Nonmonotonicity and the Scope of Reasoning

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Abstract

Circumscription, Default Logic, and Autoepistemic Logic capture aspects of the nonmonotonicity of human commonsense reasoning. However, Perlis has shown that circumscription suffers from certain counterintuitive limitations, concerning exceptions or "counterexamples" to defaults. We observe that the unfortunate limitations of circumscription are even broader than Perlis originally pointed out. Moreover, these limitations are not peculiar to circumscription; they appear to be endemic in nonmonotonic reasoning formalisms. We develop a general solution, involving restricting the "*scope*" of nonmonotonic reasoning, and show that it remedies these problems in a variety of formalisms.

Our solution has a number of attractive aspects in addition to its generality. Most importantly, no modification of the underlying for-

malisms is required, and the result is semantically compatible with existing approaches. Furthermore, the necessary machinery is intuitively plausible and, arguably, useful for other purposes. Finally, the solution is robust: it is relatively tolerant of imprecise determinations of scope.

1 Introduction

The study of nonmonotonic reasoning—reasoning that can reach conclusions that are not *strictly* entailed by what is known, and so may need to be retracted as new information is acquired—has become increasingly popular in Artificial Intelligence. On reflection, this is not surprising; much of human reasoning, both commonsense and more specialized, seems to fit this characterization. The imprimatur of logical certitude rarely factors into the decision-making process.

Whether trying to diagnose diseases from test results and symptoms or merely thinking about where one's car might be, given that it was left in the parking lot this morning, the weight of what typically is the case, and of "rules of thumb", overwhelms that of what *must* be true. The search for formal theories of how to reason with such information has yielded many promising formal systems, including Default Logic [30], Circumscription [21, 22], and Autoepistemic Logic [25].

While these formalisms provide many useful insights into nonmonotonic reasoning, each has some persistent problems that have, thus far, resisted solution. In many naturally-occurring cases, the straightforward encoding of a situation of interest and the default information associated therewith leads these formalisms for commonsense reasoning to quite unintuitive conclusions—or prevents the derivation of intuitively-obvious conclusions. In section 3, we discuss several such significant problems, and show them to manifest themselves in each of the major formalisms.

We argue that these problems are actually aspects of a single, more general, problem. This problem has more to do with the underlying understanding of the function of nonmonotonic reasoning than with the particular details of existing frameworks. We show that a simple idea, simple in its realization, avoids these problems. This not only greatly enhances the usefulness of the theories, but seems to bring them into much closer harmony with an intuitive understanding of commonsense reasoning.

2 Theories of Nonmonotonic Reasoning

Before we embark on our examination of the problems with existing theories of default reasoning, we present the briefest of recapitulations of the principal formalisms. This section aims to present the underlying technical details required to understand the essence (if not the technicalities) of later sections, rather than to provide a tutorial introduction. The unfamiliar reader is referred to the original papers, or to Etherington's book [5], for a detailed introduction, while readers comfortable with the technicalities may wish to skip directly to §3.

2.1 Circumscription

Circumscription was developed by McCarthy [21], who later refined it to make it more flexible and powerful [22]. The idea is that if some property typically holds, one should assume that as few individuals as possible lack the property. Thus, an axiom (or axiom schema) is added to the theory that has the effect of *minimizing* the set of exceptional individuals.

Semantically, this amounts to restricting attention to those models in which the set of exceptional individuals is no larger than necessary—i.e., any proper subset of exceptions would no longer satisfy the original axioms. This preference for so-called *minimal* models leads to assumptions of typicality for individuals unless there is specific reason to believe that they might be exceptional.

Given a set of axioms $A = A[P] = A[P_0, ..., P_n]$, if being exceptional can be characterized by the formula $W[P, y]$, then the circumscription schema, $CIRC$, is written as:

$$A[P'] \wedge \Big[\forall y.\ W[P', y] \rightarrow W[P, y]\Big] \longrightarrow \Big[\forall y.\ W[P, y] \rightarrow W[P', y]\Big] \qquad (1)$$

The idea is that if there is an interpretation $P'_0, ..., P'_n$ of the predicates $P_0, ..., P_n$ such that $A[P'_0, ..., P'_n]$ holds, and this interpretation imposes further restrictions on W, then we should assume that the actual interpretation for the P's satisfies those restrictions. Asserting the circumscription schema expresses the information that exceptions (W's) are rare enough to be assumed away.

$P_0, ...P_n$ are called the variable predicates. These are the predicates of the theory that are allowed to be affected by the assumptions made in the process of minimizing W. Assumptions cannot entail information about any other predicates that is not already entailed by A.

2.2 Default Logic

Reiter's Default Logic [30] provides a more direct means for specifying defaults, with more precise constraints on when they are applicable. A *default rule,* δ, has the form:

$$\frac{\alpha(\bar{x}) : \beta(\bar{x})}{\gamma(\bar{x})} \, , \tag{2}$$

where α, β, and γ are formulae whose free variables are among $\bar{x} = x_1, ..., x_n$, and are called the *prerequisite, justification,* and *conclusion* of the default, respectively. The rule δ can be read as saying that if α is believed, and β is consistent with what is believed, then γ should be believed or, informally, an α is typically a γ unless it is not a β. A *default theory,* $\Delta = (D,\ W)$, consists of a set of defaults, D, and a set of first-order formulae, W.

The defaults of a default theory induce acceptable sets of beliefs, called *extensions,* given the facts, W. Extensions are defined to be the least fixed points of an operator that includes W and satisfies as many of the defaults in D as possible. A theory may have any number of extensions, including none. Since a default theory has an inconsistent extension if and only if W is inconsistent (in which case, that is its only extension) [30, Corollary 2.2], we will subsequently restrict our attention to consistent theories.

A *normal default* is one whose prerequisite and justification are identical, and a *normal default theory* is a default theory in which each default is normal. We also distinguish the *simple abnormality theories,* in which there is a single normal default,

$$\frac{: \neg Abnormal(\bar{x})}{\neg Abnormal(\bar{x})},$$

in D. Simple abnormality theories also typically have one or more axioms of the form $\forall \bar{x}.\ \alpha(\bar{x}) \wedge \neg Abnormal(\bar{x}) \supset \gamma(\bar{x})$, where α and γ play much the same role they do in (2), although there are many subtle differences.[1]

2.3 Autoepistemic Logic

Propositional Autoepistemic Logic (AEL) was introduced by Moore [25]. The propositional case and restricted first-order versions were further studied by Levesque [18] and Konolige [13]. As in default logic, defaults can be individually represented in AEL, but they are treated as part of the language, rather than as inference rules. Defaults are represented using a modal operator, $\mathbf{L}$, where $\mathbf{L}\alpha$ means roughly "α is known" or "α is provable" in the theory in question.[2] Thus, the default that if α is known and β is consistent with everything known then γ should be inferred might be written:

$$\mathbf{L}\alpha \wedge \neg\mathbf{L}\neg\beta \supset \gamma. \tag{3}$$

Like default logic, the primary focus of AEL is normative: given an initial (or base) set of beliefs, A, about the world, the goal is to determine what set(s) of beliefs an ideal, introspective, agent should settle on. Clearly such an agent should believe all the logical consequences of her beliefs. But because the intended meaning of $\mathbf{L}\phi$ depends on the beliefs of the agent, the definition of the belief set itself becomes circular, necessitating the use of a fixed-point construction.

AEL has a model theory and a fixed-point proof-theory. For our purposes, we employ the model-theoretic approach given by Konolige [13]: A set, T, is an *autoepistemic extension* of an initial set of beliefs A iff $T = \{\phi \mid A \models_T \phi\}$, where $\models_T \mathbf{L}\phi$ iff $\phi \in T$, and otherwise $\models_T$ behaves as a standard semantic consequence relation (see [13, 25] for details).

3 "Paradoxes" of Nonmonotonic Reasoning

Default logic and circumscription have been studied for over a decade, and autoepistemic logic for nearly that long. For most of that time, it was believed that these formalisms (or at least some of them) captured the essential ideas

[1]In particular, such axioms can lead to the default conclusion of $\neg\alpha$ from $\neg\gamma$, as well as disjunctive default conclusions.

[2] Thus the $\mathbf{L}$ operator incorporates the self-referential aspect frequently found in nonmonotonic logics.

of nonmonotonic reasoning, and that it would only be a matter of time before they could be adapted to practical reasoning systems.

Recently, a variety of problems have been noticed that seem to raise fundamental doubts about these optimistic projections. Some of these—such as the "Yale Shooting Problem" [12]—now seem more indicative of the difficulty of adequately axiomatizing even a relatively simple world; others remain that seem more paradoxical, in that they turn the formalisms' basic mechanisms against them.

In the subsections below, we briefly recount four such "paradoxes" of default reasoning, and show how they affect the various theories of non-monotonic reasoning. Section 4 then argues that the observed problems can be viewed as stemming from a common root—a misapprehension, common to all the approaches, of the principles underlying this type of reasoning. Once identified, this deficiency is readily corrected with simple tools whose benefits, we believe, easily outweigh their cost.

3.1 The Lottery Paradox

The first problematic example is the "Lottery Paradox" [16, 31, 23, 27]. The lottery paradox arises in situations in which the conjunction of a set of assumptions, each reasonable individually, is inconsistent with what is known about the world. For example, in the paradigmatic case, it is usually safe to assume that any particular ticket in a lottery will not win—given the overwhelming odds against it. Assuming the lottery is "fair", however, the conjunction of such an assumption for each ticket with the fact that some ticket must win is inconsistent.[3]

To maintain consistency, some (or all) of the assumptions about tickets not winning must be foregone. Since there is no basis for determining *which* assumptions to forego, however, any is as good as another, and no assumption is unequivocally sanctioned by the formalisms. In circumscriptive terms, there are as many minimal models as there are tickets, each with a different ticket chosen as the winner. Since circumscription describes what is true in all minimal models, nothing can be assumed about the individual tickets. The most that can be assumed is that if some particular ticket wins, it will

[3] We assume the set of tickets is fixed and finite. Other, related, problems arise if not.

be the only one.[4]

In the cases of default logic and AEL, assuming the set of tickets is closed, there will be as many extensions as there are tickets. Since each determines a "reasonable" set of beliefs, there will be one reasonable belief set in which the ticket of interest wins. If this extension is chosen by a brave agent who does not consider the other extensions, it might seem as though buying lottery tickets was compatible with common sense!

3.2 Counterexample Axioms

Problems also occur when there are *counterexample axioms* [27], asserting that there are exceptions to defaults. These axioms specify the existence of individuals lacking some default property, without specifying their identities. For example, if birds were taken to fly by default, then a counterexample axiom might look like:

$$\exists x.\ Bird(x) \wedge \neg Flies(x). \tag{4}$$

Counterexample axioms are problematic for circumscription because circumscription minimizes the *number* of exceptions, but does not necessarily determine *which* individuals are exceptional. Thus, any of a number of individuals could be taken to be exceptional without changing the number of exceptions. For example, if we minimize the set of flightless birds in the theory:

$$\{\ Bird(Tweety),\quad \exists x.\ Bird(x) \wedge \neg Flies(x)\ \}, \tag{5}$$

we get no conjectures about Tweety not entailed by the original axioms. The conclusion that Tweety flies does not follow, since there are (minimal) models in which Tweety is the only bird, and hence the flightless bird required by the counterexample axiom. Even if we posit the existence of other birds different from Tweety,[5] circumscription has no way to prefer Tweety's flying to that of any other bird.

[4] Perhaps this accounts for the disappointment of some who—against the odds—*do* win, but have to share the prize.

[5] Since circumscription cannot generate new equality facts without resorting to variable terms [8], explicit inequalities are required to rule out the models where only Tweety is a bird, but she goes by various aliases.

The obvious patch is to try to somehow distinguish Tweety from the existentially-specified flightless bird, for instance by naming the latter (say Opus), and replacing the original counterexample axiom by a Skolemized version such as: $Bird(Opus) \wedge \neg Flies(Opus)$. However, $Flies(Tweety)$ still does not follow by circumscription unless the further axiom that $Tweety \neq Opus$ is adopted. But this amounts to assuming that Tweety is not the exceptional bird—which seems to obviate the circumscription.

The problem surfaces in a slightly different form in default logic. Because defaults apply only to individuals in the Herbrand Universe, rather than being universally quantified, and because there is no problem with conjectures enriching the equality theory, the default theory, Δ, with axioms (5) and the single default,

$$\frac{Bird(x)\ :\ Flies(x)}{Flies(x)}\ , \tag{6}$$

does have a unique extension in which Tweety flies, and some unnamed bird doesn't. Peculiarities arise when additional information, such as the extent of the prerequisite predicate ($Bird$, in this case), is known. So, for example, if Δ were enriched with the fact that $Tweety$ and $B_1, ..., B_n$ are the only birds, i.e., $(\forall x.\ Bird(x) \equiv x = Tweety \vee x = B_1 \vee ... \vee x = B_n)$, the resulting default theory would have $n+1$ extensions, each with exactly one (different) nonflying bird.

If, on the other hand, the exact extent of $Bird$ is not necessarily known, the default theory will result in the conjecture that all the known birds fly, but there is some other bird that does not. This is particularly bizarre when the whole domain is known (e.g., because of a domain closure axiom[6]), since the theory will have a unique extension—one that entails that an individual not known to be a bird is, in fact, a flightless bird.

For example, from (4) and the facts:

$$\{\ Bird(Tweety),\quad Animal(Charlie),\quad \forall x.\ x = Tweety \vee x = Charlie\ \}$$

the default, (6), will induce the single extension in which $Charlie$ is a flightless bird!

In the AEL case, we get similar results. One might try to represent δ as $\forall b.\ \mathbf{L}Bird(b) \wedge \neg\mathbf{L}\neg Flies(b) \rightarrow Flies(b)$ and add this rule to (5), but this

[6] A *domain-closure axiom* (DCA) [29] is a formula of the form $\forall x.\ x = t_1 \vee ... \vee x = t_n$, for some set of ground terms, $t_1, ..., t_n$.

involves quantifying into the scope of **L**. Quantifying in is not dealt with in [25] and [13]; however, Konolige [15] and Levesque [18] suggest reasonable approaches. We will not get into details but, at least in [15], the results are much like those of default logic. In particular, when the extension of $Bird$ is known and finite and $\forall x.\ Bird(x) \equiv x = Tweety \vee x = B_1 \vee ... \vee x = B_n$ is added to (5), we can instantiate b by each of $B_1, ..., B_n$, and $Tweety$ and the theory will have $n+1$ different extensions, as does the corresponding default theory.

3.3 Everything is Abnormal

Yet another inappropriate result occurs when there are defaults describing the typical values of a variety of (possibly orthogonal) properties for some class, C. If C is made up of several subclasses, each but one (say C_k) of which is atypical with respect to a different property, then the nonmonotonic formalisms will result in the conjecture that individuals known to belong to C must also belong to C_k. While there may be cases where this behaviour is appropriate, it does not accord well with many common uses of defaults.

For example, imagine that birds typically fly, sing, and build nests, except that penguins don't fly, swans don't sing, and mynahs don't build nests [28]. Now if birds must be penguins, swans, mynahs, or canaries, default reasoners of the type envisioned in the literature will assume that arbitrary birds are canaries, in order to minimize the violation of defaults! Although there may be no typical colour for birds (and hence no explicitly conflicting default), it does seem inappropriate to assume an arbitrary bird to be yellow, as would be the case if one assumed it were a canary.

Even more counterintuitively, if it turns out that *all* subclasses are atypical (e.g., canaries are abnormal by virtue of being brightly coloured), then there will suddenly no longer be any undisputed conjectures: different atypicalities will hold in different minimal models/extensions, and the theory entails that some abnormality holds in each. (Thus, e.g., learning that canaries are exceptionally coloured blocks the assumption that Tweety flies.) At best, circumscription will conjecture only that at most one atypicality holds, whatever it may be. Similarly, default logic and AEL will induce multiple extensions—one for each different way in which individuals might be atypical. In each extension, individuals will be forced to be members of one particular subclass or another. This equivocation may be an appropriate characterization of the omniscient observer's view of the global situation; however, it makes the formalisms of little use to a reasoner interested in whether a particular bird flies.

Poole [28] and others have noticed that when one tries to build a system to do nonmonotonic reasoning about a real domain, one frequently encounters situations in which *everything* is abnormal in some respect (violates some default). This suggests that the problematic handling of universal abnormality is not an isolated baroque instance where the formalisms do not perform well but, rather, symptomatic of fundamental issues that need to be addressed.

3.4 There's Nobody Here But Us Chickens[7]

Another counterintuitive aspect of some nonmonotonic formalisms is that, in their efforts to make everything as typical as possible (i.e., violate as few defaults as possible), they conjecture that exceptional classes are empty. Since membership in an exceptional class entails the violation of a default, and since the formalisms are bent on minimizing such violations, they naturally infer that exceptional classes have as few members as possible. This is both completely logical and completely nonsensical: logical because default reasoning *does* seem to involve assuming things are as normal as possible; nonsensical because the assumption that some object of interest has typical properties clearly should not rest on the absence of atypicality elsewhere in the world.

Circumscription is particularly susceptible to this anomalous behaviour. The circumscription schema explicitly states that there is no world that is less exceptional than this, and the semantics explicitly prefers those models where all exceptions are forced. Thus, for example, if we are told that penguins are birds that do not fly, that birds normally fly, and that Tweety is a bird:

$$\begin{gathered}\forall x.\ Penguin(x) \supset Bird(x) \\ \forall x.\ Penguin(x) \supset \neg Flies(x) \\ \forall x.\ Bird(x) \land \neg Abnormal(x) \supset Flies(x) \\ Bird(Tweety)\end{gathered}$$

and minimize the set of abnormal individuals, we conclude that Tweety flies, and hence is not a penguin—but we also conjecture that there are

[7] Or whatever class of birds is quintessentially prototypical.

no penguins! If, on the other hand, the objection to this conclusion is made explicit—by asserting $\exists x.\ Penguin(x)$—the result is a counterexample axiom, since the enriched theory implies $\exists x.\ Bird(x) \wedge \neg Flies(x)$, and the problems discussed earlier recur.

The obvious answer—including *Penguin* in the set of fixed predicates—does prevent the conclusion that there are no penguins. There are two problems with this method of saving penguins from "extinction", however. First, it may be very difficult to determine which predicates should be fixed and which allowed to vary. In particular, fixing a predicate in the context of one default inference may block other desirable default inferences. More seriously, the survival of the penguin is achieved at the expense of the ability to conclude that Tweety flies. With *Penguin* fixed, the strongest conjecture that can be made about Tweety is that she flies unless she is a penguin.[8] This seems unsatisfactory.

The problem is more subtle in default logic, since the effects of default reasoning are conditioned by the provability of the prerequisites of defaults, and the form of the default plays a greater role. For example, the default:

$$\frac{Bird(x)\ :\ Flies(x)}{Flies(x)}$$

will sanction the conjecture that none of the known birds are penguins, but not that there are no penguins at all. The former seems much more innocuous than the latter, although perhaps it becomes less so as the number of known birds becomes very large. On the other hand, if the set of all birds is known, the conclusion that there are no penguins will follow.

Popular alternate default logic representations can make the problem more pronounced. If the fact that birds typically fly is represented by the axiom:

$$\forall x.\ Bird(x) \wedge \neg Abnormal(x) \supset Flies(x)$$

and the default:

$$\frac{:\ \neg Abnormal(x)}{\neg Abnormal(x)} \tag{7}$$

[8] As the set of predicates we wish to protect from extinction grows, the conclusions about Tweety are correspondingly weakened.

then it will follow that no individual in the Herbrand Universe is a penguin (in other words, if there *are* any penguins, the language has no names for them). Although this conclusion is weaker than its circumscriptive counterpart,[9] it still retains the unintuitive flavour. If the theory has a domain closure axiom, or otherwise explicitly bounds the set of possible birds (or penguins), then the stronger conclusion that there are no penguins at all *will* follow.

In the AEL case, we again can get the similar behaviour. For instance, the default (7) corresponds roughly to the autoepistemic logic axiom:

$$\forall x.\ \neg \mathbf{L} \mathit{Abnormal}(x) \rightarrow \neg \mathit{Abnormal}(x) \tag{8}$$

Again, using a domain closure axiom, we can treat (8) as a schema of propositional formulae, and conclude that there are no penguins at all.

4 A Common Thread

Each of the above difficulties with extant theories of nonmonotonic reasoning can be attributed to a single root cause—overzealousness. In the attempt to capture default reasoning, a subtle twist has been introduced. One way to look at it is that the commonsense notion that such reasoning is essentially the *elimination of unforced abnormalities* or atypicalities has been replaced by the notion of the *introduction of forced normalities* or typicalities. Commonsense says you should take the possibility that something is exceptional into account only if forced to; nonmonotonic formalisms force you to consider as much as possible to be unexceptional.

Assumptions are necessary in everyday reasoning because what follows from what we *know* about the world leaves too many questions undecided. Paradoxically, the mechanisms developed to redress this shortcoming leave too *few* questions undecided. Using such tools to decide whether Tweety flies is akin to cracking walnuts with a cannon—not only are there likely to be undesired side-effects, but the meat of the matter (or the nut) may be much harder to find among the irrelevant fragments.

We frequently know *that* there are exceptional individuals without knowing *who* they are. If defaults are applied injudiciously, paradoxes are bound

[9] This weakening is inherent in the fact that a default with free variables stands for the set of its ground instances, not for its universal closure.

to arise—yet paradoxes rarely arise in people's default reasoning. It seems clear that defaults are usually not broadly applied.

The fact that reasoning is commonly *directed* seems to have been ignored. We contend that the intention of default reasoning is generally not to determine the properties of every individual in the domain, but rather those of some particular individual(s) of interest. Making assumptions is inherently risky—those assumptions might be wrong.[10] From a probabilistic viewpoint, there seems to be no advantage to making "unnecessary" assumptions. Incorporating uncertain beliefs into a belief system when those beliefs are not of direct interest is likely to be counterproductive, simply increasing the likelihood that some beliefs will have to be retracted.[11]

Reconsider the four paradoxes discussed above. In each case, the problem arises because the existence of something atypical is entailed, and default reasoning might encompass it. In the case of the lottery paradox, by considering the fate of every ticket as a suitable object for default reasoning, we are faced with the problem that some ticket must win, giving rise to numerous "preferred" models. If we could consider only the small set of tickets we might consider buying, there would be no problem with assuming that none of them would win, and we would find ourselves safely past the lottery vendor, paycheque intact. Similarly, faced with a counterexample axiom, so long as there were no reason to believe that the posited counterexample was among the individuals of interest, one could make assumptions about the interesting cases without wrestling with the identity of the counterexample. By the same token, when everything is abnormal in some aspect or other, it should be possible to reason about some small set of aspects of interest, and ignore all the other aspects. Finally, when the scope of interest does not cover whole domains, conjectures to the effect that atypical classes are empty classes would not arise.

In all the above cases, the risk associated with default reasoning whether some particular individual in the scope of interest is unexceptional is higher in some respects than the corresponding risk for straightforward default rea-

[10] We leave aside the consequences of being wrong, although these clearly must be taken into consideration eventually. For the purposes of this paper, we will circumvent questions of decision-theoretic utility, assuming all errors are equally expensive.

[11] It is conceivable that, in particular situations, there may be computational advantages to making many assumptions (e.g., sweeping generalizations) that outweigh the cost of errors. We know of no convincing examples, however, and ignore the possibility here.

soning, since the set of "preferred" theories is more restrictive. However, provided the scope of interest is sufficiently narrow vis à vis the antecedent class(es) for the defaults, relatively few additional models will be excluded, and so the risk does not seem disproportionate with that of doing default reasoning in the first place. In fact, since default conjectures are made about fewer individuals, it is intuitively reasonable to believe that the net result is more probable. If the scope is too broad, or there are indications that the exceptional cases are within the scope, of course, the advisability of making assumptions decreases proportionally.

It seems, then, that restricting the scope of default reasoning may provide a solution to the paradoxes. This "solution" may appear to amount to simply sweeping the necessary atypicalities under the proverbial rug. We suggest that the essence of default reasoning lies in sweeping our ignorance under the rug; provided the rug is large enough, we can hope that the lump will be unnoticed and unimportant.

5 Scope in Nonmonotonic Reasoning

If we are reasoning about Tweety in particular, then the fact that millions of other birds are potentially candidates for default reasoning need not enter our deliberations. The typical examples of default reasoning in the literature are of this sort, where reasoning is of narrow scope. On the other hand, the Lottery Paradox and related problems involve drawing default conclusions about an entire population (every lottery ticket or bird); this is (very) broad scope. It seems that reasoning about whole populations is more properly the domain of probability theory than of nonmonotonic logics, which seem better suited to reasoning about small numbers of cases.

We contend that this distinction has an analogue in human reasoning: people assess whether their reasoning has appropriate scope, at least when challenged by evidence of exceptions. If the scope is judged to be suitable then it seems reasonable to go ahead with a default conclusion. Otherwise—when trying to decide about an entire population of individuals, for instance—there may be concern about trusting default conclusions.

For example, when planning an airline trip involving connecting flights, one may follow the travel industry default that one hour is sufficient for connections, and assume that the first leg's flight will arrive sufficiently close to its scheduled arrival time.[12] An airline considering offering a service guarantee, however, might be prompted by the wider scope of their plans to reason about the statistics concerning their arrival records.

It is clear that the idea of making the default reasoning process dependent on the scope of interest enables intuitively-desirable conclusions in otherwise intransigent cases. In the following subsections, we show that scope restrictions can easily be added to the existing formalisms, and that the desired results can be achieved. In particular, we show that more powerful conjectures obtain in the cases we have been considering, and that appropriate notions of consistency are preserved.

We require that the scope of reasoning be suitably proscribed. We do not attempt to define this, beyond noting that the scope of interest should not include a "significant fraction" of whatever class we are drawing default conclusions about. Our approach to limiting the scope of reasoning ensures, however, that—even when this requirement is violated—performance and consistency will be no worse than that of the unscoped approaches. In addition, as will be seen in §5.5, the approach is tolerant of fairly loose determination of the scope.

Three important distinctions must be made at this point. First, by restricting scope, we are not suggesting that it is inappropriate to have defaults *about* whole (even infinite) populations—only that those defaults should not be *applied* to obtain conclusions about too large a fraction of those populations at any given instant. Second, restricting scope is a methodological, not theoretical, issue: we do not claim that there are *no* instances where widely-scoped default reasoning is appropriate. Neither do we suggest that anything like a provably optimal scope exists (except a posteriori as the narrowest scope from which some desired conclusion follows by default). It should be clear, however, that narrowly-scoped default reasoning is both empirically useful and intuitively reasonable. Third, our notion of restricted scope differs from the notion of *reference class* [20, 16, 17] in the following way: the reference class helps determine what default information to use when reasoning about an individual (the defaults associated with the smallest class for which adequate statistics are known); scope helps determine

[12] *Not* making such an assumption can provide one with the opportunity to sample the many entertainments afforded by modern airports—a consideration that some may feel outweighs the mere possibility of a missed connection!

what individuals to reason about.

The technical modifications required to limit the scope of default reasoning are simple. In some cases, existing techniques can even be used to achieve the same effect. The major contribution of this work is not sophisticated new versions of the formalisms—we consider it an advantage that developing yet another nonmonotonic formalism is unnecessary. Rather, the important result is that a simple, *uniform,* representational technique provides significant leverage on a variety of problems, not previously identified as having common roots, across a variety of formalisms.

5.1 Scoped Circumscription

Scope can be accounted for in circumscription by minimizing only within the extent of a predicate representing the scope of interest. Specifically, we minimize $W[P,y] \wedge \mathit{Scope}(y)$ rather than just $W[P,y]$. The resulting scoped instance of (1) we label $\mathit{CIRC}_{\mathit{Scope}}$: [13,14]

$$A[P', \mathit{Scope}'] \wedge \left[\forall y.\ W[P',y] \wedge \mathit{Scope}'(y) \rightarrow W[P,y] \wedge \mathit{Scope}(y)\right] \\ \longrightarrow \left[\forall y.\ W[P,y] \wedge \mathit{Scope}(y) \rightarrow W[P',y] \wedge \mathit{Scope}'(y)\right] \qquad (9)$$

Notice that the predicate *Scope* is treated as variable, in addition to the predicates in P. The reason for this is that if *Scope* were fixed it might be necessary to *prove* that none of the individuals that must be exceptional (those specified in counterexample axioms for example) are in the scope before conjecturing that nothing in the scope was exceptional. With *Scope* variable, conversely, it suffices that they not necessarily be scoped.[15] There is no difference between fixed and variable scope if *Scope* is completely determined, but it seems more reasonable to require an agent to be aware of

[13] Note that we are not introducing a new form of circumscription. Rather, whatever is circumscribed is forced to include the new predicate, *Scope*. The approach is independent of which of the major variants of circumscription is chosen.

[14] In most of the sequel, we will limit our attention to unary predicates, simply as a notational convenience. The generalization to n-ary predicates presents no technical problems.

[15] In model-theoretic terms, variable scope allows pathological models—where accidents of the scope induce scoped exceptions—to be discounted when considering minimal models; fixed scope precludes this, for example allowing the possibility that the scope may include everything not explicitly excluded.

everything she is considering than to require her to be aware of what she is not considering![16]

While variable scope seems to suffice, and the foregoing notwithstanding, it may not be necessary to rule out using fixed scope. Fixed scope typically leads to conditional conjectures to the effect that if there are "enough" individuals outside the scope, the scoped individuals are unexceptional. The notion of "narrow scope" seems to mean precisely that there are sufficiently many individuals outside the scope. It may be possible, therefore, to eliminate this conditionalization on the basis of the reasoner's judgement that the scope of reasoning is suitably narrow and, hence, according to our rationale, default reasoning is appropriate. We are currently exploring this possibility.

5.1.1 Sufficiency

Scoped circumscription is free of some limitations of its unscoped counterpart. For example, *Scope* provides a solution to the counterexample problem [27]. Given that the domain is nontrivial, and that Tweety is in the scope of concern in (5), it is possible to conclude that Tweety flies. This is true despite the presence of the counterexample axiom: $\exists x.\ Bird(x) \wedge \neg Flies(x)$. To see this, consider the following axioms, $A[\mathit{Scope}, Bird, Flies]$:

$$\Big\{Bird(Tweety),\ \exists x.\ Bird(x) \wedge \neg Flies(x),$$
$$Charlie \neq Tweety,\ \mathit{Scope}(Tweety)\Big\}$$

We introduce Charlie here because, to envisage possibilities, we need an ontology rich enough to allow the formation of various interpretations. In particular we need an object other than Tweety that we can at least imagine to be a potential flightless bird, to let Tweety off the hook. Notice, however, that Charlie's role as a "scapebird" is quite limited—we do not conclude $\neg Flies(Charlie)$, for example (nor even $Bird(Charlie)$). In fact, simply asserting $\exists x.\ x \neq Tweety$, rather than $Charlie \neq Tweety$ would suffice. Since any realistic ontology will provide many individuals, this requirement presents no particular hardship.

[16] Much like the joke, "Try really hard not to think of an elephant!", being aware of something seems to imply considering it—however briefly.

From A and (9) with $W[Bird, Flies, y]$ being $\neg Flies(y)$, $Flies(Tweety)$ follows. The appropriate instance of the scoped circumscription schema is:

$$\begin{aligned} & Bird'(Tweety) \wedge \Big[\exists x.\ Bird'(x) \wedge \neg Flies'(x)\Big] \wedge Scope'(Tweety) \\ & \quad \wedge \Big[\forall y.\ \neg Flies'(y) \wedge Scope'(y) \rightarrow \neg Flies(y) \wedge Scope(y)\Big] \\ & \quad \longrightarrow \Big[\forall y.\ \neg Flies(y) \wedge Scope(y) \rightarrow \neg Flies'(y) \wedge Scope'(y)\Big] \end{aligned}$$

Substituting $x = x$ for $Bird'(x)$, and $x = Tweety$ for $Scope'(x)$ and $Flies'(x)$, gives:

$$\begin{aligned} & Tweety = Tweety \wedge \Big[\exists x.\ x = x \wedge x \neq Tweety\Big] \wedge Tweety = Tweety \\ & \quad \wedge \Big[\forall y.\ y \neq Tweety \wedge y = Tweety \rightarrow \neg Flies(y) \wedge Scope(y)\Big] \\ & \quad \longrightarrow \Big[\forall y.\ \neg Flies(y) \wedge Scope(y) \rightarrow y \neq Tweety \wedge y = Tweety\Big]. \end{aligned}$$

The second conjunct of the antecedent holds because $Charlie \neq Tweety$, and the remaining conjuncts are trivially true. Thus $\forall x.\ \neg Flies(x) \wedge Scope(x) \supset x \neq x$, and so $\forall x.\ Flies(x) \vee \neg Scope(x)$. In other words, all non-fliers are outside the scope of reasoning, and so $Flies(Tweety)$, since we have $Scope(Tweety)$.

If *Bird* is not allowed to vary, then the ontology must provide one or more birds different from Tweety and not necessarily in the scope to allow the conjecture that Tweety flies. Regardless of the domain, however, it is possible to infer that if there is a bird not in the scope, then Tweety (and every other bird in the scope) flies. Again, a realistic ontology would guarantee this. At any rate, if this is not the case, the scope of reasoning is clearly not narrow with respect to the domain of birds.

More generally, we can show that exceptions can frequently be "assumed away".

Theorem 5.1 Let A be a first-order theory. If, for every consistent extension, A', of A by ground (in)equalities, $A' \not\vdash \exists x.\ W(P, x) \wedge Scope(x)$, then $CIRC_{Scope}[A]$ entails that all exceptional individuals are outside the scope (i.e., $\forall x.\ W(P, x) \supset \neg Scope(x)$ or, equivalently, $\forall x.\ Scope(x) \supset \neg W(P, x)$), provided all predicates are variable and A entails a domain-closure axiom.

Proof: [17]

Without loss of generality, consider only the theories, A', that completely determine the equality relation. Since it entails a DCA and completely decides the equality relation on ground terms, each such A' uniquely determines a domain and interpretation for the terms of the language. Furthermore, each model of A is a model for some A', and vice versa.

Consider a model, M of some A'. Since $A' \not\vdash \exists x.\ W(P, x) \wedge Scope(x)$, there is a model, M', with the same domain and interpretation for the terms in which $(|W(P)|_{M'} \cap |Scope|_{M'})$ is empty. Now M' is clearly a submodel of M, since the equality predicate is determined, and all other predicates are variable.

Thus each model of A has a minimal submodel in which all exceptional individuals are outside the scope. The result follows. ∎

Corollary 5.2 If A entails a domain closure axiom and decides the equality predicate for all ground terms of the language, and $A \not\vdash \exists x.\ W(P, x) \wedge Scope(x)$, then $CIRC_{Scope}[A]$ entails that all exceptional individuals are outside the scope, provided all predicates are variable.

Proof: The DCA and the fact that A decides the equality predicate mean A is the only consistent completion of its equality theory. ∎

Theorem 5.1 and Corollary 5.2 can be generalized to allow *known* scoped exceptions, without losing the ability to conjecture away *anonymous* exceptions in the scope.

Theorem 5.3 If $A \vdash W(P, \alpha_i) \wedge Scope(\alpha_i)$, for ground terms $\alpha_i \in \{\alpha_1, ..., \alpha_n\}$, and, for every consistent extension, A', of A by ground (in)equalities, $A' \not\vdash \exists x.\ x \neq \alpha_1 \wedge ... \wedge x \neq \alpha_n \wedge W(P, x) \wedge Scope(x)$, then $CIRC_{Scope}[A] \vdash \forall x.\ [x \neq \alpha_1 \wedge ... \wedge x \neq \alpha_n \wedge Scope(x)] \supset \neg W(P, x)$, provided all predicates are variable and A entails a domain-closure axiom.

[17] **Notation:** $|P|_m$ denotes the interpretation of the predicate, P, in the model, m; $\|S\|$ denotes the cardinality of the set, S.

Proof: The theorem is proved by the obvious modification of the proof of Theorem 5.1, accounting for the known exceptions. ∎

As the example at the beginning of this section shows, the sufficient restrictions in Theorems 5.1 and 5.3 and Corollary 5.2 are by no means necessary. Essentially, what is required is an ontology with "enough" distinct individuals, but in which exceptions and the scope do not depend on the ontology of the model. In particular, the result cannot be generalized to cover minimization of Ab in theories such as:

$$a \neq b,\ \ b \neq c,\ \ c \neq a,$$
$$[\forall x.\ x = a \vee x = b \vee x = c] \supset Ab(a) \wedge Scope(a)\ .$$

The model consisting of only a, b, and c must have a scoped exception (a), even though a need not be exceptional in general. The need for "domain independence" is a consequence of circumscription's inability (without use of variable terms) to produce conjectures entailing new facts about the ontology [5]. It may be possible to relax this requirement by allowing variable terms, or using some stronger form of circumscription. This we have yet to investigate.

Although the necessary conditions for effective scoped circumscription are difficult to make precise, the cases that do not yield the appropriate conclusions seem unlikely to cause problems. It seems likely that a realistic theory of a reasonably-complex problem domain will have an abundance of individuals known to be distinct from those known to be in the scope. Similarly, statements explicitly predicating exceptionalness of individuals on what else exists, or on what things are identical, seems inappropriate in many commonsense domains.[18]

It is crucial that theory not entail that anonymous exceptional individuals being claimed to exist are also in the scope; otherwise the problem resurfaces. We claim, however, that it is not reasonable for an agent to believe a default conclusion while also believing that some unknown one of the very objects of concern is a counterexample to that default. Notice that there is no problem, however, in believing that there are *known* exceptions in the scope (e.g., $Bird(Opus) \wedge \neg Flies(Opus)$).

[18] Obvious exceptions include "whodunnits" and diagnosis, where the absence of other suspects definitely affects who may be assumed to be innocent. In such situations, however, the goal is to reach a situation in which the exception (the guilty party) *is* in the scope of reasoning, and the conjecture that the culprit is outside the scope would be inappropriate anyway.

5.1.2 Consistency

We now must ask whether the resulting circumscribed theory is consistent. The question of consistency is important because inconsistency has plagued certain applications of circumscription from the beginning [8]. Etherington [5] shows that the circumscription of universal theories (i.e., those without existential quantifiers) is consistent. However, counterexample axioms take us out from under this umbrella of safety. Nevertheless, we have proved that, under appropriate conditions, the circumscribed theory is consistent, regardless of the structure of the original theory.

> **Theorem 5.4** If A has a model in which the *Scope* predicate has a finite extension, then $CIRC_{Scope}[A]$ is consistent.
>
> **Proof:** Assume A has a model, M, in which $|Scope|$ is finite, and yet $CIRC_{Scope}[A]$ is inconsistent. Clearly, every submodel of M has a proper submodel. (Otherwise, M has a minimal submodel, and A has a minimal model, which would be a model for $CIRC_{Scope}[A]$, contradicting the assumption of inconsistency.) But clearly this induces an infinite descending chain of subsets of a finite set, starting with $|W(P)|_M \cap |Scope|_M \subset |Scope|_M$, which is impossible. ∎

> **Corollary 5.5** If A entails that *Scope* is finite, then the $CIRC_{Scope}[A]$ is well-founded (every model has a minimal submodel). ∎

Well-founded theories are "well-behaved". Among other things, this means that only predicates explicitly allowed to vary may be changed—something not necessary true for arbitrary theories [5, Section 6.3]. Although it is possible to construct theories in which scope is infinite and yet still excludes infinitely many individuals, it seems reasonable to stipulate that this is not narrowly-scoped reasoning. In particular, making an infinite number of default assumptions seems somewhat extravagant. Again, we emphasize that we are not suggesting that it is inappropriate to have defaults about infinite classes, merely that it may be imprudent to actually *make* an infinite number of default assumptions.

Of course, theories of the class for which generalized circumscription is known to be consistent (e.g., universal theories [5] and separable theories [19]) also preserve consistency under scoped circumscription.

5.1.3 Protected Circumscription

The scoped circumscription schema is reminiscent of protected circumscription [24], with $\neg Scope$ as the protected predicate. There are three differences. First, our approach merely changes what is circumscribed, rather than the form of the circumscription schema. This is appealing, since it means that everything already known about circumscription applies directly to scoped circumscription. Furthermore, the ideas can be directly applied to the numerous variants of circumscription.

The second difference is that it is possible for circumscription to affect the scope, something protected circumscription generally does not do. We are not sure whether this will eventually prove useful. In fact, the minimization of a conjunction using ordinary circumscription, with one conjunct being the protecting predicate, seems to provide the functionality of protected circumscription without changing the underlying framework. Moreover, this technique allows the protecting predicate to be varied or fixed, depending on whether one wants to protect all individuals that satisfy the protecting predicate, or only those known to do so.

The third difference between our work and that on protected circumscription is that scoped circumscription derives from the idea that the scope of interest should determine the fodder for default rules. Protected circumscription provides no such intuitive basis; it is simply a mechanism to control application of defaults given some basis. Our work provides a rationale for using something like a protection mechanism—a rationale that bears significantly on a number of problems in the literature.

To reiterate, the point of this work is not that scoped reasoning could not have been achieved previously, whether by protected or ordinary circumscription. Rather, the key idea is that the notion of scope can and should play an important role in default reasoning.

It is worth mentioning that neither protected nor scoped circumscription provide absolute assurance that no changes to the protected individuals (outside the scope) will be induced. For example, given:

$$\{Scope(a),\ \neg Scope(b),\ a \neq b,\ P(a) \vee P(b)\}$$

both protected circumscription of P, protecting $\neg Scope$, and scoped circumscription will induce $\neg P(a)$, and hence $P(b)$, even though b is supposedly protected. The explicit connection between a and b induced by the disjunction is stronger than the scope restriction.

It should also be noted that the goals of protected circumscription and scoped circumscription are not mutually exclusive. There may well be individuals within the scope of our interest about whom (for one reason or another) we do not want to draw default conclusions. These individuals could be protected during circumscription in the obvious way.

5.2 Scoped Default Logic

The greater expressive power of default logic [3] means there are many more candidate methods for restricting the scope of reasoning in default logic than were available in circumscription. Possibilities include adding a *Scope* term to the prerequisite or the justification of defaults, or to default axioms (e.g., (10), (11), and (12), respectively).[19]

$$\frac{\alpha(x) \wedge Scope(x) \ : \ \beta(x)}{\beta(x)} \tag{10}$$

$$\frac{\alpha(x) \ : \ Scope(x) \supset \beta(x)}{Scope(x) \supset \beta(x)} \tag{11}$$

$$\forall x.\ \alpha(x) \wedge \neg Abnormal(x) \wedge Scope(x) \supset \beta(x)$$
$$\frac{: \ \neg Abnormal(x)}{\neg Abnormal(x)} \tag{12}$$

Other alternatives that conjoin a *Scope* term into the justification, such as

$$\frac{\alpha(x) \ : \ \beta(x) \wedge Scope(x)}{\beta(x) \wedge Scope(x)},$$

can be quickly eliminated, since they have the effect of *maximizing* rather than minimizing the scope or, when combined with a default that minimizes scope, they can induce *more,* not fewer, preferred models, and hence weaker, not stronger, conjectures.

Let us consider the approaches typified by (10), (11), and (12), in turn. (10) says that individuals known to be α's in the scope can be assumed to

[19] α and β may be arbitrary formulae in which x occurs free.

be β's; (11) says known α's can be assumed to be β's if they are scoped; (12) says scoped α's are normally β's. The distinctions are subtle—they hinge on what must be known before the default can be applied. In the first case, both α and *Scope* must be known; in the second, only α; in the third, there is no prerequisite. The three forms of the default expression are strictly ordered in terms of strength, as shown by the following theorem.

Theorem 5.6 Let D, D', and D'' be

$$\left\{\frac{\alpha(x) \wedge \beta(x) \ : \ \gamma(x)}{\gamma(x)}\right\}, \qquad \left\{\frac{\alpha(x) \ : \ \beta(x) \supset \gamma(x)}{\beta(x) \supset \gamma(x)}\right\}, \quad \text{and}$$

$$\left\{\frac{: \ \neg Abnormal(x)}{\neg Abnormal(x)}\right\},$$

respectively. If W does not mention *Abnormal* then every extension of $\Delta = (D, W)$ is contained in one for $\Delta' = (D', W)$, and every extension of Δ' is contained in one of $\Delta'' = (D'', W \cup \{\forall x.\ \alpha(x) \wedge \neg Abnormal(x) \wedge \beta(x) \supset \gamma(x)\})$.

Proof: The details are tedious, but are easily constructed from the following sketch:

1) Let E be an extension for Δ, and let GD be the generating defaults of E [30, Theorem 2.5]. Order the defaults in GD according to the natural ordering induced in [30, Theorem 2.1]. Let $GD' \subset \Delta'$ be the analogues of these defaults in Δ'. It is straightforward to show that (GD', W) has E as an extension. By semi-monotonicity [30, Theorem 3.2], Δ' has an extension containing E.

2) Again, the proof proceeds from an extension, E, for Δ', and uses the generating defaults of E. The only complications are that $GD'' \subset \Delta''$ must be used in stages, creating a series of "concentric" extensions through repeated use of semi-monotonicity, and that the fact that W does not mention *Abnormal* must be used to rule out direct contradiction of $\neg$*Abnormal* assumptions by W. ∎

It should be obvious that Theorem 5.6 generalizes directly to theories with more than one default.

These results are interesting in their own right, since they characterize the different effects of different means of expressing default information in default logic. We do not pursue the matter further, here, however. Instead, we turn to the question of whether any or all of these forms produce the type of improvements limiting scope yielded for circumscription.

5.2.1 Sufficiency

The introduction of scope, in any of the above forms, to default logic is sufficient to circumvent the lottery paradox, as the following example shows. Imagine a lottery with 10,000 tickets, $t_1, ..., t_{10,000}$, and imagine we are considering buying one of the tickets, t_{100}–t_{175}, available at the corner store. This corresponds to the default theory with axioms:

$$\forall t.\ Ticket(t) \equiv t = t_1 \vee ... \vee t = t_{10,000}$$
$$Scope(t_{100}), ..., Scope(t_{175})$$
$$\exists t.\ Ticket(t) \wedge Wins(t)$$

and the default:[20]

$$\frac{Ticket(x) \wedge Scope(x) \ : \ \neg Wins(x)}{\neg Wins(x)}$$

This theory has a unique extension in which $\neg Wins(t_{100}), ..., \neg Wins(t_{175})$, but the fate of the remaining tickets is undecided.

It is no accident that the desired result holds, as can be seen from the following theorems, which show that individuals in the scope are conjectured to be unexceptional whenever possible.

Theorem 5.7 If $W \not\vdash \exists x.\ \Phi(x) \wedge \neg\Psi(x) \wedge Scope(x)$, and

$$D = \left\{ \frac{\Phi(x) \ : \ Scope(x) \supset \Psi(x)}{Scope(x) \supset \Psi(x)} \right\},$$

then any extension, E, for $\Delta = (D, W)$ has no scoped exceptions, in the sense that if $E \vdash \Phi(\alpha)$, then $E \vdash Scope(\alpha) \supset \Psi(\alpha)$, for any ground term, α.

[20]Since *Scope* is defined in terms of ground atomic formulae, the choice of which form of scoped default to use is irrelevant.

Proof: Assume $W \not\vdash \exists x.\ \Phi(x) \wedge \neg\Psi(x) \wedge Scope(x)$, but Δ has an extension, E, such that $E \vdash \Phi(\alpha)$ but $E \not\vdash Scope(\alpha) \supset \Psi(\alpha)$. Now, there is a model, M, of W such that $M \models \forall x.\ \neg\Phi(x) \vee \neg Scope(x) \vee \Psi(x)$. Using the semantic characterization developed in [4], the set of models of E, $MOD(E)$, must be reachable from $MOD(W)$ by discarding models incompatible with $Scope(t_i) \supset \Psi(t_i)$ from a chain, C_i, of sets of models such that $C_i \models \Phi(t_i)$, where t_i is a ground term. M must be eliminated, or else $MOD(E)$ is not maximal. But M must be eliminated in a step taking all $M' \models Scope(t) \wedge \neg\Psi(t)$, for some t, from a set of models $C \models \Phi(t)$. So $M \models \Phi(t) \wedge Scope(t) \wedge \neg\Psi(t)$, a contradiction. ∎

Theorem 5.8 If $W \not\vdash \exists x.\ \Phi(x) \wedge \neg\Psi(x) \wedge Scope(x)$, and

$$D = \left\{ \frac{\Phi(x) \wedge Scope(x)\ :\ \Psi(x)}{\Psi(x)} \right\},$$

then any extension, E, for (D, W) has no scoped exceptions, in the sense that if $E \vdash \Phi(\alpha) \wedge Scope(\alpha)$, then $E \vdash \Psi(\alpha)$, for any ground term, α.

Proof: The proof is essentially identical, except for the appropriate weakenings of conditions. ∎

Theorem 5.9
Let $W' = W \cup \{\forall x.\ \Phi(x) \wedge Scope(x) \wedge \neg Abnormal(x) \supset \Psi(x)\}$. If

$$D = \left\{ \frac{:\ \neg Abnormal(x)}{\neg Abnormal(x)} \right\},$$

and $W' \not\vdash \exists x.\ Abnormal(x)$, then any extension, E, for (D, W') has no scoped exceptions, in the sense that $E \vdash \Phi(\alpha) \wedge Scope(\alpha) \supset \Psi(\alpha)$, for any ground term, α.

Proof: Since $W' \not\vdash \exists x.\ Abnormal(x)$, (D, W') has a unique extension with $\neg Abnormal(\alpha)$ for every ground term, α. ∎

Theorem 5.7 and Corollaries 5.8 and 5.9 generalize to the case where W (or W') entails the existence of scoped exceptions, provided the number of scoped exceptions entailed is not greater than the number of provably-distinct ground terms that are provably in the scope and exceptional. In this case, the extension(s) of the scoped default theory entail that each ground term is unexceptional unless W (or W') specifically entails otherwise. This can be shown by simply amending the proofs of the respective results to explicitly exclude the known exceptions from the constructions. This generalization means that scoped default reasoning continues to be useful even when there are explicitly-scoped exceptions, so long as the identity of enough of those exceptions is known.

The major difference between scoped defaults like (10) and those like (11) surfaces when the scope theory entails disjunctions such as $Scope(a) \vee Scope(b)$ without determining which disjunct holds. Defaults of the first form generally behave as though neither a nor b were in the scope, while the latter form may result in the disjunction of default conclusions; the third form ((12)) permits even more disjunction of conclusions. While there may be circumstances where this subtlety is important, at the moment none occur to us. It is important to remember that *Scope* is a purely *introspective* predicate. As such, it seems reasonable to think that an agent who believes she is either thinking about one thing or another is in fact thinking about both. This observation suggests that it may be inappropriate to allow anything but ground atomic axioms for scope; however, since the theoretical machinery permits a spectrum of views on this point, we make no such stipulation.

5.2.2 Consistency

Normal default theories always have extensions, and the lack of extensions in some non-normal theories appears to be caused by circularities among the defaults [5], rather than by overzealous assumptions. So narrowly-scoped default reasoning should not be expected to increase the coherence of default logic. It can be shown, though, that coherence is not generally *decreased,* either. In particular, for many instances of the three classes of defaults we have been considering, every extension of the scope-limited theory turns out to be a subset of an extension of the unscoped version. These results are comforting, since they mean that scoped reasoning does not lead in directions that would be rejected if the scope of reasoning were broader.

Theorem 5.10 Let $\Delta = (D, W)$ be a normal default theory, and let D' be the result of replacing each default,

$$\frac{\Phi(x)\ :\ \Psi(x)}{\Psi(x)},$$

in D with

$$\frac{\Phi(x) \wedge \mathit{Scope}(x)\ :\ \Psi(x)}{\Psi(x)}.$$

Then every extension of $\Delta' = (D', W)$ is contained in an extension of Δ.

Proof: By semi-monotonicity [30, Theorem 3.2], any applicable sequence of defaults from D leads to an extension. Strengthening the prerequisite of some of the defaults simply (at most) makes some of them inapplicable. The remainder assert the same consequent as their unscoped counterparts. But any extension for a default theory is the closure of W plus the consequents of its generating defaults. So any extension for Δ' is contained in a corresponding extension for Δ. ∎

In the following, we assume that the defaults in D do not mention *Scope*. We will also refer to the *scope subtheory of* W*,* which consists of all and only those axioms of W that involve the *Scope* predicate, and we will say that a scope axiom is *pure* if it mentions only the *Scope* predicate.

Theorem 5.11 Let $\Delta = (D, W)$ be a normal default theory, and let D' be the result of replacing each default,

$$\frac{\Phi(x)\ :\ \Psi(x)}{\Psi(x)},$$

in D with

$$\frac{\Phi(x)\ :\ \mathit{Scope}(x) \supset \Psi(x)}{\mathit{Scope}(x) \supset \Psi(x)}.$$

If W entails a DCA and decides equality, and the scope subtheory of W is pure, then every extension of $\Delta' = (D', W)$ (restricted to the language without *Scope*) is contained in an extension of Δ.

Proof: Let E' be an extension for Δ'. We construct an extension, E, for Δ. Since E' is an extension for $\Delta' = (D', W')$, we have that:

$$\begin{aligned}
E'_0 &= W, \text{ and for } i \geq 0, \\
E'_{i+1} &= Th(E'_i) \cup CONSEQUENTS(D'_i), \text{ where} \\
&\quad D'_i = \left\{ \delta \;\middle|\; \delta = \frac{\Phi(\alpha) \ :\ Scope(\alpha) \supset \Psi(\alpha)}{Scope(\alpha) \supset \Psi(\alpha)},\ \Phi(\alpha) \in E'_i, \text{ and} \right. \\
&\qquad\qquad \left. \neg\Psi(\alpha) \wedge Scope(\alpha) \notin E' \right\} \\
E' &= \bigcup_{i=0}^{\infty} E_i
\end{aligned}$$

Let D_i be the analogues of the defaults in D'_i, and construct an intermediate set, E^*, as follows:

$$\begin{aligned}
E^*_0 &= W, \text{ and for } i \geq 0, \\
E^*_{i+1} &= Th(E^*_i) \cup CONSEQUENTS(D^*_i), \text{ and} \\
E^* &= \bigcup_{i=0}^{\infty} E^*_i
\end{aligned}$$

where D^*_i is a maximal subset of D_i such that $CONSEQUENTS(D^*_i)$ is consistent with $E^*_i \cup E'$. Observe that E^* contradicts nothing in E'.

We show by induction that E^*_i has all the *Scope*-free consequents of E'_i. This is obvious for $i = 0$; assume it holds for $i \leq j$ and consider E^*_{j+1}.

We introduce some notation to simplify the discussion. Let S be the *Scope* subtheory of W, and let C'_i be the *Scope*-free ground consequences of $CONSEQUENTS(D'_i) \cup S$ not entailed by W. Without loss of generality, assume C'_i is in clausal form and contains only minimal disjunctions. Notice that decided equality and the DCA ensure that nothing is lost by considering only ground formulae. The additional requirement that the *Scope* subtheory be pure ensures that C'_i consists of only ground disjunctions of the form $\Psi(\alpha_1) \vee ... \vee \Psi(\alpha_n)$. Finally, notice that the *Scope*-free subset of E'_i is contained in $Th(W \cup C'_i)$.

For the inductive step, assume that there are some *Scope*-free facts in E'_{j+1} not contained in E^*_{j+1}. Given the foregoing, it suffices to consider minimal ground disjunctions, $\Psi = \Psi(\alpha_1) \vee ... \vee \Psi(\alpha_n)$, in

C'_{j+1}. Assume $E'_{j+1} \vdash \Psi$ and $E^*_{j+1} \not\vdash \Psi$. By hypothesis, $E^*_j \vdash \Phi(\alpha_i)$, for $i = 1, ..., n$. Since $\Psi \notin E^*_{j+1}$ it must be the case that $E^*_{j+1} \cup E' \vdash \neg\Psi(\alpha_1) \wedge ... \wedge \neg\Psi(\alpha_n)$. This contradicts the assumption that $E'_{j+1} \vdash \Psi$, since E^*_{j+1} must be consistent with E'.

By induction, E^* entails every scope-free consequence of E'. By semi-monotonicity, there is an extension for Δ containing E^* and hence containing all the scope-free consequences of E'. ∎

The following examples illustrate the need for requiring pure scope and fixed domains, by showing cases where a scoped version of a default theory has extensions not contained in any extension of its unscoped counterpart.

Example 5.1 If the scope subtheory is not pure, it is possible to link the satisfaction of the prerequisite of a default for some individual with some other individual's exclusion from the scope because of exceptions. For example, consider $\Delta = (D, W)$ and $\Delta' = (D', W)$, with

$$D = \left\{ \frac{P(x) \ : \ Q(x)}{Q(x)} \right\}, \quad D' = \left\{ \frac{P(x) \ : \ \mathit{Scope}(x) \supset Q(x)}{\mathit{Scope}(x) \supset Q(x)} \right\}, \text{ and}$$

$$W = \left\{ \begin{array}{c} P(a), \ \neg Q(a), \ \neg \mathit{Scope}(a) \supset P(b), \ \mathit{Scope}(b), \\ \forall x. \ x = a \vee x = b, \ a \neq b \end{array} \right\}.$$

Here, Δ has a unique extension, $Th(W)$, which does not contain $Q(b)$. But $Q(b)$ is in $Th(W \cup \{\neg \mathit{Scope}(a), \ P(b), \ Q(b)\})$, the only extension of Δ'. ∎

Example 5.2 To see the role of decided equality, let D and D' be as in Example 5.1, and consider $W = \{P(a), \ P(b), \ \mathit{Scope}(a), \ \neg Q(a), \neg Q(b)\}$. Again, Δ has a single extension, $Th(W)$. Δ' also has a unique extension, $Th(W \cup \{\neg \mathit{Scope}(b), \ a \neq b\})$, and $a \neq b \notin Th(W)$. Because the default in D' is applicable for b even though $W \vdash \neg Q(b)$, we can conclude $\mathit{Scope}(b) \supset Q(b)$, and hence $\neg \mathit{Scope}(b)$. From this and $\mathit{Scope}(a)$, we obtain $a \neq b$. ∎

Example 5.3 The DCA plays a similar role. If D and D' are as in Example 5.1, and $W = \{P(a), \ P(b), \ \neg Q(a), \ \neg Q(b), \ \exists x. \ \mathit{Scope}(x)\}$, then Δ' has a unique extension with $\exists x. \ x \neq a \wedge x \neq b$, but this is not in Δ's unique extension. ∎

Theorem 5.12 Let $\Delta = (D, W)$ be a simple abnormality theory, and let W' be the result of textually replacing every occurrence of $Abnormal(\alpha)$ in W (where α is an arbitrary term) with $(Abnormal(\alpha) \vee \neg Scope(\alpha))$. Then for each extension, E', of $\Delta' = (D, W')$, the *Scope*- and *Abnormal*-free formulae of E' are all contained in some extension, E, of Δ, provided W entails a DCA and decides equality, contains only positive and negative ground literals in *Scope*, and has no negative occurrences of *Abnormal* in its conjunctive normal form.

Proof: Let E' be an extension of Δ', and let W^* be the scope-free subset of $Th(W')$. Obviously, any *Scope*-free consequent of E' is a consequent of $GD(E', \Delta') \cup W^*$, given that W has a closed domain and decides equality.

Let $GD' = CONSEQUENTS(GD(E', \Delta')) \cap \{\neg Abnormal(\alpha) \mid \alpha$ *is a ground term and* $Abnormal(\alpha)$ *occurs positively in a formula in* $W^*\}$. Observe that no other default consequents can participate in the derivation of *Abnormal*-free formulae in E'.

We claim that the resulting set of normality assumptions, GD', is consistent with W. Otherwise, $W \vdash Abnormal(\alpha_1) \vee ... \vee Abnormal(\alpha_n)$ for some finite subset of GD' (by compactness). Any derivation of $Abnormal(\alpha_1) \vee ... \vee Abnormal(\alpha_n)$ requires axioms in W but not in W^*, since $W^* \not\vdash Abnormal(\alpha_1) \vee ... \vee Abnormal(\alpha_n)$. But these must also have *Abnormal* in them, since all other axioms of W (except ground literals in *Scope*) are *Scope*-free in W', and hence are in W^*.

Moreover, these axioms must involve abnormality of terms besides $\alpha_1, ..., \alpha_n$, because each of $\neg Abnormal(\alpha_1)$ through $\neg Abnormal(\alpha_n)$ has a positive counterpart in W^*. By construction, each occurrence of $Abnormal(\alpha)$ in W is paired with $\neg Scope(\alpha)$ in W'. Since *Scope* occurs in W only as ground literals, any formula in W with no *Abnormal* terms other than $Abnormal(\alpha_1)$ ... $Abnormal(\alpha_n)$ must occur in W^*.

Since *Abnormal* does not occur negatively in W, these additional *Abnormal* terms cannot be eliminated, which contradicts the assumption that the axioms containing them enter into the derivation of $Abnormal(\alpha_1) \vee ... \vee Abnormal(\alpha_n)$ from W. So $W \not\vdash Abnormal(\alpha_1) \vee ... \vee Abnormal(\alpha_n)$.

By semi-monotonicity, Δ has an extension, E, which is a superset of $W \cup GD'$. Since $W^* \subset W$, it follows that all *Scope*- and *Abnormal*-

free consequences of E' are contained in E. ∎

Complicated though it may sound, what is meant by the substitution of $(Abnormal(\alpha) \vee \neg Scope(\alpha))$ for $Abnormal(\alpha)$ is generally straightforward. The following example illustrates the result.

Example 5.4 A simple abnormality theory, W, and the corresponding W' are given in parallel columns, below:

$Bird(Tweety)$	$Bird(Tweety)$
$\forall x.\ Bird(x) \wedge \neg Abnormal(x)$ $\supset Flies(x)$	$\forall x.\ Bird(x) \wedge \neg Abnormal(x)$ $\wedge Scope(x) \supset Flies(x)$
$\forall x.\ Penguin(x) \supset Abnormal(x)$	$\forall x.\ Penguin(x)$ $\supset Abnormal(x) \vee \neg Scope(x)$
$\forall x.\ Penguin(x) \supset Bird(x)$	$\forall x.\ Penguin(x) \supset Bird(x)$
$Scope(Tweety)$	$Scope(Tweety)$
$\forall x.\ x = Tweety \vee x = B_1 \vee$ $\ldots \vee x = B_n$	$\forall x.\ x = Tweety \vee x = B_1 \vee$ $\ldots \vee x = B_n$

If

$$D = \left\{ \frac{:\ \neg Abnormal(x)}{\neg Abnormal(x)} \right\},$$

then $\Delta = (D, W)$ and $\Delta' = (D, W')$ both have unique extensions, E and E', respectively, that contain $Flies(Tweety)$ and $\neg Penguin(Tweety)$. However, E also entails that there are no penguins ($\forall x.\ \neg Penguin(x)$), whereas E' does not. ∎

The stipulations in Theorem 5.12, for the strongest form of scoped default representation, are more stringent than for either of the other cases. The prohibition of negative occurrences of *Abnormal* is necessary because otherwise it is possible construct dependencies among abnormalities of individuals, some of whom may fall outside the scope. Outright prohibition is stronger than necessary, but we do not yet have a more appropriate characterization. Restricting *Scope* to ground literals is also too Draconian. We believe the result may hold so long as *Scope* occurs only in pure formulae, but we have yet to prove this. Finally, the restriction to theories with DCA's and decided equality are necessary for the same reason they were required for Theorem 5.11, to rule out the possibility that what is exceptional might depend directly on what (else) exists.

5.2.3 Non-normal Default Theories

We have not yet begun to investigate the application of scope to non-normal default theories (those where the prerequisite and justification of the defaults need not be identical). While we expect to find limiting the scope of reasoning as profitable and as necessary in that context, it seems unlikely that it will be possible to characterize the results of adding scope so precisely.

In particular, we doubt the scoped versions of non-normal theories will always have extensions "contained in" extensions for their unscoped counterparts. This is because the property of semi-monotonicity, on which we relied heavily in the preceding sections, does not hold for non-normal theories, since defaults may undercut other defaults' conclusions. However, Etherington [2] delineates a class of theories, the "ordered theories" that are much better behaved than arbitrary non-normal theories. Ordered theories suggest themselves as a likely next target for generalizing these results.

Another possible application of non-normal theories might be to further restrict the application of defaults. Given the default

$$\frac{P(x) \ : \ \mathit{Scope}(x) \supset Q(x)}{\mathit{Scope}(x) \supset Q(x)},$$

it may be possible, for example, to conjecture $\mathit{Scope}(a) \supset Q(a)$ from $P(a)$, even though $\neg Q(a)$ is known—leading to the conclusion $\neg \mathit{Scope}(a)$ (see examples 5.2 and 5.3). We could avoid such conclusions about scope for individuals known to be exceptional by choosing either the seminormal or non-normal representation:

$$\frac{P(x) \ : \ (\mathit{Scope}(x) \supset Q(x)) \wedge Q(x)}{\mathit{Scope}(x) \supset Q(x)}, \text{ or } \frac{P(x) \ : \ (\mathit{Scope}(x) \supset Q(x)), Q(x)}{\mathit{Scope}(x) \supset Q(x)},$$

respectively, for the default. We have yet to explore such alternatives.

5.3 Scoped Autoepistemic Logic

Unscoped reasoning also presents problems in autoepistemic logic which are ameliorated by restricting the scope of reasoning. However, since a fully-quantificational first-order autoepistemic logic has not yet been formalized (but see [13, 18] for suggestions), we restrict our discussion to three examples and two theorems of limited generality.

First, consider the Lottery Paradox. As in §5.2.1, suppose we have 10,000 lottery tickets, and wish to buy one among the 176 from $t_{100} \ldots t_{175}$. The only change required for AEL is that, instead of a default rule, we use the schema:

$$\mathbf{L}Ticket(t) \wedge \mathbf{L}Scope(t) \wedge \neg\mathbf{L}Wins(t) \rightarrow \neg Wins(t)$$

where t ranges over the 10,000 ticket constants. In such a case, as with default logic, we get only one extension, in which we have $\neg Wins(t)$ for the 176 scoped tickets but not for the rest.

Next we turn to the very similar case of counterexample axioms. Assume we have an axiom specifying the existence of a non-flying bird, as in (4). We use the schema:

$$\mathbf{L}Bird(b) \wedge \mathbf{L}Scope(b) \wedge \neg\mathbf{L}\neg Flies(b) \rightarrow Flies(b)$$

to represent the default. If we also have $Scope(Tweety)$, then there is a unique extension, in which the scoped birds fly. In the case in which all birds are scoped, the problem of multiple extensions resurfaces. However, this wide scope necessarily contains anonymous exceptions, and default reasoning may therefore be inappropriate.

Third, suppose there are only three kinds of bird, Canary, Mynah, and Penguin, and that Canaries are typical but Mynahs and Penguins are not: Mynahs do not build nests and Penguins do not fly [28]. What is the intuitively desired behaviour here? We certainly do not want to conclude that all birds are canaries, despite that being the result of straightforward application of autoepistemic logic. Specifically, from the axioms:

$$\begin{gathered}\forall x.\ Mynah(x) \supset \neg Nesting(x)\\ \forall x.\ Penguin(x) \supset \neg Flies(x)\\ \forall x.\ Bird(x) \equiv Mynah(x) \vee Penguin(x) \vee Canary(x)\end{gathered}$$

and the defaults that birds typically fly and build nests, we get that there are no mynahs or penguins, i.e., all birds are canaries. Scope can help if we employ the two scope-limited schemata:

$$\begin{gathered}\mathbf{L}Bird(b) \wedge \mathbf{L}Scope(b) \wedge \neg\mathbf{L}\neg Flies(b) \rightarrow Flies(b), \quad \text{and}\\ \mathbf{L}Bird(b) \wedge \mathbf{L}Scope(b) \wedge \neg\mathbf{L}\neg Nesting(b) \rightarrow Nesting(b),\end{gathered}$$

where again b ranges over the finitely-many constants. Provided scope is narrow, there will be only one extension, in which all scoped birds are canaries, but unscoped birds are indeterminate as to species (as well as flying and nesting behaviours).[21]

Thus in each of the three examples, scoped reasoning provides us with an intuitively-plausible outcome. It would be better to have a general result, even if based on strong restrictions. For the case of strongly-grounded autoepistemic extensions (see [13, 14]), we can provide this, albeit modulo the DCA (as were the examples). For such extensions, Konolige showed the following [13, Theorem 5.5]:[22]

> **Theorem 5.13 (Konolige)** Let A be the AE transform of a default theory Δ. A set, E, is a default extension of Δ iff it is the $\mathbf{L}$-free subtheory of a strongly grounded AE extension of A. ∎

Here "the AE transform" of a default theory is obtained by replacing each default rule,

$$\frac{\alpha : M\beta_1, ..., M\beta_n}{\omega},$$

by $(\mathbf{L}\alpha \wedge \neg\mathbf{L}\neg\beta_1 \wedge ... \wedge \neg\mathbf{L}\neg\beta_n) \supset \omega$.

We begin with a sufficiency result.

> **Theorem 5.14** If W entails a domain closure axiom, $W \not\models \exists x.\ \Phi(x) \wedge \neg\Psi(x) \wedge Scope(x)$, and $D = \{\mathbf{L}\Phi(c) \wedge \mathbf{L}Scope(c) \wedge \neg\mathbf{L}\neg\Psi(c) \supset \Psi(c)\}$ is a schema over all the constants, c, of W, then no strongly grounded AE extension of $W \cup D$ contains any scoped exceptions.
>
> **Proof:** From theorem 5.13, we see that E is a default logic extension of a theory, W, with defaults of the form:
>
> $$D' = \left\{ \frac{\Phi(x) \wedge Scope(x) \ :\ \Psi(x)}{\Psi(x)} \right\}$$
>
> iff E is the $\mathbf{L}$-free subtheory of a strongly grounded AE extension of $W \cup D$. By corollary 5.8, since extensions, E, of (D', W) have no scoped exceptions, neither do strongly grounded AE extensions of $W \cup D$. ∎

[21] We have more to say on this example in §5.4.

[22] Note that we actually use the revised definition of strong groundedness from [14], avoiding the difficulties of [13].

Similarly, from theorem 5.10, we get a consistency result.

Theorem 5.15 Suppose W entails a DCA and is $\mathbf{L}$-free, and D consists of schemata of the form $\mathbf{L}\Phi(c) \wedge \neg\mathbf{L}\neg\Psi(c) \supset \Psi(c)$. Let D' be the result of replacing each schema in D by $\mathbf{L}\Phi(c) \wedge \mathbf{L}Scope(c) \wedge \neg\mathbf{L}\neg\Psi(c) \supset \Psi(c)$. Then the $\mathbf{L}$-free subtheory of any strongly-grounded AE extension of $W \cup D'$ is contained in the $\mathbf{L}$-free subtheory of a strongly-grounded AE extension of $W \cup D$.

Proof: Let D_{dl} and D'_{dl} consist of defaults of the form:

$$\frac{\Phi(c) \ : \ \Psi(c)}{\Psi(c)} \qquad \text{and} \qquad \frac{\Phi(c) \wedge Scope(c) \ : \ \Psi(c)}{\Psi(c)},$$

respectively. Then E' is the $\mathbf{L}$-free subtheory of a strongly-grounded extension of $W \cup D$ iff it is a default extension of (D'_{dl}, W) according to theorem 5.13, above. By theorem 5.10, E' is contained in a default extension, E, of (W, D_{dl}). Again by theorem 5.13, E is the $\mathbf{L}$-free subtheory of a strongly grounded extension of $W \cup D$. ∎

These results are not as broad as those above for circumscripion or default logic; however, they are suggestive of the same trend, indicating that a *Scope* predicate can be useful in AEL for treating the "paradoxes" of overzealousness (forced normalities) surveyed in §4.

5.4 Scoped Properties

Consider again the case discussed in §3.3, in which there are three types of birds (mynahs, penguins, and canaries) each of which is atypical in some regard. Recall that this situation tends to result in a nonmonotonic theory with multiple extensions or minimal models. This in turn leads to the anomaly that *no* bird can be conjectured to fly or have any of the other properties that birds typically have.

We claim that these problems are again due to the the application of default reasoning over too wide a scope of interest. In particular, all *properties* (flying, nesting, singing, and drabness) are taken to be of interest. As long as this is so, then we indeed have Poole's problem, and we claim that there is no intuitive outcome.

We contend, however, that it is more common to be concerned with only a few properties. By employing the common technique of explicitly representing aspects of typicality (c.f. [22]) it is possible to apply scope to this problem in much the same manner we have been discussing.

For instance, if we want to build a cage for a bird, we may be interested in whether it flies, and not care about nesting or drabness. Thus, we might have the axioms:

$$\begin{gathered}
\forall x.\ Bird(x) \wedge \neg Abnormal(aspect_1, x) \supset Nesting(x) \\
\forall x.\ Bird(x) \wedge \neg Abnormal(aspect_2, x) \supset Flies(x) \\
\forall x.\ Bird(x) \wedge \neg Abnormal(aspect_3, x) \supset Drab(x) \\
\forall x.\ Bird(x) \equiv Mynah(x) \vee Penguin(x) \vee Canary(x) \\
\forall x.\ Mynah(x) \supset \neg Nesting(x) \\
\forall x.\ Penguin(x) \supset \neg Flies(x) \\
\forall x.\ Canary(x) \supset \neg Drab(x)
\end{gathered}$$

and then use our favourite nonmonotonic reasoning scheme to minimize $Abnormal(a, x)$, with $aspect_2$ and the bird of interest specified to be in the scope.[23]

5.5 Expanding the Scope

Since the reasoner must determine her scope of interest before applying defaults, the question of how sensitive our approach is to changes of scope immediately arises. It seems quite possible (if not probable) that a reasoner will occasionally find that more individuals need to be considered in order to realize a goal or complete a line of reasoning. If this routinely necessitates retraction of all default conclusions reached under a narrower scope, the effort expended to that point on the solution will be wasted. Fortunately, it is frequently the case that all conjectures supported under a narrow scope are still warranted when that scope is expanded. In particular, as the following theorems show, provided the wider scope does not necessarily contain anonymous scoped exceptions to the defaults, any conjecture sanctioned under a narrow scope is also sanctioned under a wider scope.

[23]Notice that this presumes the obvious generalization of the techniques we have presented in order to deal with the joint scoping of aspects and individuals.

Theorem 5.16 Let W be a first-order theory, with a pure scope subtheory and let

$$D = \left\{ \frac{\Phi(x) \ : \ Scope(x) \supset \Psi(x)}{Scope(x) \supset \Psi(x)} \right\}.$$

Let W' be a superset of W which differs only in having additional pure *Scope* axioms. If $W \vdash \Phi(\alpha_i) \wedge Scope(\alpha_i) \wedge \neg\Psi(\alpha_i)$, for ground terms $\alpha_i \in \{\alpha_1, ..., \alpha_n\}$, and $W' \not\vdash \exists x.\ x \neq \alpha_1 \wedge ... \wedge x \neq \alpha_n \wedge [\Phi(x) \wedge Scope(x) \wedge \neg\Psi(x)]$, then Δ' has a unique extension, E', that contains the unique extension, E, for Δ.

Proof: By lemma 5.18, $\Delta = (D, W)$ and $\Delta' = (D, W')$ have at most one extension, and because they are normal, they have at least one. We show that we can construct an extension E' for Δ' such that every default in $GD(E, \Delta)$ is in $GD(E', \Delta')$, from which the result follows.

By [30, Theorem 2.1],

$$\begin{aligned}
E &= \bigcup_{i=0}^{\infty} E_i, \quad \text{where :} \\
E_0 &= W, \text{ and for } i \geq 0, \\
E_{i+1} &= Th(E_i) \cup CONSEQUENTS(D_i), \text{ where} \\
& \quad D_i = \left\{ \frac{\Phi(\alpha) \ : \ Scope(\alpha) \supset \Psi(\alpha)}{Scope(\alpha) \supset \Psi(\alpha)} \,\middle|\, \Phi(\alpha) \in E_i, \text{ and} \right. \\
& \qquad \left. (Scope(\alpha) \wedge \neg\Psi(\alpha)) \notin E \right\}.
\end{aligned}$$

Define E'_i as follows:

$$\begin{aligned}
E'_0 &= W', \text{ and for } i \geq 0, \\
E'_{i+1} &= Th(E'_i) \cup CONSEQUENTS(D'_i), \text{ where} \\
& \quad D'_i \text{ is a maximal subset of } D_i \text{ consistent with } E'_i.
\end{aligned}$$

We claim that $E_i \subseteq E'_i$ (in particular, $D_i = D'_i$) for all i. Clearly $E_0 = W \subset W' = E'_0$; if $D'_i \subset D_i$, then $W' \vdash \wedge_j \Phi(\alpha_j)$, for the ground terms α_j in D_i, and $W' \vdash \vee_j \neg(Scope(\alpha_j) \supset \Psi(\alpha_j))$. Since none of the α_j are in $\{\alpha_1, ..., \alpha_n\}$, this contradicts the assumption that $W' \not\vdash \exists x.\ x \neq \alpha_1 \wedge ... \wedge x \neq \alpha_n \wedge [\Phi(x) \wedge Scope(x) \wedge \neg\Psi(x)]$.

For the inductive step, assume that $D_j = D'_j$ for $j < k$, and hence $E_k \subseteq E'_k$. Observe that

$$E'_k = W' \cup \left\{ \Phi(\alpha) \wedge (Scope(\alpha) \supset \Psi(\alpha)) \;\middle|\; \frac{\Phi(\alpha) \;:\; Scope(\alpha) \supset \Psi(\alpha)}{Scope(\alpha) \supset \Psi(\alpha)} \in \bigcup_{j=0}^{k-1} D_j \right\}.$$

From this observation it is easily seen, by iteration of the argument used in the base case, that if $D'_k \subset D_k$, then the assumptions in the statement of the theorem are violated.

It follows by semi-monotonicity [30, Theorem 3.2] that $\bigcup_{i=0}^{\infty} E'_i$ can be extended to an extension for Δ', and it must be the only such extension. ∎

Theorem 5.17 Let W be a first-order theory, with a pure scope subtheory and let

$$D = \left\{ \frac{\Phi(x) \wedge Scope(x) \;:\; \Psi(x)}{\Psi(x)} \right\}.$$

Let W' be a superset of W which differs only in having additional pure *Scope* axioms. If $W \vdash \Phi(\alpha_i) \wedge Scope(\alpha_i) \wedge \neg\Psi(\alpha_i)$, for ground terms $\alpha_i \in \{\alpha_1, ..., \alpha_n\}$, and $W' \not\vdash \exists x.\ x \neq \alpha_1 \wedge ... \wedge x \neq \alpha_n \wedge [\Phi(x) \wedge Scope(x) \wedge \neg\Psi(x)]$, then Δ' has a unique extension, E', that contains the unique extension, E, for Δ.

Proof: The proof is obvious from that for theorem 5.16. ∎

Lemma 5.18 Let D be a normal default theory:

$$\left\{ \frac{\Phi_i(\bar{x}) \;:\; \Psi_i(\bar{x})}{\Psi_i(\bar{x})} \;\middle|\; i = 1, 2, ... \right\},$$

where $\Phi(\bar{x})$ and $\Psi(\bar{x})$ are arbitrary formulae, and let W be a first-order theory. Define I, the implications corresponding to D, by $I = \{\Phi_i(\bar{\alpha}) \supset \Psi_i(\bar{\alpha}) \mid i = 1, 2, ...; \bar{\alpha}$ a vector of ground terms$\}$. If $\Delta = (D, W)$ has more than one extension, then $W \cup I$ is inconsistent.

Proof: Let E_1 and E_2 be extensions for Δ. Since Δ is normal, E_1 and E_2 are orthogonal [30, Theorem 3.3]. By [30, Theorem 2.1], for $i \in \{1, 2\}$,

$$\begin{aligned} E_i &= \bigcup_{j=0}^{\infty} E_j^i, \quad \text{where :} \\ E_0^i &= W, \text{ and for } j \geq 0, \\ E_{j+1}^i &= Th(E_j^i) \cup \mathit{CONSEQUENTS}(D_j^i), \text{ where} \\ & \quad D_j^i = \left\{ \frac{\Phi_k(\bar{\alpha}) \ : \ \Psi_k(\bar{\alpha})}{\Psi_k(\bar{\alpha})} \;\middle|\; \Phi_k(\bar{\alpha}) \in E_j^i, \ \neg\Psi(\bar{\alpha}) \notin E_i \right\}. \end{aligned}$$

$$\text{Define } I_j^i = \left\{ \Phi_k(\bar{\alpha}) \supset \Psi_k(\bar{\alpha}) \;\middle|\; \frac{\Phi_k(\bar{\alpha}) \ : \ \Psi_k(\bar{\alpha})}{\Psi_k(\bar{\alpha})} \in D_j^i \right\}.$$

Clearly, $W \cup \bigcup_{j=1}^{r-1} I_j^i$ is logically equivalent to E_r^i, for arbitrary r.

It is easily seen that $E_i = Th(W \cup \bigcup_{j=0}^{\infty} I_j^i)$. But $\bigcup_{j=0}^{\infty} I_j^i \subset I$. Since E_1 and E_2 are orthogonal, it follows that I is inconsistent with W.[24]
∎

We conjecture that a result similar to theorems 5.16 and 5.17 holds for the other scoped default representation we have presented, but have no proof. Below, we show that scoped circumscription is also quite tolerant of expansion of the scope of interest.

Theorem 5.19 Let A and A' be first-order theories, differing only in that A' contains additional pure $Scope$ axioms. If, for every consistent extension, A'', of A' by ground (in)equalities, $A'' \not\vdash \exists x.\, W(P, x) \wedge Scope(x)$, then every $W(P, x) \wedge Scope(x)$-minimal model for A' is minimal for A, provided all predicates are variable, and A entails a domain-closure axiom.

[24] Notice that the lemma gives us a sufficient condition for the existence of a unique extension for a default theory that is considerably stronger than Reiter's condition that W be consistent with the consequents of all the defaults [30, Corollary 3.4]. This result is interesting in its own right.

Proof: Clearly, every model for A' satisfies A. Assuming A' has models (the result is trivially true otherwise) then, by theorem 5.1, every minimal model, M, for A' has $(|W(P)|_M \cap |Scope|_M) = \emptyset$, and hence is minimal for A. ∎

Since the circumscription of a theory is true in all minimal models of that theory, theorem 5.19 guarantees that circumscription over a (consistent) wider (or more-fully-specified) scope does not invalidate the results of circumscription over narrower (or less-specified) scopes—in fact, they are strengthened—provided anonymous exceptions are not conscripted into the scope.[25] Theorem 5.19 generalizes directly to the case of *known* scoped exceptions, analogously to the generalization of theorem 5.1 to theorem 5.3.

Returning to autoepistemic logic, we can get an analogous result for theories with domain closure, using theorems 5.13 and 5.17.

Theorem 5.20 Let W be a first order theory with a pure scope subtheory that entails a domain closure axiom, and let $D = \{\mathbf{L}\Phi(c) \wedge \mathbf{L}\mathit{Scope}(c) \wedge \neg\mathbf{L}\neg\Psi(c) \supset \Psi(c)\}$ be a schema over all the constants, c, of W. Let W' be a superset of W which differs only in having additional pure *Scope* axioms. If $W \vdash \Phi(\alpha_i) \wedge \mathit{Scope}(\alpha_i) \wedge \neg\Psi(\alpha_i)$, for ground terms $\alpha_i \in \{\alpha_1, ..., \alpha_n\}$, and $W' \not\vdash \exists x.\ x \neq \alpha_1 \wedge ... \wedge x \neq \alpha_n \wedge [\Phi(x) \wedge \mathit{Scope}(x) \wedge \neg\Psi(x)]$, then the **L**-free subtheory of any strongly-grounded AE extension of $W \cup D$ is contained in the L-free subtheory of a strongly-grounded AE extension of $W' \cup D$.

Proof: Let D_{dl} be the DL transform of D, i.e. D_{dl} consists of the defaults of the form:

$$\frac{\Phi(c) \wedge \mathit{Scope}(c)\ :\ \Psi(c)}{\Psi(c)},$$

for any constant c in W. By theorem 5.17, (D, W') has a unique extension, E', that contains the unique extension, E, for (D, W). By theorem 5.13, E is an extension of (D, W) iff it is the **L**-free subtheory of a strongly grounded AE extension of $W \cup D$. Similarly, E' is an

[25] But recall that the lack of a completeness result (in the general case) for any version of circumscription with an effective proof theory (and vice versa) means that some conclusions true in all minimal models may not be derivable from the circumscribed theory.

extension of (D, W') iff it is a **L**-free subtheory of a strongly grounded AE extension of $W' \cup D$. The result follows. ∎

Of course, once the scope of reasoning is expanded to the point where it necessarily includes anonymous exceptions, these guarantees of good behaviour expire. This is as it should be; once we discover that an exception is within our scope of interest, we should no longer trust our earlier assumptions about its identity.[26] In general, it is important to ensure that the scope of reasoning is always narrow, since injudicious scoping can force all exceptions to occur among a small set of unscoped individuals—something no less paradoxical than the behaviour we introduced scope to avoid.

6 Unresolved Issues

As usual, there are loose ends remaining. Before concluding, we touch on two of the most significant. The imaginative reader will no doubt think of others, probably including some that have not occurred to us.

6.1 Determining Scope

The most obvious outstanding question concerns the nature of the scope theory. How does an agent determine the scope of her reasoning, and how does that scope change as reasoning proceeds? For the purposes of this paper, and indeed in our work to date, we have assumed that the extent of the scope is given in the statement of the problem. In this, scope is treated like the goal statement in problem-solving and planning research. By the same token, just as agents must eventually be able to set their own goals, they will need to be able to figure out what interests them.

Ideally, it should be possible to determine scope from the current context, attention, and goals of the agent. Among other things, we imagine that the individuals mentioned in a query or goal statement, or attended to as the

[26] Otherwise, one could "identify" the exception by repeatedly expanding the scope by one individual at a time until the exception was necessarily in the scope. Since each step would produce the conjecture that the exception was not in the scope up to that point, the exception would have to be the last one added to the scope!

result of recent discourse or experience will be scoped. We suspect, too, that work of Halpern and Rabin [11], Halpern and McAllester [9], Halpern and Moses [10], Drapkin, Miller and Perlis [1], and Nutter [26] will be relevant. In particular the notion of an awareness set seems to have a similar spirit. We imagine "scope" to be slightly different, however—more like "of concern" or "relevant to making a decision".

An attractive aspect of our approach is that it is not catastrophic if the scope is a bit too wide. Some irrelevant default conjectures may be made if too many individuals are scoped, perhaps resulting in extra computational effort and a somewhat lower probability that all conjectures are correct. Still, given the requirement that default reasoning only be done when the scope is narrow (e.g., orders of magnitude smaller than the whole domain), even fairly gross scope determinations should be acceptable without the reintroduction of paradoxical behaviour.

It is also important to note that it is not necessary to assert what is not in the scope. By their nature, the techniques we have introduced depend on positive assertions of scope. Aside from the practical point that many fewer things will be in the scope than out of it, this has the aesthetic property that agents don't have to think about what they are not thinking about.

Similarly, so long as the "ideal" scope of interest is still narrow, including too few individuals should not be catastrophic. The conjectures obtained from scoped default reasoning may be too weak but, as we showed in §5.5, they generally will not contradict the conclusions obtained using a broader scope, provided the broader scope does not necessarily contain counterexamples. If the results of scoped reasoning are too weak to achieve some goal, this may simply be a signal that something else needs to be taken into account.

The determination of whether the scope of interest is narrow with respect to the reference population should be easier. It seems likely that agents' knowledge bases will have some sort of lower bounds on the size of classes about which they would be prepared to do default reasoning, so it should be an easy matter to determine whether the individuals they are interested in are few, relative to that number.

6.2 Equality and Domain Closure

The reader has probably noticed that unique-names and domain-closure assumptions have figured prominently in many of the results in this paper.

This is a result of the peculiar role that ontology plays in existing theories of nonmonotonic reasoning. In circumscription, what exists and which individuals coincide are strictly beyond the purview of the formalism. If there are models with pathological ontologies, they generally contribute to the set of minimal models along with the less peculiar models. Thus, if it is possible for either existence or identity to affect the results of circumscriptive reasoning, that possibility is factored into the conjectures obtained. In particular, if the properties of objects depend on the ontology—either explicitly or implicitly, through the requirement that some minimum number of individuals have a property—circumscription tends to be hamstrung.

Default logic, on the other hand, is blithely unconscious of any special status for equality. If satisfaction of a default entails that two individuals are distinct (or even the same), the distinction (or conflation) is made without special consideration. Similarly, if more defaults can be satisfied by positing the existence of some more individuals, they are assumed to exist, all philosophical tradition notwithstanding. In fact, as we have mentioned, because of the way defaults with free variables are treated, the presence of a domain-closure axiom preventing the multiplication of entities tends to induce paradoxical behaviour in unscoped theories.

If the ontology is not fixed, default conclusions about scope may induce consequences not contained in any extension of the unscoped theory. Lack of decided equality and/or DCA's seem less problematic for default logic than for circumscription, however. For the first representation for scoped defaults, fixed ontologies are not necessary; in the other cases, even when scoped representations produce conclusions not sanctioned by their unscoped counterparts, the extra conclusions seem justifiable.

What is lacking is a commonsense theory of identity. In circumscription, identity has absolute precedence over default reasoning; in default logic, identity is affected at most peripherally, as a side effect of satisfying defaults. The variety of problems presented to theories of nonmonotonic reasoning by identity suggest that there is an interesting, and virtually untouched, area of investigation waiting to be addressed.

7 Conclusions

We have pointed out common roots underlying four significant problems with existing approaches to nonmonotonic reasoning. We showed that these problems visit all the major current approaches, and argued that they were significant—rather than artificial—impediments to using these formalisms for commonsense reasoning.

Having done this, we introduced an idea—that of restricting the scope of reasoning—that provides powerful leverage on the problems. We showed that this idea has direct application in the various circumscriptive theories, in default logic, and in autoepistemic logic, and that it is similarly effective in each. Even more satisfying, we showed that what is required to achieve these benefits involves simple methodological changes, rather than development of new formalisms or new variants of existing formalisms.

We outlined how restricting the scope of nonmonotonic reasoning provides acceptable, commonsensical, solutions to the problems in question. These include the lottery paradox, the problem of anonymous exceptions to defaults, the difficulty of dealing with multiple dimensions of typicality when few, if any, individuals may be absolutely typical in every respect, and nonmonotonic formalisms' unfortunate tendency to conjecture the typicality of individuals at the expense of evacuating entire classes that might be atypical.

For each of the formalisms in question, we showed that the conclusions sanctioned by the strengthened, scope-limited, versions are generally in accord with (some subset of) the preferred models of the original theory. These results are comforting, since they mean that we have strengthened the theories, rather than simply subverting them. We also showed that appropriate notions of consistency are preserved. In fact, for at least two of the formalisms, consistency is preserved over a broader class of theories than for the corresponding unscoped representations.

The framework we developed not only avoids paradox, but also adapts naturally to goal-directed reasoning, unlike current approaches. Default assumptions are made only about particular objects of interest; this appears to be much more natural than current maximal-consistent-set approaches. We intend to explore this aspect of scoped reasoning in detail, as we believe it offers promise for the development of practical nonmonotonic reasoning systems.

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A preliminary version of this paper, with fewer results and without proofs, appears as [7], and a discussion of scoped circumscription in the context of the frame problem appears as [6].

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