

Max Grove
MG6392
HW 8

Question 7

a) 6.1.5 b - d

$$\text{b) } (13 \text{ choose } 1) * (4 \text{ choose } 3) * (12 \text{ choose } 2) * (4 \text{ choose } 1) * \\ (4 \text{ choose } 1 / (52 \text{ choose } 5)) = 88/4165 = 0.021128$$

$$\text{c) } (4 \text{ choose } 1) * (13 \text{ choose } 5) / (52 \text{ choose } 5) = 33/16660 = 0.001981$$

$$\text{d) } (13 \text{ choose } 1) * (4 \text{ choose } 2) * (12 \text{ choose } 3) * 4 * 4 * 4 / (52 \text{ choose } 5) = 352 / 833 \\ = 0.422569$$

b) 6.2.4 a - d

$$\text{a) } 1 - (39 \text{ choose } 5) / (52 \text{ choose } 5) = 1 - 2109/9520 = 7411 / 9520$$

$$\text{b) } 1 - (13 \text{ choose } 5) * 4^5 / (52 \text{ choose } 5) = 2053 / 4165$$

$$\text{C) } [2 * (13 \text{ choose } 1) * (39 \text{ choose } 4) / (52 \text{ choose } 5) \\ - (13 \text{ choose } 1) * (13 \text{ choose } 1) * (26 \text{ choose } 3)] \\ / (52 \text{ choose } 5) \\ 65351 / 99960$$

$$\text{d) } 1 - [(26 \text{ choose } 5) / (52 \text{ choose } 5)] = 9743 / 9996$$

Question 8

a) 6.3.2a - e

a) $p(A) = 6! / 7! = 1/7$

$p(B) = (7! / 2) / 7! = 1/2$

$p(C) = 5! / 7! = 1 / 42$

b) $p(A|C) = 3! * 2 / 5! = 1/10$

c) $p(B|C) = (5!/2) / 5! = 1/2$

d) $P(A|B) = (3 * 5!) / (7! / 2) = 1/7$

e) A and C are not independent. $P(A|C)$ does not equal $P(A)$. B and C are independent as $P(B|C) = P(B)$.

b) 6.3.6 b, c

b) $(1/3)^5 * (2/3)^5$

c) $(1/3) * (2/3)^9$

c) 6.4.2 a

a) $[(1/6)^6 * (1/2)] / [(1/6)^5 * (1/2) + (3/20)^4 * (1/4)^2 * (1/2)] = 40000 / 99049$

Question 9

6.5.2 a, b

a) $\{0, 1, 2, 3, 4\}$

b)
 $\{(0, (48 \text{ choose } 5) / (52 \text{ choose } 5),$
 $(1, (48 \text{ choose } 4) * (4 \text{ choose } 1) / (52 \text{ choose } 5),$
 $(2, (48 \text{ choose } 3) * (4 \text{ choose } 2) / (52 \text{ choose } 5)),$
 $(3, (48 \text{ choose } 2) * (4 \text{ choose } 3) / (52 \text{ choose } 5)),$
 $(4, (48 \text{ choose } 1) * (4 \text{ choose } 4) / (52 \text{ choose } 5))$
 $\}$

6.6.1 a

a) $E[G] = 2 * 7 / 10 = 7 / 5$

6.6.4 a, b

a) $(12 + 22 + 32 + 42 + 52 + 62) / 6 = 91 / 6$

b) $(0 + 3 * 1 + 3 * 4 + 9) / 8 = 3$

6.7.4 a

a) $(1/10) * 10 = 1$

Question 10

6.8.1 a-d

a) $(100 \text{ choose } 2) \cdot 0.01^2 \cdot 0.99^{98}$

b) $1 - [(100 \text{ choose } 0) \cdot 0.01^0 \cdot 0.99^{100} + (100 \text{ choose } 1) \cdot 0.01^1 \cdot 0.99^{99}]$

c) $100 \cdot 0.01 = 1$

d) Probability of 100 circuit boards, at least having a defect =

$$= 1 - [(50 \text{ choose } 0) \cdot 0.01^0 \cdot 0.99^{50}]$$

What is the expected number of circuit boards with defects out of the 100 made?

$$= 2 \cdot 50 \cdot 0.01 = 0.5 \cdot 2 = 1$$

There are different answers in the case with batches vs independent circuit boards as circuit boards are not independent in the batches example.

6.8.3 b

b) We reach the incorrect solution when there are at least 4 heads.

$$P(H \geq 4 \mid \text{Biased}) =$$

$$1 - \left[\binom{10}{0} \cdot 0.3^0 \cdot 0.7^{10} + \binom{10}{1} \cdot 0.3^1 \cdot 0.7^9 + \binom{10}{2} \cdot 0.3^2 \cdot 0.7^8 + \binom{10}{3} \cdot 0.3^3 \cdot 0.7^7 \right]$$