Lemma $\quad \left| \sum_{0 \le k \le t} \frac{(-t)^k}{k!} \right| \gtrsim \frac{e^t}{t^{3/2}} \quad t \to \infty$

Pf Taylor Thm w/ remainder implies

$$e^{-t} = \sum_{0 \le k \le t+1} \frac{(-t)^k}{k!} + e^{-s} \frac{t^{t+2}}{(t+2)!}$$

for some $0 < s < t+1$.

The error term is at most $\frac{t^{t+2}}{t+2!}$

So

$$\left| \sum_{0 \le k \le t} \frac{(-t)^k}{k!} \right| \ge \frac{t^{t+1}}{t+1!} - e^{-t} - \frac{t^{t+2}}{t+2!}$$

$$= \frac{t^{t+1}}{t+1!} \left(1 - \frac{t}{t+2} \right) - e^{-t}$$

$$\sim \frac{2e^t}{\sqrt{2\pi}\; t^{3/2}} \qquad t \longrightarrow \infty$$