

T.2 Table: Computations with a highly signed Dirichlet inverse function

n	Primes		Sqfree	PPower	\mathbb{S}		$g^{-1}(n)$	$\lambda(n) \operatorname{sgn}(g^{-1}(n))$	$\lambda(n)g^{-1}(n) - \hat{f}_1(n)$	$\lambda(n)g^{-1}(n) - \hat{f}_2(n)$		$G^{-1}(n)$	$G_+^{-1}(n)$	$G_-^{-1}(n)$
1	1 ¹	–	Y	N	N	–	1	1	0	0	–	1	1	0
2	2 ¹	–	Y	Y	N	–	–2	1	0	0	–	–1	1	–2
3	3 ¹	–	Y	Y	N	–	–2	1	0	0	–	–3	1	–4
4	2 ²	–	N	Y	N	–	2	1	0	–1	–	–1	3	–4
5	5 ¹	–	Y	Y	N	–	–2	1	0	0	–	–3	3	–6
6	2 ¹ 3 ¹	–	Y	N	N	–	5	1	0	–1	–	2	8	–6
7	7 ¹	–	Y	Y	N	–	–2	1	0	0	–	0	8	–8
8	2 ³	–	N	Y	N	–	–2	1	0	–2	–	–2	8	–10
9	3 ²	–	N	Y	N	–	2	1	0	–1	–	0	10	–10
10	2 ¹ 5 ¹	–	Y	N	N	–	5	1	0	–1	–	5	15	–10
11	11 ¹	–	Y	Y	N	–	–2	1	0	0	–	3	15	–12
12	2 ² 3 ¹	–	N	N	Y	–	–7	1	2	–2	–	–4	15	–19
13	13 ¹	–	Y	Y	N	–	–2	1	0	0	–	–6	15	–21
14	2 ¹ 7 ¹	–	Y	N	N	–	5	1	0	–1	–	–1	20	–21
15	3 ¹ 5 ¹	–	Y	N	N	–	5	1	0	–1	–	4	25	–21
16	2 ⁴	–	N	Y	N	–	2	1	0	–3	–	6	27	–21
17	17 ¹	–	Y	Y	N	–	–2	1	0	0	–	4	27	–23
18	2 ¹ 3 ²	–	N	N	Y	–	–7	1	2	–2	–	–3	27	–30
19	19 ¹	–	Y	Y	N	–	–2	1	0	0	–	–5	27	–32
20	2 ² 5 ¹	–	N	N	Y	–	–7	1	2	–2	–	–12	27	–39
21	3 ¹ 7 ¹	–	Y	N	N	–	5	1	0	–1	–	–7	32	–39
22	2 ¹ 11 ¹	–	Y	N	N	–	5	1	0	–1	–	–2	37	–39
23	23 ¹	–	Y	Y	N	–	–2	1	0	0	–	–4	37	–41
24	2 ³ 3 ¹	–	N	N	Y	–	9	1	4	–3	–	5	46	–41
25	5 ²	–	N	Y	N	–	2	1	0	–1	–	7	48	–41
26	2 ¹ 13 ¹	–	Y	N	N	–	5	1	0	–1	–	12	53	–41
27	3 ³	–	N	Y	N	–	–2	1	0	–2	–	10	53	–43
28	2 ² 7 ¹	–	N	N	Y	–	–7	1	2	–2	–	3	53	–50
29	29 ¹	–	Y	Y	N	–	–2	1	0	0	–	1	53	–52
30	2 ¹ 3 ¹ 5 ¹	–	Y	N	N	–	–16	1	0	–4	–	–15	53	–68
31	31 ¹	–	Y	Y	N	–	–2	1	0	0	–	–17	53	–70
32	2 ⁵	–	N	Y	N	–	–2	1	0	–4	–	–19	53	–72
33	3 ¹ 11 ¹	–	Y	N	N	–	5	1	0	–1	–	–14	58	–72
34	2 ¹ 17 ¹	–	Y	N	N	–	5	1	0	–1	–	–9	63	–72
35	5 ¹ 7 ¹	–	Y	N	N	–	5	1	0	–1	–	–4	68	–72
36	2 ² 3 ²	–	N	N	Y	–	14	1	9	1	–	10	82	–72
37	37 ¹	–	Y	Y	N	–	–2	1	0	0	–	8	82	–74
38	2 ¹ 19 ¹	–	Y	N	N	–	5	1	0	–1	–	13	87	–74
39	3 ¹ 13 ¹	–	Y	N	N	–	5	1	0	–1	–	18	92	–74
40	2 ³ 5 ¹	–	N	N	Y	–	9	1	4	–3	–	27	101	–74
41	41 ¹	–	Y	Y	N	–	–2	1	0	0	–	25	101	–76
42	2 ¹ 3 ¹ 7 ¹	–	Y	N	N	–	–16	1	0	–4	–	9	101	–92
43	43 ¹	–	Y	Y	N	–	–2	1	0	0	–	7	101	–94
44	2 ² 11 ¹	–	N	N	Y	–	–7	1	2	–2	–	0	101	–101
45	3 ² 5 ¹	–	N	N	Y	–	–7	1	2	–2	–	–7	101	–108
46	2 ¹ 23 ¹	–	Y	N	N	–	5	1	0	–1	–	–2	106	–108
47	47 ¹	–	Y	Y	N	–	–2	1	0	0	–	–4	106	–110
48	2 ⁴ 3 ¹	–	N	N	Y	–	–11	1	6	–4	–	–15	106	–121
49	7 ²	–	N	Y	N	–	2	1	0	–1	–	–13	108	–121
50	2 ¹ 5 ²	–	N	N	Y	–	–7	1	2	–2	–	–20	108	–128
51	3 ¹ 17 ¹	–	Y	N	N	–	5	1	0	–1	–	–15	113	–128
52	2 ² 13 ¹	–	N	N	Y	–	–7	1	2	–2	–	–22	113	–135
53	53 ¹	–	Y	Y	N	–	–2	1	0	0	–	–24	113	–137
54	2 ¹ 3 ³	–	N	N	Y	–	9	1	4	–3	–	–15	122	–137
55	5 ¹ 11 ¹	–	Y	N	N	–	5	1	0	–1	–	–10	127	–137
56	2 ³ 7 ¹	–	N	N	Y	–	9	1	4	–3	–	–1	136	–137

Table T.2: Computations of the first several cases of $g^{-1}(n) \equiv (\omega + 1)^{-1}(n)$ for $1 \leq n \leq 56$.

The column labeled **Primes** provides the prime factorization of each n so that the values of $\omega(n)$ and $\Omega(n)$ are easily extracted. The columns labeled, respectively, **Sqfree**, **PPower** and **\mathbb{S}** list inclusion of n in the sets of squarefree integers, prime powers, and the set \mathbb{S} that denotes the positive integers n which are neither squarefree nor prime powers. The next two columns provide the explicit values of the inverse function $g^{-1}(n)$ and indicate that the sign of this function at n is given by $\lambda(n) = (-1)^{\Omega(n)}$. Then the next two columns show the small-ish magnitude differences between the unsigned magnitude of $g^{-1}(n)$ and the summations $\hat{f}_1(n) := \sum_{k \geq 0} \binom{\omega(n)}{k} \cdot k!$ and $\hat{f}_2(n) := \sum_{k \geq 0} \binom{\omega(n)}{k} \cdot \#\{d|n : \omega(d) = k\}$. Finally, the last three columns show the summatory function of $g^{-1}(n)$, $G^{-1}(x) := \sum_{n \leq x} g^{-1}(n)$, broken down into its respective positive and negative components: $G_+^{-1}(x) := \sum_{n \leq x} g^{-1}(n) [g^{-1}(n) > 0]_\delta$ and $G_-^{-1}(x) := \sum_{n \leq x} g^{-1}(n) [g^{-1}(n) < 0]_\delta$.