Gmail

MDS <maxieds@gmail.com>

Help verifying JNT preprint (AG referred me to you for this)

45 messages

Maxie Schmidt <maxieds@gmail.com> Mon, Mar 21, 2022 at 10:59 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Can you help me to verify the contents of the JNT revision? After I gave a talk at MOBIUS ANT last week, Andrew Granville (total class act) suggested I talk to you for help with it. He said more politely than I would have that Michael Lacey is useless for this project.

They pointed out some issues with the conjecture I had. I also found a subtle technical error in the Selberg-Delange method. I have been working on little sleep since Thursday afternoon to correct things. I believe it is working out now. Moreover, numerical computations of the first and second moments I use in the new conjectured CLT statement are accurate. Though with terms involving \log\log\log x, I am still very limited computationally to check it for large values of that function.

The corrected article is the subject of my invited talk at JMM on April 9. SJM (JNT handling editor) said he will touch base with me after I give the talk to figure out how to best handle the fact that the last revision I submitted a few weeks ago (now tentatively corrected as above) has errors.

Can you help me to verify these results in the next week or two? It is very important that I get some good technical sense to second check the results before the big talk. A tentative corrected version is attached, as are my working slides for the 20 minute JMM talk.

Maxie

2 attachments

mertens-lower-bounds-2022.03.21-v2.pdf
574K

jmm2022-presentation-slides.pdf
234K

Ernie Croot <erniecroot@gmail.com> Mon, Mar 21, 2022 at 11:54 AM
To: Maxie Schmidt <maxieds@gmail.com>

I have to give a talk on Wednesday, so need to finish editing slides. Then on Thursday I will drive back to Atlanta (I'm visiting relatives in Louisville). After that I can look at it.

best,

Ernie

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Mon, Mar 21, 2022 at 12:33 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Thank you! I have had literally over 12 hours of Zoom chat time with Jeff Lagarias about organization and good mathematical writing style these last few months. He said he was doing some advising from afar on the project. Jeff would not help with verifying the technical guts of the article. His input was extremely helpful and productive.

I am going to spend the rest of today trying to copyedit the manuscript and with verifying Theorem 1.2 numerically with Sage. If I spot anything, I will make sure to send an updated version before the morning.

AG's comment was about the way I had stated the previous conjecture for C_{\Omega}(n). He said that the way it was given before would lead to a pointwise (non smooth) distribution, whereas taking the logarithm gives a normal tending CLT. It turns out that the statement should have been problematic anyway because the Selberg-Delange method proof application was, due to undersight and stupidity on my part I suppose in a gross misapplication of a Laplace transform, essentially off by k!.

Chapter 3 is where the really technical details are found. The rest should be fairly easy to spot check.

Maxie
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com>
To: Ernie Croot <erniecroot@gmail.com>
Mon, Mar 21, 2022 at 8:17 PM

Ernie,

I checked the numerical data for x <= 1000000 using the SageMath code in the next repo link. Each computation took 2-4 hours to complete. Based on preliminary results, the moments and formulas for the restricted sums of C_{\Omega}(n) such that \Omega(n)=k seem reasonable. The problem here is that we quickly hit a computational wall by the fact that these formulas minimally involve powers of \log\log x -- and the moments with \log\log\log x < 1 even at that large x.

https://github.com/maxieds/MertensFunctionComputations

I made some copy edits and proofread the technical proofs. I am going to have to let this sit for a few days. I am looking forward to your feedback. We can correspond over email as usual, or meet on BlueJeans to discuss the article when you have some time to read through it.

The updated version of the article (today's date -- specified in a way that lexicographically sorts well with a file browser -- at the version v3) is attached. Thanks again for helping to check this article. It means a lot. SJM wants to hear your praise on it before it can go back under review at JNT after the JMM talk.

Maxie
[Quoted text hidden]

---

mertens-lower-bounds-2022.03.21-v3.pdf
574K

Maxie Schmidt <maxieds@gmail.com>
To: Ernie Croot <erniecroot@gmail.com>
Fri, Mar 25, 2022 at 9:21 PM

Ernie,

I caught a few typos in the proof of Theorem 1.3. Everything else is the same.

Maxie
[Quoted text hidden]

---

mertens-lower-bounds-2022.03.25-v1.pdf
574K

Maxie Schmidt <maxieds@gmail.com> Mon, Mar 28, 2022 at 1:15 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Just checking in after the long weekend. Do you think you will be able to give me suggestions on the article this week? My JMM talk on it is April 9.

Maxie
[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Mon, Mar 28, 2022 at 6:09 PM
To: Maxie Schmidt <maxieds@gmail.com>

I was about to send some comments today, but got a little sick. I have a slight fever (teaching class today was difficult as a result0. I think it's food poisoning. Hopefully I'll be better tomorrow.
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Mon, Mar 28, 2022 at 6:11 PM
To: Ernie Croot <erniecroot@gmail.com>

Hope you feel better. Take your time until you get well.

Maxie
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Tue, Mar 29, 2022 at 11:31 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

How are you feeling today?

Maxie
[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Tue, Mar 29, 2022 at 11:50 AM
To: Maxie Schmidt <maxieds@gmail.com>

Better. Temperature went down; don't feel as queasy.
[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Tue, Mar 29, 2022 at 12:10 PM
To: Maxie Schmidt <maxieds@gmail.com>

A few general comments: first, you mention the Mertens function M(x), and then derive formulas for it in terms of G(x) and various other functions. This seems to serve as motivation for looking at these other functions (like G(x)). However, there doesn't appear to be much progress made on understanding M(x) by this route in the paper -- nor does it seem as though there is a route towards proving strong distributional bounds using the kind of estimates you present. Thus, the problems considered in the paper seem a little "artificial". Perhaps you could explain.

Second, it looks like there is quite a proliferation of auxiliary functions. When this occurs and one reads it, ones sense of meaning gets spread a little too thin (all of them can't be meaningful). This is fine if most of these are mere means to an end (to understand C_Omega and g); and almost all of them are forgotten once done reading each section. To understand M(x) you try to understand g(n) and G(x); to understand g(n) you need to understand C_Omega(n); to understand C_Omega(n) you need to understand G^(z); to understand G^(z) you need to understand F^(z); then there're the functions A_z(x) and A^hat _z(x); and F^* and F^hat; and C^hat. But I guess the main ones you focus on, as you say on page 2, are C_Omega and g(n).

I'll have more comments later.

Best wishes,

Ernie

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com>
To: Ernie Croot <erniecroot@gmail.com>
Tue, Mar 29, 2022 at 3:29 PM

Ernie,

Sorry for taking so long to get back with comments. I am exhausted, stressed out and have been spacing out with my cat for an hour or two.

Enough of what you said in the first paragraph was echoed in my Zoom discussions with Jeff Lagarias. The idea for why the new formulas for M(x) proved in Theorem 1.1 are potentially more useful is that they are a partial sum n <= x of terms whose sign is \lambda(n). Compare to the classical relation in (1.3) whose summands involve the lambda function, but are less predictably signed. Moreover, we can argue that the unsigned magnitudes of the summands in these formulas are "nicer" in the sense of the properties conjectured in Section 5. This is quickly discussed in the paragraph after the statement of Theorem 1.1 in the introduction.

Also, the link of M(x) to sums over summatory functions whose terms are signed by \lambda(n) has a few other distinctive threads worth noting. JL harshly objected to my terminology of these formulas as new "characterizations" of the Mertens function. The reasoning goes something like the following: If we look to Sarnak's conjecture on what a bear \mu(n) is to sum up and predict, and note that lambda(n) agrees with \mu(n) on a set of density ~0.61 asymptotically, then \lambda(n) should be equally difficult to deal with as a sign weight in these formulas. OTOH, some naive and optimistic grad students might argue back that \lambda(n) is completely multiplicative and so might very well be easier as an object to deal with off of the squarefree integers if we someday can pinpoint where to look. JL pointed out that some bounds on sums of products of the lambda function have been accomplished recently (Matomaki and Radziwill; Sound.) where this was not possible for the Mobius function analog. No one seems to have a consensus. I contend that these formulas need more study before conclusively throwing them into a bin called "artificial [intelligence]". This kind of reminds me of a "rigor mortis" stance of sorts whereby you can never really accomplish anything because everything you have to accomplish must to it all from the get go. All of the structure underneath the unsigned terms, vis a vis the function C_{\Omega}(n) and its nice regular properties, is very interesting. And it hasn't been looked at before from this lense.

W.R.T. the second paragraph about the proliferation of auxiliary notation throughout Section 4: So noted. I will work on reducing the problem as I make more edits to the manuscript.

Do you think I need to precisely define what a Dirichlet inverse function is right before stating (1.4) in the introduction? AG had to ask me during my talk at MOBIUS ANT what the inverse was taken with respect to.

Maxie

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com>
To: Ernie Croot <erniecroot@gmail.com>
Tue, Mar 29, 2022 at 7:12 PM

Ernie,

Ok. I gave the last draft I sent some work with respect to your second set of points about too much auxiliary notation. The result is attached. A few minor changes are found throughout. The only major substantive changes are to Proposition 3.6, where I forgot to use \log in front of the function. The argument is the same and the result is the same. Just letting you know so as not to interrupt your thought process working from the previous draft.

I am going to assume that the longer it takes for you to come back with comments, the better off I will be. So please do not rush on this. The JMM talk is not for > 1.5 weeks yet.

If you end up heading to bed, do you mind giving me a heads up? I am inclined to continue to drink beer and red bull until the feedback comes in if it will be tonight. :)

Maxie

[Quoted text hidden]

mertens-lower-bounds-2022.03.29-v1.pdf
576K

Maxie Schmidt <maxieds@gmail.com> Wed, Mar 30, 2022 at 10:49 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

I have made a few more corrections. I also added error terms to a few formulas where they were omitted to be precise and clear. I am not able to do much more without input at this point. Most recent edits are attached if you want to read a fresh copy.

Maxie

[Quoted text hidden]

mertens-lower-bounds-2022.03.30-v3.pdf
598K

Ernie Croot <erniecroot@gmail.com> Wed, Mar 30, 2022 at 11:34 AM
To: Maxie Schmidt <maxieds@gmail.com>

One thing worth mentioning is that when Andrew asked you about what
the Dirichlet Inverse of (omega + 1) means, that's something I recall
mentioning to you in the past -- that it's not a common object in the
lexicon of people who do analytic number theory. You can easily find
it on the web on Wikipedia, probably; but these pages aren't written
by analytic number theorists -- they're probably written by grad
students or something.

[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Wed, Mar 30, 2022 at 11:42 AM
To: Maxie Schmidt <maxieds@gmail.com>

Although not your goal at this point, but: if you wanted to use find
a combinatorial proof of bounds on Mertens's function, you might want
to find a formula for M(x) in terms of the error term in the prime
number theorem -- pi(x) = Li(x) + Error(x). Now, |Error(x)| is
guaranteed to be *at least* O(x^{1/2}) or something in size; and if it
is *larger* than this amount, you could hope that your formula would
indicate that M(x) is substantially larger than sqrt(x).

What happens if you substitute pi(x) = Li(x) + Error(x) into (1.6b)?
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Wed, Mar 30, 2022 at 12:13 PM
To: Ernie Croot <erniecroot@gmail.com>

>You can easily find
>it on the web on Wikipedia, probably; but these pages aren't written
>by analytic number theorists -- they're probably written by grad
>students or something.

The concept of a Dirichlet inverse for an arithmetic function f is covered early on in Apostol's classic advanced undergrad introduction to ANT. Such functions form a ring with respect to the operation of Dirichlet convolution (e.g., divisor sums). DGFs of a function expressed as f = g*h have a natural analog to Laplace transformations of convolutions as DGF[f](s) = DGF[g](s)DGF[h](s). DGFs have long been central to the study of number theoretic functions since Dirichlet (if not before) and Riemann popularized the use of complex analysis to study integer sequences back in their day. Hardy and Wright also consider the topic.

>Now, |Error(x)| is
>guaranteed to be *at least* O(x^{1/2}) or something in size; and if it
>is *larger* than this amount, you could hope that your formula would
>indicate that M(x) is substantially larger than sqrt(x).

This is basically going to be borderline insulting and much familiar rhetoric from yourself and Lacey. Do you understand the material in the paper well enough to help me check the technical proofs? If so, can you please make an effort to help me do this very soon to prepare for my talk at the large AMS JMM conference? This is important.

MDS

[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Wed, Mar 30, 2022 at 1:22 PM
To: Maxie Schmidt <maxieds@gmail.com>

yes, and you can find the definition of a frustum in high school math books -- but it doesn't mean most mathematicians remember it or use it more than once every 20 years.
[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Wed, Mar 30, 2022 at 3:24 PM
To: Maxie Schmidt <maxieds@gmail.com>

And let's see about Theorem 1.2. I don't think it's right: for k bounded as n -> infty, the n's that contribute most to the sum of C_Omega(n) will be those that are square-free (since the # of integers n <= x that are divisible by exactly k primes is ~ (x / log x) (log log x)^{k-1} / (k-1)!); and in that case, your

sum_{n <= x, Omega(n) = k} C_Omega(n) ~ k * x * (log log x)^{k-1} / log x.

Is that what your formula predicts? First, let's find G^(z), for z = (2k-1) / log log x. To find that we have to compute Gamma(1+z); well,

Gamma(1+z) ~ 1 for z near 0,

So, G^(z) ~ 1. So your formula predicts that

sum_{n <= x, Omega(n) = k} C_Omega(n) >> (x / log x) (log log x)^{2k-3/2},

which doesn't agree with my estimate above.

If you restrict to a range of k in [(log log x) / 2, 3 (log log x)/2], you can probably develop *much* simpler estimates for the sum of C_Omega(n) that your do -- in fact, the whole thing is probably *much* simpler, for wide ranges on k.

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com>
To: Ernie Croot <erniecroot@gmail.com>
Fri, Apr 1, 2022 at 11:05 PM

Ernie,

I realized that this theorem was not needed to prove the average order formula for C_{\Omega}(n). That material is removed. Please consider te attached new draft.

MDS
[Quoted text hidden]

mertens-lower-bounds-2022.04.02-v2.pdf
553K

Ernie Croot <erniecroot@gmail.com>
To: Maxie Schmidt <maxieds@gmail.com>
Sat, Apr 2, 2022 at 8:49 PM

In the new theorem 1.2 in the new draft, to obtain an estimate for sum over log C_Omega(n), I'm not sure about the estimate used in (2.3) whether it holds over a wide enough range of k -- it does hold when k is small. But you can prove this theorem a different way; and, furthermore, will get a sharper bound, with the B_0 = 1. Here's the idea (first, though, I should point out I didn't try to get the optimal bounds for things -- just the rough order):

First, find an *upper bound*: since

C_Omega(n) <= Omega(n)! <= Omega(n)^Omega(n),

taking logs we find

log C_Omega(n) <= Omega(n) log Omega(n). So,

sum_{n <= x} log C_Omega(n) <= sum_{n <= x} Omega(n) log Omega(n).

Now, all but at most x / (log x)^2 (actually, much smaller, but this will do) integers n <= x will have the property that Omega(n) < 10 log log x (this follows
by a strong enough form of the Erdos-Kac Theorem, but can be proved in a more elementary fashion); and since for all n <= x we have Omega(n) << log x, we have

sum_{n <= x} Omega(n) log Omega(n) = sum_{n <= x, Omega(n) < 10 log log x} Omega(n) log Omega(n) +
sum_{n <= x, Omega(n) >= 10 log log x} (log x) (log log x)
= sum_{n <= x, Omega(n) < 10 log log x} Omega(n) log Omega(n) + O(x).

And this is

<= (log (10 log log x)) sum_{n <= x, Omega(n) < 10 log log x} Omega(n) + O(x)
~ (log log log x)) sum_{n <= x, Omega(n) < 10 log log x} Omega(n) + O(x)
<= (log log log x) sum_{n <= x} Omega(n) + O(x)
~ x (log log log x) log log x.

Now we find a lower bound: let S be the set of integers n <= x such that

prod_{p^a || n} a! < log x.

We claim that

====

Lemma. |S| ~ x.

Proof of Lemma: We claim that every n not in S has
a square divisors d^2 > log log log log x, say; and then you use the fact that the number
of integers up to x with a square divisor of that size is << x sum_{d^2 > log log log log x} 1/d^2
which is O(x / log log log log x), the lemma is proved.

To see that such a d^2 > log log log log x must exist for n not in S, you can consider two
cases: case 1 is where

F(n) := #{p prime : p^a || n, a >= 2} < 2 log log log log n,

and case 2 is where the reverse inequality holds. If we're in case 2, then we have a
square divisor d^2 of n of the form

prod_{p^a || n, a >= 2} p^a > 2^{2 log log log log n} > log log log n,
which establishes the claim.

If we're in case 1, on the other hand, then for our n not in S,

max_{p^a || n} a! >= (prod_{p^a || n} a!)^(1 / F(n)) > (log n)^{1 / 2 log log log log n}
> log log log n.

Thus, for that prime p giving this maximal value for a! we will have that

a! > log log log n ==> a > (log log log log n) / log log log log log n
==> p^a > log log log log n, as needed (i.e. that p^a is our d^2 dividing n).

====

We have that

sum_{n <= x} log C_Omega(n) >= sum_{n in S} log C_Omega(n)
>= sum_{n in S} (Omega(n) log Omega(n) - log log n).

(Note that that last -log log n comes from the fact that

log (prod_{p^a || n} a!) < log log x.)

So, this is

>= sum_{n in S} (Omega(n) log Omega(n)) - O(x log log x).

I claim that the sum here is

~ sum_{n <= x} (Omega(n) log Omega(n)) ~ x (log log log x) log log x,

(i.e. you can replace the sum over "n in S" with "n <= x")

which would match the upper bound above. This is not hard to show, but takes yet
another little lemma.

Best wishes,

Ernie

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Wed, Apr 13, 2022 at 12:36 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

I am working to find some time to respond to your concerns in the last message. Have to spend a lot of time the next few weeks prepping for technical job interviews. I also missed my JMM talk this weekend due to a nasty flu virus...

Maxie
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Wed, Apr 13, 2022 at 5:52 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Please wade through my elementary school child handwritten response to your comments above. I believe the proof of Theorem 1.2 I gave is still correct. Response attached.

Maxie
[Quoted text hidden]

Ernie-MertensUpdates-EmailResponseToProofOfTheorem1-2.pdf
5286K

Maxie Schmidt <maxieds@gmail.com> Wed, Apr 13, 2022 at 7:06 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

I cleaned up some of the proofs to include an argument that both of the absolute constants B_0,D_0 in the last version are identically one. This simplifies the conjectures. It might also cut to the crux of your objections in the last message. Please refer to the attached PDF as the new working version of the article.

Maxie
[Quoted text hidden]

mertens-lower-bounds-2022.04.13-v1.pdf
557K

Maxie Schmidt <maxieds@gmail.com> Wed, Apr 13, 2022 at 3:09 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Yet more additions and editing produce a new draft worth datestamping. I assume it will not be a substantial burden to send you the copy since you haven't looked at the previous that carefully yet. :)

Maxie
[Quoted text hidden]

---

mertens-lower-bounds-2022.04.13-v2.pdf
563K

Maxie Schmidt <maxieds@gmail.com> Fri, Apr 15, 2022 at 10:05 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

I asked Jayadev Athreya (on my committee from UW) to read my introduction. He ran the ergodic theory seminar for years as a professor at UIUC. I figured he would be a great person to ask about the arguments about relative simplicity of my(n) versus lambda(n) vis a vis Sarnak's conjecture. His feedback is incorporated into this latest draft.

Do you mind reading this version by next Friday? Please respond.

Maxie
[Quoted text hidden]

---

mertens-lower-bounds-2022.04.15-v1.pdf
565K

Maxie Schmidt <maxieds@gmail.com> Tue, Apr 19, 2022 at 2:25 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Attached is the revised article after talking with Jayadev and JL last week. It is going to sit untouched for a while.

Can you please have the courtesy to estimate when you plan on looking at this manuscript again? Two weeks? One month? Never...

MDS
[Quoted text hidden]

---

mertens-lower-bounds-2022.04.19-v2.pdf
568K

Ernie Croot <erniecroot@gmail.com> Wed, Apr 20, 2022 at 9:50 AM
To: Maxie Schmidt <maxieds@gmail.com>

Looking at section 2.2 in the latest draft you sent, there're lots that need to be done to make this a well-written proof -- though, journals may not care, in any case (often, people don't read things carefully):

1. How do you know that (2.2) holds?

2. In (2.3) you are using some fact about the number of integers up to x having exactly k prime factors. I see the

estimate you use is written out in Appendix B. You should probably write, "Provided (2.2) holds, we will use Theorem B.2 (see appendix) to show that there is an absolute constant B0^* > 0..."

3. I agree that (2.3) is probably correct; however, what you say at the bottom of page 5 is not true. The function G~(z) is not ~1 for all k <= (3/2) loglog x. What's going on is that when k ~ loglog x, THEN you get G~((k-1) / loglog x) ~ G(1) = 1. And then in your formula for L_Omega(x), since most of the mass is coming from the values of k near to loglog x, you can assume that your B0^* is essentially 1 -- however, it's not true that for every value of k <= (3/2) loglog x you can assume you are dealing with B0^* = 1.

4. Perhaps this will also destroy the error term factor (1 + O(1 / loglog x)) you have on that theorem? You might get something a little worse -- like (1 + O(logloglog x / loglog x)) or something similar -- due to the B0^* drifting away from 1 as you move k away from loglog x.

More later...

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Fri, Apr 22, 2022 at 9:09 AM
To: Ernie Croot <erniecroot@gmail.com>

Updated draft that addresses the points above is attached.

MDS
[Quoted text hidden]

---

mertens-lower-bounds-2022.04.22-v2.pdf
565K

Ernie Croot <erniecroot@gmail.com> Wed, Apr 27, 2022 at 10:45 AM
To: Maxie Schmidt <maxieds@gmail.com>

More corrections:

1. In the statement of Theorem 1.2 on page 4 you didn't say that B0 = 1.

2. In the proof of Theorem 1.2 your justification for (2.2) is off. For example, it *could* be the case (you need to prove that it isn't...) that

(*) sum_{k >= 1} #{n <= x : Omega(n) = k, n not-square-free} log(k!)
> 100 * sum_{k >= 1} #{n <= x : Omega(n) = k, n square-free} log(k!).

(Yes, there aren't 100x as many non-square-free numbers as square-free numbers, but the weights here are log(k!), instead of 1, so you can't immediately rule out (*). You have to prove it isn't possible...)

And also the case that (again, you need to prove that it isn't...)

(**) sum_{n <= x, n square-free} log(C_Omega(n)) > 100 * sum_{n <= x, n not-square-free} log(C_Omega(n)).

(It isn't immediately obvious that this would contradict (*), since log(Omega(n)!) is not the same as log(C_Omega(n)). Instead, log(C_Omega(n)) = log(Omega(n)!) - log (prod_{p^a || n} a!), and when you sum up that log(prod_{p^a || n} a!), you might make the right-hand-side sum of (**) is 100x smaller than the left-hand-side, as claimed).

If you put (*) and (**) together, then your (2.2) can't be true. Let's see: from (**) we get that, essentially,

sum_{n <= x} log(C_Omega(n)) =apprxoimately= sum_{n <= x, n square-free} log(C_Omega(n))
= sum_{n <= x, n square-free} log(Omega(n)!) (when n square-free we have C_Omega(n) = Omega(n)!)

And then using (*) we get that this is much smaller than (by a factor of 100)

sum_{n <= x, n not-square-free} log(Omega(n)!) < sum_{n <= x} log(Omega(n)!).

Thus, we would have that

sum_{n <= x} log(C_Omega(n)) < (1/100) sum_{n <= x} log(Omega(n)!),

which contradicts (2.2).

3. There is another error in the statement of Proposition 2.3. That bound can't be right: we have

(1/n) sum_{k <= n} (log C_Omega(k))^2 is clearly smaller than (log n)^100, say (a power of log n is a very crude upper bound).

But the right-hand-side of the proposition has a sqrt(n).

More later...
[Quoted text hidden]

Ernie Croot <erniecroot@gmail.com> Thu, Apr 28, 2022 at 1:26 PM
To: Maxie Schmidt <maxieds@gmail.com>

More feedback on Proposition 2.3: as I said before, in the statement of the Proposition there is that extra sqrt(n); also, "variance" has a precise meaning in a probability context, and it is not the quantity you deal with in this writeup.

What the variance *should be* is

V(x) := sum_{n <= x} (log C_Omega(n) - E(x))^2,

where

E(x) = (1/x) sum_{n <= x} log C_Omega(n).

Also, equation (2.8) can't be right, since log C_Omega(k) < log k, which implies

V_Omega(n) = sum_{k <= n} (log C_Omega(k))^2 <= sum_{k <= n} (log k)^2 << n (log n)^2, not >> n^2 like you have.

This whole proof writeup just looks like it's too complicated and wrong.
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Fri, Apr 29, 2022 at 6:26 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

My mom passed in hospice this week. I will be in replying. Math means so little right now.

MDS

[Quoted text hidden]

Mama_Sarah.pdf
432K

Ernie Croot <erniecroot@gmail.com>
To: Maxie Schmidt <maxieds@gmail.com>
Sat, Apr 30, 2022 at 8:06 PM

Dear Maxie,

Sorry to hear that. My mother also died in hospice, back in 2019. In her case, she had Alzheimer's. I try to exercise a lot now and also eat a high-quality diet, and take supplements to slow aging prevent decline in health, so that I don't suffer a similar fate.

Best wishes,

Ernie

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com>
To: Ernie Croot <erniecroot@gmail.com>
Mon, May 2, 2022 at 9:23 PM

Ernie,

My mother was a long time smoker. She developed lung cancer. The treatments (targeted radiation therapy on her right lung and chemo) were effective at shrinking the tumor and fighting the cancer. These procedures were also effective at wracking up her body and killing her. It got to the point where further chemo would have killed her very quickly. When the doctors realized there was nothing else they could do for her, she was moved to hospice care. She died within 24 hours of that transition (medicated and asleep, fortunately). I have been a heavy smoker since 13. I vowed to quit by the time I graduate from GT. I thought it would never happen -- this did it, very, very sad to say...

My dad let me take back this old framed family photo of her. I'm keeping that, and the original signed print copy of her doctoral thesis from 1983, on my bookshelf to remember her. Those were the days when typesetting was non-computerized and had to be done painstakingly page-by-page, key-by-key. Of course she only made use of applied stats within her area, so typesetting complicated math formulas wasn't an issue.

I will follow up about the article when I have some time to look over it again.

MDS
[Quoted text hidden]

IMG_20220502_232317365.jpg
2544K

Maxie Schmidt <maxieds@gmail.com> Tue, May 3, 2022 at 1:38 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Let's see if these modifications get closer to fixing your points in the last two messages about the proof of Theorem 1.2 and Proposition 2.3.

Maxie
[Quoted text hidden]

mertens-lower-bounds-2022.05.03-v1.pdf
572K

Maxie Schmidt <maxieds@gmail.com> Tue, May 3, 2022 at 9:53 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

A few more remarks and arithmetic checking the the section on conjectures (Section 4, iirc) are contained in this version. I am going to let it sit aside for a while. Looking forward to more discussion.

MDS
[Quoted text hidden]

mertens-lower-bounds-2022.05.03-v2.pdf
573K

Maxie Schmidt <maxieds@gmail.com> Fri, May 6, 2022 at 7:10 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Here is a more up to date version. I am traveling to NM for a job interview the week of Monday May 16. I have this upcoming week to work more on this article. Do you have some time to look over it this weekend?

Maxie
[Quoted text hidden]

mertens-lower-bounds-2022.05.06-v1.pdf
484K

Maxie Schmidt <maxieds@gmail.com> Sun, May 8, 2022 at 8:24 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

I spent some time today after my thesis was looking good to update this article. The major change is that I provide a quick proof sketch of the conjecture using the Lindeberg CLT. I am still traveling the week of 5/16. I would like to be able to work on this article when I get back if you are too busy now.

Maxie
[Quoted text hidden]

mertens-lower-bounds-2022.05.08-v3.pdf
668K

Maxie Schmidt <maxieds@gmail.com> Tue, May 10, 2022 at 10:51 PM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

In full procrastination mode for something else I have to do this week, I spent a few hours doing more modifications and copyedits to the article. The result is attached. Do you have any sort of estimate as to when you might have some more time to look at it?

Also, you might want to check out the dedication to my tentative thesis draft (also attached). It's my way of paying tribute to what my mom means to me while still being able to move on and look forward. :)

Maxie
[Quoted text hidden]

2 attachments

mertens-lower-bounds-2022.05.10-v1.pdf
639K

thesis-ideas-working.pdf
3004K

Ernie Croot <erniecroot@gmail.com> Wed, May 11, 2022 at 8:15 AM

To: Maxie Schmidt <maxieds@gmail.com>

I'm away visiting my father at th

e moment, but might look at it later today.

best wishes

ernie
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Wed, May 11, 2022 at 9:40 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

In that case, let me send you the latest version. I am running some intensive (time consuming) computations of the empirical distributions of the C_{\Omega}(n) theorem. This draft includes a few minor corrections.

Thanks. Enjoy the time with family,

Maxie
[Quoted text hidden]

mertens-lower-bounds-2022.05.11-v2.pdf
632K

Maxie Schmidt <maxieds@gmail.com> Fri, May 13, 2022 at 9:55 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Here is an even more polished draft. I am traveling all next week (through Friday). Can you find some time between now and then to read the new version of the manuscript?

Maxie
[Quoted text hidden]

mertens-lower-bounds-2022.05.13-v1.pdf
632K

Ernie Croot <erniecroot@gmail.com> Mon, Jun 20, 2022 at 9:03 PM
To: Maxie Schmidt <maxieds@gmail.com>

I looked at the draft a few weeks ago, but didn't feel like writing up a long list of comments. I did, however, get around to writing up a version of your proof of Theorem 1.2. See the attached. Basically, heavy tools aren't needed, and would be over-kill.

Best wishes,

Ernie
[Quoted text hidden]

---

maxie_theorem.pdf
190K

Maxie Schmidt <maxieds@gmail.com> Tue, Jun 21, 2022 at 12:17 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Quite a bit has changed. I am attaching the most recent revised version of the manuscript. Please let me know if you will have feedback within a couple of weeks.

Maxie
[Quoted text hidden]

---

mertens-lower-bounds-2022.06.21-v1.pdf
686K

Maxie Schmidt <maxieds@gmail.com> Tue, Jun 28, 2022 at 6:42 AM
To: Ernie Croot <erniecroot@gmail.com>

Ernie,

Here is the latest version. I plan to submit it in a couple of weeks after my final defense on July 6:
https://intranet.math.gatech.edu/seminars-colloquia/series/dissertation-defense/maxie-dion-schmidt-wed-07062022-1500

If you have some time, please let me know what feedback you have.

MDS
[Quoted text hidden]

---

mertens-lower-bounds-2022.06.28-v1.pdf
781K