

1 Overview

The primary research interests of the PI are in combinatorial and analytic number theory with an emphasis on integer sequences, generating function methods, continued fractions, software development, and experimental mathematics. The proposed host institution is Penn State University (PSU) under the direction of supporting mathematician Prof. R. C. Vaughan. The expertise of the PSU faculty in both analytic and combinatorial number theory with a focus on partitions will provide the support needed to continue creative work in the areas in which the applicant (the PI; henceforth, MDS) already has 21+ publications and manuscripts. The breadth of the tenure track PSU math faculty also offers unique prospects to get involved in new work with innovative mathematicians.

2 Intellectual Merit

The work of MDS on the Mertens function, $M(x)$, over the past few years explores new unconventional connections between strong additivity and signed sums of multiplicative functions. Her research on this function connects $M(x)$ with a key unsigned auxiliary sequence and the characterization of its distribution with an explicit non-centrally normal probability distribution. A favorite quote of hers due to her NSF sponsor is reproduced as follows: *"It is evident that the primes are randomly distributed but, unfortunately, we do not know [yet] what 'random' means."* The recent work of hers on cross-correlation statistics provides new ways to express deep connections of number theoretic objects, such as the distribution of the primes and divisor sums of multiplicative functions, to other branches of mathematics such as partitions. Her plans as a postdoc are to continue to publishing work in number theory and combinatorial analysis. This includes generalizing the new connections connecting signed sums of sequences besides $\mu(n)$ to additivity with explicit probability distributions and work on the open questions in her doctoral thesis research. The project description precisely motivates her proposed work at PSU and addresses significant open problems.

3 Broader Impacts

From 2020–2021, MDS worked on a larger scale, energizing and experimental security project that was a welcome challenge and software outlet away from her pure math research over the pandemic. The project brings sophisticated new low-level embedded firmware extensions to a type of NFC penetration testing and wireless data sniffer-loggers (the Chameleon Mini) that interfaces with common NFC tags (e.g., most university student ID cards). This project afforded a heightened platform online over which to bring more awareness by users to important issues that have increasingly beleaguered the world this pandemic. In particular, the first beta testing versions became a reality the long weekend of the sad passing of inspirational US supreme court justice Ruth Bader Ginsberg on Rosh Hashanah, to whom MDS was able to pay tribute by working on this software. MDS is proud to have "hacked" for freedom with free software, as she likes to say, for days on end while cloaked in an over-sized *Black Lives Matter* shirt. The experience working on this project speaks to the power of open source software (OSS) and how MDS intends to use its reach as a postdoc.

MDS also feels that she will be able to more broadly contribute to initiatives in mathematics and other STEM fields as a NSF postdoctoral fellow at PSU. One crucial component of the proposed activities at PSU is to continue collaborating on OSS with a focus on advancing educational STEM initiatives. For example, *Prairie Learn* is a LMS that is a viable option to replace *Canvas* at many institutions. It is actively developed at a few leading universities. She has so far contributed code to enhance the `sympy` Python library parsing within the project. This enables parsing for questions posed to students in mathematics and physical sciences. MDS will actively hold a seminar focused on teaching all types of students and educators how to use and write software using OSS in mathematics as a postdoc at PSU.

MDS reflects positively on having benefited over the years from her undergraduate research experiences with Prof. Bruce Reznick and mentoring experiences with Prof. Bruce Berndt who was influential in developing her mathematical writing style. MDS reflects being taken seriously as an undergraduate pursuing original research was particularly encouraging through her mentors. She plans to pass on the wisdom she has learned by talking with these experienced career mathematicians to young motivated researchers as her own professional career moves forward. This especially applies to offering advice and influencing talented women in mathematics to develop their own early original research work as undergraduates and onward. MDS will make these plans a reality by leading seminars at PSU targeted at broader outreach by getting women and minority students at or below the graduate level involved in fulfilling extracurricular roles presenting their research and mentoring other students in math at PSU.