Angular displacement $\Delta\phi = \phi_f - \phi_i$
Avg. angular speed $\overline{\omega} = \frac{\Delta\phi}{\Delta t}$
Inst. angular speed $\omega = \frac{d\phi}{dt}$
Avg. angular acceleration $\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$
Inst. angular acceleration $\alpha = \frac{d\omega}{dt}$
$\omega = \omega_0 + \alpha t$
$\phi = \phi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega^2 = \omega_0^2 + 2\alpha(\phi - \phi_0)$
$v = v_0 + at$
$x = x_0 + v_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a(x - x_0)$
$I = \int_{body} r^2 dm$
Rotational kinetic energy: $K_r = \frac{1}{2}I\omega^2$
Translational kinetic energy: $K_t = \frac{1}{2}mv^2$
Parallel Axis Theorem: $I - I_{cm} + MD^2$
Torque: $\tau = rFsin\phi = rF_t = I\alpha$
"Rolling without slipping" occurs $\iff$
$[S_{cm} = R\phi \wedge v_{cm} = R\omega \wedge a_{cm} = R\alpha]$
Rolling motion: $K = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$
$= \frac{1}{2}(I_{cm} + MR^2)\omega^2 = \frac{1}{2}(\frac{I_{cm}}{R^2} + M)v_{cm}^2$
Angular Momentum: $L = I\omega(= rmv)$
Gravity: $F_g = G\frac{m_1 m_2}{r^2} \wedge g = \frac{GM}{R^2}$
$U_g(r) = -G\frac{m_1 m_2}{r}$
$v_{escape} = \sqrt{\frac{2GM_{planet}}{R}}$
Kepler's Laws:
a is semimajor axis ($\frac{1}{2}$ of "long diameter")
b is semiminor axis ($\frac{1}{2}$ of "short diameter")
F_1 and F_2 are foci of the ellipse, each located c distance from the other, where $c = \sqrt{a^2 - b^2}$
Ellipse is a circle $\iff a = b = R$
Kepler's 1st Law: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = c^2$
Kepler's 2nd Law: Area swept by radius vector in time Δt
$= A_1 = \frac{\Delta t}{2m}L$, where $L = r \times mv = r \times p$
Kepler's 3rd Law: $T^2 = \frac{r\pi^2}{GM}r^3$
Mass on a Spring: $m\frac{d^2x(t)}{dt^2} + kx(t) = 0$
$x(t) = Acos(\omega t + \phi)$
A is amplitude, ω is angular frequency, ϕ is phase
Spring-Mass: $\omega = \sqrt{\frac{k}{m}}$
Simple Pendulum: $\omega = \sqrt{\frac{g}{l}}$
Physical Pendulum: $\omega = \sqrt{\frac{mgL_{cm}}{I}}$
Period of Oscillation: $T = \frac{2\pi}{\omega}$
Frequency of Oscillation: $f = \frac{1}{T} = \frac{\omega}{2\pi}$ Hz (s^{-1})
Total energy of simple harmonic motion:
$K(t) = \frac{1}{2}m\omega^2 A^2 sin^2(\omega t + \phi)$
$U(t) = \frac{1}{2}kA^2 cos^2(\omega t + \phi)$
$E_{net}(t) = \frac{1}{2}kA^2$
For simple pendulum,
$F_{tan} = -mgsin(\phi) \wedge \alpha = -\frac{g}{l}sin(\phi)$
For *small* ϕ, we can say that $sin(\phi) \approx \phi$
Dampered Harmonic Oscillator: $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$
$\implies x(t) = Ae^{-\frac{b}{2m}t}cos(\omega t + \phi)$
$\implies \omega = (\omega_0^2 - (\frac{b}{2m})^2)^{0.5}$
$\implies \omega_0 = \sqrt{\frac{k}{m}}$, which is the "natural frequency"
For $b > 2\sqrt{mk}$, x decays to 0 without a single oscillation
We call this an "overdamped oscillator"
The underdamped oscillator is:
$x(t) = e^{-\gamma t}acos(\omega_0 t - \alpha)$
For a damped driven oscillator,
$A = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{b\omega}{m})^2}}$