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On the Origins of the Universe and the Nature of the Cosmological Singularity

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Abstract: I assume quantum mechanics applies to the entire universe and solve the Schrödinger equation exactly. I make three fundamental assumptions: 1) that the wave function of the universe exists (Everett's principle), 2) that the universe has zero-total energy (Guth's principle), and 3) that the *form* of the laws of physics remain unchanged on all scales (Turok's principle). The observational consequences are carefully worked out, and among them is a shocking conclusion: quantum gravity has no separation of scale. The problem of time is resolved by a flat space holographic duality between a timeless theory in Euclidean 3-space without a spin-2 field and an emergent theory in $3 + 1$ spacetime dimensions with a spin-2 field. The unitary evolution of the wave function of the universe explains the microscopic origins of spacetime, matter, black holes, dark energy, and the Bekenstein-Hawking area entropy law. Among the observable consequences is a spectacular prediction: black holes are dark energy composite objects, implying a cosmological coupling that violates the separation of scales.

Contents

1 Motivation

Our universe appears to have originated from a curvature singularity that obliterates the effective field theory (EFT) description of space, time, matter, and energy [1]. The cosmos seems to have been born into a spatially flat and thermal state of low entropy [2]. This grand puzzle demands new physics.

The standard EFT lore of beginning with classical fields and quantizing them has neither resulted in a theory of quantum gravity (QG) nor is it capable of describing physics at $t = 0$. The quantum-first program—the idea that spacetime and fields emerge from the wavefunction of the universe—has recently been proposed by Carroll [3, 4]. The Everettian view is austere: if the universe, as a whole, is quantum mechanical, then it must always obey the Schrödinger equation [5].

The central theme of this program is the Chewian bootstrapping [6] of the quantum-first program. How far can one get by demanding the self-consistency of Everettian quantum mechanics as it applies to the universe as a whole? In what is to come, I gain analytic control of the universal wave function through the inclusion of two additional general principles and work out the consequences.

Everett's principle of quantum universality

The wavefunction of the Universe exists and evolves unitarily, **always** obeying the Schrödinger equation:

$$\hat{H}\Psi = i\partial_t\Psi \tag{1.1}$$

This is to say: there exists no Heisenberg cut between quantum and classical systems. The universe, as a whole, is quantum. The wave function of the universe does not collapse; there exists no external observer that makes measurements from outside the system. The universe exists as a closed system independent of the observer and obeys the Schrödinger equation always. Everett's principle fulfills Einstein's desiderata of physical realism.

There are no special points or singularities in the state space of quantum mechanics (QM). The most general solution to the Schrödinger equation (with time-independent Hamiltonian $\hat{H}$) is written in terms of energy eigenstates:

$$\Psi(t) = \sum_n r_n e^{i(\theta_n - E_n t)} |E_n\rangle \, , \tag{1.2}$$

where the constant real parameters (r_n, θ_n) parameterize a circle $S^1 \in \mathbb{C}$ that defines the nth energy eigenstate $|E_n\rangle$. The circle is geodesically complete. There are neither singularities nor initial conditions in the state space. Each eigenstate is merely rotated by a complex phase with time extending infinitely into the past and future. Pure quantum states of n-level systems are points on the surface of an $(n-1)$-dimensional Riemann sphere $\mathbb{C}P^{n-1}$. (Strictly speaking, this clean geometric interpretation works for finite-dimensional Hilbert spaces. As it turns out, this can be generalized to the infinite-dimensional case in a non-trivial way.)

Universal Democracy as a guiding principle

Confronting inconsistencies in the description of natural law with general principles is a great tradition in fundamental physics and, historically, one of the most successful [7]. The most basic conflict between QM and general relativity (GR) is their mutually inconsistent definitions of energy (resp. time) [8]. Energy in GR is defined only for geometries that asymptotically approach a well-defined metric tensor at infinity. In QM, energy is defined by the time-translation symmetry of the Hamiltonian. Further still, the time reparameterization invariance of GR means the Hamiltonian $\hat{H}$ vanishes on physical states with compact geometries. This is a clue.

The Wheeler-DeWitt (WdW) equation, which follows from working in the ADM formalism of GR (where the Hamiltonian vanishes), is nonrenormalizable and generally pathological [9–12]. In the many decades since its formulation, numerous solutions (most notably the Hartle-Hawking no-boundary proposal [13, 14] and Vilenkin's tunneling proposal [15, 16]) have been put forth with little success [17–19]. In a detailed and rigorous analysis by Turok and friends, there are supple clues hinting at a deep simplicity of the universal wavefunction that has yet to be unraveled [17]. Echoing the philosophy of Geoffrey Chew and Neil Turok: Nature has somehow found the most economical way of instantiating self-consistency.

The conceptual theme of this work is the generalization of Chew's "nuclear democracy" to cosmology. I assume reductionism works in *both* directions. The guiding principle of **Universal Democracy** is this: the Universe, as a whole, has a Lemaîtrian particle-like representation governed by general principles required for the self-consistency of quantum universality. This research program can therefore be defined as a formal nomology, or deductive-nomological model [20] where the fundamental object is a finite set of axioms: $\Omega = \{\omega_i\}_{i=0,N}$, from which one attempts to derive all physical phenomena (with $i = 0$ as quantum universality).

By far, the simplest and most elementary inconsistency between GR and QM are their mutually inconsistent definitions of time and energy. The study of energy conservation in GR has a long and controversial history [21–24]. It is generally accepted among relativists and cosmologists that energy is conserved only locally. There exists no (non-trivial) global conservation of energy in GR [21]. A global non-conservation of energy is inconsistent with quantum universality. If QM is to apply everywhere and always, then energy must be conserved in cosmology. What is more, the simplest way to explain the origins of the universe is to assume it has zero-total energy. This is the trivial realization of energy conservation in GR: the positive mass-energy of the universe annihilates with its negative gravitational potential. A universe from nothing was first proposed by Tryon [25] and later explored by Guth in inflationary cosmologies [26, 27].

The Hamiltonian vanishes in gravity. This begs the question: can one flip the zero-energy postulate on its head and find gravity in QM as a consequence?

Guth's zero-energy postulate

The total energy of the Universe is exactly zero. The Hamiltonian annihilates the universal quantum state. The Schrödinger equation becomes time-independent and assumes the form:

$$\hat{H}\Psi = 0 \tag{1.3}$$

where Ψ is the wavefunction of the universe and $\hat{H}$ is the universal Hamiltonian. Time is, therefore, not fundamental and must somehow emerge.

How one proceeds to find new principles is largely a matter of taste. Guth's postulate brings us halfway to calculation. We have two moves: guess the potential V in the Hamiltonian or the form of Ψ. The former is an assumption of the field content of the theory. The latter is an assumption of the *form* of the laws of physics. Assuming the form of Ψ is most general and plucks a particular potential V out of an ill-defined landscape of unbounded size. There are infinitely more ways of guessing V and solving for Ψ than guessing the form of Ψ and solving for V. Echoing Turok: the Universe as a whole—like a point particle—is simple, and simple phenomena demand simple explanations. Without assuming any particular field content, what principle ensures the microscopic degrees of freedom have kinetic and potential energy eigenstates that exactly cancel?

Let us now quest in the pursuit of clues. These clues are the materials we forge our general principles from—and from these principles, we calculate.

Compendium of clues

Conformal symmetry and the separation of scales

The study of scale symmetry in physics has very old roots [28]. The classical action of the standard model is consistent with scale symmetry if the Higgs mass term is dropped, suggesting its mass could emerge from the vacuum expectation value (vev) of a new scalar field in a fully scale-invariant theory [29]. Scale invariance lies at the heart of the renormalization group (RG) flow. It is the critical insight enabling the standard model to make relevant predictions, despite sweeping its infinities to "irrelevant" scales below some cutoff. There is no fundamental principle that explains why renormalization works or why it fails for gravity.

Assuming the scales separate into relevant and irrelevant regimes is motivated by calculation. The assumption is incredibly powerful at low-energy scales and enables calculation in the standard model of particle physics. The EFT paradigm, however, has always been a low-energy approximation to a deeper theory, a theory valid on *all scales*. Remarkably, there is observational evidence that gravity violates the separation of scales. The fact that the cosmological constant Λ enters local galaxy dynamics through a universal acceleration scale $a_0 \approx cH_0 \approx c^2\Lambda^{1/2}$ is startling [30]. How is it that something as small as a galaxy "knows" about the dS radius associated with the whole of the observable universe? The data tell us that gravity has no separation of scale and is somehow Machian. This is a clue.

Deeper still, there is every reason to believe gravity must violate the separation of scales to remain consistent with unitarity. One can see this from the *split property*. In non-gravitational theories, the algebra of observables on a Cauchy slice can be separated into a subalgebra on the bounded region and a commuting subalgebra associated with its complement [31, 32]. As Raju and friends beautifully demonstrate, the split property does not hold in gravity, i.e., gravity must violate the separation of scales to maintain consistency with unitarity [33–35]. Their papers show where Hawking went wrong with the black hole information loss paradox. For gravity to be reconciled with quantum theory, all physical information of the universe must live at the boundaries of the world [36, 37]. The *holography of information*—together with the *observation* of the UV/IR mixing of galaxy dynamics through (the completely nonlocal) acceleration scale a_0—are profound clues to the quantum nature of gravity.

UV/IR mixing, T-duality, and CPT symmetry

UV/IR mixing appears to be a model-independent feature of quantum gravity. Numerous thought experiments on quantum uncertainty with gravity (including many stringy constructions [40, 41]) lead to a generalized uncertainty principle (GUP) that mixes small, and large scales [38, 39]. UV/IR mixing in quantum gravity can take many forms, e.g., GUPs, non-commutative spacetime geometries, and the T-duality [42]. Across the theoretical landscape, there are instances of UV/IR mixing occurring in an unusually diverse number of places: e.g., in little string theory [43], in topological quantum field theories (TQFTs) like Fracton qubit models [44], and, most surprisingly, in CPT (or mirror) symmetric cosmological models [45–47]. Given the observational data in the context of the theoretical landscape, one must ask: is semiclassical gravity a valid approximation in the low curvature regime? In a rigorous series of papers by Turok and friends [17, 18], the answer appears to be an emphatic **no**. A quantum de Sitter (dS) spacetime is in doubt from uncontrollable divergences arising from the UV and IR mixing. Curiously enough, the Swampland program is coming to the same conclusion: meta-stable dS vacua do not appear to exist in quantum gravity [48]. One must wonder if the Hubble tension [49] (and others [1]) are data telling supporting the view that it makes no sense to quantize dS space, that nature is quantum from the get-go, that dS is a classical approximation to a UV/IR complete quantum cosmology. Herein lies a deep clue: quantum field theory tells us that the dark energy is dominated by the UV vacuum energy, yet it is also what determines the IR cutoff by limiting our causal horizon and thus the largest scale anyone shall ever see [17].

More surprising still, in a recent series of beautiful papers by Turok and Boyle, the Friedmann equation (with radiation, non-relativistic matter, a cosmological constant, and arbitrary spatial curvature) is symmetric under the exchange of early-time (radiation dominated) and late-time (dark energy dominated) phases [45, 50, 51]. This UV/IR duality implies the existence of a CPT symmetric (or mirror) universe where the Bang is a surface of CT symmetry [45–47, 50, 51]. The general solution for the scale factor $a(\tau)$, with conformal time τ, is an elliptic meromorphic function that is doubly periodic in the complex τ-plane, with one period oriented along the real τ-axis and the other is oriented along the imaginary τ-axis. The general solution to the Friedmann equation in fully realistic cosmologies is thus a wrapping of the complex 2-torus. The periodicity in imaginary time allows one to calculate the thermodynamic gravitational temperature and entropy of the spacetime, just as Gibbons and Hawking did for black holes and de Sitter space [52, 53]. Remarkably, Turok and Boyle found that the gravitational entropy favors spatially flat, homogeneous, and isotropic universes with a small positive cosmological constant. These are profound clues to the microscopic structure of quantum gravity.

It is difficult not to draw a comparison between Turok and Boyle's solution to the Friedmann equation with a string winding around the 2-torus (the simplest Calabi-Yau manifold). This result, together with the interplay between the UV and the IR discovered in earlier works [17, 18, 54, 55], seem to be converging along some of the same ideas in the Swampland program [56]. In string theory, the extreme limits are dual: pulling field space to asymptotic infinity gives a tower of light states. Pulling field space to zero gives a tower of heavy states. The T-duality (e.g., inverting the radii of the torus a string wraps around) predicts the extreme limits are the same, the heavy state is a composite of the light state and goes like $m \sim \Lambda^{\alpha}$ for some power α [56, 57]. In other words, the universe in the far future (the deep IR) is dual with its far past (the deep UV).

Holography and the unreasonable ubiquity of power-law phenomena

The primordial fluctuations in the cosmic microwave background are (nearly) scale-invariant and admit a power-law form: $A_s f^{n_s - 1}$ [2]. The energy densities of the field content in the observable universe scales as a power-law: $\rho_i = \rho_{i0} a^{-n_i}$ for some fixed n_i. Power-like phenomena are ubiquitous in thermal systems near a phase transition. Many such systems all share the same scale-invariant limit under the RG flow. In the AdS/CFT correspondence, a holographic gauge gravity duality (and one of our deepest clues to the nature of quantum gravity), the equations of motion have power-like solutions [58, 59]. Further still, in biadjoint scalar field theory, a gauge gravity duality known as the double copy, the equations of motion have power-like solutions [60]. Power laws are unreasonably ubiquitous in the character of physical law [61]. They form what I call the **inescapable pattern**.

And, finally, to reinforce what is to come: if there exists no Heisenberg cut, then the *form* of the laws of physics do not change with scale. The universe, on all scales, is quantum.

2 Turok's principle of fundamental scale invariance

The form of the laws of physics remains unchanged at all scales. The outcomes of experiments do not change as the size of the universe changes. This is to say the laws are geodesically complete and that Nature has no fundamental scale. It follows that Ψ is scale-invariant and takes the form of a power-law: $\Psi \sim a^n$, where a is the universal radius of curvature (or scale factor) and n is an arbitrary parameter assumed (at the moment) to be an integer. One may now immediately write down the following timeless Schrödinger equation:

$$\left(\frac{-\hbar^2}{2M} \frac{\partial^2}{\partial a^2} + \frac{\hbar^2}{2M} g^{ij} R_{ij} \right) \Psi = 0 \tag{2.1}$$

where M is the mass of the universe in the Lemaîtrian picture, $R_n = g^{ij} R_{ij} = \frac{n(n-1)}{a^2}$ is the scalar curvature of the n-sphere, g_{ij} is its metric tensor, and R_{ij} the Ricci curvature tensor. For simplicity set $\hbar = 1$, define $\partial \equiv \frac{\partial}{\partial a}$, and multiply the factor of $1/2M$ out. The Hamiltonian $\hat{H}_n = -\partial^2 + R_n$ is a scalar curvature operator: $\hat{H}\Psi = 0$ corresponds to a spatially flat universe.

Equation (2.1) is far from the whole story, yet one reads off an unambiguous prediction: **the Universe is spatially flat**. The equivalence between curvature and energy was the conceptual starting point for Einstein's GR. As it turns out, the aforementioned principles provide a deeper explanation for the equivalence.

The cosmological information paradox

The timeless equation is a beautiful paradox. The equation occupies a liminal space between geometry and quantum mechanics. The Hamiltonian can be interpreted as 'living' at the asymptotic boundary of a flat n-sphere with an infinite radius. As stated, Ψ is real; it has no complex phase. There is neither a global $U(1)$ phase symmetry nor a Born rule. The equation is valid for both signs of n, but the squared-modulus $|\Psi|^2$ diverges for $n > 0$ as $a \to \infty$ and for $n < 0$ as $a \to 0$. Most intriguingly, under a change of variables, the equation is dual to the reduced radial equation of a *free particle* in elementary quantum mechanics. Turok's principle breathes fire to the particle-universe duality that is Universal Democracy.

The timeless equation has two equivalent interpretations. On the one hand, it describes a zero-energy (infinitely degenerate) Lemaîtrian point-like particle, or Cosmon, with charge M and position a from the origin, sourcing equal and opposite scalar potentials that self-annihilate. On the other, it describes a charge M sourcing the scalar curvatures R of an n-sphere and an n-hyperboloid, each with scale a that annihilate. The fundamental space forms: the n-spheres, hyperbolic n-space, and Euclidean n-space all emerge from eq. (2.1). There is neither a fixed number of spatial dimensions nor a concept of time, only a (non-unitary) energy-curvature relation $\hat{H}$. Turok's principle further implies the corresponding Hilbert space of states is infinite-dimensional. Nature has no fundamental scale; the universe can have any arbitrary size. This gives one hope in recovering Lorentz invariance and (approximately) local quantum field theories with gravity.

There is a trivial Z_2 symmetry $a \to -a$ that becomes important later in the story. Now observe the following non-trivial Z_2 action: sending $n \to -n$ sends $\Psi(a) \to \Psi(1/a)$ and takes the Hamiltonian to $\hat{H}_{n+1} = -\partial^2 + R_{n+1}$, where $R_{n+1} = \frac{n(n+1)}{a^2}$ is the scalar curvature of an $(n+1)$-sphere. The exchange symmetry $-n \leftrightarrow n+1$ and its consequence of taking $\Psi(a)$ to $\Psi(1/a)$ is reminiscent of the T-duality in string theory. Further still, insisting this additional Z_2 action is a symmetry of the system, together with the perspective that the Hamiltonian lives at the asymptopia of an infinite sphere, plus the duality between *position* and *scale*—screams celestial holography [62, 63].

The timeless equation is not quite consistent with quantum universality: there is no unitary evolution of the wave function. Information is lost at the singularity *and* asymptotic infinity. This leads to the conjecture that the resolution to the cosmological information paradox is a quantum theory of spacetime. Or as I like to call it, the physics of nothing at $t = 0$.

Stringy dualities and the problem of time

The duality between eq. (2.1) and the reduced radial equation is rather curious. There is a direct correspondence between R_{n+1} and the effective centrifugal potential with $n \leftrightarrow l$ where l is the centrifugal term. The potential is repulsive for $l > 0$. The timeless equation is surreptitiously suggestive of *acceleration through an internal rotational degree of freedom.* This is a pertinent clue: without time, there must exist some other degree of freedom to integrate over for unitary evolution. What is more, there is a pair of states Ψ_n and Ψ_{n+1} that must become complexified. Spinors (pairs of complex numbers) facilitate spacetime computations in general. There always exists a map from spinors of order n to tensors of order $2n$.

A Calabi-Yau is nothing but a $2n$-dimensional manifold whose holonomy group either reduces to or is contained in $SU(n)$. These spaces are usually defined by solutions of the Dirac equation without potential: $D\Psi = 0$, where D is the Dirac operator. The non-trivial Z_2 symmetry implies $n = 2$ and that eq. (2.1) takes a spinorial form. Our universe has three spatial and a single time dimension. To restore unitarity to eq. (2.1) the dimension n must be fixed. The case of $n = 2$ is special. This case corresponds to $SU(2)$, the double cover of $SO(3)$, the rotational symmetry group of Euclidean 3-space. The quantum states correspond to the potential curvatures of the 2-sphere and 3-sphere. These spheres are uniquely related by the Hopf fibration and arise naturally in the description of two-level systems (i.e., qubits) [64]. Given the Hamiltonian $\hat{H}_n$ and its partner $\hat{H}_{n+1}$, there is exactly one principal fiber bundle—the Hopf bundle—where the base space and total space differ by a single dimension. Intriguingly, the $Z_2 : n \to -n$ action uniquely provides the base space S^2 and the total space $S^3 \cong SU(2)$. The Hopf bundle is a $U(1)$-fiber over S^3, so the only missing ingredient is the $S^1 \cong U(1)$ fiber. It is economical to identify this fiber with the missing global $U(1)$ phase symmetry.

Quantum information all the way down?

The fiber bundle structure of a qubit is the Hopf fibration. The proposed postulates yield a Hamiltonian of a timeless system living at the asymptotic boundary, $S^3(a = \infty) \cong \mathbb{R}^3$ and

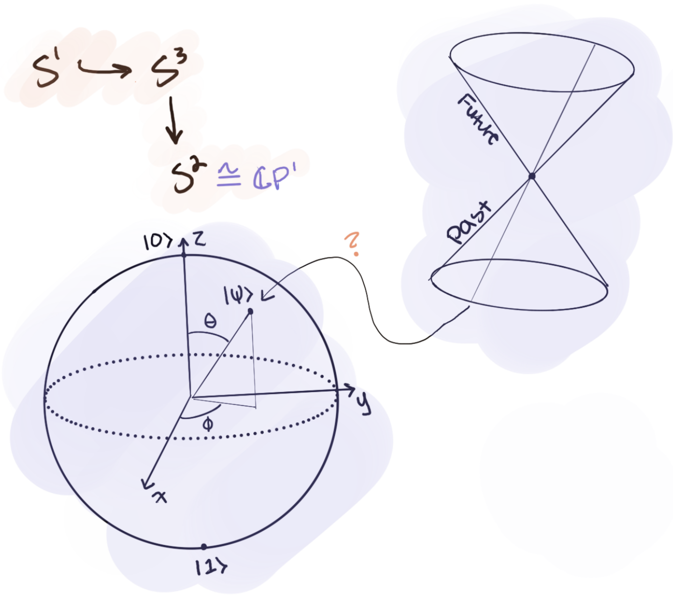

Figure 1. The curious duality between two-level systems (i.e., qubits) and Minkowski space. Bundle theoretically, the Bloch or Riemann sphere is a $U(1)$-fiber over a unit 3-sphere in two complex dimensions (i.e., the Hopf bundle). With the exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, one can identify Minkowksi space with the Riemann sphere. Flat spacetime, therefore, has the same fiber bundle structure of quantum information.

a non-trivial Z_2 symmetry, $n \to -n$ that leaves asymptotic infinity interchangeable with the cosmological singularity. With a $U(1) = e^{i\phi}$ phase, the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$ is unlocked. The 3-sphere therefore 'lives' on the surface of S^2 by the identification of phase circles in S^3 with points on S^2. There always exists a representation of $S^3 \hookrightarrow \mathbb{C}^2$ which implies the existence of a complex 2-spinor:

$$\Psi(a,\phi) = \begin{pmatrix} \psi_n \\ \psi_{n+1} \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{2.2}$$

with $a = \pm\sqrt{x_1^2 + x_2^2 + x_3^2}$ and an angle ϕ that parametrizes the S^1, mapping points in the state space to points on a 2-sphere in $\mathbb{R}^3$. If our S^2 and S^3 are unit spheres, one can identify them as the Bloch sphere and Riemann sphere, respectively. The mapping from the unit 3-sphere in a two-dimensional complex Hilbert space $\mathbb{C}^2$ (otherwise known as the complex projective line $\mathbb{C}P^1$ or the complex plane $\mathbb{C}$ with a point at infinity) to the Bloch sphere is the Hopf fibration, with each ray of spinors mapping to a point on the Bloch sphere. Turok's principle tells us our spheres can have any arbitrary size, but there is an obvious problem: all geodesic paths terminate at $S^3(a=0)$. Curvature divergences in GR are effectively moved to the wavefunction in eq. (2.1). The equation is not fully consistent with Turok's principle, hence the paradox.

The problem of time is deeply puzzling, but consider the following clue: the fundamental representations of the Lorentz group contain a pair of complex 2-spinors $\mathbf{2}$ and $\overline{\mathbf{2}}$, i.e., two fermionic directions. The tensor product $\mathbf{2} \otimes \overline{\mathbf{2}} = \mathbf{3} \oplus \mathbf{1}$ is the adjoint representation and can be identified with either Minkowski space or a spin-1 field. This is the motivation of a supersymmetry between spatial and fermionic directions. One can see this by exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, which allows one to identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$. The celestial sphere can therefore be identified with complex 2-spinors modulo a rescaling and mapped to the Riemann sphere. One can interpret this as time emerging in the adjoint representation from a pair of fermionic directions in the fundamental representation.

The duality between $\hat{H}_n$ and its partner $\hat{H}_{n+1}$ is seductively supersymmetric for $n = 2$. To see this consider the following. The Hopf fibration has as the global symmetry group $U(2) = SU(2) \ltimes U(1)$, which *locally* decomposes into the direct product $SU(2) \times U(1)$. The action of the unitary group $U(2)$ on $\mathbb{C}^2$ leaves the 3-sphere invariant, sending fibers to fibers as it commutes with the $U(1)$ action [64]. This story seems to be leading to some notion of *superholography*: the existence of a fermionic theory without gravity in N dimensions equivalent to a bosonic theory in $N+1$ dimensions with gravity (big N being the embedding dimension of the spheres). It is only in the $N = 3$ case can one identify the Hopf bundle: the S^1 fiber is the global phase symmetry, R_n is the scalar curvature of the base space S^2, and R_{n+1} is the scalar curvature of the total space S^3.

3 Holography of information, T-duality, and the Bang as a mirror

The cosmological information paradox is more puzzling still. How can it be that something comes from nothing? We exist in a world with time and energy, so how can a timeless equation of 'motion' give rise to the richness we see? A consistent version of eq. (2.1) cannot be the final story. Space has emerged holographically, but time, energy, and matter must somehow too emerge. What breathes fire into the void and animates it into a violent sea of fluctuating quantum fields? If one thinks about this problem long enough, there must be a pair of equations of motion: one on the asymptotic boundary without time, with two-component spinor solutions, and the other in the bulk, with time, and four-component spinor solutions that decompose into a *pair* of two-component spinors. (Recall the fundamental representation of the Lorentz group is a pair of complex 2-spinors). The map between the boundary (the fundamental rep., i.e., fermions) and the bulk (the adjoint rep., i.e., bosons) is the Hopf fibration. The fibration has symmetry group $U(2)$, a compact presentation of Minkowski space [65].

Something from nothing is inconsistent with unitarity. An ultimate beginning is thus inconsistent with Everett's principle of quantum universality. This leads to the inescapable conclusion: the dynamic field content must be (approximately) local and relative to not one, but *two* observers: a t and $-t$ pair that self-annihilate at the timeless boundary. Stated another way, the full $Z_2 \times Z_2$ symmetry demands the existence of a mirror world that annihilates with the one we observe. The boundary equation instantiates a pair of two-component spinor solutions to the bulk equations of motion: one that evolves forward in time (with positive energy) and the other that evolves backward in time (with negative energy). More beautiful still, Turok's principle leads one to believe the Bang is a mirror (a surface of CT symmetry) that separates an infinite sea of positive energy states from an equally infinite sea of negative energy states, i.e., the Dirac sea.

The geometry of physical information

Recall our first clue: the Hamiltonian vanishes for compact geometries only. This fact, together with the fact that information is lost at infinity and zero, implies a compactification with a transition map that identifies zero and infinity as the same point. That is, the inclusion of a point at infinity in the ambient space with a non-trivial topology that "glues" the singularity to asymptotic infinity. Collecting the clue forms, the geometry of physical information (GPI) satisfies the following constraints.

- The boundary Hilbert space of states is a smooth, complex, and compact manifold.
- The Hopf fibration acts as a duality map between the boundary and bulk state spaces.
- The fibration between the boundary and the bulk has a global symmetry group: $U(2) = SU(2) \ltimes U(1)$ that decomposes *locally* into the direct product $SU(2) \times U(1)$ in the bulk.
- There exists a transition map (or T-duality) that identifies the cosmological singularity with the boundary at infinity.
- The manifold has a global left-right (or mirror) symmetry.
- The space is conformally flat.

A natural starting point is the one-point compactification of the reals, i.e., the real projective line $\mathbb{R} \cup \{\infty\} = \mathbb{R}P^1$. Take two circles $S^1_a \cong \mathbb{R}P^1$ and $S^1_\beta \cong \mathbb{R}P^1$, orient them in opposite directions, embed them in separate complex planes $\mathbb{C}$, and glue each point together. Define this new space as $\mathbb{R}P^{(1,1)} := (\mathbb{R}P^1 \times \mathbb{R}P^1)/ \sim_\pi$ with the equivalence relation $(a, \beta) \sim_\pi (1/a, 1/\beta)$. See Fig. 2 for a drawing. The scale factor a and the inverse $1/a$ are real numbers, while β and $1/\beta$ are imaginary. As we soon prove, $\mathbb{R}P^{(1,1)} \cong T^2$ where the 2-torus $T^2 = S^1_a \times S^1_\beta$ is the conformal Clifford torus.

$\mathbb{R}P^{(1,1)}$ is a spin manifold. Rotate a vector in $\mathbb{R}P^{(1,1)}$ by 2π and it becomes negative. The "vectors" in $\mathbb{R}P^{(1,1)}$ are not vectors at all; they are spinors. Our construction identifies the scale circle S^1_a with the circle S^1_β. It is known $\mathbb{R}P^1$ is the boundary at infinity of hyperbolic 2-space $\mathbb{H}^2$. Hence $\mathbb{R}P^{(1,1)}$ is the boundary at infinity of the upper *and* lower halves of $\mathbb{H}^2$. The implied light cone structure of $\mathbb{R}P^{(1,1)}$ is pictured in the upper left panel of Fig. 3. The boundary of future and past infinity are identified, implying a T^2 topology that double covers the suppressed S^2 sitting inside the light cone. The double sheetedness follows from the oppositely oriented fibers $S^1_a = \exp(\pm i\phi)$ and $S^1_\beta = \exp(\mp i\phi)$. For convenience one labels the circle oriented from 0 to ∞ as the complex phase $e^{i\phi_L}$, and the circle oriented from ∞ to 0 as $e^{i\phi_R}$ with $\phi_R = -\phi_L$, as depicted in the left panel of Fig 2. Curvature vanishes everywhere, so this 2-torus is flat and is embedded in $\mathbb{C}^2$. It is obvious β must be the coolness or inverse temperature. When the universe has zero

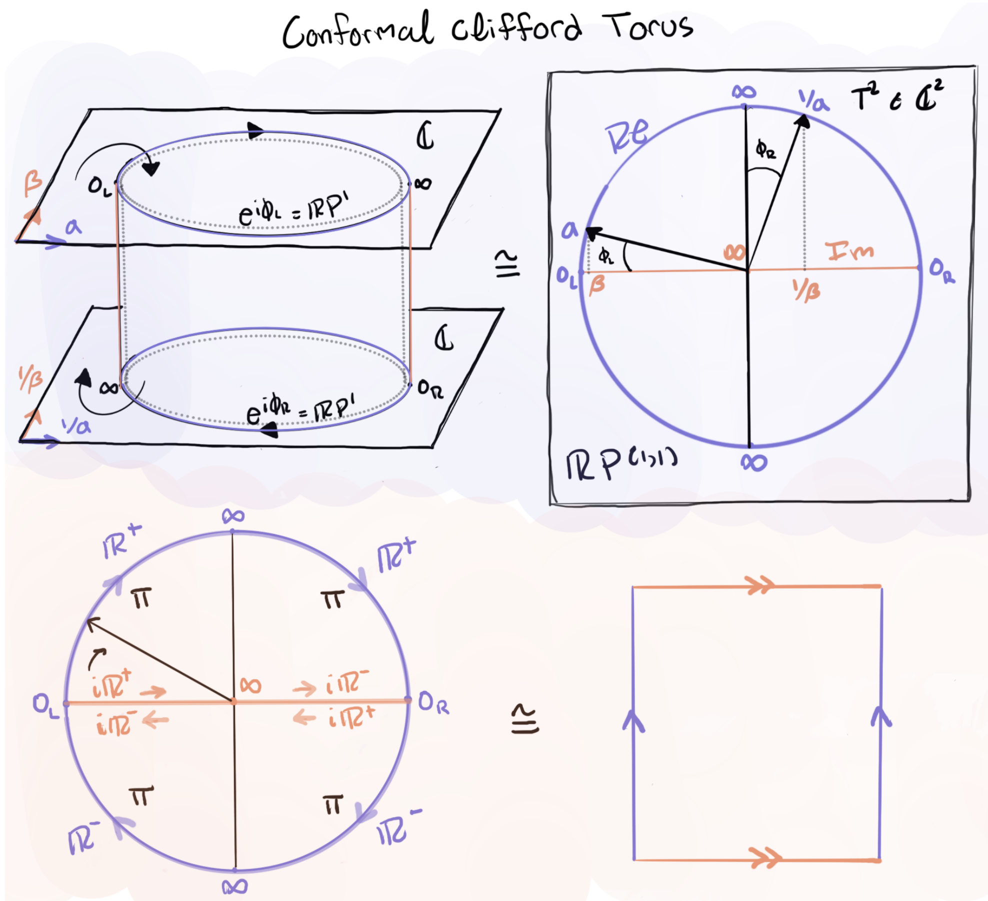

Figure 2. Upper panel: construction of the conformal Clifford torus by embedding two oppositely oriented real projective lines $S^1_a \cong \mathbb{R}P^1$ and $S^1_\beta \cong \mathbb{R}P^1$ in separate complex planes $\mathbb{C}$, followed by the identification $(a, \beta) \sim_\pi (1/a, 1/\beta)$ where $\sim_\pi$ denotes (a, β) can be rotated into $(1/a, 1/\beta)$ by a rotation of π. Lower panel: $\mathbb{R}P^{(1,1)}$ is a spin manifold (a rotation of 2π sends $a \to -a$). The line that runs from 0_L to ∞ and back to 0_R is really a circle. The left and right-hand sides of the line at infinity live in separate complex planes $\mathbb{C}$ (the L and R subscripts on zero are for booking keeping only). The conformal Clifford torus can also be seen as two congruent tori tiling $\mathbb{C}$ with the fundamental parallelogram.

size it is infinitely hot—this is the cosmological singularity. It is now obvious that $\beta = it$ must be true. Spacetime curls into a conformal torus and is identified with the global $U(2)$ symmetry of the Hopf fibration between the curvature Hamiltonians $\hat{H}_n$ and $\hat{H}_{n+1}$. The cosmological singularity $(a, \beta) = (0, 0)$ is a conformal zero: cosmological evolution is the wrapping of the conformal Clifford torus.

Another way of thinking about the infinite Clifford torus: define a finite Clifford torus as the Cartesian product of the closed imaginary interval $[0, \beta]/\sim$ and the closed real interval $[0, a]/\sim$, with an equivalence relation $\sim$ that glues the endpoints together, i.e., the fundamental parallelogram $I_a \times I_\beta$. Each interval is a circle of diameter a and β. In the limit $\beta \to \pm\infty$ and $a \to \pm\infty$, each circle is equivalent to $\mathbb{R}P^1$. Hence, with the S^2 suppressed, $\mathbb{R}P^{(1,1)}$ is the boundary at infinity of all past and future light cones.

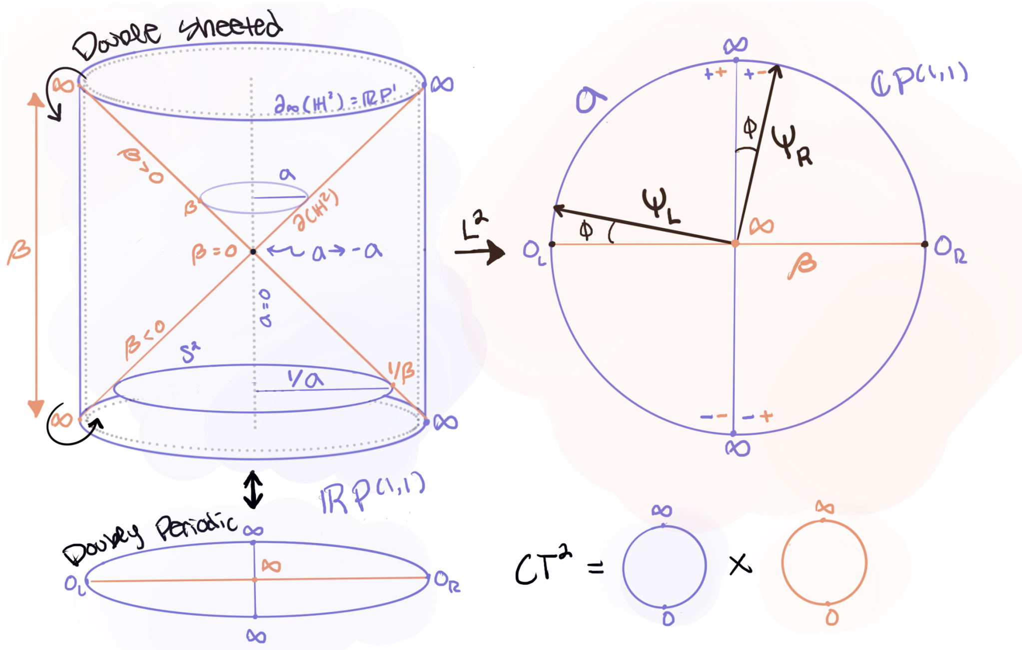

Figure 3. Left panel: $\mathbb{R}P^{(1,1)}$ is the boundary at future and past infinity with an implied two-sheeted light cone structure. The S^2 is suppressed: along every value of β is an S^2 of radius a. The future and past boundaries at infinity are identified: the S^2 expands through future infinity to the infinite past, contracts to a conformal zero, and turns inside-out, sending $a \to -a$. Identifying a full covering of the light cone as a rotation of 2π and it is clear the double-periodicity of $\mathbb{R}P^{(1,1)}$ is the same as the double-sheetedness of the cylinder. Right panel: $\mathbb{C}P^{(1,1)}$ is topologically identical to $\mathbb{R}P^{(1,1)}$. Each spinor component $\psi \in \mathbb{C}P^{(1,1)}$ describes the quantum state of the S^2. Hence $\psi \in L^2(S^2_a \times S^1_\beta)$. Each quadrant of $\mathbb{C}P^{(1,1)}$ has a cigar shape that pinches off to zero at $a = 0$ and becomes infinite in radius at $a = \infty$.

Definition 3.1 (Conformal Clifford Torus)**.** Let the Conformal Clifford Torus be defined as follows:

$$CT^2 \equiv \lim_{a \to \infty} S^1_a \times_{Z_2 \times Z_2} \lim_{\beta \to \infty} S^1_\beta \tag{3.1}$$

where $CT^2 \hookrightarrow \mathbb{C}^2$ with the Cartesian product $\times_{Z_2 \times Z_2}$ denoting the symmetry $a \to -a$ and $\beta \to -\beta$.

At every point along β, there is an $S^2(a)$ sitting inside the light cone. Over the surface of $S^2(a)$ is the Hopf bundle that encodes a 3-sphere living in $\mathbb{C}^2$. Normalizing every spinor in $\mathbb{R}P^{(1,1)}$ to unit length defines a Hilbert space (see the right panel of Fig 3) where the left-handed state vector ψ_L maps to points on the surface of $S^2(a)$ and the right-handed state vector ψ_R maps to points on the surface of $S^2(1/a)$. To see how a local Minkowski spacetime emerges in the bulk from $\mathbb{C}P^{(1,1)}$, consider the following. By the exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, one may identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$ (the symmetry group of the Hopf fibration),

$$q = \sigma^\mu x_\mu = I x_0 + \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3. \tag{3.2}$$

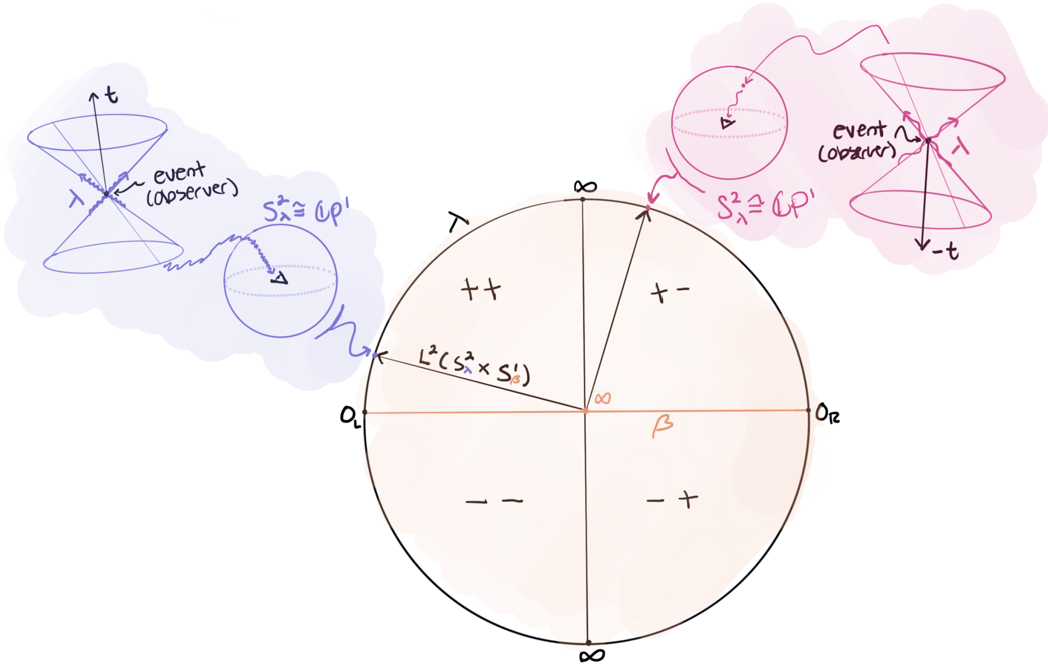

Figure 4. The bulk interpretation of $\mathbb{C}P^{(1,1)}$ goes as follows: at every point along the space is a celestial sphere $S^2_\lambda \hookrightarrow \mathbb{R}^3$ with a characteristic scale λ and the equivalence relation $a \sim \lambda a$ (for all real numbers $\lambda \neq 0$) that normalizes the 2-sphere to unit length. One can always find a λ, normalize the base space to unity, and identify the Bloch sphere with the celestial sphere.

Here the Lorentz metric norm is $||x_\mu|| = 2\det(q)$. From this identification one finds that x_μ is lightlike precisely if there is a complex 2-spinor $\mathbf{2}$ such that $q = \mathbf{2\overline{2}}$. The celestial sphere can therefore be identified with complex 2-spinors modulo a rescaling. This identifies the celestial sphere S^2 with the Riemann sphere $\mathbb{C}P^1$. Hence the claim spacetime emerges from the global $U(2)$ symmetry of the fibration between the scalar curvature Hamiltonians $\hat{H}_n$ and $\hat{H}_{n+1}$, which takes 2-component spinor states Ψ_i on the timeless boundary to 4-component spinor states Ψ_μ in the dynamic bulk.

The Ψ_i are elements of the square-integrable space $L^2(S^2_a \times S^1_\beta)$. At each point along $\mathbb{C}P^{(1,1)}$ (except at zero and infinity) there is a 2-sphere with the equivalence relation $a \sim \lambda a$ (for all real numbers $\lambda \neq 0$) that normalizes the 2-sphere to unit length. The relativistic quantum information of the boundary contains all possible future and past light cones of an expanding flat space bulk. From the god's eye view, everything that can happen, happens everywhere all at once in $\mathbb{C}P^{(1,1)}$. There is no Ptolemaic center of the Hilbert space; no state granted privilege of reality over others. The universe is a perfect democracy.

The following theorem proves the torus (pictured as the cylinder with each side identified) in the left panel of Fig. 3 is homeomorphic to $\mathbb{R}P^{(1,1)}$. This theorem is the first step in establishing a flat space holographic correspondence between the boundary $\mathbb{C}P^{(1,1)}$ and a bulk Minkowski space.

Theorem 1 (Correspondence). $\mathbb{R}P^{(1,1)} \cong CT^2$, *and without loss of generality,* $\mathbb{C}P^{(1,1)} \cong CT^2$.

Proof. See figure 3. Beginning at the cosmological singularity $(a, \beta) = (0, 0)$, move up to the boundary and expand the S^2 to future infinity. Wrap around to the boundary at past infinity and contract S^2 back to the cosmological singularity. Crossing the singularity, turn the S^2 **inside-out** by sending $a \to -a$, and return to future infinity. Repeat the previous two actions and finally end at the cosmological singularity turning the S^2 **outside-in**. Identify each action with a rotation of π. The actions *crossing* $0_R \leftrightarrow$ *turning inside-out* and *crossing* $0_L \leftrightarrow$ *turning outside-in*. Thus the double-sheetedness of the cylinder is the same as the double-periodicity of $\mathbb{R}P^{(1,1)}$. This identification completes the proof. □

Remark 3.1. It is easy to verify from Fig. 3 that the pair (a, β) have sign sequence $(++, +-, -+, --)$ corresponding to the double covering of S^2 (each entry is a rotation of π). For $\beta < 0$, the S^2 contracts towards the cosmological singularity and decreases in entropy as the system becomes more energetic. The contracting phase is a perfect mirror image of the expanding phase. This is a consequence of the system's derived $Z_2 \times Z_2$ symmetry.

The boundary equations of motion

The conformal Clifford torus is a spin manifold. Thus the differential operator on the boundary is constructed with the Clifford algebra $C\ell_3(\mathbb{R})$, which is isomorphic to the Lie algebra $\mathfrak{su}(2) = \{i\sigma_1, i\sigma_2, i\sigma_3\}$ that spans the space of observables of the two-dimensional complex Hilbert space ($\Psi \in \mathbb{C}^2$) representing half-integer spin along the ith coordinate axis in $\mathbb{R}^3$. The boundary equations of motion are therefore governed by the Dirac-spin operator:

$$D = i(\sigma^1\partial_1 + \sigma^2\partial_2 + \sigma^3\partial_3) \tag{3.3}$$

where the σ's are the Pauli matrices. The boundary equations of motion, therefore, take the most general form:

$$(D^2 + V)\Psi = 0 \tag{3.4}$$

where the potential V becomes a real symmetric 2×2 matrix and Ψ is a complex 2-spinor. In the limit that $V \to 0$, the boundary equation reduces to the Dirac equation and its solutions to harmonic spinors. There are two equivalent ways of solving the boundary equations: one geometrical and the other algebraic. Our approach has not changed: we constrain the form of Ψ from symmetry and solve for V. The most general form the components of Ψ can take follows from enforcing the $Z_2 \times Z_2$ symmetry:

$$\psi_n \sim a^n + (-a)^n, \qquad \psi_{n+1} \sim a^{-n} + (-a)^{-n}. \tag{3.5}$$

It is now clear from figures 3 and 4 that $\phi = n\pi$ where n is continuous and lies in the interval $[0, 2)$. It immediately follows that

$$\Psi(a, \phi) = \begin{pmatrix} (1 + e^{i\phi})a^{\phi/\pi} \\ (1 + e^{-i\phi})a^{-\phi/\pi} \end{pmatrix} \tag{3.6}$$

which is a solution to eq. 3.4 in the reduced radial (or scale) basis where $D^2 = -\partial^2 I_2$, $\partial = \partial/\partial a$, and $V = diag(R_n, R_{n+1})$. The Hamiltonian is simply $\hat{H} = diag(\hat{H}_n, \hat{H}_{n+1})$. I refer to Ψ as a *biconformal spinor* and claim it is the fundamental representation of Minkowski space. The components ψ have the following modular invariance: $\psi(a^{-1},\phi)\psi^*(a^{-1},\phi) = \psi(a,-\phi)\psi^*(a,-\phi)$, which is the same as $Z(T) = Z(1/T)$ where Z is the partition function and T is the temperature (see e.g., the conformal bootstrap [66]). The Hamiltonian vanishes, so $Z = e^S$ where S is the entropy of the boundary. The biconformal spinor is a pure state, i.e., $S = 0$ and hence $Z = 1$ with infinite degeneracy. There are **no initial conditions**; unitary evolution is simply a phase rotation of Ψ.

The algebra of physical information

Another way of finding the biconformal spinor is purely algebraically. Every point in $\mathbb{R}^3$ has the following matrix representation:

$$Q = \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3 \tag{3.7}$$

where the σ's are the generators of $SU(2)$. Each eigenvalue is the Euclidean l^2-norm $a = \pm||\mathbf{x}||_2$. So $\mathrm{tr}(Q) = a + (-a) = 0$. Given the power-laws $\psi^+ = a^n$ and $\psi^- = a^{-n}$ with $n \in [0, 2)$, the left and right-handed states are computed as

$$\psi_L = \mathrm{tr}\ \psi^+(Q) = (1 + e^{in\pi})a^n \tag{3.8}$$

and

$$\psi_R = \mathrm{tr}\ \psi^-(Q) = (1 + e^{-in\pi})a^{-n} \tag{3.9}$$

One interprets Q as a qubit embedded in $\mathbb{R}^3$ with an equivalence relation $a \sim \lambda a$ for all real numbers $\lambda \neq 0$ that identifies the 2-sphere (of any arbitrary size) as the Bloch sphere. One can always find a λ such that λa has unit norm and identify the 2-sphere as the Bloch sphere $\mathbb{C}P^1$, hence why I claim the biconformal spinor is the fundamental representation of Minkowski space.

The wave function of the universe is profoundly beautiful, but there is an obvious problem. We observe a world with an arrow of time, so how can the wave function of the universe be in a pure state? The answer is that the boundary equations of motion are only half the story. There are bulk equations of motion that describe the dynamics of the *observable universe*. Holography tells us the boundary and the bulk equations are the same in a highly non-trivial way. For internal consistency, the bulk equations of motion must contain a massless spin-2 field coupled to a spin-1 Minkowski background. This is the subject of the companion paper.

4 Defining the cosmological Hilbert space of states

Definition 4.1 (Clifford-Hilbert space)**.** Let the Clifford-Hilbert space of states (or Clifford space) $\mathcal{C} := \mathcal{H} = \mathbb{C}P^{(1,1)}$ be defined as:

$$\langle\Psi|\Psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\langle\psi_L|\psi_L\rangle \\ \langle\psi_R|\psi_R\rangle\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ 1\end{pmatrix} \tag{4.1}$$

such that

$$\Psi \in L^2\left(\frac{1}{\sqrt{2}}S_a^1 \times \frac{1}{\sqrt{2}}S_\beta^1\right) \tag{4.2}$$

where the conformal Clifford torus is normalized with a factor of $1/\sqrt{2}$ such that it sits at the center of the unit 3-sphere embedded in $\mathbb{C}^2$. From the Heegaard splitting property of compact oriented 3-manifolds, the 3-sphere divides into two congruent solid tori $S_a^1 \times D_\beta^1$. It further follows from the fiber bundle structure that

$$\psi_L, \psi_R \in L^2(S_a^2 \times S_\beta^1), \tag{4.3}$$

where phase rotations of ψ_L and ψ_R correspond to expanding and contracting the celestial sphere along the thermal circle S_β^1, respectively. The thermal circle implies the Hawking temperature,

$$T \sim \frac{H}{2\pi} \tag{4.4}$$

where H is the acceleration of the celestial sphere with a sign that distinguishes expanding from collapsing states. Negative temperatures occur for systems with finite accessible states [51], and implies the Hilbert space of states factorizes into a globally infinite and *locally* finite-dimensional system.

Lemma 2 (Biconformal Unitarity)**.** *Integration of $\psi_L\psi_L^*$ and $\psi_R\psi_R^*$ w.r.t to ϕ over a rotation of 2π, $\psi_L\psi_L^*$ w.r.t a from 0 to λ_∞, and $\psi_R\psi_R^*$ w.r.t a from λ_0 to ∞, converges.*

Proof.

$$\begin{aligned} \langle L|L\rangle &:= \int_0^{\lambda_\infty}\int_0^{2\pi} d\phi da\ \psi_L\psi_L^* \\ &= 4\pi\int_0^{\lambda_\infty} da\left(a^4-1\right)\left(\frac{1}{\ln a}\cdot\frac{\pi^2+8\ln^2 a}{\pi^2+4\ln^2 a}\right) < \infty \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} \langle R|R\rangle &:= \int_{\lambda_0}^{\infty}\int_0^{2\pi} d\phi da\ \psi_R\psi_R^* \\ &= 4\pi\int_{\lambda_0}^{\infty} da\left(1-\frac{1}{a^4}\right)\left(\frac{1}{\ln a}\cdot\frac{\pi^2+8\ln^2 a}{\pi^2+4\ln^2 a}\right) < \infty \end{aligned} \tag{4.6}$$

where one interprets the "cut-offs" λ_0 and λ_∞ as the Compton wavelengths of the masses m_0 and m_∞, respectively. The above integrals converge. Unitary evolution of the wavefunction through the phase angle ϕ is established. □

The boundary equations of motion are then given as $M^{-1}(D^2+V)\Psi = 0$ where $M = diag(m_0, m_\infty)$. This equation is in natural units. It has no dimensions. One can multiply both sides by $M/\hbar^2$ and send $a \to a/\lambda_\alpha$ (where $\alpha = 0, \infty$ are the conformal indices) to make it dimensionless. The dimensionlessness of the boundary equations of motion implies the existence of natural units where $M^{-1} = I_2$. It follows m_0 and m_∞ are built out of the fundamental constants.

Something quite interesting happened when Ψ picked up the relative phase $e^{i\phi}$. The parameter $n = \phi/\pi$ is now fractional and no longer the topological dimension. The topological dimension is instead given by $d = 2\pi/\pi = 2$. In gravity, mass (energy) is charge. The symmetry group of the Hopf bundle is $U(2)$, so m_0 and m_∞ can be seen as the quantized charges in the fundamental representation of dimension d. It is clear m_0 must be the Planck mass and λ_0 the Planck length, so m_0 can not be fundamental. It is a composite of m_∞ and energetically favored to decay, or Hawking evaporate into m_∞. The wave function of the universe has a $U(1)^2 = diag(e^{i\phi_L}, e^{i\phi_R})$ global phase symmetry, i.e., the maximal torus of $U(2)$, the conformal Clifford torus. The fundamental charge is thus m_∞.

The theory predicts Euclidean 3-space is gravitationally charged with m_∞, i.e., every unit Planck volume λ_0^3 has charge m_∞. To ensure eq. (3.4) remains dimensionless send $a \to \frac{a}{\lambda_0}$ and $a \to \frac{a}{\lambda_\infty}$ in ψ_L and ψ_R, respectively. A remarkable duality then follows:

$$\psi_L(\lambda_\infty, \phi)\psi_L^*(\lambda_\infty, \phi) = \psi_R(\lambda_0, \phi)\psi_R^*(\lambda_0, \phi). \tag{4.7}$$

The boundary at infinity, like the cosmological singularity, has a horizon. Alas, one arrives at another unambiguous prediction. Unitarity demands the existence of a pair of horizons: $S^2(a = \lambda_0)$ in the UV and $S^2(a = \lambda_\infty)$ in the IR. The horizon at λ_0 is dual to the horizon at λ_∞. **The gravitational charge m_∞ defines the boundaries of the observable universe, beyond which no measurement can be made.** Everett's principle of quantum universality—when bootstrapped with Guth's zero-energy principle and Turok's principle of scale invariance—*predicts* an accelerated expansion with an information speed limit.

Nature may not have a fundamental scale, but the observer does. Everything that can be empirically known of the universe is bounded by a finite number of bulk states:

$$Z_{\text{obs}} \sim e^{2\pi\mathcal{R}} \tag{4.8}$$

where

$$\mathcal{R} = \frac{\lambda_\infty}{\lambda_0}, \tag{4.9}$$

defines the dimensionless radius of the conformal horizon that hides the singularity and its dual at infinity. The entropy of the cosmic event horizon is then

$$S_{\text{CEH}} \sim 2\pi\mathcal{R}, \tag{4.10}$$

which is the arc length between λ_0 and λ_∞ in $\mathbb{C}P^{(1,1)}$. This number defines the fundamental limit to how much information an observer can extract from the observable universe. Observers exist at the center of closed celestial spheres, bound to interact with at most $e^{S_{\text{CEH}}}$ qubits. The Hilbert space of states, $\mathcal{H} = \mathcal{H}_{\text{env}} \otimes \mathcal{H}_{\text{obs}}$, separates into an infinite and finite-dimensional piece where $dim\ \mathcal{H}_{\text{obs}} \leq Z_{\text{obs}}$. The implied **Universal Relativity** is this: everything that can be known of the universe is relative to a conformal horizon that hides the cosmological singularity and its dual at future infinity.

The extreme heavy charge m_0, the Planck mass, is the mass of a black hole whose Compton wavelength is equal to the Schwarzschild radius. The dark energy is the extreme light charge m_∞. The heavy state m_0 is not fundamental: it is a composite of m_∞.

Interchanging L and R states is equivalent to holding them fixed and rotating the singularity into infinity. This is the T-duality. Remarkably, string theory makes a similar prediction [57]. In the extreme limits, the stringy dualities take over. Pulling field space to asymptotic infinity gives a tower of light states. Pulling field space to zero, one obtains a tower of heavy states. The T-duality predicts the extreme limits are the same; the heavy state is a composite of the light state, $m_\infty \sim \Lambda$. The unitary evolution of the wave function is like a string wrapping around the torus.

Potential curvature as quantum geometry

The potential matrix V is only indirectly related to the classical scalar curvatures of the 2-sphere and 3-sphere. The quantum scalar curvatures are given as:

$$R_d \equiv V_L(a, \phi) = \frac{\hbar^2}{2m_0 a^2}\left[\frac{\phi}{\pi}\left(\frac{\phi}{\pi} - 1\right)\right] \tag{4.11}$$

and

$$R_{d+1} \equiv V_R(a, \phi) = \frac{\hbar^2}{2m_\infty a^2}\left[\frac{\phi}{\pi}\left(\frac{\phi}{\pi} + 1\right)\right], \tag{4.12}$$

respectively. The potential curvature V_L is no longer positive always: it is attractive for $0 < \phi < \pi$, repulsive when $\pi < \phi < 2\pi$, and is zero at $\phi = \pi$. On the other hand, V_R is always repulsive with m_∞ sourcing positive quantum curvature. The left-hand potential V_L has a decaying attractive phase when $0 < \phi < \pi$ that goes to zero at $\phi = \pi$. This could provide an explanation for the Hubble tension. However, more work needs to be done to understand better the observational consequences of these potentials and their coupling to the scale factor.

Another observation is this: since $m_\infty << m_0$, the field configuration rolls down V_R many orders of magnitude faster than V_L. The right-hand potential acts like an inflationary potential, blowing the scale factor up to its most probable value λ_∞. Further still, it is clear from the ϕ-coupling of the potentials that m_0 is energetically favored to decay into m_∞. The 'world densities' $\rho_0(a) = \int_0^{2\pi} \psi_L \psi_L^* d\phi$ and $\rho_\infty(a) = \int_0^{2\pi} \psi_R \psi_R^* d\phi$ with $\phi = \phi(\beta) = \phi(it)$ integrated out, are perfect mirror images of each other. Most of ρ_0 is near the IR horizon $S^2(a = \lambda_\infty)$ and decays to zero at $S^2(a = 0)$. Most of ρ_∞ is at the UV horizon $S^2(a = \lambda_0)$ and decays to zero at $S^2(a = \infty)$. The state vectors ψ_L and ψ_R describe expanding and contracting states, respectively.

Consequently, the celestial 2-sphere is no longer classical. It has become intrinsically quantum. The phase angle $\phi = n\pi$ implies the metric tensor g_{ij} is promoted to a "continuous" tensor $K_g(i, j)$, i.e., a Hilbert-Schmidt integral operator acting on a continuous row vector f:

$$(\hat{K}_g f)(i) = \int K_g(i, j) f(j) dj \tag{4.13}$$

where $i, j \in [0, 2)$. The same is true of the Ricci curvature tensor. A reasonable guess is that the tensor product gives the corresponding quantized scalar curvature:

$$R = g^{(i,j)} R_{(i,j)} \equiv \hat{K}_g \otimes (\hat{K}_R f)(i, j) = \int\int K_g(i, k) K_R(j, l) f(i, j) dk dl \tag{4.14}$$

where one assumes $f(i,j) = f_g(i)f_R(j)$ with $i,j,k,l \in [0,2)$ labeling the continuous indices. Generalizing the procedure to four spacetime dimensions: take $\mu,\nu \in [0,4)$ with maximum phase angle $\phi = 4\pi$. The quantum metric is thus coupled to the phase of the universal wave function. How one obtains analytic control of the right-hand side of the above integral operator and solves for the quantum metric $K_g(i,j)$ is a central open problem of the program.

5 Echoes of a Hot Thermal Bang

One can see from the potential curvatures that the heavy gravitational charge m_0 rapidly decays into the light gravitational charge m_∞. The Planck particle m_0 Hawking bangs into standard model particles, and the Cosmonium, m_∞. The Cosmon has negative kinetic energy and thus a negative pressure that accelerates the decay products of m_0 towards the IR horizon.

The Planck particle m_0 is the smallest possible black hole. From 3, the scale factor a can be identified with the black hole radius in the bulk, where the inverse temperature β is nothing but the inverse Hawking temperature T. A duality, therefore, exists between the components of Ψ and the wavefunction of the black hole and its time-reversed image, the white hole. The cosmological and black hole information paradoxes are one and the same.

We know information is conserved since Ψ is a pure state. There is no information paradox: $S = 0$ on the boundary where $t = 0$. All physical information lives on the surface of the celestial S^2, so it follows in the bulk representation, where time emerges, the entropy $S \sim A$, where A is the surface area of the S^2. This explains the origin of the arrow of time. Further still, the bulk representation of the black hole obeys the Bekenstein-Hawking area entropy relation and, therefore, must be contained in the boundary representation Ψ for self-consistency.

Express the densities $\rho_0(a)$ and $\rho_\infty(a)$ as a function of the surface area of the celestial S^2 with the scale transformation $a \to \sqrt{\pi}a$:

$$\rho_0(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_0^2}\right)^{\phi/\pi} d\phi \tag{5.1}$$

and

$$\rho_\infty(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_\infty^2}\right)^{-\phi/\pi} d\phi. \tag{5.2}$$

such that $A = 4\pi a^2$ and the spin $s = \pm 1/2$. The quantum 2-sphere has intrinsic half-integer spin. Unitary evolution wraps the S^2 around the conformal Clifford torus. The normalization constants

$$N_0 = \frac{1}{\sqrt{\int_0^{A_\infty} \rho_0 dA}} \text{ and } N_\infty = \frac{1}{\sqrt{\int_{A_0}^{\infty} \rho_\infty dA}}, \tag{5.3}$$

count the reciprocal square root number of states of the S^2 relative to the conformal horizon $\mathcal{R} = \frac{\lambda_\infty}{\lambda_0}$. Here $\rho = \rho(A)$, $A_\infty = 4\pi\lambda_\infty^2$, and $A_0 = 4\pi\lambda_0^2$. The squared-modulus $\rho = |\psi|^2$

is *not* a probability density; it is the 'world density' of the *gravitationally charged Dirac sea*. Said another way, ρ is a world density of universal states with global charge m_∞ and a conformal horizon $\mathcal{R}$ that is the same in all worlds. The number of bulk states $Z_{\text{obs}} \sim e^{S_{\text{CEH}}}$ where $S_{\text{CEH}} \neq 0$ is the entropy of the cosmic event horizon. Solutions to the bulk equations of motion are mixed states. The mixing of states, i.e., the factorization $\mathcal{H} = \mathcal{H}_{\text{env}} \otimes \mathcal{H}_{\text{obs}}$, arises from the entanglement between the observable universe $\mathcal{H}_{\text{obs}}$ and the unobservable multiverse $\mathcal{H}_{\text{env}}$ beyond the cosmic event horizon at R_{CEH}. Indeed, one can re-express the world densities as a function of the horizon entropy instead of area:

$$\rho_\alpha(S) = (4\pi N_\alpha \lambda_\alpha)^2 \int_0^{2\pi} S_\alpha^{\phi/\pi} \cos^2\left(\frac{\phi}{2}\right) d\phi \tag{5.4}$$

where the black hole entropy is identified:

$$S_\alpha = \frac{s^2 A}{\lambda_\alpha^2} = \frac{A}{4\lambda_\alpha^2} \tag{5.5}$$

with the conformal indices $\alpha = 0, \infty$. ($\alpha = \infty$ is the $\mathcal{R} \to 1/\mathcal{R}$ dual.) The above is nothing but the Bekenstein-Hawking area entropy formula. Finally, one can normalize to One:

$$1 = \int_0^{\pi\lambda_\infty^2} \rho_0(S) dS = \int_{\pi\lambda_0^2}^{\infty} \rho_\infty(S) dS. \tag{5.6}$$

The Bekenstein-Hawking area entropy law is a geometric consequence of the fact that $\psi \in L^2(S_a^2 \times S_\beta^1)$. In Hawking and Gibbons's original paper, they were able to compute the action precisely because the Euclidean section is non-singular: the entropy can be evaluated on a region of a spacetime manifold M bounded by some surface $r = r_0 > 2M$, whose boundary ∂M has compact topology $S^2 \times S^1$ (i.e., a 2-sphere cross periodic time) [52]. Since the Ricci scalar vanishes in the Schwarzschild metric, the action is determined entirely by the Gibbons-Hawking-York boundary term, meaning the action is an integral over the boundary $S^2 \times S^1$. The squared modulus $\rho = |\psi|^2$ is nothing but an integral over the boundary $S_a^2 \times S_\beta^1$ and precisely why one can write $\rho = \rho(S)$ with $S = \frac{A}{4\lambda^2}$. The microscopic theory explains the origin of the area entropy law: it is the entanglement entropy between $\mathcal{H}_{\text{env}}$ and $\mathcal{H}_{\text{obs}}$ separated by a closed quantum celestial sphere with radius R_{CEH} and spin $s = \pm 1/2$. One is now free to import Jacobson's argument [67] and convince oneself that GR proper can be *derived* from the wave function of the universe.

We can use holography to calculate the radius of the cosmological event horizon by applying the black hole area entropy law to the observable universe:

$$\begin{aligned} \frac{\pi R_{\text{CEH}}^2}{\lambda_0^2} &= 2\pi \frac{\lambda_\infty}{\lambda_0} \\ R_{\text{CEH}} &= \sqrt{2\lambda_0 \lambda_\infty} \\ &= \sqrt{\frac{2}{\Lambda}} \\ &\approx 14.26 \text{ billion ly} \end{aligned} \tag{5.7}$$

where the cosmological constant is identified as

$$\Lambda = \frac{1}{\lambda_0 \lambda_\infty}. \tag{5.8}$$

It is straightforward to calculate m_∞ and verify that

$$\frac{m_\infty}{m_0} = \Lambda \lambda_0^2. \tag{5.9}$$

The vacuum energy density (in reduced Planck units) is then

$$\tilde{\rho}_\infty = \rho_\Lambda = \frac{m_\infty c^2}{\lambda_0^3}. \tag{5.10}$$

Holographic derivations of the dark energy equation of state, the Planck mass m_0, and the dynamic bulk equations of motion are given in the proceeding companion paper. The small positive value of the cosmological constant is thus a consequence of the unitary evolution of the wave function of the universe. One can take this consequence further still. The theory has no fundamental scale: the mass matrix M can be multiplied out of $\hat{H}\Psi = 0$. It can therefore have any arbitrary value such that $\mathcal{R}$ remains invariant. Geometrically this is seen as a rotation of λ_0 into λ_0' and λ_∞ into λ_∞', such that $\mathcal{R} = \mathcal{R}'$. Now $r = \lambda_0'$ is the Compton wavelength of a black hole of arbitrary mass m_0'—a composite of m_∞'. **Unitarity, therefore, requires all black holes to be dark energy composite objects.** Since all massive objects dilute proportional to the inverse volume a^{-3} of Euclidean 3-space, black holes must have a mass coupling $M \sim a^3$ to be a dark energy species. This observational consequence will soon provide the first experimental test of Everettian quantum gravity.

To conclude this paper, we draw one final consequence to help us conceptualize how black holes have dark energy interiors. Return to Fig. 3. The singularity $a = 0$ is T-dual to future infinity in $\mathbb{C}P^{(1,1)}$. The black hole interior is thus time-like. When a star collapses into a black hole, it travels around future infinity into the infinite past and back to the cosmological singularity, or Cosmon, from which it came.

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