



MDS &lt;maxieds@gmail.com&gt;

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## Journal placement of a proof of unboundedness of the sort-scaled Mertens Function

26 messages

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**Maxie Schmidt** <maxieds@gmail.com>  
To: Steven J Miller <sjm1@williams.edu>

Fri, Jun 5, 2020 at 4:05 PM

Steven,

I'm following up on my questions for you over LinkedIn messaging from earlier this week that you requested I talk with you about over email correspondence.

I have been doing some work with new formulas for the Mertens function,  $M(x) := \sum_{n \leq x} \mu(n)$ , or the summatory function of the Mobius function. To the best of my knowledge, it is still an open conjecture as to whether  $M(x) / \sqrt{x}$  grows without bound in the limit supremum (infimum) senses. That is while there is substantial conditional evidence that supports this conjecture, I believe that no absolute proof has yet been given of the unboundedness of this scaled function along infinitely tending subsequences. However, I believe I can prove that the limit supremum (as  $x \rightarrow \infty$ ) of  $|M(x)| \log x / \sqrt{x}$  is unbounded. I have attached the title page of a proof attempt I am actively seeing through and lively at work editing right now.

My question for you is about topical journal placement for a correct proof of the property I have claimed. Would JNT field a review for an article that builds up to the proof of this result based on new methods? In 2009, some work by Soundararajan (see [ArXiv preprint link](#)) that proves new improvements on upper bounds on  $M(x)$  assuming the RH made it into *Annals*. Is a result like I believe I can prove of that quality?

It would be much farther into the summer before I can provide much more in the way of details about the proof, or even produce a more informative draft to document my method for you.

Sincerely,

Maxie D. Schmidt



**mertens-lower-bounds-AbstractTitlePage-2020.06.05-v2.pdf**  
243K

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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Fri, Jun 5, 2020 at 4:46 PM

in the queue  
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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Sat, Jun 6, 2020 at 6:27 AM

I believe this would be a significant result -- I'm not aware of growth rate results.

If you submit to JNT we would quickly determine if it is appropriate to send to a referee.

My suggestion is that as you are writing do not try to get optimal bounds; trade quality of result for easiness of reading.

On Fri, 5 Jun 2020, Maxie Schmidt wrote:

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**Maxie Schmidt** <maxieds@gmail.com>  
To: michael lacey <lacey.michaelt@gmail.com>

Sat, Jun 6, 2020 at 7:03 AM

One opinion.

Maxie  
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**Maxie Schmidt** <maxieds@gmail.com>  
To: Steven J Miller <sjm1@williams.edu>

Sat, Jun 6, 2020 at 9:04 AM

Steven,

Thank you for the quick response. My question is whether given a proof of a result like this that tends to a more general interest base than just number theorists, should I shoot high and try to get a (much better proofread draft than I have now) reviewed positively at a top tier journal like Annals? In principle, that sort of recognition would elevate any pending postdoc applications substantially beyond the publications I already have so far. Assuming postdocs have any funding these next few years.

I will be in touch later this summer about the article. I am still revising and checking my proofs carefully for correctness and corner cases.

With respect to your comment on optimal bounds: This has been my approach almost since the start of the new material that led to this paper starting earlier this year. Refinements are readily the next thing to study. Right now it's crucial that I can show my method and sub-optimal bounds are correct clearly without convoluting the argument. There are some natural upper bounds on summatory functions I can take (e.g.,  $\log\log x$ , rather than something like  $(\log x)^2$  that would yield the classical result). The logic to proving the inequalities hold in the first case is not too bad, where in the second, it will have to be very involved, to say the least.

Thanks again for the suggestions and heads up. Let me know if anything strikes you this summer about  $M(x)/\sqrt{x}$  type unboundedness problems.

Maxie  
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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Sat, Jun 6, 2020 at 9:10 AM

we do have JNT Prime for top papers, those at the level of compositio, and you could submit and ask it to be considered there or not at all (you can email it to me before a formal submission)

if you are able to get it in Annals that will contribute far more to your career in terms of publication, but if you truly are teh first to show limsup diverges you'll be fine. one issue is how long it takes to referee; I would hope if you have a proof that looks like a legitimate contender any journal would move on it quickly (we would)

hope this helps  
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**michael lacey** <lacey.michaelt@gmail.com>  
To: Maxie Schmidt <maxieds@gmail.com>

Sat, Jun 6, 2020 at 10:54 AM

Hi: Two things

He stressed "My suggestion is that as you are writing do not try to get optimal bounds; trade quality of result for easiness of reading" That is what I have been trying to stress as well.

Second, you are aiming for something weaker than disproof of Merten's conjecture. So, the interest in your result would be more in the direction of "how a sequence  $x_j$  can you find so that  $M(x_j) \gg (\sqrt{x_j})/\log x_j$  AND  $x_j$  grows at some some "slow" rate."

It is not clear from their response that they understood that point.

--- ML

Best regards,

Michael Lacey

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**Maxie Schmidt** <maxieds@gmail.com>

Thu, Aug 6, 2020 at 3:17 AM

To: Steven J Miller <sjm1@williams.edu>

Steven,

I apologize for using LinkedIn to contact you. It's an artifact of once upon a time needing a software job that I stay in touch that way. It periodically pings me on my phone, which is why I finally got the message you sent. I actually meant to touch base with you about the Mertens function article I emailed about last time.

I am working with my adviser, Michael Lacey, to improve some bounds that I had in the works this fall. I will emphasize that my work on this topic connects canonical additive functions,  $\omega(n)$  and  $\Omega(n)$ , in a central way to  $M(x)$ . I have also been able to prove a few Erdos-Kac like analogs for similar sequences that I use to express  $M(x)$  in my proof. This manuscript should be very informative and correct, but I'm still working out kinks. The argument is very technical.

I am hoping to be in con tact with you soon about its publication. Does the offer still hold to send you a pre-submission draft for suggestions still hold? I have also started a source-code repository on my (private for now) GitHub page that helps with computations, e.g., computing  $L(x)$  (the related summatory function of  $\lambda(n)$ ) for very large  $x$  with more efficient algorithms. This is in line with what you suggested about a prior JNT submission with extensive computational support and interpretations.

Maxie

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**Steven Miller** <sjm1@williams.edu>

Thu, Aug 6, 2020 at 3:32 AM

To: Maxie Schmidt <maxieds@gmail.com>

Cc: Steve Miller <sjm1@williams.edu>

Absolutely, please feel free to send

Sent from my iPad

On Aug 6, 2020, at 5:17 AM, Maxie Schmidt <[maxieds@gmail.com](mailto:maxieds@gmail.com)> wrote:

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**Maxie Schmidt** <[maxieds@gmail.com](mailto:maxieds@gmail.com)>  
To: Steven Miller <[sjm1@williams.edu](mailto:sjm1@williams.edu)>

Thu, Aug 6, 2020 at 3:40 AM

Thank you. It might be some time, but I will definitely be in touch.

Maxie

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**Maxie Schmidt** <[maxieds@gmail.com](mailto:maxieds@gmail.com)>  
To: Steven Miller <[sjm1@williams.edu](mailto:sjm1@williams.edu)>

Tue, Aug 18, 2020 at 10:00 PM

Steven,

It occurs to me that beyond that which I contacted you for about the Mertens function this year, we have spent very little time talking mathematics. I read up on some of your interests online.

One of the topics I have become interested in since I came to GA Tech around 2017 is NFC pentesting and hardware development. I have a [popular application for Android](#) ([Play Store installer](#)) that controls and logs the [Chameleon Mini](#) boards. I have volunteered myself on one of the open source sites related to developing firmware for these boards (with an onboard ATmega128\* chip) to finally make the fabled, much requested DESFire NFC tag support functional on the RevG class of these devices. These sophisticated little 3DES/AES++ crypto enabled chips are basically the ubiquitous components used to authenticate most school ids these days with door locks, readers, vending transactions, and anything else they can be swiped to validate. Very fun testing project with your ID and a droid device for experimental purposes (ask me). It also would be a good benchmark that could lead to security-related publications as I recently found this year that some physical authentication systems bypass (inadvertently, it seems) the crypto-enabled mechanisms these chips are so powerful for -- big vulnerability, sans any need for an actual cryptographic attack scheme.

As it turns out, not such a trivial task to get a DESFire stack fully working, but I have most of the software ready to go. The problem I am running into is inadequate testing materials. Basically, I think I am going to functionally need one of the pricier Proxmark devices (as opposed to the cheaper USB NFC reader/writer combo sticks I have been suffering through off of Amazon with libnfc) and these are priced at around ~\$320. Not exorbitant as far as most research funding goes in this area, but out of my personal range as a student. I wonder if you might be interested in getting more involved in the development process for the DESFire firmware with me (or at least hearing more about it, and the challenges it poses)?

I have a currently private GitHub repo where I am housing my partially working source code for now. If you have an account, I can add you to it as a collaborator on GitHub. The net-net of this, more so than for a publication on this (which is also a prospect) is straight up bragging rights. This stack is notoriously difficult to get functional on the small memory limited AVR chips, and in addition to that, the protocol is so nuanced and scarcely documented without NDA signed to Phillips corporation/NXP that the complexity there is yet another challenge to put in place. I am confident in my coding skills, at any rate.

I will be up a while longer tonight if you have time to chat. Let me know.

Maxie

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**Steven J Miller** <[sjm1@williams.edu](mailto:sjm1@williams.edu)>  
To: Maxie Schmidt <[maxieds@gmail.com](mailto:maxieds@gmail.com)>

Tue, Aug 18, 2020 at 10:05 PM

sadly I don't have a github

while I used to program in many environments, now it's mostly just mathematica

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**Maxie Schmidt** <maxieds@gmail.com>

Tue, Aug 18, 2020 at 10:29 PM

To: Steven J Miller <sjm1@williams.edu>

Would you have an undergrad research mentoring opportunities that might focus well on applied crypto? Part of the reason I wanted to reach out to you is since I know you are so prolific with this sort of engagement. I could stand to get my hands dirty (so to speak) working with students outside of GA Tech. Ideally, I would like to find an academic position when I graduate. I can send you a better list of my software resume if you are interested. If you just don't have the time, I understand.

Maxie

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**Steven Miller** <sjm1@williams.edu>

Wed, Aug 19, 2020 at 1:48 AM

To: Maxie Schmidt <maxieds@gmail.com>, Steve Miller <sjm1@williams.edu>

Right now I have to avoid taking on any new commitments. In addition to my regular job, sitting on the school committee, and the standard summer research experience for undergraduates I am also doing to polymath research experiences for undergraduates, and of course at a moments notice might have my kids school canceled and be homeschooling

Sent from my iPad

On Aug 19, 2020, at 12:29 AM, Maxie Schmidt <maxieds@gmail.com> wrote:

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**Maxie Schmidt** <maxieds@gmail.com>

Wed, Aug 19, 2020 at 10:56 AM

To: Steven Miller <sjm1@williams.edu>

Thanks, Steven. Everyone is making sacrifices in this unprecedented pandemic. No worries.

Stay safe.

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**Steven J Miller** <sjm1@williams.edu>

Wed, Aug 19, 2020 at 11:21 AM

To: Maxie Schmidt <maxieds@gmail.com>

sadly no

a LOT of people are making sacrifices

some are trying to take advantage of the situation, which pisses me off

we set up a polymath reu and had 200 - 300 students who had nothing now do math research....

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**Maxie Schmidt** <maxieds@gmail.com>

Wed, Aug 19, 2020 at 11:26 AM

To: Steven J Miller <sjm1@williams.edu>

I'm not sure I understand the complaint well. My intent earlier was to see what else you are interested in these days, and share something I thought would be relevant that I have been up to working on. If you recall from the Young Mathematicians Conference in 2012, the closest I recall we ever got to talking math was when you handed me your business card and told me to contact you if I find anything else related to the (my poorly pronounced) Hurwitz zeta function. I still have that somewhere in my notebooks.

No harm, no foul from my end.

I also do a lot of math-related software ended research. It funds my RA at GA Tech, for example. You can check out my pinned GitHub page repos for some examples. You might consider signing up and getting involved, even if you do not spend much time contributing free and open source code.

Maxie

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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Wed, Aug 19, 2020 at 11:30 AM

you misunderstand me

I am saying that there are many people who are all hands on deck and are voluntareeing / doing more

I've been trained and already done a 6 hour shift to do covid testing on campus

there are however many people who are doing the minimum, adn that minimum is low in places

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**Maxie Schmidt** <maxieds@gmail.com>  
To: Steven J Miller <sjm1@williams.edu>

Wed, Aug 19, 2020 at 11:35 AM

I get the volunteering spirit. I am just personally not that extroverted and willing to put myself out there at risk in a covid pandemic right now. But you have the right spirit about this.

My dad for example, donates more to local food banks right now. I will (now, thanks to your reminder) try to donate a box of masks to a local food bank to make sure they have the ability to utilize these key preventative tools in service the public that needs them.

I would do or give more if I had the ability, but I make peanuts as a grad student. Not sure how to help.

Maxie

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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Wed, Aug 19, 2020 at 11:42 AM

there are many ways to step up

I provided masks for the school district and the police at the start -- I had friends in china send me masks, and I passed those on

fortunately the mask crisis is gone

I just tabulated how much I've spent in the local sandwich shop since the pandemic started. from going maybe 2-3 times a year I've now spent a very good amount in getting lunches for my family

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**Maxie Schmidt** <maxieds@gmail.com>  
To: Steven J Miller <sjm1@williams.edu>

Wed, Aug 19, 2020 at 12:16 PM

Hmm. I contacted the local food bank to see if they can make good use of my masks, distributing them to those in need, or providing them to volunteers serving the community. Haven't heard back yet.

Atlanta doesn't really have the same food tie for me that I had with pizza and gyros at uiuc years ago. Jimmy John's has some good new non kosher menu items though.

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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Wed, Aug 19, 2020 at 12:57 PM

we're supporting one of the local baseball leagues to give kids something to do

we bought \$100 in tickets, don't want to win, jsut give the money back

the problem is we keep winning! now instead of just giving the money back to the league we'll put a tab at the concession stand for everyone at the next game....

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**Maxie Schmidt** <maxieds@gmail.com>  
To: Steven J Miller <sjm1@williams.edu>

Wed, Aug 19, 2020 at 1:15 PM

Ok. I get it.

I gave the local synogogue my child hood drum set last year.

Contributing to free and open source software is another important [way.to](#) me that I can give back.

I have a meeting, and a Bluetooth stack to write today.

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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Wed, Aug 19, 2020 at 1:23 PM

I don't do software, but all of my lectures are free and I post on my homepage and facebook

have done a lot of math enrichment for my kids and shared those

did make slides, takes more time, but better videos

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**Maxie Schmidt** <maxieds@gmail.com>  
To: Steven J Miller <sjm1@williams.edu>

Wed, Aug 19, 2020 at 1:27 PM

What is your view opinion on Montgomery and Vaughan. I have gotten a lot out of it so far. Covers a lot of ground. Much more modern than Davenport, right?

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**Steven J Miller** <sjm1@williams.edu>  
To: Maxie Schmidt <maxieds@gmail.com>

Wed, Aug 19, 2020 at 1:32 PM

I like reading several for different perspectives

I'd also throw in serre's 'a course in arithmetic'

and ellison and ellison 'prime numbers'

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