A Theory of the Universal Wave Function and the Origins of the Cosmos

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I assume quantum mechanics applies to the entire universe and solve the Schrödinger equation exactly, obtaining a unique closed-form wave function of the universe. Quantum information and quantum geometry emerge as one from the bootstrap. The geometry of quantum information is central to the problem of time in quantum cosmology, which is resolved by a flat space holographic duality between a timeless theory in Euclidean 3-space without a spin-2 field and a theory in 3 + 1 spacetime dimensions with an emergent spin-2 field. Among the observable consequences are two spectacular predictions: the dark energy is a quantum scalar curvature field that decays with the scale factor like a^{-2}, and all black holes are dark energy composite objects with a mass function that grows like a^3.

I. MOTIVATION

The cosmos seems to have been born into a spatially flat and thermal state of low entropy, originating from a curvature singularity beyond the effective field theory (EFT) description of space, time, matter, and energy [1]. This grand puzzle demands new physics. This research program aims to discover a unifying principle of the physics of creation and its observable consequences.

The mythos of our shared humanity is a liminal space of creation stories. The origin of the universe is the ultimate question and a timeless puzzle. The philosophy of this research program is a form of monism that hypothesizes all of reality can be derived from a single unifying mathematical principle—an ancient idea. But physicists need more than philosophy and mathematics. We need empirical data, and the empirical data is conclusive: the universe is quantum.

Understanding the nature of the cosmological singularity requires a complete theory of quantum gravity. Indeed, the conceptual problems in the foundations of physics at extremely early times do not begin at the Planck scale [2]. The Copenhagen interpretation of quantum mechanics (QM), which assumes a Heisenberg cut between quantum and classical systems for wave function collapse, breaks down when the scale of the universe is on the order of an atom. QM in the 'laboratory frame' applied to the early universe as a whole requires the troubling assumption of an external classical observer to prepare the initial state of the cosmos. I argue Copenhagen can not be the final formulation of quantum mechanics. Indeed, a compendium of clues points elsewhere.

One of the most consequential experiments in the foundations of physics is the Aharonov–Bohm effect [3–6]. How can a charged particle's gauge potential possibly couple to the complex phase of a nonphysical probability wave? Is the measured *change* in phase not an indirect measurement of the physical existence of the wave function? There is still more evidence of the reality of the quantum state. The Pusey–Barrett–Rudolph (PBR) no-go theorem has been experimentally confirmed: either the quantum state is an objective physical state, or the assumption of preparation independence is violated, and the experimenter has no free-will in the choice of measurement settings [7–9]. But if the wave function is physical, where do probability amplitudes and the Born rule come from? The measurement problem, the problem of the external observer, and the problem of time in quantum cosmology are all deeply related [10]. These foundational conceptual issues can not be ignored in a fundamental theory. To understand the quantum origins of space and time, we must first understand the true nature of quantum mechanics.

The most cogent formulation of QM with a physical wave function is Everett's seminal work: *The Theory of a Universal Wave Function* [11, 12]. Everett profoundly impacted Wheeler and DeWitt, inspiring the Wheeler-DeWitt (WdW) equation and the field of quantum cosmology. The development of the canonical quantum gravity program of WdW and what became popularized as the many worlds interpretation diverged. Work in the foundations of QM became orthogonal to quantum gravity.

On the one hand, Everettian QM is done almost exclusively in the non-relativistic limit. Space and time are not just classical. They are Newtonian! The creation of new worlds (i.e., branching) happens with respect to a universal clock in strictly finite-dimensional Hilbert spaces. What happens if we replace the Schrödinger equation with a relativistic wave equation in the Everttian framework and work in a covariant fashion with an infinite-dimensional Hilbert space?

On the other hand, WdW *assumes* general relativity (GR) as an input to quantize, and out comes an ill-defined infinite dimensional partial differential equation with pathological consequences. There are no known unitary solutions of WdW [13]. What is more, the Hamiltonian vanishes on physical states with compact geometries in general covariant theories, meaning there is an infinite number of possible boundary conditions to impose on the equation and, therefore, an infinite number of possible solutions. Vacuum solutions are not unique. But if the universal wave function contains all physical information of the universe, there can only be One.

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General covariance is necessary, but is it sufficient? To the best of human knowledge, nature is quantum from the beginning. Quantization schemes—like coordinate systems—are an artifact of formalism. This guiding principle of *quantum covariance* is a radical constraint. But without intuition of the quantum world at the moment of creation, how is one to even begin?

The standard lore in the foundations of physics tells us degrees of freedom at higher energies (smaller distances) are more fundamental than those at lower energies (larger distances). At the heart of the Wilsonian EFT framework is the assumption that the macroscopic dynamics of a system are independent of the microscopic details. But is this true for gravity? Is the observation of the cosmological constant entering local galaxy dynamics through a universal acceleration scale $a_0 \approx cH_0 \approx c^2\Lambda^{1/2}$ [14–16] merely a numerical coincidence or a profound clue into the nature of quantum gravity? How is it that something as small as a galaxy "knows" about the de Sitter (dS) radius associated with the whole of the observable universe? UV/IR mixing appears to be a model-independent feature of quantum gravity, with examples emerging across string theory and beyond [13, 17–32]. This begs the question: does gravity violate the separation of scales?

Perhaps moving deep into the IR, at the scale of the entire universe, is no less fundamental than working in the UV. Indeed, this perspective is supported by the data. EFT predicts the dark energy is dominated by the UV vacuum energy (the Planck energy). Yet, it is observed to be astonishingly smaller (by 120 orders of magnitude), meaning it sets the IR cutoff that determines the largest scale anyone shall ever see. Motivated by this compendium of clues, I assume reductionism works in both directions. This *universal democracy*, a generalization of Chew's "nuclear democracy" to cosmology, suggests the quantum degrees of freedom of the entire universe *are* the fundamental UV degrees of freedom. Intriguingly, this view was discovered at the end of the second superstring revolution with holography [33–39]. In some deeply mysterious way, the fundamental degrees of freedom exist at the boundary of spacetime—at infinity—where a covariant definition of energy is well-defined in both quantum mechanics (QM) and GR.

The guiding principles of quantum covariance and universal democracy motivate the quantum-first program recently proposed by Carroll, the idea that spacetime and quantum fields emerge from the wavefunction of the universe [41, 42]. Carroll and friends [43] approach this program in a non-covariant fashion, leaving a clear gap in the literature. In general covariant theories of quantum gravity, the Hamiltonian annihilates the universal quantum state. Time drops out of the Schrödinger equation, meaning all information of the physical universe lives at future infinity, where time stands still. A notion consistent with the experimental realization of the Page-Wooters mechanism in continuous time [40].

In this paper, I report a minimal bootstrapping of quantum universality. No relic classical baggage is assumed. To the author's knowledge, this is the first attempt to write down the quantum state of the universe that is not some semiclassical approximation like the Hartle-Hawking no-boundary proposal [44, 45], Vilenkin's tunneling proposal [46, 47], or numerous minisuperspace models [48, 49], which all assume the WdW equation as a starting point [50].

Modulo continuity, no assumptions of spacetime, matter, or energy are made and put in by hand. Instead, the One fundamental assumption is as follows.

Everett's principle of quantum universality

The wavefunction of the universe exists and evolves unitarily, **always** obeying the Schrödinger equation:

$$\hat{H}\Psi = i\partial_t\Psi. \tag{1}$$

A corollary of quantum universality is this: nature has no fundamental scale—the *form* of the laws of physics remains unchanged on all scales.

From this elementary assumption, everything that follows is pure deduction. The research program is a formal nomology or deductive-nomological model that aims to derive relativistic principles from quantum universality, write down a relativistic cosmological Schrödinger equation, and explore its consequences.

II. QUANTUM UNIVERSALITY

Quantum universality implies there is no Heisenberg cut between quantum and classical systems. The Everettian view is austere. The universe, on all scales, is quantum. If nature has no fundamental scale, the mass term M in the Schrödinger equation must multiply out, implying the

Everett-Wheeler-DeWitt equation

$$\hat{H}\Psi = 0. \tag{2}$$

Neither time nor mass-energy is fundamental. This further implies the wavefunction of the universe is scale-invariant: $\Psi \sim a^n$ where a is the cosmological scale factor, and n is an arbitrary integer. One can now solve for the potential V and write down the following one-dimensional EWD equation:

$$\left(-\frac{\partial^2}{\partial a^2} + \frac{n(n-1)}{a^2}\right)\Psi = 0. \tag{3}$$

The above equation is not quite consistent with the axioms of QM since Ψ can not be normalized. But observe the following. The potential V is the scalar curvature R of an n-sphere with radius a, the kinetic term is the scalar curvature of an n-hyperbola, and the right-hand side of the equation is the scalar curvature of Euclidean n-space. From this minimal conformal bootstrap, the fundamental space forms are derived!

GR assumes the equivalence between energy and curvature. Here, it is a consequence. A scalar curvature singularity is at $V(a=0)$. Since the Hamiltonian annihilates the universal quantum state, the scalar curvature singularity cancels with the kinetic term, yielding a zero-total energy universe with global spatial flatness, thus providing a path to solving the cosmological singularity and flatness problems. In GR, the dark energy is proportional to the constant scalar curvature of dS space:

$$R = \frac{d(d-1)}{L^2} = \frac{2d}{d-2}\Lambda \tag{4}$$

where d is the spacetime dimension and L is the radius. However, there is no unitary evolution of Ψ—a common problem for the WdW equation and its approximations. The timeless equation occupies a liminal space between geometry and quantum mechanics. Under a change of variables, eq. (3) is the reduced radial equation of a zero-energy free particle, i.e., the simplest one-dimensional time-independent Schrödinger equation in elementary QM. There is a direct correspondence between $V(a)$ and the effective centrifugal potential $V_{\text{eff}}(r)$ with $n \leftrightarrow -l$, where l is the centrifugal term. A consequence that echoes Mach's principle and violates decoupling.

The physical picture is that of duality. The universe, as an (almost-quantum) whole, can be seen as a Lemaîtrian point-like particle, or Cosmon, with charge M and position a from the origin, sourcing a repulsive potential that accelerates the Cosmon away from the origin and *decays to zero at infinity.* Seen from the geometric view, it describes a global charge M sourcing the scalar curvatures of an n-sphere and an n-hyperboloid that annihilates to flat space. The change of variables establishes a duality between *position* and *scale*. It follows from eq. (3) that the preferred basis is the scale basis. The equation is timeless, so one can interpret the Hamiltonian as describing a celestial sphere at future infinity (a kind of celestial holography [54, 55]) dual to a theory in one timelike (and lightlike) dimension higher. The equation is coordinate-independent, meaning general covariance is a consequence of quantum universality. The number of spatial dimensions $n+1$ is also unfixed.

Can we bootstrap our way to a unitary instantiation of the timeless equation and explain *why* we observe only three spatial dimensions? Yes, we can—with symmetry. There is a trivial $\mathbb{Z}_2$ action $a \to -a$ and a non-trivial $\mathbb{Z}_2$ action $n \to -n$. The former is a symmetry of the system; the latter is not. Suppose we demand the latter is a symmetry of the system. The action $n \to -n$ gives the potential $R_{n+1} = \frac{n(n+1)}{a^2}$ and is interpreted as the scalar curvature of an $(n+1)$-sphere. The potential then becomes,

$$V \sim \begin{pmatrix} R_n & 0 \\ 0 & R_{n+1} \end{pmatrix} \tag{5}$$

implying Ψ is promoted to a pair of complex numbers, i.e., a spinor. A complex 2-spinor solution, together with the consequence of conformal flatness, implies an underlying $SU(n=2)$ symmetry, forcing the self-consistent equations of motion to take the unique form of a

relativistic cosmological wave equation

$$(D^2 + \hat{V})\Psi = 0. \tag{6}$$

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry is a fundamental symmetry of the boundary theory and will continuously reappear in the story.

The Dirac-spin operator

$$D = i(\sigma^1\partial_1 + \sigma^2\partial_2 + \sigma^3\partial_3), \tag{7}$$

where the σ^i are the Pauli matrices. In the cosmological setting, the preferred basis is the scale basis:

$$\partial_1 = \partial_2 = \partial_3 = \partial_a.$$

In the limit $\hat{V} \to 0$, we have $D^2\Psi = D\Psi = 0$, i.e., the Dirac equation. For large-scale factors, solutions to $\hat{H}\Psi = 0$ are effectively harmonic spinors, and the underlying space is a Calabi-Yau (spin) manifold.

The Dirac equation is a *conformal* 2-spinor relativistic wave equation for the base manifold $M = \mathbb{R}^3$. The physical interpretation is a (massless) spin-1/2 field over $\mathbb{R}^3$ with a wave function $\psi : \mathbb{R}^3 \to \mathbb{C}^2$, where the components specifies the probability amplitude for the field configuration at each point in $\mathbb{R}^3$ to be in the spin-up or spin-down state. The physical interpretation of the cosmological wave equation is a spin-1/2 field over $\mathbb{R}^3$ where the components are the probability amplitudes of the field configuration at each point in $\mathbb{R}^3$ to be in the expanding or contracting state. Two deductions are immediate.

First, probability takes a strictly frequentist meaning: in a spatially infinite universe, everything that can happen does; some things happen more than others. Consider the state of the observable universe at t_{now}. At some arbitrarily large distance beyond our causal horizon is a copy of our observable universe at time $t_{\text{now}} + \delta t$, in our future. *In the God's eye view, everything that can happen happens everywhere all at once.* Time is not fundamental in a universe with no fundamental scale. This is the origin of probability in QM.

Second, the potential scalar curvature V must be quantized. To drive the contracting state, an attractive or negative (quantum) scalar curvature component must emerge in $\hat{V}$. In solving the cosmological wave equation, we learn how to quantize geometry. As will soon become apparent, general relativity is a logical consequence of QM at its most minimal.

Let us take a moment to absorb the observable consequences. Starting from the simplest formulation of QM, we have deduced 1) there is an equivalence between energy and curvature, 2) the universe, as a whole, is flat with three spatial directions that are either 3) accelerating outward (expanding) or accelerating inward (contracting), scaling proportional to a^{-2}.

We have also learned of a deep geometric connection between fermions and spatial directions. Space and fermions have emerged from a conformal bootstrap of quantum universality. And where there are spinors, there is spacetime. A map always exists between spinors of order n and tensors of order $2n$. We deduced $n = 2$, so it stands to reason that a path must exist to derive the dynamic field content of our world from the first principle.

A. Geometry of quantum information

Before quantization, the components of V are the scalar curvatures of the 2-sphere and 3-sphere, respectively. The Hopf fibration uniquely relates these spheres: one can always think of the 3-sphere as a $U(1)$-fiber over the 2-sphere [51]. The fiber bundle structure of the qubit is the Hopf fibration, and the symmetry group of the qubit is $SU(2)$. Quantum information naturally follows from quantum universality. The mapping from the unit 3-sphere in a two-dimensional complex Hilbert space $\mathbb{C}^2$ (otherwise known as the complex projective line $\mathbb{C}P^1$ or the complex plane $\mathbb{C}$ with a point at infinity) to the Bloch (or Riemann) sphere is the Hopf fibration. There is a direct correspondence between each spinor in $\mathbb{C}^2$ and a point on the Bloch sphere. Schematically, we write the fibration as

$$\begin{array}{ccc} S^1 & \hookrightarrow & S^3 \\ & & \downarrow \\ & & S^2 \end{array}$$

i.e., embedding a 1-sphere in the 3-sphere 'wraps' the 3-sphere around the 2-sphere. The S^3 lives on the surface of S^2 by the identification of phase circles in S^3 with points on S^2. Every possible state of a two-level system (i.e., qubit) lives on the surface of $S^2 \cong \mathbb{C}P^1$. The system under question (the entire universe) is not a two-level system. The components of Ψ are not simply elements of $\mathbb{C}P^1$. There is more structure that captures the intrinsic quantum nature of the S^2. Nevertheless, the geometry of quantum information is a foundational starting point in solving eq. (6) and emerging time and quantum fields from Ψ. We know the components of the spinor are not real-valued power laws. They are complex and have a global $U(1)$ phase symmetry. This means the potential will take a slightly different form from eq. (5), encoding the *quantum* S^3 as a $U(1)$-fiber over the *quantum* S^2.

The Hopf fibration has a symmetry group: the action of the unitary group $U(2)$ on $\mathbb{C}^2$ leaves the S^3 invariant. It thus descends to an action on the S^2 by ordinary rotations, carrying fibers into fibers as it commutes with the $U(1)$ action [51]. An even less known fact is that $\tilde{M} = U(2)$ is a conformal compactification of Minkowski space with a "lightcone and 2-sphere at infinity" [52]. What is more, by the exceptional spin isomorphism $Spin(3,1) \cong SL(2,\mathbb{C})$, one may identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$,

$$X = \sigma^\mu x_\mu = I x_0 + \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3. \tag{8}$$

Here the Lorentz metric norm is $||x_\mu|| = \det(X)$. From this identification one finds that x_μ is lightlike precisely if there is a complex 2-spinor $\mathbf{2}$, modulo a rescaling, such that $X = \mathbf{2} \otimes \overline{\mathbf{2}}$. This identifies the celestial sphere (the space of lines in the light cone that pass through a point in Minkowski space) with the Riemann sphere, i.e., $S^2 \cong \mathbb{C}P^1$. The symmetry group of the simplest non-trivial fiber bundle *is* compactified Minkowski space! *Timelike and lightlike directions are derived from the Hopf bundle symmetry of the qubit.* A map always exists between rays of lightcone and points on the surface of the Bloch sphere. Seen from another perspective, the fundamental representations of the Lorentz group contain a pair of complex 2-spinors $\mathbf{2}$ and $\overline{\mathbf{2}}$, i.e., two fermionic directions. Using the Littlewood–Richardson rule, the tensor product $\mathbf{2} \otimes \overline{\mathbf{2}} = \mathbf{3} \oplus \mathbf{1}$ is the adjoint representation and can be identified with either Minkowski space or a spin-1 field. This is the motivation for a supersymmetry between spatial and fermionic directions.

B. Quantum geometry

The "inner mechanism" of holography appears to be the simplest non-trivial (the exceptional) fibration between the spheres, i.e., where the base space S^2 is one dimension lower than the total space S^3. But what, exactly, is a quantum sphere? What does it mean for geometry to be quantum? More precisely, what is the form of the quantum metric tensor? What new mathematical tools does one need to bootstrap a unique solution eq. (6)? Is a unique vacuum solution the key to building up an intuition of quantum geometry and writing down a complete theory of quantum gravity?

The $\mathbb{Z}_2$ symmetry $n \to -n$ leaves future infinity interchangeable with the cosmological singularity by taking $\Psi(a)$ to $\Psi(1/a)$. This duality symmetry implies a compactification with a transition map that identifies zero and infinity as the same point, i.e., the inclusion of a point at infinity with a non-trivial topology that "glues" the singularity to future infinity. The quantum state Ψ

belongs to some conformally flat and compact spin manifold in $\mathbb{C}^2$. The simplest such manifold is the Clifford torus. So perhaps our target space has the topology of a 2-torus embedded in $\mathbb{C}^2$ with a non-trivial conformal spin structure. After some thought, the only such construction is as follows.

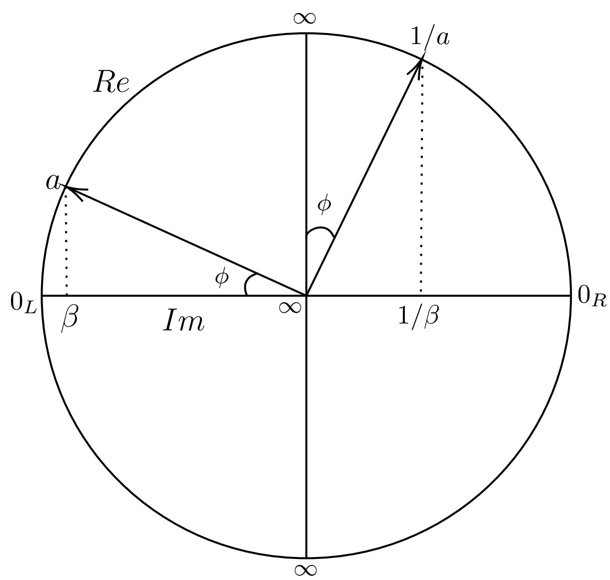

FIG. 1. The conformal Clifford torus $\mathcal{T}^2$. The scale factor a goes around the 'line at infinity' and is real, while the imaginary axis β goes through the line at infinity. The left-hand side is defined by the pair $(a, \beta) \in \mathbb{C}$, whereas the right-hand side is defined by $(1/a, 1/\beta) \in \mathbb{C}$. The entire object lives in $\mathbb{C}^2$ with the identification $(a, \beta) \sim_\pi (1/a, 1/\beta)$ where $\sim_\pi$ denotes (a, β) can be rotated into $(1/a, 1/\beta)$ by a rotation of π. Above the imaginary axis, the scale factor is positive; below, it is negative. A rotation from 0_L to 0_R takes 2π radians and sends $a \to -a$. Thus, the space is, by construction, a spin manifold. $\beta \in i\mathbb{R}^+$ from 0_L to ∞ and $1/\beta \in i\mathbb{R}^-$ from ∞ to 0_R. Both zeros are identified. The imaginary axis that runs through ∞ is the thermal circle, S^1_β. Thus, each quadrant has the sign sequence $(++, +-, --, -+)$, which is independent of the choice of clockwise or counterclockwise rotation and is nothing but the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry.

The following deductions are immediate. The physical interpretation of β is the inverse temperature. The phase angle $\phi = \phi(\beta)$ is a scalar temperature field. Topologically, $\mathcal{T}^2 = S^1_a \times S^1_\beta$ where S^1_β is the thermal circle. The thermal circle implies the de Sitter or Hawking temperature:

$$T \sim \frac{H}{2\pi} \tag{9}$$

where H is the Hubble constant. The values of the quantum state Ψ (the amplitudes) are pairs of points on $\mathcal{T}^2$, meaning the configuration space of all possible amplitudes is conformally flat and thermal.

We can geometrically bootstrap a unique vacuum solution to the cosmological wave equation from $\mathcal{T}^2$. Use the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry and assemble the EWD equation and its dual, $\hat{H}_n\psi = 0$ and $\hat{H}_{n+1}\psi = 0$, into a single pre-spinorial equation:

$$\begin{pmatrix} -\partial_a^2 + R_n & 0 \\ 0 & -\partial_a^2 + R_{n+1} \end{pmatrix} \begin{pmatrix} \psi_n \\ \psi_{n+1} \end{pmatrix} = 0. \tag{10}$$

We determined the space of states $\mathcal{T}^2$ has the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry, so it follows Ψ too is invariant under this action. The most general solution to the above is

$$\psi_n \sim a^n + (-a)^n, \qquad \psi_{n+1} \sim a^{-n} + (-a)^{-n}. \tag{11}$$

When n is odd, we have the trivial solution $\psi_n = \psi_{n+1} = 0$. The components are not complex-valued, that is if n is an integer. Identifying the phase angle $\phi = n\pi$ with $0 \le n < 2$, we obtain a unique

wave function of the universe

$$\Psi(a, \phi) = \begin{pmatrix} (1 + e^{i\phi}) a^{\phi/\pi} \\ (1 + e^{-i\phi}) a^{-\phi/\pi} \end{pmatrix}. \tag{12}$$

The quantum scalar curvature operator is then

$$\hat{V}(a, \phi) = \frac{\phi}{\pi a^2} \begin{pmatrix} \frac{\phi}{\pi} - 1 & 0 \\ 0 & \frac{\phi}{\pi} + 1 \end{pmatrix}. \tag{13}$$

The cosmological wave equation is unlike any other in the foundations of physics. It was solved without initial conditions. The initial data is unified with the first principle—the *form* of the laws of physics is the same on all scales, meaning the conformal zero that is the cosmological singularity is identified with future conformal infinity at $\phi = \pi$. It is of profound mathematical beauty that Ψ vanishes at $\phi = \pi$ by Euler's identity,

$$\Psi(a, \pi) = \begin{pmatrix} (1 + e^{i\pi}) a \\ (1 + e^{-i\pi}) a^{-1} \end{pmatrix} = 0. \tag{14}$$

The quantum vacuum state Ψ is pure, meaning the entropy is zero:

$$Z = 1 = e^S \Rightarrow S = 0. \tag{15}$$

The contracting state is a perfect mirror image of the expanding state. There is only One state in the cosmological Hilbert space. *At $t = 0$, the universe is spatially flat and in a thermal state of zero entropy and zero energy!* At $t > 0$, the dual dynamical theory with a spin-2 field is required.

Conceptually, the next great challenge is moving between the timeless God's-eye view and the dynamic ant's-eye view. On a technical level, this reduces to the origin of the Born rule, the unitary evolution of the universal quantum state Ψ, and the nature of the quantum metric tensor. Let us begin our journey with symmetry. The global phase symmetry $U(1)^2$ of Ψ is the maximal torus of $U(2)$. Multiplication of Ψ by

$$U(1)^2 = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix} \tag{16}$$

takes the components of Ψ to their complex conjugates $\overline{\Psi}$ by a phase rotation. Unitary evolution is simply a phase rotation. It is rather obvious that integrating over $\mathcal{T}^2$ (see fig. 1) means integrating Ψ (with components living in the upper quadrants) and $\overline{\Psi}$ (with components living in the lower quadrants). The Born rule follows from the double covering of conformal infinity. As it turns out, the double covering of fig. (1) is a pair of Clifford tori squeezed at a common point where past infinity, future infinity, and spacelike infinity, $I^-_{L,R} = I^+_{L,R} = I^0_{L,R} = \Psi = 0$, with a differentiable structure similar to the $U(2)$ compactification of Minkowski space, visualized in fig. 8 of [52]. A key difference in the quantum picture of Minkowksi space is the cyclic timelike direction of $\mathcal{T}^2$ is imaginary and hence isomorphic to inverse temperature. The bond between time, causality, quantum information, and geometry is deeper still.

1. *The algebra of physical information*

Another way of calculating Ψ is purely algebraically. Every point in $\mathbb{R}^3$ has the following matrix representation:

$$Q = \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3 \tag{17}$$

where the σ^i are the Pauli matrices. The eigenvalues are $\pm a$ and $\mathrm{tr}(Q) = a + (-a) = 0$. Using the spectral theorem to compute the function of a matrix, the left and right-handed states are computed as

$$\psi_L = \mathrm{tr}\ Q^{\frac{\phi}{\pi}} = (1 + e^{i\phi}) a^{\frac{\phi}{\pi}} \tag{18}$$

and

$$\psi_R = \mathrm{tr}\ Q^{-\frac{\phi}{\pi}} = (1 + e^{-i\phi}) a^{\frac{\phi}{\pi}} \tag{19}$$

where $0 \leq \phi < 2\pi$. One interprets Q as a qubit embedding of $\mathbb{R}^3$ with an equivalence relation $a \sim \lambda a$ for all real numbers $\lambda \neq 0$ that identifies the 2-sphere (of any arbitrary size) as the Bloch sphere. One can always find a λ such that λa has unit norm and identify the 2-sphere as the Bloch sphere. The physical interpretation is this: every point on the Bloch sphere is a light ray of the celestial sphere with characteristic wavelength λ. Thus, every point of $\mathcal{T}^2$ is both a qubit and a celestial sphere. From the ant's-eye view, the cosmological Hilbert space is the configuration space of all possible celestial spheres that can surround the observer. From the God's-eye view, it is a *qubit continuum* (or Q-continuum) where every point of $\mathcal{T}^2$ (except at $\Psi = 0$) is a qubit.

2. *The form of the quantum metric tensor*

Recall what we learned from the geometry of quantum information: Minkowski space has a compactification $\tilde{M} = U(2)$ and maps to the celestial sphere as a pair of 2-spinors by the exceptional spin isomorphism $Spin(3,1) \cong SL(2,\mathbb{C})$. The quantum celestial (Riemann) sphere has a quantum metric tensor. Knowing the quantum scalar curvature operator $\hat{V}(a,\phi)$ is enough to deduce the *form* of the quantum metric operator. The metric tensor g_{ij} for the n-sphere is an $n \times n$ matrix. Since the topological dimension n is now a fractional quantity, the metric tensor is promoted to a continuous matrix, i.e., an integral kernel operator $\hat{K}_g$ that acts on a 2-spinor:

$$\hat{K}_g := \int d\overline{\Psi} K_g(\Psi, \overline{\Psi}). \tag{20}$$

Recall that in QFT, Dirac 4-spinors are *reducible* representations that can always be decomposed into a pair of *irreducible* Weyl 2-spinors. The story here is a timeless boundary, a theory of a pair of irreducible 2-spinors with neither gravity nor observers, somehow dual to a dynamical theory of 4-spinors with gravity and observers. The form and role of this operator will become more precise when we derive the bulk dynamical theory. To do this, we must first understand the unitary evolution of the boundary theory.

C. Cosmological Hilbert space of states

The simplest inner-product space over $\mathcal{T}^2$ is chiral with a conformal horizon $\mathcal{R} = \frac{\lambda_\infty}{\lambda_0}$ that places a fundamental limit on the number of qubits that are, in principle, observable. The cosmological Hilbert space of states $\mathcal{H}(\mathcal{T}^2)$ is defined as:

$$\begin{aligned}
\langle L|L\rangle &:= \int_0^{\lambda_\infty} \int_0^{2\pi} d\phi da\ \psi_L \overline{\psi}_L \\
&= 4\pi \int_0^{\lambda_\infty} da\, (a^4 - 1) \left(\frac{1}{\ln a} \cdot \frac{\pi^2 + 8\ln^2 a}{\pi^2 + 4\ln^2 a} \right) \\
&< \infty
\end{aligned}$$

and

$$\begin{aligned}
\langle R|R\rangle &:= \int_{\lambda_0}^{\infty} \int_0^{2\pi} d\phi da\ \psi_R \overline{\psi}_R \\
&= 4\pi \int_{\lambda_0}^{\infty} da \left(1 - \frac{1}{a^4}\right) \left(\frac{1}{\ln a} \cdot \frac{\pi^2 + 8\ln^2 a}{\pi^2 + 4\ln^2 a} \right) \\
&< \infty,
\end{aligned}$$

where the "cut-offs" λ_0 and λ_∞ are the Compton wavelengths of the masses m_0 and m_∞, respectively. It is clear that λ_0 is the Planck length and m_0 is the Planck mass, the mass of the smallest possible black hole. In gravity, the charge is mass. There is a global $U(1)^2$ phase symmetry, meaning the vacuum has a unique global charge m with units of mass. A massive global charge *does not* break the conformal symmetry. The cosmological wave equation is 2nd-order with dimensions of the Schrodinger equation. There is a natural mass matrix $M = diag(m_0, m_\infty)$ that enters eq. (6) by multiplication of M^{-1}. The total energy is zero, meaning the charge is conformal.

Integrating over all possible phases (all possible paths), one obtains the *world densities* $\rho_0(a)$ and $\rho_\infty(a)$, which are perfect mirror images of each other. Why define the universal wave function as an element of a normed Hilbert space? Because there is no supernatural observer that exists outside of the universe bestowed with the decision to choose the initial conditions of the universe. All initial conditions and, hence, all paths consistent with the symmetries of the system are allowed. There is no Ptolemaic center in the Hilbert space of states—no first cause. This is the origin of the "many worlds" and why quantum mechanics has the structure that it does. Quantum mechanics is the ultimate realization of the Copernican principle.

D. The Bang

The charge $m_0 = M_{pl}$ can not be fundamental as it radiates away with Hawking temperature $T \sim g/2\pi$, where g is the surface gravity of the horizon at $r = \lambda_0 = L_{pl}$. Another way to see why this is true is from $\hat{V}$ (see eq. 13). The Planck mass m_0 sources the left-hand potential V_L which has a decaying attractive phase for $0 < \phi < \pi$ and is zero at $\phi = \pi$, meaning V_L decays into the right-hand potential V_R. Thus, one concludes m_0 is not fundamental and must be a composite of m_∞. The right-hand potential V_R is always repulsive, with m_∞ sourcing positive quantum curvature that energetically favors an inflationary-like event and then slowly rolls like a^{-2} to zero at future infinity. The ultra-light gravitational charge determines the largest scale the observer shall ever see. Future infinity has a horizon at λ_∞, explaining why the charge m_∞ is small. (A calculation of this number will appear in future work.)

Nature has no fundamental scale, but the observer does. To ensure eq. (6) remains dimensionless send $a \to \frac{a}{\lambda_0}$ and $a \to \frac{a}{\lambda_\infty}$ in ψ_L and ψ_R, respectively. A remarkable duality then follows:

$$\psi_L(\lambda_\infty, \phi)\overline{\psi}_L(\lambda_\infty, \phi) = \psi_R(\lambda_0, \phi)\overline{\psi}_R(\lambda_0, \phi). \tag{21}$$

The UV horizon at λ_0 and the IR horizon at λ_∞ are dual. UV/IR mixing is required for unitarity! Unitary evolution is a phase rotation of ψ_L from $a = 0_L$ through $a = \lambda_\infty^L$ to $a = \lambda_0^R$ (still labeled as ψ_L) to $a = 0_R$. The UV and IR scales place a fundamental limit on what is, in principle, observable. From the ant's-eye view, the observer can access at most $\mathcal{R}$ qubits (modulo some constant). *Observers are finite-dimensional Hilbert spaces embedded in* $\mathcal{H}(\mathcal{T}^2)$.

The rabbit hole is deeper still. The vacuum has a conserved charge m_∞, meaning the mass density $\rho = m_\infty/\lambda_0^3$ of empty space is a conserved quantity. But if m_0 is a composite of m_∞, the black hole mass function must violate decoupling and scale like a^3 since mass density dilutes like a^{-3} in an expanding Euclidean 3-space. This means two competing mechanisms set up the Bang: 1) the decay of m_0 into m_∞ and 2) the inflation of m_0 into larger black holes as space accelerates in all directions. The physical picture of the moment before time is a perfect *conformal* lattice of quantum black holes. From the uncertainty principle, the quantum black holes do not all decay into m_∞ at the same rate. There will always be a fraction that swells to astrophysical sizes, seeding structure formation in the early universe.

To rewind the universe back to $t = 0$ and hit play, we must derive the local dynamical equations of motion.

E. The dynamic bulk

For the remainder of the paper, I will use the mostly minus metric signature $(+ - --)$ and natural units $c = G = \hbar = 1$. Greek indices $\mu, \nu = 0, 1, 2, 3$ index spacetime, lowercase Latin indices $i, j = 1, 2, 3$ index space, and upper case $I, J = 0, 1$ index 2-spinors in the fundamental representation. I use a 4-spinor notation where ψ^μ denotes a column 4-spinor and ψ_μ the conjugate transpose. One lower and upper repeated index implies summation.

The boundary equations of motion, eq. (6), are defined by the Clifford algebra $C\ell_3(\mathbb{R})$. It follows from the exceptional spin isomorphism that the bulk equations of motion are defined by the Dirac algebra $C\ell_{3,1}(\mathbb{C})$, with the simplest holographic dictionary given by

$$D \leftrightarrow \not{D}, \quad \hat{V} \leftrightarrow \hat{V}_{\mu\nu} \tag{22}$$

where $\not{D} = i\gamma^\mu\partial_\mu$ is the Dirac operator and the potential $V_{\mu\nu}$ is a second-rank tensor. The square of the Dirac operator $\not{D}^2 = \partial^\mu\partial_\mu I_4$, and the bulk Hamiltonian becomes

$$\hat{H}_{\mu\nu} = -\nabla^2 I_4 + V_{\mu\nu}. \tag{23}$$

One can then write down the

dynamical cosmological wave equation

$$\hat{H}_{\mu\nu}\psi^\nu = E^2\psi^\nu \tag{24}$$

$$(-\nabla^2 I_4 + V_{\mu\nu})\psi^\nu = -\partial_t^2\psi^\nu \tag{25}$$

$$\partial^\mu\partial_\mu\psi^\nu + V_{\mu\nu}\psi^\nu = 0. \tag{26}$$

The bulk equations of motion are four coupled relativistic wave equations second-order in time. Fields live in representations of the Lorentz group. The potential $V_{\mu\nu}$ decomposes into the following classical irreducible representations of the Lorentz group: an antisymmetric tensor $F_{\mu\nu} = -F_{\nu\mu}$, a symmetric ($H_{\mu\nu} = H_{\nu\mu}$) and traceless ($H^\mu{}_\mu = 0$) tensor $H_{\mu\nu}$ (not to be confused with the Hamiltonian), and a scalar trace component:

$$V_{\mu\nu} = F_{\mu\nu} + H_{\mu\nu} + X^\mu{}_\mu. \tag{27}$$

Under the appropriate local phase (gauge) invariance conditions, $H_{\mu\nu}$ is a massless spin-2 field—the graviton! If we set the scalar trace component $X^{\mu}{}_{\mu} = 1/\lambda_C^2$ where λ_C is the Compton wavelength and introduce some dimensionful coupling constants, the bulk equations of motion

$$\partial^\mu \partial_\mu \psi^\nu + \alpha F_{\mu\nu}\psi^\nu + \beta H_{\mu\nu}\psi^\nu + m^2\psi^\nu = 0, \tag{28}$$

reduce to four Klein-Gordon equations coupled by m^2 in the limit $\alpha \to \beta \to 0$. There is no imaginary number i in the above equation, meaning ψ^ν can either be real-valued and transform like a 4-vector or complex-valued and transform like a 4-spinor.

The timeless boundary severely restricts the dynamic bulk. The boundary theory is defined by a pair of 2-spinors, so the above wave equation must decompose into a pair of 2-spinor wave equations in the reducible or fundamental representation.

Recall that the Hopf bundle has $U(2)$, a compact representation of Minkowski space, as its symmetry group. Locally $U(2)$ is diffeomorphic to the direct product $G = SU(2) \times U(1)$. Up to a local diffeomorphism, the compact gauge group G, together with the emergence of the antisymmetric tensor $F = F_{\mu\nu}$, implies the existence of Yang-Mills equations, with a gauge connection A on G. The connection A is locally a one-form on spacetime, and the curvature or field strength tensor is the two-form: $F = dA + A \wedge A$. It follows the antisymmetric component of $V_{\mu\nu}$ can be expanded into the electroweak field strength tensors (pre spontaneous symmetry breaking) with $U(1)$ and $SU(2)$ gauge connections A_μ and $\{W^i_\mu\}_{i=1,2,3}$, respectively.

Generally speaking, mixing spinor and tensor indices always results in a conflict of notation. Keep in mind when we work with 4-spinors, we do so in the reducible representation, meaning when we move to the fundamental representation (a pair of 2-spinors with the Pauli algebra representation of the field strength tensors), we no longer have the issue of mixing tensor and spinor indices. The limitation is in formalism only—not the physics.

I show in appendix [A] if one restricts themselves to the electromagnetic sector with an agreed-upon index notation, it is not hard to show $F_{\mu\nu}\psi^\nu \to e^{i\phi}F_{\mu\nu}\psi^\nu$ under a local $U(1)$ gauge transformation with the covariant derivative.

F. Quantum gravity

In semiclassical gravity, the simplest metric is Minkowksi, with some quantum fluctuations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{29}$$

where $h_{\mu\nu}$ ($\equiv H_{\mu\nu}$) is a symmetric and traceless rank-2 tensor. From eq. (20), it was deduced that the quantum metric tensor is an integral kernel operator; hence, so too is the spin-2 field as it appears in the bulk equations of motion. An educated guess on the form of this operator is an indefinite integral operator of the following form:

$$H_{\mu\nu} \to H = \int d\psi_\mu h(\psi^\mu, \psi_\nu) \tag{30}$$

such that the kernel is a complex symmetric

$$h(\psi^\mu, \psi_\nu) = \overline{h(\psi_\nu, \psi^\mu)} := h(\psi^\nu, \psi_\mu), \tag{31}$$

traceless trace-class operator

$$\mathrm{tr}\ H = \int d\psi_\mu h(\psi^\mu, \psi_\mu) = 0. \tag{32}$$

The classical irreducible representation of the Lorentz group, $H_{\mu\nu}$, is promoted to a unique class of Hilbert-Schmidt integral operators where

$$\int_\Omega \int_\Omega d\psi_\mu |h(\psi^\mu, \psi_\nu)|^2 d\psi^\nu < \infty \quad (\mu \neq \nu) \tag{33}$$

with the domain Ω determined from the boundary theory. Gravity is exceptional in that it acts on the quantum state as an integral operator, whereas the other forces act on the quantum state as differential operators. The classical irrep. $H_{\mu\nu}$ of the Lorentz group is fully quantized and makes eq. (24) a highly non-linear partial differential equation, where $H\psi^\nu = U(\psi^\nu)$ is a quantum gravitational potential and self-interaction term.

The first step in demonstrating a holographic duality between the timeless boundary and the dynamic bulk is to match the energy spectrum of the bulk and boundary theories. The boundary theory is based on the energy E, which is zero, whereas the bulk is based on the squared energy E^2, which is non-zero and has a negative energy solution. Cosmologically, this implies a CPT-symmetric universe consistent with the proposal of Turok & Boyle [29].

The conformal invariance of the boundary is locally broken from the ant's-eye perspective of the bulk observer. So, how can it be that these two theories are equivalent? How is a dynamical non-linear differential equation secretly a non-dynamical linear one? There must exist a highly non-trivial map, like a Fourier transform, between equations (6) and (24) for self-consistency.

The 4-spinor ψ^ν in eq. (24) decomposes into a pair of 2-spinors χ_L and χ_R, respectively, that satisfies two coupled non-linear relativistic wave equations

$$\left(-\sigma^\mu \sigma^\nu \partial_\mu \partial_\nu I_2 + V\right)\chi_L = 0 \tag{34}$$

$$\left(-\overline{\sigma}^\mu \overline{\sigma}^\nu \partial_\mu \partial_\nu I_2 + V^\dagger\right)\chi_R = 0 \tag{35}$$

where $\sigma^\mu = (I_2, \sigma^i)$, $\sigma^\nu = (-I_2, \sigma^j)$, $\overline{\sigma}^\mu = (I_2, -\sigma^i)$, and $\overline{\sigma}^\nu = (-I_2, -\sigma^j)$, i.e., the generators of $U(2)$, and the potential $V = V_{IJ}$ contains the same information as $V_{\mu\nu}$ by recasting the field strength tensors in the Pauli matrix representation with the Clifford algebra $C\ell_{3,0}(\mathbb{R})$, otherwise known the Pauli algebra. Recall what we learned from the geometry of quantum information: Minkowski space has a compact presentation $U(2)$, and the fundamental representation of the Lorentz group contains two fermionic directions. Unpacking the above, we have,

$$\left(\partial^{\mu}\partial_{\mu}I_2 + \alpha F + \beta H + m^2\right)\chi_L = 0 \tag{36}$$

$$\left(\partial^{\mu}\partial_{\mu}I_2 + \alpha F^{\dagger} + \beta H^{\dagger} + m^2\right)\chi_R = 0 \tag{37}$$

In the limit that $\alpha \to \beta \to 0$, the above reduces to the Klein-Gordon equation. Thus, one concludes equation (24) is the adjoint or *reducible* representation where the above is the *irreducible* or fundamental representation.

In the scale basis and in the limit $V_{\mu\nu} \to 0$, the identification $a \leftrightarrow \sigma$ (proper space) and $t \leftrightarrow \tau$ (proper time) of the string worldsheet, the fundamental representation reduces to the equations of motion of the bosonic string in conformal gauge. The implied supersymmetry here is of a more fundamental nature than string theory without quantum covariance. All bosonic fields (including Minkowski space as a spin-1 field) are dual to a pair of spinors on the timeless boundary. The fundamental representation *is* the boundary at future infinity. This explains why the LHC has not seen superpartners: they represent redundant unphysical degrees of freedom that only appear in perturbative constructions without quantum covariance.

I write down the bulk field equations in the fundamental representation with the electromagnetic 2-form and the spin-2 integral operator calculated explicitly in appendix [B].

1. *Holographic transform*

The Dirac field can be written as a sum of chiral Weyl fields. The boundary theory has no timelike direction, so its solutions obviously can not be a linear combination of the non-linear states χ_L and χ_R. The holographic transform maps non-linear dynamical bulk states to linear timeless linear states on the boundary. The quantum Minkowski metric operator $\hat{\eta}$ takes χ_L and χ_R,

$$\widehat{\chi}(a,\phi) = \int_{-\infty}^{\infty} \eta_{IJ}(a,t;\phi)\chi^J(a,t)dt \tag{38}$$

to solutions of the boundary theory:

$$\Psi \sim \widehat{\chi_L} + \widehat{\chi_R}. \tag{39}$$

The quantum metric $\hat{\eta} = \eta_{IJ}$ is a spinorial integral operator that integrates out the timelike direction and linearizes the chiral fields such that linear combinations are solutions to the boundary equations of motion. The quantum metric operator establishes an observer-dependent time evolution, a *causal reference frame* for bulk observers. This causal reference frame acts as an atlas for locating bulk events in the boundary Hilbert space—the God's eye view. The integral operator formalism is like a path integral over trajectories not in space but in time, i.e., an integral over all possible rates an observer's clock can tick, meaning the quantum metric is an observer-dependent field and a field-theoretic generalization of the Page-Wooters mechanism [40].

Understanding this holographic transform at a deeper level requires 1) a better understanding of the spinorial integral operator algebra that encodes the quantum geometry and 2) a first-principles calculation of m_∞, which is essential to gauging and solving eq. (24).

III. CAUSAL IMPLICATIONS

The 'many worlds' in Everettian QM are causally disconnected when a classical global time coordinate is assumed. It is clear that if time is not fundamental, the classical notions of causality break down. When time drops out of the Schrödinger equation, the Hilbert space becomes causally indefinite. The conformal Clifford torus has no causal ordering—lightcones can be superimposed. Remarkably, in the first of its kind, the quantum switch experiment has demonstrated *causal indefiniteness* [63, 64]. Since there is no global time coordinate, there must exist a *causal reference frame* [65–68] for each event that establishes an observer-dependent time to describe the evolution of quantum subsystems. I expect causal reference frames to be key in gauge fixing and solving eq. (24).

If quantum theory is truly universal and if UV/IR mixing is a general feature of the quantum universe, what are the implications of causal indefiniteness? Can exotic UV/IR couplings that violate EFT reasoning enable the quantum metric tensor to be technologically exploited? Looking at equations (34) - (39), it is tantalizingly possible to engineer the quantum metric tensor with some highly tuned matter-antimatter reaction. This begs the question: does quantum universality allow travel between the many worlds?

If $\eta_{IJ}(a,t;\phi)$ establishes a causal reference frame that acts as an atlas for locating a local patch of the bulk on the boundary, then can the observer's causal reference frame be phase rotated into an adjacent world with slightly different initial conditions? From the constraints on the spin-2 integral operator, we know $h(\psi^\mu,\psi_\nu)$ is a traceless Hermitian matrix. The simplest possible (though not unique) choice is then:

$$h(\psi^\mu,\psi_\nu) = \psi_\mu\overline{\psi_\nu} - \psi_\mu\overline{\psi_\mu}. \tag{40}$$

Here, I use outer product notation by lowering all indices on spinors to avoid ambiguity with the standard Einstein notation where $\psi_\mu\overline{\psi_\mu} := diag(\psi_0\overline{\psi}_0, \psi_1\overline{\psi}_1, ...)$. The spin-2 integral operator is now manifestly gauge invariant, meaning we can place yet another constraint on the Minkowski metric operator: under a local $U(1)$ phase transformation

$$\eta_{IJ}(a,t;\phi) \to e^{i\phi(a,t)}\eta_{IJ}(a,t;\phi'), \tag{41}$$

the *form* of the metric operator $\hat{\eta}$ is invariant. The phase angle ϕ in the universal wave function is coordinate-independent, and the phase symmetry is global. To preserve the global phase symmetry of the boundary, local phase transformations of the bulk take $\phi \to \phi'$ on the boundary, where the initial data of possible Bangs are related by the global $U(1)$ phase symmetry.

The *relative phase* of the universal wave function is coupled to the potential (quantum scalar) curvature (eq. 13) and the quantum metric tensor (eq. 38) through the global $U(1)$ phase angle ϕ. This is strikingly similar to the Aharonov–Bohm effect. Indeed, looking at eq. (24) and taking $\alpha \to 0$ and $\partial_\mu \to D_\mu := \partial_\mu - ieA_\mu$, where A_μ is the local $U(1)$-gauge connection or electromagnetic 4-potential, e is the charge of the electron, and m is the electron mass:

$$D^\mu D_\mu \psi^\nu + \beta \int d\psi_\mu h(\psi^\mu, \psi_\nu)\psi^\nu + m^2\psi^\nu = 0, \qquad (42)$$

the general relativistic wave equation becomes fully gauge-invariant. (I provide the de Sitter limit in appendix C.) In the absence of an electromagnetic field $F_{\mu\nu}$, the electromagnetic 4-potential still exists and couples to the unified wave function of the electron and graviton.

In quantum field theory, we are taught the gauge degrees of freedom are redundant. The internal symmetries are an artifact of our description of nature and not physical. But this is not so. The gauge degrees of freedom connect the coordinate-dependent phase angle $\phi(a, t)$ in the bulk to a coordinate-independent phase angle ϕ' on the boundary that encodes the initial data of an arbitrary Bang. Since no unphysical axes exist in the configuration space (i.e., the boundary), a gauge transformation (when appropriately gauge fixed with a casual reference frame) acts as a shear along these extra dimensions. It is, therefore, at least in principle, possible to phase shift the observer's causal reference frame into a world with different initial conditions. *Everettian field theory does not prohibit a traversable multiverse.*

This begs the profound question: can we manipulate the quantum metric with the precision we manipulate electrons and warp spacetime into shapes of our design? Will humankinds one day, in other worlds out there, find their way to us? Will we one day find our way to them? If quantum universality is true and the multiverse exists, there must exist worlds where future humans master the quantum nature of spacetime and travel to "past-adjacent" light cones—a form of time travel in the multiverse. But if this were possible, then where are they?

For most of human history, we have been a deeply divided species. So much so that we have developed nuclear arsenals that may annihilate us all. But if the deepest truth of quantum mechanics is that we are all One, connected in profound and mysterious ways that we have yet to understand, there is hope for a future beyond our wildest imagination. A future where science and spirituality need not be separate. A civilization of humankinds cocreating as One people—in the liminal spaces, between worlds—past, present, and future.

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Appendix A: Gauging the bulk equations of motion

1. Notation

I use the mostly minus metric signature $(+---)$ with the standard Einstein notation and natural units $\hbar = c = G = 1$. Greek indices index spacetime, lowercase Latin indices index space, and uppercase Latin indices index 2-spinors. One lower and upper repeated index implies summation.

Recall that ψ^ν can transform as either a 4-vector or a 4-spinor. Mixing spinor and tensor indices is messy, but it can be done. I use the following convention: for a covariant 2-form multiplied by a 4-spinor,

$$F_{\mu\nu}\psi^\nu \longrightarrow \psi^\nu = \begin{pmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} \quad \psi_\nu = \begin{pmatrix} \overline{\psi}_0 & \overline{\psi}_1 & \overline{\psi}_2 & \overline{\psi}_3 \end{pmatrix}$$

where lowering the index on the 4-spinor denotes the conjugate transpose. (The same index convention is used for 2-spinors.) For a contravariant 2-form multiplied by a 4-spinor, the convention is inverted. Note the difference between raising and lowering indices of a 4-spinor and 4-vector. Generally, this is a problem as conflicts of notation will arise. However, for what we are trying to calculate, so long as we are consistent in the notation, all is well.

When propagating the product rule, one runs into the following conflict:

$$\partial_\mu A_\nu \psi^\nu \stackrel{?}{=} A_\nu \partial_\mu \psi^\nu + \psi^\nu \partial_\mu A_\nu \qquad (A1)$$

$$\equiv A_\nu \partial_\mu \psi^\nu + (\partial_\mu A_\nu)\psi^\nu \qquad (A2)$$

The second term on the first line conflicts with our 4-spinor notation. Left multiplication should have a lowered index denoting the transpose for a row 4-spinor. However, we do not want to conjugate the elements of the 4-spinor or raise the index on the 4-vector for consistency with Einstein notation. Instead, we can use parentheses to denote the product rule has already been applied with right-multiplication of ψ^ν. To make the calculations easier on the eyes, I will adopt the following convention,

$$\psi^\nu \partial_\mu A_\nu \equiv (\partial_\mu A_\nu)\psi^\nu \tag{A3}$$

where left-multiplication of ψ^ν followed by two lowered indices denotes the product rule has already been applied and to move ψ^ν to the right for contraction of the repeated index.

2. Adjoint representation

Let's ignore the $H_{\mu\nu}$ term in eq. (28) and focus on the electromagnetic field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{A4}$$

where the four-potential $A_\mu = (A_0, -A_1, -A_2, -A_3)$. The field strength tensor is antisymmetric, so $F_{\mu\mu} = 0$, meaning there are only six independent degrees of freedom. Computing each component yields:

$$F_{01} = -\frac{\partial A_1}{\partial x_0} - \frac{\partial A_0}{\partial x_1} = E_1 \tag{A5}$$

$$F_{02} = -\frac{\partial A_2}{\partial x_0} - \frac{\partial A_0}{\partial x_2} = E_2 \tag{A6}$$

$$F_{03} = -\frac{\partial A_3}{\partial x_0} - \frac{\partial A_0}{\partial x_3} = E_3 \tag{A7}$$

$$F_{12} = -\frac{\partial A_2}{\partial x_1} + \frac{\partial A_1}{\partial x_2} = -B_3 \tag{A8}$$

$$F_{13} = \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1} = B_2 \tag{A9}$$

$$F_{23} = -\frac{\partial A_3}{\partial x_2} + \frac{\partial A_2}{\partial x_3} = -B_1 \tag{A10}$$

Taking the scalar trace component $X^\mu_\mu = m^2$ and inserting $F_{\mu\nu}$ explicitly in equation eq. (28) gives:

$$\Box \begin{pmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} + \alpha \begin{pmatrix} 0 & \frac{\partial A_1}{\partial x_0} & \frac{\partial A_2}{\partial x_0} & \frac{\partial A_3}{\partial x_0} \\ -\frac{\partial A_1}{\partial x_0} & 0 & -\frac{\partial A_2}{\partial x_1} & \frac{\partial A_1}{\partial x_3} \\ -\frac{\partial A_2}{\partial x_0} & \frac{\partial A_2}{\partial x_1} & 0 & -\frac{\partial A_3}{\partial x_2} \\ -\frac{\partial A_3}{\partial x_0} & -\frac{\partial A_1}{\partial x_3} & \frac{\partial A_3}{\partial x_2} & 0 \end{pmatrix} \begin{pmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} + \alpha \begin{pmatrix} 0 & \frac{\partial A_0}{\partial x_1} & \frac{\partial A_0}{\partial x_2} & \frac{\partial A_0}{\partial x_3} \\ -\frac{\partial A_0}{\partial x_1} & 0 & \frac{\partial A_1}{\partial x_2} & -\frac{\partial A_3}{\partial x_1} \\ -\frac{\partial A_0}{\partial x_2} & -\frac{\partial A_1}{\partial x_2} & 0 & \frac{\partial A_2}{\partial x_3} \\ -\frac{\partial A_0}{\partial x_3} & \frac{\partial A_3}{\partial x_1} & -\frac{\partial A_2}{\partial x_3} & 0 \end{pmatrix} \begin{pmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} + m^2 \begin{pmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} = 0.$$

Good. Four coupled KG equations are in the limit $\alpha \to 0$. We have two derivatives acting on ψ^ν from $\Box$ and one derivative from the 2-form $F_{\mu\nu}$ with a local U(1) gauge-connection A_μ that acts on the 4-spinor by the product rule:

$$\begin{aligned} F_{\mu\nu}\psi^\nu &= \partial_\mu A_\nu \psi^\nu - \partial_\nu A_\mu \psi^\nu \\ &= A_\nu \partial_\mu \psi^\nu + \psi^\nu \partial_\mu A_\nu - A_\mu \partial_\nu \psi^\nu - \psi^\nu \partial_\nu A_\mu. \end{aligned} \tag{A11}$$

Promoting the derivative to the gauge-covariant derivative,

$$\partial_\mu \to \partial_\mu - ieA_\mu, \quad \partial_\nu \to \partial_\nu - ieA_\nu \tag{A12}$$

the field strength tensor is invariant,

$$F_{\mu\nu} = (\partial_\mu - ieA_\mu)A_\nu - (\partial_\nu - ieA_\nu)A_\mu \tag{A13}$$

$$= \partial_\mu A_\nu - ieA_\mu A_\nu - \partial_\nu A_\mu + ieA_\nu A_\mu \tag{A14}$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{A15}$$

where the last step follows from the fact that $A_\mu A_\nu = A_\nu A_\mu$, i.e., the vector potential commutes with itself $[A_\mu, A_\nu] = 0$. This is true because the 4-position is commutative: $[x_\mu, x_\nu] = 0$. (The covariant derivative D_μ in non-commutative geometries will always violate Lorentz invariance with a trace anomaly $F^\mu_\mu \neq 0$.)

3. The U(1) gauge transformation

Let $\phi = \phi(x, y, z, t)$ be the local phase angle, i.e., a scalar field. A $U(1)$ gauge transformation,

$$\psi^\nu \to e^{i\phi}\psi^\nu \tag{A16}$$

with the vector potential transforming as

$$A_\mu \to A_\mu - \frac{1}{e}(\partial_\mu\phi), \quad A_\nu \to A_\nu - \frac{1}{e}(\partial_\nu\phi), \tag{A17}$$

and the covariant derivative

$$\partial_\mu \to \partial_\mu - ieA_\mu - i(\partial_\mu\phi), \quad \partial_\nu \to \partial_\nu - ieA_\nu - i(\partial_\nu\phi). \tag{A18}$$

Propagating the product rule

$$\begin{aligned}
F_{\mu\nu}e^{i\phi}\psi^\nu &= \partial_\mu A_\nu e^{i\phi}\psi^\nu - \partial_\nu A_\mu e^{i\phi}\psi^\nu && \text{(A19)}\\
&= A_\nu\partial_\mu e^{i\phi}\psi^\nu + e^{i\phi}\psi^\nu\partial_\mu A_\nu - A_\mu\partial_\nu e^{i\phi}\psi^\nu - e^{i\phi}\psi^\nu\partial_\nu A_\mu && \text{(A20)}\\
&= e^{i\phi}A_\nu\partial_\mu\psi^\nu + A_\nu\psi^\nu\partial_\mu e^{i\phi} + e^{i\phi}\psi^\nu\partial_\mu A_\nu - e^{i\phi}A_\mu\partial_\nu\psi^\nu - A_\mu\psi^\nu\partial_\nu e^{i\phi} - e^{i\phi}\psi^\nu\partial_\nu e^{i\phi} && \text{(A21)}\\
&= e^{i\phi}(A_\nu\partial_\mu\psi^\nu + \psi^\nu\partial_\mu A_\nu - A_\mu\partial_\nu\psi^\nu - \psi^\nu\partial_\nu A_\mu) + A_\nu\psi^\nu\partial_\mu - A_\mu\psi^\nu\partial_\nu e^{i\phi} && \text{(A22)}\\
&= e^{i\phi}F_{\mu\nu}\psi^\nu + A_\nu\psi^\nu\partial_\mu e^{i\phi} - A_\mu\psi^\nu\partial_\nu e^{i\phi}, && \text{(A23)}
\end{aligned}$$

we know $F_{\mu\nu}$ is invariant under the covariant derivative, so we isolate the last two terms and see they cancel under the covariant derivative:

$$\begin{aligned}
A_\nu\psi^\nu\partial_\mu e^{i\phi} &= A_\nu\psi^\nu(\partial_\mu - ieA_\mu - i(\partial_\mu\phi))e^{i\phi} && \text{(A24)}\\
&= A_\nu(i(\partial_\mu\phi)e^{i\phi} - ieA_\mu e^{i\phi} - i(\partial_\mu\phi)e^{i\phi})\psi^\nu \quad \text{(from } A3) && \text{(A25)}\\
&= -ieA_\nu A_\mu e^{i\phi}\psi^\nu && \text{(A26)}\\
-A_\mu\psi^\nu\partial_\nu e^{i\phi} &= -A_\mu\psi^\nu(\partial_\nu - ieA_\nu - i(\partial_\nu\phi))e^{i\phi} && \text{(A27)}\\
&= -A_\mu(i(\partial_\nu\phi)e^{i\phi} - ieA_\nu e^{i\phi} - i(\partial_\nu\phi)e^{i\phi})\psi^\nu \quad \text{(from } A3) && \text{(A28)}\\
&= ieA_\mu A_\nu e^{i\phi}\psi^\nu. && \text{(A29)}
\end{aligned}$$

Thus, the field strength tensor acting on a 4-spinor transforms as

$$F_{\mu\nu}\psi^\nu \to e^{i\phi}F_{\mu\nu}\psi^\nu \tag{A30}$$

under the $U(1)$ gauge transformation $\psi^\nu \to e^{i\phi}\psi^\nu$.

Appendix B: The fundamental representation

In this section, I explicitly write down the bulk equations of motion in the fundamental representation. Unsuppressing the indices of equations (36) and (37), we have

$$\partial^\mu\partial_\mu\chi^J + \alpha F_{IJ}\chi^J + \beta H\chi^J + m^2\chi^J = 0 \tag{B1}$$

$$\partial^\mu\partial_\mu\zeta^J + \alpha F^\dagger_{IJ}\zeta^J + \beta H^\dagger\zeta^J + m^2\zeta^J = 0 \tag{B2}$$

where we relabeled the left-handed spinor $\chi_L := \chi^J$ and the right-handed spinor $\chi_R := \zeta^J$.

1. Electromagnetic field

Using the standard Pauli matrix formalism

$$F_{\mu\nu} \to F_{IJ} = \sigma^l_{IJ}E_l + i\sigma^l_{IJ}B_l = \begin{pmatrix} E_3 & E_1 - iE_2 \\ E_1 + iE_2 & -E_3 \end{pmatrix} + i\begin{pmatrix} B_3 & B_1 - iB_2 \\ B_1 + iB_2 & -B_3 \end{pmatrix}, \tag{B3}$$

the electromagnetic 2-form becomes a first-order differential operator:

$$F_{IJ} = \sigma^l_{IJ}(-\partial_0 A_l - \partial_l A_0) + \frac{i}{2}\sigma^l_{IJ}\epsilon_{lmn}(\partial^m A^n - \partial^n A^m), \tag{B4}$$

where ϵ_{lmn} is the 3-dim Levi-Civita tensor and σ^l_{IJ} are the Pauli matrices.

2. Gravitational field

The boundary theory provides an explicit recipe for quantizing the classical spin-2 field $H_{\mu\nu}$ by promoting it to a unique class of Hilbert-Schmidt integral operators:

$$H = \int d\chi_I h(\chi^I, \chi_J) \tag{B5}$$

that acts on a 2-spinor χ^J with a complex symmetric kernel,

$$h(\chi^I, \chi_J) = \overline{h(\chi_I, \chi^J)} := h(\chi^J, \chi_I) \tag{B6}$$

such that H is a traceless trace-class operator:

$$\text{tr } H = \int d\chi_I h(\chi^I, \chi_I) = 0 \tag{B7}$$

where it is assumed the integration constant is zero. The simplest (and non-unique) choice of a kernel is,

$$h(\chi^I, \chi_J) = \begin{pmatrix} 0 & \chi_0\overline{\chi_1} \\ \chi_1\overline{\chi_0} & 0 \end{pmatrix} \tag{B8}$$

such that

$$\int_\Omega \int_\Omega d\chi_I |h(\chi^I, \chi_J)|^2 d\chi^J < \infty \quad (I \neq J) \tag{B9}$$

with the domain Ω defined by the boundary theory. Evaluating the above integral (before normalizing the boundary), we have

$$\begin{aligned}
\int_\Omega \int_\Omega d\chi_I |h(\chi^I, \chi_J)|^2 d\chi^J &= \int_{\psi_R(a=\lambda_0,\phi=\pi)}^{\psi_L(a=\lambda_\infty,\phi=0)} \int_{\psi_R(a=\lambda_0,\phi=\pi)}^{\psi_L(a=\lambda_\infty,\phi=0)} d\chi_I |h(\chi^I, \chi_J)|^2 d\chi^J && \text{(B10)}\\
&= \int_0^{2\mathcal{R}} \int_0^{2\mathcal{R}} d\chi_I |h(\chi^I, \chi_J)|^2 d\chi^J && \text{(B11)}\\
&= \int_0^{2\mathcal{R}} \int_0^{2\mathcal{R}} \begin{pmatrix} d\overline{\chi_1} & d\overline{\chi_0} \end{pmatrix} \begin{pmatrix} |\chi_0|^2|\chi_1|^2 & 0 \\ 0 & |\chi_1|^2|\chi_0|^2 \end{pmatrix} \begin{pmatrix} d\chi_0 \\ d\chi_1 \end{pmatrix} && \text{(B12)}\\
&= \int_0^{2\mathcal{R}} \int_0^{2\mathcal{R}} |\chi_0|^2|\chi_1|^2 d\overline{\chi_1} d\chi_0 + \int_0^{2\mathcal{R}} \int_0^{2\mathcal{R}} |\chi_1|^2|\chi_0|^2 d\overline{\chi_0} d\chi_1 && \text{(B13)}\\
&= \frac{128\mathcal{R}^6}{9} && \text{(B14)}
\end{aligned}$$

where $\mathcal{R} = \frac{\lambda_\infty}{\lambda_0}$. We absorb the normalization constant inherited from the boundary in the dimensionful constant β that determines the strength of the self-interaction term:

$$\begin{aligned}
H\chi^J = \frac{3\beta}{8\sqrt{2}\mathcal{R}^3} \int d\chi_I h(\chi^I, \chi_J)\chi^J &= \frac{3\beta}{8\sqrt{2}\mathcal{R}^3} \int \begin{pmatrix} d\overline{\chi_0} & d\overline{\chi_1} \end{pmatrix} \begin{pmatrix} 0 & \chi_0\overline{\chi_1} \\ \chi_1\overline{\chi_0} & 0 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} && \text{(B15)}\\
&= |\chi_0|^2 \frac{3\beta}{8\sqrt{2}\mathcal{R}^3} \int \chi_1 d\overline{\chi_1} + |\chi_1|^2 \frac{3\beta}{8\sqrt{2}\mathcal{R}^3} \int \chi_0 d\overline{\chi_0} && \text{(B16)}\\
&= \frac{3\beta}{8\sqrt{2}\mathcal{R}^3} |\chi_0|^2|\chi_1|^2 && \text{(B17)}
\end{aligned}$$

where it is assumed all integration constants are zero. To compute the interaction strength, we must first find a way to calculate λ_∞ and normalize the boundary theory. The spin-2 integral operator acting on the spinor field

$$H\chi^J = U(\chi_0, \chi_1) \tag{B18}$$

is the potential of a gravitational harmonic oscillator with a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry—a fundamental symmetry of the boundary. Not only is the above manifestly gauge invariant, it is gauge-independent: the spin-2 integral operator transforms as $H\chi^J \to H\chi^J$ under the $U(1)$ gauge transformation $\chi^J \to e^{i\phi}\chi^J$. An interesting observation is this: if we set $\chi_0 = \chi_1$ we have a $|\chi|^4$ theory, similar to the potential in a complex φ^4 theory like the Higgs potential. We can expand around $h(\chi^I, \chi_J)$ and add more terms such that we always have even powers of the spinor field. Is the Higgs mechanism derivable from quantum gravity?

3. Bulk field equations

Putting everything together (and absorbing the Hilbert-Schmidt normalization factor in β), the gauged bulk field equations are

$$D^\mu D_\mu \chi^J + \alpha_1 \sigma^l_{IJ} E_l \chi^J + i\alpha_2 \sigma^l_{IJ} B_l \chi^J + \beta|\chi_0|^2|\chi_1|^2 + m^2\chi^J = 0 \tag{B19}$$

$$D^\mu D_\mu \zeta^J + \alpha_1 \sigma^l_{IJ} E_l \zeta^J - i\alpha_2 \sigma^l_{IJ} B_l \zeta^J + \beta|\zeta_0|^2|\zeta_1|^2 + m^2\zeta^J = 0 \tag{B20}$$

(with dimensionful coupling constants $\alpha_1 \sim [V \cdot L]^{-1}$, $\alpha_2 \sim [A \cdot L]^{-1}$, and $\beta \sim [L]^{\frac{5}{2}}$), subject to the constraint of eq. (38), the quantum metric:

$$\Psi = \int_{-\infty}^{\infty} \eta_{IJ}\chi^J dt + \int_{-\infty}^{\infty} \eta^\dagger_{IJ}\zeta^J dt \tag{B21}$$

where local gauge transformations in the bulk correspond to global gauge transformations on the boundary. Solving the above system of equations requires judicious gauge fixing with a deeper understanding of the causal reference frame that is the quantum metric. A strategy for solving the above is to first Wick rotate (which is fully justified by the boundary theory) and use Hopf coordinates.

Each of the above field equations has a positive and negative energy solution. The solution space of the coupled spinor fields has the energy quartet $(\pm E, \pm E)$, which is nothing but the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry and is interpreted as a pair of charged fermions oscillating forwards and backward in time. In some sense, points in quantum spacetime are a pair of spinor fields oscillating in imaginary cyclic time.

Herein lies a great clue: the boundary theory violates decoupling, so bulk solutions must exist where the UV mixes with the IR—exotic scenarios where quantum distortions of spacetime may be observable from a two-fermion-two-antifermion interaction. *The future of particle physics may, therefore, not involve building larger colliders to accelerate protons closer to c but smaller, compact linear colliders that collide matter and antimatter ions of mass m at low accelerations deep in the IR.*

Appendix C: The De Sitter limit and its dual

The calculation of the dS limit is similar in spirit to Jacobson's [83] and Verlinde's [84] thermodynamic derivation of GR. From the God's-eye view, the universal wavefunction is pure, and the entropy is zero. To rigorously calculate the entropy of the bulk as seen from the ant's-eye view, we need analytic control over the bulk field equations. I leave this to future work. However, if one assumes bulk observers can only access

$$Z_{\text{obs}} = e^S \sim e^{2\pi\mathcal{R}} \tag{C1}$$

qubits, where $\mathcal{R} = \frac{\lambda_\infty}{\lambda_0}$ defines the dimensionless radius of the conformal horizon that hides the cosmological singularity and its dual at future infinity, then we can find dS space in the following limit. First, take note that the boundary spinor components $\psi_L, \psi_R \in L^2(S^2_a \times S^1_\beta)$. Now express the densities $\rho_0(a)$ and $\rho_\infty(a)$ (given in II C) as a function of the surface area of the 2-sphere with the scale transformation $a \to \sqrt{\pi}a$:

$$\rho_0(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_0^2}\right)^{\phi/\pi} d\phi \tag{C2}$$

and

$$\rho_\infty(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_\infty^2}\right)^{-\phi/\pi} d\phi \tag{C3}$$

where $A = 4\pi a^2$ and the spin $s = \pm 1/2$. One can re-express the above as functions of the Bekenstein-Hawking entropy (the entropy from the ant's-eye perspective) by the identification:

$$S_\alpha = \frac{s^2 A}{\lambda_\alpha^2} = \frac{A}{4\lambda_\alpha^2} \tag{C4}$$

with the conformal indices $\alpha = 0, \infty$ where $\alpha = \infty$ is the $\mathcal{R} \to 1/\mathcal{R}$ dual. If it seems odd that this can be done, it should not. The black hole entropy is a geometric consequence of integrating over a boundary with an $S^2 \times S^1$ topology in Hawking and Gibbon's original paper [85]. The trick to constructing the dS limit is to fix the dark energy as a constant by setting

$$\Lambda = \frac{1}{\lambda_0 \lambda_\infty}. \tag{C5}$$

We now have two equivalent definitions of entropy, as seen from the ant's-eye view, and can calculate the radius of the cosmic event horizon in the semiclassical limit:

$$\begin{aligned} \frac{\pi R_{\text{CEH}}^2}{\lambda_0^2} &= 2\pi \frac{\lambda_\infty}{\lambda_0} \\ R_{\text{CEH}} &= \sqrt{2\lambda_0 \lambda_\infty} \\ &= \sqrt{\frac{2}{\Lambda}} \\ &\approx 14.26 \text{ billion ly.} \end{aligned} \tag{C6}$$

It is now straightforward to calculate the vacuum energy density (in reduced Planck units):

$$\rho_\Lambda = \rho_\infty = \frac{m_\infty c^2}{\lambda_0^3}. \tag{C7}$$

Now consider a flat acceleration surface with $2\pi k_B$ entropy per unit (Planck) area $1/\lambda_0^2$. Now multiply this quantity by the Hawking temperature $T(g)/\lambda_0$ per unit length of the accelerating surface. The result is the pressure

$$p_0 = \left(\frac{c^2}{G\lambda_0}\right) g, \tag{C8}$$

where the quantity in parentheses is an *area density* with units [mass][area]$^{-1}$. The physical construction is an accelerating membrane, like the surface of a glowing soap bubble. The above equation is the ideal gas law $pV = k_B T$ where $V = \lambda_0^3$ is the unit volume swept out by the membrane. The above is a thermodynamic derivation of $F = m_0 g$, Newton's second law. Now consider the dual acceleration surface with $4\pi k_B$ entropy per unit R_{CEH}^2 and repeat the above steps to obtain:

$$p_\infty = \left(\frac{\Lambda \hbar}{c\lambda_0}\right) g, \tag{C9}$$

which is just $F = m_\infty g$ when multiplied by λ_0^2 on both sides. The above pressures obey the ideal gas law, so we can construct the stress-energy tensor $T_{\mu\nu} = diag(\rho_\infty, p_\infty, p_\infty, p_\infty)$. In the UV limit,

$$\lim_{g \to -c^2/\lambda_0} T_{\mu\nu} = \begin{pmatrix} \rho_\Lambda & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{C10}$$

one obtains the vacuum stress-energy tensor $T_{\mu\nu}^{(\text{vac})}$. The sign on g distinguishes the expanding and contracting states. Indeed, the first fact we learned from the bootstrap is that energy and curvature are equivalent—the fundamental *assumption* of GR. There are not many options to equate $T^{(\text{vac})}_{\mu\nu}$ to curvature on the left-hand side. The second fact we learned is that the *form* of the laws of physics are independent of the chosen coordinate system—no coordinate system was used to solve eq. (2). The third fact we learned is that the fundamental charge of the vacuum, the gravitational charge m_∞, is equivalent to inertial mass. These deductions are none other than the equivalence principle and general covariance! Given these deductions, the simplest equations that follow are none other than Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} \tag{C11}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, and $\kappa = 1$ in natural units. Quantum mechanics—when assumed universal—naturally contains GR.

There is then the dual vacuum stress-energy tensor for the contracting state. As before, we can construct a stress-energy tensor $T_{\mu\nu} = diag(\rho_0, p_0, p_0, p_0)$ for an ideal gas. In the IR limit,

$$\lim_{g\to c^2/\lambda_\infty} T_{\mu\nu} = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \tag{C12}$$

one obtains a bizarre substance with the Planck energy density and a positive vacuum energy pressure $p = \Lambda$ (with the minus sign absorbed) that satisfies the equation of state for dark matter: $w = p/\rho_0 \approx 0$. Are

$$G_{\mu\nu} = \kappa T^{(\text{dual-vac})}_{\mu\nu} \tag{C13}$$

the classical field equations for dark matter? Does this explain how the cosmological constant Λ enters local galaxy dynamics through a universal acceleration scale $a_0 \approx cH_0 \approx c^2\Lambda^{1/2}$, and how something as small as a galaxy "knows" about the dS radius associated with the whole of the observable universe? Is the dark matter graviball solutions to the bulk field equations?

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[85] The Bekenstein-Hawking area entropy law is a geometric consequence of the fact that $\psi \in L^2(S_a^2 \times S_\beta^1)$. The action is computable in Hawking and Gibbons's original paper because the Euclidean section is non-singular: the entropy is evaluated on a region of a spacetime manifold bounded by some surface $r = r_0 > 2M$ with compact topology $S^2 \times S^1$ (i.e., a 2-sphere cross periodic time) [61]. The Ricci scalar vanishes in the Schwarzschild metric, so the action is determined only by the Gibbons-Hawking-York boundary term and is thus an integral over the boundary $S^2 \times S^1$. The world density $\rho = |\psi|^2$ is nothing but an integral over the boundary $S_a^2 \times S_\beta^1$ and precisely why one can write $\rho = \rho(S)$ with $S = \frac{A}{4\lambda^2}$.