1 My background and direction as a professional researcher

My journey from an undergraduate at the University of Illinois (UIUC) to now, where I will graduate with my Ph.D. in mathematics from the Georgia Institute of Technology (GA Tech) in 2022, is long and was difficult for me. One constant that has always been with me over this time period has been my enthusiasm and drive to work on mathematics. I have always had an excellent support system over the years by keeping my immediate family around me. In 2005–2006, when I first assumed academic leave from UIUC as a freshmen in computer science, Hurricane Katrina struck New Orleans and the Gulf Coast. As a result of that historic natural disaster and the structural engineering catastrophe that ensued, my grandmother's house was submerged under water for weeks. I traveled with my mom by car from the Midwest to try to recover what remained of her life and belongings. I ended up receiving a fateful challenge and inspiration from another family member.

My late uncle talked with me about what I was interested in studying, and then used his home office inkjet printer to assemble a stack of six to ten printouts of top ranked general mathematics journal websites with their submission guidelines for authors. I took him literally when he issued me the kind directive to "*[here] go work on this*" as a challenge to publish my own original research in mathematics. I started with the material from my high school copy of the graduate *Concrete Mathematics* textbook [8] and gradually read every survey and book on the prime numbers that I could find online (*cf.* [17]). The effort and emphasis on personal work and discovery paid dividends after I returned to UIUC as a double major in mathematics and computer science. I received the Barry M. Goldwater scholarship in 2010. My first peer-reviewed publication was also accepted into a respected journal that year.

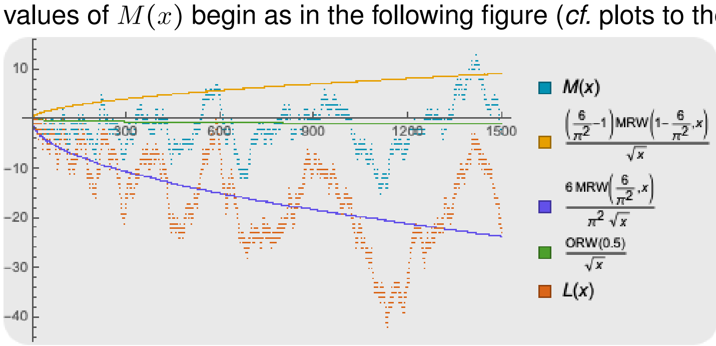
2 Research objectives, methodology and significance of proposed work

2.1 Diversity of my past accomplishments and publications

My research combines number theory, combinatorics and software development. More broadly, I have interests in studying combinatorial and analytic number theory, in applied cryptography and embedded computer hardware, and in software engineering. I have been funded as a graduate research assistant for the last three years or so developing open source software in applied mathematical biology at GA Tech. My active peer-reviewed publication list is diverse with now over twenty-one entries, as is my public profile of open source software projects, each of which reflect the breadth and depth of my combined research areas. Reviewers can find the detailed bibliography sections starting on page 6 for complete reference information.

2.2 Recent work in analytic number theory

History of the Mertens function (or partial sums of the Möbius function). The function $\mu(n)$ is deeply connected to the distribution of the prime numbers and the Riemann Hypothesis (RH) through the Euler product representation of the Riemann zeta function where it is convergent [2]. The Mertens function, $M(x) := \sum_{n \leq x} \mu(n)$, forms partial sums of the classical Möbius function for $x \geq 1$ [18, A008683; A002321]. We interpret the probability that any integer $n \geq 2$ has an even (or odd) number of prime factors so that we can view the function $\lambda(n) := (-1)^{\Omega(n)}$ as a randomized "*digital coin flip*" of sorts [18, A008836; A002819]. Since $\mu(n) \neq 0$ if and only if n is squarefree where $\mu(n) = \lambda(n)$ whenever n is squarefree, this description facilitates modeling the partial sums, $M(x)$, as a $\{-1, 0, +1\}$-weighted random walk on the integers. The value of the Mertens function indicates the position of a particle starting at zero after x steps whose directions are simulated in this quasi-randomized way according to the values of $\mu(n)$, or equivalently, by the distribution of the number of distinct prime factors of any $n \geq 2$. The sequence of slow growing oscillatory values of $M(x)$ begin as in the following figure (*cf.* plots to the left; legend on the right):



A comparison of the plots of $M(x)$, $L(x) := \sum_{n \leq x} \lambda(n)$, and three simulated random walks for $1 \leq x \leq 1500$. We define a sequence of i.i.d. random variables, $\{B_k(p)\}_{k \geq 1}$, such that $\mathbb{P}[B_k(p) = +1] = 1 - \mathbb{P}[B_k(p) = -1] = p$ for $p \in (0,1)$. The two random walk variants we consider are defined by the sums $\mathrm{ORW}(p,x) := \sum_{k=1}^{x} B_k(p)$ and $\mathrm{MRW}(p,x) := \sum_{k=1}^{x} B_k(p)\,[\mu(k) \neq 0]_\delta$ for $x \geq 1$.

An inverse Mellin transform leads to the exact expression of $M(x)$ for any $x > 0$ given by the statement of the following theorem of Titchmarsh [21]. It relates the Mertens function to the non-trivial zeros of the Riemann zeta function for any $x > 0$. That is, assuming the RH, there is an infinite sequence $\{T_k\}_{k\geq 1}$ satisfying $k \leq T_k \leq k+1$ for each $k \geq 1$ such that for any $x > 0$

$$M(x) = \lim_{k\to\infty} \sum_{\substack{\rho:\zeta(\rho)=0 \\ |\Im(\rho)|<T_k}} \frac{x^\rho}{\rho\zeta'(\rho)} - 2 + \sum_{n\geq 1} \frac{(-1)^{n-1}}{n(2n)!\zeta(2n+1)} \left(\frac{2\pi}{x}\right)^{2n} + \frac{\mu(x)}{2} [x \in \mathbb{Z}^+]_\delta .$$

The RH is equivalent to showing that $M(x) = O\left(x^{\frac{1}{2}+\epsilon}\right)$ for any $0 < \epsilon < \frac{1}{2}$. There is a rich history to the original statement of the Mertens conjecture which asserted that $|M(x)| < C\sqrt{x}$ for some absolute constant $C > 0$. The conjecture was first verified by F. Mertens himself for $C = 1$ and all $x < 10^4$ without the benefit of modern computation [10]. Since its beginnings in 1897, the Mertens conjecture has been disproved by computational methods using non-trivial simple zeta function zeros with comparatively small imaginary parts in the famous paper by A. M. Odlyzko and H. J. J. té Riele [16]. More recent attempts at bounding $M(x)$ naturally consider determining the rates at which the function $M(x)x^{-\frac{1}{2}}$ grows with or without bound along infinite subsequences in the limit supremum and infimum senses [13]. Non-computational breakthrough progress on bounding or determining significant properties underlying the distribution of the Mertens function has been slow with the exception of two major papers over the last twenty years or so [15, 19].

My recent work with the Mertens function. My preprint manuscript [46] is accepted into the *Journal of Number Theory* this year. I rigorously prove a more combinatorial and additively structured new characterization of $M(x)$ by considering an auxiliary signed sequence, $g^{-1}(n)$, and its corresponding partial sums, $G^{-1}(x)$. These comparatively regular sequences have not yet been studied in connection with the Mertens function. We precisely define the Dirichlet inverse function $g^{-1}(n) := (\omega+1)^{-1}(n)$ for all $n \geq 1$ where $\omega(n)$ is the strongly additive function that counts the number of *distinct* prime factors of n [18, A001221; A001222]. The first few values of $g^{-1}(n)$ are given by [18, A341444]. An exact formula for $g^{-1}(n)$ is

$$g^{-1}(n) = \lambda(n) \times \sum_{d|n} \mu^2\left(\frac{n}{d}\right) C_{\Omega(d)}(d), n \geq 1,$$

where [18, A008480]

$$C_{\Omega(n)}(n) = \begin{cases} 1, & \text{if } n = 1; \\ (\Omega(n))! \times \prod_{p^\alpha \| n} \frac{1}{\alpha!}, & \text{if } n \geq 2. \end{cases}$$

For large x and $n \leq x$, we can then associate a natural combinatorial significance to the magnitude of the distinct values of $g^{-1}(n)$ that depends directly on the exponent patterns in the prime factorizations of the integers in $\{2, 3, \ldots, x\}$ viewed as multisets. Proof methods adapted in the spirit of Montgomery and Vaughan's work [14] allow us to uniquely reconcile the property of strong additivity with signed sums of multiplicative functions through our new characterization of $M(x)$ by the partial sums of $g^{-1}(n)$. We prove that for an absolute constant $B_0 > 0$ the average order of the unsigned sequence satisfies

$$\frac{1}{n} \times \sum_{k\leq n} |g^{-1}(k)| = \frac{6B_0}{\pi^2}(\log n)^2\sqrt{\log\log n}(1+o(1)), \text{ as } n \to \infty.$$

There is an analog to the famous Erdős-Kac theorem [4, 5] characterizing the distribution of the distinct values of $\omega(n)$ for all $n \leq x$ at large x. Let $\sigma_x(C) := \sqrt{\log\log x}$ and $\mu_x(C) := \log\log x - \log\left(\frac{\sqrt{2\pi}A_0}{\zeta(2)(1+P(2))}\right)$ where $P(s)$ is the *prime zeta function* [7] and $A_0 > 0$ is an absolute constant. Our theorem states that for any fixed $Y > 0$, we have uniformly for any $-Y \leq y \leq Y$ as $x \to \infty$ that

$$\frac{1}{x}\#\left\{3 \leq n \leq x : \frac{|g^{-1}(n)|}{(\log n)\sqrt{\log\log n}} - \frac{6}{\pi^2 n(\log n)\sqrt{\log\log n}} \times \sum_{k\leq n} |g^{-1}(k)| \leq y\right\} = \Phi\left(\frac{\frac{\pi^2 y}{6} - \mu_x(C)}{\sigma_x(C)}\right) + o(1). \quad (1)$$

The function $\Phi(z) = \frac{1}{\sqrt{2\pi}} \times \int_{-\infty}^{z} e^{-\frac{t^2}{2}} dt$ is the CDF of the standard normal distribution. Let the partial sums of $g^{-1}(n)$ be defined as follows [18, A341472]: $G^{-1}(x) := \sum_{n\leq x} g^{-1}(n) = \sum_{n\leq x} \lambda(n)|g^{-1}(n)|$. We prove for all $x \geq 1$ the characterizations that

$$\begin{aligned} M(x) &= G^{-1}(x) + G^{-1}\left(\left\lfloor\frac{x}{2}\right\rfloor\right) + \sum_{1\leq k<\frac{x}{2}} G^{-1}(k)\left[\pi\left(\left\lfloor\frac{x}{k}\right\rfloor\right) - \pi\left(\left\lfloor\frac{x}{k+1}\right\rfloor\right)\right], \\ M(x) &= G^{-1}(x) + \sum_{p\leq x} G^{-1}\left(\left\lfloor\frac{x}{p}\right\rfloor\right), \end{aligned} \quad (2)$$

where $\pi(x) := \sum_{p\leq x} 1$ is the classical prime counting function [18, A000720]. My approach to $M(x)$ using the λ-sign-weighted distribution in (1) and the summatory function $G^{-1}(x)$ via (2) offers new hope and methodology to characterizing the asymptotics of this classical function. Indeed, these auxiliary sequences have a decidedly more additive nature with clearer structure and more insightful properties on inspection. There is a direct link between the summatory function $L(x)$ of the completely multiplicative function $\lambda(n)$ and the distribution of $|g^{-1}(n)|$. Hence, we see another non-classical concrete link between the distributions of $L(x)$ and $M(x)$. The new results in this article then connect the distributions of $L(x)$, an explicitly identified probability distribution, and $M(x)$ as $x \to \infty$. Formalizing the properties of the distribution of $L(x)$ is viewed as a problem that is equally as difficult as understanding the distribution of $M(x)$ well along infinite subsequences of large integers (*cf.* [11]).

Future related work, open problems and conjectures. I plan on extending the work for this manuscript in a few directions as a postdoctoral fellow. The following is a list of open conjectures about the asymptotic behavior of $M(x)$ at large x and along infinite subsequences I wish to make progress towards proving:

(M-A) We have that
$$\liminf_{x\to\infty} \frac{M(x)}{\sqrt{x}} = -\infty; \quad \text{and} \quad \limsup_{x\to\infty} \frac{M(x)}{\sqrt{x}} = +\infty;$$

(M-B) There is an infinite sequence of integers $\{x_n\}_{n\geq 1}$ along which $M(x_n) = 0$ for all $n \geq 1$. That is, the Mertens function takes on arbitrarily small absolute values infinitely often. We have also conjectured that $M(x)$ changes sign infinitely often through the conjectures in **(M-A)**.

(M-C) Let $\mathcal{M}_-(x) := \#\{n \leq x : M(n) < 0\}$ denote the set of positive integers less than x for which the Mertens function is negative. As $x \to \infty$, the values of $M(x)$ assume a pronounced negative bias insomuch as the density
$$\frac{\mathcal{M}_-(x)}{x} > \frac{3}{5}(1 + o(1)), \text{ as } x \to \infty.$$
We remark on how interesting this property is by noting that the asymptotic densities of the respective sets of squarefree integers for which $\mu(n) = \pm 1$ are proven to be equal:
$$\lim_{x\to\infty} \frac{1}{x} \times \#\{n \leq x : \mu(n) = +1\} = \lim_{x\to\infty} \frac{1}{x} \times \#\{n \leq x : \mu(n) = -1\} = \frac{3}{\pi^2} \approx 0.303964.$$

I have started work to generalize the new discoveries for the Mertens function case to characterize partial sums of arithmetic functions. The aim is to develop a standardized methodology by which we can associate the distribution of another strongly additive function in analog to the role $\omega(n)$ plays with $M(x)$ given any multiplicative function $f(n)$ or any $h(n)$ whose Dirichlet series, $\sum_{n\geq 1} h(n)n^{-s}$ convergent for $\Re(s) > \sigma_a(h) \geq \sigma_c(h)$, satisfies certain properties and is hence suitably well enough behaved.

2.3 Formal generating function methods in number theory and combinatorial analysis

Background and significant research experience. My peer-reviewed research profile in mathematics since 2010 includes nearly two dozen articles written on the following topics: generalized Stirling numbers, polynomials and factorial functions, continued fraction representations for sequence generating functions (OGFs) leading to new congruences and properties modulo any $h \geq 2$, and varied methods of transformations of formal integer sequence OGFs. I have published in top journals in general mathematics and within my subject areas including articles in the *Journal of Number Theory*, the *American Mathematical Monthly*, *Acta Artithmetica*, and the *Ramanujan Journal*. I will become a co-author with Prof. Ernie Croot at GA Tech in the near future on a preprint to be submitted to the *Annals of Mathematics* on the p-divisibility of the central binomial coefficients. In 2019, I published a survey article on generating function transformation methods in a special issue of the journal *Axioms* focused on mathematical analysis and its applications.

My doctoral thesis work at Georgia Tech. My collaboration and coauthored work with M. Merca from 2016–2018 led to several theorems that characterize so-termed matrix-based *factorization theorems* for Lambert series generating functions (LGFs) that provide unexpected new identities for multiplicative functions. This forms the basis for the work in my doctoral thesis at GA Tech under supervision of Prof. Michael Lacey and Prof. Josephine Yu. The work with Merca grew out of a common interest in matrix-based formulas from my *Acta Arithmetica* article [39] to express the LGFs that formally enumerate the divisor sums $\sum_{d|n} f(d)$ that are characteristic of many multiplicative functions:
$$\sum_{n\geq 1} \frac{f(n)q^n}{1-q^n} = \frac{1}{(q;q)_\infty} \times \sum_{n\geq 1} \left(\sum_{k=1}^{n} s_{n,k} f(k)\right) q^n = \sum_{m\geq 1} \left(\sum_{d|m} f(d)\right) q^m, |q| < 1. \tag{3}$$
In (3), the lower triangular sequence $s_{n,k} = [q^n](q;q)_\infty \times \frac{q^k}{1-q^k} = s_o(n,k) - s_e(n,k)$ where the functions $s_e(n,k)$ and $s_o(n,k)$ denote the number of k's in all partitions of n into an even (respectively odd) number

of distinct parts. The matrix $s_{n,k}$ is invertible with $s_{n,k}^{-1} = \sum_{d|n} p(n-k)\mu\left(\frac{n}{d}\right)$. Hence, there is a concrete link connecting both the structures underneath multiplicative functions and the distinctively more additive theory of partitions that is apparent from the LGF factorization theorem results and its corollaries given in print.

Natural generalizations. A natural question for future work on these topics is why (restricted) partition functions are such a natural fit in connecting the generating functions that enumerate multiplicative functions within the context of (3)? Suppose that we consider more general special convolution-type sums of the following form: $(f \boxdot_{\mathcal{D}} g)(n) := \sum_{k=1}^{n} f(k)g(n+1-k)\mathcal{D}(n,k)$. An open problem is to find the corresponding "canonically best" pairing of sequences to expand a matrix-based factorization theorem in analog to (3). In the last, more expository, section of my doctoral dissertation, I consider quantitative metrics in the form of sequence cross-correlation statistics for the qualitative observation of "good (or most revealing, or interesting) fit" witnessed between multiplicative and partition theoretic functions in the LGF case above. In particular, if the lower triangular kernel function $\mathcal{D}(n,k)$ is invertible, then we define

$$\operatorname{Corr}(n;\mathcal{C},\mathcal{D}) := \frac{1}{n} \times \frac{\sum_{k=1}^{n} |c_k(\mathcal{C})\mathcal{D}^{-1}(n,k)|}{\sqrt{\left(\sum_{k=1}^{n} c_k(\mathcal{C})^2\right)\left(\sum_{k=1}^{n} \mathcal{D}^{-1}(n,k)^2\right)}} \tag{4}$$

Open questions. Given a fixed kernel $\mathcal{D}$, what is the optimal OGF $\mathcal{C}(q)$ with integer (or rational) coefficients and where $\mathcal{C}(0) := 1$ that generates the sequence $\{c_n(\mathcal{C})\}_{n\geq 0}$ corresponding to a minimal possible correlation statistic, $\operatorname{Corr}(n;\mathcal{C},\mathcal{D})$, defined in (4)? Preliminary theorems I prove in my thesis combined with numerical results suggest that, indeed, in the LGF case, the optimal correlation statistic defined above corresponds to taking the factorizing OGF as the *infinite q-Pochhammer symbol*, $\mathcal{C}(q) := (q;q)_\infty = \prod_{i\geq 1}(1-q^i)$ [1, 9], so that we obtain the observed expansions involving partition functions.

Ties to existing work and modern literature in number theory. The ongoing study of this type of cross-correlation statistic in future work is an active and fruitful method for understanding complicated sums of these types that I will continue to pursue in my postdoctoral work. There is already a vast body of modern work in number theory that motivates semi-standardized ways to quantify relationships between functions and sequences we study via correlation based statistics. There is historically relevant literature about using statistical analysis to motivate studying the structure underneath number theoretic objects. For example, the non-trivial zeros of the Riemann zeta function have been related and bounded via pair correlation formulas. Results in analytic number theory that make sense of the distribution of the non-trivial zeros of $\zeta(s)$ originated in the work of Montgomery. Subsequent follow-up work that collectively builds on Montgomery's contributions in the context of L-functions, Gaussian unitary ensemble (GUE), random matrix theory applications and associated correlation statistics is famously due to Hejal, Rudnick, Sarnak and Odlyzko [3].

3 Broader impacts of proposed activities, future goals and career trajectory

3.1 Long-term career goals, research objectives and methodology

My long-term professional goal is to become a top-tier research mathematician. In making this happen, I can advance and make progress on important topics of merit and broad interest in my areas of study. I strive to continue to contribute high quality open source software, educationally literate publicly available source code advancing STEM areas, and to grow myself as a professional software developer. I seek to find interesting intersections and interplay between these fields while making a difference to others through my engagement working on this new research. A career goal of mine is to never stagnate by always keeping things fresh, open and interesting. I will continue to dare to take on the uncommon risk of enjoying challenging research problems that I encounter within new fields and sub-branches of mathematics. I will pursue stimulating research problems with cross-disciplinary applications in mathematics, computer science and OSS. I intend to publish the results of my research on these topics in peer-reviewed journals, present the results through talks at professional conferences, and make the research broadly available for education, teaching, and other purposes in venues such as the web. As woman working within my areas of study, I will continue to promote new learning and research mentorship opportunities to encourage diversity and help to bridge the gender gap for the under-represented talented women in these fields. I will make these plans a reality by leading seminars at PSU targeted at broader outreach by getting women and minority students at or below the graduate level involved in fulfilling extracurricular roles presenting their research and mentoring other students in math at PSU.

3.2 Active work on open source software projects

I am an active user of the *open source software* (OSS) based platforms Linux and OpenBSD since my teenage years. I was exposed to the availability, education, superior documentation and freely available source to high-quality production software that runs the backbone of all large computer networks early on by teaching myself how to use these systems. This experience provides me insight, grounded philosophy and a great passionate love for OSS. I have gone to significant efforts to donate time to and develop publicly available OSS. This work contributing to OSS is both as an extension of my formal studies as a graduate student and to facilitate growth of my skill set as a professional software engineer. I actively develop and maintain over a dozen public cross-platform OSS projects on *GitHub* written in the C/C++, Java, Python and assembly languages, among others. I will actively hold a seminar focused on teaching all types of students and educators how to use and write software using OSS in mathematics as a postdoc at PSU.

My responsibilities working with Prof. Christine Heitsch as the *gtDMMB* mathematical biology group's graduate RA and software engineer on record at GA Tech have allowed me to gain experience through hundreds of hours back-porting and extending OSS. This work that enables research and experimentation in mathematical and computational biology. In 2021, we co-authored an application note publication in *BioInformatics* focused on the *RNAStructViz* graphical tool for visualizing RNA secondary structures [40]. We are planning to publish other OSS I have developed with her group online this year providing Python bindings, or a Python accessible scripting interface, to the historic *GTFold* software [20] that provides computationally efficient predictions of RNA secondary structures. Another subset of my applied research interests in OSS since relocating to GA Tech in 2017 is focused on security software (utilities and libraries) on contactless NFC smartcard and wireless RFID hardware technology.

3.3 Teaching profile, philosophy and influences

I aim to interact with students by an active learning approach that includes combining new technologies into the classroom. I also seek to promote a flexible, friendly and open learning environment for students coming from diverse backgrounds to engage with me as the instructor from within. While working on computational geometry projects funded by Prof. Jayadev Athreya at the University of Washington from 2016–2017, I was offered an unforgettable opportunity to take part in mentoring advanced undergraduates in mathematics by teaching a self-created junior-level topics course. The course outline focused on getting students hands-on experience with experimental mathematics, gap distributions and spatial statistics by visualizing substitution tilings of the plane in the Python programming language while practicing standardized agile software development methodologies. My enthusiasm for teaching students has grown as I have developed more techniques to overcome shyness in large groups. I was promoted to be the first head TA of *Integral Calculus* in the Fall of 2018 at GA Tech. I accepted the opportunity to teach a section of integral calculus as the instructor of record over the summer of 2021. This is an exciting learning experience for me to administer a large course and to expand my CV with more expansive requisite professional experience in academia.

4 How the NSF fellowship at PSU will help me in my career development

I have elected sponsorship for the postdoctoral fellowship from Prof. R. C. Vaughan at Penn State University (PSU). One of the primary benefits and foremost reasons I chose to pursue PSU as my host institution is that I will get to work with world class professors and mathematicians at the top of their respective fields. The modern analytic number theory textbook by H. Montgomery and Prof. Vaughan is a key inspiration and influence on my work with the Mertens function, $M(x)$, summarized in Section 2.2. Once I began chatting and communicating mathematics with Prof. Vaughan online this year in 2021, I felt an immediate intellectual connection and personal interest formed by talking with him about my work. I have so far learned new techniques in analytic number theory from him and have quickly acquired the benefit of several historical reference points with respect to $M(x)$. Another PSU faculty that I will have available to support me is George E. Andrews. Prof. Andrews is a former president of the AMS, a prolific expert that has championed the theory of partitions throughout his career, and an admirable cataloger of the great collective notebooks of the nineteenth century mathematical genius, S. Ramanujan. More tenure track faculty at PSU whose research interests are compatible with mine that I have been in contact with include A. J. Yee and A. Malik. The combined faculty expertise, academic mentorship and professional development at PSU is thus an excellent fit, and so a recipe for success, for me to continue the proposed future research from Section 2. My background and experience in symbolic computation and experimental mathematics will add breadth to the faculty at PSU that sets my goals and career trajectory apart from other distinguished applicants.

B Bibliography and software contributions

B.1 Citations to external works

[1] Andrews, G. E. *The Theory of Partitions (Encyclopedia of Mathematics and its Applications)*. Cambridge University Press, 1984.
[2] Apostol, T. M. *Introduction to Analytic Number Theory*. Springer–Verlag, 1976.
[3] Barrett O., Firk F.W.K., Miller S.J., Turnage-Butterbaugh C. *From Quantum Systems to L-Functions: Pair Correlation Statistics and Beyond*. In: Nash, Jr. J., Rassias M. (eds) Open Problems in Mathematics. Springer, Cham. (2016)
[4] Billingsley, P. On the central limit theorem for the prime divisor function. *Amer. Math. Monthly*, 76(2):132–139, 1969.
[5] Erdős, P. and Kac, M. The Gaussian errors in the theory of additive arithmetic functions. *American Journal of Mathematics*, 62(1):738–742, 1940.
[6] Flajolet, P. and Sedgewick, R. Analytic Combinatorics. Cambridge University Press, 2009.
[7] C. E. Fröberg. On the prime zeta function. *BIT Numerical Mathematics*, 8:87–202, 1968.
[8] Graham, R. L., Knuth, D. E. and Patashnik, O. *Concrete Mathematics: A Foundation for Computer Science*. Addison-Wesley, 1994.
[9] Hardy, G. H. and Wright E. M., editors. *An Introduction to the Theory of Numbers*. Oxford University Press, 2008.
[10] Havil, J. *Gamma: Exploring Euler's constant*. Princeton University Press, 2003.
[11] Humphries, P. The distribution of weighted sums of the Liouville function and Pólya's conjecture. *J. Number Theory*, 133:545–582, 2013.
[12] Iwaniec, H. and Kowalski, E. *Analytic Number Theory*, volume 53. AMS Colloquium Publications, 2004.
[13] Kotnik, T. and van de Lune, J. On the order of the Mertens function. *Exp. Math.*, 2004.
[14] Montgomery, H. L. and Vaughan, R. C. *Multiplicative Number Theory: I. Classical Theory*. Cambridge, 2006.
[15] Ng, N. The distribution of the summatory function of the Móbius function. *Proc. London Math. Soc.*, 89(3):361–389, 2004.
[16] Odlyzko, A. M. and te Riele, H. J. J. Disproof of the Mertens conjecture. *J. Reine Angew. Math.*, 1985.
[17] Ribenboim, P. *The new book of prime number records*. Springer, 1996.
[18] Sloane, N. J. A. The Online Encyclopedia of Integer Sequences, 2021.
[19] Soundararajan, K. Partial sums of the Möbius function. *J. Reine Angew. Math.*, 2009(631):141–152, 2009.
[20] Swenson, M. S., Anderson, J., Ash, A., Gaurav, P., Sukos, Z., Bader, D. A., Harvey, S. C., and Heitsch, C. E. *GTfold: Enabling parallel RNA secondary structure prediction on multi-core desktops*. BMC Research Notes. 5(1): 341, 2012.
[21] Titchmarsh, E. C. *The theory of the Riemann zeta function*. Oxford University Press, second edition, 1986.

B.2 Peer-reviewed publications of the PI

[22] Merca, M. and Schmidt, M. D. *A partition identity related to Stanley's theorem*. Amer. Math. Monthly 125 **10**: 929–933 (2018).
[23] Merca, M. and Schmidt, M. D. *Factorization theorems for generalized Lambert series and applications*. Ramanujan J. **51**: 391–419 (2020).
[24] Merca, M. and Schmidt, M. D. *Generating special arithmetic functions by Lambert series factorizations*. Contrib. Discrete Math. 14 **(1)**: 31–45 (2019).
[25] Merca, M. and Schmidt, M. D. *The partition function $p(n)$ in terms of the classical Möbius function*. Ramanujan J. **49**: 87–96 (2019).
[26] Mousavi, H. and Schmidt, M. D. *Factorization theorems for relatively prime divisor sums, GCD sums and generalized Ramanujan sums*. Ramanujan J. **54**: 309–341 (2021).
[27] Schmidt, M. D. *A computer algebra package for polynomial sequence recognition*. Illinois IDEALS (2014).
[28] Schmidt, M. D. *A short note on integral transformations and conversion formulas for sequence generating functions*. Axioms Special Issue on Mathematical Analysis and Applications II 8 **2**, 62 (2019).
[29] Schmidt, M. D. *Combinatorial identities for generalized Stirling numbers expanding f-factorial functions and the f-harmonic numbers*. J. Integer Seq. 21 **18.2.7** (2018).
[30] Schmidt, M. D. *Combinatorial sums and identities involving generalized divisor functions with bounded divisors*. Integers 20 **A85** (2020).

[31] Schmidt, M. D. *Continued fractions and q-series generating functions for the generalized sum-of-divisors functions*. J. Number Theory 180: 579–605 (2017).
[32] Schmidt, M. D. *Continued Fractions for Square Series Generating Functions*. Ramanujan J. **46**: 795–820 (2018).
[33] Schmidt, M. D. *Generating function transformations related to polylogarithm functions and the k-order harmonic numbers*. Online J. Anal. Comb. 12 **2** (2017).
[34] Schmidt, M. D. *Exact formulas for the generalized sum-of-divisors functions*. Integers 21 **A19** (2021).
[35] Schmidt, M. D. *Generalized j-factorial functions, polynomials, and applications*. J. Integer Seq. 13 **10.6.7** (2010).
[36] Schmidt, M. D. *Jacobi-type continued fractions and congruences for binomial coefficients modulo integers $h \geq 2$*. Integers 18 **A46** (2018).
[37] Schmidt, M. D. *Jacobi-type continued fractions for the ordinary generating functions of generalized factorial functions*. J. Integer Seq. 20 **17.3.4** (2017).
[38] Schmidt, M. D. *New congruences and finite difference equations for generalized factorial functions*. Integers 18 **A78** (2018).
[39] Schmidt, M. D. *New recurrence relations and matrix equations for arithmetic functions generated by Lambert series*. Acta Arith. 181 (2017): 355-367.
[40] Schmidt, M. D., Kirkpatrick, A., and Heitsch, C. *RNAStructViz: graphical base pairing analysis*. Bioinformatics **197** (2021).
[41] Schmidt, M. D. *Square series generating function transformations*. J. Inequal. Spec. Funct. 8 **2** (2017).
[42] Schmidt, M. D. *Zeta series generating function transformations related to generalized Stirling numbers and partial sums of the Hurwitz zeta function*. Online J. Anal. Comb. 13 **158**. (2018).

B.3 Additional manuscripts by the PI

[43] Schmidt, M. D. *A catalog of interesting and useful Lambert series identities*. Preprint (2020). arXiv/2004.02976.
[44] Schmidt, M. D. *A computer algebra package for polynomial sequence recognition*. Preprint (2016). arXiv/1609.07301.
[45] Schmidt, M. D. *Factorization theorems for Hadamard products and higher-order derivatives of Lambert series generating functions*. Preprint (2017). arXiv/1712.00608.
[46] Schmidt, M. D. *New characterizations of partial sums of the Möbius function*. Preprint (2021). arXiv/2102.05842.
[47] Merca, M. and Schmidt, M. D. *New factor pairs for factorizations of Lambert series generating functions*. Preprint (2017). arXiv/1706.02359.
[48] Schmidt, M. D. *Pair correlation and gap distributions for substitution tilings and generalized Ulam sets in the plane*. Preprint (2017). arXiv/1707.05509.

B.4 DESFire security software project references (described in the Project Summary)

Chameleon Mini crypto mod firmware extension: A modification of the stock Chameleon Mini firmware sources to enable cryptographically secure and integrity checked binary data uploads onto the device.
`github/maxieds/ChameleonCryptoModFirmware`
Chameleon Mini Live Debugger (CMLD): An interactive NFC logging interface for Android OS phones that interfaces to Chameleon Mini hardware over USB. Over 500 active users on the Google *Play Store*.
`github/maxieds/ChameleonMiniLiveDebugger`
DESFire emulation support for the Chameleon Mini: The Chameleon Mini is a hardware tool for NFC debugging, card emulation, security testing, reconnaissance, and general purpose low-level data logging for contactless RFID cards like university IDs. This work enables embedded emulation support for the complex and proprietary Mifare DESFire type NFC tags on recent Chameleon Mini devices.
`github/emsec/ChameleonMini`
`github/maxieds/ChameleonMiniDESFireStack`
Preprint manuscript: Schmidt, M. D. *A recent open source embedded implementation of the DESFire specification designed for on-the-fly logging with NFC based systems*. Preprint (2021).

B.5 STEM supportive and educational software

GTFold Python: Python bindings and library to modernize and extend for the historical set of *GTFold* command line utilities for use with Python. It is a scientific computing project to facilitate experimentation with RNA structures in computational biology. The source code will be released publicly on GitHub in late 2021.

Mathematically-oriented Unix fortune utility mod: A math-related add-on package providing terminal-based text to be displayed on the command line in the form of Unix fortune cookie wisdom. It features a custom *Concrete Math* book style upper case Σ summation ASCII-art graphic.

`github/maxieds/math-fortune-mod`

Mertens function manuscript computational supplement: Facilitates computations with and exploration of the Mertens function, $M(x)$, in both *SageMath* and *Mathematica*. Software and supporting documentation written to accompany the publication of [46].

`github/maxieds/MertensFunctionComputations`

OptiKey "Big Hacker" keyboard extensions: Open source code and documentation that makes typing programming languages on-screen for users with disabilities more accessible. These extensible "Big Hacker" encoded keyboards are designed to simplify on-screen entry of programming languages. This task otherwise requires scrolling through a cell-phone-style nested set of keyboard screens to enter a single line of code in C++, Perl or Python.

Partitions into parts package: An extendable and expository Mathematica demo package for computing the number of partitions of a positive integer n into parts of the form $pt + a$ for p prime and $0 \leq a < p$.

`github/maxieds/PartitionsIntoParts`

Prairie Learn contributor: Prairie Learn is an open source *learning management system* (or LMS) that is a viable option to replace usage of the popular *Canvas* LMS at many universities. It is actively developed at UBC and UIUC and is used on a private server form at UC Berkeley. I have so far contributed code to enable custom function names, symbolic constants, custom-defined operator symbols, and documentation available for use with `sympy` Python library parsing of internal `pl-symbolic-input` elements. This pull request enables crucial parsing for questions in calculus, mathematics and physics by enabling custom function names and symbolic constants.

`github/PrairieLearn/PrairieLearn`

RNAStructViz: A cross-platform GUI-based application to visualize and compare RNA secondary structures that commonly arise in mathematical biology applications. See the application note in [40].

`github/gtDMMB/RNAStructViz/wiki`

Sage and Mathematica special sequence formula recognition packages: UIUC MS thesis software in both Mathematica (original) and Sage (extended). Designed to recognize formulas for sequences involving special combinatorial primitives and functions.

`github/maxieds/GuessPolynomialSequences`

`github/maxieds/sage-guess`

WXML tilings Python library: I was offered an unforgettable opportunity by Jayadev Athreya over 2016–2017 to take part in mentoring advanced undergraduates in mathematics. The course outline focused on getting students hands-on experience with experimental mathematics methodology, gap distributions and spatial statistics and visualizing substitution tilings of the plane in the Python programming language.

`github/maxieds/WXMLTilingsHOWTO`

B.6 Other significant open source software

Android file picker light library: A file and directory chooser widget library for Android OS that focuses on presenting an easy to configure lightweight UI. Designed from the top down to work with newer Android 10 and 11 (API 29+) platforms in the future.

`github/maxieds/AndroidFilePickerLight`

Homebrew live streamer: A customizable, roll-your-own solution for live A/V recording to an Android phone device. It is also used with live media streaming to Facebook and YouTube for a transparent, open source non-proprietary application to perform the media streaming. The application was written to covertly record a private memento of a special three hour Smashing Pumpkins concert in Atlanta from 2018.

`github/maxieds/HomeBrewLiveStreamer`

Mifare classic tool library: A Java and Android OS library wrapper around the functionality of the popular *Mifare Classic Tool* (MCT) application for Android phones.

`github/maxieds/MifareClassicToolLibrary`

`github/maxieds/ChameleonMiniUSBInterface`