Humboldt University Berlin
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Statistics (M.Sc.)

The Dynamics of Wealth & Income Inequality: Assessment of Temporal Relationships and Influence of GDP & Interest Rates

Term Paper

submitted by
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Contents

1 Introduction

> "There may be wide difference of opinion as to the significance of a very unequal distribution of wealth, but there can be no doubt as to the importance of knowing whether the present distribution is becoming more or less unequal." (Lorenz, 1905, p. 209)

Economists have made great strides in measuring economic inequality, generating large-scale data bases of global inequality based on quantitative measures of inequality (Chancel et al., 2022). Transparency on the distribution of income and wealth is essential to enable a fact-based discussion, as Lorenz (1905) recognized.

Economic inequality is high in the United States, both when assessing income and wealth inequality. The top 10% receives 45.5% of the total income (Chancel et al., 2022). In contrast, the bottom 50% earns just 13.3%. The economic divide is even more apparent when examining wealth inequality, where the top 10% owns above 70% of the nation's wealth, whereas the bottom half owns 1.5% of the total wealth.

While there has been progress in the research of consequences of wealth and income inequality (e.g., Kondo et al., 2009; Pickett & Wilkinson, 2015; Oishi et al., 2011; Oishi & Kesebir, 2015; Sommet et al., 2018) and its descriptive measurement, less work has been done in regard to modeling the long-run dynamics of economic inequality using time-series analysis. One of the barriers was access to high-quality datasets, with the administrative tax data by Saez and Zucman (2016), Piketty and Saez (2003) and survey data (Kuhn et al., 2020) now paving the way for new analyses. In addition, much research was conducted only in regard to one specific source of economic inequality, most often on income inequality, thereby neglecting the role of wealth inequality as an economic inequality dimension. Finally, research that aimed to explain macroeconomic forces involved in shaping these dynamics has only in roughly the past decade begun to assess a broader variety of factors involved in shaping the dynamics of inequality (e.g., Jännti Jenkins, 2010; Berisha & Meszaros, 2020). Building on this strand of research, this seminar paper aims to contribute to the literature on U.S. postwar inequality by modeling income and wealth inequality and including key macroeconomic variables such as *gross domestic product* (GDP) and *interest rates* in an attempt to help explain the trajectories partially.

2 Theoretical Background

This section is divided into three parts to outline the rationale of the paper and develop the hypotheses sequentially. First, the measurement of economic inequality will be described shortly. Second, trends in the development of U.S. income and wealth inequality will be explained. Finally, a set of macroeconomic factors and their potential role in regard to economic inequality will be addressed.

2.1 Definition & Measurement of Inequality

Researching economic inequality requires a set of conceptual decisions on how to properly define and measure inequality, boiling down to essential questions such as "Inequality of what?", "Inequality of whom?", and "Inequality over which time horizon?".

Starting with the former, it can be argued that while the traditional economic research has strongly focussed on (a) income and to a lesser extend (b) wealth as primary foci of inquiry (e.g., Atkinson, 2015). Scholars have pointed out that economic inequality can be viewed much broader (for an overview see de Muro, 2016; Sen, 1997). For practical reasons, this thesis will focus solely on dimensions with particularly well-established findings and data sources, namely income and wealth inequality.

Next to the the dimension of choice, the researcher is also confronted with selecting the appropriate unit of observation, offering a spectrum of granularity ranging from individuals to countries as well as the time period (De Muro, 2009; Jenkins & van Kerm, 2009).

Economists have developed a range of measures since Lorenz started the discussion in 1905. According to Jenkins and van Kerm (2009) commonly used inequality measures include the (1) coefficient of variation (i.e., standard deviation divided by the mean), (2) the variance of the logs, which ensures scale invariance of the variance, (3) percentile ratios (e.g., P90:P10, P90:50), (4) the Gini coefficient, (5) Generalized Entropy indices (Cowell & Kuga, 1981) such as the Theil index, and (6) the Atkinson family of inequlity measures (Atkinson, 1970). Measures (4)-(6) show essential properties defined for inequality measures (see Jenkins & Van Kerm, 2009 for details).

Due to its common use (e.g., as core dimension in the *World Inequality Report* and simplicity of calculation, Chancel et al., 2022), the Gini coefficient is selected as measure for this term paper. The Gini coefficient is computed as follows (Gini, 1912):

$$G(Y) = 1 - 2\int_0^1 L(p;Y)\,dp, \tag{2.1}$$

where $G(Y)$ ranges from 0 (absolute equality) to 1 (absolute inequality). It represents the ratio formed between the area enclosed by the Lorenz curve and the perfect line of equality to the overall area under this line. After having set the scene by outlining what economic inequality entails and how it is measured, the focus will shift towards concrete findings on the development of inequality in the U.S.

2.2 Economic Inequality in the U.S.

The following section is structured into two parts, shedding light first on recent findings in U.S. income inequality, before proceeding with empirical insights on U.S. wealth inequality.

2.2.1 Income Inequality

Piketty and Saez (2003) found that income inequality demonstrated a U-shaped pattern in the time period between 1913 and 2002, with a substantial drop in the world war period (due to large tax increases to finance the wars, mostly). Examining the top percentile (P99:100) of the income distribution, they found that the share of capital income share, which formed most of the income in this group, declined substantially over time and wage as well as entrepreneurial income started forming the majority of income at the very top.

In contrast, P90:95 and P95:99 relied on labor income for the majority of their income. Also, Kuhn et al. (2020) showed that the income shares in the lower parts of the distribution shrink, representing a hollowing-out of the American middle class. Turning to the role of income inequality in long-range inequality development, it is hypothesized that income inequality is temporally associated with wealth inequality. Obviously, wealth accumulation is driven by savings rate, and savings rate has been shown to be higher in households with higher lifetime income (Dynan et al., 2004). Consequently it is assumed:

H_1: Wealth inequality increases as income inequality grows.

2.2.2 Wealth Inequality

Recently, research has amassed a plethora of new facts on wealth inequality, highlighting that wealth inequality has risen sharply since the 1980s. Across multiple datasets (i.e., Survey of Consumer Finances (SCF) and the updated SCF+, Kuhn et al., 2020; capitalized income data from tax records, Saez Zucman, 2016; and the Forbes 400 ranking; Dolan et al., 2022), findings indicate similar developments for wealth inequality (Zucman, 2019):

Wealth inequality followed a U-shaped evolution over the past 100 years when examining combined tax return data since 1913, with wealth inequality high in the early 20th century, with a drop from 1929 to 1978, and a growth period since then. Wealth inequality growth is mostly attributable to the increase of wealth concentration in the top 0.1% of the population, whereas the wealth of the bottom 90% decreased since the 1980s. Wealth captured by the top percentile is approximately 40% across all data sources (Zucman, 2019).

Also, it has been shown that wealth portfolios of households at different parts of the wealth distribution differ substantially (Kuhn et al., 2020), with the top 10% households having a large share of stocks and business as well as bonds and liquid and other financial assets, whereas for the less rich households, housing and non-financial assets are the majority of assets.

Turning to the potential influence of wealth on income inequality, stock and business assets are assumed to have a higher rate of return than housing assets. Also, excess wealth can act as a buffer from economic downturns and personal crises, allowing individuals to pursue a long-term strategy (e.g., through investments in human capital). Consequently, it is hypothesized that:

H_2: Income inequality grows as wealth inequality increases.

2.3 Macroeconomic Factors and Economic Inequality

Having examined the the core inequality dimensions and their potential interplay, now macroeconomic factors that are hypothesized to shape the temporal dynamics of the previous inequality dimensions are examined. This section starts by outlining how economic growth influences economic inequality, before describing the findings and potential role of interest rates.

2.3.1 Gross Domestic Product

There has been a wealth of research on factors influencing the income distributions (e.g., Atkinson, 1997; Jännti & Jenkins, 2010), much focussed on the role of GDP. Kuznet posited that the association between GDP and income inequality follows an inverse-U-relation, first increasing inequality and then as markets mature, inequality decreases again (Kuznet, 1955). Empirical tests of the association between the two dimensions have undergone a series of methodological paradigm shifts (for reviews see Dominices et al., 2008; Neves & Silva, 2013), ranging from cross-country regression studies (Alesina & Rodrik, 1994; Clarke, 1995; Deininger & Squire, 1998) over panel regression model (e.g., Li & Zou, 1998; Forbes, 2000; Barro, 2000) to a third wave of studies questioning specific assumptions. Evidence has been mixed so far, especially in the panel studies (Neves & Silva, 2013).

Focussing on the role of the developmental stage of a country, Castelló-Climent (2010) found a negative impact of income inequality on growth in less-developed countries and a positive one in the higher-income economies. In line with these predictions, Gobbin and Rayp (2008) found that economic growth is positively affected by income inequality in the U.S. in a time series analysis.

Turning to the role of GDP on economic inequality, there is much less research available (for an exception, see e.g., Stiglitz, 2012). When the economy grows, richer individuals are assumed to be able to capture large shares of the gain through their larger exposure to the financial market. Since the U.S. are a developed market, it is assumed that:

H_3: Wealth inequality is associated with GDP.

H_4: Income inequality is influenced by GDP.

2.3.2 Interest Rates

In the more recent past, economists have turned towards the role of interest rates in shaping the distribution of income. Coibion et al. (2012) elaborated on potential mechanisms for a link between interest rates and inequality, of which a few are relevant for income and wealth inequality. First of all, households differ in their sources of income, with richer households receiving more income from businesses and lower income households more from wages or transfers. Since business income grows more than wages after monetary expansion, this could increase expansion. Second, richer households which are more active on the financial markets might be more affected by monetary expansion than those who are less integrated to the financial markets.

In their time-series analysis they find that contractionary monetary policy shocks increase U.S. income inequality (Coibion et al., 2012). However, Berisha et al. (2018) found a negative relationship between interest rates and income inequality. The difference is likely to be attributable to methodological issues such as the fact that the upper income in Coibon et al. (2012) is defined by the 90th percentile income, whereas Berisha et al. (2018) rely on top percentile measures, and those two household groups have significant different income sources. In addition, Berisha and Meszaros (2020) find that wealth inequality rises as interest rates decreases, a finding they explain with the fact that wealthier households hold the majority of stocks, which are boosted by low interest rates. In line with these findings, it is expected that:

H_5: Income inequality is influenced by interest rates.

H_6: Wealth inequality is associated with interest rates.

3 Methods

In this section, all methodological aspects of the term paper will be presented. First, the econometric background is explained. Second, the data sample and measures. Third, the econometric problem is formalized and finally, the analytical strategy will be outlined.

3.1 Econometric Background

In the following subsections, all the necessary econometric background will be outlined.

3.1.1 Vector Autoregressive Processes

The VAR model represents a generalisation of the univariate autoregressive model for modeling a vector of time series data (Hyndman & Athanasopoulos, 2021; Sims, 1980). Whereas some key time series models such as ARIMA model unidirectional relationships, the VAR models allows modeling of multidirectional relationships. Assuming an observed time series of K dimensions $y_t = (y_{1t}, ..., y_{kt})$, the VAR process of order p can be expressed as follows (Lütkepohl, 2013):

$$y_t =_t + B_1 x_{t-1} + ... + B_p x_{t-p} + u_t, \tag{3.1}$$

where the deterministic term μ_t can represent either a linear trend ($\mu_t = \mu_0 + \mu_{1t}$), a constant ($\mu_t = \mu_0$) or be zero ($\mu_t = 0$). B_i ($i = 1, .., p$) is a ($K \times K$) parameter matrix. The vector u_t=($u_{1t}, ..., u_{kt}$)' is k-dimensional vector of residuals and a zero mean white noise process. The covariance matrix $E(u_t u_t^{\prime}) = \Sigma_u$. Consequently, $u_t \overset{i.i.d}{\sim} (0, \Sigma_u)$. An alternative representation using lag operators $B(L)$ in the form of $B(L) = I_k - B_1 L - ... - B_p L^p$, can be written as (Lütkepohl, 2013):

$$B(L) x_t = u_t \tag{3.2}$$

Depending on the order of the time series variables, variables may be written in first differences using the difference operator $\Delta y_t = (y_t - y_{t-1})$ to ensure the stationarity of the model. The reduced form VAR(p) can be estimated by a range of estimation methods, including *ordinary least squares* (OLS), *maximum-likelihood* (ML), and *Bayesian estimation methods* (Lütkepohl, 2013). In the appendix, the statistical background of estimation via OLS is included.

3.1.2 VARX

Unlike the VAR model, the VARX model does not treat all variables as endogenous variables. Rather, some other observable variables (so called *exogenous variables*), which are determined outside of the system of interest are also modeled. This *conditional model* has the following structural form (Lütkepohl, 2005; Tsay, 2013):

$$Ay_t = A_1^* y_{t-1} + ... + A_p^* y_{t-p} + B_0^* x_t B_1^* x_{t-1} + ... + B_s^* x_{t-s} + w_t, \tag{3.3}$$

where $y_t = (y_{1t}, ..., y_{Kt})'$ represents a vector of endogenous variables with dimension K, $x = (x_{1t}, ..., x_{Mt})'$ is a vector of exogenous variables with dimension M, A is of dimension $(K \times K)$ and contains the instantaneous relationships between the endogenous variables, $A_{\underset{i}{*}}$ and $B_{\underset{j}{*}}$ are of dimension $(K \times K)$ and $(K \times M)$ and contain the coefficients for the endogenous and exogenous variables. The error term is assumed to represent white noise. The reduced form is obtained by multiplying the equation with A^{-1} (assuming the inverse of A exists).

The coefficient matrices can be estimated via *General Least Squares* (GLS). Since the Σ_u is unknown, it can be estimated via the EGLS estimator under standard assumptions (Lütkepohl, 2005). The appendix includes the methodological background of the VARX estimation procedure.

Having delineated the mathematical background of the present analysis, the next sections will cover the dataset and measures used as part of the analysis.

3.2 Dataset & Measures

In the following sections, the dataset and the associated measures that were used in the present assignment paper are described. The core dataset is the *SCF+* dataset by Kuhn et al. (2020) which is a survey-based source of long-run trends in the distribution of income and wealth in the United States.

The pooled dataset ranges from 1950 to 2016 and contains triannual measurements. Pooling was conducted using the pooling variable in the dataset to match the frequency of the time series of the pre-1983 dataset with the frequency in the post-1983 period. Imputation one of five available imputations was chosen for this analysis. Overall, $n = 102.304$ household measurements are contained in the dataset. There were 3 missing time periods in the dataset (years 1974, 1980, 1986), which were imputed for income and wealth inequality measures separately using linear interpolation (for more sophisticated multivariate imputation see packages *Amelia* by King et al., 2001 and *mtsdi* by Junger & Ponce de Leon, 2015). Linear interpolation showed good results in comparison between different algorithms for univariate time series imputation (Moritz et al., 2015).

In the following paragraphs, the measures that were used in the present study are described. In line with the definition of Kuhn et al. (2020), *income* is defined as the sum of wages and salaries, income from professional practice and self-employment, rental income, interest, dividends, transfer payments, as well as business and farm income. *Wealth* is measured as the aggregated value of the household balance sheet, i.e., assets minus debt. Assets include liquid assets, housing, bonds, stocks and business equity, mutual funds, life insurance (in cash value), defined-contribution plans, other real-estate, and cars (Kuhn et al., 2020). Income and wealth data were derived from from Kuhn et al. (2020). The *Gini coefficient* is a statistical measure for *inequality* (Gini, 1912). The Gini coefficient ranges from 0, indicating perfect equality, and 1, indicating one individual holds all of the target dimension. The data is collected from the appendix of Kuhn et al. (2020). The *GDP per capita* was gathered from the U.S. Bureau of Economic Analysis on January 30, 2022. The data contains the annual GDP per capita in Dollars without seasonal adjustments. Aiming for a comparable time series as in the inequality measures, only those observations matching the tri-annual years were maintained in the GDP time series. *Interest rates* are collected from the International Monetary Fund on February 13th, 2022. The monthly data was averaged and years without corresponding values in the inequality time series were dropped.

Having outlined the dataset properties and measures, the following section will formalize the specific technical problem in econometric terms.

3.3 Econometric Problem

Two models will be fitted, starting with a basic VAR(p) model of order p describing the relationship between income and wealth inequality. In a next step, an enhanced model is estimated including macroeconomic variables to improve the predictive value. The basic VAR(p) model can be expressed in mathematical terms as follows ($\forall\ t = 1950, 1953, ..., 2016$; adapted from Lütkepohl, 2005):

$$\hat{y}_t = v + A_1 y_{t-1} + ... + A_p, y_{t-p}, \tag{3.4}$$

where y_t=$(y_{1t}, y_{2t})'$ is a (2×1)-dimensional vector of the endogenous variables U.S. household income inequality and wealth inequality, A_i are estimated (2×2)-dimensional coefficient matrices, $v = (v_1, v_2)'$ is a fixed (2×1)-dimensional intercept vector.

The second model is a VARX model including the macroeconomic variables GDP and interest rates. These two variables will be examined for exogeneity and then appropriately modeled in the following reduced form VARX(p, s) model ($\forall\ t = 1950, 1953, ..., 2016$; adapted from Lütkepohl, 2005):

$$\hat{y}_t = v + A_1 y_{t-1} + ... + A_p, y_{t-p} + B_0 x_t + ... + B_s x_{t-s}, \tag{3.5}$$

where y_t=$(y_{1t}, ..., y_{Kt})'$ is a K-dimensional vector of endogenous variables (incl. income inequality, wealth inequality and further endogenous macroeconomic variables), x_t=$(x_{1t}, ..., x_{Mt})'$ is a M-dimensional vector of exogenous macroeconomic variables, $A_i := A^{-1}A_i^*$ (i = 1,...,p), $B_i := A^{-1}B_j^*$ (j = 0, 1, ...,s) are the matrix products of the matrix A_i of instantaneous relationships between the endogenous variables and the matrices A_i^* and B_j^* are the (2×2)-dimensional estimated coefficient matrices of the structural form.

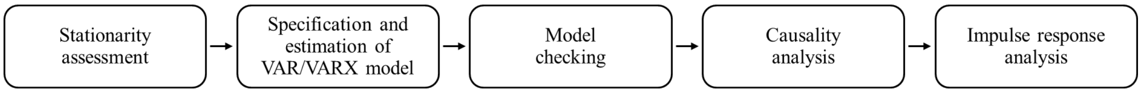
The problem can be considered solved appropriately, if the transformations to the data as well as the model parameters are chosen in such a way, that the model residuals resemble Gaussian white noise.

3.4 Analytical Strategy

In the following section, the analytical strategy is outlined, explaining some of the methodological choices in the modeling process in more detail. Figure 1 shows the steps followed during the modeling.

Figure 1

Analytical approach process structure.



First, stationarity of the time-series was assessed using the *Augmented Dickey-Fuller test* (*ADF*; Dickey & Fuller, 1979; Dickey & Fuller, 1981; Said & Dickey, 1984; for detailled statistical background of residual tests, see the appendix) as well as through visual plots of *autocorrelations* (ACFs) and *partial correlations* (PCFs). The objective of this step was to infer the order of the used time series and ensure the time series exhibit stationarity, which is a requirement for the application in the VAR/VARX model (Hyndman Athanasopoulos, 2021).

Second, the order of the model was determined via information criteria. Both the *Akaike Information Criterion* (AIC; Akaike, 1973) and the *Schwarz Criterion* (SC; Schwarz, 1978) were used in this step (see appendix for formulas).

In addition, the time series were tested for co-integration using *Johansen's cointegration test* (Johansen, 1995). Then, the VAR(p) and the VARX(p, s) multivariate time series models were fitted to the data via the *vars* (Pfaff, 2008) and the *MTS* package (Tsay, 2013), respectively. The VAR was estimated via package default estimation methods, namely OLS equation by equation, whereas the VARX model was estimated via OLS. The macroeconomic variables were allocated to the VARX model either as endogenous or exogenous variables. Following Lütkepohl (2005), exogeneity was assumed if the exogenous variable observations x_t, x_{t-1},...,x_{t-s} are independent of the error term w_t. To test this requirement, bivariate VAR(p) models were fitted and Granger-causality between the variable and the included inequality dimension assessed. After an initial VAR and VARX model were estimated, a restricted version was estimated with a threshold t-value of two.

Third, model checking was performed. Specifically, model residuals were examined for white noise characteristics (for statistical background of all tests see appendix). For the VAR model, the multivariate *Breusch-Godfrey-test* (Breusch, 1978; Godfrey, 1978) was used to test for the presence of no auto-correlation. The BG-test was chosen over the adjusted Portmanteau-test since the lag order p was low and it was shown that the Lagrange Multiplier tests are more suitable for small values of lag order p (Lütkepohl, 2005). The multivariate *ARCH-test* assessed if residuals exhibit heterogeneity (Engle, 1982; Hamilton, 1994). Furthermore, the *Jarque-Bera-test* (Bera Jarque, 1980; Bera Jarque, 1981; Jarque Bera, 1987) was used to test for deviations of multivariate normality. In addition, a visual assessment of residual plots, squared residual plots (as indicators of heterogeneity), histograms (for normality), and ACF plots (for remaining autocorrelation) was conducted for both models. Due to differences in the r packages used for both models, the VARX model checking used slightly different tests. The univariate JB-test was used for residuals of all endogenous variables instead of the multivariate version and the Hosking-test (Hosking, 1980) to test for remaining autocorrelation.

Fourth, Granger-causality (Granger, 1969) was assessed to determine if income and wealth inequality are temporally related.

Finally, the impulse response analysis (Lütkepohl, 2005) were conducted including bootstrap with 100 runs, in which the response of one variable to a sudden but temporary change in another variable was analyzed. The impact of exogenous variables in the VARX model was assessed by examining the residual covariance matrix and individual coefficient parameters (due to technical limitations).

4 Results

Table 1

Descriptive statistics

Time Series	M	SD	CV	min(year)	max(year)
Median Household Income	56117.0	15219.7	27.1%	28895.0	83360.0
Median Household Wealth	135656.0	88355.8	65.1%	40920.0	351110.0
Income Inequality	48.3	5.2	10.8%	42.0	58.0
Wealth Inequality	80.1	2.5	3.1%	76	86.0
GDP per Capita	21165.0	18677.8	88.2%	1976.0	57592.0
Interest Rates	4.4%	2.7%	61.7%	0.7%	11.9%

Note. M = mean, SD = standard deviation, CV = coefficient of Variation

The descriptive statistics for the time series data are shown in *Table 1*. Visual charts of the time series are shown in the appendix. The data shows that the U.S. inequality was decreasing for both income and wealth until roughly 1977, after which a period of strong increase in inequality started. The median income and wealth show that for the time after the financial crisis, a sharp drop of income and wealth happened, which is not yet reflected in the inequality measures. Additionally, a close inspection of the data shows that the variation of wealth inequality is rather very small, and somewhat larger for income inequality. The macroeconomic time series vary stronger descriptively.

In the sections that follow, results for time series stationarity tests are described, before determining the order parameters of the models. Afterwards, the best models are fitted to the data and white noise characteristics of residuals are tested. Finally, Granger causality and impulse response analysis results are presented.

Table 2

Augmented Dickey-Fuller test results

	Constant with Trend		Constant w/0 Trend		None	
Criterion	Levels	1st Diff	Levels	1st Diff	Levels	1st Diff
Gini Income	-2.96**	-2.53**	.09	-2.58**	1.95*	-1.83*
Gini Wealth	-.93	-4.93***	.15	-2.74**	1.39	-2.69**
Interest Rates	-2.11*	-4.41***	-.87	-3.77***	-.72	-3.80***
GDP	-1.50	-3.89***	2.90**	.50	-1.37	1.17

Note. $^{***}\,p \leq .01,^{**}\,p \leq .05,^{*} \leq .10$; optimal leg length based on the Akaike Information Criterion (AIC).

Test results for the ADF are shown in *Table 2*. Results indicate that stationarity is ensured most likely with deterministic terms including a constant with trend. Choosing the mentioned deterministic terms and assuming an α-level of 5%, the null hypothesis of unit root is not rejected. Since a trend is obviously present in the Gini income time series (also see Figure A1 in appendix), first order differencing was applied. For the other three time series, the null hypothesis of unit root is rejected at an α-level of 5%, indicating that first order differencing is required to ensure stationarity. ACF/PACF plots for the four time series are included in the appendix. The ACFs for all four time series show large autocorrelations that are linearly decreasing and turning negative at later lags, supporting the decision to apply differencing at least once. The PCF plots show a significant first lag for all four time series, indicating that the large number of significant autocorrelations found in the ACFs are merely due to the propagation of the autocorrelations at lag 1. For both income inequality and interest rates, the 10th lag of the PCF is close to significance.

Table 3 displays the results of different VAR models that were fitted to the data, and the corresponding AIC and SC values for different parameters lag values p. Results indicate that the most suitable bivariate VAR model has $p = 6$ lags (based on both AIC and SC). In contrast, the multivariate VAR model has $p = 3$ lags (also indicated by both the AIC and SC), although the maximum lag number

Table 3

Lag selection results for the VAR models

	VAR_{bv}		VAR_{mv}	
Lags	$AIC(n)$	$SC(n)$	$AIC(n)$	$SC(n)$
1	1.91	2.30	18.46	19.65
2	1.82	2.40	19.10	21.08
3	1.88	2.65	**12.09**	**14.86**
4	1.75	2.72	-Inf	- Inf
5	1.69	2.85	-	-
6	**0.25**	**1.61**	-	-

Note. AIC = Akaike Information Criterion, SC = Schwarz Criterion, Best-performing model highlighted in bold based on lowest AIC/BIC values; VAR_{bv} model: *gini income*, *gini wealth*, VAR_{mv} model: *gini income*, *gini wealth*, *interest rates*, *gdp per capita.*

Table 4

Maximum Eigenvalue test result (for cointegration)

	VAR_{bv}				VAR_{mv}			
	Statistic	10pct	5pct	1pct	Statistic	10pct	5pct	1pct
$r \leq 3$	-	-	-	-	5.04	10.49	12.25	16.26
$r \leq 2$	-	-	-	-	16.90	16.85	18.96	23.65
$r \leq 1$	16.14	10.49	12.25	16.26	18.55	23.11	25.54	30.34
$r = 0$	42.41	16.85	18.96	23.65	46.73	29.12	31.46	36.65

Note. VAR_{bv} model: *gini income*, *gini wealth*, VAR_{mv} model: *gini income*, *gini wealth*, *interest rates*, *gdp per capita.*

was restrained by the sample size to $p_{max} = 4$ (vs. $p_{max} = 6$ for the bivariate model). The VARX(3, 3) model fits the data best based on the AIC, whereas the (0, 2) model is indicated by the SC (for technical reasons of the used package MTS, only the ranks are shown for VARX). Since the VAR_{mv} model also supports three endogenous lags, the AIC solution is chosen for the $VARX$ model.

Cointegration test results are shown in *Table 4*. For the bivariate VAR model, one can conclude that there are $r = 2$ cointegration relationships on an α-level of 5%. Consequently, a VAR model is estimated. For the multivariate model, a cointegration rank of $r = 1$ is assumed. This indicates that a *vector error correction model* would be the appropriate model class to model the long-term equillibrium between the time series. Due to the low sample size and many parameters that need to be estimated, a VAR or VARX model is chosen. The exo- or endogeneity of the macroeconomic time series variables dictate the specific model class to be used in the next steps.

To determine if the macroeconomic time series are exogenous, they are tested for Granger causality. If income inequality or wealth inequality Granger-cause the respective macroeconomic variable, they are not exogenous (Engle et al., 1983). Results for Granger-causality are depicted in *A1* in the appendix. The data indicates that interest granger-cause wealth inequality. Consequently, interest rates will be modeled as endogenous variable. Neither income nor wealth inequality Granger-cause GDP, so the dimension will be modeled as exogenous variables. Consequently, a VARX(3, 3) model with the number of dimensions K_{endo}=3 and K_{exo}=1 is chosen.

The restricted VAR(6)-model is presented below: ($\forall\ t = 1950, 1953, ...2016$)

$$\begin{bmatrix}\hat{y}_{1t}\\ \hat{y}_{2t}\end{bmatrix} = \begin{bmatrix}0\\ -3.16^{***}\end{bmatrix} + \begin{bmatrix}.11^{***}\\ .26^{***}\end{bmatrix} * t + \begin{bmatrix}-.54^{**} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_{1,t-1}\\ y_{2,t-1}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & -.34^{**}\end{bmatrix}\begin{bmatrix}y_{1,t-2}\\ y_{2,t-2}\end{bmatrix} + \begin{bmatrix}0 & -.73^{**}\\ 0 & -.41^{**}\end{bmatrix}\begin{bmatrix}y_{1,t-3}\\ y_{2,t-3}\end{bmatrix} + \begin{bmatrix}0 & -.83^{**}\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_{1,t-4}\\ y_{2,t-4}\end{bmatrix} + \begin{bmatrix}0 & 0\\ -.24^{**} & 0\end{bmatrix}\begin{bmatrix}y_{1,t-6}\\ y_{2,t-6}\end{bmatrix} \quad (4.1)$$

The restricted VARX(3,3)-model is presented below (for unrestricted model including standard errors see appendix; $\forall\ t = 1950, 1953, ..., 2016$):

$$\begin{bmatrix}\hat{y}_{1t}\\ \hat{y}_{2t}\\ \hat{y}_{3t}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}-.49^{*} & 0 & 0\\ 0 & 0 & -.36^{*}\\ -.47^{**} & -.68^{**} & -.90^{***}\end{bmatrix}\begin{bmatrix}y_{1,t-1}\\ y_{2,t-1}\\ y_{3,t-1}\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & -.30 & -.39^{**}\\ -.39^{**} & 0 & -.48^{**}\end{bmatrix}\begin{bmatrix}y_{1,t-2}\\ y_{2,t-2}\\ y_{3,t-2}\end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 & .31^{*} \\ -.41^{**} & -.43^{*} & 0 \\ 0 & -.64^{**} & 0 \end{bmatrix} \begin{bmatrix} y_{1,t-3} \\ y_{2,t-3} \\ y_{3,t-3} \end{bmatrix} + 10^3 \begin{bmatrix} 0 \\ .36^{**} \\ 0 \end{bmatrix} \begin{bmatrix} x_{1,t-0} \end{bmatrix} + 10^3 \begin{bmatrix} -.42^{**} \\ .69^{***} \\ 0 \end{bmatrix} \begin{bmatrix} x_{1,t-1} \end{bmatrix} \quad (4.2),$$

where $^{***}p \leq .01, ^{**}p \leq .05, ^{*}p \leq .10$.

VAR coefficients are examined in regard to their support of hypothesis, although Granger-causality tests and IRFs where possible are the primary tools to asssess the influence.

H_1 concerned the influence of income inequality on wealth inequality. The data does not support the hypothesis, with no lag being significant at an α-level of 5%. H_2 predicted that income inequality is influenced by wealth inequality. Indeed, the 3^{rd} lag (a = -.73, SE = .32, p = .04) and 4^{th} lag (a = -.83, SE = .34, p = .03) of wealth inequality were significantly associated with income inequality. H_3 concerned the influence of GDP on wealth inequality. Indeed, there is a significant 1^{st} lag of GDP (b = -.42, SE = .16, $p \leq .05$). H_4 assumed that GDP influences wealth inequality. In support of the hypothesis, there is a significant influence of GDP at lag zero (b = .36, SE = .16, $p \leq .05$) and at the 1^{st} lag (b = .69, SE = .18, $p \leq .01$).H_5 predicted that interest rates influence income inequality. On an α-level of 5%, the 3^{rd} lag (b = .31, SE = .17, $p \geq .05$) is not significant. H_6 stated that interest rates influence wealth inequality. Supporting the hypothesis, the 2^{nd} lag (b = -.39, SE = .15, $p \geq .05$) show a significant influence.

The model diagnostic results are described in the following section. The VAR(6) model shows no deviation from multivariate normality (JB_{mv}=.88, p = .93) and no signs of heteroskedasticity ($ARCH_{mv}$ = 33.00, p = .91). However, diagnostic tests indicate some remaining serial correlation ($BG_{mv} = 32.00, p = .04$). A visual inspection of ACF plots shows no significant autocorrelations for both income and wealth inequality, even though the third ACF lag for wealth inequality is close to being significant. Consequently, it is assumed that the model provides a limited, but decent approximation to the data. Furthermore, all the moduli of the eigenvalues of the companion matrix are below the threshold value one, indicating stability (see *Table 7*). Visual plots of the distribution of residuals, squared residuals, autocorrelations, and histograms can be accessed in the appendix. The plots for the VAR(6) residuals show no violations of assumptions.

The VARX(3,3) model tests indicate no signs of heteroskedasticity ($ARCH_{mv}$

Table 5

Stability

	1	2	3	4	5	6	7	8	9	10	11	12
Eigen values	.95	.95	.85	.85	.84	.83	.83	.82	.82	.78	.00	.00

$= 2.62$, $p = .45$), no deviation from multivariate distributions for each time series (JB_{y_1}=1.02, $p = .60$, JB_{y_2}=.66, p =.72, JB_{y_3}=1.83, $p = .40$), and no remaining autocorrelation (PT_{mv}=120.24, p =.20). A visual inspection of ACFs also shows no significant lags. Plots of squared residuals show some spikes in year 1999 for income inequality, and year 1977 for interest rates. Histograms show no substantial deviation from normality. To conclude, the VARX(3,3) model is assumed to be a reasonable fit to the data.

Next, interpretative analyses of the VAR(6) model are described and contributions of the exogenous variables in the VARX(3,3) model are assessed. Neither does income inequality Granger-cause wealth inequality ($F_{(df_1=6,df_2=4)}$=1.12, $p = .48$) nor vice versa ($F_{(df_1=6,df_2=4)}$=.97, $p = .53$), thereby not supporting H_2 and H_4. *Figure 3* and *Figure 4* show the reaction of wealth inequality to a shock in income inequality and vice versa. In line with the Granger causality results, there is no strong response of wealth inequality to an external shock of income inequality, rather the series fluctuates erratically around the mean. Income inequality has a similar impulse response, but fluctuates even stronger around the mean. A comparison of the residual covariances demonstrates:

$$\hat{\Sigma}_{VAR} = \begin{bmatrix} 5.94 & .56 & .21 \\ .56 & 3.69 & -.64 \\ .21 & -.64 & 2.89 \end{bmatrix} \geq \begin{bmatrix} 2.75 & .02 & -.00 \\ .02 & .91 & -.29 \\ -.00 & -.29 & 1.72 \end{bmatrix} = \hat{\Sigma}_{VARX} \tag{4.3}$$

Evidently, the residual covariances of the VAR(3) model are much larger than those of the VAR(3,3) model, supporting the coefficient estimate results for GDP mentioned above.

Figure 2

Response of Wealth Inequality by Income Inequality

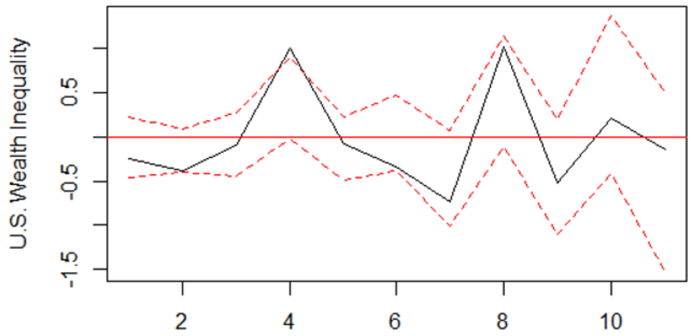
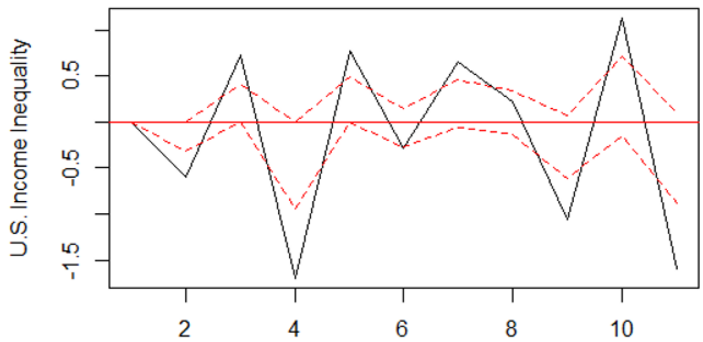


Figure 3

Response of Income Inequality by Wealth Inequality



5 Discussion

This paper contributes to the existing literature on U.S. postwar income and wealth inequality by using a multivariate time series approach on the recent SCF+ dataset by Kuhn et al. (2020). Building on recent findings in regard to trends in economic inequality, it was predicted that income inequality Granger-causes wealth inequality and vice versa. Contrary to predictions, neither did income inequality Granger-cause wealth inequality nor vice versa. In line with this finding, no persistent effects of shocks of one variable on the other could be identified. Also supporting these findings, there were no significant lags of income inequality on wealth inequality. However, there were some significant lag effects of wealth inequality on income inequality (i.e., 3^{rd} & 4^{th} lag).

To extend previous research in the field, effects of economic growth (via GDP) and interest rates on those two types of inequality were examined. In line with predictions, GDP reduced the residual error covariance matrix, thereby improving the explanation of the endogenous variables and showed a significant influence on both income inequality and wealth inequality. Surprisingly, the direction of the effect was negative for income inequality. In addition, interest rates appear to be shaped by income inequality in the mid-term (i.e., 3^{rd} lag), and, surprisingly, was influenced by income and wealth inequality (at 1^{st} lag by both, 2^{nd} for income inequality only and 3^{rd} for wealth inequality only). Consequently, data indicated that interest rates are an endogenous variable and are shaped by wealth and income inequality.

With these results, this paper contributes to a more refined understanding of income and wealth inequality. First of all, this paper used time series models based on a high-quality dataset to examine the long-term dynamics of wealth and income inequality, thereby avoiding methodological weaknesses of earlier studies on wealth and income inequality. Second, it used a multidimensional operationalization of economic inequality, showcasing the different explanatory factors for wealth and income inequality. The results of the present paper underscore, that wealth- and income-inequality appear to be self-influencing, although on slightly different time horizons, indicating different mechanisms. Surprisingly, interest rates only influenced income inequality in the mid-term, and did not influence wealth inequality (although it was influenced by wealth inequality lags). One potential explanation could be that the model could not capture long-run dynamics of interest rates on wealth inequality due to the low sample size and the limited number of maximal lags. Interest rates might require more time to unfold their effect on wealth accumulation substantially. In line with predictions, the model results show that the influence of GDP on economic inequality appears to depend on the type of inequality. Whereas increases in GDP tend to reduce income inequality, it increases wealth inequality. The richer households might be able to capture economic grow. One potential explanation for the reduction of income inequality could be that in phases of economic upturn new jobs are created and unemployed persons could find new employment opportunities easier.

This paper offers various implications for policy makers. First, the results underscore the role of economic growth in increasing wealth inequality. Through appropriate redistribute measures (e.g., progressive taxation), policy makers could ensure that economic gains are more evenly distributed, thereby avoiding a growing wealth divide in the population. Second, the central bank should be cautious in their use of interest rates to stimulate the economy, since an expansionary monetary policy (i.e., lowering interest rates) could excaberate wealth inequality. Finally, policy makers could aim to increase financial literacy in low-wealth districts to change household wealth composition in the lower parts of the wealth distribution. A stronger exposure to financial markets could ensure that the less wealthy could also benefit from economic gains.

The empirical results reported herein should be considered in light of some limitations. Firstly, the sample size can be considered a major limitation of the present thesis. Due to the low sample size, the maximum number of lags was limited for both the bivariate, but also the multivariate model. As a consequence, the long-term influence of macroeconomic variables could not be assessed for technical reasons. Since vector autoregressive models are heavily parametrized, a sufficient sample size is crucial (Lütkepohl, 2005). In this particular case, also the cointegration was not modeled using a vector error correction model to limit the set of estimated parameters.

Secondly, althrough no limitation in the strict sense, the operationalization of inequality should be kept in mind when interpreting results. The Gini coefficient was computed based on all household data. Much of research in regard to trends in economic inequality is looking at specific percentiles in the upper parts of the distribution (e.g., Piketty Saez, 2003). Besides, the research was limited to one specific operationalization of inequality, thereby restricting the robustness of the findings.

Finally, the VAR and VARX modeling approach is an atheoretic approach, where a large number of variables and lags are tested rather opportunistically (Hyndman & Athanasopoulos, 2021). Coefficient interpretation in VAR models is impaired (Hyndman Athanasopoulos), therefore hypothesis tests relying on coefficient estimates should be interpreted with caution. The decomposition of forecast errors into orthogonal structural shocks and with an economic meaning is crucial for any meaningful causal inference based on these models (Killian, 2013). Indeed, the atheoretical style of macroeconomics was criticized strongly in the past (e.g., Cooley Leroy, 1985), stressing the limited ability of VAR models in supporting causal statements.

Future work could aim to test the long-range dynamics by addressing the methodological impairments mentioned before. Specifically, researchers could compute inequality measures for specific parts of the wealth and income distribution to assess their unique temporal relationships. In addition, new studies could test different measures of inequality to establish even stronger validity of the findings (e.g., see Jenkins & van Kerm, 2009). Furthermore, future work could establish a set of restrictions based on economic theory and thereby follow a more targeted epistemic approach, for example by using Structural Vector Autoregressive Models (Lütkepohl, 2005).

To conclude, this thesis adds to the literature on U.S. postwar inequality by using time series-modeling to a multidimensional operationalization of inequality. As the findings illustrate, the temporal dynamics of economic inequality are complex and far from being solved. Due to the foundational nature of inequality for cohesion in society, there will be further opportunities to untangle the long-run dynamics of inequality development.

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B Appendix

1. Descriptive Plots

Figure A1

Time Series Plots for the Period $t = 1950, 1953, ..., 2016$.

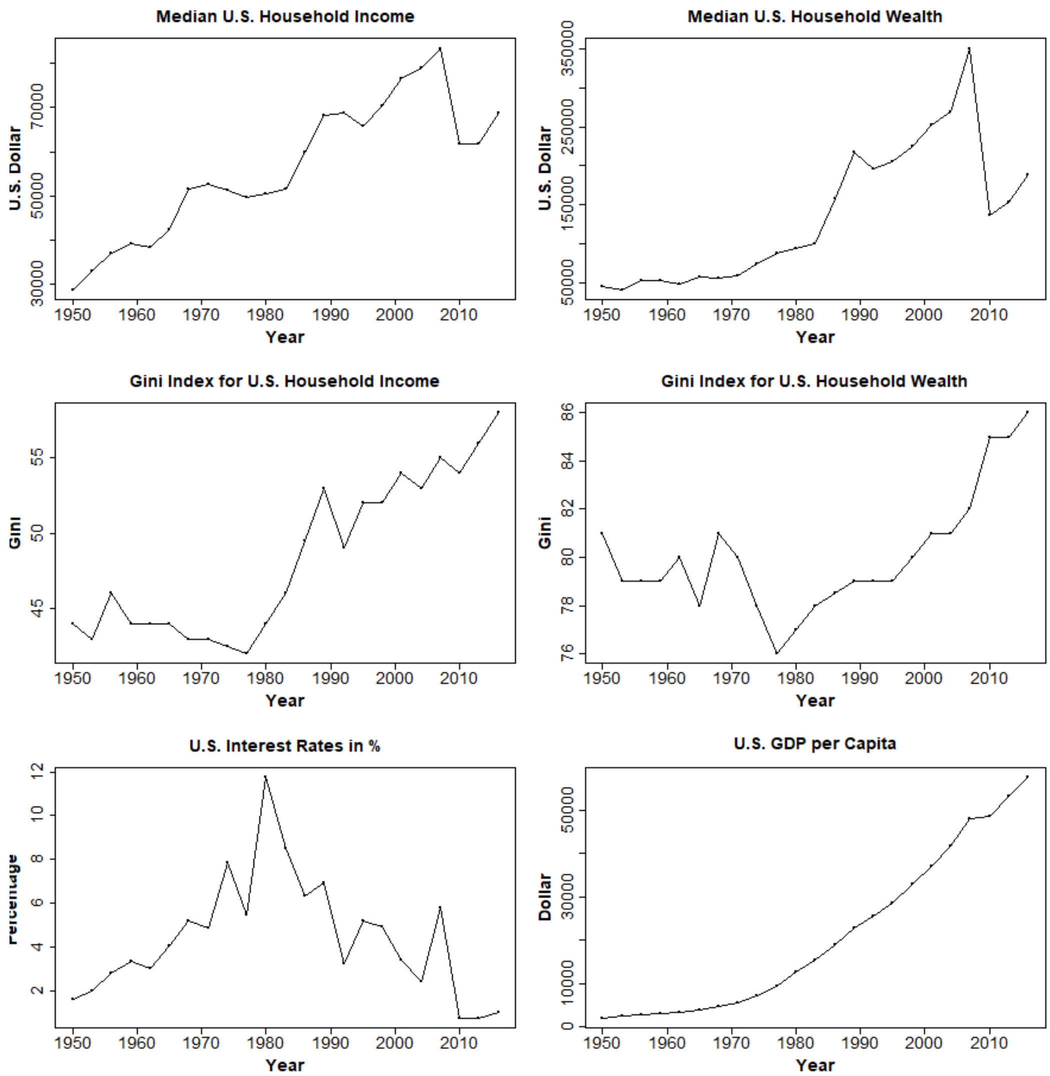


Figure A2

ACF/PCF Plots for the Time Series Before Differencing $t = 1950, 1953, ..., 2016.$

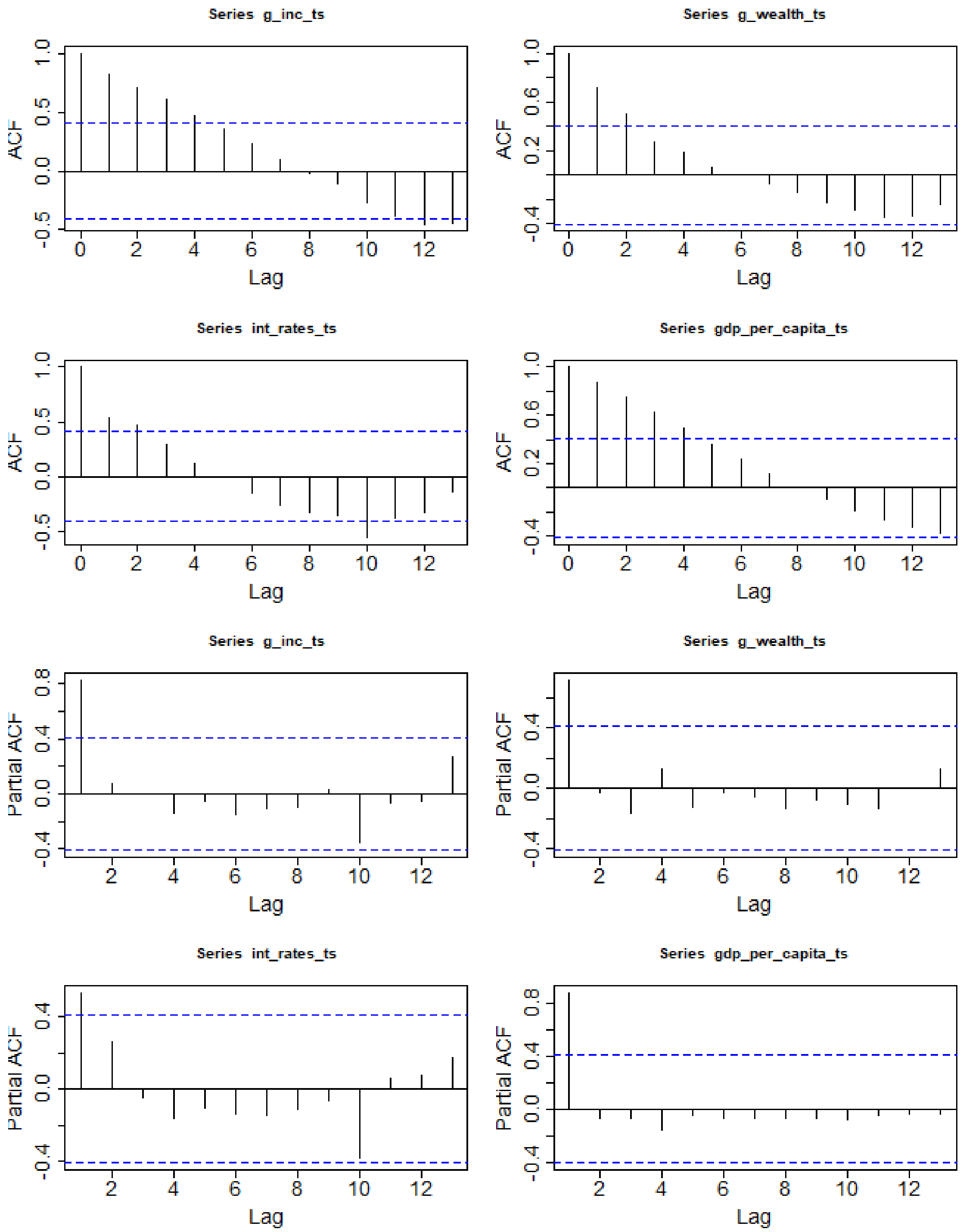


Figure A3

Residual Plots for the VAR(6) Model in the Period $t = 1950, 1953, ..., 2016.$

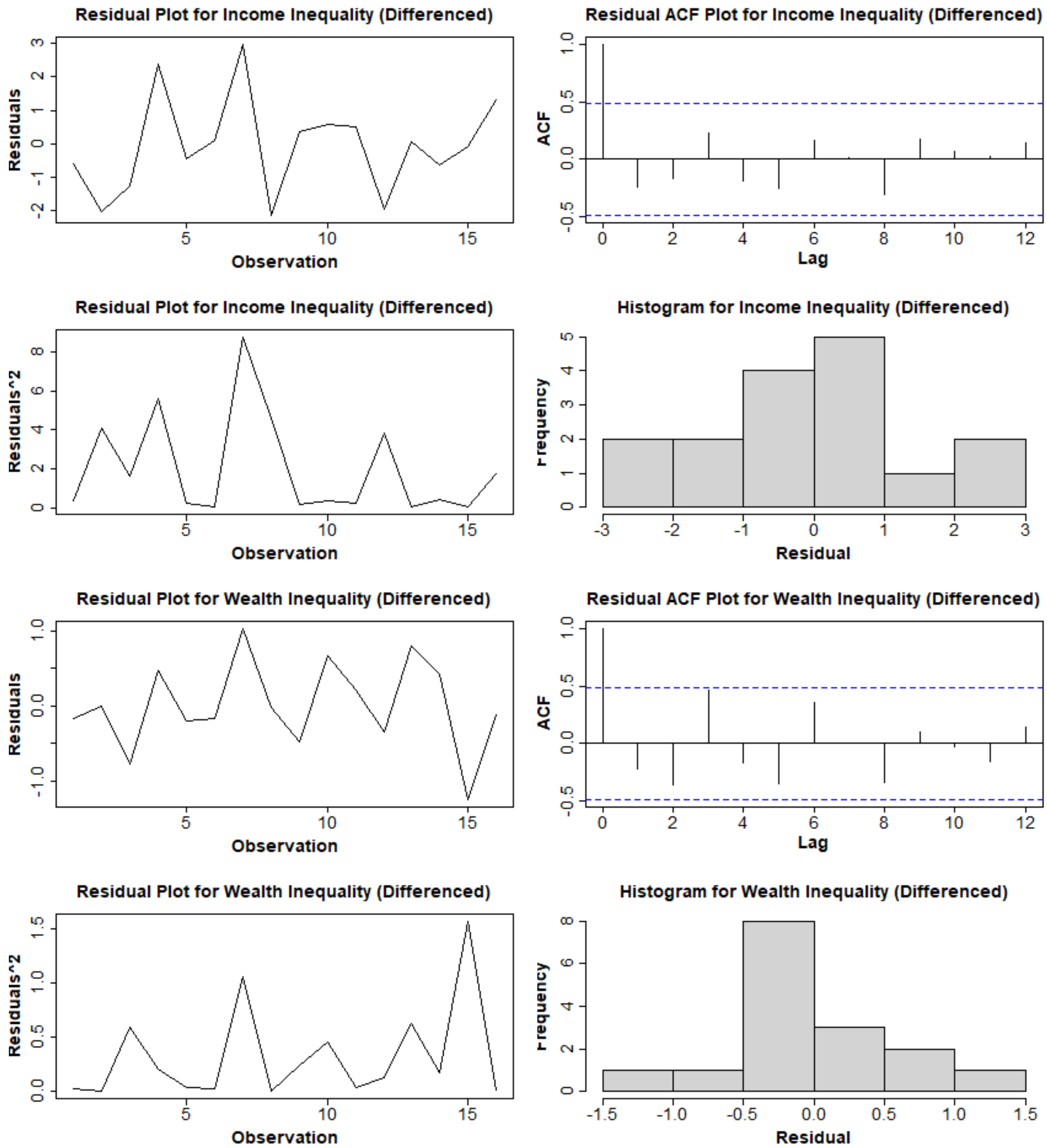
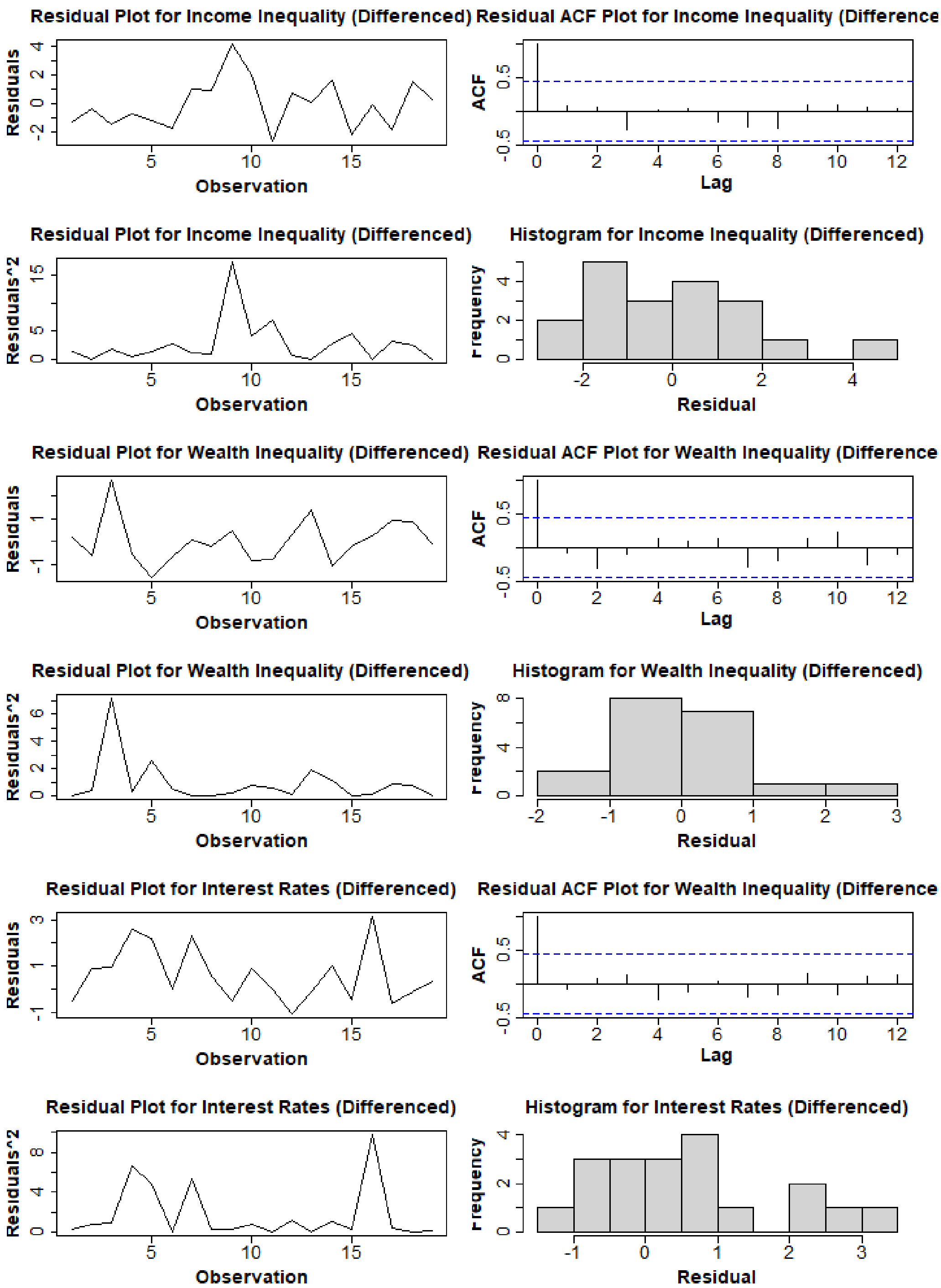


Figure A4

Residual Plots for the VARX(3,3) Model in the Period $t = 1950, 1953, ..., 2016.$



2. Additional Analysis Results

Table 5

Exogeneity Assessment of Macroeconomic Variables (Granger causality)

	Statistic	p-value
Gini income → Interest rates	.19	.91
Gini wealth → Interest rates	**3.88**	**.02**
Gini income → GDP	.37	.78
Gini wealth → GDP	2.03	.14

Note. Identified Granger-causalities highlighted in bold; based on VAR(3) model with $K = 2$ dimensions.

Table A2

Estimation Results of VARX(3,3) Model

Parameter	Estimates			SE			$t-Statistic$		
v_0'	-1.16	.37	1.06	.68	.53	.60	-1.71	.70	1.78
A_1^*	-.41*	.25	-.28	.17	.21	.24	-1.95	1.19	-1.17
	-.04	-.20	-.56	.13**	.16	.19	-.31	-1.05	-2.95
	-.27	-.40*	-.65**	.15	.19	.21	-1.80	-2.10	-3.09
A_2^*	-.03	-.30	.19	.18	.25	.24	-.17	-1.20	-1.06
	-.27*	-.40*	-.65***	.14	.19	.19	-1.93	-2.11	-3.42
	-.63***	-.67**	-.68***	.16	.22	.21	-3.94	-3.05	-3.24
A_3^*	-.15	-.31	.47**	.21	.24	.15	-.71	-1.29	3.13
	-.60***	-.56**	-.11	.16	.18	.12	-3.75	-3.11	-.92
	-.36*	-.69***	-.02	.18	.21	.14	-2.00	-3.29	-.14
$B_0^* * 10^{-3}$	.41	.62*	.38	.26	.25	.25	1.58	2.48	1.52
$B_1^* * 10^{-3}$	-.67***	.62**	.38*	.20	.20	.19	-3.35	3.10	2.00
$B_2^* * 10^{-3}$	.09	.14	-.28	.23	.23	.22	.39	.61	-1.27

Note. $^{***}\, p \leq .01^{**}\, p \leq .05^{*} \leq .10$ $(df = p - q = 22 - 12 = 10)$.

Table A3

Estimation Results of Restricted VARX(3,3) Model

Parameter	Estimates			SE			$t - Statistic$		
$v_0^{'}$	.00	.00	.00	-	-	-	-	-	-
A_1^*	-.49*	.00	.00	.23	-	-	-2.13	-	-
	.00	.00	-.36*	-	-	.16	-	-	-2.25
	-.47**	-.68**	-.90***	.18	.23	.18	-2.61	-2.96	-5.00
A_2^*	.00	.00	.00	-	-	-	-	-	-
	.00	.00	-.39**	-	-	.15	-	-	-2.60
	-.39**	.00	-.48**	.17	-	.18	-2.29	-	-2.66
A_3^*	.00	.00	.31*	-	-	.17	-	-	1.82
	-.41**	-.43*	.00	.14	.20	-	-2.93	-2.15	-
	.00	-.64**	.00	-	.27	-	-	-2.37	-
$B_0^* * 10^{-3}$	.00	.36**	.00	-	.14	-	-	2.57	-
$B_1^* * 10^{-3}$	-.42**	.69***	.00	.16	.18	-	-2.63	3.83	-
$B_2^* * 10^{-3}$	.00	.00	.00	-	-	-	-	-	-

Note. $^{***}\, p \leq .01^{**}\, p \leq .05^{*}\, \leq .10$ ($df = p - q = 22 - 12 = 10$).

3. Theoretical Background of Tests

In the following subsection, a range of tests that need to be implemented in the course of the multivariate time series modeling are described in terms of their econometric background. First, the *augmented Dickey-Fuller-test* (*ADF*; Dickey & Fuller, 1979; Dickey & Fuller, 1981; Said & Dickey, 1984) is a test to assess stationarity of time series. The ADF tests the following hypothesis:

H_0: $p - 1 = 0$ (random walk with drift)

or

H_0: $p - 1 < 0$ (trend stationary with $ß \neq 0$)

The test statistic is as follows:

$$t = \frac{\hat{p} - 1}{\hat{\sigma}_{\hat{p}}(\mathrm{B.1})}$$

The ADF estimates the following regression:

$$\Delta Y_t = (1 - \rho)Y_{t-1} + \mu + ßt + \sum_{j=1}^{\rho-1} ß_j \Delta Y_{t-j} + u_t, \tag{B.2}$$

where Δ is a time series, μ is a drift and $ßt$ a linear trend, U_t is a white noise process. The number of maximal lags j can be selected using information criteria, which will be described next.

Two popular criteria are the *Akaike information criterion (AIC)* and the *Schwarz criterion (SC)*. The formulas for the metrics are shown in Table A4 (Lütkepohl, 2005):

When comparing both metrics, it is evident that the SC penalizes models more heavily for their complexity (Bishop, 2006). In contrast, the AIC does not penalize for model complexity as much, thereby favoring models with larger lags.

There are a range of tests to assess white noise characteristics of residuals of a VAR(p) model, which, if tests validate those characteristics, indicate a proper model specification and enable further testing (e.g., Granger causality). Those tests will be described below. The *Breusch-Godfrey (BG) Lagrange-Multiplier* statistic tests for residual autocorrelation in a VAR model (Breusch, 1978; Godfrey, 1978). The BG test statistic is computed via the auxiliary regression (Lütkepohl, 2005; Pfaff, 2008):

$$\hat{\boldsymbol{u}}_t = A_1 y_{t-1} + ... + A_p y_{t-p} + CD_t + B_1 \hat{\boldsymbol{u}}_{t-1} + ... + B_h \hat{\boldsymbol{u}}_{t-h} + \epsilon_t \tag{B.3}$$

Table A4

Information Criteria Used for VAR and VARX Model Order Selection.

Criterion	Formula
1. AIC(p)	$ln\|\tilde{\Sigma}_u(p)\| + \frac{2mK^2}{T}$
2. SC(p)	$ln\|\tilde{\Sigma}_u(p)\| + \frac{lnT}{T}mK^2$

Note. AIC = Aikake information criterion, SC = Schwarz Criterion

p = order of VAR model, T = sample size, K = dimension of time series,

$\widetilde{\Sigma}_u$ = estimate of white noise covariance matrix.

The test hypothesis is H_0: $B_1 = ... = B_h = 0$ against the alternative hypothesis H_1: $B_j \neq 0$ for at least one $i \in \{1, 2, ..., h\}$. The test statistic is computed based on this auxiliary regression as follows (Pfaff, 2008):

$$LM(h) = T(K - tr(\widetilde{\Sigma}_R^{-1}\Sigma_\epsilon)), \tag{B.4}$$

where h is the number of lags, $\widetilde{\Sigma}_R^{-1}$ and Σ_ϵ are the restricted and the unrestricted residual covariance matrix of the estimated models, respectively. The test statistic is asymptotically χ^2-distributed with hK^2 degrees of freedom (Lütkepohl, 2005; Pfaff, 2008), where K describes the number of dimensions in the model. If the LM statistic is bigger than the critical χ^2-value for a given significance level, the null hypothesis of no auto-correlation is rejected.

An alternative test for autocorrelation is the *Hosking test* (Hosking, 1970). The statistic is a generalization of the univariate Portmanteau test for autocorrelation. Its test statistic is:

$$P' = n^2 \sum_{r=1}^{s} (n - r)^{-1} tr(\hat{C}_r^T \hat{C}_0^{-1} \hat{C}_r^T \hat{C}_0^{-1}), \tag{B.5}$$

where s refers to the number of lags tested, $\hat{C}_r$ is the residual autocovariance matrix, $\hat{C}_0 = \hat{\Sigma}$. The statistic is χ^2-distributed with $m^2(s-p-q)$ degrees of freedom, where m is the number of dimensions in the model, p the number of autoregressive components and q the number of additional potential moving average components.

The *Jarque-Bera* test (Bera Jarque, 1980; Bera Jarque, 1981; Jarque Bera, 1987) is used to test the residuals of a VAR(p) model for normality. The multivariate version of the test is based on residuals which are standardized using a Cholesky decomposition of the variance-covariance matrix of centered residuals (Pfaffer, 2008). The test statistic for the multivariate VAR(p) model follows a $\chi^2(2K)$ distribution and is defined as follows (Pfaffer, 2008):

$$JB_{mv} = s_3^2 + s_4^2, \tag{B.6}$$

where the multivariate skewness s_3^2 and the kurtosis test s_4^2 are the metrics below (Pfaffer, 2008):

$$s_3^2 = T\mathbf{b}_1^{'}\mathbf{b}_1/6 \tag{B.7}$$

$$s_4^2 = T(\mathbf{b}_2^{'} - \mathbf{3}_K)'(\mathbf{b}_2 - \mathbf{3}_K)/24, \tag{B.8}$$

where $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ represent the third and fourth non-central moment vector of the standardized residuals $\hat{\boldsymbol{u}}_{\boldsymbol{t}}^{\boldsymbol{s}} = \widetilde{P} - (\hat{\boldsymbol{u}}_{\boldsymbol{t}}^{\boldsymbol{s}} - \overline{\hat{\boldsymbol{u}}}_{\boldsymbol{t}}^{\boldsymbol{s}})$ and $\widetilde{P}$ is a lower triangular matrix coming from the Choleski decomposition (i.e., $\widetilde{P}\widetilde{P}' = \widetilde{\Sigma}_{\boldsymbol{u}}$; Pfaffer, 2008).

The *ARCH test* can be used to examine the heteroskedasticity of residuals (Engle, 1982; Hamilton, 1994). The regression equation below is the foundation for the ARCH-test:

$$vech(\hat{\boldsymbol{u}}_t\hat{\boldsymbol{u}}_t^{'}) = \mathbf{ß_0} + B_1 vech(\hat{\boldsymbol{u}}_{t-1}\hat{\boldsymbol{u}}_{t-1}^{'}) + ... + B_q vech(\hat{\boldsymbol{u}}_{t-q}\hat{\boldsymbol{u}}_{t-q}^{'}) + v_t, \tag{B.9}$$

where v_t represents a spherical error process, *vech* is a column stacking operator for symmetric matrices that piles columns from the main diagonal descending. $\mathbf{ß_0}$ is of dimension $\frac{1}{2}K(K+1)$. The coefficient matrix B_i (i = 1,...,q) is $(\frac{1}{2}K(K+1) \times \frac{1}{2}K(K+1))$-dimensional (Pfaff, 2008). The following hypothesis is tested:

H_0: $B_1 = B_2 = ... = B_q = 0$
vs. H_1: $B_i \neq 0\,(\forall\, i = 1, ..., q)$

The test statistic is defined as follows:

$$VARCH_{LM}(q) = \frac{1}{2}TK(K+1)R_m^2, \tag{B.10}$$

with

$$R_m^2 = 1 - \frac{2}{K(K+1)}tr(\hat{\Omega}\hat{\Omega}_0^{-1}), \tag{B.11}$$

where $\hat{\Omega}$ is the covariance matrix of the regression outlined above. The test statistic is $\chi^2(qK^2(K+1)^2/4)$-distributed. The univariate case is a special case of the above mentioned test (Pfaff, 2008).

Granger (1981) and Engle Granger (1987) introduced the concept of *cointegrated* processes. Variables in a K-dimensional process y_t are cointegrated of order (d, b) if all components of y_t are $I(d)$, but there exists a linear combination $z_t := ß'y_t$ with $ß = (ß_1, .., ß_K)' \neq 0$. As a consequence, z_t is $I(d-b)$. The cointegration rank can be assessed by examining a sequence of hypotheses:

$H_0 = rk(\Pi) = 0$, H_0: $rk(\Pi) = 1$,..., H_0: $rk(\Pi) = K-1$,

where Π comes from a Vector Error Correction model such as the following (Lütkepohl, 2005):

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{p-1}\Delta y_{t-p+1} + u_t, \tag{B.12}$$

where y_t is a time series process of dimension K, $rk(\Pi) = r$ with $0 \leq r \leq K$, $\Gamma_j(\forall j = 1, ..., p-1)$ is of dimension $(K \times K)$ and represents parameter matrices, u_t is a white noise error term, assuming no deterministic terms (for other deterministic terms, see Lütkepohl, 2005). For a H_0: $rk(\Pi) = r_0$, the test statistic is:

$$\lambda_{LR}(r_0, r_1) = 2[ln\, l(r_1] - ln\, l(r_0)] = -T\sum_{i=r_0+1}^{r_1} ln(1-\lambda_i, \tag{B.13}$$

where $l(r_i)$ refers to the maximum of the Gaussian likelihood function for the cointegration rank r_i.

The test series mentioned above is terminated once a null hypothesis cannot be rejected for the first time. The tested rank in this iteration represents the cointegration rank. There are two types of cointegration test, the *trace* and the *eigevalue* test (Lütkepohl, 2005). The paper uses the eigenvalue statistic $\lambda_{LR}(r_0, r_0+1)$, which tests the hypothesis H_0: $rk(\Pi) = r_0$ vs. $H1$: $rk(\Pi) = r_0 + 1$ (Lütkepohl, 2005). The statistic converges to a limiting distribution which is a functional of a $(K - r_0)$-dimensional Wiener process, for which critical values can be found in Johanson (1995).

VAR models can be analyzed in different ways to examine the interplay between the variables. In the following subsection, two interpretations of the VAR models are explained. First, Granger (1969) formalized an idea of causality which is applicable in VAR modeling. At its core, the idea is that if a variable x affects a variable z, x should support in generating valid predictions for z (Lütkepohl, 2005). Assuming y_t is a VAR(p) process, *Granger non-causality* is defined as (Lütkepohl, 2005):

$$z_t(1|y_s|s \leq t) = z_t(1|z_s|s \leq t)(\text{B.14})$$

Equivalently, the VAR(p) model can be assessed for significance of lag coefficients with H_0: $\theta_{i,12} = 0(\forall\, i = 1, 2, ..., p)$ in the equation:

$$y_t = \begin{pmatrix} z_t \\ x_t \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \sum_{n=1}^{p} \begin{pmatrix} A_{i,11} & A_{i,12} \\ A_{i,21} & A_{i,22} \end{pmatrix} \begin{pmatrix} z_{t-i} \\ x_{t-i} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \tag{B.15}$$

Second, an *impulse response analysis* can show the reaction of one variable to an impulse in another variable of the system. Quantifying the effect of an exogenous shock is at the core of impulse response analysis. In the Moving Average (MA)-representation of the VAR(p) model (Lütkepohl, 2005):

$$Y_t = \boldsymbol{\mu} + \sum_{i=0}^{\infty} A^i U_{t-i}, \tag{B.16}$$

where Y_t is characterized in terms of past and present innovation vector U_t and the mean $\boldsymbol{\mu}$. The MA coefficients contain the impulse responses of the modeled system (Lütkepohl, 2005). The parameter $_{jk,i}$ describes the reaction of the j-th variable to a shock in the k-th variable i periods ago (assuming no effect by other shocks to the system).

4. Estimation

The reduced form VAR(p) can be estimated by a range of estimation methods, including *ordinary least squares* (OLS), *maximum-likelihood* (ML), and *Bayesian estimation methods* (Lütkepohl, 2013). In the following, estimation via OLS will be outlined. The reduced form equation (3.4) can be rewritten in the following form, when assuming a drift as a deterministic term:

$$Y = BZ + U, \tag{B.17}$$

where $Y := (y_1, ..., y_T)$ is $(K \times T)$-dimensional, $B := (v, A_1, ..., A_p)$ is of dimension $(K \times (Kp+1))$, $Z_t := (1, y_t, ..., y_{t-p+1})'$ is $((Kp+1)) \times 1)$-dimensional, $Z := (Z_0, ..., Z_{T-1})$ is of dimension $((Kp{+}1)) \times T)$, $U := (u_1, ..., u_T)$ is $(K{\times}T)$-dimensional, and finally, $\boldsymbol{y} := vec(Y)$ is $(KT \times 1)$-dimensional. The VAR(p) model can be estimated consistently through OLS, equation by equation or multivariate (Lütkepohl, 2005). The LS estimator for a VAR(p) model is (adapted from Lütkepohl, 2005):

$$\hat{\boldsymbol{\beta}} = (\hat{ß}_1, ..., \hat{ß}_p) = ((ZZ')^{-1}Z_K)\boldsymbol{y} \tag{B.18}$$

The reduced form VARX(p, s) model can be estimated in the following way. Assuming we have a regression equation including endogenous and exogenous variables as follows (Lütkepohl, 2005):

$$y_t = AY_{t-1} + BX_{t-1} + B_0x_t + u_t, \tag{B.19}$$

where $A := [A_1, .., A_p]$, $B := [B_1, ..., B_p]$, $Y_t := [y_t, ..., y_{t-p+1}]'$, $X_t := [x_t, ..., x_{t-s+1}]$', u_t represents white noise with a nonsingular covariance matrix Σ_u. Parameter restrictions are possible and consequently a matrix R and a vector γ exist such that:

$$ß := R\gamma \tag{B.20}$$

In compact notation, we get
$$Y = ßZ + U, \tag{B.21}$$

where $Y := [y_1, ..., y_T]$, $Z := \begin{bmatrix} Y_0, ..., Y_{T-1} \\ X_0, ..., X_{T-1} \\ x_1, ..., x_T \end{bmatrix}$, and $U := [u_1, ..., u_T]$.

Parameters A, B, B_0 can be estimated via *General Least Squares* (GLS). Since the Σ_u is unknown, it can be estimated as $\check{\Sigma}_u = \frac{\check{U}\check{U}'}{T}$ (Lütkepohl, 2005). Assuming standard