

Max Grove, HW1 MG6392 eng.v.edu

Question 1

$$A.1) 10011011_2 =$$

| | | | | | | | |
|--|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 2^7 | 2^6 | 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 |
| x 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| <hr/> | | | | | | | |
| 128 + 16 + 8 + 2 + 1 = 155 ₁₀ | | | | | | | |

$$A.2) 456_7 =$$

| | | |
|----------------------------------|-------|-------|
| 4 | 5 | 6 |
| 7^2 | 7^1 | 7^0 |
| x 49 | 7 | 1 |
| <hr/> | | |
| 196 + 35 + 6 = 237 ₁₀ | | |

$$A.3) 38A_{16} =$$

| | | |
|------------------------------------|--------|--------|
| 3 | 8 | A=10 |
| 16^2 | 16^1 | 16^0 |
| x 256 | 16 | 1 |
| <hr/> | | |
| 768 + 128 + 10 = 906 ₁₀ | | |

$$A.4) 2214_5 =$$

| | | | |
|---|-------|-------|-------|
| 2 | 2 | 1 | 4 |
| 5^3 | 5^2 | 5^1 | 5^0 |
| x 125 | 25 | 5 | 1 |
| <hr/> | | | |
| 250 + 50 + 5 + 4 = 309 309 ₁₀ | | | |

$$B.1) 69_{10} =$$

| | | | | | | | |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 2^7 | 2^6 | 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| <hr/> | | | | | | | |
| 64 + 4 + 1 = 69 | | | | | | | |

$$= 1000101_2$$

$$B.2) 485_{10} =$$

| | | | | | | | | | |
|-----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 2^9 | 2^8 | 2^7 | 2^6 | 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 |
| 512 | 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| <hr/> | | | | | | | | | |
| 256 + 128 + 64 + 32 + 4 + 1 = 485 | | | | | | | | | |

$$= 111100101_2$$

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Question 1, continued

$$B.3) 6D1A_{16} = 0110 \ 1101 \ 0001 \ 1010 = 110110100011010_2$$

(based on table presented in lectures)

$$C.1) 1101011_2 = 0110 \ 1011 = 6B_{16}$$

(based on the table presented in lectures)

$$C.2) 895_{10} = \frac{0}{16^3} \frac{3}{16^2} \frac{7}{16^1} \frac{F}{16^0} = 37F_{16}$$

4096 256 16 1

$$\begin{array}{r} 3 \text{ R } 127 \\ 256 \overline{) 895} \end{array}$$

$$\begin{array}{r} 7 \text{ R } 15 \\ 16 \overline{) 127} \end{array} \quad 15 = F$$

Question 2

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 1 & & 1 & & 1 \\
 7 & 5 & 6 & 6 & 8 \\
 + & 4 & 5 & 1 & 5 & 8 \\
 \hline
 1 & 4 & 3 & 0 & 3 & 8
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 1 & & 1 & & \\
 7 & A & 6 & 6 & 10 \\
 + & 4 & 5 & C & 5 & 16 \\
 \hline
 C & 0 & 2 & B & 16
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccccccc}
 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1_2 \\
 + & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1_2 \\
 \hline
 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0_2
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 2 & 4 & 1 & 1 \\
 3 & 0 & 2 & 2_5 \\
 - & 2 & 4 & 3 & 3_5 \\
 \hline
 0 & 0 & 3 & 4 & 5
 \end{array}
 \end{array}$$

Question 3

$$A.1) 124_{10} = 0 \frac{1}{2^6} \frac{1}{2^5} \frac{1}{2^4} \frac{1}{2^3} \frac{1}{2^2} \frac{0}{2^1} \frac{0}{2^0} = 01111100_{8b2c}$$

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$
 $64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$
 $64 + 32 + 16 + 8 + 4 = 124$

$$A.2) -124_{10} = \begin{array}{r} 01111100 \\ + 00000100 \\ \hline 10000000 \end{array} \rightarrow 10000100_{8b2c}$$

$$A.3) 109_{10} = 0 \frac{1}{2^6} \frac{1}{2^5} \frac{0}{2^4} \frac{1}{2^3} \frac{1}{2^2} \frac{0}{2^1} \frac{1}{2^0} = 01101101_{8b2c}$$

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$
 $64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$
 $64 + 32 + 8 + 4 + 1 = 109$

$$A.4) -79_{10} =$$

$$\sim 79_{10} = 0 \frac{1}{2^6} \frac{0}{2^5} \frac{0}{2^4} \frac{1}{2^3} \frac{1}{2^2} \frac{1}{2^1} \frac{1}{2^0}$$

$64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$
 $64 + 8 + 4 + 2 + 1 = 79$

$$\begin{array}{r} 01001111 \\ + 00110001 \\ \hline 10000000 \end{array} \rightarrow -79 = 10110001_{8b2c}$$

$$B.1) \frac{1}{16} \frac{1}{8} \frac{1}{4} \frac{1}{2} \frac{0}{1}$$

$16 \quad 8 \quad 4 \quad 2 \quad 1$
 $16 + 8 + 4 + 2 = 30_{10}$

$$B.2) \begin{array}{r} 01100110 \\ + 00011010 \\ \hline 10000000 \end{array} \rightarrow 11010$$

$16 \quad 8 \quad 4 \quad 2 \quad 1$
 $16 + 8 + 2 = 26 \therefore 11100110_{8b2c} = -26_{10}$

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Question 3 continued

$$B.3) 00101101_{862C} = \begin{array}{cccccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$
$$32 + 8 + 4 + 1 = 45_{10}$$

$$B.4) 10011110_{862C} \rightarrow \begin{array}{r} 11111 \\ 00011110 \\ + 01100010 \\ \hline 10000000 \end{array} \rightarrow 1100010$$

64 32 16 8 4 2 1

$$64 + 32 + 2 = 98 \therefore$$

$$10011110_{862C} = -98_{10}$$

Question 4: 1.2.4 b, c; 1.3.4 b, d

1) 1.2.4 b) $\neg(p \vee q)$

| <u>p</u> | <u>q</u> | <u>$\neg(p \vee q)$</u> |
|----------|----------|------------------------------------|
| T | T | F |
| T | F | F |
| F | T | F |
| F | F | T |

1.2.4 c) $r \vee (p \wedge \neg q)$

| <u>p</u> | <u>q</u> | <u>r</u> | <u>$p \wedge \neg q$</u> | <u>$r \vee (p \wedge \neg q)$</u> |
|----------|----------|----------|-------------------------------------|--|
| T | T | T | F | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

1.3.4 b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

| <u>p</u> | <u>q</u> | <u>$p \rightarrow q$</u> | <u>$q \rightarrow p$</u> | <u>$(p \rightarrow q) \rightarrow (q \rightarrow p)$</u> |
|----------|----------|-------------------------------------|-------------------------------------|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

1.3.4 d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

| <u>p</u> | <u>q</u> | <u>$p \leftrightarrow q$</u> | <u>$p \leftrightarrow \neg q$</u> | <u>$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$</u> |
|----------|----------|---|--|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | T | F | T |

Question 5 : 1.2.7 b-e; 1.3.7 b-e; 1.3.9 c,d

$$1.2.7b) (B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$$

$$1.2.7c) B \vee (D \wedge M)$$

$$1.3.7b) (S \vee Y) \rightarrow P$$

$$c) P \rightarrow Y$$

$$d) P \leftrightarrow (S \wedge Y)$$

$$e) P \rightarrow (S \vee Y)$$

$$1.3.9c) C \rightarrow P$$

$$d) C \rightarrow P$$

Question 6 : i. 3.6b-d; 1.3.10 c-f

1.3.6b) If Joe is eligible for the honors program,
then they must maintain a B average.

c) If Rajiv can go on the roller coaster,
then he is at least 4 feet tall

d) If he is at least 4 feet tall,
then Rajiv can go on the rollercoaster

1.3.10 c) $(p \vee r) \longleftrightarrow (q \wedge r)$

$p = T$ $(T \vee r) \longleftrightarrow (F \wedge r)$

$q = F$ $T \longleftrightarrow F$

$r = ?$ False

d) $(p \wedge r) \longleftrightarrow (q \wedge r)$

$(T \wedge r) \longleftrightarrow (F \wedge r)$

$(T \wedge r) \longleftrightarrow F$

unknown

e) $p \rightarrow (r \vee q)$

$T \rightarrow (r \vee F)$

unknown

f) $(p \wedge q) \rightarrow r$

$(T \wedge F) \rightarrow r$

$F \rightarrow r$

True

Question 7 : 1.4.5 section b-d

1.4.5b) $\neg j \rightarrow (l \vee \neg r) \stackrel{?}{=} (r \wedge \neg l) \rightarrow j$

| j | l | r | $(l \vee \neg r)$ | $(r \wedge \neg l)$ | $\neg j \rightarrow (l \vee \neg r)$ | $(r \wedge \neg l) \rightarrow j$ |
|-----|-----|-----|-------------------|---------------------|--------------------------------------|-----------------------------------|
| T | T | T | T | F | $F \rightarrow T = T$ | $F \rightarrow T = T$ |
| T | T | F | T | F | $F \rightarrow T = T$ | $F \rightarrow T = T$ |
| T | F | T | F | T | $F \rightarrow F = T$ | $T \rightarrow T = T$ |
| T | F | F | T | F | $F \rightarrow T = T$ | $F \rightarrow T = T$ |
| F | T | T | T | F | $T \rightarrow T = T$ | $F \rightarrow F = T$ |
| F | T | F | T | F | $T \rightarrow T = T$ | $F \rightarrow F = T$ |
| F | F | T | F | T | $T \rightarrow F = F$ | $T \rightarrow T = T$ |
| F | F | F | T | F | $T \rightarrow T = T$ | $F \rightarrow F = T$ |

\therefore logically equivalent

1.4.5c) $j \rightarrow \neg l \stackrel{?}{=} \neg j \rightarrow l$

| j | l | $j \rightarrow \neg l$ | $\neg j \rightarrow l$ |
|-----|-----|------------------------|------------------------|
| T | T | $T \rightarrow F = F$ | $F \rightarrow T = T$ |
| T | F | $T \rightarrow T = T$ | $F \rightarrow F = T$ |
| F | T | $F \rightarrow F = T$ | $T \rightarrow T = T$ |
| F | F | $F \rightarrow T = T$ | $T \rightarrow F = F$ |

\therefore not logically equivalent

d) $(r \vee \neg l) \rightarrow j \stackrel{?}{=} j \rightarrow (r \wedge \neg l)$

| j | l | r | $(r \vee \neg l)$ | $(r \wedge \neg l)$ | $(r \vee \neg l) \rightarrow j$ | $j \rightarrow (r \wedge \neg l)$ |
|-----|-----|-----|-------------------|---------------------|---------------------------------|-----------------------------------|
| T | T | T | T | F | $T \rightarrow T = T$ | $T \rightarrow F = F$ |
| T | T | F | T | F | $T \rightarrow T = T$ | $T \rightarrow F = F$ |
| T | F | T | T | T | $T \rightarrow T = T$ | $T \rightarrow T = T$ |
| T | F | F | T | F | $T \rightarrow T = T$ | $T \rightarrow F = F$ |
| F | T | T | T | F | $T \rightarrow F = F$ | $F \rightarrow F = T$ |
| F | T | F | T | F | $F \rightarrow F = T$ | $F \rightarrow F = T$ |
| F | F | T | T | T | $T \rightarrow F = F$ | $F \rightarrow T = T$ |
| F | F | F | T | F | $T \rightarrow F = F$ | $F \rightarrow F = T$ |

\therefore not logically equivalent

Question 8 : 1.5.2 c, f, i ; 1.5.3 c, d

$$1.5.2c) \begin{aligned} (p \rightarrow q) \wedge (p \rightarrow r) &\equiv p \rightarrow (q \wedge r) \\ (\neg p \vee q) \wedge (\neg p \vee r) &\equiv \neg p \vee (q \wedge r) \\ (\neg p \vee q) \wedge (\neg p \vee r) &\equiv (\neg p \vee q) \wedge (\neg p \vee r) \end{aligned}$$

$$1.5.2f) \begin{aligned} \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg q \\ \neg p \wedge \neg(\neg p \wedge q) \\ \neg p \wedge (\neg \neg p \vee \neg q) \\ (\neg p \wedge \neg \neg p) \vee (\neg p \wedge \neg q) \\ (F \vee (\neg p \wedge \neg q)) \\ (\neg p \wedge \neg q) &\equiv (\neg p \wedge \neg q) \end{aligned}$$

$$1.5.2i) \begin{aligned} (p \wedge q) \rightarrow r &\equiv (p \wedge \neg r) \rightarrow \neg q \\ \neg(p \wedge q) \vee r &\equiv \neg(p \wedge \neg r) \vee \neg q \\ \neg p \vee \neg q \vee r &\equiv \neg p \vee \neg \neg r \vee \neg q \\ \neg p \vee \neg q \vee r &\equiv \neg p \vee \neg q \vee r \end{aligned}$$

$$1.5.3c) \begin{aligned} \neg r \vee (\neg r \rightarrow p) \\ \neg r \vee (\neg \neg r \vee p) \\ (\neg r \vee r) \vee p \\ T \vee p \\ T \end{aligned}$$

$$1.5.3d) \begin{aligned} \neg(p \rightarrow q) \rightarrow \neg q \\ \neg(\neg p \vee q) \rightarrow \neg q \\ (\neg \neg p \wedge \neg q) \rightarrow \neg q \\ (p \wedge \neg q) \rightarrow \neg q \\ \neg(p \wedge \neg q) \vee \neg q \\ \neg p \vee \neg \neg q \vee \neg q \\ \neg p \vee (q \vee \neg q) \\ \neg p \vee T \\ T \end{aligned}$$

Question 9: 1.6.3 c, d; 1.7.4 b-d

1.6.3 c) $\exists x (x = x^2)$

d) $\forall x (x \leq x^2)$

Correct Answer: $\forall x (x \leq x^2 + 1)$

1.7.4 b) $\forall x (\neg S(x) \vee W(x))$

c) $\forall x (S(x) \rightarrow \neg W(x))$

d) $\exists x (S(x) \wedge W(x))$

Question 10: 1.7.9 c-i ; 1.9.2 b-i

1.7.9 c) True. When $x = a$: $\exists x ((x = c) \rightarrow P(x))$

$$a = c \rightarrow P(a)$$

$$F \rightarrow T$$

$$T$$

d) True. When $x = e$: $\exists x (Q(x) \wedge r(x))$

$$Q(e) \wedge r(e)$$

$$T \wedge T$$

$$T$$

e) True

f) True. When $x = a, b, c, d, e \Rightarrow \forall x ((x \neq b) \rightarrow Q(x)) = T$

g) False. When $x = c$: $\forall x (P(x) \vee R(x))$

$$P(c) \vee R(c)$$

$$F \vee F$$

$$F$$

h) True.

i) $\exists x (Q(x) \vee R(x))$. True. When $x = a$, $Q(a) \vee R(a)$

$$T \vee F$$

$$T$$

1.9.2 b) True ($x = 1$)

c) True ($y = 1$)

d) False

e) False

f) True ($y = 1$)

g) False ($P(2, 2)$)

h) True ($Q(2, 1)$)

i) True

Question 11 : 1.10.4 c-g ; 1.10.7 c-f ; 1.10.10 c-f

1.10.4 c) $\exists x \exists y ((x+y) = xy)$

d) $\forall x \forall y ((x > 0) \wedge (y > 0) \rightarrow (x \div y > 0))$

e) $\forall x ((x < 1) \wedge (x > 0) \rightarrow (1 \div x > 1))$

f) $\neg \exists x \forall y (x < y)$

g) $\forall x \exists y ((x \neq 0) \rightarrow (xy = 1))$

1.10.7 c) $\exists x (N(x) \wedge D(x))$

d) $\forall x (D(x) \rightarrow P(\text{Sam}, x))$

e) $\exists x \forall y (N(x) \wedge P(x, y))$

f) $\exists x \forall y [(N(x) \wedge D(x)) \wedge ((x \neq y \wedge N(y)) \rightarrow \neg D(y))]$

1.10.10 c) $\forall x \exists y (T(x, y) \wedge (y \neq \text{Math 101}))$

d) $\exists x \forall y ((y \neq \text{Math 101}) \rightarrow T(x, y))$

e) $\forall x \exists y \exists z ((x \neq \text{Sam}) \rightarrow ((y \neq z) \wedge T(x, y) \wedge T(x, z)))$

f) $\exists y \exists z \forall a [(y \neq z) \wedge T(\text{Sam}, y) \wedge T(\text{Sam}, z)] \wedge [(a \neq y) \wedge (a \neq z) \rightarrow \neg T(\text{Sam}, a)]$

Question 12 : 1.8.2 b-e ; 1.9.4 c-e

$$1.8.2 b) \forall x (P(x) \vee D(x))$$

$$\neg \forall x (P(x) \vee D(x))$$

$$\exists x \neg (P(x) \vee D(x))$$

$$\exists x (\neg P(x) \wedge \neg D(x))$$

There is a patient who was not given medication and not given placebo.

$$c) \exists x (D(x) \wedge M(x))$$

$$\neg \exists x (D(x) \wedge M(x))$$

$$\forall x \neg (D(x) \wedge M(x))$$

$$\forall x (\neg D(x) \vee \neg M(x))$$

Every patient was not given medication, or ^{did} not had migraines, or both of those

$$d) \forall x (P(x) \rightarrow M(x))$$

$$\neg \forall x (P(x) \rightarrow M(x))$$

$$\exists x \neg (P(x) \rightarrow M(x))$$

$$\exists x \neg (\neg P(x) \vee M(x))$$

$$\exists x \neg \neg P(x) \wedge \neg M(x) = \exists x (P(x) \wedge \neg M(x))$$

There is a patient who took placebo and did not have migraines.

$$e) \exists x (M(x) \wedge P(x))$$

$$\neg \exists x (M(x) \wedge P(x)) = \forall x \neg (M(x) \wedge P(x))$$

$$\forall x (\neg M(x) \vee \neg P(x))$$

Every patient either didn't have migraines, or didn't take placebo, or both of those.

$$1.9.4c) \quad \exists x \forall y (P(x,y) \rightarrow Q(x,y))$$

$$\neg \exists x \forall y$$

$$\forall x \exists y \neg (P(x,y) \rightarrow Q(x,y))$$

$$\forall x \exists y \neg (\neg P(x,y) \vee Q(x,y))$$

$$\forall x \exists y \neg \neg P(x,y) \wedge \neg Q(x,y)$$

$$\forall x \exists y P(x,y) \wedge \neg Q(x,y)$$

$$d) \quad \exists x \forall y (P(x,y) \leftrightarrow P(y,x))$$

$$\neg \exists x \forall y ([P(x,y) \rightarrow P(y,x)] \wedge [P(y,x) \rightarrow P(x,y)])$$

$$\neg \exists x \forall y ([\neg P(x,y) \vee P(y,x)] \wedge [\neg P(y,x) \vee P(x,y)])$$

$$\forall x \exists y \neg ([\neg P(x,y) \vee P(y,x)] \wedge [\neg P(y,x) \vee P(x,y)])$$

$$\neg [\neg P(x,y) \vee P(y,x)] \vee \neg [\neg P(y,x) \vee P(x,y)]$$

$$[\neg \neg P(x,y) \wedge \neg P(y,x)] \vee [\neg \neg P(y,x) \wedge \neg P(x,y)]$$

$$\forall x \exists y [P(x,y) \wedge \neg P(y,x)] \vee [P(y,x) \wedge \neg P(x,y)]$$

$$e) \quad \neg \{ \exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y) \}$$

$$\neg \exists x \exists y P(x,y) \vee \neg \forall x \forall y Q(x,y)$$

$$\forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)$$