Triadic Ego Domination in Social Network Analysis

Moses A. Boudourides

Visiting Associate Professor
Computer Science Department
Haverford College
Haverford, PA

Moses.Boudourides@cs.haverford.edu

The Social Media Research Winter School 2022
Virtual

12:00 pm EDT, Monday, January 17, 2022

- **Triangles (in a Undirected Graph)**
- **Structural Holes**
- **Local Clustering**
- **Local Bridges**
- **Ego Networks**
- **Minimum Dominating Sets (Domination Number)**

Georg Simmel, Paul Holland (& Sam Leinhardt), Ronald Burt, Duncan Watts & Steven Strogatz





General Notions, I

Let G a undirected graph (social network) with adjacency matrix A. Assume that the diameter of G is greater than two. Let i be a node with its degree denoted as d_i.

- The **number of triangles** having i as a vertex (such triangles will be said **incident** to i) is denoted as $\tau(i)$:

$$\tau(i) = \frac{1}{2}\{A^3\}_{ii}.$$

- We say that node i is a **clustered node**, if $\tau(i) > 0$, and **traversing node**, otherwise.
- If $d_i > 1$, the (**local**) **clustering coefficient** of node i is:

$$C(i) = \frac{2\tau(i)}{d_i(d_i - 1)}.$$

- We say that an edge (tie) in G is a (**local**) **bridge**, if this edge is not a side of any triangle in the graph.

General Notions, II

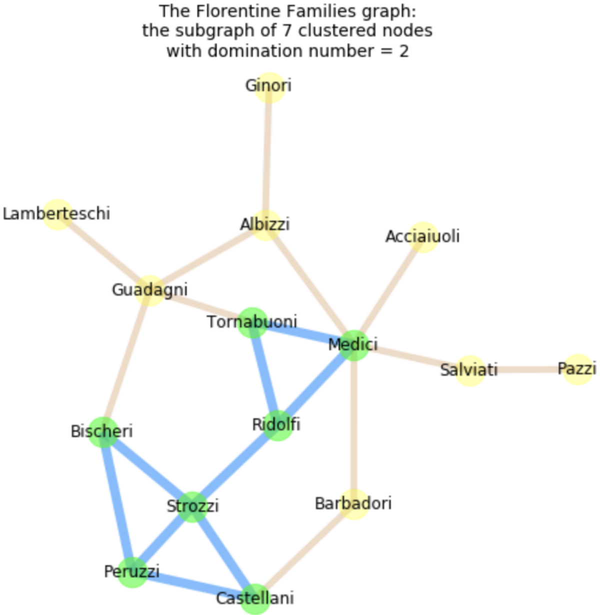
- If G is complete, all nodes are clustered.
- If G is either a tree or a k–cycle graph (where $k > 3$) or a bipartite graph or a graph without triangles, all nodes are traversing.
- If i is a clustered node (i.e., $\tau(i) > 0$), node i can be identified to the **ego** of a nontrivial **ego network** comprising ego i together with its neighbors, called **alters**, which should span $\tau(i)$ edges (ties) of the graph. Thus, the **redundancy** of ego i is (c.f. Burt, Borgatti):

$$\text{redundancy}(i) = \frac{2\tau(i)}{d_i} = (d_i - 1)C(i).$$

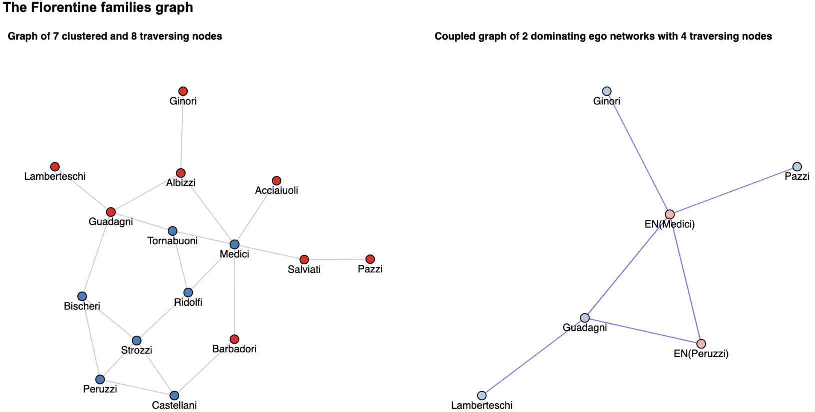
General Notions, III

- Denoting by $\tau = \frac{1}{3}\sum \tau(i) = \frac{1}{6}\mathrm{Tr}(A^3)$ the total number of triangles in the G, if $\tau > 0$, the total number of all (possible) ego networks is in the interval $[3, 3\tau]$. *Highly clustered graphs contain many nontrivial ego networks!*
- **How can we select a minimum number of ego networks such that the nodes of all the remaining ego networks are alters to the former?**
- *This is how*: Take the subgraph of all clustered nodes and compute the **domination number** γ_c of this subgraph. Then there exist γ_c **dominating nontrivial ego networks** in the graph and any node outside them, if it exists, it is necessarily a traversing broker.
- Hence, aggregating nodes inside the dominating nontrivial ego networks and aggrgating nodes among these ego networks and all the outside traversing nodes yields a compression of G into a smaller (weighted) graph of dominating nontrivial ego networks and (possibly existing) traversing brokers.

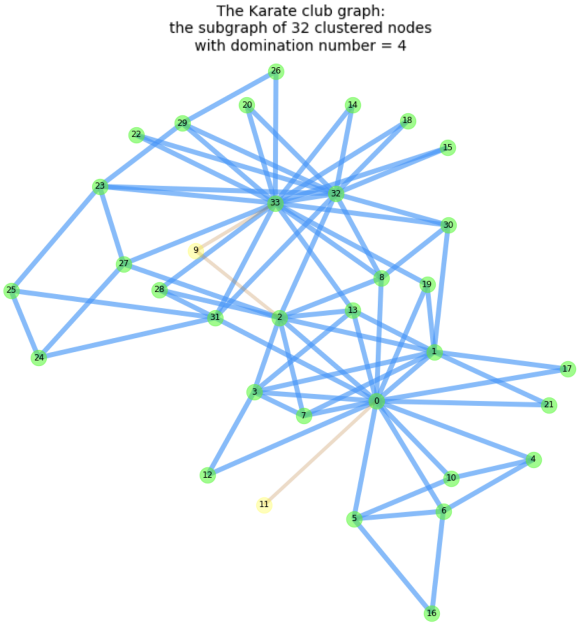
The Florentine Families Graph, I



The Florentine Families Graph, II



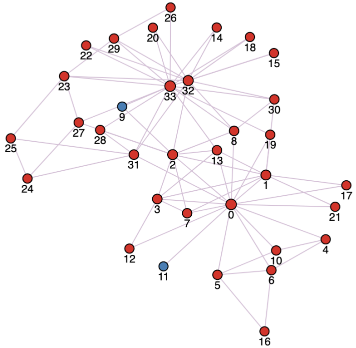
The Karate Club Graph, I



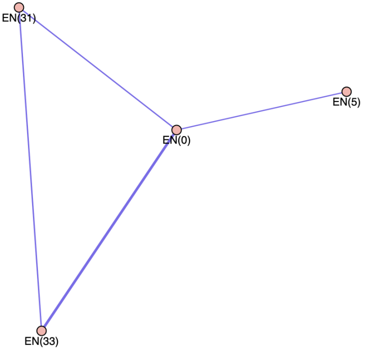
The Karate Club Graph, II

The Karate club graph

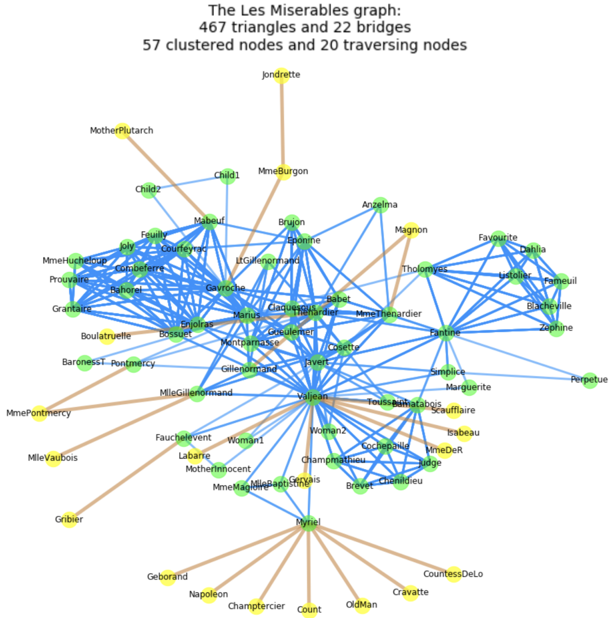
Graph of 32 clustered and 2 traversing nodes



Coupled graph of 4 dominating ego networks with 0 traversing nodes



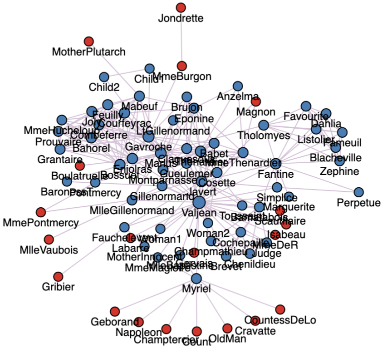
Les Miserables Graph, I



Les Miserables Graph, II

Les Miserables graph

Graph of 57 clustered and 20 traversing nodes



Coupled graph of 5 dominating ego networks with 12 traversing nodes