Max Grove, HW1 MG6392 e ny v. edv

Question1

A.1) 10011011_2 =

1	0	0	1	1	0	1	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
x 128	64	32	16	8	4	2	1

$128 + 16 + 8 + 2 + 1 = 155_{10}$

A.2) 456_7 =

4	5	6
7^2	7^1	7^0
x 49	7	1

$196 + 35 + 6 = 237_{10}$

A.3) $38A_{16}$ =

3	8	A = 10
16^2	16^1	16^0
x 256	16	1

$768 + 128 + 10 = 906_{10}$

A.4) 214_5

2	2	14
5^3	5^2	5^1 5^0
x 125	25	5 1

$250 + 50 + 5 + 4 = 309_{10}$

B.1) 69_{10} =

0	1	0	0	0	1	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

$64 + 4 + 1 = 69$

$= 1000101_2$

B.2) 485_{10} =

0	1	1	1	1	0	0	1	0	1
2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
512	256	128	64	32	16	8	4	2	1

$256 + 128 + 64 + 32 + 4 + 1 = 485$

$= 111100101_2$

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Question 1, continued

B.3) $6D1A_{16}$ = 0110 1101 0001 1010 = 1101101000110102
(based on table presented in lectures)

C.1) 1101011_2 = 0110 1011 = $6B_{16}$
(based on the table presented in lectures)

C.2) 895_{10} = $\frac{0}{16^3}\ \frac{3}{16^2}\ \frac{7}{16^1}\ \frac{F}{16^0}_{16}$ = $37F_{16}$

4096 256 16 1

$256\overline{)895}$ 3 R127

$16\overline{)127}$ 7 R15

15 = F

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Question 2

1. $75_8 + 45_8$

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ 1} \\ 75_8 \\ +\ 45_8 \\ \hline 143_8 \end{array}$$

2. $10110011_2 + 00001101_2$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1\ 1\ 1\ 1} \\ 10110011_2 \\ +\ 00001101_2 \\ \hline 11000000_2 \end{array}$$

3. $7A66_{16} + 45C5_{16}$

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 7A66_{16} \\ +\ 45C5_{16} \\ \hline C02B_{16} \end{array}$$

4. $3082_5 - 2433_5$

$$\begin{array}{r} {\scriptstyle 2\ 4\ 11\ 1} \\ 3\not{0}\not{8}2_5 \\ -\ 2433_5 \\ \hline 0034_5 \end{array}$$

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Question 3

A.1) 124_{10} = 0 1 1 1 1 1 0 0 = 0111 1100$_{8b2c}$

2^6 2^5 2^4 2^3 2^2 2^1 2^0

64 32 16 8 4 2 1

64+32+16+8+4 = 124

A.2) -124_{10} =

1 1 1 1 1

0 1 1 1 1 1 0 0

+ 0 0 0 0 0 1 0 0 → 1 0 0 0 0 1 0 0$_{8b2c}$

1 0 0 0 0 0 0 0

A.3) 109_{10} = 0 1 1 0 1 1 0 1 = 0110 1101$_{8b2c}$

2^6 2^5 2^4 2^3 2^2 2^1 2^0

64 32 16 8 4 2 1

64+32+8+4+1 = 109

A.4) -79_{10} =

$\sim 79_{10}$ = 0 1 0 0 1 1 1

64 32 16 8 4 2 1

64+ 8+4+2+1 = 79

1 1 1 1 1 1 1

0 1 0 0 1 1 1 1

+ 0 0 1 1 0 0 0 1 → -79 = 1 0 1 1 0 0 0 1$_{8b2c}$

1 0 0 0 0 0 0 0

B.1) 1 1 1 1 0

16 8 4 2 1

16+8+4+2 = 30_{10}

1 1 1 1 1

B.2) 0 1 1 0 0 1 1 0

+ 0 0 0 1 1 0 1 0 → 1 1 0 1 0

1 0 0 0 0 0 0 0 16 8 4 2 1

16+8+2 = 26 ∴ 1 1 1 0 0 1 1 0$_{8b2c}$ = -26_{10}

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Question 3 continued

B.3) 00101101_{8b2c} = $\frac{1}{32}\ \frac{0}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{0}{2}\ \frac{1}{1}$

$32 + 8 + 4 + 1 = 45_{10}$

B.4) 10011110_{8b2c} → 00011110

$+\ 01100010$ → 1100010

10000000 64 32 16 8 4 2 1

$64+32+2 = 98$ ∴

$10011110_{8b2c} = -98_{10}$

Question 4: 1.2.4 b, c; 1.3.4 b, d

1) 1.2.4 b) $\neg(p \vee q)$

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

1.2.4 c) $r \vee (p \wedge \neg q)$

p	q	r	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

1.3.4 b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

1.3.4 d) $(p \longleftrightarrow q) \oplus (p \longleftrightarrow \neg q)$

p	q	$p \longleftrightarrow q$	$p \longleftrightarrow \neg q$	$(p \longleftrightarrow q) \oplus (p \longleftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5 : 1.2.7 b-e; 1.3.7 b-e ; 1.3.9 c, d

1.2.7 b) $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

1.2.7 c) $B \vee (D \wedge M)$

1.3.7 b) $(S \vee y) \rightarrow P$

c) $P \rightarrow y$

d) $P \leftrightarrow (S \wedge y)$

e) $P \rightarrow (S \vee y)$

1.3.9 c) $C \rightarrow P$

d) $C \rightarrow P$

Question 6 : 1.3.6 b-d ; 1.3.10 c-f

1.3.6 b) If Joe is eligible for the honors program, then they must maintain a B average.

c) If Rajiv can go on the roller coaster, then he is at least 4 feet tall

d) If he is at least 4 feet tall, then Rajiv can go on the rollercoaster

1.3.10 c) $(p \vee r) \leftrightarrow (q \wedge r)$

$p = T$ $q = F$ $r = ?$

$(T \vee r) \leftrightarrow (F \wedge r)$

$T \leftrightarrow F$

False

d) $(p \wedge r) \leftrightarrow (q \wedge r)$

$(T \wedge r) \leftrightarrow (F \wedge r)$

$(T \wedge r) \leftrightarrow F$

unknown

e) $p \rightarrow (r \vee q)$

$T \rightarrow (r \vee F)$

unknown

f) $(p \wedge q) \rightarrow R$

$(T \wedge F) \rightarrow r$

$F \rightarrow r$

True

Question 7 : 1.4.5 section b-d

1.4.5 b) $\neg j \rightarrow (\ell \vee \neg r) \stackrel{?}{\equiv} (r \wedge \neg \ell) \rightarrow j$

j	ℓ	r	$(\ell \vee \neg r)$	$(r \wedge \neg \ell)$	$\neg j \rightarrow (\ell \vee \neg r)$	$(r \wedge \neg \ell) \rightarrow j$
T	T	T	T	F	F → T = T	F → T : T
T	T	F	T	F	F → T = T	F → T : T
T	F	T	F	T	F → F = T	T → T : T
T	F	F	T	F	F → T = T	F → T : T
F	T	T	T	F	T → T = T	F → F : T
F	T	F	T	F	T → T = T	F → F : T
F	F	T	F	T	T → F = F	T → F : F
F	F	F	T	F	T → T = T	F → F : T

∴ logically equivalent

1.4.5 c) $j \rightarrow \neg \ell \stackrel{?}{\equiv} \neg j \rightarrow \ell$

j	ℓ	$j \rightarrow \neg \ell$	$\neg j \rightarrow \ell$
T	T	T → F = F	F → T = T
T	F	T → T = T	F → F = T
F	T	F → F = T	T → T = T
F	F	F → T = T	T → F = F

∴ not logically equivalent.

d) $(r \vee \neg \ell) \rightarrow j \stackrel{?}{\equiv} j \rightarrow (r \wedge \neg \ell)$

j	ℓ	r	$(r \vee \neg \ell)$	$(r \wedge \neg \ell)$	$(r \vee \neg \ell) \rightarrow j$	$j \rightarrow (r \wedge \neg \ell)$
T	T	T	T	F	T → T = T	T → F = F
T	T	F	F	F	F → T = T	T → F = F
T	F	T	T	T	T → T = T	T → T = T
T	F	F	T	F	T → T = T	T → F = F
F	T	T	T	F	T → T = T	F → F = T
F	T	F	F	F	F → F = T	F → F = T
F	F	T	T	T	T → F = F	F → T = T
F	F	F	T	F	T → F = F	F → F = T

∴ not logically equivalent.

Question 8: 1.5.2 c, f, i; 1.5.3 c, d

1.5.2 c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(\neg p \lor q) \land (\neg p \lor r) \equiv \neg p \lor (q \land r)$

$(\neg p \lor q) \land (\neg p \lor r) \equiv (\neg p \lor q) \land (\neg p \lor r)$

1.5.2 f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg p \land \neg(\neg p \land q))$

$\neg p \land (\neg\neg p \lor \neg q)$

$(\neg p \land \neg\neg p) \lor (\neg p \land \neg q)$

$(\neg p \land p) \lor (\neg p \land \neg q)$

$F \lor (\neg p \land \neg q)$

$(\neg p \land \neg q) \equiv (\neg p \land \neg q)$

1.5.2 i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$\neg(p \land q) \lor r \equiv \neg(p \land \neg r) \lor \neg q$

$\neg p \lor \neg q \lor r \equiv \neg p \lor \neg\neg r \lor \neg q$

$\neg p \lor \neg q \lor r \equiv \neg p \lor \neg q \lor r$

1.5.3 c) $\neg r \lor (\neg r \to p)$

$\neg r \lor (\neg\neg r \lor p)$

$(\neg r \lor r) \lor p$

$T \lor p$

T

1.5.3 d) $\neg(p \to q) \to \neg q$

$\neg(\neg p \lor q) \to \neg q$

$(\neg\neg p \land \neg q) \to \neg q$

$(p \land \neg q) \to \neg q$

$\neg(p \land \neg q) \lor \neg q$

$\neg p \lor \neg\neg q \lor \neg q$

$\neg p \lor (q \lor \neg q)$

$\neg p \lor T$

T

Question 9: 1.6.3 c, d ; 1.7.4 b-d

1.6.3 c) $\exists x(x = x^2)$

d) $\forall x(x \neq x^2)$

1.7.4 b) $\forall x(\neg S(x) \wedge W(x))$

c) $\forall x(S(x) \rightarrow \neg W(x))$

d) $\exists x(S(x) \wedge W(x))$

Question 10: 1.7.9 c-i ; 1.9.2 b-i

1.7.9 c) True. When $x=a$: $\exists x((x=c)\rightarrow P(x))$

$a=c \rightarrow P(a)$

$F \rightarrow T$

T

d) True. When $x=e$: $\exists x(Q(x)\wedge r(x))$

$Q(e)\wedge r(e)$

$T \wedge T$

T

e) True

f) True. When $x=a,b,c,d,e \Rightarrow \forall x((x\neq b)\rightarrow Q(x)) = T$

g) False. When $x=c$: $\forall x(P(x)\vee R(x))$

$P(c)\vee R(c)$

$F \vee F$

F

h) True.

i) $\exists x(Q(x)\vee R(x))$. True. When $x=a$, $Q(a)\vee R(a)$

$T \vee F$

T

1.9.2 b) True $(x=1)$

c) True $(y=1)$

d) False

e) False

f) True $(y=1)$

g) False $(P(2,2))$

h) True $(Q(2,1))$

i) True

Question 11 : 1.10.4 c-g ; 1.10.7 c-f ; 1.10.10 c-f

1.10.4 c) $\exists x \exists y ((x+y) = xy)$

d) $\forall x \forall y ((x \neq 0) \wedge (y > 0) \rightarrow (x \div y > 0))$

e) $\forall x ((x < 1) \wedge (x > 0)) \rightarrow (1 \div x > 1))$

f) $\neg \exists x \forall y (x < y)$

g) $\forall x \exists y ((x \neq 0) \rightarrow (xy = 1))$

1.10.7 c) $\exists x (N(x) \wedge D(x))$

d) $\forall x (D(x) \rightarrow P(Sam, x))$

e) $\exists x \forall y (N(x) \wedge P(x, y))$

f) $\exists x \forall y [(N(x) \wedge D(x)) \wedge ((x \neq y) \wedge N(y)) \rightarrow \neg D(y)]$

1.10.10 c) $\forall x \exists y (T(x,y) \wedge (y \neq Math\ 101))$

d) $\exists x \forall y ((y \neq Math\ 101) \rightarrow T(x,y))$

e) $\forall x \exists y \exists z ((x \neq Sam) \rightarrow ((y \neq z) \wedge T(x,y) \wedge T(x,z)))$

f) $\exists y \exists z \forall a ([(y \neq z) \wedge T(Sam,y) \wedge T(Sam,z)] \wedge [((a \neq y) \wedge (a \neq z)) \rightarrow \neg T(Sam,a)])$

Question 12 : 1.8.2 b-e ; 1.9.4 c-e

1.8.2b) $\forall x (P(x) \vee D(x))$
$\neg \forall x (P(x) \vee D(x))$
$\exists x \neg (P(x) \vee D(x))$
$\exists x (\neg P(x) \wedge \neg D(x))$
There is a patient who was not given medication and not given placebo.

c) $\exists x (P(x) \wedge M(x))$
$\neg \exists x (P(x) \wedge M(x))$
$\forall x \neg (P(x) \wedge M(x))$
$\forall x (\neg P(x) \vee \neg M(x))$
Every patient was not given medication, or did not had migraines, or both of those

d) $\forall x (P(x) \rightarrow M(x))$
$\neg \forall x (P(x) \rightarrow M(x))$
$\exists x \neg (P(x) \rightarrow M(x))$
$\exists x \neg (\neg P(x) \vee M(x))$
$\exists x \neg\neg P(x) \wedge \neg M(x) = \exists x (P(x) \wedge \neg M(x))$
There is a patient who took placebo and did not have migraines.

e) $\exists x (M(x) \wedge P(x))$
$\neg \exists x (M(x) \wedge P(x)) = \forall x \neg (M(x) \wedge P(x))$
$\forall x (\neg M(x) \vee \neg P(x))$
Every patient either didn't have migraines, or didn't take placebo, or both of those.

d) $\exists x \forall y \; (P(x,y) \iff P(y,x))$

$\neg \exists x \forall y \; ([P(x,y) \rightarrow P(y,x)] \wedge [P(x,y) \rightarrow P(x,y)])$

e) $\neg \{ \exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y) \}$

$\neg \exists x \exists y P(x,y) \vee \neg \forall x \forall y Q(x,y)$

$\neg \exists x \forall y [\neg P(x,y) \vee P(y,x)] \wedge [\neg P(y,x) \vee P(x,y)]$

$\forall x \exists y \neg ([\neg P(x,y) \vee P(y,x)] \wedge [\neg P(y,x) \vee P(x,y)])$

$\neg [\neg P(x,y) \vee P(y,x)] \vee \neg [\neg P(y,x) \vee P(x,y)]$

$[\neg \neg P(x,y) \wedge \neg P(y,x)] \vee [\neg P(y,x) \wedge \neg P(x,y)]$

$\forall x \exists y \; [P(x,y) \wedge \neg P(y,x)] \vee [P(y,x) \wedge \neg P(x,y)]$

1.9.4c) $\exists x \forall y \; (P(x,y) \rightarrow Q(x,y))$

$\neg \exists x \forall y$

$\forall x \exists y \; \neg (P(x,y) \rightarrow Q(x,y))$

$\forall x \exists y \; \neg (\neg P(x,y) \vee Q(x,y))$

$\forall x \exists y \; \neg \neg P(x,y) \wedge \neg Q(x,y)$

$\forall x \exists y \; P(x,y) \wedge \neg Q(x,y)$