Montgomery Blair Math Tournament Online Round

June 13-17, 2020

This round consists of **45** questions. You will have from **June 13 to June 17** to complete the round. Point values for questions are based on the number of solves; see the submission site for more details.

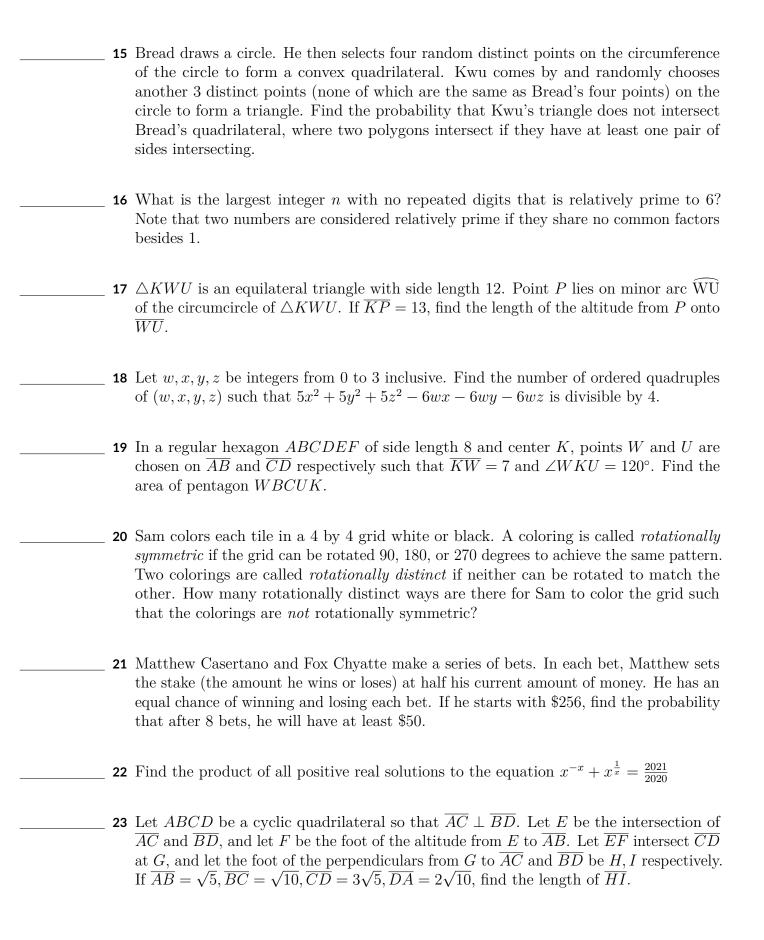
Problems **41-45** were released on **June 16** as optional enrichment problems. Please note that these problems will not contribute to the final team score. However, you can still submit answers to these problems and receive feedback for them.

Submissions and Information:

https://online.mbmt.mbhs.edu

1 Chris has a bag with 4 black socks and 6 red socks (so there are 10 socks in total). Timothy reaches into the bag and grabs two socks without replacement. Find the probability that he will not grab two red socks.
 2 Daniel, Clarence, and Matthew split a \$20.20 dinner bill so that Daniel pays half of what Clarence pays. If Daniel pays \$6.06, what is the ratio of Clarence's pay to Matthew's pay?
 3 Square \overrightarrow{ABCD} has a side length of 1. Point E lies on the interior of $ABCD$, and is on the line \overrightarrow{AC} such that the length of \overline{AE} is 1. Find the shortest distance from point E to a side of square $ABCD$.
 4 Ken has a six sided die. He rolls the die, and if the result is not even, he rolls the die one more time. Find the probability that he ends up with an even number.
5 Fuzzy draws a segment of positive length in a plane. How many locations can Fuzzy place another point in the same plane to form a non-degenerate isosceles right triangle with vertices consisting of his new point and the endpoints of the segment?
6 Given that $\sqrt{10} \approx 3.16227766$, find the largest integer n such that $n^2 \leq 10,000,000$.
 7 Let $S = \{1, 2, 3,, 12\}$. How many subsets of S , excluding the empty set, have an even sum but not an even product?

8	Let $\triangle ABC$ be inscribed in circle O with $\angle ABC = 36^{\circ}$. D and E are on the circle such that \overline{AD} and \overline{CE} are diameters of circle O . List all possible positive values of $\angle DBE$ in degrees in order from least to greatest.
9	Consider a regular pentagon $ABCDE$, and let the intersection of diagonals \overline{CA} and \overline{EB} be F . Find $\angle AFB$.
10	Mr. Squash bought a large parking lot in Utah, which has an area of 600 square meters. A car needs 6 square meters of parking space while a bus needs 30 square meters of parking space. Mr. Squash charges 2.50 per car and 7.50 per bus, but Mr. Squash can only handle at most 60 vehicles at a time. Find the ordered pair (a, b) where a is the number of cars and b is the number of buses that maximizes the amount of money Mr. Squash makes.
11	There are 8 distinct points on a plane, where no three are collinear. An ant starts at one of the points, then walks in a straight line to each one of the other points, visiting each point exactly once and stopping at the final point. This creates a trail of 7 line segments. What is the maximum number of times the ant can cross its own path as it walks?
12	Find the number of ways to partition $S = \{1, 2, 3, \dots, 2020\}$ into two disjoint sets A and B with $A \cup B = S$ so that if you choose an element a from A and an element b from B , $a + b$ is never a multiple of 20. A or B can be the empty set, and the order of A and B doesn't matter. In other words, the pair of sets (A, B) is indistinguishable from the pair of sets (B, A) .
13	How many ordered pairs of positive integers (a, b) are there such that a right triangle with legs of length a, b has an area of p , where p is a prime number less than 100?
14	Mr. Schwartz has been hired to paint a row of 7 houses. Each house must be painted red, blue, or green. However, to make it aesthetically pleasing, he doesn't want any three consecutive houses to be the same color. Find the number of ways he can fulfill his task.



24 Nashan randomly chooses 6 positive integers a, b, c, d, e, f . Find the probability that
$2^a + 2^b + 2^c + 2^d + 2^e + 2^f$ is divisible by 5.

25 Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Find the sum of all positive integer solutions to

$$\left| \frac{n^3}{27} \right| - \left| \frac{n}{3} \right|^3 = 10.$$

- **26** Let $\triangle MBT$ be a triangle with $\overline{MB} = 4$ and $\overline{MT} = 7$. Furthermore, let circle ω be a circle with center O which is tangent to \overline{MB} at B and \overline{MT} at some point on segment \overline{MT} . Given $\overline{OM} = 6$ and ω intersects \overline{BT} at $I \neq B$, find the length of \overline{TI} .
- 27 The perfect square game is played as follows: player 1 says a positive integer, then player 2 says a strictly smaller positive integer, and so on. The game ends when someone says 1; that player wins if and only if the sum of all numbers said is a perfect square. What is the sum of all n such that, if player 1 starts by saying n, player 1 has a winning strategy? A winning strategy for player 1 is a rule player 1 can follow to win, regardless of what player 2 does. If player 1 wins, player 2 must lose, and vice versa. Both players play optimally.
- **28** Consider the system of equations

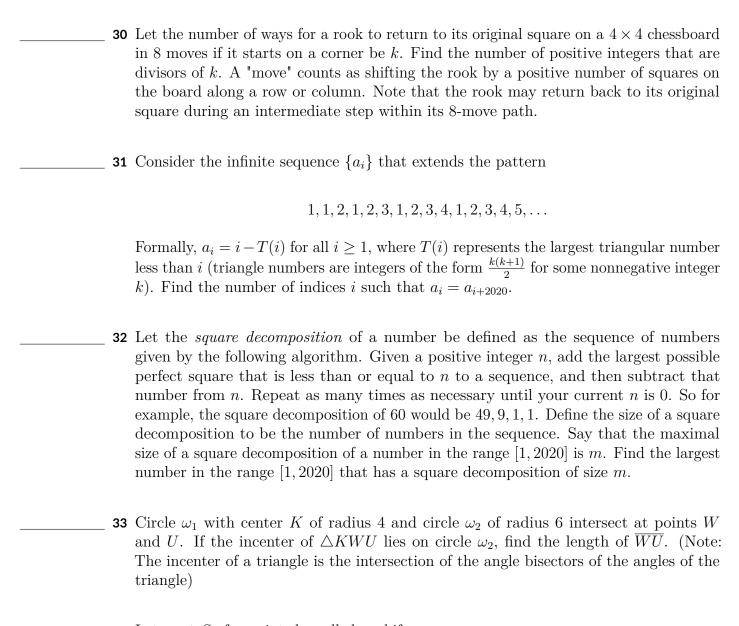
$$a + 2b + 3c + \ldots + 26z = 2020$$

 $b + 2c + 3d + \ldots + 26a = 2019$
 \vdots
 $y + 2z + 3a + \ldots + 26x = 1996$
 $z + 2a + 3b + \ldots + 26y = 1995$

where each equation is a rearrangement of the first equation with the variables cycling and the coefficients staying in place. Find the value of

$$z + 2y + 3x + \dots + 26a.$$

29 The center of circle ω_1 of radius 6 lies on circle ω_2 of radius 6. The circles intersect at points K and W. Let point U lie on the major arc \widehat{KW} of ω_2 , and point I be the center of the largest circle that can be inscribed in $\triangle KWU$. If KI + WI = 11, find $KI \cdot WI$.



- **34** Let a set S of n points be called cool if:
 - All points lie in a plane
 - No three points are collinear
 - There exists a triangle with three distinct vertices in S such that the triangle contains another point in S strictly inside it

Define g(S) for a cool set S to be the sum of the number of points strictly inside each triangle with three distinct vertices in S. Let f(n) be the minimal possible value of g(S) across all cool sets of size n. Find

$$f(4) + \cdots + f(2020) \pmod{1000}$$

