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HW 2

Question 5.

a(1) 1.12b

$$\begin{array}{l} p \to (q \wedge r) \\ \neg q \\ \hline \therefore \neg p \end{array}$$

1. p → (q ∧ r) : Hypothesis
2. p → q : Simplification 1
3. ¬ q : Hypothesis
4. ¬ p : Modus Tollens 2, 3

a(1) 1.12e

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \neg q \\ \hline \therefore r \end{array}$$

1. (p v q) : Hypothesis
2. (¬p v r) : Hypothesis
3. (q v r) : Resolution 1, 2
4. ¬ q : Hypothesis
5. r : Disjunctive syllogism 3, 4

a(2) 1.12.3c

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

1. p v q : Hypothesis
2. ¬ ¬ p v q : Double negation, 1
3: ¬ p → q : Conditional Identity, 2
4. ¬ p: Hypothesis
5. q : Modus ponens, 3, 4

a(3) 1.2.5c

I will buy a new car and a new house only if I get a job.
I am not going to get a job.

∴ I will not buy a new car.

p : I will buy a new car
q : I will buy a new house
r : I get a job

$(p \land q) \to r$
$\neg r$

$\therefore \neg p$

Invalid argument. When p = True, q = False, r = False:
1. $(p \land q) \to r$: Hypothesis
    $F \to (T \land F)$
    True
2. $\neg r$: Hypothesis
    $\neg$ False
    True
3. $\neg P$
    $\neg$ True
    False

Both hypotheses are true and conclusion is false.

a(3) 1.12.5d

I will buy a new car and a new house only if I get a job.
I am not going to get a job.
I will buy a new house.

∴ I will not buy a new car.

p : I will buy a new car
q : I will buy a new house
r : I get a new job

$(p \land q) \to r$
$\neg r$
q

$\therefore \neg p$

1. $(p \land q) \to r$: Hypothesis
2. $\neg (p \land q) \lor r$: Conditional Identity, 1
3. $\neg r$: Hypothesis
4. $\neg (p \land q)$: Disjunctive Syllogism 2, 3
5. $\neg p \lor \neg q$: DeMorgans Law 4
6. q : Hypothesis
7. $\neg \neg q$: Double negation 6
8. $\neg p$: Disjunctive Syllogism 7

The argument is valid.

b(1) 1.13.3b

$\exists x\, (P(x) \lor Q(x))$
$\exists x\, \neg Q(x)$

$\therefore \exists x\, P(x)$

Invalid: Both hypotheses are true and conclusion is false in the below example

	P	**Q**	**P v Q**	**¬ Q**
a	F	T	F v T = T	F
b	F	F	F v F = F	T

b(2) 1.13.5d

> Every student who missed class got a detention.
> Penelope is a student in the class.
> Penelope did not miss class.
> ______________________________
> Penelope did not get a detention.

P(x) : x student missed class
Q(x) : x student got a detention

Vx (P(x) -> Q(x))
Penelope, a student in the class
¬ P(Penelope)
____
∴ ¬ Q(Penelope)

Invalid Argument.

	P(x)	**Q(x)**	**P(x) → Q(x)**
Penelope	F	T	F → T = T
Adrian	T	T	T → T = T
Alex	F	F	F → F = F

b(2) 1.13.5e

> Every student who missed class or got a detention did not get an A.
> Penelope is a student in the class.
> Penelope got an A.
> ______________________________
> Penelope did not get a detention.

P(x) : x student missed class

Q(x) : x student got a detention
R(x) : x student got an A

Vx ((P(x) v Q(x)) → ¬ R(x)
Penelope, a student in the class
R(Penelope)
∴ ¬ Q(Penelope)

1. Vx ((P(x) v Q(x)) → ¬ R(x) : Hypothesis
2. Penelope, a student in the class : Hypothesis
3. (P(Penelope) v Q(Penelope)) → ¬ R(Penelope) : Universal Instantiation 1, 2
4. ¬ (P(Penelope) v Q(Penelope) v ¬ (R(Penelope)) : Conditional Identity 3
5. (¬ P(Penelope) ∧ ¬ Q(Penelope)) v ¬ R(Penelope) : Demorgans 4
6. ¬ Q(Penelope) v ¬ R(Penelope) : Simplification 5
7. R(Penelope) : Hypothesis
8. (¬ Q(Penelope) v ¬ R(Penelope)) ∧ R(Penelope) : Conjunction 6, 7
9. (¬ Q(Penelope) ∧ R(Penelope)) v (R(Penelope) ∧ ¬ R(Penelope)) : Distributive Law 8
10. (¬ Q(Penelope) ∧ R(Penelope)) v True : Complement 9
11. (¬ Q(Penelope) ∧ R(Penelope)) : Domination Law 10
12. ¬ Q(Penelope) : Simplification 11

This argument is valid.

Question 6

2.4.1d

Prove: The product of two odd integers is an odd integer.
(If n is odd integer and p is odd integer, then n * p is odd integer)

Proof: Let n and p be s odd integers
Since n is odd, n = 2k + 1, for some integer k.
Since p is odd, p = 2m + 1, for some integer m.
Plugging n and p into n*p yields:
(2k + 1) * (2m + 1) =
4km + 2k + 2m + 1 =
2(2km + k + m) + 1.
Since k and m are integers, (2km + k + m) is also an integer, say r.
Since n * p = 2r + 1, n * p is odd. $\blacksquare$

2.3.4b

Prove: If x is a real number and x <= 3,
then 12 - 7x + x^2 >= 0 =
then x^2 - 7x + 12 >= 0

Proof: Let x be a real number <= 3.
Since x <= 3, 0 <= (3-x). Adding 1 to both sides:
4-x >= 1.
Since 4-x >= 1, 4-x > 0.
Multiplying (4-x) * (3-x) >= 0 yields:
12 - 7x + x^2 >= 0. $\blacksquare$

<u>Question 7.</u>

2.5.1d

Prove: For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.
Proof by contrapositive.
Assume: n is an even integer.
Therefore, $n = 2(k)$ for some integer k.
Plugging in n, we get $(2k)^2 - 2(2n) + 7 =$
$4k^2 - 4n + 7 =$
$4k^2 - 4n + 6 + 1 =$
$2\ (2k^2 - 2n + 3) + 1$.
Since k is an integer, $(2k^2 - 2n + 3)$ is an integer, say r
Therefore, we have $2r + 1$, so $n^2 - 2n + 7$ is odd. $\blacksquare$

2.5.4a

For every pair of real numbers x and y, if $x^3 + xy^2 <= x^2 * y + y^3$, then $x <= y$
Proof by contrapositive
Assume: $x >= y$
We multiply each side by $(x^2 + y^2)$
$x * (x^2 + y^2) >= y * (x^2 + y^2)$
$x^3 + yx^2 >= x^2 * y + y^3$
Therefore, we have have proved by contrapositive. $\blacksquare$

2.5.4b

For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$
Proof by contrapositive
Assume: $x <= 10$ and $y <= 10$
If x and y are both smaller than 10, $x + y$ must be $<= 20$.
Therefore, $x + y > 20$. $\blacksquare$

2.5.5c

For every nonzero real number x, if x is irrational, then $1/x$ is also irrational.
Proof by contrapositive
Assume: Let x be a nonzero real number.
Since $1/x$ is a real number and is not irrational,
$1/x$ is rational.
Let $1/x = n/k$, where n and k are integers and k does not equal 0
Taking the reciprocal, $x = k/n$.
Since x is the ratio of two integers and since $k = x * n$ and k does not equal 0 (therefore n does not equal 0), x is real and rational. $\blacksquare$

Question 8

2.6.6c

Proof: The average of 3 real numbers is greater than or equal to at least one of the numbers.

$(x + y + z) / 3 >= x$ or $(x + y + z) / 3 >= y$ or $(x + y + z) / 3 >= z$

Proof by contradiction.

Assume there is a group of 3 numbers such that the average of the three real numbers is not greater than or equal to at least one of the numbers.

$(x + y + z) / 3 < x$ and $(x + y + z) / 3 < y$ and $(x + y + z) / 3 < z$

Adding up all the terms, we get:

$3 * (x + y + z) / 3 < (x + y + z)$

$(x + y + z) < (x + y + x)$

No sum of three numbers can be less than itself.

If we subtract $(x+y+z)$ from each side, we get:

$0 < 0$, which is incorrect.

Thus, the average of 3 real numbers is greater than or equal to at least one of the numbers. ■

2.6.6d

Proof: There is no smallest number. For every x, there exists a $y < x$.

Proof by contradiction

Assume there is a smallest number. That implies for an x, there does not exist a y less than x. If x is a real number, $y = (x-1)$ is a real number. Since there is no y less than x, $x < y$. This equals $x < x-1 = 0 < -1$. This is incorrect so, we have proven by contradiction. ■

<u>Question 9</u>
2.7.2b
If 2 integers x and y have the same parity, then x + y is even.
Proof by cases.
Case 1: x and y are both odd. If x is odd and n is an integer, $x = 2n + 1$. If y is odd and k is an integer, then $y = 2k + 1$. $X + y = 2n + 2k + 1 + 1 = 2n + 2k + 2 = 2(n + k + 1)$. If n and k are integers, then $d = n + k + 1$ is an integer, which means $x + y = 2d$ when they are both odd. This implies x + y is even.

Case 2: x and y are both even. If x is even and n is an integer, $x = 2n$. If y is even and k is an integer, then $y = 2k$. $X + y = 2n + 2k = 2(n+k)$. If n and k are both integers, then $d = n + k$ is an integer. This means $x + y = 2d$ when they are both even. This implies x + y is even.

Since both cases evaluate to x + y being even and the cases cover the domain of real numbers (If x and y have the same parity, they either are both odd or both even), if x and y have the same parity, then x + y is even. ■