Notation and conventions

The next listing provides a glossary of common notation, conventions and abbreviations employed throughout the article.

Symbols	**Definition**
$\gg, \ll, \asymp$	For functions A, B, the notation $A \ll B$ implies that $A = O(B)$. Similarly, for $B \geq 0$ the notation $A \gg B$ implies that $B = O(A)$. When we have that $A, B \geq 0$, $A \ll B$ and $B \ll A$, we write $A \asymp B$.
$\approx, \sim$	We write that $f(x) \approx g(x)$ if $\lvert f(x) - g(x)\rvert \ll 1$ as $x \to \infty$. Two arithmetic functions $A(x), B(x)$ satisfy the relation $A \sim B$ if $\lim_{x\to\infty} \frac{A(x)}{B(x)} = 1$.
$\chi_{\mathbb{P}}(n), P(s)$	The indicator function of the primes equals one if and only if $n \in \mathbb{Z}^+$ is prime, and is zero-valued otherwise. For any $s \in \mathbb{C}$ such that $\operatorname{Re}(s) > 1$, we define the prime zeta function to be the Dirichlet generating function (DGF) defined by $P(s) = \sum_{n\geq 1} \frac{\chi_{\mathbb{P}}(n)}{n^s}$. The function $P(s)$ has an analytic continuation to the half-plane $\operatorname{Re}(s) > 0$ through the formula $P(s) = \sum_{k\geq 1} \frac{\mu(k)}{k} \log\zeta(ks)$ with poles at the reciprocal of each positive integer and a natural boundary at the line $\operatorname{Re}(s) = 0$.
$C_k(n), C_{\Omega(n)}(n)$	The sequence is defined recursively for integers $n \geq 1$ and $k \geq 0$ as follows: $$C_k(n) := \begin{cases} \delta_{n,1}, & \text{if } k = 0; \\ \sum_{d\mid n} \omega(d) C_{k-1}\left(\frac{n}{d}\right), & \text{if } k \geq 1. \end{cases}$$ It represents the multiple (k-fold) convolution of the function $\omega(n)$ with itself. The function $C_{\Omega(n)}(n)$ has the DGF $(1 - P(s))^{-1}$ for $\operatorname{Re}(s) > 1$.
$[q^n]F(q)$	The coefficient of q^n in the power series expansion of $F(q)$ about zero when $F(q)$ is treated as the ordinary generating function (OGF) of some sequence, $\{f_n\}_{n\geq 0}$. Namely, for integers $n \geq 0$ we define $[q^n]F(q) = f_n$ whenever $F(q) := \sum_{n\geq 0} f_n q^n$.
$\varepsilon(n)$	The multiplicative identity with respect to Dirichlet convolution, $\varepsilon(n) := \delta_{n,1}$, defined such that for any arithmetic function f we have that $f * \varepsilon = \varepsilon * f = f$ where the operation $*$ denotes Dirichlet convolution (see definition below).
$f * g$	The Dirichlet convolution of any two arithmetic functions f and g is denoted by the divisor sum $(f * g)(n) := \sum_{d\mid n} f(d) g\left(\frac{n}{d}\right)$ for $n \geq 1$.
$f^{-1}(n)$	The Dirichlet inverse f^{-1} of an arithmetic function f exists if and only if $f(1) \neq 0$. The Dirichlet inverse of any f such that $f(1) \neq 0$ is defined recursively by $f^{-1}(n) = -\frac{1}{f(1)} \times \sum_{\substack{d\mid n \\ d>1}} f(d) f^{-1}\left(\frac{n}{d}\right)$ for $n \geq 2$ with $f^{-1}(1) = f(1)^{-1}$. When it exists, this inverse function is unique and satisfies $f^{-1} * f = f * f^{-1} = \varepsilon$.
$g^{-1}(n), G^{-1}(x)$	The Dirichlet inverse function, $g^{-1}(n) = (\omega + 1)^{-1}(n)$ with corresponding summatory function $G^{-1}(x) := \sum_{n\leq x} g^{-1}(n)$ for $x \geq 1$.

Symbols	**Definition**
$[n=k]_{\delta}$,$[\texttt{cond}]_{\delta}$	The symbol $[n=k]_{\delta}$ is a synonym for $\delta_{n,k}$ which is one if and only if $n=k$, and is zero otherwise. For boolean-valued conditions, `cond`, the symbol $[\texttt{cond}]_{\delta}$ evaluates to one precisely when `cond` is true, and to zero otherwise.
$\lambda(n), L(x)$	The Liouville lambda function is the completely multiplicative function defined by $\lambda(n) := (-1)^{\Omega(n)}$. Its summatory function is defined by the partial sums $L(x) := \sum_{n \leq x} \lambda(n)$ for $x \geq 1$.
$\mu(n), M(x)$	The Möbius function defined such that $\mu^2(n)$ is the indicator function of the squarefree integers $n \geq 1$ where $\mu(n) = (-1)^{\omega(n)}$ whenever n is squarefree. The Mertens function is the summatory function defined for all integers $x \geq 1$ by $M(x) := \sum_{n \leq x} \mu(n)$.
$\Phi(z)$	For $z \in \mathbb{R}$, we take the cumulative density function of the standard normal distribution to be denoted by $\Phi(z) := \frac{1}{\sqrt{2\pi}} \times \int_{-\infty}^{z} e^{-\frac{t^2}{2}} dt$.
$\nu_p(n)$	The valuation function that extracts the maximal exponent of p in the prime factorization of n, e.g., $\nu_p(n) = 0$ if $p \nmid n$ and $\nu_p(n) = \alpha$ if $p^{\alpha} \\| n$ for $p \geq 2$ prime, $\alpha \geq 1$ and $n \geq 2$.
$\omega(n)$,$\Omega(n)$	We define the strongly additive function $\omega(n) := \sum_{p \mid n} 1$ and the completely additive function $\Omega(n) := \sum_{p^{\alpha} \\| n} \alpha$. This means that if the prime factorization of $n \geq 2$ is given by $n := p_1^{\alpha_1} \times \cdots \times p_r^{\alpha_r}$ with $p_i \neq p_j$ for all $i \neq j$, then $\omega(n) = r$ and $\Omega(n) = \alpha_1 + \cdots + \alpha_r$. We set $\omega(1) = \Omega(1) = 0$ by convention.
$\pi_k(x), \widehat{\pi}_k(x)$	For integers $k \geq 1$, the function $\pi_k(x)$ denotes the number of $2 \leq n \leq x$ with exactly k distinct prime factors: $\pi_k(x) := \#\{2 \leq n \leq x : \omega(n) = k\}$. Similarly, the function $\widehat{\pi}_k(x) := \#\{2 \leq n \leq x : \Omega(n) = k\}$ for $x \geq 2$ and fixed $k \geq 1$.
$Q(x)$	For $x \geq 1$, we define $Q(x)$ to be the summatory function indicating the number of squarefree integers $n \leq x$. That is, $Q(x) := \sum_{n \leq x} \mu^2(n)$ where $Q(x) = \frac{6x}{\pi^2} + O(\sqrt{x})$.
$W(x)$	For $x, y \in \mathbb{R}_{\geq 0}$, we write that $x = W(y)$ if and only if $xe^x = y$. This function denotes the principal branch of the multi-valued Lambert W function defined on the non-negative reals.
$\zeta(s)$	The Riemann zeta function is defined by $\zeta(s) := \sum_{n \geq 1} \frac{1}{n^s}$ when $\operatorname{Re}(s) > 1$, and by analytic continuation to any $s \in \mathbb{C}$ with the exception of a simple pole at $s = 1$ of residue one.