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Research Interests

My research interests are primarily in combinatorial and analytic number theory and recursive combinatorics with an emphasis on generating function methods, combinatorial structures, software development, and experimental mathematics. I am always open to exploring new interesting problems.

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Graduate Research Plan
Jonathan Gerhard

We were playing a game. For a graph G , define a configuration of G to be an assignment of integers to every vertex, which we think of as placing some number of chips on each vertex. We fire a vertex v by sending a chip along each edge adjacent to v , and consider two configurations equivalent if one can be obtained by the other through chip-firing. The set of configurations that sum to 0 modulo this equivalence forms a finite abelian group $K(G)$ called the critical group of G .¹

During the summer of 2015, I did a research project describing the structure of the critical group of the square rook's graph.⁸ Once the year started, I quickly became enthralled in a new application of a continuing research project relating three objects associated to a q -Weil polynomial $f \in \mathbb{Z}[T]$ (where q is a prime power) with a numerical equality:

- (1) $|A_q(\mathbb{P}_n, f)|$: The size of the isogeny class of principally polarizable ordinary abelian varieties of dimension 3 over \mathbb{F}_q with characteristic polynomial of Frobenius f .
- (2) $h_q(f, \chi)$: The ratio of class numbers of the totally imaginary degree 6 number field $K = \mathbb{Q}(T)(f)$ and its totally real cubic subfield K^+ .
- (3) $v_n(f) \prod_{i=1}^n v_i(f)$: The product over all rational primes l of the relative frequency of f as the characteristic polynomial of an element of $\text{GL}_3(\mathbb{F}_l)$ and an Archimedean term coming from the Sata-Tate measure.

This research is inspired by work done on $\text{GL}_2(\mathbb{F}_p)$ and abelian varieties of dimension 2 by Achter and Williams⁹ and work on $\text{GL}_2(\mathbb{F}_p)$ and elliptic curves by Gekhtman.⁵

1. MY RESEARCH PLAN

Could it be? Recently, I began to suspect a connection between my two research projects. Just as we define a divisor of an algebraic curve, we can define a divisor of a graph. We see that every configuration of G is in this sense a divisor of G , and the resulting Jacobian group (degree 0 divisors modulo linear equivalence) is the critical group.⁷ This leads to the first part of my research plan: finding a connection between graphs and principally polarizable abelian varieties.

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1. INTELLECTUAL MERIT

To me, the beauty of mathematics comes from its interconnections. Consider the following combinatorial problem:

Take a list of numbers $1, 2, \dots, n$. Choose two (x, y) at random, remove them from the list, and add back in the number $x + y + xy$. Iterating this process ends with one number. What is that number?

We could proceed by induction on n , or we could take a route that is more fun. We can define a generating function $f(x) = (1 + x)(1 + 2x) \dots (1 + nx)$, and notice that by Vieta's formulas with symmetric functions, $f(1) - 1 = (n + 1)! - 1$ is the number we end up with! Further, thinking of this as an Eulerian polynomial, this process actually counts the number of lattice points in the n -dimensional cuboid with side lengths $1, 2, \dots, n$. Just like that, we have a beautiful connection between combinatorics, algebra, and geometry.

This philosophy has guided me throughout my research experiences. My first opportunity to do research came freshman year, when I approached a professor, Dr. Laura Tschman, asking for help with computing the Alexander-Conway polynomial of a figure-eight knot. After our discussion, she invited me to participate in a research project using 3D-printing to demonstrate knot invariants.

A few years later, in the spring of 2016, I participated in the Math in Moscow program. In addition to being the optimal opportunity to practice my Russian, this program also allowed me to take graduate courses I could not have taken at MIT. One of the most beautiful topics I encountered while there was Algebraic Topology. Remembering my first research project, I

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