BSc Notes

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Abstract. asdf

1 Theoretical Background

1.1 Relations & Predicates

Notation For a binary relation $R \subseteq X \times Y$ we consider the following notations to be semantically equivalent:

$$R(x, y) = xRy := (x, y) \in R$$

For that reason, relationships and predicates should be thought of as interchangeable.

Composition of Relations We define the composition of two binary relations $R \subseteq X \times Y$ and $S \subseteq Y \times X$ as:

$$S \circ R = \{(x, y) | \exists z \in Y : xRz \land zSy\}$$

Transitive Closures of Relations: R^+, R^* For a binary relation R on a set X ($R \subseteq X \times X$) we define its *transitive closures* R^+ and R^* as

$$\begin{aligned} R^0 &:= \{(x, x) | x \in X\} \\ R^1 &:= R \\ R^{n+1} &:= R \circ R^n \\ R^+ &:= \bigcup_{1 \leq n} R^n \\ R^* &:= \bigcup_{0 \leq n} R^n = R^0 \cup R^+ \end{aligned}$$

1.2 Mereologoy

Parthood [1]

Definition 1 (Parthood). *The binary relation* partOf *and defined:*

$$\mathsf{partOf} \subseteq X \times Y$$

and literally means that one element of set X *may be component or part of an element of set* Y*. We do not make any assumption of the relationship between both sets. However, we assume* partOf *to be ...*

1. ***reflexive*** $\forall x \in X, : \mathsf{partOf}(x, x)$
2. ***antysymmetric*** $\forall (x, y) \in X \times Y : \mathsf{partOf}(x, y) \wedge \mathsf{partOf}(y, x) \Rightarrow x = y$
3. ***transitive*** $\forall x, y, z \in L : \mathsf{partOf}(x, y) \wedge \mathsf{partOf}(y, z) \Rightarrow \mathsf{partOf}(x, z)$

Axiom 1 (M4) *content...*

Corollary 1 (Transitive Closures of PartOf). *For the transitve closures*

$$\mathsf{partOf}^{+}, \mathsf{partOf}^{*} \in L \times L$$

of partOf *holds:*

1. $\mathsf{partOf}^{+} \subset \mathsf{partOf}^{*}$
2. $\forall x, y \in L : x \neq y \wedge \mathsf{partOf}(x, y) \Rightarrow \mathsf{partOf}^{+}(x, y)$
3. $\forall x, y \in L : \mathsf{partOf}^{+}(x, y) \Rightarrow \mathsf{partOf}^{*}(x, y)$
4. $\forall x \in L : \mathsf{partOf}^{*}(x, x)$

Proper Parts, Overlaps & Underlaps

Definition 2 (Proper Part).

$$\forall x \in X, y \in Y : \mathsf{properPartOf}(x, y) :\Leftrightarrow \mathsf{partOf}(x, y) \wedge \neg \mathsf{partOf}(y, x)$$

Definition 3 (Overlap).

$$\forall x \in X, y \in Y : \mathsf{overlaps}(x, y) :\Leftrightarrow \exists z(\mathsf{partOf}(z, x) \wedge \mathsf{partOf}(z, y))$$

Definition 4 (Underlap).

$$\forall x \in X, y \in Y : \mathsf{underlaps}(x, y) :\Leftrightarrow \exists z(\mathsf{partOf}(x, z) \wedge \mathsf{partOf}(y, z))$$

Definition 5 (Sum).

$$\begin{aligned} \mathsf{underlaps}(x, y) &\Rightarrow \exists u \forall v(\mathsf{overlaps}(v, u) \Leftrightarrow (\mathsf{overlaps}(v, x) \vee \mathsf{overlaps}(v, y))) \\ &\Rightarrow \exists u \forall v(\mathsf{overlaps}(v, u) \Leftrightarrow (\mathsf{overlaps}(v, x) \vee \mathsf{overlaps}(v, y))) \end{aligned}$$

1.3 Mereotopologoy

1.4 Grammars & Languages

2 A Mereologoy for Languages

3 Weblinks

1. https://plato.stanford.edu/entries/mereology/
2. https://de.wikipedia.org/wiki/Hasse-Diagramm
3. http://ontology.buffalo.edu/smith/articles/mereotopology.htm
4. https://en.wikipedia.org/wiki/Ontology_(information_science)
5. https://en.wikipedia.org/wiki/Mereology
6. https://en.wikipedia.org/wiki/Mereotopology
7. https://en.wikipedia.org/wiki/Meronomy
8. https://en.wikipedia.org/wiki/Connected_space
9. https://en.wikipedia.org/wiki/First-order_logic
10. https://userpages.uni-koblenz.de/~softlang/lao/
11. https://de.wikipedia.org/wiki/Korrespondenz_(Mathematik)
12. http://www.theoretische-informatik.com/relationen.php
13. https://en.wikipedia.org/wiki/Natural_deduction
14. https://en.wikipedia.org/wiki/Order_theory
15. https://de.wikipedia.org/wiki/Ordnungsrelation
16. https://en.oxforddictionaries.com/definition/meronym
17. https://www.w3.org/TR/rdf-primer/
18. https://www.w3.org/2001/sw/BestPractices/OEP/SimplePartWhole/
19. https://www.w3.org/2004/02/skos/
20. https://plato.stanford.edu/entries/set-theory/ZF.html
21. https://de.wikipedia.org/wiki/Zermelo-Fraenkel-Mengenlehre
22. https://en.wikibooks.org/wiki/Set_Theory/Zermelo-Fraenkel_Axiomatic_Set_Theory
23. https://en.wikibooks.org/wiki/Set_Theory/Sets
24. https://en.wikibooks.org/wiki/Set_Theory/Relations
25. https://de.wikipedia.org/wiki/Relation_(Mathematik)
26. https://de.wikipedia.org/wiki/Ordnungsrelation
27. https://de.wikibooks.org/wiki/Mathematik:_Analysis:_Grundlagen:_Relationen
28. https://en.wikipedia.org/wiki/Fragment_(logic)

References

1. Varzi, A.C.: Parts, wholes, and part-whole relations: The prospects of mereotopology. Data Knowl. Eng. 20(3), 259–286 (1996), http://dx.doi.org/10.1016/S0169-023X(96)00017-1