Reducing Reactive to Strong Bisimilarity

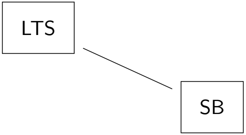
Bachelor's thesis

Max Pohlmann

TU Berlin

June 8, 2021

Outline



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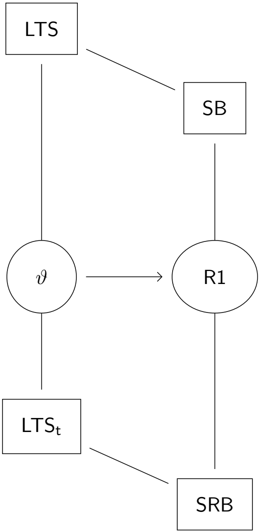

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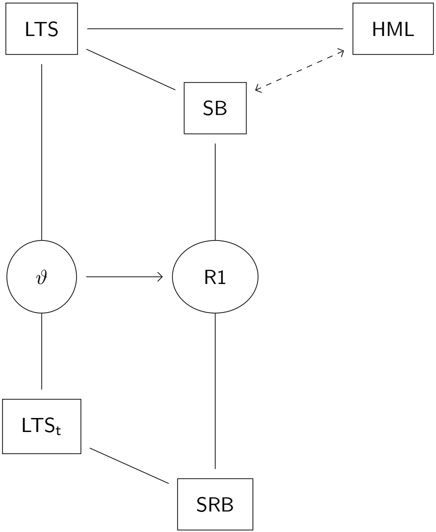

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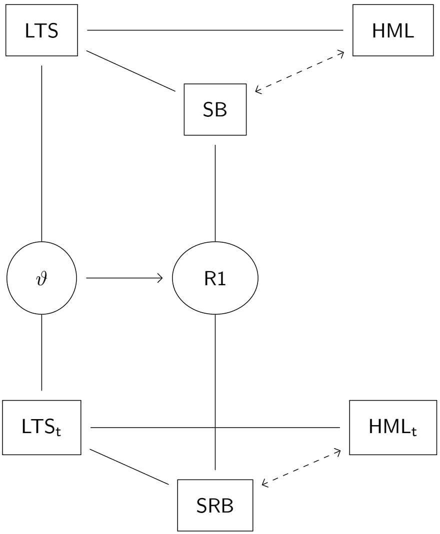

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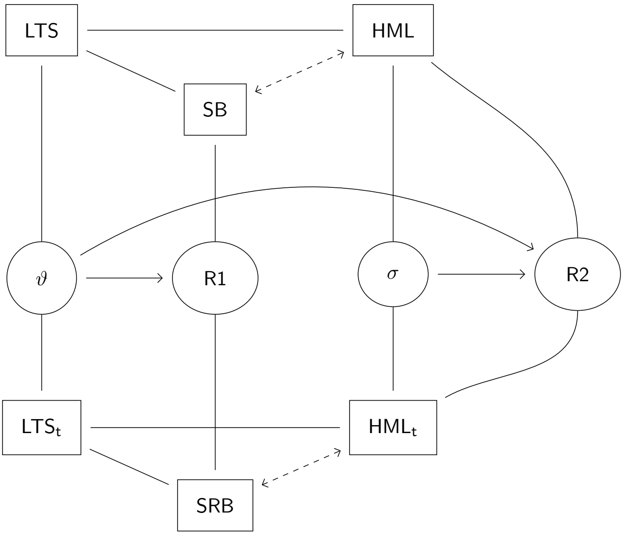
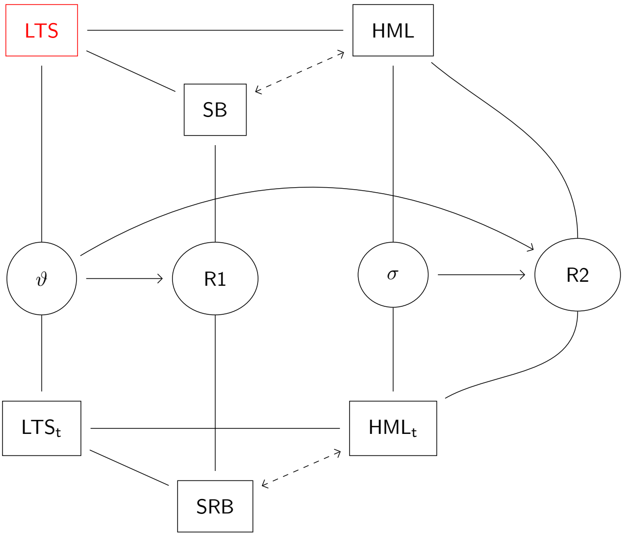


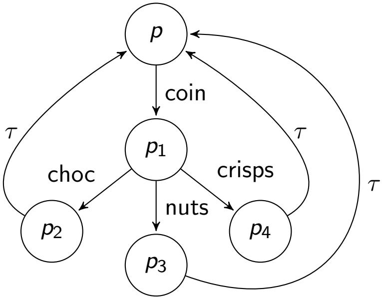
Labelled Transition Systems



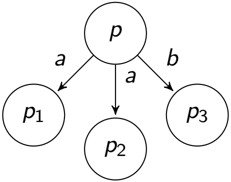
Labelled Transition Systems

- labelled directed graph
- reactive system:
  behaviour depends on continuous interaction with environment
- e.g. a machine and a user

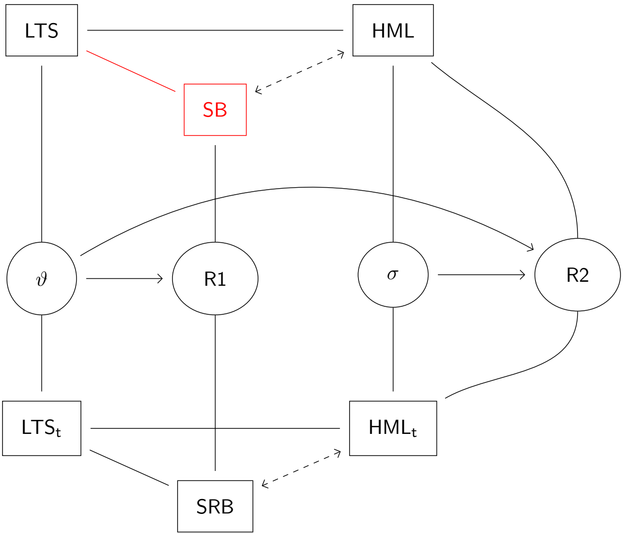
Labelled Transition Systems



Labelled Transition Systems



Strong Bisimilarity



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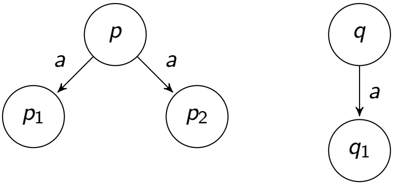
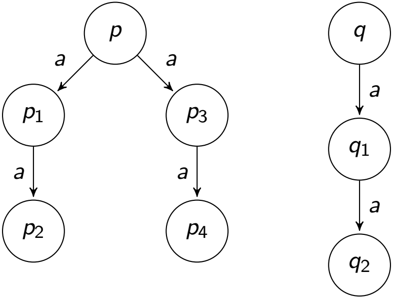


$$p \leftrightarrow q$$

Strong Bisimilarity



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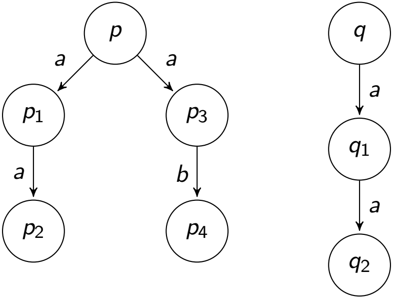
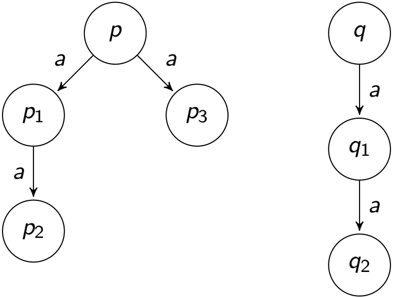


$$p \not\leftrightarrow q$$

Strong Bisimilarity



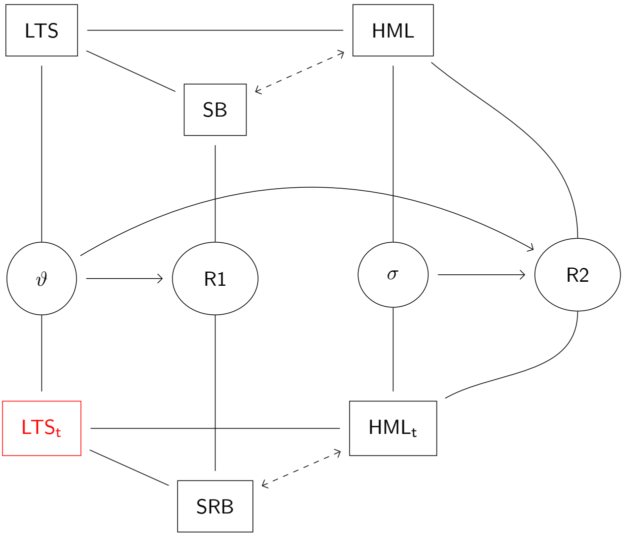
$$p \not\leftrightarrow q$$

Strong Bisimilarity

$p \leftrightarrow q$ if and only if:

$\forall \alpha, p'$ with $p \xrightarrow{\alpha} p'$. $\exists q'$ with $q \xrightarrow{\alpha} q'$ and $p' \leftrightarrow q'$, and
$\forall \alpha, q'$ with $q \xrightarrow{\alpha} q'$. $\exists p'$ with $p \xrightarrow{\alpha} p'$ and $p' \leftrightarrow q'$.

Labelled Transition Systems with Time-Outs



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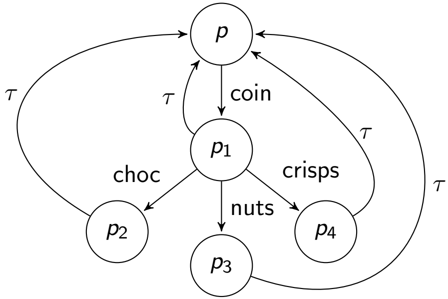
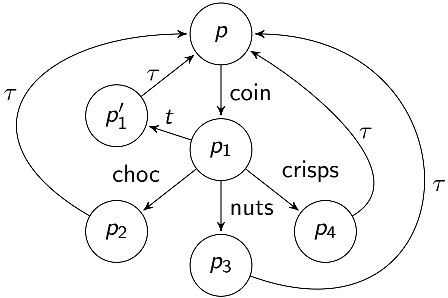


Labelled Transition Systems with Time-Outs



Labelled Transition Systems with Time-Outs

- in each given moment, there is a fixed set of actions that the environment allows

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- if a system state has a transition that is currently allowed, it will be performed immediately

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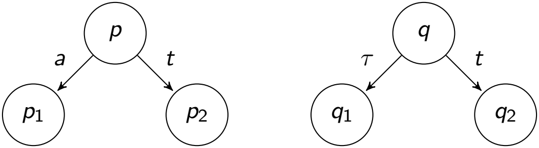
- in each given moment, there is a fixed set of actions that the environment allows
- if a system state has a transition that is currently allowed, it will be performed immediately
- only when no non–time-out transition is allowed by the environment, a state may perform a t-transition

Labelled Transition Systems with Time-Outs



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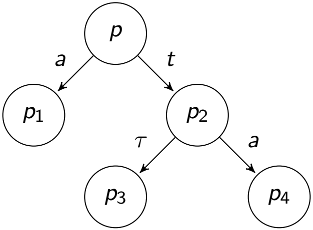

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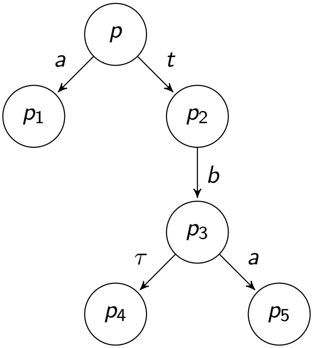
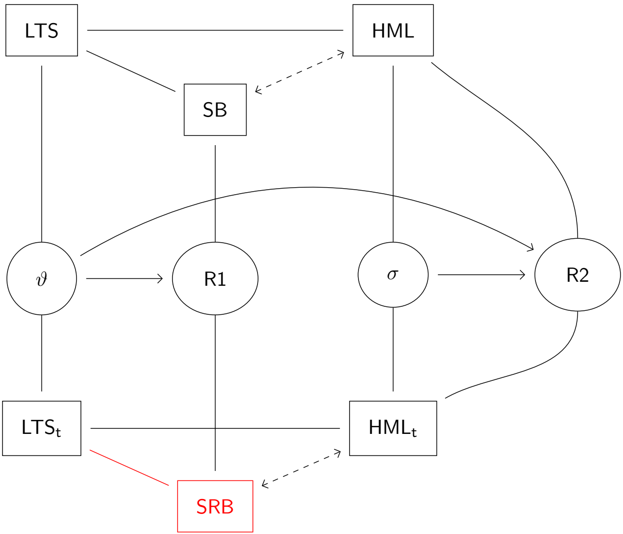


Strong Reactive Bisimilarity



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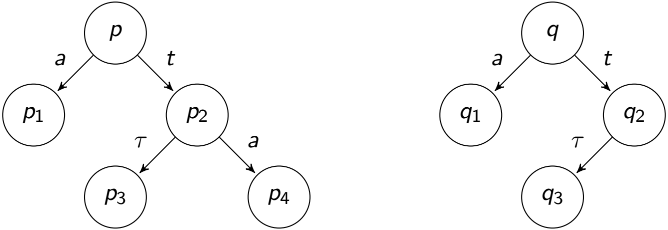


$$p \leftrightarrow_r q$$

Strong Reactive Bisimilarity

A *strong reactive bisimulation* is a symmetric relation

$$\mathcal{R} \subseteq (Proc \times P(A) \times Proc) \cup (Proc \times Proc),$$

such that, for all $(p, q) \in \mathcal{R}$:

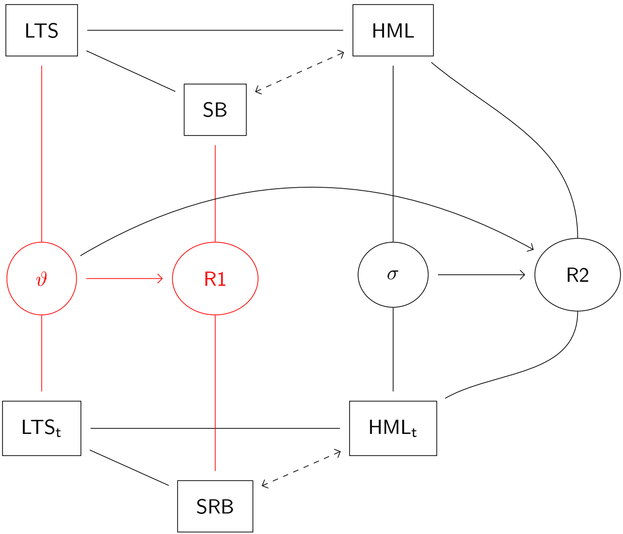
1. if $p \xrightarrow{\tau} p'$, then there exists a q' such that $q \xrightarrow{\tau} q'$ and $(p', q') \in \mathcal{R}$,
2. $(p, X, q) \in \mathcal{R}$ for all $X \subseteq A$,

and for all $(p, X, q) \in \mathcal{R}$:

3. if $p \xrightarrow{a} p'$ with $a \in X$, then there exists a q' such that $q \xrightarrow{a} q'$ and $(p', q') \in \mathcal{R}$,
4. if $p \xrightarrow{\tau} p'$, then there exists a q' such that $q \xrightarrow{\tau} q'$ and $(p', X, q') \in \mathcal{R}$,
5. if $\mathcal{I}(p) \cap (X \cup \{\tau\}) = \emptyset$, then $(p, q) \in \mathcal{R}$, and
6. if $\mathcal{I}(p) \cap (X \cup \{\tau\}) = \emptyset$ and $p \xrightarrow{t} p'$, then there exists a q' such that $q \xrightarrow{t} q'$ and $(p', X, q') \in \mathcal{R}$.

$(\mathcal{I}(p) := \{\alpha \mid p \xrightarrow{\alpha} \wedge \alpha \neq t\})$

Reducing Reactive to Strong Bisimilarity



Reducing Reactive to Strong Bisimilarity

For an $\mathrm{LTS_t}$ $\mathbb{T}$ with $\mathbb{T} = (\mathit{Proc}, \mathit{Act}, \rightarrow)$,
let $\mathbb{T}_\vartheta$ be an LTS with $\mathbb{T}_\vartheta = (\mathit{Proc}_\vartheta, \mathit{Act}_\vartheta, \rightarrow_\vartheta)$.

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$$\text{Goal:} \quad p \leftrightarrow_r q \iff \vartheta(p) \leftrightarrow \vartheta(q),$$

with $p, q \in \mathit{Proc}$ and $\vartheta(p), \vartheta(q) \in \mathit{Proc}_\vartheta$.

Reducing Reactive to Strong Bisimilarity

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$$\mathit{Proc}_\vartheta = \{\vartheta(p) \mid p \in \mathit{Proc}\} \cup \{\vartheta_X(p) \mid p \in \mathit{Proc} \wedge X \subseteq (\mathit{Act} \setminus \{\tau, t\})\}$$

Reducing Reactive to Strong Bisimilarity

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$$\mathit{Proc}_\vartheta = \{\vartheta(p) \mid p \in \mathit{Proc}\} \cup \{\vartheta_X(p) \mid p \in \mathit{Proc} \land X \subseteq (\mathit{Act} \setminus \{\tau, t\})\}$$

$$\mathit{Act}_\vartheta = \mathit{Act} \cup \{t_\varepsilon\} \cup \{\varepsilon_X \mid X \subseteq (\mathit{Act} \setminus \{\tau, t\})\}$$

Reducing Reactive to Strong Bisimilarity

For an LTS_t $\mathbb{T}$ with $\mathbb{T} = (\mathit{Proc}, \mathit{Act}, \rightarrow)$,
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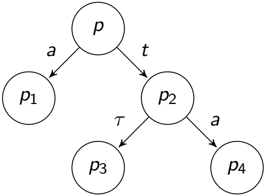
$$(1)\ \frac{}{\vartheta(p) \xrightarrow{\varepsilon_X}_\vartheta \vartheta_X(p)}\ X \subseteq A \qquad (2)\ \frac{p \xrightarrow{\tau} p'}{\vartheta(p) \xrightarrow{\tau}_\vartheta \vartheta(p')}$$

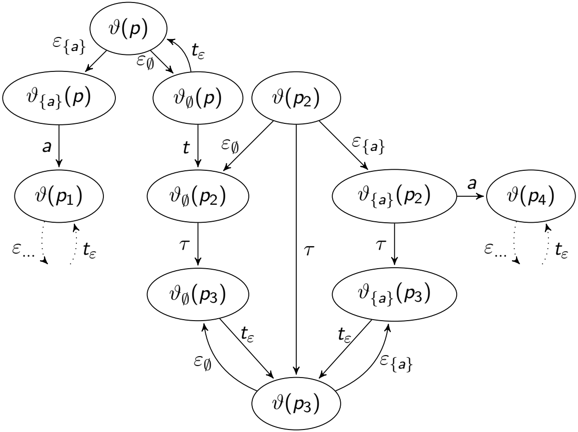
$$(3)\ \frac{p \stackrel{\alpha}{\nrightarrow} \text{ for all } \alpha \in X \cup \{\tau\}}{\vartheta_X(p) \xrightarrow{t_\varepsilon}_\vartheta \vartheta(p)}$$

$$(4)\ \frac{p \xrightarrow{a} p'}{\vartheta_X(p) \xrightarrow{a}_\vartheta \vartheta(p')}\ a \in X \qquad (5)\ \frac{p \xrightarrow{\tau} p'}{\vartheta_X(p) \xrightarrow{\tau}_\vartheta \vartheta_X(p')}$$

$$(6)\ \frac{p \stackrel{\alpha}{\nrightarrow} \text{ for all } \alpha \in X \cup \{\tau\} \quad p \xrightarrow{t} p'}{\vartheta_X(p) \xrightarrow{t}_\vartheta \vartheta_X(p')}$$

Reducing Reactive to Strong Bisimilarity





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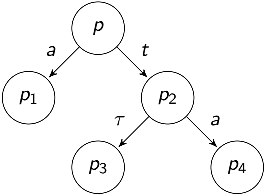
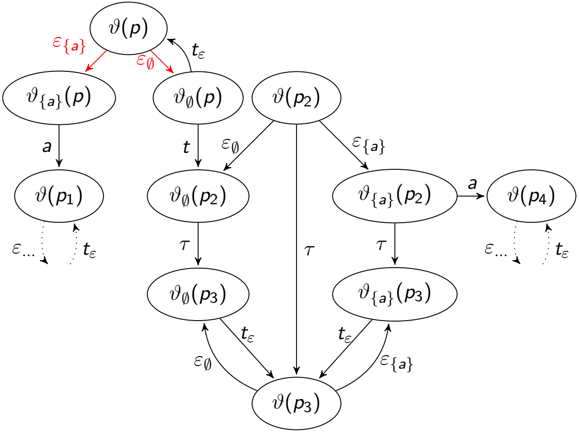
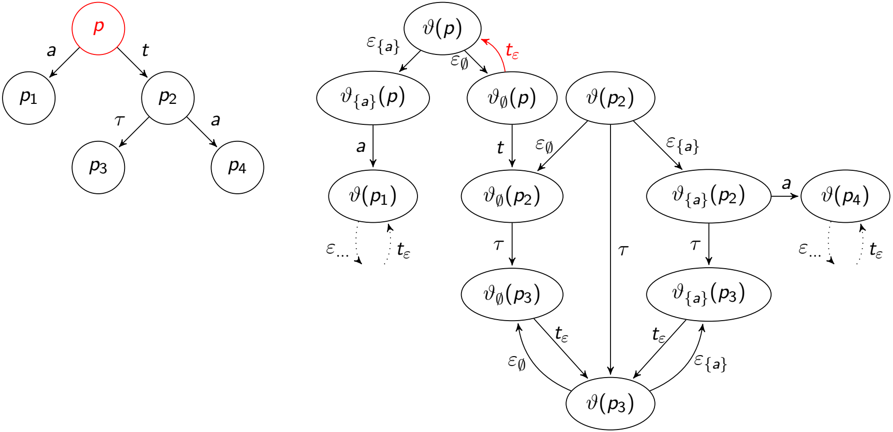




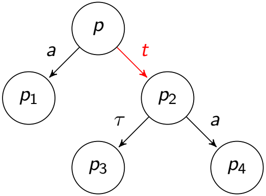
$$(1)\ \frac{}{\vartheta(p) \xrightarrow{\varepsilon_X}_{\vartheta} \vartheta_X(p)}\ X \subseteq A$$

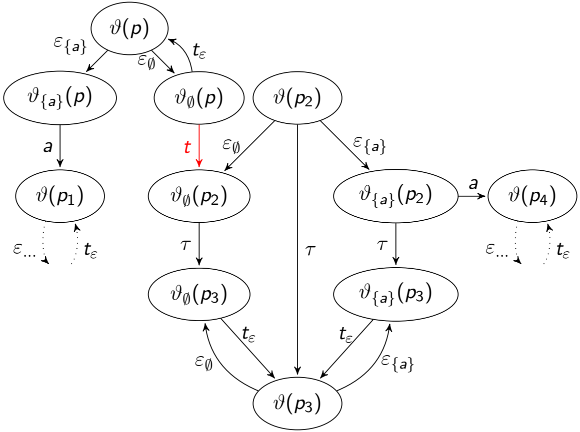
Reducing Reactive to Strong Bisimilarity



(3) $\dfrac{p \stackrel{\alpha}{\not\rightarrow} \text{ for all } \alpha \in X \cup \{\tau\}}{\vartheta_X(p) \xrightarrow{t_\varepsilon}_\vartheta \vartheta(p)}$

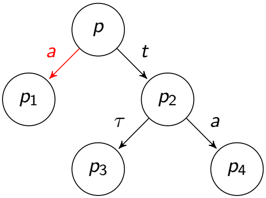
Reducing Reactive to Strong Bisimilarity

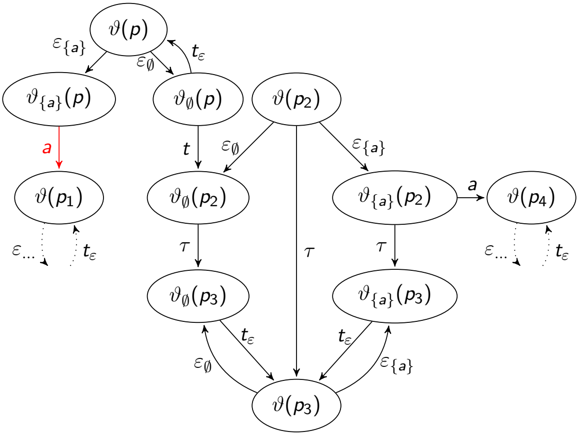




$$(6)\ \frac{p \overset{\alpha}{\not\longrightarrow} \text{ for all } \alpha \in X \cup \{\tau\} \qquad p \xrightarrow{t} p'}{\vartheta_X(p) \xrightarrow{t}_{\vartheta} \vartheta_X(p')}$$

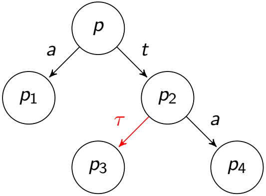
Reducing Reactive to Strong Bisimilarity

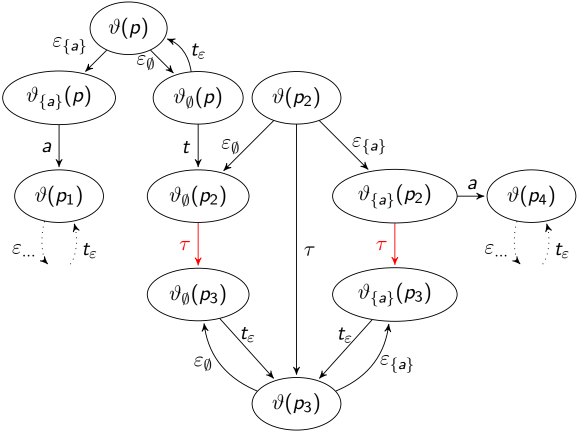




$$(4)\ \frac{p \xrightarrow{a} p'}{\vartheta_X(p) \xrightarrow{a}_\vartheta \vartheta(p')}\ a \in X$$

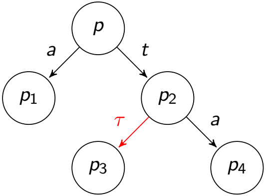
Reducing Reactive to Strong Bisimilarity

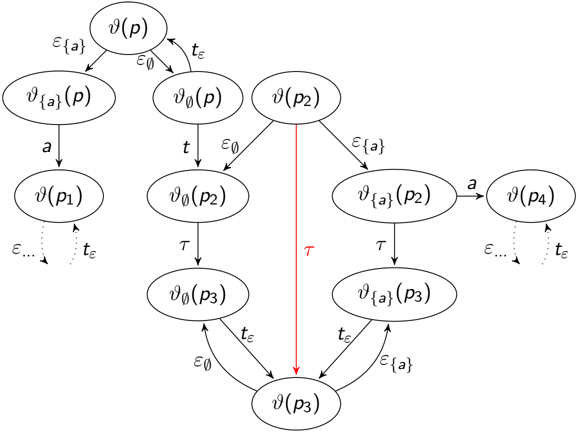




(5) $\dfrac{p \xrightarrow{\tau} p'}{\vartheta_X(p) \xrightarrow{\tau}_{\vartheta} \vartheta_X(p')}$

Reducing Reactive to Strong Bisimilarity





$$(2)\ \frac{p \xrightarrow{\tau} p'}{\vartheta(p) \xrightarrow{\tau}_{\vartheta} \vartheta(p')}$$

Reducing Reactive to Strong Bisimilarity

Theorem

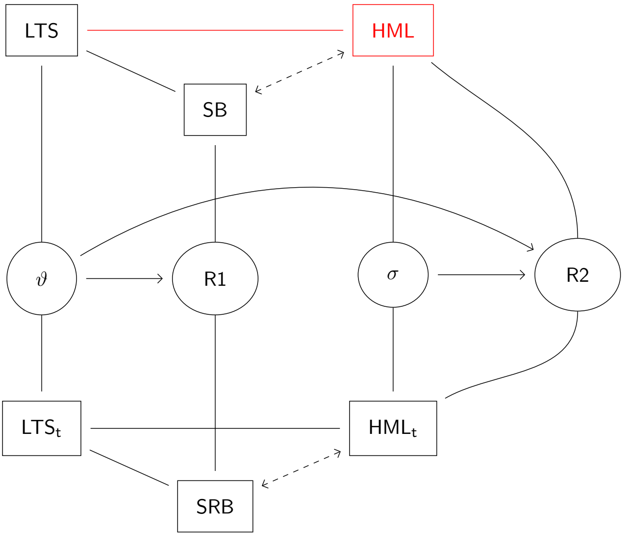
For an LTS_t $\mathbb{T}$ with $\mathbb{T} = (Proc, Act, \rightarrow)$,
let $\mathbb{T}_\vartheta$ be an LTS with $\mathbb{T}_\vartheta = (Proc_\vartheta, Act_\vartheta, \rightarrow_\vartheta)$ defined as above.

Then we have, for all $p, q \in Proc$:

$$p \leftrightarrow_r q \iff \vartheta(p) \leftrightarrow \vartheta(q),$$

$$p \leftrightarrow_r^X q \iff \vartheta_X(p) \leftrightarrow \vartheta_X(q).$$

Hennessy-Milner Logic



Hennessy-Milner Logic

$$\varphi ::= tt \mid \varphi_1 \ \wedge \ \varphi_2 \mid \neg\varphi \mid \langle\alpha\rangle\varphi$$

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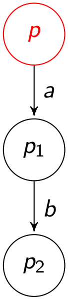

$$\langle a\rangle tt \wedge \neg(\langle b\rangle tt)$$

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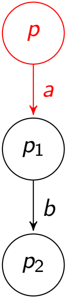

$$\langle a \rangle tt \wedge \neg(\langle b \rangle tt)$$

$$\langle a \rangle tt$$

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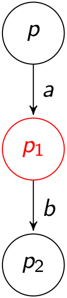

$$\langle a\rangle tt \wedge \neg(\langle b\rangle tt)$$

$$tt$$

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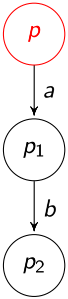

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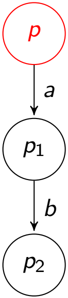
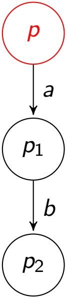


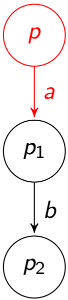
$$p \vDash \langle a\rangle tt \wedge \neg(\langle b\rangle tt)$$

Hennessy-Milner Logic



$$\langle a \rangle (\neg (\langle b \rangle tt))$$

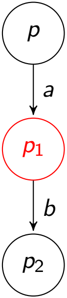
Hennessy-Milner Logic



$$\langle a \rangle (\neg(\langle b \rangle tt))$$

$$\langle a \rangle (\dots)$$

Hennessy-Milner Logic



$$\langle a \rangle (\neg(\langle b \rangle tt))$$

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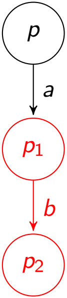
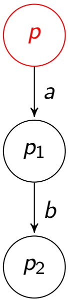
Hennessy-Milner Logic



$$\langle a \rangle (\neg(\langle b \rangle tt))$$

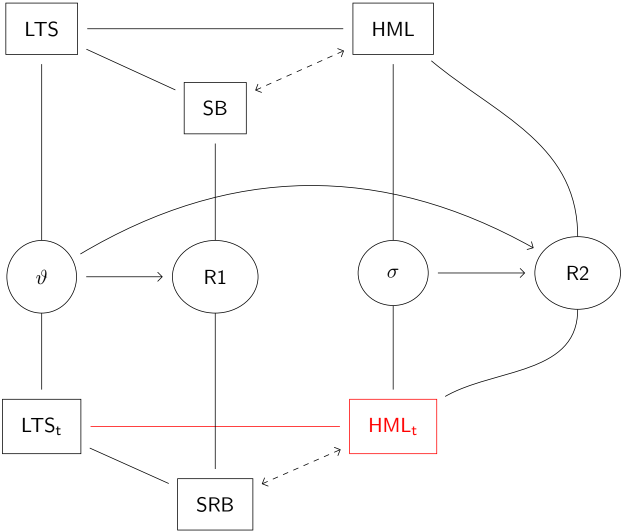
$$\neg(\langle b \rangle tt) \quad \lightning$$

Hennessy-Milner Logic



$$p \not\models \langle a \rangle (\neg(\langle b \rangle tt))$$

Hennessy-Milner Logic with Time-Outs



Hennessy-Milner Logic with Time-Outs

$$p \models \langle X \rangle \varphi$$

Hennessy-Milner Logic with Time-Outs

$$p \models \langle X \rangle \varphi$$

$$p \models \langle t \rangle_X \varphi$$

Hennessy-Milner Logic with Time-Outs

$$p \vDash \langle X \rangle \varphi$$

$$p \vDash_X \varphi$$

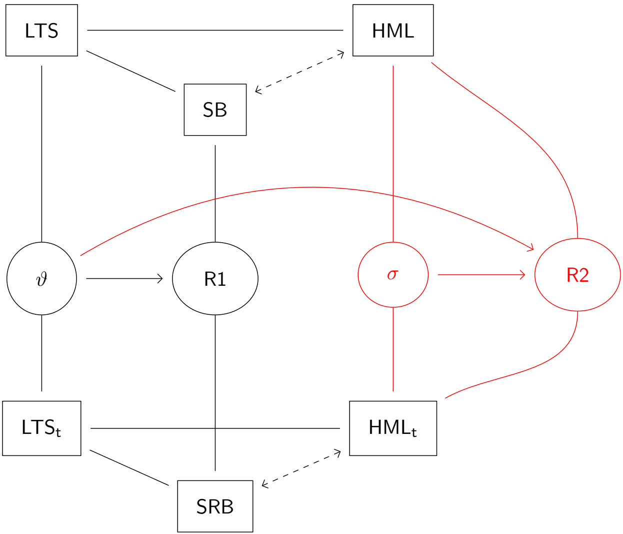
Hennessy-Milner Logic with Time-Outs

$$
\begin{array}{lll}
p \vDash \bigwedge_{i\in I} \varphi_i & & \text{if} \quad \forall i \in I.\, p \vDash \varphi_i \\
p \vDash \neg\varphi & & \text{if} \quad p \nvDash \varphi \\
p \vDash \langle\alpha\rangle\varphi & \text{with } \alpha \in A \cup \{\tau\} & \text{if} \quad \exists p'.\, p \xrightarrow{\alpha} p' \wedge p' \vDash \varphi \\
p \vDash \langle X\rangle\varphi & \text{with } X \subseteq A & \text{if} \quad \mathcal{I}(p) \cap (X \cup \{\tau\}) = \emptyset \wedge \\
 & & \qquad \exists p'.\, p \xrightarrow{t} p' \wedge p' \vDash_X \varphi
\end{array}
$$

$$
\begin{array}{lll}
p \vDash_X \bigwedge_{i\in I} \varphi_i & & \text{if} \quad \forall i \in I.\, p \vDash_X \varphi_i \\
p \vDash_X \neg\varphi & & \text{if} \quad p \nvDash_X \varphi \\
p \vDash_X \langle a\rangle\varphi & \text{with } a \in A & \text{if} \quad a \in X \wedge \exists p'.\, p \xrightarrow{a} p' \wedge p' \vDash \varphi \\
p \vDash_X \langle\tau\rangle\varphi & & \text{if} \quad \exists p'.\, p \xrightarrow{\tau} p' \wedge p' \vDash_X \varphi \\
p \vDash_X \varphi & & \text{if} \quad \mathcal{I}(p) \cap (X \cup \{\tau\}) = \emptyset \wedge p \vDash \varphi
\end{array}
$$

$(\mathcal{I}(p) := \{\alpha \mid p \xrightarrow{\alpha} \wedge \alpha \neq t\})$

Reducing $\mathrm{HML_t}$ to HML formula satisfaction



Reducing $\mathrm{HML_t}$ to HML formula satisfaction

Goal: σ : ($\mathrm{HML_t}$ formulas) $\longrightarrow$ (HML formulas), such that:

$$p \models \varphi \iff \vartheta(p) \models \sigma(\varphi)$$

Reducing $\mathrm{HML_t}$ to HML formula satisfaction

Let σ : ($\mathrm{HML_t}$ formulas) $\longrightarrow$ (HML formulas) be recursively defined by

$$
\begin{aligned}
\sigma(\textstyle\bigwedge_{i\in I}\varphi_i) &= \textstyle\bigwedge_{i\in I}\sigma(\varphi_i) && \\
\sigma(\neg\varphi) &= \neg\,\sigma(\varphi) && \\
\sigma(\langle\tau\rangle\varphi) &= \langle\tau\rangle\,\sigma(\varphi) && \\
\sigma(\langle\alpha\rangle\varphi) &= \langle\alpha\rangle\,\sigma(\varphi) \vee && \\
&\quad\ \langle\varepsilon_A\rangle\langle\alpha\rangle\,\sigma(\varphi) \vee && \\
&\quad\ \langle t_\varepsilon\rangle\langle\varepsilon_A\rangle\langle\alpha\rangle\,\sigma(\varphi) && \text{if } \alpha\in A \\
\sigma(\langle\alpha\rangle\varphi) &= \mathit{ff} && \text{if } \alpha\notin A\cup\{\tau\} \\
\sigma(\langle X\rangle\varphi) &= \langle\varepsilon_X\rangle\langle t\rangle\,\sigma(\varphi) \vee && \\
&\quad\ \langle t_\varepsilon\rangle\langle\varepsilon_X\rangle\langle t\rangle\,\sigma(\varphi) && \text{if } X\subseteq A \\
\sigma(\langle X\rangle\varphi) &= \mathit{ff} && \text{if } X\not\subseteq A
\end{aligned}
$$

Reducing HML_t to HML formula satisfaction

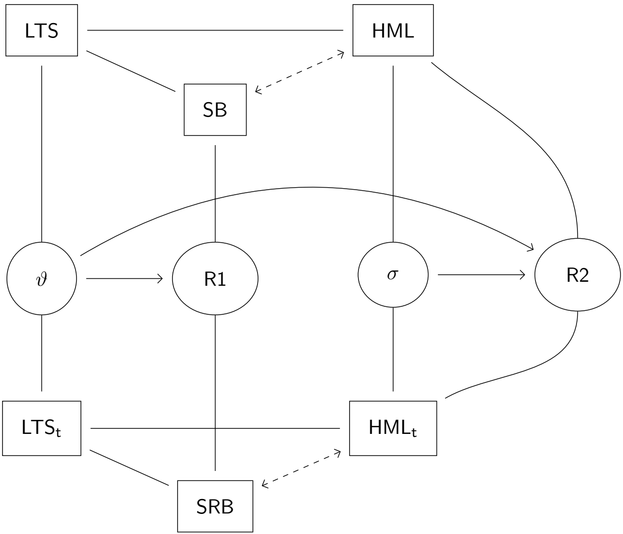
Theorem

For some LTS_t $\mathbb{T}$, let $\mathbb{T}_\vartheta$ and σ be defined as above. Then, for all $p \in Proc$ and $\varphi : HML_t$ formulas, we have:

$$p \models \varphi \iff \vartheta(p) \models \sigma(\varphi),$$

$$p \models_X \varphi \iff \vartheta_X(p) \models \sigma(\varphi).$$

That's all, folks!



Main Resource:
van Glabbeek, Rob. "Reactive Bisimulation Semantics for a Process Algebra with Time-Outs." arXiv preprint arXiv:2008.11499 (2020).

My Thesis on GitHub:
https://github.com/maxpohlmann