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HW 5: Math Questions

Question 3

a) 4.1.3: b,c
- b) Not a function. X cannot be -2 or 2
- c) Is a function. Range is [0, Infinity)

b) 4.1.5 b, d, h, i, l
- b) {4, 9, 16, 25}
- d) {0, 1, 2, 3, 4, 5}
- h) { (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }
- i) { (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4) }
- l) { ∅, {2}, {3}, {2,3} }

Question 4

I. a) 4.2.2 c, g, k

c) One-to-one, but not onto. There is no integer x such that $h(x) = 7$.

g) One-to-one, but not onto. There is no integer y such that $f(x, y) = f(1, 3)$

k) Neither one-to-one nor onto. $f(1, 3) = f(2, 1) = 5$. There is no such x and y such that $f(x,y) = 1$

b) 4.2.4 b, c, d, g

b) Neither one-to-one nor onto. $f(001) = f(101) = 101$. There is no such x such that $f(x) = 001$

c) One-to-one and onto

d) One-to-one, but not onto. There is no such x such that $f(x) = 0000$

g) Neither one-to-one nor onto. $f(\{1,2\}) = f(\{2\}) = \{2\}$. There is no such X such that $f(X) = \{1\}$

II. Give an example of a function that is ______ from **Z** to $\mathbf{Z}^+$

a) One to one but not onto.

$$f(x) = \begin{cases} -2x + 2, & x \leq 0 \\ 2x + 1, & x > 0 \end{cases}$$

b) Onto, but not one to one

$$f(x) = |x| + 1$$

c) One-to-one and onto

$$f(x) = \begin{cases} -2x + 1, & x \leq 0 \\ 2x + 1, & x > 0 \end{cases}$$

d) neither one-to-one nor onto

$$f(x) = |x| + 2$$

Question 5

a) 4.3.2 c, d, g, i

c) Inverse is well-defined. $f^{-1}(x) = (y-3) / 2$

d) Inverse is not well defined. Function is not one-to-one

g) Inverse is reversing the bits of the input string. I.e. f(011) = 110. $f^{-1}(011) = 110$.

$f^{-1}(f(011)) = 011$

i) $f^{-1}(x, y) = (x-5, y+2))$

b) 4.4.8 c, d

c) $f \circ h = f(h(x)) = 2(x^2 + 1) + 3 = 2x^2 + 2 + 3 = 2x^2 + 5$

d) $h \circ f = (2x + 3)^2 + 1 = 4x^2 + 12x + 9 + 1 = 4x^2 + 12x + 10$

c) 4.4.2 b-d

b) $(f \circ h)(52) = (\text{ceiling}(52/5))^2 = (\text{ceiling } (10.4))^2 = 11^2 = 121$

c) (g o h o f)(4) = 2^[ceiling((4^2) / 5)] = 2^[ceiling(16 / 5)] = 2^[ceiling (3.2)]

= 2^4 = 2^4 = 16

d) $h \circ f = \lceil \frac{x^2}{5} \rceil$

d) 4.4.6 c - e

c) (h o f)(110) = h(f(110)) = h(110) = 111

d) (h o f) range is {101, 111}

e) (g o f) range is { 001, 101, 011, 111 }

e) 4.4.4 c, d

c) Is it possible that f is not one-to-one and g o f is one-to-one?

We will prove by contraposition.

If g o f is not one-to-one, then f must be one-to-one.

If g o f is not one-to-one, then every g(f(x)) must be different.

Thus, $g(f(x_1)) \neq g(f(x_2))$

Thus, $f(x_1) \neq f(x_2)$

Thus, f is not one-to-one, which is in contraposition to our assumption that g o f is not one-to-one.

d) Is it possible that g is not one-to-one and g o f is one-to-one.

Yes - in the diagram below, f is one-to-one on its domain, and the domain of f onto g (or g o f) is one-to-one, but the full domain of g is not one-to-one. $g(y_3) = g(y_4)$

$f(x_1) = y_1$	$g(y_1) = z_1$
$f(x_2) = y_2$	$g(y_2) = z_2$
$f(x_3) = y_3$	$g(y_3) = z_3$
	$g(y_4) = z_3$