We have that

$$\#\{3 \le N \le x : \Omega(N) \le \ell\ell N\} = \frac{x}{2} + O\left(\frac{x}{\sqrt{\ell\ell x}}\right)$$

→ note the bound is flipped in your comment

Using the reversed inequa. form for the func $B(x,r)$, $1 \le r < 2$ and $x \ge 2$, it should be that

$$B(x,1) \ll x$$

$$\Rightarrow \frac{1}{x}|N_\omega(x)| \ll 1$$

Really, it still gives the correct result:

$$\left|\frac{N_\omega(x)}{P_\omega(x)}\right|^{-1} = O\left(\frac{1}{\sqrt{\ell\ell x}}\right) = o(1)$$

So, I guess I need to write at the end of the proof that

$$1+o(1) \ll \left|\frac{P_\omega(x)}{A_\omega(x)}\right| \ll 1+o(1),$$

but the idea is the same: You capture the dominant asymp. main term by only summing over the $\pi_k(x)$ in the range $1 \le k \le \ell\ell x$ where we get the nice uniform bounds. That is what I use in Section 4 that is important here.