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HW 7
MG6392

Question 3
a) 8.2.2 b:

$f(n) = n^3 + 3n^2 + 4$
$f(n) <= n^3 + 3n^3 + 4n^3$
$f(n) <= 8n^3 = c*n^3$ when $c = 8$ and $n >= n_0$, when $n_0 = 1$
Thus, $f(n) = O(n^3)$

$f(n) >= n^3 = c*n^3$, when $c = 1$ and $n >= n_0$, when $n_0 = 1$
Thus, $f(n) = \text{Theta}(n^3)$. Since $f(n) = \text{Theta}(n^3) = O(n^3)$, $f(n) = \text{Omega}(n^3)$

b) 8.3.5 a - e

a) This algorithm uses two identifiers (i and j) in a sequence of numbers (*a*) along with an identifier p. It first searches with i for the final instance from the start of *a* where a_i is >= p. j searches for the final instance from the end of *a* where a_j is < p. It will swap a_i and a_j. The loop is begun again, until all the elements less than p are in the front partition of the sequence, and all the elements greater than p are in the back partition of the sequence.

b) The total number of times (i := i + 1) or (j := j +1) is n-1 for a sequence of length n.

c) The total number of times the swap sequence depends on the items in the sequence. It is maximized when all the numbers greater than p come before all the numbers less than p in the sequence. In this instance, the swap is executed (floor)(n / 2) times. The swap function is run 0 times when all the numbers less than p come before the numbers greater than p in the sequence.

d) The lower bound of the time complexity is for the two while loops to be executed n-1 times and the swap to be executed (floor)(n/2) times. This would equate to Omega(n) complexity.

e) The upper bound is the total number of operations, which is at most n-1 + (floor)(n/2), which equates to O(n).

Question 4

a) 5.1.2 b,c

b) |Special Chars| = 4
|Digits| = 10
|Letters| = 26
String length 7 = 40^7
String length 8 = 40^8
String length 9 = 40^9
Total possibilities = $40^7 + 40^8 + 40^9$

c) First character cannot be a letter
String length 7 = $14*40^6$
String length 8 = $14*40^7$
String length 9 = $14*40^8$
Total possibilities = $14*40^6 + 14*40^6 + 14*40^6$

b) 5.3.2 a

a) 3 possibilities for the first character. 2 possibilities for each next character as they cannot be the one preceding it.
$= 3 * 2^9 = 1536$

c) 5.3.3 b, c

b) $10*9*8*26^4$
c) 10*9*8*26*25*24*23

d) 5.2.3 a, b

a) For the function f : B^9 -> E_{10}, if x belongs to B^9, then f(x) is obtained by counting the number of 1s in x, and appending a 1 if the count is odd and appending a 0 if the count is even. f is one to one because if f(x) = f(y), then the first 9 bits of f(x) and f(y) are also the same, which implies that x = y. f is onto since for any y belonging to E_{10}, the first nine bits x belongs to B^9 and f(x) = y. It is possible to create any outcome in E_{10}.

b) Since f is a bijection, $|B^9| = |E_{10}|$. $|B_9| = 2^9 = |E_{10}|$

Question 5

a) 5.4.2 a, b
  a) $2*10^4 = 20000$
  b) 2 * 10 * 9 * 8 * 7 = 10080

b) 5.5.3 a-g
  a) $2^{10} = 1025$
  b) $2^7 = 125$
  c) $2^7 + 2^8 = 384$
  d) $4 * 2^6 = 256$
  e) 10 choose 6 = 210
  f) 9 choose 6 = 84
  g) 5 choose 1 * 5 choose 3 = 50

c) 5.5.5 a
  a) (30 choose 10) * (35 choose 10)

d) 5.5.8 c-f
  c) 26 choose 5 = 65780
  d) 13 * (48 choose 1) * (4 choose 4) = 624
  e) 13 * (4 choose 2) * 12 * (4 choose 3) = 3,744
  f) (13 choose 5) * 4^5

e) 5.6.6 a, b
  a) (44 choose 5) * (56 choose 5)
  b) 44 * 43 * 56 * 55 = 5827360

Question 6

a) 5.7.2 a, b

- a) (52 choose 5) - (39 choose 5)
- b) (52 choose 5) - ((13 choose 5) * 4^5)

b) 5.8.4 a, b

- a) 5^{20}
- b) $\frac{20!}{(4!)^5}$

Question 7
a) 0, as the cardinality of the target set is less than the cardinality of the domain.

b) 5! = 120

c) P(6,5) = 720

d) P(7, 5) = 7*6*5*4*3 = 2520