The Everettian Conformal Bootstrap

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(Dated: August 10, 2023)

I assume quantum mechanics applies to the entire universe and solve the Schrödinger equation exactly, obtaining a unique closed-form wavefunction of the universe. Quantum information and quantum geometry emerge as one from the bootstrap. The geometry of quantum information is central to the problem of time in quantum cosmology, which is resolved by an emergent flat space holographic duality between a timeless theory in Euclidean 3-space without a spin-2 field, and a theory in 3 + 1 spacetime dimensions with a spin-2 field. Among the observable consequences is a spectacular prediction: black holes are dark energy composite objects, implying all black holes grow in mass proportional to the third power of the cosmological scale factor. Another consequence is the de Sitter limit has a dual solution: a dual vacuum stress-energy tensor with the dark matter equation of state.

I. INTRODUCTION

The quantum-first program—the idea that spacetime and quantum fields emerge from the wavefunction of the Universe—has recently been proposed by Carroll [1, 2]. Everett's *Principle of Quantum Universality* is radically conservative: assume the Schrödinger equation is valid on all scales and throw away the collapse postulate [3]. Everettian quantum mechanics (QM) has been described by Max Jammer, a philosopher of science, as "one of the most daring and most ambitious theories ever constructed in the history of science" [4].

Emerging the world from quantum universality *alone* is a daunting task. How can it be that the world of dynamical spacetime and quantum fields follows from a single unifying principle? Carroll and Singh approach this problem by trying to find the most useful factorization of Hilbert space into quasi-classical subsystems with only the Hamiltonian and an initial state [5]. Since no preferred factorization is assumed, there is no preferred basis other than the eigenstates of the Hamiltonian, which is fully specified by its eigenspectrum (and the initial state). In other words, they attempt to derive the world from nothing more than a set of energy eigenvalues by decomposing Hilbert space into the most "information-rich" tensor factors. As ingenious as their approach is, it comes with classical baggage: a classical global time coordinate is assumed.

In this *Letter*, I report a minimalist bootstrapping of Everett's principle, and what I claim is the logical conclusion of Everettian QM. No relic classical baggage is assumed. The exact form of the wavefunction of the Universe and some of its *observable consequences* are derived. To the author's knowledge, this is the first attempt to write down the quantum state of the Universe that is not some semiclassical approximation like the Hartle-Hawking no-boundary proposal [6, 7], Vilenkin's tunneling proposal [8, 9], or numerous minisuperspace models, which all assume the Wheeler-DeWitt (WdW) equation as a starting point [10].

No assumptions of space, time, matter, or energy are made and put in by hand. In what is to follow, I argue quantum universality implicitly assumes nature has no fundamental scale—that the *form* of the laws of physics remains unchanged on all scales. From this elementary assumption, everything that follows is pure deduction.

II. BOOTSTRAPPING THE MULTIVERSE

Quantum universality implies there is no Heisenberg cut between quantum and classical systems. The Everettian view is austere. The Universe, on all scales, is quantum: the form of the laws of physics remains unchanged on all scales. If nature has no fundamental scale, the mass term M in the Schrödinger equation must multiply out, implying

$$\hat{H}\Psi = 0. \tag{1}$$

Neither time nor mass-energy is fundamental. This further implies the wavefunction of the universe is scale-invariant: $\Psi \sim a^n$ where a is the cosmological scale factor, and n is an arbitrary integer. One can now solve for the potential V and write down the following timeless equation,

$$\left(-\frac{\partial^2}{\partial a^2} + \frac{n(n-1)}{a^2}\right)\Psi = 0. \tag{2}$$

Clearly, Ψ can not be normalized. The above equation is not quite consistent with the axioms of QM. But observe the following: the potential is the scalar curvature of an n-sphere with radius a, the kinetic term is the scalar curvature of an n-hyperbola, and the right-hand side of the equation is the scalar curvature of Euclidean n-space. From this simple conformal bootstrap, the fundamental space forms emerge. There is a scalar curvature singularity at $V(a=0)$, but since the Hamiltonian annihilates the universal quantum state, the scalar curvature singularity cancels with the kinetic term, yielding a zero-total energy universe that implies global spatial flatness, a prediction consistent with current observations [11].

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There is, however, no unitary evolution of Ψ. The timeless equation occupies a liminal space between geometry and quantum mechanics. Can we bootstrap our way to a consistent instantiation of the timeless equation? Yes, we can—with symmetry. There is a trivial $\mathbb{Z}_2$ action $a \to -a$ and a non-trivial $\mathbb{Z}_2$ action $n \to -n$. The former is a symmetry of the system; the latter is not. Suppose we demand the latter is a symmetry of the system. The action $n \to -n$ gives the potential $R_{n+1} = \frac{n(n+1)}{a^2}$ and is interpreted as the scalar curvature of an $(n+1)$-sphere. The potential then becomes,

$$V \sim \begin{pmatrix} R_n & 0 \\ 0 & R_{n+1} \end{pmatrix} \tag{3}$$

implying Ψ is promoted to a pair of complex numbers, i.e., a spinor. A complex 2-spinor solution, together with the consequence of conformal flatness, implies an underlying $SU(n=2)$ symmetry, forcing the self-consistent equations of motion to take the unique form:

$$(D^2 + V)\Psi = 0 \tag{4}$$

where

$$D = i(\sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3) \tag{5}$$

is the Dirac-spin operator and the σ^i are the Pauli matrices. In the limit $V \to 0$, we have $D^2\Psi = D\Psi = 0$, i.e., the Dirac equation. So for large scale factors, solutions to $\hat{H}\Psi = 0$ are effectively harmonic spinors, and the underlying space is a Calabi-Yau (spin) manifold.

Naively, the components of Ψ correspond to the 2-sphere and 3-sphere, respectively. These spheres are uniquely related by the Hopf fibration: one can always think of the 3-sphere as a $U(1)$-fiber over the 2-sphere [12]. But there is a caveat. We know the components of the spinor are not real-valued power laws. They are complex. This means the potential will take a slightly different form from eq. (3) with the spinor components encoding the *quantized* 3-sphere as a $U(1)$-fiber over the *quantized* 2-sphere, such that $a = \sqrt{x_1^2 + x_2^2 + x_3^2}$. It is implicitly assumed the fibration between the quantum spheres is preserved and that the $U(1)$ fiber is the complex phase that complexifies the power-law components of Ψ.

Let us take a moment to absorb what we have learned. Starting from the simplest formulation of QM, we have deduced 1) the universe, as a whole, has three spatially flat directions, 2) there is an equivalence between energy and curvature, and 3) there is a deep geometric connection between fermions and spatial directions. Space and fermions have emerged from a conformal bootstrap of Everettian quantum mechanics. And where there are spinors, there is spacetime. A map always exists between spinors of order n and tensors of order $2n$. We deduced $n = 2$, so it stands to reason there must exist a path to emerge time and energy. What is more, the fiber bundle structure of the qubit is the Hopf fibration, and the symmetry group of the qubit is $SU(2)$. Quantum information emerges from quantum universality!

A. geometry of quantum information

Quantum universality implies the world can be organized into two unique views: the "ant's-eye" view, where observers experience the flow of time, and the "God's-eye" view, where everything that can happen happens everywhere all at once. From the God's-eye perspective, the world is unified as one eternal and unchanging quantum object, the Universal wave function. Envisioning the future of fundamental physics, Frank Wilczek wrote in 2016, "To me, ascending from the ant's-eye view to the God's eye-view of physical reality is the most profound challenge for fundamental physics in the next 100 years" [13]. Alas, this is precisely the challenge we face. And there are subtle clues at the intersection of geometry and quantum information that lead the way.

The mapping from the unit 3-sphere in a two-dimensional complex Hilbert space $\mathbb{C}^2$ (otherwise known as the complex projective line $\mathbb{C}P^1$ or the complex plane $\mathbb{C}$ with a point at infinity) to the Bloch sphere is the Hopf fibration, with each ray of spinors mapping to a point on the Bloch sphere. Schematically we write the fibration as $S^1 \hookrightarrow S^3 \to S^2$, i.e., embedding a 1-sphere in the 3-sphere 'wraps' the 3-sphere around the 2-sphere. The S^3 therefore lives on the surface of S^2 by the identification of phase circles in S^3 with points on S^2. Every possible state of a two-level system (i.e., qubit) lives on the surface of $S^2 \cong \mathbb{C}P^1$. The system under question (the entire Universe) is not a two-level system. The components of Ψ are not simply elements of $\mathbb{C}P^1$. There is more structure that captures the intrinsic quantum nature of the 2-sphere. Nevertheless, the geometry of quantum information is a foundational starting point in solving eq. (4) and emerging time and quantum fields from Ψ.

The Hopf fibration has a symmetry group: the action of the unitary group $U(2)$ on $\mathbb{C}^2$ leaves the S^3 invariant and carries fibers into fibers as it commutes with the $U(1)$ action, and thus descends to an action on the S^2 by ordinary rotations [12]. By the exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, one may identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$,

$$X = \sigma^\mu x_\mu = I x_0 + \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3. \tag{6}$$

Here the Lorentz metric norm is $||x_\mu|| = 2\det(X)$. From this identification one finds that x_μ is lightlike precisely if there is a complex 2-spinor $\mathbf{2}$ such that $X = \mathbf{2}\overline{\mathbf{2}}$. The *celestial sphere* can therefore be identified with complex 2-spinors modulo a rescaling, i.e., $S^2 \cong \mathbb{C}P^1$. Minkowski space has the same fiber bundle structure as quantum information! There always exists a map between rays of the lightcone and points on the surface of the Bloch sphere. Seen from another perspective, the fundamental representations of the Lorentz group contain a pair of complex 2-spinors $\mathbf{2}$ and $\overline{\mathbf{2}}$, i.e., two fermionic directions. The tensor product $\mathbf{2} \otimes \overline{\mathbf{2}} = \mathbf{3} \oplus \mathbf{1}$ is the adjoint representation and can be identified with either Minkowski space or a spin-1 field. This is the motivation for a supersymmetry between spatial and fermionic directions. One can interpret this as time emerging in the adjoint representation from a pair of fermionic directions in the fundamental representation.

B. quantum celestial holography

Under a change of variables, eq. (2) is the reduced radial equation of a zero-energy free particle in elementary quantum mechanics. There is a direct correspondence between R_{n+1} and the effective centrifugal potential with $n \leftrightarrow l$ where l is the centrifugal term. The non-trivial action $n \to -n$ sends $\Psi(a) \to \Psi(1/a)$ and takes the Hamiltonian to $\hat{H}_{n+1} = -\partial_a^2 + R_{n+1}$. On the one hand, eq. (2) describes a zero-energy Lemaîtrian point-like particle, or Cosmon, with charge M and position a from the origin, sourcing equal and opposite kinetic and potential energies that self-annihilate. On the other, it describes a charge M sourcing the scalar curvatures of an n-sphere and an n-hyperboloid that annihilate to flat space. The change of variables establishes a duality between *position* and *scale*. It follows the preferred basis of eq. (4) is the reduced radial or scale basis where $D^2 = -\partial_a^2 I_2$ and I_2 is the 2-dim identity matrix. The Hamiltonian then takes the form,

$$\hat{H} = \begin{pmatrix} \hat{H}_2 & 0 \\ 0 & \hat{H}_3 \end{pmatrix} \tag{7}$$

where the subscripts denote the dimension of the base space S^2 and total space S^3 of the Hopf bundle, respectively. The equation is timeless, so one can interpret the above Hamiltonian as describing a (quantum) celestial sphere at null infinity, dual to a theory in one time dimension higher, with gravity. The Everettian conformal bootstrap is leading us to quantum celestial holography through the geometry of quantum information. (See, e.g., [14–17] for a review of semiclassical celestial holography.) The "inner mechanism" of holography appears to be the simplest non-trivial fibration between the spheres, i.e., where the base space S^2 is one dimension lower than the total space S^3. But what, exactly, is a quantum sphere? What does it mean for geometry to be quantum? More precisely, what is the form of the quantum metric tensor? One can answer this question by solving eq. (4) exactly.

C. quantum geometry

The $\mathbb{Z}_2$ symmetry $n \to -n$ leaves asymptotic infinity interchangeable with the cosmological singularity by taking $\Psi(a)$ to $\Psi(1/a)$. This symmetry implies a compactification with a transition map that identifies zero and infinity as the same point, i.e., the inclusion of a point at infinity with a non-trivial topology that "glues" the singularity to asymptotic infinity. The quantum state Ψ belongs to 1) some conformally flat and compact spin manifold, i.e., a Calabi-Yau (with potential) embedded in $\mathbb{C}^2$ with 2) a transition map that identifies zero and infinity as the same point. The simplest and most symmetric Calabi-Yau that meets the first requirement is the Clifford torus. So perhaps our target space has the topology of a 2-torus embedded in $\mathbb{C}^2$ with some spin structure.

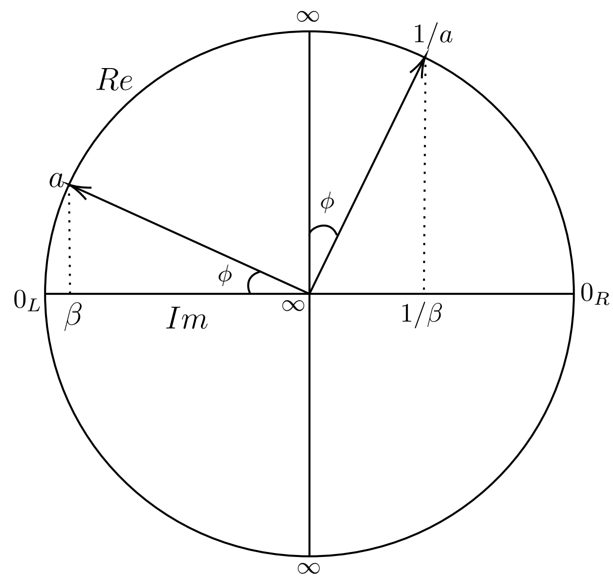

FIG. 1. The conformal Clifford torus $\mathcal{CT}^2$. The scale factor a goes around the 'line at infinity' and is real, while the imaginary axis β goes through the line at infinity. The left-hand side is defined by the pair $(a, \beta) \in \mathbb{C}$, whereas the right-hand side is defined by $(1/a, 1/\beta) \in \mathbb{C}$. The entire object lives in $\mathbb{C}^2$ with the identification $(a, \beta) \sim_\pi (1/a, 1/\beta)$ where $\sim_\pi$ denotes (a, β) can be rotated into $(1/a, 1/\beta)$ by a rotation of π. Above the imaginary axis, the scale factor is positive, below it is negative. A rotation from 0_L to 0_R takes 2π radians and sends $a \to -a$. Thus, the space is, by construction, a spin manifold. The sign of β depends on the orientation of rotation: rotating from $0_L \to \infty$ for positive a and $\beta \in i\mathbb{R}^+$ with $1/\beta \in i\mathbb{R}^-$. Rotate from $0_L \to \infty$ for negative a and the signs for β and $1/\beta$ flip. Thus each quadrant has the sign sequence $(++, +-, -+, --)$, which is independent of the choice of clockwise or counterclockwise rotation and is nothing but the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry.

Assembling eq. (4) with the potential in eq. (3) we have,

$$\begin{pmatrix} -\partial_a^2 + R_n & 0 \\ 0 & -\partial_a^2 + R_{n+1} \end{pmatrix} \begin{pmatrix} a^n \\ a^{-n} \end{pmatrix} = 0. \tag{8}$$

We determined eq. (4) has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. By also requiring the invariance of Ψ under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ action, one has enough data to bootstrap the solution to eq. (4), fix the potential V, and write down the quantum scalar curvatures. Demanding the components of Ψ have the $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariance implies

$$\psi_n \sim a^n + (-a)^n, \qquad \psi_{n+1} \sim a^{-n} + (-a)^{-n} \tag{9}$$

The components are not complex-valued, that is if n is an integer. Promoting the exponent to a continuous variable $n \in [0, 2)$ and the bootstrap yields:

$$\Psi(a, \phi) = \begin{pmatrix} (1 + e^{i\phi})a^{\phi/\pi} \\ (1 + e^{-i\phi})a^{-\phi/\pi} \end{pmatrix} \tag{10}$$

which is a solution to eq. (4) in the scale basis with the potential

$$V(a, \phi) = \frac{\phi}{\pi a^2} \begin{pmatrix} \frac{\phi}{\pi} - 1 & 0 \\ 0 & \frac{\phi}{\pi} + 1 \end{pmatrix} \tag{11}$$

where $\phi = n\pi$. How is one to interpret this solution? Well, knowing the exact form of the quantum state is enough to "reverse engineer" the underlying spin manifold, i.e., fig. (1). And knowing the quantum scalar curvature is enough to deduce the *form* of the quantum metric tensor. The metric tensor g_{ij} for the n-sphere is an $n \times n$ matrix. Reading off the quantum state Ψ, the metric tensor is promoted to a *continuous* matrix, i.e., an integral kernel operator $K_g(i, j)$ acting on a continuous row vector f:

$$(\hat{K}_g f)(i) = \int K_g(i, j) f(j) dj. \tag{12}$$

The notion of a metric kernel will become more precise once the geometry of the underlying spin manifold is understood, and, in particular, how a dynamic bulk emerges from it.

One can stare at the conformal Clifford torus $\mathcal{CT}^2$ in fig. (1) and convince themselves it is a candidate Hilbert space, with the rays the state vectors $\Psi = (\psi_L, \psi_R)$, if, that is, they can be normalized (they can). A longstanding conceptual issue with Everettian QM is the meaning of the Born rule and probabilities in a physically real theory. Can we use the geometry of $\mathcal{CT}^2$ to derive the Born rule and normalize Ψ? What does it mean for the wavefunction of the Universe to evolve unitarily without time? It is obvious from figures (1) and (2) that unitary evolution is a phase rotation.

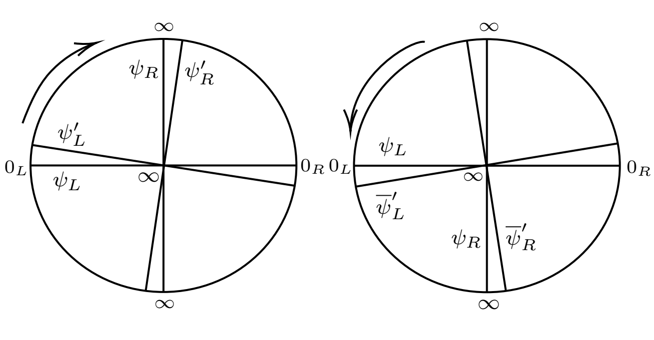

FIG. 2. Left-hand side: clockwise rotation of the state vectors leaves the imaginary unit i unchanged. Right-hand side: counterclockwise rotation of the state vectors sends $i \to -i$, conjugating the state vectors.

However, note that $\lim_{a\to\infty} \Psi = (\infty, 0)$ and that $\lim_{a\to 0} \Psi = (0, \infty)$. Rotating the ψ_L into infinity and ψ_R into zero results in a divergence. This implies the existence of an IR and UV scale ψ_L and ψ_R do not rotate past, respectively. The topology of the target Hilbert space $\mathcal{CT}^2$ is not a circle; two of the four dimensions of the conformal Clifford torus are suppressed. There is another direction the state vectors can travel along for unitary evolution, that is if the UV and IR scales mix. One thinks of the upper two quadrants of $\mathcal{CT}^2$ as labeled by the rays ψ_L and ψ_R and the bottom quadrant labeled by the conjugates $\overline{\psi}_L$ and $\overline{\psi}_R$. The inner-product space is given by

$$\begin{aligned} \langle L|L\rangle &:= \int_0^{\lambda_\infty} \int_0^{2\pi} d\phi da\ \psi_L \overline{\psi}_L \\ &= 4\pi \int_0^{\lambda_\infty} da\,(a^4 - 1)\left(\frac{1}{\ln a} \cdot \frac{\pi^2 + 8\ln^2 a}{\pi^2 + 4\ln^2 a}\right) < \infty \end{aligned}$$

and

$$\begin{aligned} \langle R|R\rangle &:= \int_{\lambda_0}^{\infty} \int_0^{2\pi} d\phi da\ \psi_R \overline{\psi}_R \\ &= 4\pi \int_{\lambda_0}^{\infty} da\left(1 - \frac{1}{a^4}\right)\left(\frac{1}{\ln a} \cdot \frac{\pi^2 + 8\ln^2 a}{\pi^2 + 4\ln^2 a}\right) < \infty \end{aligned}$$

which defines the boundary Hilbert space of states $\mathcal{H} = \mathcal{CT}^2$. The born rule $\rho = |\psi|^2$ follows from taking the left and right-hand states as the product state of the upper and lower quadrants of $\mathcal{CT}^2$ and integrating ϕ over a rotation of 2π and the scale factor a over a bounded interval. One interprets the "cut-offs" λ_0 and λ_∞ as the Compton wavelengths of the masses m_0 and m_∞, respectively. There is thus a natural mass matrix $M = diag(m_0, m_\infty)$ that enters eq. (4) by multiplication of M^{-1}. Despite the fact that there is a natural choice of M, the Hamiltonian still annihilates the quantum state—there is no fundamental scale—eq. (4) must be dimensionless. This implies the existence of natural units where $M^{-1} = I_2$, i.e., m_0 and m_∞ are built out of the fundamental constants.

To ensure eq. (4) remains dimensionless send $a \to \frac{a}{\lambda_0}$ and $a \to \frac{a}{\lambda_\infty}$ in ψ_L and ψ_R, respectively. A remarkable duality then follows:

$$\psi_L(\lambda_\infty, \phi)\overline{\psi}_L(\lambda_\infty, \phi) = \psi_R(\lambda_0, \phi)\overline{\psi}_R(\lambda_0, \phi). \tag{13}$$

The UV scale λ_0 and the IR scale λ_∞ are dual. Unitary evolution is a phase rotation of ψ_L from $a = 0_L$ through $a = \lambda_\infty^L$ to $a = \lambda_0^R$ (still labeled as ψ_L) to $a = 0_R$. The UV and IR scales place a fundamental limit on what is, in principle, observable. It is clear λ_0 is the Planck length and m_0 is the Planck mass, i.e., the smallest possible black hole. So what is the ultra-light mass m_∞ that sets the largest scale the observer shall ever see?

Nature has no fundamental scale, but the observer does. In the ant's-eye view, i.e., from the perspective of observers living in $\mathcal{CT}^2$, there is a fundamental limit to the physical information any apparatus can collect. In some sense, the observer is simply a finite number of qubits.

We have operationally derived the Born rule but have not explained what probability means in Everettian QM. By understanding probability in the multiverse, can we answer more precisely what quantum geometry is and emerge spacetime and quantum fields?

D. the algebra of physical information

Another way of calculating Ψ is purely algebraically. Every point in $\mathbb{R}^3$ has the following matrix representation:

$$Q = \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3 \tag{14}$$

where the σ's are the generators of $SU(2)$. Each eigenvalue is the Euclidean l^2-norm, $a = \pm||\mathbf{x}||_2$ and $\mathrm{tr}(Q) = a + (-a) = 0$. The left and right-handed states are computed as

$$\psi_L = \mathrm{tr}\ Q^n = (1 + e^{in\pi})a^n \tag{15}$$

and

$$\psi_R = \mathrm{tr}\ Q^{-n} = (1 + e^{-in\pi})a^{-n} \tag{16}$$

where $n \in [0, 2)$. One interprets Q as a qubit embedded in $\mathbb{R}^3$ with an equivalence relation $a \sim \lambda a$ for all real numbers $\lambda \neq 0$ that identifies the 2-sphere (of any arbitrary size) as the Bloch sphere. One can always find a λ such that λa has unit norm and identify the 2-sphere as the Bloch sphere. In other words, there is a qubit at every point in $\mathcal{CT}^2$. A deductive consequence of the Everettian conformal bootstrap is the concept of a *qubit continuum* (or Q-continuum) where points of a manifold are qubits. Such models are non-trivial generalizations of n-level quantum systems and naturally resemble the structure of quantum fields in the continuum limit of lattice qubit models [18, 19]. Intriguingly, lattice qubit models display UV/IR mixing from global subsystem symmetries [20–22], similar in spirit to the UV/IR mixing that was encountered in the previous section. To answer our previous question: probability densities in the multiverse are physical densities of a quantum information continuum (which becomes conceptually explicit in section II F). This answer begs a new question: are qubit manifolds a window into the nature of quantum geometry?

Recall from the section II A that the symmetry group of the Hopf fibration is $U(2)$. By the exceptional spin isomorphism $\mathrm{Spin}(3,1) \cong \mathrm{SL}(2,\mathbb{C})$, one may identify the points x_μ in Minkowski space with the infinitesimal generators of $U(2)$. In other words, points on the Bloch sphere can always be mapped to rays of the lightcone. Thus every point of $\mathcal{CT}^2$ is also a celestial sphere. The boundary equation of motion, eq. (4), is defined by the Clifford algebra $C\ell_3(\mathbb{R})$. It follows from the exceptional spin isomorphism that the bulk equations of motion are defined by the Dirac algebra $C\ell_{3,1}(\mathbb{C})$, with the simplest holographic dictionary given by

$$D \leftrightarrow \not{D}, \quad V \leftrightarrow V_{\mu\nu} \tag{17}$$

where $\not{D} = i\gamma^\mu \partial_\mu$ is the Dirac operator and $V_{\mu\nu}$ is a second-rank tensor that decomposes into the following classical irreducible representations of the Lorentz group: an antisymmetric tensor $F_{\mu\nu} = -F_{\nu\mu}$, a symmetric ($H_{\mu\nu} = H_{\nu\mu}$) and traceless ($H^\mu{}_\mu = 0$) tensor $H_{\mu\nu}$, and a scalar trace component $X \equiv X^\mu{}_\mu$. Under the appropriate local phase (gauge) invariance conditions, $H_{\mu\nu}$ is a massless spin-2 field—the graviton. The bulk equations of motion then take the following form:

$$\begin{aligned}
(\not{D}^2 + V_{\mu\nu})\Psi &= (-\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu + V_{\mu\nu})\,\Psi \\
&= \left(\eta^{\mu\nu}\partial_\mu\partial_\nu + F_{\mu\nu} + H_{\mu\nu} + X^\mu{}_\mu\right)\Psi \\
&= \left(\partial^\mu\partial_\mu + g_1 B_{\mu\nu} + g_2 W_{\mu\nu} + g_3 H_{\mu\nu} + m^2\right)\Psi \\
&= \left(\Box + g_1(\partial^\mu B^\nu - \partial^\nu B^\mu) + g_2(\partial^\mu W_i^\nu - \partial^\nu W_i^\mu + g_w \epsilon^{ijk} W_j^\mu W_k^\nu) + g_3 H_{\mu\nu} + m^2\right)\Psi \\
&= 0
\end{aligned} \tag{18}$$

The action of the unitary group $U(2)$ on $\mathbb{C}^2$ leaves the 3-sphere invariant and carries fibers into fibers as it commutes with the $U(1)$ action, and thus descends to an action on the 2-sphere by ordinary rotations. Locally $U(2)$ is diffeomorphic to the direct product $G = SU(2) \times U(1)$. The compact gauge group G, together with the emergence of the antisymmetric tensor $F = F_{\mu\nu}$, implies the existence of Yang-Mills equations, where A denotes the gauge connection on G. The connection A is locally a one-form on space-time. Thus the curvature or field strength tensor is the two-form: $F = dA + A \wedge A$. The classical field equations are therefore given by the Yang-Mills equation: $d_A \star F = 0$, where d_A is the gauge-covariant extension of the exterior derivative and $\star$ is the Hodge duality operator. Thus the antisymmetric tensor $F_{\mu\nu}$ is expanded into the electromagnetic and weak field strength tensors in eq. (18). The electroweak interactions have emerged with the graviton from the Q-continuum!

E. quantum gravity

In semiclassical gravity, the simplest metric is Minkowksi with some quantum fluctuations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{19}$$

where $h_{\mu\nu}$ is a symmetric and traceless rank-2 tensor. From eq. (12), it was deduced that the quantum metric tensor is an integral kernel operator, hence so too is the spin-2 field as it appears in the bulk equations of motion:

$$H_{\mu\nu} \to \int d\Psi K_h(\Psi, \overline{\Psi}). \tag{20}$$

The classical irreducible representation of the Lorentz group, $H_{\mu\nu}$, is promoted to a Hilbert-Schmidt integral kernel:

$$\int\int d\Psi d\overline{\Psi} |K_h(\Psi, \overline{\Psi})|^2 < \infty, \tag{21}$$

that is a symmetric

$$K_h(\Psi, \overline{\Psi}) = K_h(\overline{\Psi}, \Psi), \tag{22}$$

traceless trace-class operator:

$$tr K_h = \int d\Psi K_h(\Psi, \Psi) = 0. \tag{23}$$

Gravity is exceptional in that it acts on the quantum state as an integral kernel operator, whereas the other forces act on the quantum state as differential operators. The classical irrep. $H_{\mu\nu}$ of the Lorentz group is fully quantized and makes eq. (18) a highly non-linear differential equation, where $H_{\mu\nu}\Psi \sim V_g(\Psi)$ can be thought of as a quantum gravitational potential, and some complicated function of the 4-spinor field Ψ that arises from quantum gravitational self-interactions. In the spin-free limit where $g_1 \to g_2 \to g_3 \to 0$, eq. (18) reduces to the Klein-Gordon equation:

$$\left(\Box + m^2\right) \Psi = 0, \tag{24}$$

hence why the scalar trace component $X^{\mu}_{\ \mu} = m^2$ was chosen. The coupling constants g_i are such that the fields have units of squared mass: the bulk equations of motions are four (gauge-coupled) non-linear Klein-Gordon equations. Because gravity is the weakest force, in almost all cases, the quantum gravitational backreaction is negligible at low energies, except for exotic scenarios that violate decoupling.

Eq. (18) is a formidable beast to solve. What are the appropriate boundary conditions? Once an appropriate boundary condition and a spin-2 kernel are chosen, one needs to update the 4-gradient ∂_μ such that the equation is gauge invariant and then fix the gauge. Because the equation is a) non-linear and b) contains multiple gauge fields, these procedures are significantly more complicated than in quantum field theory (QFT) without gravity.

Despite the troubles eq. (18) faces, it is utterly remarkable something so complicated can be derived from something as simple as the principle of quantum universality. The deductive conformal bootstrap takes Everettian QM and spins out a full-fledged theory of quantum gravity, albeit not a very well-understood one.

The first step in demonstrating a holographic duality is to match the energy spectrum of the bulk and boundary theories. The boundary theory is based on the energy E, which is zero, whereas the bulk is based on the squared energy E^2, which is non-zero and has a negative energy solution. The conformal invariance of the boundary is locally broken from the ant's-eye perspective of the bulk observer. So how can it be that these two theories are equivalent? How is a dynamical non-linear differential equation secretly a non-dynamical linear one? There must be a highly non-trivial duality between equations (4) and (18) for self-consistency.

The 4-spinor Ψ in eq. (18) decomposes into a pair of 2-spinors with equal and opposite energies, χ_L and χ_R, respectively, that satisfies two coupled non-linear Weyl-like equations

$$\left(-\sigma^\mu \sigma^\nu \partial_\mu \partial_\nu + V\right) \chi_L = 0 \tag{25}$$

$$\left(-\overline{\sigma}^\mu \overline{\sigma}^\nu \partial_\mu \partial_\nu + \overline{V}\right) \chi_R = 0 \tag{26}$$

where $\sigma^\mu = (I_2, \sigma^i)$ and $\sigma^\nu = (-I_2, \sigma^i)$, i.e., the generators of $U(2)$. Recall what we learned from the geometry of quantum information: Minkowski space has a compact presentation $U(2)$, and the fundamental representation of the Lorentz group contains two fermionic directions. The potential V can be re-written to contain the same information as $V_{\mu\nu}$ by recasting the field strength tensors in the standard Pauli matrix representation with the Clifford algebra $C\ell_{3,0}(\mathbb{R})$, otherwise known as the algebra of physical space:

$$\left(\Box + g_1 F + g_2 W + g_3 H + m^2\right) \chi_L = 0 \tag{27}$$

$$\left(\Box + g_1 \overline{F} + g_2 \overline{W} + g_3 \overline{H} + m^2\right) \chi_R = 0 \tag{28}$$

In the limit that $g_1 \to g_2 \to g_3 \to 0$, the above reduces to the Klein-Gordon equation. Thus we conclude equation (18) is the adjoint representation where the above is the fundamental representation.

The Dirac field can be written as a sum of chiral Weyl fields. The boundary theory has no timelike direction, so its solutions obviously can not be a linear combination of the non-linear states χ_L and χ_R. However, the quantum metric tensor is missing from our recipe. One can reasonably conjecture that the dynamic non-linear bulk is mapped to the timeless linear boundary by applying the quantum metric tensor K_g (also a Hilbert-Schmidt integral operator) to the states χ_L and χ_R,

$$\chi'(a,\phi) = \int_{-\infty}^{\infty} K_g(a,\phi;a',t)\chi(a',t)dt \tag{29}$$

such that

$$\Psi \sim \chi'_L + \chi'_R \tag{30}$$

is a solution to $(D^2+V)\Psi = 0$. The quantum metric tensor integrates out the t direction and linearizes the chiral fields such that their linear combinations are solutions to the boundary equations of motion. One interpretation of K_g is a propagator of all possible isometries between a phase rotation in $\mathcal{CT}^2$ and the observer-dependent time evolution of a local patch in the bulk. The integral operator formalism is like a path integral over trajectories not in space but in time, i.e., an integral over all possible rates an observer's clock can tick. Understanding the holographic dictionary at a deeper level requires a better understanding of the integral operator algebra that encodes the quantum geometry, which is essential to gauging and solving eq. (18). Nevertheless, one does not need to solve the equation to derive precise observational numbers.

F. quantum cosmology

Negative energy solutions can be reinterpreted as positive energy solutions traveling backward in time. If we head back to fig. (1), it is clear the phase rotation of ψ_L and ψ_R describe expanding and contracting states, respectively. The contracting state is a perfect mirror image of the expanding state, as can be seen when plotting $\rho_0(a) = \int_0^{2\pi} \psi_L\overline{\psi}_L d\phi$ and $\rho_\infty(a) = \int_0^{2\pi} \psi_R\overline{\psi}_R d\phi$. **The Born rule is the density of worlds with scale factor a in the multiverse.** This mirror symmetry implies ψ_R is, in some sense, the time-reversed image of ψ_L. However, to be clear, the Universe is not in some *definite* state at some time t, at least not from the God's-eye view where everything that can happen happens everywhere all at once.

The imaginary axis β in fig. (1) does not have the topology of $i\mathbb{R}$; it runs from zero through infinity and back to zero. Recall two of the four dimensions are suppressed. The imaginary axis is, in fact, the thermal circle S^1_β. One can see this clearly from the fact that at the cosmological singularity $a = 0$, the imaginary variable reads $\beta = 0$. Ergo, β is the inverse temperature. The equivalence of inverse temperature and cyclic imaginary time, expressed through the formula $\beta = it$, is manifest from the geometry of $\mathcal{CT}^2 \cong S^1_a \times S^1_\beta$. One can therefore justify using the Wick rotation to simplify eq. (18). Quantum gravity is fundamentally Euclidean. The thermal circle implies the de Sitter or Hawking temperature:

$$T \sim \frac{H}{2\pi} \tag{31}$$

where H is the Hubble constant. The boundary state Ψ has the global phase symmetry $U(1)^2 = diag(e^{-i\phi}, e^{i\phi})$, i.e., the maximal torus of $U(2)$. The action of $U(1)^2$ takes the components of Ψ to their complex conjugates. In gravity, the charge is mass (energy), so the conserved charge must be the gravitational charge m_∞. The charge $m_0 = M_{pl}$ can not be fundamental as it radiates away with Hawking temperature $T \sim g/2\pi$, where g is the surface gravity of the horizon at $r = \lambda_0 = L_{pl}$. Restoring the inverse mass matrix $M^{-1} = diag(1/m_0, 1/m_\infty)$ to $V(a,\phi)$, and it is clear m_0 decays into m_∞. The left-hand potential V_L is suppressed by a factor of $\phi/\pi - 1$ whereas V_R is amplified by a factor of $\phi/\pi + 1$. Thus one concludes m_0 is not fundamental and must be a composite of m_∞. **The gravitational charge m_∞ defines the boundaries of the observable universe, beyond which no measurement can be made.**

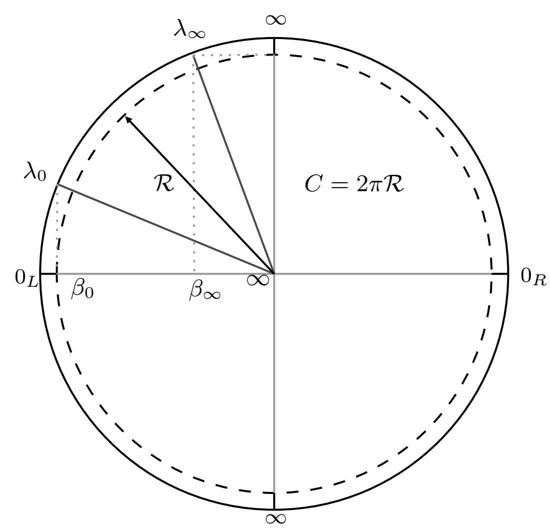

FIG. 3. From the ant's-eye view in $\mathcal{CT}^2$, the conformal horizon hides the cosmological singularity and its dual at future infinity. The dimensionless radius $\mathcal{R}$ bounds all observables to the energy scale $m_\infty \leq m \leq m_0$. Naturally, β_0 is the inverse Planck temperature, and β_∞ is the time of heat death, defined by the moment the universe becomes so dilute no measurement can be made.

Everything that can be empirically known of the universe is bounded by a finite number of qubits:

$$Z_{\text{obs}} \sim e^{2\pi\mathcal{R}} \tag{32}$$

where

$$\mathcal{R} = \frac{\lambda_\infty}{\lambda_0} \tag{33}$$

defines the dimensionless radius of the conformal horizon that hides the cosmological singularity and its dual at future infinity. The entropy of the cosmic event horizon is

$$S_{\text{CEH}} \sim 2\pi\mathcal{R}, \tag{34}$$

which is the circumference of the conformal horizon pictured as the dashed circle in fig. (3).

Recall from the algebra of physical information (API) that ψ_L and ψ_R are computed by embedding a 2-sphere at each point in $\mathbb{R}^3$ with radius a, and normalizing the 2-sphere to unit length (i.e., to the Bloch sphere) with a characteristic scale λ. The scale λ is identified as the wavelength of light emitted through the Hawking temperature.

One can calculate the location of the cosmic event horizon as witnessed by the ant from the God's-eye view. The spinor components $\psi_L, \psi_R \in L^2(S_a^2 \times S_\beta^1)$. Express the densities $\rho_0(a)$ and $\rho_\infty(a)$ as a function of the surface area of the 2-sphere with the scale transformation $a \to \sqrt{\pi}a$:

$$\rho_0(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_0^2}\right)^{\phi/\pi} d\phi \tag{35}$$

and

$$\rho_\infty(A) = 4\pi^2 \int_0^{2\pi} \cos^2\left(\frac{\phi}{2}\right)\left(\frac{s^2 A}{\lambda_\infty^2}\right)^{-\phi/\pi} d\phi. \tag{36}$$

such that $A = 4\pi a^2$ and the spin $s = \pm 1/2$. One can re-express the above as functions of the Bekenstein-Hawking entropy (the entropy from the ant's-eye perspective) by the identification:

$$S_\alpha = \frac{s^2 A}{\lambda_\alpha^2} = \frac{A}{4\lambda_\alpha^2} \tag{37}$$

with the conformal indices $\alpha = 0, \infty$. ($\alpha = \infty$ is the $\mathcal{R} \to 1/\mathcal{R}$ dual.) We now have two equivalent definitions of entropy as seen from the ant's-eye view and can calculate the radius of the cosmic event horizon:

$$\begin{aligned} \frac{\pi R_{\text{CEH}}^2}{\lambda_0^2} &= 2\pi \frac{\lambda_\infty}{\lambda_0} \\ R_{\text{CEH}} &= \sqrt{2\lambda_0 \lambda_\infty} \\ &= \sqrt{\frac{2}{\Lambda}} \\ &\approx 14.26 \text{ billion ly} \end{aligned} \tag{38}$$

where the cosmological constant is identified as $\Lambda = \frac{1}{\lambda_0 \lambda_\infty}$. It is straightforward to calculate m_∞ and verify that $\frac{m_\infty}{m_0} = \Lambda \lambda_0^2$ where the vacuum energy density (in reduced Planck units) is given by $\rho_\Lambda = \rho_\infty = \frac{m_\infty c^2}{\lambda_0^3}$.

The presence of the cosmic event horizon R_{CEH} means there is a natural factorization of the cosmological Hilbert space of states from the ant's-eye view: $\mathcal{H}$ separates into an infinite and finite-dimensional piece $\mathcal{H}_{\text{env}} \otimes \mathcal{H}_{\text{obs}}$, where *dim* $\mathcal{H}_{\text{obs}} \leq Z_{\text{obs}}$. The implied *universal relativity* is this: everything that can be empirically known of the universe is relative to a conformal horizon that hides the cosmological singularity and its dual at future infinity. From the God's-eye view, there is nothing to factorize! There are no subsystems that interact with each other at some *definite* moment in time. On the boundary, everything that can happen happens everywhere all at once. The boundary Hilbert space has only one state: $Z = e^S = 1$. The boundary state Ψ is pure: $S = 0$. Any ray in $\mathcal{CT}^2$ can be rotated into any other ray by a conformal transformation. Nature has no fundamental scale. The continuum of 2-spheres of radius a that were normalized to unity for identification with the Bloch sphere are all the same state, *from the God's-eye view*. In the bulk, from the ant's-eye view, the conformal horizon hides information from the observer. The hidden information is the entanglement entropy of the cosmic event horizon that separates the observable universe from the multiverse. Observers live at the center of a quantum 2-sphere with radius R_{CEH} and spin-1/2.

G. dark energy holes & the dS limit

The Bekenstein-Hawking area entropy law is a geometric consequence of the fact that $\psi \in L^2(S_a^2 \times S_\beta^1)$. In Hawking and Gibbons's original paper, they were able to compute the action precisely because the Euclidean section is non-singular: the entropy can be evaluated on a region of a spacetime manifold M bounded by some surface $r = r_0 > 2M$, whose boundary ∂M has compact topology $S^2 \times S^1$ (i.e., a 2-sphere cross periodic time) [23]. Since the Ricci scalar vanishes in the Schwarzschild metric, the action is determined entirely by the Gibbons-Hawking-York boundary term, meaning the action is an integral over the boundary $S^2 \times S^1$. The squared modulus $\rho = |\psi|^2$ is nothing but an integral over the boundary $S_a^2 \times S_\beta^1$ and precisely why one can write $\rho = \rho(S)$ with $S = \frac{A}{4\lambda^2}$. But what are the microstates that the Bekenstein-Hawking area entropy counts?

Nature has no fundamental scale: the fundamental mass matrix M can be multiplied out of $\hat{H}\Psi = 0$. It can therefore have any arbitrary value such that $\mathcal{R}$ remains invariant in all worlds. Geometrically, this is seen as a rotation of λ_0 into λ_0' and λ_∞ into λ_∞', such that $\mathcal{R} = \mathcal{R}'$. Now $r = \lambda_0'$ is the Compton wavelength of a black hole of arbitrary mass m_0', a composite of m_∞'. Since all massive objects dilute proportional to the inverse volume a^{-3} of Euclidean 3-space, all black holes must have a mass coupling $m \sim a^3$ to be a dark energy species. **This observational consequence will soon provide the first experimental test of the multiverse.**

We have predicted the radius of the cosmic event horizon, so where is the de Sitter limit with the dark energy equation of state $w = p/\rho_\Lambda = -1$? Is there an explicit holographic construction where the charge m_∞ is a microstate of m_0?

Consider a flat acceleration surface with $2\pi k_B$ entropy per unit (Planck) area $1/\lambda_0^2$. Now multiply this quantity by the Hawking temperature per unit length of the accelerating surface:

$$\frac{T}{\lambda_0} = \frac{\hbar g}{2\pi c k_B \lambda_0}, \quad (39)$$

where g is the acceleration. The result is the pressure

$$p_0 = \left(\frac{c^2}{G\lambda_0}\right) g, \quad (40)$$

where the quantity in parentheses is an *area density* with units [mass][area]$^{-1}$. The physical construction is an accelerating membrane, like the surface of a glowing soap bubble. The above equation is the ideal gas law $pV = k_B T$ where $V = \lambda_0^3$ is the unit volume swept out by the membrane. The above is nothing but $F = m_0 g$, Newton's second law! Now consider the dual acceleration surface with $4\pi k_B$ entropy per unit R_{CEH}^2 and repeat the above steps to obtain:

$$p_\infty = \left(\frac{\Lambda\hbar}{c\lambda_0}\right) g, \quad (41)$$

which is just $F = m_\infty g$ when multiplied by λ_0^2 on both sides. The above pressures obey the ideal gas law, so we can construct the stress-energy tensor $T_{\mu\nu} = diag(\rho_\infty, p_\infty, p_\infty, p_\infty)$. In the UV limit,

$$\lim_{g\to -c^2/\lambda_0} T_{\mu\nu} = \begin{pmatrix} \rho_\Lambda & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \quad (42)$$

one obtains the vacuum stress-energy tensor $T_{\mu\nu}^{(\mathrm{vac})}$. The sign on g distinguishes the expanding and contracting states. Indeed, the first thing we learned from the bootstrap is that energy and curvature are equivalent. There are not many options to equate $T_{\mu\nu}^{(\mathrm{vac})}$ to curvature on the left-hand side. The simplest way to do this is none other than Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \quad (43)$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, and $\kappa = 8\pi G$. GR is a *deductive* consequence of quantum universality. There is then the dual vacuum stress-energy tensor for the contracting state. As before, we can construct a stress-energy tensor $T_{\mu\nu} = diag(\rho_0, p_0, p_0, p_0)$ for an ideal gas. In the IR limit,

$$\lim_{g\to c^2/\lambda_\infty} T_{\mu\nu} = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \quad (44)$$

one obtains a bizarre substance with the Planck energy density and a positive vacuum energy pressure (with the minus sign absorbed)

$$p = \frac{\Lambda}{8\pi G}, \quad (45)$$

that satisfies the equation of state for dark matter: $w = p/\rho_0 \approx 0$. Are

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{(\mathrm{dual\text{-}vac})} \quad (46)$$

the classical field equations for dark matter? Does this explain how the cosmological constant Λ enters local galaxy dynamics through a universal acceleration scale $a_0 \approx cH_0 \approx c^2\Lambda^{1/2}$ [24], and how something as small as a galaxy "knows" about the dS radius associated with the whole of the observable universe? Are dark energy and the dark matter emergent properties of the universal quantum state Ψ, and is UV/IR mixing a general feature of Everettian quantum gravity?

If nature has no fundamental scale, is eq. (18) valid for astrophysical modeling? Can we use the equation to understand the dynamics of black holes, as suggested by the boundary equations of motion? What new and potentially exotic phenomena emerge from coupling quantum gravity with the electroweak force and solving eq. (18)? How do the strong interactions enter the picture? Is the double-copy a clue [25]? What are the conceptual and technical foundations to fixing the spin-2 integral kernel operator K_h, gauge fixing eq. (18), and mapping its solutions to the boundary with the quantum metric tensor K_g? How does one solve for K_g?

Much work is to be done.

III. IMPLICATIONS

The 'many worlds' in Everettian QM are causally disconnected when a classical global time coordinate is assumed. It is clear that if time is not fundamental, the classical notions of causality break down. When time drops out of the Schrödinger equation, the Hilbert space becomes causally indefinite. The conformal Clifford torus has no causal ordering—light cones can be superimposed. Remarkably, in the first of its kind, the quantum switch experiment has demonstrated *causal indefiniteness* [26, 27]. Since there is no global time coordinate, there must exist a *causal reference frame* [28–31] for each event that establishes an observer-dependent time to describe the evolution of quantum subsystems. I expect causal reference frames to be key in gauge fixing and solving eq. (18).

If quantum theory is truly universal and if UV/IR mixing is a general feature of the quantum universe, what are the implications of causal indefiniteness? Can exotic UV/IR couplings that violate EFT reasoning together with quantum non-causality enable the quantum metric tensor to be technologically exploited? Looking at equations (25) - (30), it is, at least in principle, possible to engineer the quantum metric tensor with some highly tuned matter-antimatter reaction. This begs the question: does quantum universality allow travel between the many worlds? At face value, there does not appear to be any inconsistencies with a traversable multiverse. If quantum universality is true and the multiverse exists, then there must exist worlds where future humans master the quantum nature of spacetime and travel to "past-adjacent" light cones—a form of time travel in the multiverse. But if this were possible, then where are they?

ACKNOWLEDGMENTS

Thank you Sabine Hossenfelder for writing your pellucid book, *Lost in Math: How Beauty Leads Physics Astray*, which course-corrected this research program to what it is today. Thank you Prof. Niayesh Afshordi for the productive conversations and words of encouragement. Thank you Prof. David Peak, for the many correspondences on this work, the helpful feedback, and, most importantly, the words of encouragement. Lastly, thank you mom (Jodi Benowitz) for always being there when I needed you.

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