Cover Letter for Journal of Number Theory Submission (2021)

Article Title: *New characterizations of the summatory function of the Möbius function*
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Dear JNT Editors,

I am submitting an original manuscript with work I have done over the past couple of years. The manuscript contains new results that characterize, exactly express and also bound the Mertens function, M(x), or the summatory function of the Möbius function. The new results define a combinatorially motivated sequence corresponding to a Dirichlet inverse function that is defined by convolutions of a canonical strongly additive function. The sign of the terms of this new arithmetic function is given by the Liouville lambda function. The distribution of its unsigned values is characterized by a central limit theorem type result I proved that is an effective analog to the famous and much celebrated Erdős-Kac theorems. The summatory function of the sequence directly, and also when discretely convolved with the prime counting function, provides new exact formulas that characterize M(x) and its asymptotic properties at large x.

Some forms of the auxiliary sequences I work with in the article have been studied before in the references. However, to date no work has connected these sequences with M(x) nor has characterized their distributions by exact probability distributions. Thus, this characterization of the Mertens function is new and has not been studied carefully in the context of these sums. Note that while we do not prove substantial improvements on existing known bounds for M(x) or L(x), this work is still significant as it concretely connects weighted sums involving the Liouville lambda function to M(x), it connects the distribution of well-known additive functions to M(x) via an Erdős-Kac type theorem analog, and it characterizes and reconciles M(x) with the distribution of primes in a combinatorially relevant way. This new characterization may be employed to lead to a breakthrough on asymptotic bounds for these classical functions in future work.

The new results in my manuscript are rigorously proved rather than experimentally obtained by computation. That being said, calculations of these new sequences with the computer algebra systems `SageMath` and `Mathematica` has provided me with much insight and intuition behind the formal proofs in the article. As such, I have released a supplementary computational reference which is available as open-source software at https://github.com/maxieds/MertensFunctionComputations.

I have been in periodic contact with JNT editor Steven J. Miller over the last year over email about this article and my work related to its topics. He has offered me many valuable insights and feedback focusing on motivating these results, and in preparing the introduction section for readers. I do not have a problem if he handles the submission unless there is a conflict of interest by this past contact about the article. Please note that while I did receive some financial support to work on this research in 2020, no specific grant is cited for that funding source.

Thank you very much for your time. I appreciate the JNT considering my article for the publication review process.

Sincerely,

Maxie Dion Schmidt