Max Grove
MG6392
HW 6
Question 5

a) Show $5n^3 + 2n^2 + 3n = \text{Theta}(n^3)$

$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$
$5n^3 + 2n^2 + 3n \leq 10n^3$
$5n^3 + 2n^2 + 3n \leq c*n^3$
Thus, for $n_0 = 1$ and $c = 10$, for any $n \geq n_0$, $5n^3 + 2n^2 + 3n = O(n^3)$

$5n^3 + 2n^2 + 3n \geq 5n^3$
$5n^3 + 2n^2 + 3n \geq c*n^3$
Thus, for $n_0 = 1$ and $c = 5$, for any $n \geq n_0$, $5n^3 + 2n^2 + 3n = \text{Omega}(n^3)$

Since $5n^3 + 2n^2 + 3n = O(n^3) = \text{Omega}(n^3)$, $5n^3 + 2n^2 + 3n = \text{Theta}(n^3)$

b) $\text{sqrt}(7n^2 + 2n - 8) = \text{Theta}(n)$

$\text{sqrt}(7n^2 + 2n - 8) \leq \text{sqrt}(7n^2 + 2n) \leq \text{sqrt}(7n^2 + 2n^2)$
$\text{sqrt}(7n^2 + 2n - 8) \leq \text{sqrt}(9n^2)$
$\text{sqrt}(7n^2 + 2n - 8) \leq 3n$
$\text{sqrt}(7n^2 + 2n - 8) \leq cn$
Thus, for $n_0 = 1$ and $c = 3$, for any $n \geq n_0$, $\text{sqrt}(7n^2 + 2n - 8) = O(n)$

$\text{sqrt}(7n^2 + 2n - 8) \geq \text{sqrt}(7n^2 + 2n - 8n)$
$\text{sqrt}(7n^2 + 2n - 8) \geq \text{sqrt}(7n^2 - 6n) \geq \text{sqrt}(7n^2 - 6n^2)$
$\text{sqrt}(7n^2 + 2n - 8) \geq \text{sqrt}(n^2)$
$\text{sqrt}(7n^2 + 2n - 8) \geq n$
$\text{sqrt}(7n^2 + 2n - 8) \geq cn$
Thus, for $n_0 = 1$ and $c = 1$, for any $n \geq n_0$, $\text{sqrt}(7n^2 + 2n - 8) = \text{Omega}(n)$

Since $\text{sqrt}(7n^2 + 2n - 8) = O(n) = \text{Omega}(n)$, $\text{sqrt}(7n^2 + 2n - 8) = \text{Theta}(n)$