A \cup (B \cap \overline{A})$, Subtraction Law 1

3. $(A \cup B) \cap (A \cup \overline{A})$, Distributive 2

4. $(A \cup B) \cap U$, Complement Law 3

5. $(A \cup B)$, Identity Law 4