**Journal of Number Theory (JNT)**
**Manuscript Revision (April 2021)**
*New characterizations of the summatory function of the Möbius function*

Dear editors and reviewers,

I have done my best to edit and correct all previous "typographical" flaws that now indicate what I must surely have meant in the first submission. There is, of course, the possibility that I have missed something along the way in making these changes to my original manuscript. Please know that I am open to detailed feedback and suggestions by the referee moving forward.

I have also been communicating with Gergő Nemes of the Alfréd Rényi Institute of Mathematics online for the past few months. He has helped me to fill in some gaps and to pin down, polish and rigorously prove some lemmas (in the appendix section) that provide necessary asymptotics for certain sums in my proofs used in the body of the article. He is satisfied with the writeup, and with the way I have given him credit for that expertise communicated in the article, and with the new citations to his original research articles needed to establish these results. The man is a friendly mathematician worth meeting sometime and is an expert on the intricate asymptotics of the incomplete gamma function, including the asymptotic relations that dominate its particular limiting behavior for the required parameter tendencies at work in this article.

I honestly believe based on some more recent combinatorial efforts I have looked at to sum the function `$G^{-1}(x)$` that the work in this article could eventually lead to a breakthrough on the Mertens function. Namely, I eventually see (somewhere down the line) that this new work can be used to prove that `$\limsup_{x \rightarrow \infty} M(x) / \sqrt{x} = +\infty$` or `$\liminf_{x \rightarrow \infty} M(x) / \sqrt{x} = -\infty$` using methods that do not directly in this case depend on contour integrals over the Riemann zeta function in the complex plane. This interpretation is there, definitely as in all classical senses it must be, though this methodology has notably stalled progress on this type of problem for ages for the usual sets of complicated reasons we would expect. I do not have rigorous proofs of this claim nor expectation yet. That said, I believe with all my heart and thought process that this manuscript getting published in JNT this year will be a big progress marker for both myself and the journal in the coming years. As such, please bear with me in correcting any more "typographical" type errors (as characterized above) that you still find hidden in this work. :)

Thank you to the editors, and to the anonymous referee for the encouragement I received in getting quick feedback about the initial form of the manuscript several months ago. I respect your time very much, and hope that the revised manuscript attached to this submission is worth waiting for and reviewing carefully this time.

Sincerely,

Maxie Dion Schmidt
Ph.D. Candidate, Georgia Institute of Technology, 2022