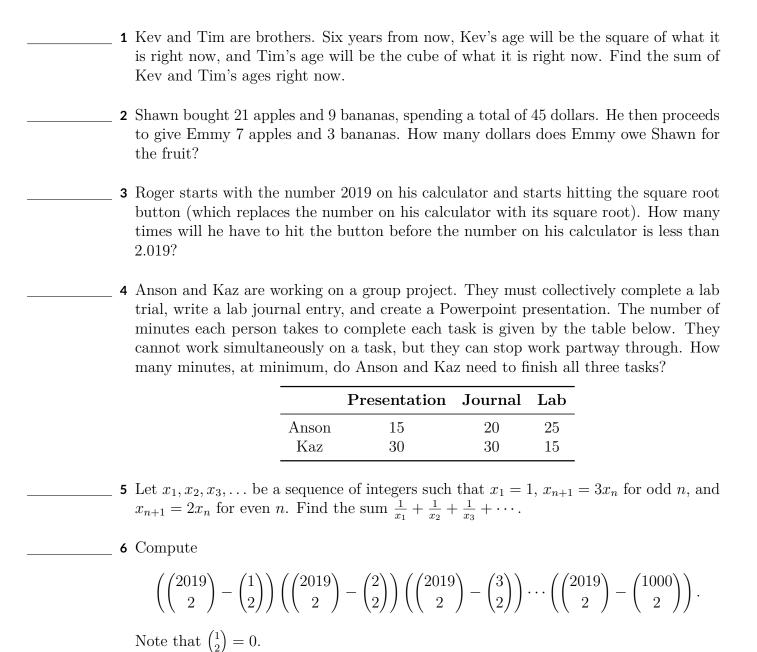# MBMT Algebra Round — Leibniz

March 30, 2019

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 Kev and Tim are brothers. Six years from now, Kev's age will be the square of what it is right now, and Tim's age will be the cube of what it is right now. Find the sum of Kev and Tim's ages right now.

2 Shawn bought 21 apples and 9 bananas, spending a total of 45 dollars. He then proceeds to give Emmy 7 apples and 3 bananas. How many dollars does Emmy owe Shawn for the fruit?

3 Roger starts with the number 2019 on his calculator and starts hitting the square root button (which replaces the number on his calculator with its square root). How many times will he have to hit the button before the number on his calculator is less than 2.019?

4 Anson and Kaz are working on a group project. They must collectively complete a lab trial, write a lab journal entry, and create a Powerpoint presentation. The number of minutes each person takes to complete each task is given by the table below. They cannot work simultaneously on a task, but they can stop work partway through. How many minutes, at minimum, do Anson and Kaz need to finish all three tasks?

	Presentation	Journal	Lab
Anson	15	20	25
Kaz	30	30	15

5 Let $x_1, x_2, x_3, \ldots$ be a sequence of integers such that $x_1 = 1$, $x_{n+1} = 3x_n$ for odd n, and $x_{n+1} = 2x_n$ for even n. Find the sum $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots$.

6 Compute

$$\left(\binom{2019}{2} - \binom{1}{2}\right)\left(\binom{2019}{2} - \binom{2}{2}\right)\left(\binom{2019}{2} - \binom{3}{2}\right)\cdots\left(\binom{2019}{2} - \binom{1000}{2}\right).$$

Note that $\binom{1}{2} = 0$.

7 Compute

$$2^{2^0} + \sqrt{2^{2^1} + \sqrt{2^{2^2} + \cdots}}.$$

8 Let $f_1(x) = x/4$, $f_2(x) = x/2 + 1/4$, and $f_3(x) = x/4 + 3/4$. There exist positive integers $n_1, n_2, \ldots \in \{1, 2, 3\}$ and real numbers $x_1, x_2, \ldots$ with $|x_i| < 2019$ such that, for all positive integers k,

$$f_{n_1}(f_{n_2}(\cdots f_{n_k}(x_k)\cdots)) = \frac{1}{3}.$$

Find $n_1 + n_2 + \cdots + n_{2019}$.