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Megamodel-driven Traceability Recovery & Exploration of Correspondence & Conformance Links

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vorgelegt von

Maximilian Meffert

Erstgutachter:	Prof. Dr. Ralf Lämmel Institut für Informatik
Zweitgutachter:	Msc. Johannes Härtel Institut für Informatik

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Zusammenfassung

TBD.

Abstract

TBD.

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Background

This section summarizes the necessary background topics of the thesis. Each topic is introduced as independent as possible, interrelation is done during synthesis of hypotheses for this thesis in chapter 4. However, some of the following sections are sorted in a way that one may be based on its predecessor. In particular sections on the necessary mathematical background share definitions. Introducing them repeatedly would be redundant.

3.1 Predicate Logic

3.2 Relations

This section introduces the necessary aspects of mathematical relations for this thesis. The concept of relations is a generalization of semantic dependencies between two or more mathematical objects. This section is based on [2].

Relations are based on set-theory. We also introduce the necessary constructs of set-theory in order to clarify terminology and notation. A set is a collection of well distinguishable mathematical objects. Objects in a set are called elements of the set. A set does not contain two or more identical elements. The notation $x \in X$ denotes that x is an element of the set X. The symbol $\emptyset$ denotes the *empty set*, which contains no elements. The symbol Ω denotes the universal set, which contains all elements.

Definition 1 (Inclusion) *Let X and Y be a sets. Y* includes *X if and only if:*

$$X \subset Y :\Leftrightarrow \forall x[x \in X \rightarrow x \in Y] \Leftrightarrow \forall x[x \notin X \vee x \in Y] \tag{3.1}$$

Then X is called subset *of Y and Y is called* superset *of X. For an arbitrary set $Z \neq \emptyset$, the statement $\emptyset \subset Z$ is always true, respectively $Z \subset \emptyset$ is always false.*

We also define the opposite property: Y does not include X if and only if:

$$X \not\subset Y :\Leftrightarrow \exists x[x \in X \wedge x \notin Y] \tag{3.2}$$

Definition 2 (Union) *Let X and Y be a sets.*

$$X \cup Y := \{x | x \in X \vee x \in Y\} \tag{3.3}$$

$X \cup Y$ is called union *of X and Y.*

Definition 3 (Intersection) *Let X and Y be a sets.*

$$X \cap Y := \{x | x \in X \wedge x \in Y\} \tag{3.4}$$

$X \cap Y$ is called intersection *of X and Y.*

Definition 4 (Power-Set) *Let X be a set, then the* power-set *of X is defined as:*

$$\mathcal{P}(X) := \{Y | Y \subset X\} \tag{3.5}$$

The definition of inclusion provides an order for power-sets. So we may compare two sets A and B in the sense of *smaller* and *larger*, i.e.:

$$A \text{ is smaller than } B \Leftrightarrow A \subset B \tag{3.6}$$

$$A \text{ is larger than } B \Leftrightarrow B \subset A \tag{3.7}$$

$$A \text{ is the smallest subset of } B \Leftrightarrow \forall C \in \mathcal{P}(B) : A \subset C \tag{3.8}$$

$$A \text{ is the largest subset of } B \Leftrightarrow \forall C \in \mathcal{P}(B) : C \subset A \tag{3.9}$$

Definition 5 (Upper & Lower Bound) *Let Ω be a universe, $X \in \mathcal{P}(\Omega)$ be a set in the universe and $A \subset \mathcal{P}(U), A \neq \emptyset$, non-empty subsets in the universe.*

$$X \textit{ is an } \text{upper bound} \textit{ for } A :\Leftrightarrow \forall Y \in A : Y \subset X \tag{3.10}$$

$$X \textit{ is a } \text{lower bound} \textit{ for } A :\Leftrightarrow \forall Y \in A : X \subset Y \tag{3.11}$$

We also define:

$$\mathbf{U}_A := \{U \in \mathcal{P}(\Omega) | \forall Y \in A : Y \subset U\} \tag{3.12}$$

$$\mathbf{L}_A := \{L \in \mathcal{P}(\Omega) | \forall Y \in A : L \subset Y\} \tag{3.13}$$

as sets of all upper/lower bounds for A.

Because our definition of upper and lower bounds is based on power-sets, existence is guaranteed: Given an arbitrary set S, the S and $\emptyset$ are always elements of $\mathcal{P}(S)$. For each element Y of a non-empty selection $A \subset \mathcal{P}(S)$ of the power-set, $Y \subset S$ and $\emptyset \subset Y$ holds. So S is an upper and $\emptyset$ is a lower bound for A.

Definition 6 (Supremum & Infimum) *Let Ω be a universe, $X \in \mathcal{P}(\Omega)$ be sets in the universe and $A \subset \mathcal{P}(\Omega), A \neq \emptyset$ a non-empty selection of sets in the universe. If*

$$X = \sup A := \bigcup_{Y \in A} Y :\Leftrightarrow X \in \mathbf{U}_A \wedge \forall U \in \mathbf{U}_A : X \subset U \tag{3.14}$$

$$X = \inf A := \bigcap_{Y \in A} Y :\Leftrightarrow X \in \mathbf{L}_A \wedge \forall L \in \mathbf{L}_A : L \subset X \tag{3.15}$$

then X is called supremum/infimum *for A.*

Existence for supremum and infimum is guaranteed, because upper and lower bounds exist as shown above. Thus, for any non-empty selection $A \subset \mathcal{P}(S)$ of a power-set, $\mathbf{U}_A$ and $\mathbf{L}_A$ are not empty. So we need to proof, that $X = \bigcup_{Y \in A} Y$ respectively $X = \bigcap_{Y \in A} Y$ are in fact the smallest upper and the largest lower bound. Or in other words: Does another bound $X' \in \mathbf{U}_A$ respectively $X' \in \mathbf{L}_A$ exist with $X' \neq X$ and $X' \subset X$ respectively $X \subset X'$?

1. Supremum: We assume $X' \in \mathbf{U}_A$ with $X' \neq X$ and $X' \subset X$ for $X = \bigcup_{Y \in A} Y$ exists, then an element $x \in X$ exists, which is not element of X'. Because X

is the union of all sets in selection A, x must be element of at least one of its sets. However, then X' cannot include sets containing x. Thus, X' cannot be an upper bound for A and $X = \sup A$.

2. Infimum: We assume $X' \in \mathbf{L}_A$ with $X' \neq X$ and $X \subset X'$ for $X = \bigcap_{Y \in A} Y$ exists, then an element $x \in X'$ exists, which is not element of X. Because X is the intersection of all sets in selection A, x cannot be element of at least one of its sets. However, then X' must include sets containing x. Thus, X' cannot be a lower bound fo A and $X = \inf A$.

Supremum and Infimum are unique for any non-empty selection of sets in a universe and can be obtained by its union respectively its intersection. [2]

Definition 7 (Cartesian Product) *Let U be a universe and $X_n \in \mathcal{P}(U)$ sets with $i = 1...n, n \in \mathbb{N}$, then:*

$$X_1 \times ... \times X_n := \{(x_1, ..., x_n)\} \tag{3.16}$$

is called Cartesian product.

Definition 8 (Relation) *A relation is a subset of a Cartesian product:*

$$R \subset X_1 \times ... \times X_n \tag{3.17}$$

The relation of only two sets is called binary relation. *Instead of writing $(x, y) \in R$ we may also use the shorter notation xRy.*

An arbitrary relation $R \subset A \times B$ is called *homogeneous* if $A = B$, otherwise it is called *heterogeneous*. However, an arbitrary relation $R \subset A \times B$ is also homogeneous in the sense of $R \subset A \times B \subset (A \cup B) \times (A \cup B)$. For the remainder of this section we focus on homogeneous relations unless noted otherwise.

In order to clarify our notation, when we are specifically working with relations instead of ordinary sets, we use the symbols $\sqsubset$ for inclusion,$\not\sqsubset$ for non-inclusion, $\sqcup$ for union and $\sqcap$ for intersection, i.e.:

$$R \sqsubset S :\Leftrightarrow \forall x, y[(x, y) \in R \rightarrow (x, y) \in S] \tag{3.18}$$

$$R \not\sqsubset S :\Leftrightarrow \exists x, y[(x, y) \in R \wedge (x, y) \notin S] \tag{3.19}$$

$$R \sqcup S := \{(x, y)|(x, y) \in R \vee (x, y) \in S\} \tag{3.20}$$

$$R \sqcap S := \{(x, y)|(x, y) \in R \wedge (x, y) \in S\} \tag{3.21}$$

Furthermore, we use $\mathcal{R}(A)$ to denote the set of all homogeneous relations in set A and the symbols $\mathcal{O}$ and $\mathcal{A}$ to denote the empty relation and the universal relation:

$$\mathcal{R}(A) := \{R|R \subset A \times A\} \tag{3.22}$$

$$(\mathcal{O} := \emptyset) \subset A \times A \qquad \Leftrightarrow \forall R \in \mathcal{R}(A)[\mathcal{O} \sqsubset R] \tag{3.23}$$

$$(\mathcal{A} := A \times A) \subset A \times A \qquad \Leftrightarrow \forall R \in \mathcal{R}(A)[R \sqsubset \mathcal{A}] \tag{3.24}$$

Definition 9 (Composition of Binary Relations) *Let $R, S \in \mathcal{R}(A)$. Then $R \circ S \in \mathcal{R}(A)$ is defined*

$$R \circ S = RS := \{(r, s) \in A \times A|\exists x \in A : (r, x) \in R \wedge (x, s) \in S\} \tag{3.25}$$

and called composition *or* multiplication *of R and S. Instead of writing $R \circ S$ we also write simply RS.*

In conjunction with $\sqsubset$ composition is monotone:

$$\forall P, Q, R \in \mathcal{R}(A) : P \sqsubset Q \rightarrow R \circ P \sqsubset R \circ Q \tag{3.26}$$

$$\forall P, Q, R \in \mathcal{R}(A) : P \sqsubset Q \rightarrow P \circ R \sqsubset Q \circ R \tag{3.27}$$

- ($\Rightarrow$) Following the definition of composition, it can easily be observed, if Q includes P, all elements of P are also in Q:

$$\forall x, y : (x, y) \in R \circ P \tag{3.28}$$

$$\rightarrow \exists z : (x, z) \in R \wedge (z, y) \in P \overset{P \sqsubset Q}{\rightarrow} \exists z : (x, z) \in R \wedge (z, y) \in Q \tag{3.29}$$

$$\rightarrow (x, y) \in R \circ Q \tag{3.30}$$

An analogous deduction can be shown for the right hand side composition of R.

- ($\Leftarrow$) The opposite direction can be proven indirectly, assuming $R \circ P \sqsubset R \circ Q$ holds, but $P \sqsubset Q$ does not:

Definition 10 (Identity Relation) *The relation $\mathcal{I} \in \mathcal{R}(A)$*

$$\mathcal{I} := \{(a,b) \in A \times A | a = b\} = \{(a,a) | a \in A\} \subset A \times A \tag{3.31}$$

is called identity relation.

$(\mathcal{R}(A), \circ, \mathcal{I})$ is a monoid, i.e. for all relations in $\mathcal{R}(A)$ composition is associative and $\mathcal{I}$ serves as it's identity element:

$$(Q \circ R) \circ S = Q \circ (R \circ S) \tag{3.32}$$

$$R \circ \mathcal{I} = \mathcal{I} \circ R = R \tag{3.33}$$

Also, $\mathcal{O}$ serves as absorbing element for composition:

$$R \circ \mathcal{O} = \mathcal{O} \circ R = \mathcal{O} \tag{3.34}$$

Because $(\mathcal{R}(A), \circ, \mathcal{I})$ is a monoid, we can define exponentiation:

Definition 11 (Exponentiation of Relations) *Let $R \in \mathcal{R}(A)$ and $n \in \mathbb{N}$.*

$$R^0 := \mathcal{I} \tag{3.35}$$

$$R^n := R \circ R^{n-1} \tag{3.36}$$

Consider the following example:

$$A = \{a,b,c,d\} \tag{3.37}$$

$$R = \{(a,b),(a,c),(c,d)\} \tag{3.38}$$

$$R^0 = \{(a,a),(b,b),(c,c),(d,d)\} \tag{3.39}$$

$$R^1 = R \circ R^0 = R \circ \mathcal{I} = \{(a,b),(a,c),(c,d)\} \tag{3.40}$$

$$R^2 = R \circ R^1 = R \circ R = \{(a,d)\} \tag{3.41}$$

$$R^3 = R \circ R^2 = \mathcal{O} \tag{3.42}$$

$$R^4 = R^5 = R^6 = \ldots = R \circ \mathcal{O} = \mathcal{O} \tag{3.43}$$

Definition 12 (Reflexivity) *A relation $R \in \mathcal{R}(A)$ is called:*

$$\text{reflexive} :\Leftrightarrow \forall a \in A : (a,a) \in R \tag{3.44}$$
$$\text{irreflexive} :\Leftrightarrow \forall a \in A : (a,a) \notin R \tag{3.45}$$

Definition 13 (Reflexive Closure) *Let $R \in \mathcal{R}(A)$. Then*

$$R^\circ := \inf\{S|R \sqsubset S \wedge S \textit{ reflexive }\} = R \sqcup \mathcal{I} \tag{3.46}$$

is called reflexive closure *of R.*

The reflexive closure of a homogeneous relation R is the infimum or largest lower bound of the set $A = \{S|R \sqsubset S \wedge S \text{ reflexive }\}$ containing all reflexive relations, which include R. The smallest reflexive relation is $\mathcal{I}$. The smallest relation including R is R itself. So, for an arbitrary relation $R' \in A$ the inclusions $R \sqsubset R'$, $\mathcal{I} \sqsubset R'$ and $(R \sqcup \mathcal{I}) \sqsubset R'$ hold. Thus $R \sqcup \mathcal{I}$ is a lower bound for A and $(R \sqcup \mathcal{I}) \sqsubset \inf A$ holds. Vice versa, $R \sqcup \mathcal{I}$ is an element of A and any relation $R'' \sqsubset (R \sqcup \mathcal{I})$ does either not include R or is not reflexive. Thus $R \sqcup \mathcal{I}$ is also the smallest relation in A and $\inf A \sqsubset (R \sqcup \mathcal{I})$. From $(R \sqcup \mathcal{I}) \sqsubset \inf A$ and $\inf A \sqsubset (R \sqcup \mathcal{I})$ follows $\inf A = (R \sqcup \mathcal{I})$. [2]

Definition 14 (Symmetry) *A relation $R \in \mathcal{R}(A)$ is called:*

$$\text{symmetric} :\Leftrightarrow \forall a,b \in A : (a,b) \in R \Rightarrow (b,a) \in R \tag{3.47}$$
$$\text{asymmetric} :\Leftrightarrow \forall a,b \in A : (a,b) \in R \Rightarrow (b,a) \notin R \tag{3.48}$$
$$\text{antisymmetric} :\Leftrightarrow \forall a,b \in A : (a,b) \in R \wedge (b,a) \in R \Rightarrow a = b \tag{3.49}$$

Definition 15 (Transitivity) *A relation $R \in \mathcal{R}(A)$ is called:*

$$\text{transitive} :\Leftrightarrow R^2 \sqsubset R \tag{3.50}$$
$$:\Leftrightarrow \forall a,b,c \in A : (a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R \tag{3.51}$$
$$\text{intransitive} :\Leftrightarrow R^2 \not\sqsubset R \tag{3.52}$$
$$:\Leftrightarrow \forall a,b,c \in A : (a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \notin R \tag{3.53}$$

The equivalent characterization of transitivity is $R^2 \sqsubset R$ is easily obtained:

$$\forall a, b, c \in A : (a, b) \in R \wedge (b, c) \in R \Rightarrow (a, c) \in R \tag{3.54}$$
$$\Leftrightarrow \forall a, c \in A : (a, c) \in RR \rightarrow (a, c) \in R \tag{3.55}$$
$$\Leftrightarrow \forall a, c \in A : (a, c) \in R^2 \rightarrow (a, c) \in R \tag{3.56}$$
$$\Leftrightarrow R^2 \sqsubset R \tag{3.57}$$

If $\mathcal{T}(A) := \{R \in \mathcal{R}(A) | R^2 \sqsubset R\}$ is the set over all transitive relations of set A, it's infimum $I := \inf \mathcal{T}(A)$ is also transitive. Assuming it is not, at least one element exists, which is in I^2, but not in I. Because I is the infimum of $\mathcal{T}(A)$, all transitive relations R must include it. Thus the same element is in R^2, but not in R. However, this is a contradiction, because R is transitive and all elements in R^2 must be in R.

$$\neg(I^2 \sqsubset I) \Leftrightarrow \forall x, y : \neg[(x, y) \in I^2 \rightarrow (x, y) \in I] \tag{3.58}$$
$$\Leftrightarrow \forall x, y : (x, y) \in I^2 \wedge (x, y) \notin I \tag{3.59}$$
$$\Leftrightarrow \forall x, y \exists z : (x, z) \in I \wedge (z, y) \in I \wedge (x, y) \notin I \tag{3.60}$$
$$\Leftrightarrow \forall x, y \exists z : (x, z) \in R \wedge (z, y) \in R \wedge (x, y) \notin R \tag{3.61}$$
$$\Leftrightarrow \forall x, y : (x, y) \in R^2 \wedge (x, y) \notin R \tag{3.62}$$
$$\Leftrightarrow \forall x, y : \neg[(x, y) \in R^2 \rightarrow (x, y) \in R] \tag{3.63}$$
$$\Leftrightarrow \neg(R^2 \sqsubset R) \lightning \tag{3.64}$$

Definition 16 (Transitive Closure) *Let $R \in \mathcal{R}(A)$ and $i \in \mathbb{N}$. Then*

$$R^+ := \inf\{S | R \sqsubset S \wedge S \textit{ transitive }\} = \sup\{R^i | i \geq 1\} \tag{3.65}$$

is called transitive closure *of R.*

The transitive closure of a relation is the infimum or greatest lower bound of the set $A = \{S | R \sqsubset S \wedge S \text{ transitive }\}$ containing all transitive relations, which include R. Because A contains only transitive relations, its infimum $I = \inf A$ is also transitive.

$PR \sqsubset QS$

$\Leftrightarrow \forall x, y : (x, y) \in PR \to (x, y) \in QS$

$\Leftrightarrow \forall x, y \exists z : (x, z) \in P \wedge (z, y) \in R \to (x, z) \in Q \wedge (z, y) \in S$

$\Leftrightarrow \forall x, y \exists z : (x, z) \notin P \vee (z, y) \notin R \vee [(x, z) \in Q \wedge (z, y) \in S]$

$\Leftrightarrow \forall x, y \exists z : [(x, z) \notin P \vee (z, y) \notin R \vee (x, z) \in Q] \wedge [(x, z) \notin P \vee (z, y) \notin R \vee (z, y) \in S]$

$\Leftrightarrow \forall x, y \exists z : [(z, y) \notin R \vee [(x, z) \notin P \vee (x, z) \in Q]] \wedge [(x, z) \notin P \vee [(z, y) \notin R \vee (z, y) \in S]]$

$\Leftrightarrow \forall x, y \exists z : [(z, y) \notin R \vee [(x, z) \in P \to (x, z) \in Q]] \wedge [(x, z) \notin P \vee [(z, y) \in R \to (z, y) \in S]]$

$\Leftrightarrow \forall x, y \exists z : [(z, y) \notin R \vee P \sqsubset Q] \wedge [(x, z) \notin P \vee R \sqsubset S]$

$PR \not\sqsubset QS$

$\Leftrightarrow \exists x, y : (x, y) \in PR \wedge (x, y) \notin QS$

$\Leftrightarrow \exists x, y, z : [(x, z) \in P \wedge (z, y) \in R] \wedge \neg[(x, z) \in Q \wedge (z, y) \in S]$

$\Leftrightarrow \exists x, y, z : [(x, z) \in P \wedge (z, y) \in R] \wedge [(x, z) \notin Q \vee (z, y) \notin S]$

$\Leftrightarrow \exists x, y, z : [(x, z) \in P \wedge (z, y) \in R \wedge (x, z) \notin Q] \vee [(x, z) \in P \wedge (z, y) \in R \wedge (z, y) \notin S]$

$\Leftrightarrow \exists x, y, z : [P \not\sqsubset Q \wedge (z, y) \in R] \vee [(x, z) \in P \wedge R \not\sqsubset S]$

$\Leftrightarrow \forall x, y, z : \neg([P \not\sqsubset Q \wedge (z, y) \in R] \vee [(x, z) \in P \wedge R \not\sqsubset S])$

$\Leftrightarrow \forall x, y, z : [P \sqsubset Q \vee (z, y) \notin R] \wedge [(x, z) \notin P \vee R \sqsubset S]$

$\Leftrightarrow \forall x, y, z : [[P \sqsubset Q \vee (z, y) \notin R] \wedge (x, z) \notin P] \vee [[P \sqsubset Q \vee (z, y) \notin R] \wedge R \sqsubset S]$

$\Leftrightarrow \forall x, y, z : [P \sqsubset Q \wedge (x, z) \notin P] \vee [(z, y) \notin R \wedge (x, z) \notin P] \vee [P \sqsubset Q \wedge R \sqsubset S] \vee [(z, y) \notin R \wedge R \sqsubset S]$

$\Leftrightarrow \forall x, y, z : [P \sqsubset Q \wedge (x, z) \notin P] \vee [(x, y) \notin PR] \vee [P \sqsubset Q \wedge R \sqsubset S] \vee [(z, y) \notin R \wedge R \sqsubset S]$

Definition 17 (Transitive-Reflexive Closure) *$R^* := R^+ \cup I$ is called reflexive-transitive closure*

Definition 18 (Order Relation) *content...*

3.3 Mereology

Mereology is the logical study on the semantics of parthood. *"As a formal theory, mereology is simply an attempt to set out the general principles underlying the relationships between a whole and its constituent parts [...]"* [3]. Achille C. Varzi describes a collection of formal theories, i.e. sets of distinct axioms, of mereology in [3], which will be summarized in this section.
ToDo: add paragraph with applications of mereology in computer science

The first part of this section will just introduce the axioms of mereology. Then, at the end of this section, we will use these axiom to build some of the theories described in [3]. Note, that the terms relation, relationship and predicate may be used synonymously throughout this section.

3.3.1 Parthood

First we define the intuitive notion of the parthood relationship:

Definition 19 (partOf) *Let x and y objects of interest. We define:*

$$x \textsf{ partOf } y :\Leftrightarrow x \text{ is a constituent part of } y \tag{3.66}$$

We further assume, that partOf satisfies the following properties:

$$\text{(P1)} \quad x \textsf{ partOf } x \quad \text{(Reflexivity)} \tag{3.67}$$

$$\text{(P2)} \quad x \textsf{ partOf } y \land y \textsf{ partOf } x \rightarrow x = y \quad \text{(Antisymmetry)} \tag{3.68}$$

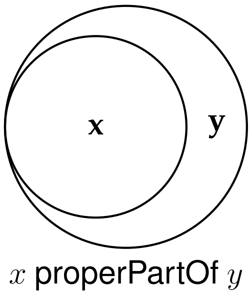
$$\text{(P3)} \quad x \textsf{ partOf } y \land y \textsf{ partOf } z \rightarrow x \textsf{ partOf } z \quad \text{(Transitivity)} \tag{3.69}$$

Thus, partOf induces a partial order of things.

However, since the reflexive parthood may be to week for some cases, we also define a stricter, irreflexive parthood relationship as follows:

$$x \textsf{ properPartOf } y :\Leftrightarrow x \textsf{ partOf } y \land \neg(y \textsf{ partOf } x) \quad \text{(Proper Part)} \tag{3.70}$$

Proper parthood induces a strict partial order of things.



This Venn-style diagram depicts a schematic illustration of Proper Part: x is certainly a part of y, however y is not a part of x.

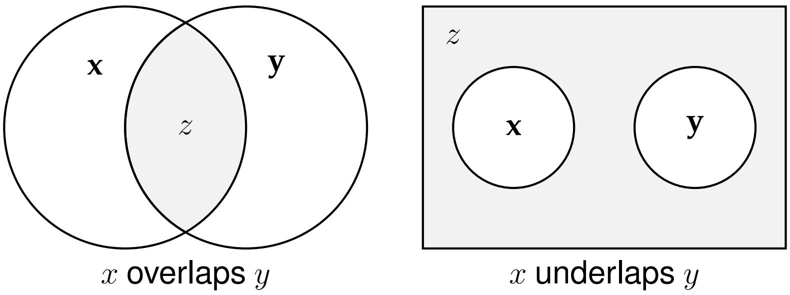
Figure 3.1 A schematic depiction of Proper Part

In addition to the relationships above we introduce the following predicates in order to provide a more concise notation:

$$x \text{ overlaps } y :\Leftrightarrow \exists z : z \text{ partOf } x \wedge z \text{ partOf } y \qquad \text{(Overlap)} \tag{3.71}$$

$$x \text{ underlaps } y :\Leftrightarrow \exists z : x \text{ partOf } z \wedge y \text{ partOf } z \qquad \text{(Underlap)} \tag{3.72}$$

Overlap models situations, where two things share at least on distinct part. Underlap models situations, where two things are part of the same distinct thing. Figure 3.2 illustrates both in a schematic fashion.



These Venn-style diagrams depict schematic illustrations of Overlap & Underlap:

- x overlaps y: x and y share a distinct part z, which is emphasized as gray area.
- x underlaps y: x and y are both parts of z, which is emphasized as gray area.

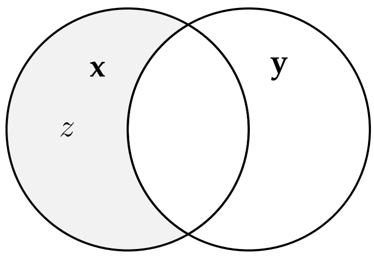
Figure 3.2 A schematic depiction of Overlap & Underlap

3.3.2 Supplementation

The fourth axiom, which can be assumed in an universe described by means of mereology, is the supplementation axiom. It models the effects of situations more precisely, where one thing is not part of another. Namely, if this is the case, a third thing may exist, which is part of the former but not part of the latter.

$$\text{(P4)} \qquad \neg(x \textsf{ partOf } y) \rightarrow \exists z(z \textsf{ partOf } x \wedge \neg(z \textsf{ overlaps } y)) \tag{3.73}$$

The supplementation axiom reads: If a thing x is not part of another thing y, then at least one part of x does not share further parts with y. Figure 3.3 depicts a schematic example of the supplementation axiom.



This Venn-style diagram exemplifies the supplementation axiom: x is not part of y. z is emphasized as gray area. x contains z, but z shares no further parts with y.

Figure 3.3 A schematic example of the Supplementation Axiom

ToDo: add paragraph explaining "strong" and "weak" supplementation

One can also observe from figure 3.3, that the supplementation axiom can be interpreted as analogue to set-theoretic difference.

3.3.3 Sum, Product & Difference

The next three axioms allow for the notions of sum, product and difference in a mereological context.

3.3.3.1 Sum

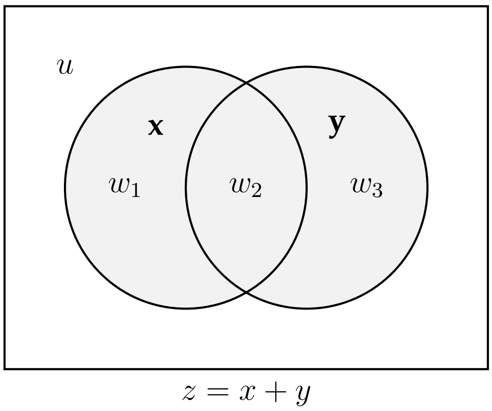
The fifth axiom, which can be assumed in an universe described by means of mereology, is the sum axiom. It models the intuitive notion, where all parts of one thing and all parts of another thing are exactly the constituent parts of a third thing.

$$\text{(P5)} \quad x \text{ underlaps } y \rightarrow \exists z \forall w (w \text{ overlaps } z \leftrightarrow (w \text{ overlaps } x \vee w \text{ overlaps } y)) \tag{3.74}$$

The sum axiom reads: If things x and y are two parts of the same thing, then another thing z exist, which only shares parts with things, which in turn share parts with x or y. Then z can be interpreted as the sum of x and y. This notion is captured by following definition of the term $z = x + y$:

$$z = x + y :\Leftrightarrow \forall w (w \text{ overlaps } z \leftrightarrow (w \text{ overlaps } x \vee w \text{ overlaps } y)) \tag{3.75}$$

Figure 3.4 depicts a schematic example of the sum axiom.



This Venn-style diagram exemplifies the sum axiom: x and y are part of u. $z = x + y$ is emphasized as gray area. All w_i share parts with z, if and only if they share parts with x or y.

Figure 3.4 A schematic example of the Sum Axiom

One can also observe similarities between the sum axiom and set-theoretic union from figure 3.4 and the definition 3.75.

3.3.3.2 Product

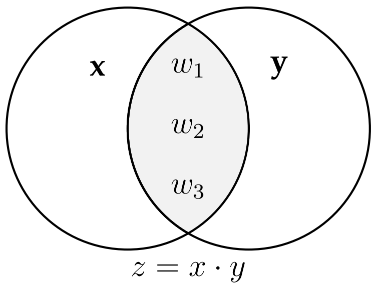
The sixth axiom, which can be assumed in an universe described by means of mereology, is the product axiom. It models the intuitive notion, where all things, which are parts of two things at the same time, are exactly the constituent parts of a third thing.

$$\text{(P6)} \qquad \begin{aligned} &x \text{ overlaps } y \\ &\rightarrow \exists z \forall w (w \text{ partOf } z \leftrightarrow (w \text{ partOf } x \wedge w \text{ partOf } y)) \end{aligned} \tag{3.76}$$

The sum axiom reads: If things x and y share at least one part, then another thing z exists, which only consists of parts, which in turn are parts of x and y at the same time. Then z can be interpreted as the product of x and y. This notion is captured by the following definition of the term $z = x \cdot y$:

$$z = x \cdot y :\Leftrightarrow \forall w (w \text{ partOf } z \leftrightarrow (w \text{ partOf } x \wedge w \text{ partOf } y)) \tag{3.77}$$

Figure 3.5 depicts a schematic example of the product axiom.



This Venn-style diagram exemplifies the product axiom: x and y share arts. $z = x \cdot y$ is emphasized as gray area. All w_i are part of z, if and only if they are part of x and y.

Figure 3.5 A schematic example of the Product Axiom

One can also observe similarities between the sum axiom and set-theoretic intersection from figure 3.5 and the definition 3.77.

3.3.3.3 Difference

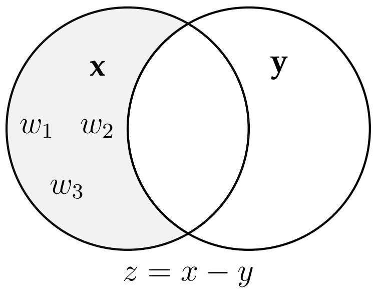
The seventh axiom, which can be assumed in an universe described by means of mereology, is the difference axiom. It models the intuitive notion, where things may be split, so that their constituent parts are regrouped into disjoint things, which share no parts.

$$\text{(P7)} \qquad \begin{aligned} &\exists z(z \text{ partOf } x \wedge \neg(z \text{ overlaps } y)) \\ &\rightarrow \exists z \forall w(w \text{ partOf } z \leftrightarrow (w \text{ partOf } x \wedge \neg(w \text{ overlaps } y))) \end{aligned} \tag{3.78}$$

The difference axiom reads: If for things x and y a supplementary thing exists, which is part of x, but shares no parts with y, then another thing z exists, consisting of parts, which are also part of x, but share no parts with y. Then z can be interpreted as the difference between x and y, in that order. This notion is captured by the following definition of the term $z = x - y$:

$$z = x - y :\Leftrightarrow \forall w(w \text{ partOf } z \leftrightarrow (w \text{ partOf } x \wedge \neg(w \text{ overlaps } y))) \tag{3.79}$$

Figure 3.6 depicts a schematic example of the difference axiom.



This Venn-style diagram exemplifies the difference axiom: x is not part of y and parts of x exists, which do not share parts with y. $z = x - y$ is emphasized as gray area. All w_i are part of z, if and only if they are part of x and do not share further parts with y.

Figure 3.6 A schematic example of the Difference Axiom

One can also observe again similarities between the sum axiom and set-theoretic difference from figure 3.6 and the definition 3.79.

<u>ToDo:</u> Outline dependencies with supplementation axiom

3.3.4 Universal Top, Complement & Bottom

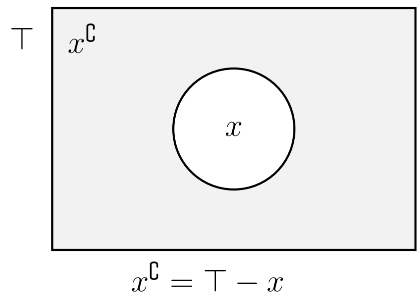
The sum axiom (P5) at 3.74 gives immediate rise to the idea, that all things can be summed up to an universal thing and all things are part of it. This notion is captured by the top axiom:

$$\exists\top\forall x(x \ \textsf{partOf} \ \top) \qquad \text{(Top)} \qquad (3.80)$$

Having an universe also facilitates the definition of a universal or absolute complement:

$$x^{\complement} := \top - x \qquad \text{(Complement)} \qquad (3.81)$$

Figure 3.7 depicts a schematic illustration of an universal top and a complement.



This Venn-style diagram illustrates universal top and a complement: Top $\top$ is the rectangle containing everything. The complement $x^{\complement}$ of x is emphasized as gray area.

Figure 3.7 A schematic illustration of Universal Top & Complement

An universal top renders the underlap relationship as defined at 3.72 trivially true, thus the mereological sum of two things as defined at 3.75 can never be undefined. In an algebraic sense, an universal top also provides an absorbing element for mereolgical sum and a neutral element for the mereoligcal product as defined at 3.77:

$$\top = \top + x = x + \top \qquad (3.82)$$

$$x = \top \cdot x = x \cdot \top \qquad (3.83)$$

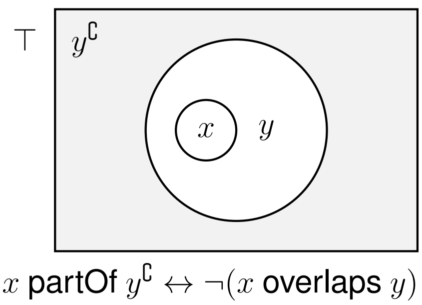
Properties of the mereological complement can in part be observed from figure 3.7. Obviously, in addition to the definition of complements, the following identity also holds:

$$x = \top - x^{\mathsf{C}} \tag{3.84}$$

Less obvious properties may be the following equivalences incorporating complement, parthood and overlap:

$$\begin{aligned} &\forall x \forall y (x \ \textsf{partOf} \ y^{\mathsf{C}} \leftrightarrow \neg(x \ \textsf{overlaps} \ y)) \\ &\forall x \forall y (x \ \textsf{partOf} \ y \leftrightarrow \neg(x \ \textsf{overlaps} \ y^{\mathsf{C}})) \end{aligned} \tag{3.85}$$

Figure 3.8 shows a schematic illustration of this equivalence. Both equations fol-



This Venn-style diagram illustrates the equivalence incorporating complement, parthood and overlap: Top $\top$ is the rectangle containing everything. x is a proper part of y. The complement y^{C} of y is emphasized as gray area. x shares no parts with y^{C}.

Figure 3.8 A schematic illustration the equivalence incorporating Complement, Parthood & Overlap

low directly from the definition of mereological difference by simple deduction, since x partOf $\top$ is always true:

$$\begin{aligned} x^{\mathsf{C}} = \top - x &\Leftrightarrow \forall y (y \ \textsf{partOf} \ x^{\mathsf{C}} \leftrightarrow x \ \textsf{partOf} \ \top \wedge \neg(y \ \textsf{overlaps} \ x)) \\ &\Leftrightarrow \forall y (y \ \textsf{partOf} \ x^{\mathsf{C}} \leftrightarrow \neg(y \ \textsf{overlaps} \ x)) \end{aligned} \tag{3.86}$$

$$\begin{aligned} x = \top - x^{\mathsf{C}} &\Leftrightarrow \forall y (y \ \textsf{partOf} \ x \leftrightarrow x \ \textsf{partOf} \ \top \wedge \neg(y \ \textsf{overlaps} \ x^{\mathsf{C}})) \\ &\Leftrightarrow \forall y (y \ \textsf{partOf} \ x \leftrightarrow \neg(y \ \textsf{overlaps} \ x^{\mathsf{C}})) \end{aligned} \tag{3.87}$$

A direct corollary is the identity of complement involution:

$$(x^{\mathtt{C}})^{\mathtt{C}} = x \tag{3.88}$$

This identity is deduced by substitution of the equivalence above at 3.86 and 3.87:

$$\begin{aligned}(x^{\mathtt{C}})^{\mathtt{C}} = T - x^{\mathtt{C}} &\Leftrightarrow \forall y(y \textsf{ partOf } (x^{\mathtt{C}})^{\mathtt{C}} \leftrightarrow \neg(y \textsf{ overlaps } x^{\mathtt{C}})) \\ &\Leftrightarrow \forall y(y \textsf{ partOf } (x^{\mathtt{C}})^{\mathtt{C}} \leftrightarrow y \textsf{ partOf } x) \\ &\Leftrightarrow (x^{\mathtt{C}})^{\mathtt{C}} = x\end{aligned} \tag{3.89}$$

This property allows for a refinement of the definition for mereological difference, which again shows another similarity with set-theoretic difference:

$$z = x - y = x \cdot y^{\mathtt{C}} \tag{3.90}$$

The proof is the same simple deduction as above:

$$\begin{aligned}z = x - y &\Leftrightarrow \forall w(w \textsf{ partOf } z \leftrightarrow w \textsf{ partOf } x \wedge \neg(w \textsf{ overlaps } y)) \\ &\Leftrightarrow \forall w(w \textsf{ partOf } z \leftrightarrow w \textsf{ partOf } x \wedge w \textsf{ partOf } y^{\mathtt{C}}) \\ &\Leftrightarrow z = x \cdot y^{\mathtt{C}}\end{aligned} \tag{3.91}$$

The notion of an universal top gives immediate rise to the question, whether a converse thing, an universal bottom, exists. The universal bottom for parthood satisfies:

$$\exists \bot \forall x(\bot \textsf{ partOf } x) \qquad \text{(Bottom)} \tag{3.92}$$

An universal bottom renders the overlap relationship as defined at 3.71 trivially true, thus the mereological product of two things can never be undefined. In an algebraic sense, an universal bottom provides an absorbing element for the mereological product and a neutral element for the mereological sum:

$$\bot = \bot \cdot x = x \cdot \bot \tag{3.93}$$

$$x = \bot + x = x + \bot \tag{3.94}$$

Moreover, it makes mereological difference sound. With an universal bottom the difference $x - x$ of only one thing is now possible:

$$\begin{aligned} z = x - x \Leftrightarrow z &= x \cdot x^{\complement} \\ &\Leftrightarrow \forall y(y \text{ partOf } z \leftrightarrow y \text{ partOf } x \wedge y \text{ partOf } x^{\complement}) \\ &\Leftrightarrow z = \bot \end{aligned} \tag{3.95}$$

The only thing, which is part of x and its complement at the same time, is the universal bottom. Given this fact, now complements of top and bottom can also be determined:

$$\top^{\complement} = \top - \top = \bot \tag{3.96}$$

$$\bot^{\complement} = \top - \bot = \top - \top^{\complement} = \top \cdot (\top^{\complement})^{\complement} = \top \cdot \top = \top \tag{3.97}$$

However, the notion of an universal bottom or null thing is more controversial than the notion of an universal top [3].
<u>ToDo:</u> Outline why bottom is disputed. Perhaps, if bottom and atoms are assumed, bottom would be the only "real" atom.

3.3.5 Unrestricted Fusion

3.3.6 Atomic Parts

3.3.7 Mereology Theories

3.4 Formal Languages & Grammars

3.4.1 Context-Free Languages & Grammars

3.5 Traceability

[4] [1] TBD.

3.5.1 Traceability Relationship

3.5.2 Traceability Link

3.5.3 Traceability Recovery

3.5.4 Traceability Exploration

3.6 Ontologies

TBD.

3.7 Megamodeling

TBD.

3.7.1 MegaL

3.7.1.1 MegaL/Xtext

3.8 Program Analysis

TBD.

3.9 XML Data Binding

TBD.

3.9.1 Java Architecture for XML Binding (JAXB)

3.10 Object Relational Mapping

TBD.

3.10.1 Java Persistence API (JPA)

3.10.2 Hibernate

3.11 Another Tool For Language Recognition (ANTLR)

Chapter 4

Hypotheses

TBD.

4.1 Fragments

4.2 Correspondence

4.3 Conformance

Chapter 5

Methodology

TBD.

5.1 The 101companies Human Resource Management System

Description of the 101companies Human Resource Management System

5.2 Link Proper Part Ratio

The ratio between all proper parts of two artifacts and the proper parts of the same artifacts in a relationship.

$$\pi_{R,A_1,A_2} = \frac{|\{(p_1, p_2) \in R : p_1 \text{ properPartOf } A_1 \wedge p_2 \text{ properPartOf } A_2\}|}{|\{p : p \text{ properPartOf } A_1 \vee p \text{ properPartOf } A_2\}|}$$

Chapter 6

Requirements

TBD.

R1 asdf

R2 asdf

R3 asdf

R4 asdf

Chapter 7

Design

TBD.

Chapter 8

Implementation

TBD.

8.1 Context-Free Grammar Fragmentation

8.2 Name Correspondence Heuristic

Heuristics are quick and "simple" methods for finding good approximate solutions for complex problems. The Name Correspondence Heuristic determines correspondence between artifacts simply by finding similarities of names in those artifacts.

TBD.

Chapter 9

Results

TBD.

Chapter 10

Conclusion

TBD.

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