CS 230: Section 3

Backpropagation

Game Plan

- Logistic regression w/ one example
- Logistic regression w/ multiple examples
- Backpropagation exercises
- Questions

Logistic Regression

One Example

Forward Pass

- *What are the equations for the logistic regression forward pass?*

Forward Pass

- *What are the equations for the logistic regression forward pass?*

$$z = wx + b$$

Forward Pass

- *What are the equations for the logistic regression forward pass?*

$$z = wx + b$$

$$\hat{y} = \sigma(z) = a$$

Forward Pass

- *What are the equations for the logistic regression forward pass?*

$$z = wx + b$$

$$\hat{y} = \sigma(z) = a$$

$$L = -(y \log(\hat{y}) + (1 - y) \log(1 - \hat{y}))$$

Dimensionality

- *What are the dimensions for the variables?*

Dimensionality

- *What are the dimensions for the variables?*

$$x = (n, 1), w = (1, n), b = (1,1)$$

Dimensionality

- *What are the dimensions for the variables?*

$$x = (n, 1), w = (1, n), b = (1,1)$$

$$z = (1,1), a = (1,1), L = (1,1)$$

Backpropagation

- *What are the backprop equations for the parameters?*

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial w}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial w}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial b}$$

Backpropagation

$$L = -y \log a - (1 - y) \log(1 - a)$$

$$\frac{\partial L}{\partial a} = ?$$

Backpropagation

$$L = -y \log a - (1 - y) \log(1 - a)$$

$$\frac{\partial L}{\partial a} = -\frac{y}{a} + \frac{1 - y}{1 - a}$$

Backpropagation

$$a = \sigma(z)$$

$$\frac{\partial a}{\partial z} = ?$$

Backpropagation

$$a = \sigma(z)$$

$$\frac{\partial a}{\partial z} = (a)(1 - a)$$

Backpropagation

$$z = wx + b$$

$$\frac{\partial z}{\partial w} = ?$$

$$\frac{\partial z}{\partial b} = ?$$

Backpropagation

$$z = wx + b$$

$$\frac{\partial z}{\partial w} = x$$

$$\frac{\partial z}{\partial b} = ?$$

Backpropagation

$$z = wx + b$$

$$\frac{\partial z}{\partial w} = x$$

$$\frac{\partial z}{\partial b} = 1$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial w}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial b}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)\frac{\partial a}{\partial z}\frac{\partial z}{\partial w}$$

$$\frac{\partial L}{\partial b} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)\frac{\partial a}{\partial z}\frac{\partial z}{\partial b}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)(a)(1-a)\frac{\partial z}{\partial w}$$

$$\frac{\partial L}{\partial b} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)(a)(1-a)\frac{\partial z}{\partial b}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)(a)(1-a)x^T$$

$$\frac{\partial L}{\partial b} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)(a)(1-a)(1)$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial L}{\partial w} = (a - y)x^T$$

$$\frac{\partial L}{\partial b} = a - y$$

Logistic Regression

Many Examples

Forward Pass

- *What are the equations for the logistic regression forward pass?*

Forward Pass

- *What are the equations for the logistic regression forward pass?*

$$z = wx + b$$

Forward Pass

- *What are the equations for the logistic regression forward pass?*

$$z = wx + b$$

$$\hat{y} = \sigma(z) = a$$

Forward Pass

- *What are the equations for the logistic regression forward pass?*

$$z = wx + b$$

$$\hat{y} = \sigma(z) = a$$

$$J = -\sum_{i=1}^{m} (y^{(i)} \log\left(\hat{y}^{(i)}\right) + \left(1 - y^{(i)}\right) \log\left(1 - \hat{y}^{(i)}\right))$$

Dimensionality

- *What are the dimensions for the variables?*

Dimensionality

- *What are the dimensions for the variables?*

$$x = (n, m), w = (1, n), b = (1,1) \rightarrow (1, m)$$

Dimensionality

- *What are the dimensions for the variables?*

$$x = (n, m), w = (1, n), b = (1,1) \rightarrow (1, m)$$

$$z = (1, m), a = (1, m), J = (1,1)$$

Backpropagation

- *What are the backprop equations for the parameters?*

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial w}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial w}$$

$$\frac{\partial J}{\partial b} = \frac{\partial J}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial b}$$

Backpropagation

$$J = -\sum_{i=1}^{m} \left(y^{(i)} \log\left(a^{(i)}\right) + \left(1 - y^{(i)}\right) \log\left(1 - a^{(i)}\right)\right)$$

$$\frac{\partial J}{\partial a^{(i)}} = ?$$

Backpropagation

$$J = -\sum_{i=1}^{m} (y^{(i)} \log\left(a^{(i)}\right) + \left(1 - y^{(i)}\right) \log\left(1 - a^{(i)}\right))$$

$$\frac{\partial J}{\partial a^{(i)}} = -\sum_{i=1}^{m} \frac{y^{(i)}}{a^{(i)}} - \frac{1 - y^{(i)}}{1 - a^{(i)}}$$

Backpropagation

$$a^{(i)} = \sigma(z^{(i)})$$

$$\frac{\partial a^{(i)}}{\partial z^{(i)}} = ?$$

Backpropagation

$$a^{(i)} = \sigma(z^{(i)})$$

$$\frac{\partial a^{(i)}}{\partial z^{(i)}} = (a^{(i)})(1 - a^{(i)})$$

Backpropagation

$$z^{(i)} = \sum_{j=0}^{n-1} w_j X_j^{(i)}$$

$$\frac{\partial z^{(i)}}{\partial w} = ?$$

Backpropagation

$$z^{(i)} = \sum_{j=0}^{n-1} w_j X_j^{(i)}$$

$$\frac{\partial z^{(i)}}{\partial w_p} = ?$$

Backpropagation

$$z^{(i)} = \sum_{j=0}^{n-1} w_j X_j^{(i)}$$

$$\frac{\partial z^{(i)}}{\partial w_p} = X_p^{(i)}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial a^{(i)}} \frac{\partial a^{(i)}}{\partial z^{(i)}} \frac{\partial z^{(i)}}{\partial w}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w_p} = \frac{\partial J}{\partial a^{(i)}} \frac{\partial a^{(i)}}{\partial z^{(i)}} \frac{\partial z^{(i)}}{\partial w_p}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w_p} = \sum_{i=1}^{m} (a^{(i)} - y^{(i)}) \frac{\partial z^{(i)}}{\partial w_p}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w_p} = \sum_{i=1}^{m} \left(a^{(i)} - y^{(i)}\right) X_p^{(i)}$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w_p} = (A - Y)X_p^T$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial w} = (A - Y)X^T$$

Backpropagation

- *What are the backprop equations for the parameters?*

$$\frac{\partial J}{\partial b} = (A - Y)\mathbf{1}$$

Backprop Exercices

[On the Board]

Questions?