A Conformal Bootstrap of the Universal Wave Function

Maya Benowitz*
(Dated: April 30, 2024)

I assume quantum mechanics applies to the entire universe and solve the Schrödinger equation exactly, obtaining a unique closed-form conformal wave function of the universe. Quantum information and quantum geometry emerge as one from the bootstrap. The geometry of quantum information is central to the problem of time in quantum cosmology, which is resolved by a flat space holographic duality between a timeless theory in Euclidean 3-space without a spin-2 field and a theory in 3 + 1 spacetime dimensions with an emergent spin-2 field. Among the observable consequences is a spectacular prediction: black holes are dark energy composite objects, implying all black holes grow in mass proportional to the third power of the cosmological scale factor. Another consequence is the de Sitter limit has a dual solution: a dual vacuum stress-energy tensor with the dark matter equation of state.

I. INTRODUCTION

The cosmos seems to have been born into a spatially flat and thermal state of low entropy [1], originating from a curvature singularity beyond the effective field theory (EFT) description of space, time, matter, and energy [2]. This grand puzzle demands new physics. Understanding the nature of the cosmological singularity requires a complete theory of quantum gravity. The philosophy of this research program does not take the view that resolving the cosmological singularity is an application of quantum gravity—rather, that quantum gravity is what happens when the singularity is resolved. To establish our footing in this grandiose project, let us begin our journey with strings.

From the perspective of string theory, the fundamental degrees of freedom are assumed to be 2-dimensional conformal fields X^μ living on a minimal surface in spacetime, i.e., the worldsheet [3]. The famous Polyakov (or sigma) action of the classical string is

$$S_\sigma = -\frac{T}{2}\int d^2\sigma\sqrt{-\gamma}\gamma^{ab}g_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}, \qquad (1)$$

where T is the string tension, $g_{\mu\nu}$ is the spacetime (or target space) metric, and γ_{ab} is the internal metric on the worldsheet with coordinates τ (proper time) and σ (proper length) of the string. The string metric γ_{ab} is an auxiliary field living on a submanifold (the worldsheet) of the target space. From the Poincaré invariance of S_σ, intervals between events and places on the worldsheet are preserved under diffeomorphisms, meaning infinitesimal distances on the worldsheet have no physical significance. Thus, under local conformal rescalings (i.e., Weyl transformations) $\gamma_{ab} \to e^{\phi(\sigma,\tau)}\gamma_{ab}$, the action remains unchanged. It follows from the Weyl invariance of the worldsheet that the Hamiltonian and stress-energy tensor vanish, implying $g_{\mu\nu}$ must satisfy the source-free Einstein field equations (EFEs) [4]. General relativity (GR) is a low-energy EFT of the classical relativistic string—from the symmetries of the worldsheet!

The story becomes significantly more complicated for the quantized string. Without the conformal symmetry of the worldsheet, GR is no longer a low-energy approximation to the string. The Weyl symmetry of classical degrees of freedom seldom ever survives quantization. One can conjecture that no classical action with Weyl symmetry (in 3 + 1 spacetime dimensions) will have an effective action that does not break the symmetry. This appears to be a generic consequence of quantizing classical degrees of freedom [5]. Unsurprisingly, there is a quantum obstruction to the Weyl symmetry of the Polyakov action. The generators of conformal transformations on the worldsheet satisfy the Virasoro commutation relation:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{d}{12}(m^3-m)\delta_{m+n,0}, \qquad (2)$$

the second term is a quantum anomaly that indicates a breakdown of the conformal symmetry of the worldsheet with central charge (or critical dimension) d. In bosonic string theory, the anomaly cancels in $d = 26$, and for superstring theory, it cancels in $d = 10$, leading to the infamous compactification of extra dimensions in string theory. The situation is somewhat precarious as the number of allowed compactifications is now believed to be bounded above by a landscape of $\mathcal{O}(10^{272,000})$ possible solutions [6]. The situation is worse still. Because the total energy vanishes for compact geometries in generally covariant theories, there is no way to use energy as a criterion to distinguish between compactifications. Searching for metastable de Sitter (dS) vacua in the landscape is a fiendishly difficult enterprise, a sentiment expressed by the Swampland program [7, 8].

General covariance is necessary, but is it sufficient? Is the generic obstruction to conformal symmetry in quantum theories a symptom of a deeper problem? Nature, to the best of human knowledge, is quantum from the beginning. Quantization schemes—like coordinate systems—are an artifact of formalism. To solve the cosmological singularity, we must move beyond the perturbative framework of guessing a classical action with Weyl invariance, varying it for the equations of motion, and con-

* mayabenowitz@gmail.com

structing the quantum state with Euclidean path integrals. This guiding principle of *quantum covariance* is a radical constraint. But without intuition of the quantum world at the moment of creation, how is one to even begin?

The standard lore in the foundations of physics tells us degrees of freedom at higher energies (smaller distances) are more fundamental than those at lower energies (larger distances). At the heart of the Wilsonian EFT framework is the assumption that the macroscopic dynamics of a system are independent of the microscopic details. But is this true for gravity? Is the observation of the cosmological constant entering local galaxy dynamics through a universal acceleration scale $a_0 \approx cH_0 \approx c^2\Lambda^{1/2}$ [9] merely a numerical coincidence or a profound clue into the nature of quantum gravity? How is it that something as small as a galaxy "knows" about the dS radius associated with the whole of the observable universe? UV/IR mixing appears to be a model-independent feature of quantum gravity, with examples emerging across string theory and beyond [10–24]. This begs the question: does gravity violate the separation of scales?

Perhaps moving deep into the IR—at the scale of the entire universe—is no less fundamental than working in the UV. Indeed, this perspective is supported by the data. EFT predicts the dark energy is dominated by the UV vacuum energy (the Planck energy), yet it is observed to be astonishingly smaller (by 120 orders of magnitude), meaning it sets the IR cutoff that determines the largest scale anyone shall ever see. Motivated by this compendium of clues, I assume reductionism works in both directions. This *universal democracy*, a generalization of Chew's "nuclear democracy" to cosmology, suggests the quantum degrees of freedom of the entire universe *are* the fundamental UV degrees of freedom. Intriguingly, this view was discovered at the end of the second superstring revolution with holography [25–31]. In some deeply mysterious way, the fundamental degrees of freedom exist at the boundary of spacetime—at infinity—where a covariant definition of energy is well-defined in both quantum mechanics (QM) and GR.

The guiding principles of quantum covariance and universal democracy motivate the quantum-first program recently proposed by Carroll, the idea that spacetime and quantum fields emerge from the wavefunction of the Universe [32, 33]. *Everett's Principle of Quantum Universality* is radically conservative: assume the Schrödinger equation is valid on all scales and throw away the collapse postulate [34, 35]. Everett's intuition is supported by the data: in the very early universe, no Heisenberg cut was possible. The Copenhagen interpretation breaks down near the bang. Lest we become priests, preaching a classical supernatural observer prepared the initial state of the quantum universe.

In general covariant theories of quantum gravity, the Hamiltonian annihilates the universal quantum state, and time drops out of the Schrödinger equation. All information of the physical universe lives at future infinity—where time stands still. In this paper, I report a minimal bootstrapping of quantum universality. No relic classical baggage is assumed. To the author's knowledge, this is the first attempt to write down the quantum state of the universe that is not some semiclassical approximation like the Hartle-Hawking no-boundary proposal [37, 38], Vilenkin's tunneling proposal [39, 40], or numerous minisuperspace models, which all assume the Wheeler-DeWitt (WdW) equation as a starting point [41]. No assumptions of space, time, matter, or energy are made and put in by hand. In what is to follow, I argue a corollary of quantum universality is this: nature has no fundamental scale—the *form* of the laws of physics remains unchanged on all scales. From this elementary assumption, everything that follows is pure deduction.

II. QUANTUM UNIVERSALITY

Quantum universality implies there is no Heisenberg cut between quantum and classical systems. The Everettian view is austere. The universe, on all scales, is quantum. If nature has no fundamental scale, the mass term M in the Schrödinger equation must multiply out, implying

$$\hat{H}\Psi = 0. \tag{3}$$

Neither time nor mass-energy is fundamental. This further implies the wavefunction of the universe is scale-invariant: $\Psi \sim a^n$ where a is the cosmological scale factor, and n is an arbitrary integer. One can now solve for the potential V and write down the following timeless equation:

$$\left(-\frac{\partial^2}{\partial a^2} + \frac{n(n-1)}{a^2}\right)\Psi = 0. \tag{4}$$

Clearly Ψ can not be normalized. The above equation is not quite consistent with the axioms of QM. But observe the following. The potential is the scalar curvature of an n-sphere with radius a, the kinetic term is the scalar curvature of an n-hyperbola, and the right-hand side of the equation is the scalar curvature of Euclidean n-space. From this simple conformal bootstrap, the fundamental space forms emerge. There is a scalar curvature singularity at $V(a = 0)$. Since the Hamiltonian annihilates the universal quantum state, the scalar curvature singularity cancels with the kinetic term, yielding a zero-total energy universe that implies global spatial flatness, a prediction consistent with current observations.

There is no unitary evolution of Ψ. A problem indeed. The timeless equation occupies a liminal space between geometry and quantum mechanics. Can we bootstrap our way to a consistent instantiation of the timeless equation? Yes, we can—with symmetry. There is a trivial $\mathbb{Z}_2$ action $a \to -a$ and a non-trivial $\mathbb{Z}_2$ action $n \to -n$. The former is a symmetry of the system; the latter is not. Suppose we demand the latter is a symmetry of the system. The action $n \to -n$ gives the potential $R_{n+1} = \frac{n(n+1)}{a^2}$ and is interpreted as the scalar curvature of an $(n+1)$-sphere. The potential then becomes,

$$V \sim \begin{pmatrix} R_n & 0 \\ 0 & R_{n+1} \end{pmatrix} \tag{5}$$

implying Ψ is promoted to a pair of complex numbers, i.e., a spinor. A complex 2-spinor solution, together with the consequence of conformal flatness, implies an underlying $SU(n=2)$ symmetry, forcing the self-consistent equations of motion to take the unique form:

$$(D^2 + V)\Psi = 0 \tag{6}$$

where

$$D = i(\sigma^1\partial_1 + \sigma^2\partial_2 + \sigma^3\partial_3) \tag{7}$$

is the Dirac-spin operator and the σ^i are the Pauli matrices. In the limit $V \to 0$, we have $D^2\Psi = D\Psi = 0$, i.e., the Dirac equation. So for large scale factors, solutions to $\hat{H}\Psi = 0$ are effectively harmonic spinors and the underlying space is a Calabi-Yau (spin) manifold.

Naively, the components of Ψ correspond to the 2-sphere and 3-sphere, respectively. The Hopf fibration uniquely relates these spheres: one can always think of the 3-sphere as a $U(1)$-fiber over the 2-sphere [42]. But there is a caveat. We know the components of the spinor are not real-valued power laws. They are complex. This means the potential will take a slightly different form from eq. (5) with the spinor components encoding the *quantum* 3-sphere as a $U(1)$-fiber over the quantum 2-sphere. It is implicitly assumed the fibration between the quantum spheres is preserved and that the $U(1)$ fiber is the complex phase that complexifies the power-law components of Ψ.

Let us take a moment to absorb what we have learned. Starting from the simplest formulation of QM, we have deduced 1) the universe, as a whole, has three spatially flat directions, 2) there is an equivalence between energy and curvature, and 3) there is a deep geometric connection between fermions and spatial directions. Space and fermions have emerged from a conformal bootstrap of Everettian QM. And where there are spinors, there is spacetime. A map always exists between spinors of order n and tensors of order $2n$. We deduced $n = 2$, so it stands to reason there must exist a path to emerge time and energy. Moreover, the fiber bundle structure of the qubit is the Hopf fibration, and the symmetry group of the qubit is $SU(2)$. Quantum information naturally follows from quantum universality.

A. Geometry of quantum information

Quantum universality implies the world can be organized into two unique views: the "ant's-eye" view, where observers experience the flow of time, and the "God's-eye" view, where everything that can happen happens everywhere all at once. Envisioning the future of fundamental physics, Frank Wilczek wrote in 2016, "To me, ascending from the ant's-eye view to the God's eye-view of physical reality is the most profound challenge for fundamental physics in the next 100 years" [43]. Alas, this is precisely the challenge we face. And there are subtle clues at the intersection of geometry and quantum information that lead the way.

The mapping from the unit 3-sphere in a two-dimensional complex Hilbert space $\mathbb{C}^2$ (otherwise known as the complex projective line $\mathbb{C}P^1$ or the complex plane $\mathbb{C}$ with a point at infinity) to the Bloch sphere is the Hopf fibration. There is a direct correspondence between each spinor in $\mathbb{C}^2$ and a point on the Bloch sphere. Schematically we write the fibration as $S^1 \hookrightarrow S^3 \to S^2$, i.e., embedding a 1-sphere in the 3-sphere 'wraps' the 3-sphere around the 2-sphere. The S^3 therefore lives on the surface of S^2 by the identification of phase circles in S^3 with points on S^2. Every possible state of a two-level system (i.e., qubit) lives on the surface of $S^2 \cong \mathbb{C}P^1$. The system under question (the entire Universe) is not a two-level system. The components of Ψ are not simply elements of $\mathbb{C}P^1$. There is more structure that captures the intrinsic quantum nature of the 2-sphere. Nevertheless, the geometry of quantum information is a foundational starting point in solving eq. (6) and emerging time and quantum fields from Ψ.

The Hopf fibration has a symmetry group: the action of the unitary group $U(2)$ on $\mathbb{C}^2$ leaves the S^3 invariant. It thus descends to an action on the 2-sphere by ordinary rotations, carrying fibers into fibers as it commutes with the $U(1)$ action. By the exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, one may identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$,

$$X = \sigma^\mu x_\mu = Ix_0 + \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3. \tag{8}$$

Here the Lorentz metric norm is $||x_\mu|| = 2\det(X)$. From this identification one finds that x_μ is lightlike precisely if there is a complex 2-spinor $\mathbf{2}$ such that $X = \mathbf{2}\overline{\mathbf{2}}$. The *celestial sphere* can, therefore, be identified with complex 2-spinors modulo a rescaling, i.e., $S^2 \cong \mathbb{C}P^1$. Minkowski space has the same fiber bundle structure as quantum information! There always exists a map between rays of the lightcone and points on the surface of the Bloch sphere. Seen from another perspective, the fundamental representations of the Lorentz group contain a pair of complex 2-spinors $\mathbf{2}$ and $\overline{\mathbf{2}}$, i.e., two fermionic directions. The tensor product $\mathbf{2} \otimes \overline{\mathbf{2}} = \mathbf{3} \oplus \mathbf{1}$ is the adjoint representation and can be identified with either Minkowski space or a spin-1 field. This is the motivation for a supersymmetry between spatial and fermionic directions.

Is eq. (6) stringy? In the quantum covariant approach, the fundamental object of study is not the worldsheet but the Hilbert space of states. And where there is stringyness, there are duality symmetries.

B. Quantum celestial holography

Under a change of variables, eq. (4) is the reduced radial equation of a zero-energy free particle in elementary QM. There is a direct correspondence between R_{n+1} and the effective centrifugal potential with $n \leftrightarrow l$, where l is the centrifugal term. The non-trivial action $n \to -n$ sends $\Psi(a) \to \Psi(1/a)$ and takes the Hamiltonian to $\hat{H}_{n+1} = -\partial_a^2 + R_{n+1}$. On the one hand, eq. (4) describes a zero-energy Lemaîtrian point-like particle, or Cosmon, with charge M and position a from the origin, sourcing equal and opposite kinetic and potential energies that annihilate. On the other, it describes a central charge M sourcing the scalar curvatures of an n-sphere and an n-hyperboloid that annihilate to flat space. (The point-particle picture is not consistent, an observation to keep in mind.) The change of variables establishes a duality between *position* and *scale*. It follows the preferred basis of eq. (6) is the reduced radial or scale basis where $D^2 = -\partial_a^2 I_2$ and I_2 is the 2-dim identity matrix. The Hamiltonian then takes the form,

$$\hat{H} = \begin{pmatrix} \hat{H}_2 & 0 \\ 0 & \hat{H}_3 \end{pmatrix} \tag{9}$$

where the subscripts denote the dimension of the base space S^2 and total space S^3 of the Hopf bundle, respectively. The equation is timeless, so one can interpret the above Hamiltonian as describing a (quantum) celestial sphere at future null infinity, dual to a theory in one time dimension higher, with gravity. This Everettian conformal bootstrap leads us to quantum celestial holography through the geometry of quantum information. (See, e.g., [44, 45] for a review of semiclassical celestial holography.) The "inner mechanism" of holography appears to be the simplest non-trivial fibration between the spheres, i.e., where the base space S^2 is one dimension lower than the total space S^3. But what, exactly, is a quantum sphere? What does it mean for geometry to be quantum? More precisely, what is the form of the quantum metric tensor? One can answer this question by solving eq. (6) exactly.

C. Quantum geometry

The $\mathbb{Z}_2$ symmetry $n \to -n$ leaves asymptotic infinity interchangeable with the cosmological singularity by taking $\Psi(a)$ to $\Psi(1/a)$. This duality symmetry implies a compactification with a transition map that identifies zero and infinity as the same point, i.e., the inclusion of a point at infinity with a non-trivial topology that "glues" the singularity to future infinity. The quantum state Ψ belongs to 1) some conformally flat and compact spin manifold, i.e., a Calabi-Yau (with potential) embedded in $\mathbb{C}^2$ with 2) a T-like duality that identifies the cosmological singularity and future infinity as the same point. The simplest and most symmetric Calabi-Yau that meets the first requirement is the Clifford torus. So perhaps our target space has the topology of a 2-torus embedded in $\mathbb{C}^2$ with a non-trivial conformal spin structure.

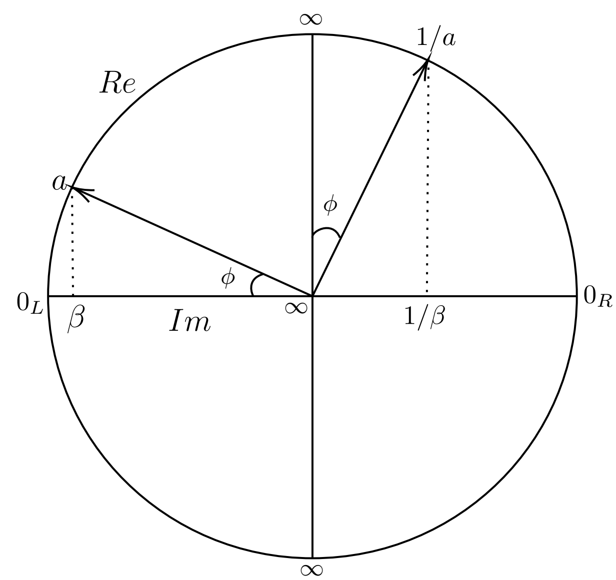

FIG. 1. The conformal Clifford torus $\mathcal{CT}^2$. The scale factor a goes around the 'line at infinity' and is real, while the imaginary axis β goes through the line at infinity. The left-hand side is defined by the pair $(a, \beta) \in \mathbb{C}$, whereas the right-hand side is defined by $(1/a, 1/\beta) \in \mathbb{C}$. The entire object lives in $\mathbb{C}^2$ with the identification $(a, \beta) \sim_\pi (1/a, 1/\beta)$ where $\sim_\pi$ denotes (a, β) can be rotated into $(1/a, 1/\beta)$ by a rotation of π. Above the imaginary axis, the scale factor is positive; below, it is negative. A rotation from 0_L to 0_R takes 2π radians and sends $a \to -a$. Thus, the space is, by construction, a spin manifold. The sign of β depends on the orientation of rotation: rotating from $0_L \to \infty$ for positive a and $\beta \in i\mathbb{R}^+$ with $1/\beta \in i\mathbb{R}^-$. Rotate from $0_L \to \infty$ for negative a and the signs for β and $1/\beta$ flip. Thus each quadrant has the sign sequence $(++, +-, -+, --)$, which is independent of the choice of clockwise or counterclockwise rotation and is nothing but the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry.

Assembling eq. (6) with the potential in eq. (5) we have,

$$\begin{pmatrix} -\partial_a^2 + R_n & 0 \\ 0 & -\partial_a^2 + R_{n+1} \end{pmatrix} \begin{pmatrix} a^n \\ a^{-n} \end{pmatrix} = 0. \tag{10}$$

We determined eq. (6) has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. By also requiring the invariance of Ψ under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ action, one has enough data to bootstrap the solution to eq. (6) and solve for the potential V and write down the quantum scalar curvatures. Demanding the components of Ψ have the $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariance implies

$$\psi_n \sim a^n + (-a)^n, \qquad \psi_{n+1} \sim a^{-n} + (-a)^{-n} \tag{11}$$

The components are not complex-valued, that is if n is an integer. Promoting the exponent to a continuous variable $n \in [0, 2)$ and the bootstrap yields:

$$\Psi(a, \phi) = \begin{pmatrix} (1 + e^{i\phi}) a^{\phi/\pi} \\ (1 + e^{-i\phi}) a^{-\phi/\pi} \end{pmatrix} \tag{12}$$

which is a solution to eq. (6) in the scale basis with the potential

$$V(a,\phi) = \frac{\phi}{\pi a^2}\begin{pmatrix} \frac{\phi}{\pi}-1 & 0 \\ 0 & \frac{\phi}{\pi}+1 \end{pmatrix} \tag{13}$$

where $\phi = n\pi$. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ duality symmetry fixes the Calabi-Yau compactification, i.e., fig. (1), and, therefore, determines the unique solution to $\hat{H}\Psi = 0$. How is one to interpret the above solution? Well, knowing the quantum scalar curvature $V(a,\phi)$ is enough to deduce the *form* of the quantum metric tensor. The metric tensor g_{ij} for the n-sphere is an $n \times n$ matrix. Since the topological dimension n is now a fractional quantity, the metric tensor is promoted to a continuous matrix, i.e., an integral kernel operator $K_g(i,j)$ acting on a complex 2-spinor f:

$$(\hat{K}_g f)(i) = \int K_g(i,j) f(j) dj. \tag{14}$$

The form and role of this operator will become more precise later in the story.

One can stare at the conformal Clifford torus $\mathcal{CT}^2$ in fig. (1) and convince themselves it is a candidate Hilbert space, with the rays the pair of 2-spinors Ψ and $\overline{\Psi}$. A long-standing conceptual issue with Everettian QM is the derivation of the Born rule and the role of probabilities in a physically real theory. Can we use the geometry of $\mathcal{CT}^2$ to derive the Born rule, normalize Ψ, and learn about quantum geometry? What does it mean for the wavefunction of the universe to evolve unitarily without time? It is obvious from figures (1) and (2) that unitary evolution is a phase rotation:

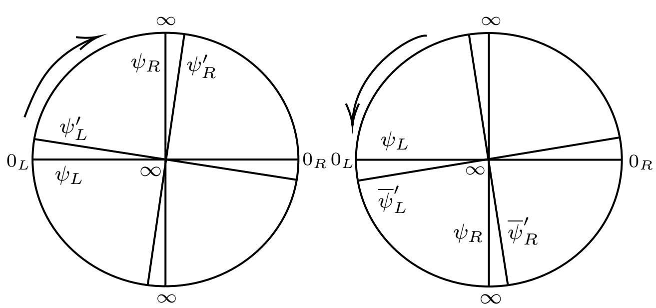

FIG. 2. Left-hand side: clockwise rotation of the spinors leaves the imaginary unit i unchanged. Right-hand side: counterclockwise rotation of the state vectors sends $i \to -i$, conjugating the spinors.

Note that $\lim_{a\to\infty}\Psi = (\infty, 0)$ and that $\lim_{a\to 0}\Psi = (0,\infty)$. Rotating the ψ_L into infinity and ψ_R into zero results in a divergence. This implies the existence of an IR and UV scale ψ_L and ψ_R do not rotate past, respectively. The topology of the target Hilbert space $\mathcal{CT}^2$ is not a circle; two of the four dimensions of the conformal Clifford torus are suppressed. There is another direction the spinors can travel along for unitary evolution, that is if the UV mixes with the IR. The unitary evolution of Ψ is like a non-critical string winding around a torus, not in physical space but the *state space of scales.*

D. Observers in the multiverse

After some thought, the only possible inner-product space over $\mathcal{CT}^2$ is chiral:

$$\begin{aligned}\langle L|L\rangle &:= \int_0^{\lambda_\infty}\int_0^{2\pi} d\phi da\ \psi_L\overline{\psi}_L \\ &= 4\pi\int_0^{\lambda_\infty} da\,(a^4-1)\left(\frac{1}{\ln a}\cdot\frac{\pi^2+8\ln^2 a}{\pi^2+4\ln^2 a}\right) < \infty\end{aligned}$$

and

$$\begin{aligned}\langle R|R\rangle &:= \int_{\lambda_0}^{\infty}\int_0^{2\pi} d\phi da\ \psi_R\overline{\psi}_R \\ &= 4\pi\int_{\lambda_0}^{\infty} da\left(1-\frac{1}{a^4}\right)\left(\frac{1}{\ln a}\cdot\frac{\pi^2+8\ln^2 a}{\pi^2+4\ln^2 a}\right) < \infty,\end{aligned}$$

define the timeless Hilbert space of states. The born rule $\rho = |\psi|^2$ follows from integrating over all possible phases: $\rho_0(a) = \int_0^{2\pi}\psi_L\overline{\psi}_L d\phi$ and $\rho_\infty(a) = \int_0^{2\pi}\psi_R\overline{\psi}_R d\phi$ (the same phase one integrates over for the Euclidean path integral). The densities are real physical quantities. They are the density of worlds with scale factor a for all possible initial conditions. The frequentist interpretation of probability is forced upon us: everything that can happen does; some things happen more than others.

One interprets the "cut-offs" λ_0 and λ_∞ as the Compton wavelengths of the masses m_0 and m_∞, respectively. There is thus a natural mass matrix $M = diag(m_0, m_\infty)$ that enters eq. (6) by multiplication of M^{-1}. Despite the fact there is a natural choice of M, the Hamiltonian still annihilates the quantum state. There is no fundamental scale. Equation (6) must be dimensionless, implying the existence of natural units where $M^{-1} = I_2$, i.e., m_0 and m_∞ depend only on fundamental constants.

To ensure eq. (6) remains dimensionless send $a \to \frac{a}{\lambda_0}$ and $a \to \frac{a}{\lambda_\infty}$ in ψ_L and ψ_R, respectively. A remarkable duality then follows:

$$\psi_L(\lambda_\infty,\phi)\overline{\psi}_L(\lambda_\infty,\phi) = \psi_R(\lambda_0,\phi)\overline{\psi}_R(\lambda_0,\phi). \tag{15}$$

The UV scale λ_0 and the IR scale λ_∞ are dual. Unitary evolution is a phase rotation of ψ_L from $a = 0_L$ through $a = \lambda_\infty^L$ to $a = \lambda_0^R$ (still labeled as ψ_L) to $a = 0_R$. The UV and IR scales place a fundamental limit on what is, in principle, observable. It is clear λ_0 is the Planck length and m_0 is the Planck mass, i.e., the smallest possible black hole. The ultra-light mass, m_∞, determines the largest scale the observer shall ever see. *Quantum universality predicts a small positive vacuum energy (eq. 13.) that accelerates the expansion of space and decays to zero at future infinity.*

Nature has no fundamental scale, but the observer does. In the ant' s-eye view, there is a fundamental limit to the number of qubits the observer can collect. One concludes that the observer can only access a finite number of qubits determined by the value of the vacuum energy. The smaller this value is, the more qubits the observer can access and the larger the observable universe.

E. The timeless boundary

The universal wave function has no initial conditions. The timeless Schrödinger equation was solved from global symmetries. Why define the universal wave function as an element of a normed Hilbert space? Because there is no supernatural observer that exists outside of the universe bestowed with the decision to choose the initial conditions of the universe. All initial conditions and, hence, all paths consistent with the symmetries of the system are allowed. There is no Ptolemaic center to the Hilbert space of states—no beginning to the universe. This is the origin of the many worlds and why quantum mechanics has the structure that it does. Quantum mechanics is the Copernican principle in the limit of all physically realizable events.

Another way of calculating Ψ is purely algebraically. Every point in $\mathbb{R}^3$ has the following matrix representation:

$$Q = \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3 \tag{16}$$

where the σ^i are the Pauli matrices. Each eigenvalue is the Euclidean l^2-norm and $\mathrm{tr}(Q) = a + (-a) = 0$. Using the spectral theorem to compute the function of a matrix, the left and right-handed states are computed as

$$\psi_L = \mathrm{tr}\ Q^n = (1 + e^{in\pi})a^n \tag{17}$$

and

$$\psi_R = \mathrm{tr}\ Q^{-n} = (1 + e^{-in\pi})a^{-n} \tag{18}$$

where $n \in [0, 2)$. One interprets Q as a qubit embedding of $\mathbb{R}^3$ with an equivalence relation $a \sim \lambda a$ for all real numbers $\lambda \neq 0$ that identifies the 2-sphere (of any arbitrary size) as the Bloch sphere. (One can always find a λ such that λa has unit norm and identify the 2-sphere as the Bloch sphere.) One can assign a qubit for each possible scale of the universe, i.e., there is a qubit at every point in $\mathcal{CT}^2$. This algebra of physical information describes a *qubit continuum* (or Q-continuum) where the points of a manifold are qubits. Such models are nontrivial generalizations of n-level quantum systems and naturally resemble the structure of quantum fields in the continuum limit of lattice qubit models [46, 47]. Intriguingly, lattice qubit models display UV/IR mixing from global subsystem symmetries [48–50], similar in spirit to the UV/IR mixing that was encountered in the previous section.

F. The dynamic bulk

Recall from the section II A that the symmetry group of the Hopf fibration is $U(2)$. By the exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, one may identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$. In other words, points on the Bloch sphere can always be mapped to rays of the lightcone. Thus every point of $\mathcal{CT}^2$ is also a celestial sphere. The cosmological Hilbert space is the configuration space of all possible celestial spheres that can surround the observer.

The boundary equations of motion, eq. (6), are defined by the Clifford algebra $C\ell_3(\mathbb{R})$. It follows from the exceptional spin isomorphism that the bulk equations of motion are defined by the Dirac algebra $C\ell_{3,1}(\mathbb{C})$, with the simplest holographic dictionary given by

$$D \leftrightarrow \not{D}, \quad V \leftrightarrow V_{\mu\nu} \tag{19}$$

where $\not{D} = i\gamma^\mu \partial_\mu$ is the Dirac operator and $V_{\mu\nu}$ is a second-rank tensor that decomposes into the following classical irreducible representations of the Lorentz group: an antisymmetric tensor $F_{\mu\nu} = -F_{\nu\mu}$, a symmetric ($H_{\mu\nu} = H_{\nu\mu}$) and traceless ($H^\mu_{\ \mu} = 0$) tensor $H_{\mu\nu}$, and a scalar trace component $X \equiv X^\mu_{\ \mu}$. Under the appropriate local phase (gauge) invariance conditions, $H_{\mu\nu}$ is a massless spin-2 field—the graviton! The bulk equations of motion then take the following form:

$$\begin{aligned}
(\not{D}^2 + V_{\mu\nu})\Psi &= \left(-\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu + V_{\mu\nu}\right)\Psi \\
&= \left(\eta^{\mu\nu}\partial_\mu\partial_\nu + F_{\mu\nu} + H_{\mu\nu} + X^\mu_{\ \mu}\right)\Psi \\
&= \left(\partial^\mu\partial_\mu + g_1 B_{\mu\nu} + g_2 W_{\mu\nu} + g_3 H_{\mu\nu} + m^2\right)\Psi \\
&= \left(\Box + g_1(\partial_\mu B_\nu - \partial_\nu B_\mu) + g_2(\partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_w \epsilon_{ijk} W^j_\mu W^k_\nu) + g_3 H_{\mu\nu} + m^2\right)\Psi \\
&= 0
\end{aligned} \tag{20}$$

The scalar trace component $X^\mu_{\ \mu} = 1/\lambda_C^2$ where λ_C is the Compton wavelength (in natural units $c = \hbar = 1$). The action of the unitary group $U(2)$ on $\mathbb{C}^2$ leaves the 3-sphere invariant. Locally $U(2)$ is diffeomorphic to the direct product $G = SU(2) \times U(1)$. Up to a local diffeomorphism, the compact gauge group G, together with the emergence of the antisymmetric tensor $F = F_{\mu\nu}$, implies the existence of Yang-Mills equations, with a gauge connection A on G. The connection A is locally a one-form on space-time. The curvature or field strength tensor is the two-form: $F = dA + A \wedge A$. The classical field equations are therefore given by the Yang-Mills equation: $d_A \star F = 0$, where d_A is the gauge-covariant extension of the exterior derivative and $\star$ is the Hodge duality operator. Thus, the antisymmetric tensor $F_{\mu\nu}$ is expanded into the electromagnetic and weak field strength tensors in eq. (20). The electroweak interactions with gravity emerge in the bulk!

G. Quantum gravity

In semiclassical gravity, the simplest metric is Minkowksi, with some quantum fluctuations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{21}$$

where $h_{\mu\nu}$ is a symmetric and traceless rank-2 tensor. From eq. (14), it was deduced that the quantum metric tensor is an integral kernel operator; hence, so too is the spin-2 field as it appears in the bulk equations of motion. An educated guess on the form of this operator is an indefinite integral operator of the following form:

$$H_{\mu\nu} \to \int d\psi_\mu h(\psi^\mu, \psi_\nu). \tag{22}$$

Here, I use the notation where ψ^μ, which denotes each element of the 4-spinor Ψ and $H_{\mu\nu} \equiv h_{\mu\nu}$. A lowered index ψ_μ denotes the elements of the conjugate transpose of Ψ. The classical irreducible representation of the Lorentz group, $H_{\mu\nu}$, is promoted to a unique class of Hilbert-Schmidt integral operators where

$$\int\int d\psi^\nu d\psi_\mu h(\psi^\mu, \psi_\nu) h(\psi_\nu, \psi^\mu) < \infty, \tag{23}$$

(with bounds to be determined from the boundary theory) such that the kernel is a complex symmetric

$$h(\psi^\mu, \psi_\nu) = \overline{h(\psi_\nu, \psi^\mu)} := h(\psi^\nu, \psi_\mu), \tag{24}$$

traceless trace-class operator

$$h^\mu{}_\mu = \int d\psi_\mu h(\psi^\mu, \psi_\mu) = 0. \tag{25}$$

Gravity is exceptional in that it acts on the quantum state as an integral operator, whereas the other forces act on the quantum state as differential operators. The classical irrep. $H_{\mu\nu}$ of the Lorentz group is fully quantized and makes eq. (20) a highly non-linear differential equation, where $H_{\mu\nu}\Psi \sim V_g(\Psi)$ can be thought of as a quantum gravitational potential, and some complicated function of the 4-spinor field Ψ that arises from quantum gravitational self-interactions. In the spin-free limit where $g_1 \to g_2 \to g_3 \to 0$, eq. (20) reduces to the Klein-Gordon equation:

$$\left(\Box + m^2\right)\Psi = 0. \tag{26}$$

The dimensionful coupling constants g are such that the fields have units of $[Length]^{-2}$: the bulk equations of motions are four (gauge-coupled) non-linear Klein-Gordon equations. Because gravity is the weakest force, in almost all cases, the quantum gravitational backreaction is negligible at low energies, except for exotic scenarios that violate decoupling (a hint of what is to come).

The first step in demonstrating a holographic duality between the timeless boundary and the dynamic bulk is to match the energy spectrum of the bulk and boundary theories. The boundary theory is based on the energy E, which is zero, whereas the bulk is based on the squared energy E^2, which is non-zero and has a negative energy solution. The conformal invariance of the boundary is locally broken from the ant's-eye perspective of the bulk observer. So, how can it be that these two theories are equivalent? How is a dynamical non-linear differential equation secretly a non-dynamical linear one? There must exist a highly non-trivial map, like a Fourier transform, between equations (6) and (20) for self-consistency.

The 4-spinor Ψ in eq. (20) decomposes into a pair of 2-spinors with equal and opposite energies, χ_L and χ_R, respectively, that satisfies two coupled non-linear Weyl-like equations

$$\left(-\sigma^\mu\sigma^\nu\partial_\mu\partial_\nu + V\right)\chi_L = 0 \tag{27}$$

$$\left(-\overline{\sigma}^\mu\overline{\sigma}^\nu\partial_\mu\partial_\nu + \overline{V}\right)\chi_R = 0 \tag{28}$$

where $\sigma^\mu = (I_2, \sigma^i)$ and $\sigma^\nu = (-I_2, \sigma^j)$, i.e., the generators of $U(2)$. Recall what we learned from the geometry of quantum information: Minkowski space has a compact presentation $U(2)$, and the fundamental representation of the Lorentz group contains two fermionic directions. The potential V can be re-written to contain the same information as $V_{\mu\nu}$ by recasting the field strength tensors in the standard Pauli matrix representation with the Clifford algebra $C\ell_{3,0}(\mathbb{R})$, otherwise known as the algebra of physical space or the Pauli algebra:

$$\left(\Box + g_1 B + g_2 W + g_3 H + m^2\right)\chi_L = 0 \tag{29}$$

$$\left(\Box + g_1 \overline{B} + g_2 \overline{W} + g_3 \overline{H} + m^2\right)\chi_R = 0 \tag{30}$$

In the limit that $g_1 \to g_2 \to g_3 \to 0$, the above reduces to the Klein-Gordon equation. Thus, we conclude equation (20) is the adjoint representation where the above is the fundamental representation.

In the limit $V_{\mu\nu} \to 0$, and with the identification $a \leftrightarrow \sigma$ and $t \leftrightarrow \tau$ in the scale basis, the fundamental representation reduces to the equation of motion of the bosonic string in conformal gauge. The implied supersymmetry here is of a more fundamental nature than string theory without quantum covariance. All bosonic fields (including Minkowski space as a spin-1 field) are dual to a pair of spinors on the timeless boundary. The fundamental representation *is* the boundary at future infinity. This explains why the LHC has not seen superpartners: they represent redundant unphysical degrees of freedom that only appear in perturbative constructions without quantum covariance.

The Dirac field can be written as a sum of chiral Weyl fields. The boundary theory has no timelike direction, so its solutions obviously can not be a linear combination of the non-linear states χ_L and χ_R. The holographic transform maps dynamic non-linear bulk states to timeless linear states on the boundary by applying the quantum metric operator $\hat{\eta}$ to the states χ_L and χ_R,

$$\widehat{\chi}(a, \phi) = \int_{-\infty}^{\infty} \eta^i{}_j(a, t; \phi) \chi^j(a, t) dt \tag{31}$$

such that

$$\Psi \sim \widehat{\chi_L} + \widehat{\chi_R} \tag{32}$$

is a solution to $(D^2 + V)\Psi = 0$. Here, I use the indices $i, j = 0, 1$ for the fundamental representation where $\eta_{ij} = \eta^i{}_j(a, t; \phi)$ is a diagonal matrix of kernels. The quantum metric $\hat{\eta}$ is a tensorial integral operator that integrates out the timelike direction and linearizes the chiral fields such that linear combinations are solutions to the boundary equations of motion. The quantum metric operator establishes an observer-dependent time evolution, a *causal reference frame* for bulk observers. This causal reference frame acts as an atlas for locating bulk events on the boundary. The integral operator formalism is like a path integral over trajectories not in space but in time, i.e., an integral over all possible rates an observer's clock can tick.

Understanding this holographic transform at a deeper level requires a better understanding of the tensorial integral operator algebra that encodes the quantum geometry, which is essential to gauging and solving eq. (20). Nevertheless, one does not need to solve the system of equations to derive precise observational numbers.

H. Quantum cosmology

Negative energy solutions can be reinterpreted as positive energy solutions traveling backward in time. If we head back to fig. (1), it is clear the phase rotation of ψ_L and ψ_R describe expanding and contracting states, respectively. The contracting state is a perfect mirror image of the expanding state, as can be seen when plotting $\rho_0(a) = \int_0^{2\pi} \psi_L \overline{\psi}_L d\phi$ and $\rho_\infty(a) = \int_0^{2\pi} \psi_R \overline{\psi}_R d\phi$. This mirror symmetry implies ψ_R is the time-reversed image of ψ_L. However, to be clear, the Universe is not in some *definite* state at some time t, at least not from the God's-eye view where everything that can happen happens everywhere all at once.

The imaginary axis β in fig. (1) does not have the topology of $i\mathbb{R}$; it runs from zero through infinity and back to zero. Recall two of the four dimensions are suppressed. The imaginary axis is the thermal circle S^1_β. One can see this clearly: at the cosmological singularity $a = 0$, the imaginary variable reads $\beta = 0$. Ergo, β is the inverse temperature. The equivalence of inverse temperature and cyclic imaginary time, expressed through the formula $\beta = it$, is manifest from the geometry of $\mathcal{CT}^2 \cong S^1_a \times S^1_\beta$. One can, therefore, justify using the Wick rotation to simplify eq. (20). Quantum gravity is fundamentally Euclidean, explaining why the Euclidean path integral formalism works so well. The thermal circle implies the de Sitter or Hawking temperature:

$$T \sim \frac{H}{2\pi} \tag{33}$$

where H is the Hubble constant. The boundary state Ψ has the global phase symmetry $U(1)^2 = diag(e^{-i\phi}, e^{i\phi})$, i.e., the maximal torus of $U(2)$. The action of $U(1)^2$ takes the components of Ψ to their complex conjugates by a phase rotation. The charge is mass (energy) in gravity, so the conserved charge must be the gravitational charge m_∞. The charge $m_0 = M_{pl}$ can not be fundamental as it radiates away with Hawking temperature $T \sim g/2\pi$, where g is the surface gravity of the horizon at $r = \lambda_0 = L_{pl}$. Thus, one concludes m_0 is not fundamental and must be a composite of m_∞.

One might say m_0 "Hawking Bangs" into m_∞. Let us briefly return to (13). The potential curvatures $V_L \sim 1/m_0 a^2$ and $V_R \sim 1/m_\infty a^2$. Here V_L is attractive for $0 < \phi < \pi$, is zero at $\phi = \pi$, and repulsive when $\pi < \phi < 2\pi$. On the other hand, V_R is always repulsive, with m_∞ sourcing positive quantum curvature that energetically favors an inflationary-like event and then *slowly rolls to zero at the boundary at future infinity.* The gravitational charge m_∞ defines the boundaries of the observable universe, beyond which no measurement can be made.

Everything that can be empirically known of the universe is bounded by a finite number of qubits:

$$Z_{\text{obs}} \sim e^{2\pi\mathcal{R}} \tag{34}$$

where

$$\mathcal{R} = \frac{\lambda_\infty}{\lambda_0} \tag{35}$$

defines the dimensionless radius of the conformal horizon that hides the cosmological singularity and its dual at future infinity. The entropy of the cosmic event horizon is

$$S_{\text{CEH}} = \frac{1}{2} C = 2\pi\mathcal{R}, \tag{36}$$

i.e., one-half the circumference of the conformal horizon pictured as the dashed circle in fig. (3). The factor of 1/2 comes from the fact that there is a double counting of states from the double cover of Euclidean space. One can calculate the location of the cosmic event horizon as witnessed by the ant from the God's-eye view. The spinor components $\psi_L, \psi_R \in L^2(S^2_a \times S^1_\beta)$. Express the densities

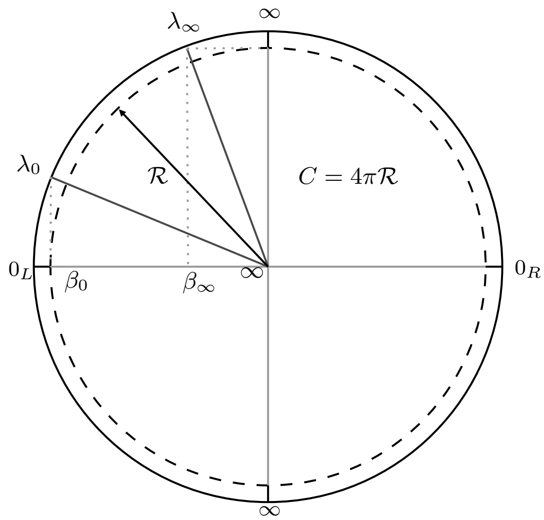

FIG. 3. From the ant's-eye view in $\mathcal{CT}^2$, the conformal horizon hides the cosmological singularity and its dual at future infinity. The dimensionless radius $\mathcal{R}$ bounds all observables to the energy scale $m_\infty \leq m \leq m_0$. Naturally, β_0 is the inverse Planck temperature, and β_∞ is the time of heat death, defined by the moment the universe becomes so dilute no measurement can be made.

$\rho_0(a)$ and $\rho_\infty(a)$ as a function of the surface area of the 2-sphere with the scale transformation $a \to \sqrt{\pi}a$:

$$\rho_0(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_0^2}\right)^{\phi/\pi} d\phi \tag{37}$$

and

$$\rho_\infty(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_\infty^2}\right)^{-\phi/\pi} d\phi. \tag{38}$$

such that $A = 4\pi a^2$ and the spin $s = \pm 1/2$. One can re-express the above as functions of the Bekenstein-Hawking entropy (the entropy from the ant's-eye perspective) by the identification:

$$S_\alpha = \frac{s^2 A}{\lambda_\alpha^2} = \frac{A}{4\lambda_\alpha^2} \tag{39}$$

with the conformal indices $\alpha = 0, \infty$. ($\alpha = \infty$ is the $\mathcal{R} \to 1/\mathcal{R}$ dual.) We now have two equivalent definitions of entropy as seen from the ant's-eye view and can calculate the radius of the cosmic event horizon:

$$\begin{aligned} \frac{\pi R_{\text{CEH}}^2}{\lambda_0^2} &= 2\pi \frac{\lambda_\infty}{\lambda_0} \\ R_{\text{CEH}} &= \sqrt{2\lambda_0\lambda_\infty} \\ &= \sqrt{\frac{2}{\Lambda}} \\ &\approx 14.26 \text{ billion ly} \end{aligned} \tag{40}$$

where the cosmological constant is identified as $\Lambda = \frac{1}{\lambda_0\lambda_\infty}$. It is straightforward to calculate m_∞ and verify that $\frac{m_\infty}{m_0} = \Lambda\lambda_0^2$ where the vacuum energy density (in reduced Planck units) is given by $\rho_\Lambda = \rho_\infty = \frac{m_\infty c^2}{\lambda_0^3}$.

The presence of the cosmic event horizon R_{CEH} means there is a natural factorization of the cosmological Hilbert space of states from the ant's-eye view: $\mathcal{H}$ separates into an infinite and finite-dimensional piece $\mathcal{H}_{\text{env}} \otimes \mathcal{H}_{\text{obs}}$, where $dim\ \mathcal{H}_{\text{obs}} \leq Z_{\text{obs}}$. The implied universal relativity is this: everything that can be empirically known of the universe is relative to a conformal horizon that hides the cosmological singularity and its dual at future infinity. From the God's-eye view, there is nothing to factorize. There are no subsystems that interact with each other at some *definite* moment in time. On the boundary, everything that can happen happens everywhere all at once. The boundary state Ψ is pure: $S = 0$. The cosmological Hilbert space, therefore, has only one state: $Z = e^S = 1$. Any ray in $\mathcal{CT}^2$ can be rotated into any other ray by a conformal transformation. In the bulk, from the ant's-eye view, the conformal horizon hides information from the observer. The hidden information is the entanglement entropy of the cosmic event horizon that separates the observable universe from the multiverse.

The fiber bundle structure of two entangled qubits is the quaternionic Hopf fibration: $S^3 \hookrightarrow S^7 \to S^4$. Fascinatingly, $S^7 \cong Spin(6)/SU(3)$. The Hilbert space of any 2-qubit state can be mapped to points on the S^4 with unit norm. Thus, one inserts the gluon field strength tensors in the bulk equations of motion by 'turning on' the entanglement degrees of freedom across the conformal Clifford torus. In what is to come, it is natural to interpret the S^4 as the Lorentzian 4-sphere of de Sitter space and the total space $S^7 \in \mathbb{C}^4$ as the space of 4-spinor solutions to the bulk equations of motion.

I. The dS and dual-dS limit

The Bekenstein-Hawking area entropy law is a geometric consequence of the fact that $\psi \in L^2(S_a^2 \times S_\beta^1)$. The action is computable in Hawking and Gibbons's original paper because the Euclidean section is non-singular: the entropy is evaluated on a region of a spacetime manifold bounded by some surface $r = r_0 > 2M$ with compact topology $S^2 \times S^1$ (i.e., a 2-sphere cross periodic time) [51]. The Ricci scalar vanishes in the Schwarzschild metric, so the action is determined only by the Gibbons-Hawking-York boundary term and is thus an integral over the boundary $S^2 \times S^1$. The world density $\rho = |\psi|^2$ is nothing but an integral over the boundary $S_a^2 \times S_\beta^1$ and precisely why one can write $\rho = \rho(S)$ with $S = \frac{A}{4\lambda^2}$.

We have predicted the radius of the cosmic event horizon, so where is the de Sitter limit with the dark energy equation of state $w = p/\rho_\Lambda = -1$? Is there an explicit holographic construction where the charge m_∞ is a microstate of m_0? Consider a flat acceleration surface with $2\pi k_B$ entropy per unit (Planck) area $1/\lambda_0^2$. Now multiply this quantity by the Hawking temperature $T(g)/\lambda_0$ per unit length of the accelerating surface. The result is the pressure

$$p_0 = \left(\frac{c^2}{G\lambda_0}\right) g, \tag{41}$$

where the quantity in parentheses is an *area density* with units [mass][area]$^{-1}$. The physical construction is an accelerating membrane, like the surface of a glowing soap bubble. The above equation is the ideal gas law $pV = k_B T$ where $V = \lambda_0^3$ is the unit volume swept out by the membrane. The above is a thermodynamic derivation of $F = m_0 g$—Newton's second law. The fundamental UV degrees of freedom can thus be interpreted as *non-critical (accelerating) membranes.* In perturbative string theory, only the critical string has conformal symmetry, not particles, membranes, or other extended objects. The fundamental object of study here is not the world volume but the *world density.*

Now consider the dual acceleration surface with $4\pi k_B$ entropy per unit R^2_{CEH} and repeat the above steps to obtain:

$$p_\infty = \left(\frac{\Lambda\hbar}{c\lambda_0}\right) g, \tag{42}$$

which is just $F = m_\infty g$ when multiplied by λ_0^2 on both sides. The above pressures obey the ideal gas law, so we can construct the stress-energy tensor $T_{\mu\nu} = diag(\rho_\infty, p_\infty, p_\infty, p_\infty)$. In the UV limit,

$$\lim_{g \to -c^2/\lambda_0} T_{\mu\nu} = \begin{pmatrix} \rho_\Lambda & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{43}$$

one obtains the vacuum stress-energy tensor $T^{(\mathrm{vac})}_{\mu\nu}$. The sign on g distinguishes the expanding and contracting states. Indeed, the first thing we learned from the bootstrap is that energy and curvature are equivalent. There are not many options to equate $T^{(\mathrm{vac})}_{\mu\nu}$ to curvature on the left-hand side. The second thing we learned is that the *form* of the laws of physics are independent of the chosen coordinate system—no coordinate system was used to solve eq. (3). The third thing we learned is that the fundamental charge of the vacuum, the gravitational charge m_∞, is equivalent to inertial mass. These deductions are none other than the equivalence principle, general covariance, and a universal limit to the rate information can propagate in the universe, determining what is, in principle, observable. Given these deductions, the simplest equations that follow are none other than Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \tag{44}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, and $\kappa = 1$ in natural units. Quantum mechanics—when assumed universal—naturally contains GR.

There is then the dual vacuum stress-energy tensor for the contracting state. As before, we can construct a stress-energy tensor $T_{\mu\nu} = diag(\rho_0, p_0, p_0, p_0)$ for an ideal gas. In the IR limit,

$$\lim_{g \to c^2/\lambda_\infty} T_{\mu\nu} = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \tag{45}$$

one obtains a bizarre substance with the Planck energy density and a positive vacuum energy pressure $p = \Lambda$ (with the minus sign absorbed) that satisfies the equation of state for dark matter: $w = p/\rho_0 \approx 0$. Are

$$G_{\mu\nu} = \kappa T^{(\text{dual-vac})}_{\mu\nu} \tag{46}$$

the classical field equations for dark matter? Does this explain how the cosmological constant Λ enters local galaxy dynamics through a universal acceleration scale $a_0 \approx cH_0 \approx c^2\Lambda^{1/2}$ [9], and how something as small as a galaxy "knows" about the dS radius associated with the whole of the observable universe? Are dark energy and the dark matter emergent properties of the universal quantum state Ψ? Is cosmological coupling a general feature of quantum gravity? We know m_0 is the mass of the smallest possible black hole and a composite object of the source, m_∞, of the dark energy. If the form of the laws of physics does not change with scale, then the mass of all astrophysical black holes are scaled-up versions of m_0. Since all massive objects dilute proportional to the inverse volume a^{-3} of Euclidean 3-space, all black holes must have a mass coupling $m \sim a^3$ to be a dark energy species. **This observational consequence will soon provide the first experimental test of quantum gravity.**

Is eq. (20) valid for astrophysical modeling? Can we use the equation to understand the dynamics of black holes, as required by the boundary equations of motion? What new and potentially exotic phenomena emerge from coupling quantum gravity with the electroweak and strong forces? What is the commutation relation on the 4-spinor field? How does one choose the spin-2 integral kernel operator, gauge fix eq. (20), and solve for the quantum metric tensor $\eta_{\mu\nu}(a, t; \phi)$?

Much work is to be done.

III. CAUSAL IMPLICATIONS

The 'many worlds' in Everettian QM are causally disconnected when a classical global time coordinate is assumed. It is clear that if time is not fundamental, the classical notions of causality break down. When time drops out of the Schrödinger equation, the Hilbert space becomes causally indefinite. The conformal Clifford torus has no causal ordering—light cones can be superimposed. Remarkably, in the first of its kind, the quantum switch experiment has demonstrated *causal indefiniteness* [53, 54]. Since there is no global time coordinate, there must exist a causal reference frame [55–58] for each event that establishes an observer-dependent time to describe the evolution of quantum subsystems. I expect causal reference frames to be key in gauge fixing and solving eq. (20).

If quantum theory is truly universal and UV/IR mixing is a general feature of the quantum universe, what are the implications of causal indefiniteness? Can exotic UV/IR couplings that violate EFT reasoning together with quantum non-causality enable the quantum metric tensor to be technologically exploited? Looking at equations (27) - (32), it is, at least in principle, possible to engineer the quantum metric tensor with some highly tuned matter-antimatter reaction. This begs the question: does quantum universality allow travel between the many worlds? At face value, there do not appear to be any inconsistencies with a traversable multiverse. If quantum universality is true and the multiverse exists, there must exist worlds where future humans master the quantum nature of spacetime and travel to "past-adjacent" light cones—a form of time travel in the multiverse. But if this were possible, then where are they?

ACKNOWLEDGMENTS

Thank you, Sabine Hossenfelder, for writing your pellucid book, *Lost in Math: How Beauty Leads Physics Astray*, which course-corrected this research program to what it is today. Thank you, Niayesh Afshordi, for the insightful conversations, words of encouragement, and advocacy. Thank you, Will Kinney, Cliff Burgess, Thomas Van Riet, and others on Physics Twitter, for the delicious conversations that contributed to this work. A special thank you to David Peak for the many correspondences on this work over the years, the helpful feedback, and, most importantly, the words of encouragement. Thank you, Neil Turok, for many of your lectures and ideas that have deeply inspired me. Lastly, thank you, Mom (Jodi Benowitz), for always being there when I needed you.

[1] P. A. R. Ade et al. *(Planck), Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13

[2] P.J.E. Peebles, Anomalies in physical cosmology, *Annals Phys.* **447** (2022) 159-169.

[3] J. Polchinski, *String theory*, *Cambridge University Press*, 2005.

[4] D. Friedan, Nonlinear models in $2+\epsilon$ dimensions, *Phys. Rev. Lett.*, **45(13)** (1980) 1057– 1060.

[5] Z. Komargodski and A. Schwimmer, On renormalization group flows in four dimensions. *J. High Energ. Phys.* **99** (2011)

[6] W. Taylor and Y.N. Wang, The F-theory geometry with most flux vacua. *J. High Energy Phys.* **164** (2015)

[7] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, *De Sitter Space and the Swampland*, arXiv:1806.08362.

[8] N. Agmon et. al., *Lectures on the string landscape and the Swampland*, arXiv:2212.06187

[9] M. Milgrom, *MOND–a pedagogical review*, arXiv:astro-ph/0112069.

[10] L. Susskind, String theory and the principles of black hole complementarity, *Phys. Rev. Lett.* **71** (1993) 2367-2368.

[11] L. Susskind, Strings, black holes and Lorentz contraction, *Phys. Rev. D* **49** (1994) 6606-6611.

[12] S. Hossenfelder, Minimal Length Scale Scenarios for Quantum Gravity, *Living Rev. Rel.* **16** (2013) 2.

[13] R. Israel et. al., Stringy Horizons and UV/IR Mixing, *JHEP* **11** (2015) 164.

[14] R.J., Adler, P. Chen, and D.I Santiago, The Generalized Uncertainty Principle and Black Hole Remnants, *Gen. Relativ. Gravit.* **33** (2001) 2100-2108.

[15] R.J., Adler and D.I., Santiago, On gravity and the uncertainty principle, *Mod. Phys. Lett.* **14** (1999) 1371.

[16] J. Feldbrugge, J.L. Lehners, and N. Turok, No rescue for the no boundary proposal: Pointers to the future of quantum cosmology, *Phys. Rev. D* **97** (2018) 023509.

[17] J. Feldbrugge, J.L. Lehners, and N. Turok, Inconsistencies of the New No-Boundary Proposal, *Universe* **4** (2018) 100.

[18] A. M. Polyakov, De Sitter space and eternity, *Nucl. Phys. B* **797** (2008) 199–217.

[19] A. M. Polyakov, Infrared instability of the de Sitter space, arXiv:1209.4135

[20] L. Boyle and N. Turok, Two-Sheeted Universe, Analyticity and the Arrow of Time, arXiv:2109.06204

[21] L. Boyle, F. Kieran, and N. Turok, CPT-Symmetric Universe, *Phys. Rev. Lett.* **121** (2018) 251301.

[22] L. Boyle, F. Kieran, and N. Turok, The Big Bang, CPT, and neutrino dark matter, *Annals Phys.* **438** (2022) 168767.

[23] N. Afshordi, Dark Energy, Black Hole Entropy, and the First Precision Measurement in Quantum Gravity, 1003.4811

[24] C. Prescod-Weinstein, N. Afshordi, et al., Stellar Black Holes and the Origin of Cosmic Acceleration, *Phys. Rev. D* **80** (2009) 043513

[25] J.M. Maldacena, The Large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2** (1998) 231–252.

[26] L. Susskind, The World as a hologram, *J. Math. Phys.*, **36** (1995) 6377–6396.

[27] G. 't Hooft, The Holographic principle: Opening lecture, *Subnucl. Ser.* **37** (2001) 72–100.

[28] A. Laddha, et. al., The Holographic Nature of Null Infinity, *SciPost Phys.* **10** (2021) 041.

[29] A. Laddha, et. al., Squinting at massive fields from infinity, arXiv:2207.06406

[30] S. Raju, Failure of the split property in gravity and the information paradox, *Class. Quant. Grav.* **39** (2022) 064002.

[31] V. Hubeny, The AdS/CFT Correspondence, *Class. Quant. Grav.* **32** (2015) 124010.
[32] S.M. Carroll, *Reality as a vector in Hilbert space*, *Springer* (2022).
[33] S.M. Carroll, *Mad-dog everettianism: Quantum mechanics at its most minimal*, *Springer* (2019).
[34] H. Everett, *The theory of the universal wave function, In The many-worlds interpretation of quantum mechanics Princeton University Press* (2015) 1-140.
[35] M. Jammer. *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective*, *Wiley* (1974)
[36] Carroll, S. M., & Singh, A. (2021). Quantum mereology: Factorizing Hilbert space into subsystems with quasiclassical dynamics. *Phys. Rev. A, 103 (2)*, 022213.
[37] J.B. Hartle and S.W. Hawking, Wave Function of the Universe, *Phys. Rev. D* **28** (1983) 2960–2975.
[38] J.B. Hartle, S.W. Hawking, and T. Hertog, The Classical Universes of the No-Boundary Quantum State, *Phys. Rev. D* **77** (2008) 123537.
[39] A. Vilenkin, Creation of Universes from Nothing, *Phys. Lett. B* **117** (1982) 25–28.
[40] A. Vilenkin, The Birth of Inflationary Universes, *Phys. Rev. D* **27** (1983) 2848.
[41] B.S. DeWitt, Quantum Theory of Gravity. I. The Canonical Theory, *Phys. Rev.* **160** (1967) 1113.
[42] H.K. Urbantke, The Hopf fibration—seven times in physics, *J. Geom. Phys.* **46** (2003) 125-150.
[43] F. Wilczek, Physics in 100 years, *Physics Today.* **69** (2016) 32–39
[44] A.M. Reclariu, Lectures on Celestial Holography, arXiv:2107.02075
[45] S. Pasterski, M. Pate, and A.M. Raclariu, Celestial Holography, arXiv:2111.11392
[46] A. Paramekanti, L. Balents and M. P. Fisher, Ring exchange, the exciton bose liquid, and bosonization in two dimensions, *Physical Review B* **66** (2002) 054526.
[47] R. M. Nandkishore and M. Hermele, Fractons, *Annual Review of Condensed Matter Physics* **10** (2019) 295–313.
[48] N. Seiberg and S.-H. Shao, Exotic Symmetries, Duality, and Fractons in 2+1-Dimensional Quantum Field Theory, arXiv:2003.10466.
[49] N. Seiberg and S.-H. Shao, Exotic U(1) Symmetries, Duality, and Fractons in 3+1-Dimensional Quantum Field Theory, *SciPost Phys.* **9** (2020) 046.
[50] Y. You, J. Bibo, T. L. Hughes and F. Pollmann, Fractonic critical point proximate to a higher-order topological insulator: How does uv blend with ir?, arXiv:2101.01724.
[51] G. W. Gibbons and S. W. Hawking, Action Integrals and Partition Functions in Quantum Gravity, *Phys. Rev. D* **15** (1977) 2752-2756.
[52] C. White, The double copy: gravity from gluons, *Contemp. Phys.* **59** (2018) 109.
[53] G. Rubino et al., Experimental verification of an indefinite causal order, *Sci. Adv.* **3** (2017) e1602589 .
[54] G. Chiribella, et al., Quantum computations without definite causal structure, *Physical Review A* (2017).
[55] Li, Y. Indefinite causality. *Nature Phys* **13** (2017) 419.
[56] P.A. Guérin and Č. Brukner, Observer-dependent locality of quantum events, *New J. Phys.* (2018) **20** 103031.
[57] M. Zych et al., Bell's theorem for temporal order. *Nat Commun* (2019) **10**, 3772.
[58] E. Castro-Ruizm, F. Giacomini, and Č. Brukner, Dynamics of Quantum Causal Structures, *Physical Review X*, (2018) **8** 1.
[59] S. Hossenfelder, *Lost in Math: How Beauty Leads Physics Astray*, *Basic Books* (2018).
[60] E. Anderson, The Problem of Time in Quantum Gravity, arXiv:1009.2157
[61] A. O. Barvinsky, Unitarity approach to quantum cosmology, *Phys. Rept.* bf 230 (1993) 237.
[62] A. Y. Kamenshchik, Time in quantum theory, the Wheeler-DeWitt equation and the Born-Oppenheimer approximation, arXiv:1809.08083v1
[63] J. Feldbrugge, J.L. Lehners, and N. Turok, No smooth beginning for spacetime, *Phys. Rev. Lett.* **119** (2017) 171301.
[64] H.O. Ukavwe, The Role of Deductive Reasoning in the Deductive-Nomological Model of Scientific Explanation, *Nasara Journal of Philosophy* (2018) 13-19.
[65] Misner, Thorne, and Wheeler (MTW), *Gravitation*, chapters 7, 20, and 25.
[66] H. Roos, Independence of local algebras in quantum field theory, *Comm. Math. Phys.* **16** (1970) 238–246.
[67] R. Haag, *Local quantum physics: Fields, particles, algebras*, Springer (1992).
[68] M. Kamionkowski and A. Riess, The Hubble tension and early dark energy, arXiv:2211.04492.
[69] N. Turok and L. Boyle, Gravitational entropy and the flatness, homogeneity and isotropy puzzles, arXiv:2201.07279.
[70] N. Turok and L. Boyle, Thermodynamic solution of the homogeneity, isotropy and flatness puzzles (and a clue to the cosmological constant), arXiv:2210.01142.
[71] J. M. Bardeen, B. Carter and S. W. Hawking, The Four laws of black hole mechanics, *Commun. Math. Phys.* **31** (1973) 161-170.