Personal Statement Essay

Essay Prompt:

Describe any personal, professional, or educational experiences or situations that have contributed to your preparation and desire to pursue advanced study in science, technology, engineering, or mathematics. Describe your leadership potential, and how you see yourself currently or in the future contributing to research, education, and innovations in science and engineering. Discuss your career aspirations and some goals you hope to achieve. Note: The personal statement is not a repeat of your research statement. What is important is the content, not the length.

NSF Fellows are expected to become globally engaged knowledge experts and leaders who can contribute significantly to research, education, and innovations in science and engineering. The purpose of this essay is to demonstrate your potential to satisfy this requirement. Your ideas and examples do not have to be confined necessarily to the discipline that you have chosen to pursue.

Overview

For this essay response I will focus on how my personal research interests have evolved since I first enrolled for study at the University of Illinois. I will also address how the support offered by the NSF GRFP will assist me in pursuing my research and graduate studies.

Motivation for Research in Computer Science and Mathematics

I want a career working on topics that are both important and interesting. I have always been drawn to the forms and properties of recurrence relations. I was introduced to the subject through study of the recursively–defined sequence of the Fibonacci numbers, F_n. A surprising and well-known result for the seemingly simple Fibonacci numbers is that the numbers can be generated in order through the closed–form of Binet's formula. In the summer before my first semester at the University of Illinois I became interested in finding forms related to the set of prime numbers. Inspired by the unusual result of Binet's formula for the Fibonacci numbers, I started challenging myself to find such a non–intuitive result for the fascinating sequence of primes. The explorations of this topic first sparked my interest in developing research of my own and has led me to consider many other topics such as by providing the original motivation for considering the topics in my 2010 publication related to expansions of generalized factorial functions as well as motivation for the research considered in my *Proposed Research* essay.

In my sabbatical from the University of Illinois (2005–2009) I had medical issues that prevented me from continuing in formal academic studies for an extended period of time. I knew that my interests would eventually lead me to graduate studies and I pursued research offers from professors to work on their projects. Prior to my time away from a campus environment, I had never had so much free time to devote to my own software projects and personal interests in Mathematics research. During this time I was able to focus my efforts almost exclusively on my own work. It was through this experience that I found a strong passion for my own independent and original research. I have decided that I can look forward to giving the same full–time intensity to my studies that is required by a rigorous Ph.D. program in my fields of study. This realization has strengthened my dedication to a career in both Computer Science and Mathematics research.

Intellectual Merit: Research, Publications, and Prior Studies

Since returning to my undergraduate studies at the University of Illinois in I have completed a dual degree program in both Computer Science in the College of Engineering and in Mathematics in the College of Liberal Arts and Sciences. In May of 2012 I was awarded both a Bachelor of Science degree in Computer Science with institutional honors of *Highest Honors* and a B.S. degree for Mathematics with institutional honors of *Cum Laude* and departmental honors of *Highest Distinction MATH*. In 2010, I received the Barry M. Goldwater scholarship. The scholarship is awarded to current undergraduate students who demonstrate strong research potential and who are committed to a Ph.D. degree in fields of Mathematics, Engineering, and other scientific disciplines. I also published an article in the *Journal of Integer Sequences* (*JIS*) in 2010 based on a significant subset of my independent Mathematics research conducted in my time away from formal academic studies over 2005–2009. A review of the *JIS* publication appeared on *MathSciNet* in 2011 as article `MR2659223 (2011h:05009)`.

Research Interests and Broader Impacts

My particular areas of interest combine Mathematics and Computer Science with emphasis on number theory, enumerative combinatorics, and software development. I intend to conduct research and publish in my fields of study, both working on my personal research in my free time and as a professional in industry. I will continue to publish significant findings in professional journals and other peer–reviewed scholarly outlets. As a female in my fields of study, I will also continue to pursue challenging research problems with cross-disciplinary applications in Mathematics, Computer Science, and other engineering sciences.

In addition to my publication in the *Journal of Integer Sequences* (*JIS*) in 2010, I have submitted an article based on so–termed "square series" generating function transformations to the Journal of *Integral Transforms and Special Functions* where it is still currently under review. I have recently participated in the *Young Mathematicians Conference* (*YMC*, supported by NSF grant DMS-0841054) in the Summer of 2012 by giving a report talk presentation based on a subset of the original 54–page *JIS* article research, including a summary of some of my more recent (generalized) harmonic–number–related results related to these topics. I have also met with several professors at the University of Illinois about the new results in my Summer 2012 REGS research in the Department of Mathematics related to the forms of Euler sums and Dirichlet series (sponsored by NSF grant DMS 08-38434 as EMS-MCTP: Research Experience for Graduate Students). I intend on submitting this research for publication later on this Fall in the *International Journal Number Theory*, where Bruce Berndt, one of my letter writers for the NSF GRFP application, is a member of the editorial board. I have also independently developed two separate open–source–software projects where the program source code is freely available in the project webpages for purposes including educational study.

Motivations for Applying for NSF GRFP Support of My Graduate Studies at the University of Illinois

I am currently a graduate student in the Computer Science Ph.D. Program at the University of Illinois at Urbana–Champaign where I hold a teaching assistant position for the CS 173 *Discrete Structures* course. I chose to attend the University of Illinois for my graduate studies for its strong academic programs in both of my primary undergraduate areas of study and for the university's strengths in other engineering sciences. In particular, the University of Illinois has world-class research faculty in Computer Science, areas of Mathematics including Number Theory and Combinatorics, and other scientific disciplines. Additionally, the University of Illinois has significant funding available for performing scientific research including resources such as the university's development of cutting-edge supercomputer technology.

I feel that the support from the NSF GRFP will allow me to focus more fully on my research, publications, and graduate studies at the University of Illinois. The fellowship award will also allow me the flexibility to pursue more research–based graduate coursework more quickly in the timeline of my program of study for the Ph.D. program. For example, the focus of my Math 597 *Reading Course* studies this Fall is to continue the research from my research this summer and to complete a manuscript of this research to be submitted for publication this Fall. The fellowship award and tuition support will allow me to focus more completely on continuing along this line of research work.

Previous Research Essay

Essay Prompt:

Describe any scientific research activities in which you have participated, such as experience in undergraduate research programs, or research experience gained through summer or part-time employment or in work-study programs, or other research activities, either academic or job-related. Explain the purpose of the research and your specific role in the research, including the extent to which you worked independently and/or as part of a team, and what you learned from your research experience. Describe how you disseminated your results (i.e. conference, symposium, publication). In your statement, distinguish between undergraduate and graduate research experience. If you provided a complete list of publications and presentations in the Education and Work Experience section of the application, it is acceptable to list the highlights of your publication and presentation record in this essay. Any list of publications and presentations included in this essay counts towards the page limit of this essay.

If you have no direct research experience, describe any activities that you believe have prepared you to undertake research.

Introduction: An Approach to Primes of the Form 4k + 1

The formulation of exact closed–form functions that generate the prime numbers ordered over the positive integers is a task that has engaged mathematicians for centuries. In the 18^{th} century, the prominent mathematician A. Legendre proved that there is no rational algebraic function that generates only primes. As recently as in the last twenty years, progress on the subject produced proofs that there are polynomials in many variables that do in fact generate the primes exactly, as well as lengthy lists of Diophantine equations that guarantee primality. In practice these results are complex and still do not produce the desired closed–form to compute the sequence of primes.

I have always been drawn to the forms and properties of recurrence relations. I was introduced to the subject through study of the sequence of the Fibonacci numbers, F_n, whose form is classically defined by the recurrence $F_n = F_{n-1} + F_{n-2}$. A surprising and well-known result for the seemingly simple Fibonacci numbers is that the numbers can be generated in order through Binet's formula for the sequence as $F_n = \left[\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n\right]/2^n\sqrt{5}$. In the summer before my first semester at the University of Illinois I became consumed with the pursuit of forms related to the set of prime numbers, $\mathbb{P} = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$. Inspired by the unusual result of Binet's formula for the Fibonacci numbers, I started challenging myself to find such a non–intuitive result for the fascinating sequence of primes. The explorations of this topic first sparked my interest in developing research of my own.

My explorations of the divisibility properties of the prime number sequence led me to consider the general formula, $\mathrm{J}_x(y)$, given in (1).

$$\mathrm{J}_x(y) = -\frac{1}{x}\left(2\ (x-1)^{\frac{(y-1)}{2}} - 2 + x\ y\right) \tag{1}$$

The construction that yields the special case that happens to be significant in the formal definition of a "*nearly prime*" integer sequence, $\mathbb{J}_5$, is given in the following equation.

$$\begin{aligned} \mathbb{J}_5 &= \{y \in \mathbb{Z} \mid\ y \text{ divides } \mathrm{J}_5(y) \text{ (i.e. } \mathrm{J}_5(y)/y \in \mathbb{Z})\} \\ &= \{5\} \cup \{13, 17, 29, 37, 41, 53, \ldots\} \cup \{341, 561, 1729, 2701, \ldots\} \end{aligned} \tag{2}$$

The form of the subset of primes of the form $p = 4k + 1$, denoted $\mathbb{P}_{4k+1}$, and $\mathbb{J}_5$ in (2) are interesting and, as it turns out, are surprisingly closely related. In particular, the following properties are apparent:

1. The sequence (ordered set) of primes $\mathbb{P}_{4k+1} = \{5, 13, 17, 29, 37, 41, 53, 61, 73\ldots\}$ is a strict subset of $\mathbb{J}_5 \cup \{5\}$, i.e. all of the elements $p \in \mathbb{P}_{4k+1}$ are contained in $\mathbb{J}_5 \cup \{5\}$.
2. The distribution of non–prime (quasi–prime) elements in $\mathbb{J}_5$ is sparse, i.e., most of the elements of $\mathbb{J}_5$ are primes of the form $p = 4k + 1$. In this sense the sequence is considered "*nearly prime*" in that most elements contained in the set are prime.

My work with the set $\mathbb{J}_5$ has led me to consider many other topics in my research and served as the original motivation for the approach formulated in my *Proposed Research* essay to find the values where $\mathrm{J}_5(y)/y$ is integer–valued (that is, $y \in \mathbb{J}_5$).

Other Research Experience and Publications

The research for my publication in the *Journal of Integer Sequences* (*JIS*) grew out of the prime–related formula in (1) by considering the factorial function expansions of binomial coefficients resulting from the expansion of $\mathrm{J}_5(y)$ by the binomial theorem. Since the original *JIS* publication in 2010, a review of the article stating that the research presented is excellent and fit as a starting point for further study in enumerative combinatorics appeared on *Math-*

SciNet cited as article `MR2659223 (2011h:05009)`. I presented a report talk this summer based on a subset of the research from the article at the *Young Mathematicians Conference* held at The Ohio State University from July 27–29 of 2012. In my talk at the conference, I also included a summary of some of my more recent work on generalized harmonic number sequences originally considered by the *JIS* article. The generalized results include the work on my CS 499 *Senior Thesis* project from 2011 where I considered the forms of more general zeta function series. The research for the thesis project is based on recursively–defined triangular coefficients that are generalize the form of the coefficient definitions in the *JIS* article and that are used to expand the partial sums corresponding to a generalized class of infinite series of the form $\sum_k t^k / f(k)^p$. The classical Riemann and Hurwitz zeta functions are special cases of this generalized series form. The topic of my project for the Math 496 honors course in *Mathematical Research* from 2010, which I also plan to submit for publication in another peer–reviewed journal, is related to the multiple factorial functions explored in the *JIS* article. In this case, the factorial function expansions are considered through recursively–defined continued fraction series that generate the products of these functions.

In the Fall of 2011, I submitted an original research article on "*square series*" generating function transformations to the journal of *Integral Transforms and Special Functions* where it is still under review. Finally, I participated in the salaried graduate REGS research experience at the University of Illinois in the Summer of 2012. I will submit the results of the research to the *International Journal of Number Theory* for publication this Fall.

Software Projects and Related Research Experience

The following is a chronological listing of the more substantial software projects that I have been involved in since I first enrolled at the University of Illinois at Urbana–Champaign.

Classnotes Optical Character Recognition Application (2006–Present)

Classnotes is a GUI–based (Graphical User Interface) Optical Character Recognition (OCR) application that is designed to generate plaintext output from scanned images of non–standard font printouts. The algorithm employed by the program is similar to trainable Bayesian e–mail spam filters, except that instead of finding patterns in text, the program operates on gridded image data. The C++ source code for the program is available on the project's webpage at `http://classnotes.sourceforge.net`. I developed the application in my free time as an extension of my Computer Science studies using at least two original algorithms created by myself for the project.

Math Pattern Hunter Project (2006–Present)

The project is a math–related program written in C++ that attempts to find closed–form formulas and other identities for input integer and rational sequences by means of programmed and configurable brute force. The source code for the program can be downloaded from the project's webpage at `http://pattern-hunter.sourceforge.net`. I developed the software in my free time as an extension of my mathematics research and am the only developer for the project.

Intel / Lockheed Martin Scholars Program (2005–2006)

The program is a salaried research experience at the University of Illinois at Urbana–Champaign for the Laser Charger project. My responsibilities involved development of software to process and recognize input video and microcontroller programming to control electronic peripherals. I learned quite a bit about C and low–level Linux programming through the project.

Proposed Research Title

Prompts:

The title should be brief and informative. It should describe in succinct terms your proposed research, reflecting the contents of your proposal. Include a list of key words, and do not use abbreviations and chemical formulas (in 255 characters or less). This title will be used for searching research topics using the key words you supply.

Use key words to describe the proposed research (in 50 characters or less).

Proposed Research Title:

An Approach to Determining the Form of Sequences Over the Integers Through Expansions of the Periodic Zeta Function With Applications to Subsequences of the Prime Numbers

Short Research Title:

Determining Sequence Formulas Using the Periodic Zeta Function With Prime Number Applications

Keywords: (TODO: 50 Character Limit)

Periodic zeta function, Riemann zeta function, polylogarithm function, prime numbers, twin primes

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Proposed Research Essay

Essay Prompt:

In a clear, concise, and original statement, present a complete plan for a research project that you plan to pursue during the Fellowship Tenure.

Your statement should demonstrate your understanding of research design and methodology and explain the relationship to your previous research, if any.

Format: Introduction and problem statement, hypothesis, methods to test hypothesis, anticipated results or findings, expected significance and broader impacts, and a short list of important literature citations. If you have not formulated a research plan, your statement should include a description of a research topic that interests you and how you might conduct research on that topic.

In addition to review of the Intellectual Merit and Broader Impacts of your proposal, research topics discussed in your proposed plan must be in fields within NSF's mission.

Introduction

Context and Prime Number Applications

The study of the prime numbers, $\mathbb{P} = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$, is fundamental to nearly all branches of number theory. Prime number results are important to many other disciplines such as computer and physical sciences, with applications to public–key cryptography, integer factorization problems, pseudo–random number generation, hash tables, and quantum mechanics. Since prime numbers are involved in the applied theory of so many scientific fields, understanding the prime sequence in more detail will yield immediate applications to further theoretical developments in these fields of study – even far outside of the interests of pure mathematical research.

Open Problems in Number Theory and Integer Sequences Research

One famous open question in number theory concerns whether there are infinitely–many twin primes, i.e. prime pairs of the form $(p, p+2)$. The problem of determining whether there are infinitely–many primes of the form n^2+1 is also open. These and many other problems in integer sequences research can be posed as questions of whether a real–valued function for the sequence is integer–valued. The approach formulated in the next sections attempts to identify the $p \in \mathbb{N}$ where an application–dependent function, $\mathrm{W}(p)$, is integer–valued through application of my existing research results and known properties of the periodic zeta function.

Motivation

Congruences for Subsequences of the Prime Numbers

Many prime–related sequences are defined through congruence formulas in established theorems related to modular arithmetic over the integers. Perhaps the most well–known of such theorems is Wilson's theorem given in (3) and Clement's theorem concerning twin primes given in (4):

$$(p-1)! \equiv -1 \pmod{p} \iff p \text{ prime} \tag{3}$$

$$4\left[(p-1)!+1\right] \equiv -p \pmod{p(p+2)} \iff p,\ p+2 \text{ prime.} \tag{4}$$

The primality conditions given by the previous theorem statements can each be posed as a question of whether a corresponding rational function of p is integer–valued at a set $p \in \mathbb{N}$.

Special Properties of the Periodic Zeta Function

Consider the following definition of the polylogarithm function, $\mathrm{Li}_s(z)$, given in (5) where $s > 1$ is real–valued. The corresponding definitions of the Riemann zeta function, $\zeta(s) := \mathrm{Li}_s(1)$, and the periodic zeta function, $F(x,s) := \mathrm{Li}_s(\exp(2\pi\imath x))$ are special cases of the series in (5). [3, cf. §25.14; §25.13]

$$\mathrm{Li}_s(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^s} \tag{5}$$

The form of the periodic zeta function, $F(x,s)$, satisfies the particularly interesting property that

$$F(x,s) = \zeta(s) \quad \iff \quad \exp(2\pi\imath x) = 1 \quad \iff \quad x \in \mathbb{Z}$$

where $\zeta(s)$ is known in closed–form as $\zeta(2m) = (2\pi)^{2m}|B_{2m}|/2(2m)!$ whenever $s := 2m$ is a positive even integer and where B_k is a Bernoulli number.

Problem Statement and Relations to Previous Research

Precise Problem Statement in Terms of the Periodic Zeta Function

The problem for determining the integer values, k, of a fixed (real or rational–valued) function for the general sequence case, $\mathbb{S}$, may then be phrased as

$$\begin{aligned} S(k) \in \mathbb{S} \quad \iff \quad W_{\mathbb{S}}(k) \in \mathbb{Z} \quad & \iff \quad F(W_{\mathbb{S}}(k), 2) = \zeta(2) = \frac{\pi^2}{6} \\ & \iff \quad F(W_{\mathbb{S}}(k), 4) = \zeta(4) = \frac{\pi^4}{90}. \end{aligned} \tag{6}$$

Note that the function $W_{\mathbb{S}}(p)$ in (6) need not be directly tied to theorems of modular arithmetic over the integers, though particular prime–related cases of (6) can be phrased through the theorems given in (3) and (4).

Applications of the 2012 REGS Research Related to Polylogarithm Functions

One application of my existing research is given by the following non–recursive, closed–form series for the polylogarithm function from (5) in terms of the r–order harmonic numbers, $H_n^{(r)} := \sum_{k=1}^n k^{-r}$, is the following [1, cf. §2]:

$$\begin{aligned} \mathrm{Li}_2(z) &= \sum_{j=1}^\infty \left[\frac{(-1)^{j-1}}{2j!}\left(H_j^2 + H_j^{(2)}\right)\right] \frac{z^j j!}{(1-z)^{j+1}} \\ \mathrm{Li}_4(z) &= \sum_{j=1}^\infty \left[\frac{(-1)^{j-1}}{24j!}\left(H_j^4 + 6H_j^2 H_j^{(2)} + 3\left(H_j^{(2)}\right)^2 + 8H_j H_j^{(3)} + 6H_j^{(4)}\right)\right] \frac{z^j j!}{(1-z)^{j+1}}. \end{aligned} \tag{7}$$

My approach to the problem phrased in (6) through the periodic zeta function, $F(x,s)$, is to apply the results of my REGS research cited in (7) to the respective special case forms of $F(x,s) := \mathrm{Li}_s(\exp(2\pi\imath x))$.

Expected Significance and Broader Impacts

Open Problems Addressed by the Research

The next equations follow from the theorems in (3) and (4) and are related to the problem statement in (6) for each of the noted functions, $W_{\mathbb{S}}(p)$.

$$W_{\mathrm{TP}}(p) := \left[4(p-1)! + 4 + p\right]/p(p+2) \in \mathbb{Z} \quad \iff \quad p,\ p+2 \text{ are prime} \tag{8}$$

$$W_{\mathrm{NSQ}}(n) := \left[(n^2)! + 1\right]/(n^2+1) \in \mathbb{Z} \quad \iff \quad n^2+1 \text{ is prime} \tag{9}$$

$$W_{\mathrm{Fib}}(p) := \left[(F_p - 1)! + 1\right]/F_p \in \mathbb{Z} \quad \iff \quad p^{th} \text{ Fibonacci number is prime} \tag{10}$$

The open problems concerning the infinitude of the twin primes and of primes of the form n^2+1 can be posed as determining whether there are infinitely–many natural numbers such that the respective results in (8) and (9) hold. The problem of finding the distribution of prime elements of the Fibonacci number sequence (and of other sequence forms similarly) can be studied through the result in (10).

Intention for Publication and Broad Dissemination of Results

I intend to publish the results of my research on these topics in peer–reviewed journals, present the results through talks at professional conferences, and make the research broadly available for educational, teaching, and other purposes in venues such as the web.

References and Literature Citations

[1] V. Adamchik. On Stirling numbers and Euler sums. *J. Comput. Appl. Math.*, 79(1):119–130, 1997.

[2] P. Flajolet and B. Salvy. Euler sums and contour integral representations. *Experimental Mathematics*, 7(1), 1998.

[3] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, editors. *NIST Handbook of Mathematical Functions*. Cambridge University Press, 2010.