



A Voronoi-based Algorithm for **Medial Axis** Extraction

(Case Study: Watershed Delineation from River Networks)

By

Mehran Ghandehari



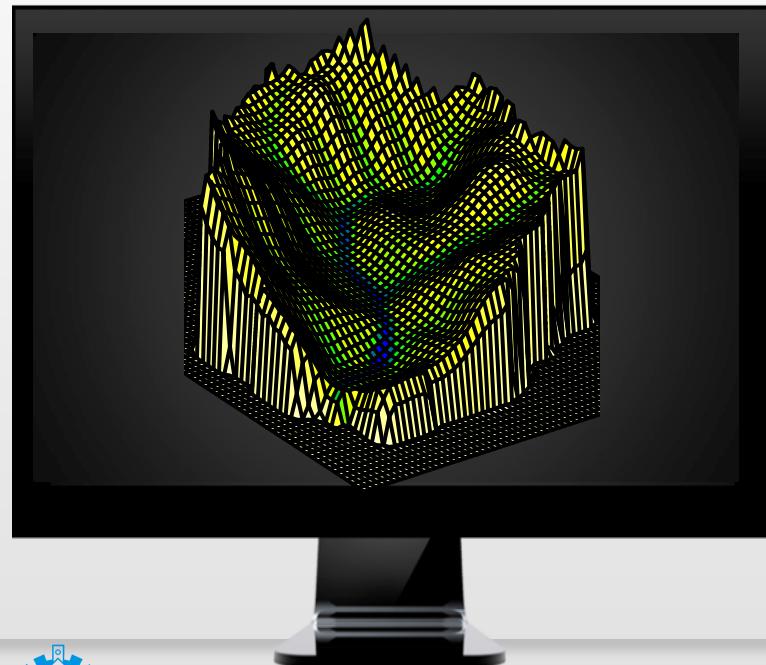
Supervisors

Dr. Farid Karimipour



A Voronoi-based Algorithm for Medial Axis Extraction

Case Study: Watershed Delineation from River Networks



Computational Geometry

GIS

Hydrology



Outline

1. Introduction



- Medial Axis
 - Theory
 - Algorithms
 - Applications

2. Geometric Definitions



- Delaunay Triangulation, Voronoi Diagram, Sampling, Curve Reconstruction

3. Voronoi-based Algorithms



- Semi Continues Voronoi-based Algorithms

4. Proposed Method



- A Stable Voronoi-based Algorithm for Medial Axis Extraction through Labeling Sample Points

5. Case Study



- Watershed Delineation
- Results
- Evaluation

6. Conclusions & Future Work



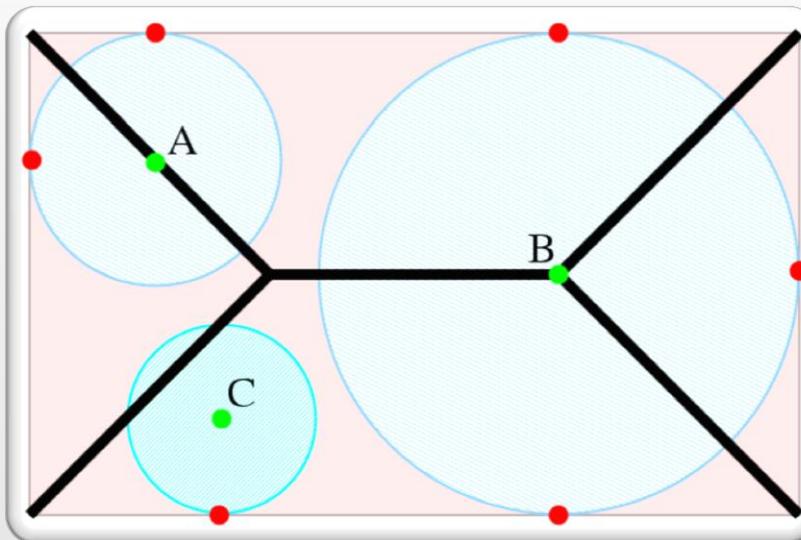
- Summary
- Conclusions
- Future Work



1

•Introduction

Medial Axis (Definition)



Medial Axis (Algorithms)

1. Discrete
2. Continuous (Exact)
3. Semi-continuous



Medial Axis (Algorithms)

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2. Continuous (Exact)
3. Semi-continuous

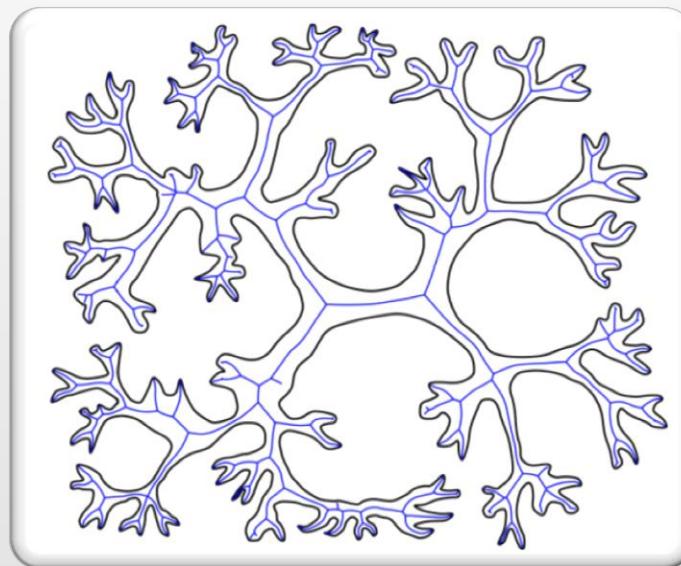


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



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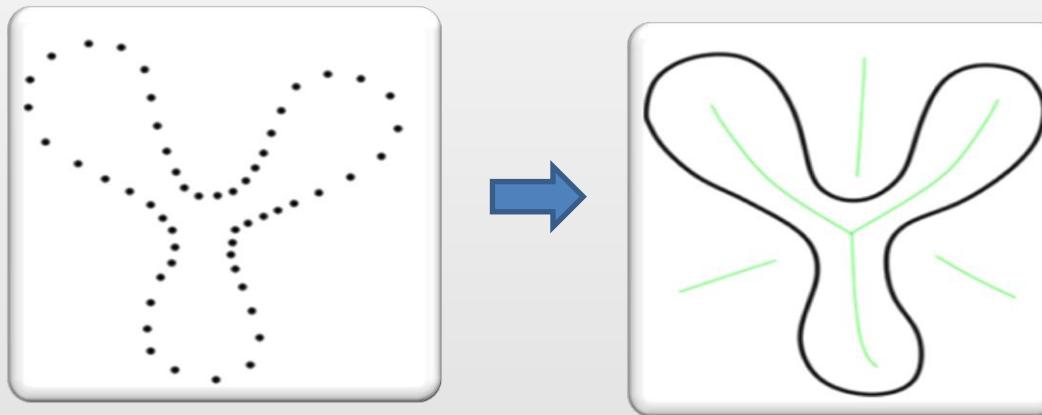


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



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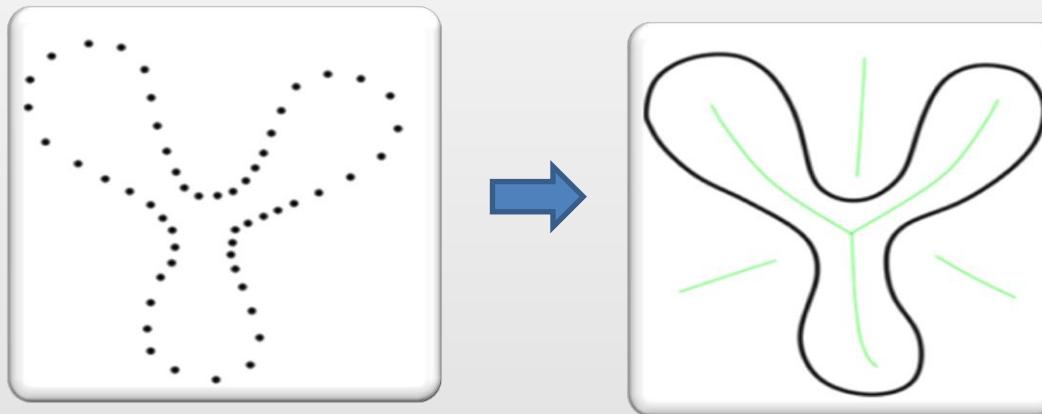


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Medial Axis (Algorithms)

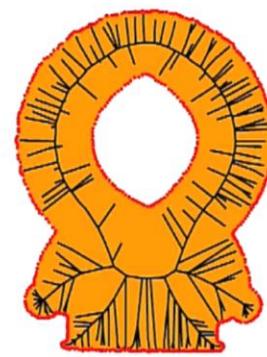
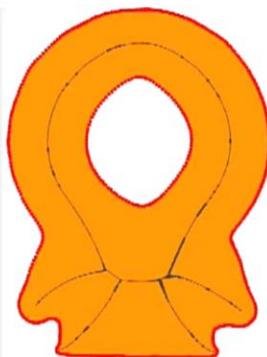
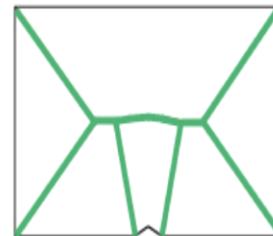
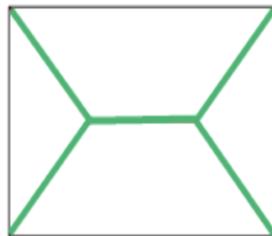
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A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Medial Axis (Filtering)



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



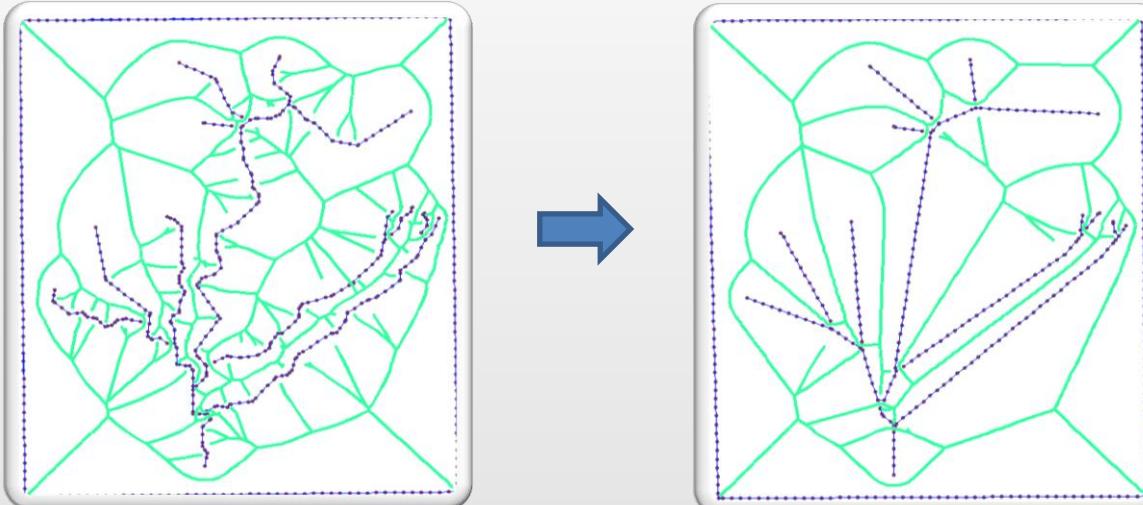
Medial Axis (Filtering)

- Pre-processing (simplifying)
- Post-processing (pruning)



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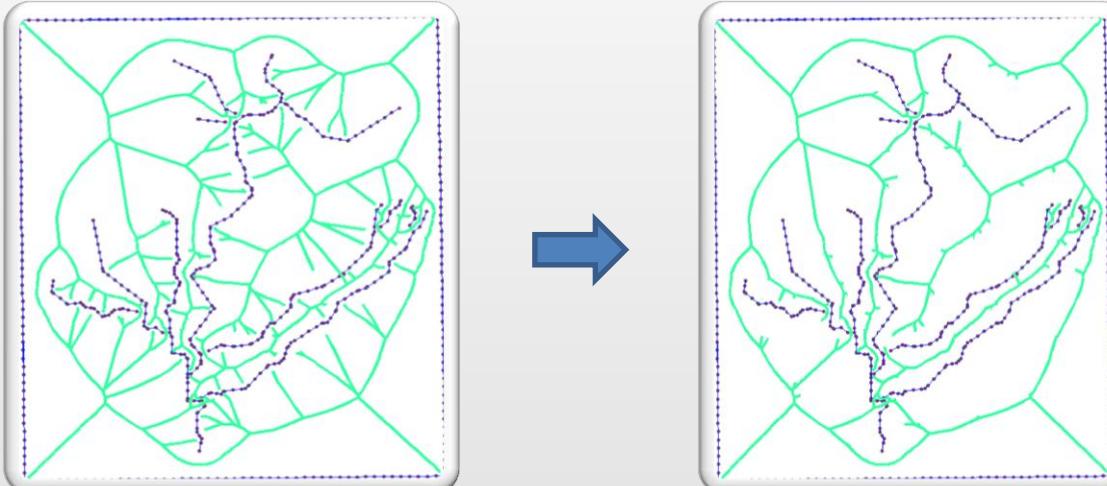


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



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- Post-processing (pruning)

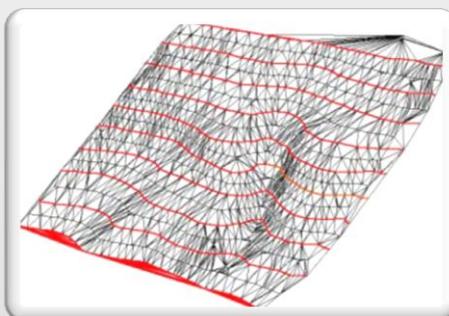
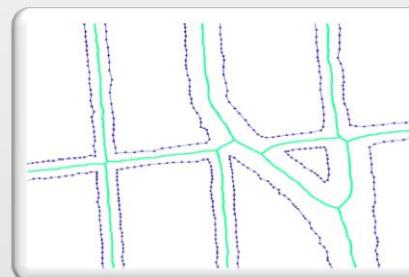
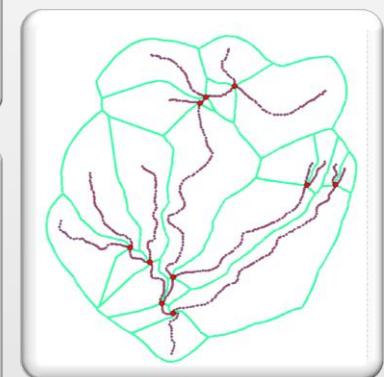
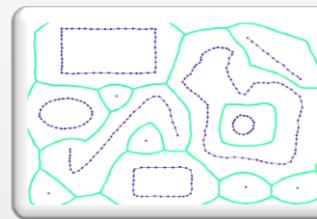
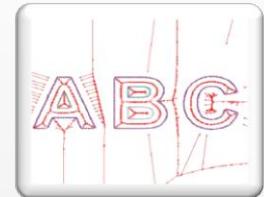


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Medial Axis (Applications in GIS)

- Object separation
- Text recognition and topology from scan maps
- Centerline extraction
- Map generalization
- Terrain modeling
- Watershed delineation

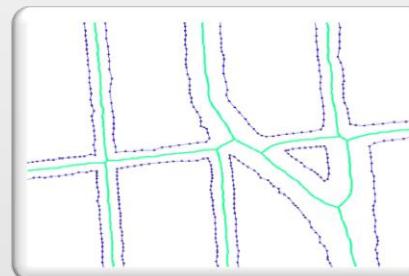
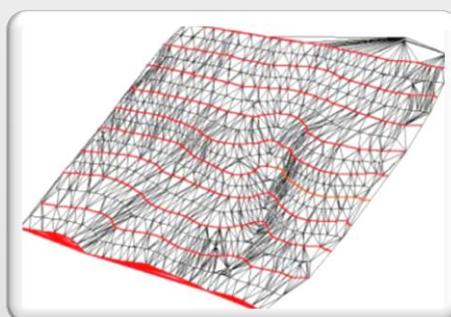
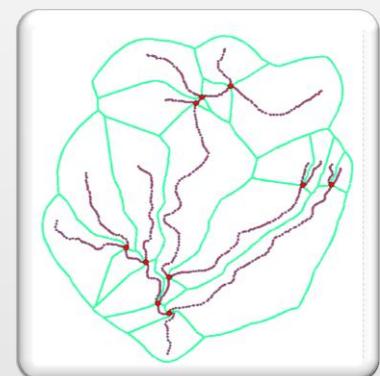
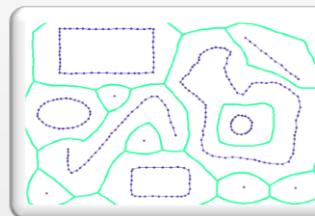
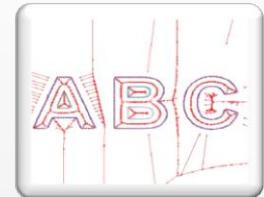


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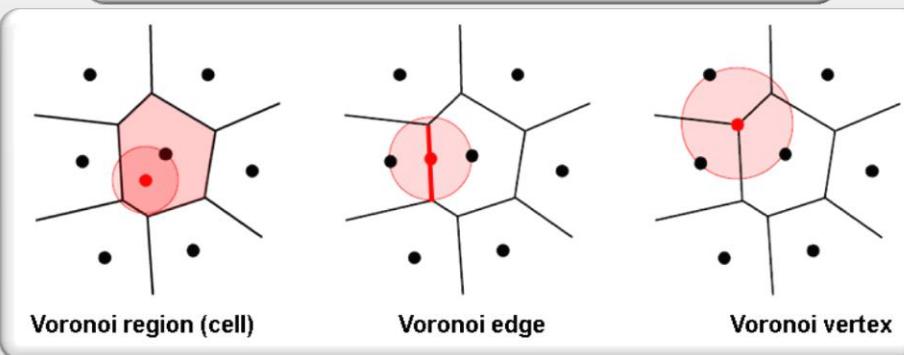
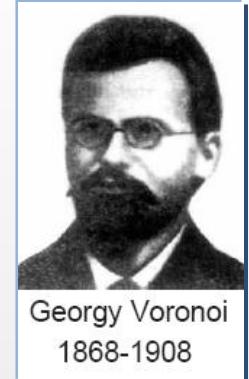
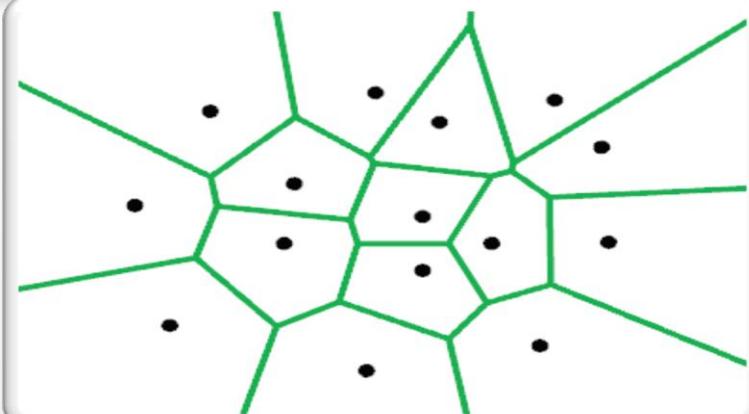
1

•Introduction

2

•Geometric Definitions

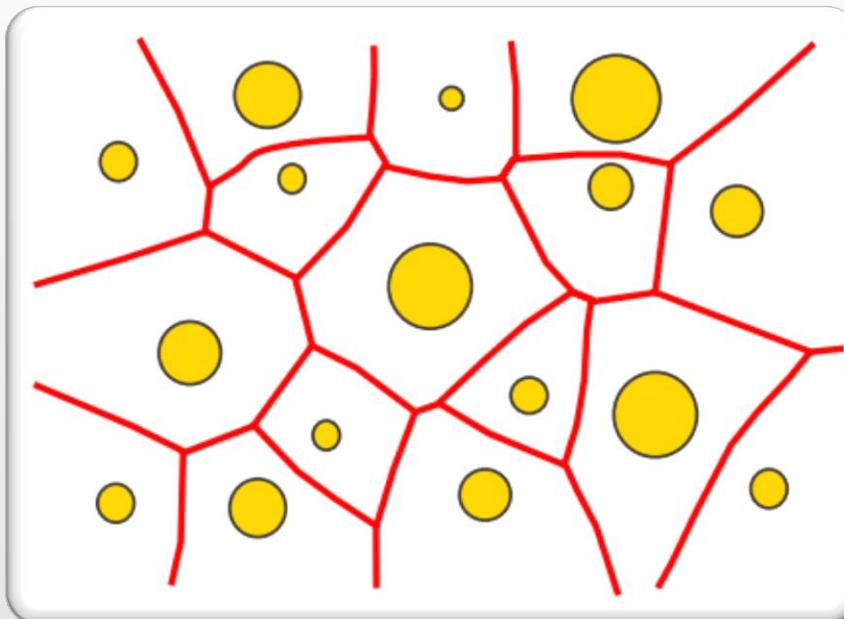
Voronoi Diagram



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



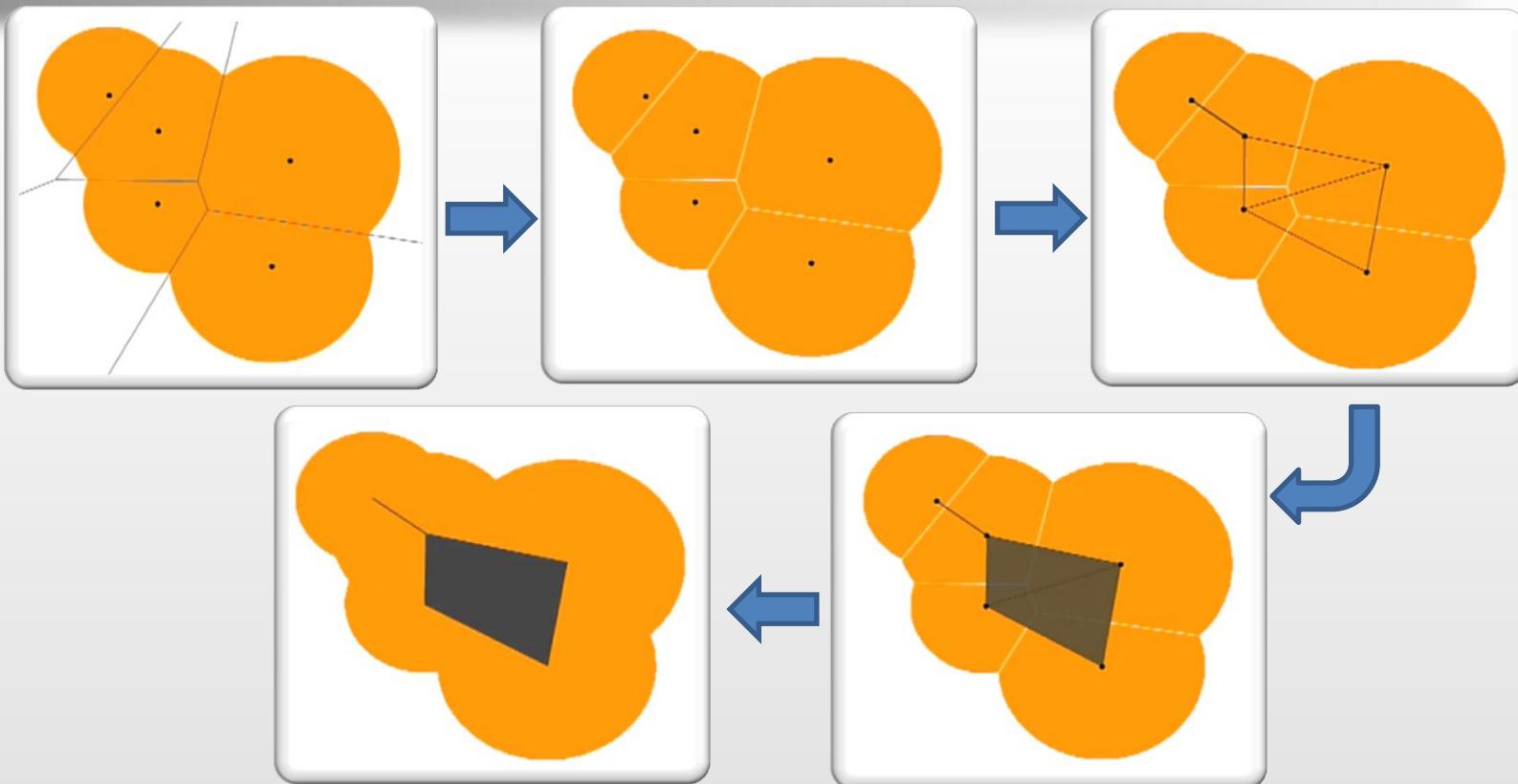
Weighted Voronoi Diagram



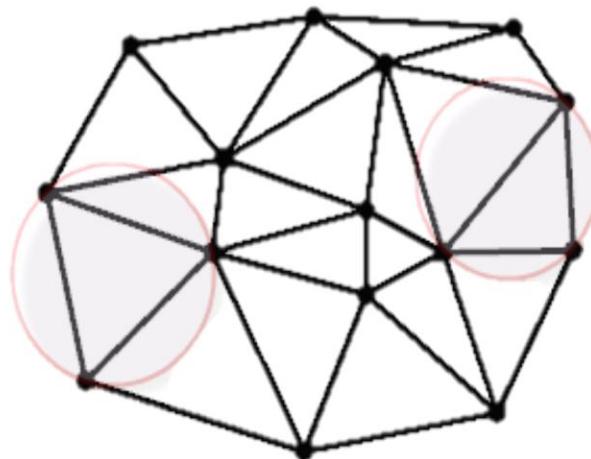
A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Dual of Union of Balls



Delaunay Triangulation



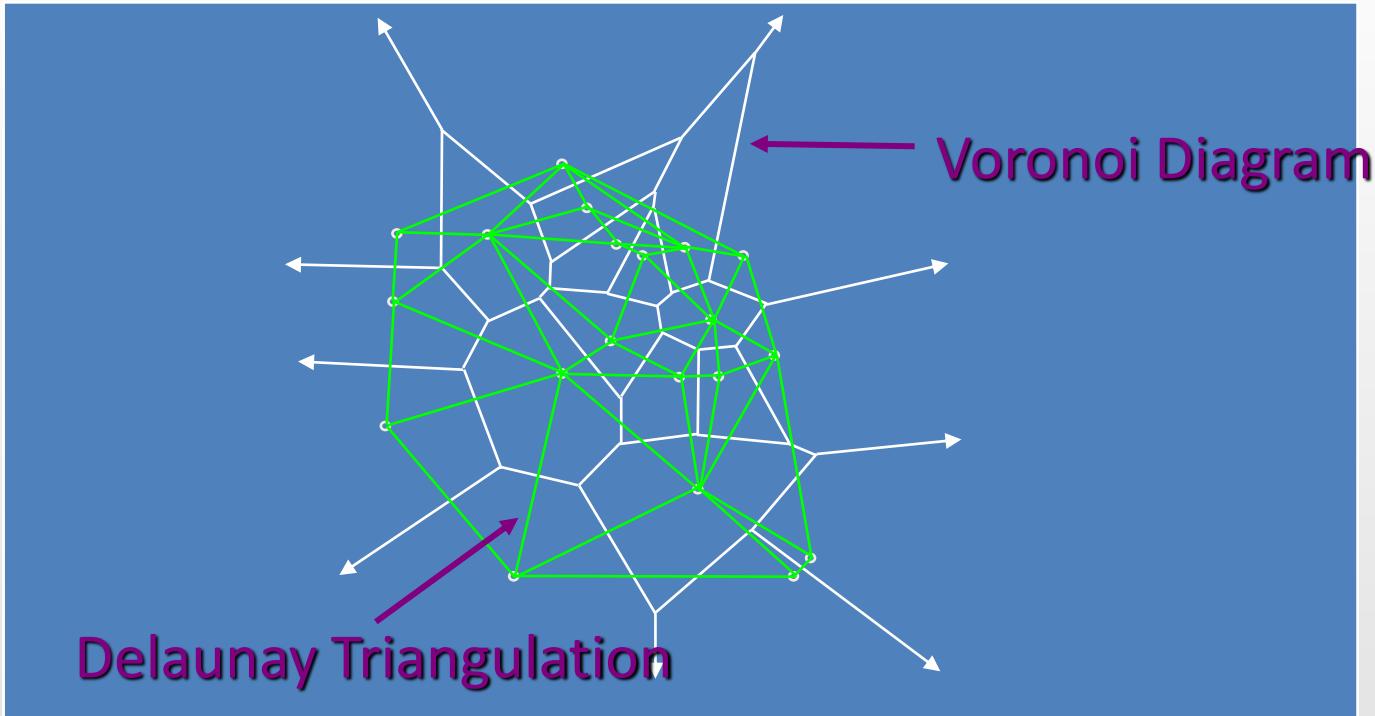
B.N. Delaunay
1890-1980



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



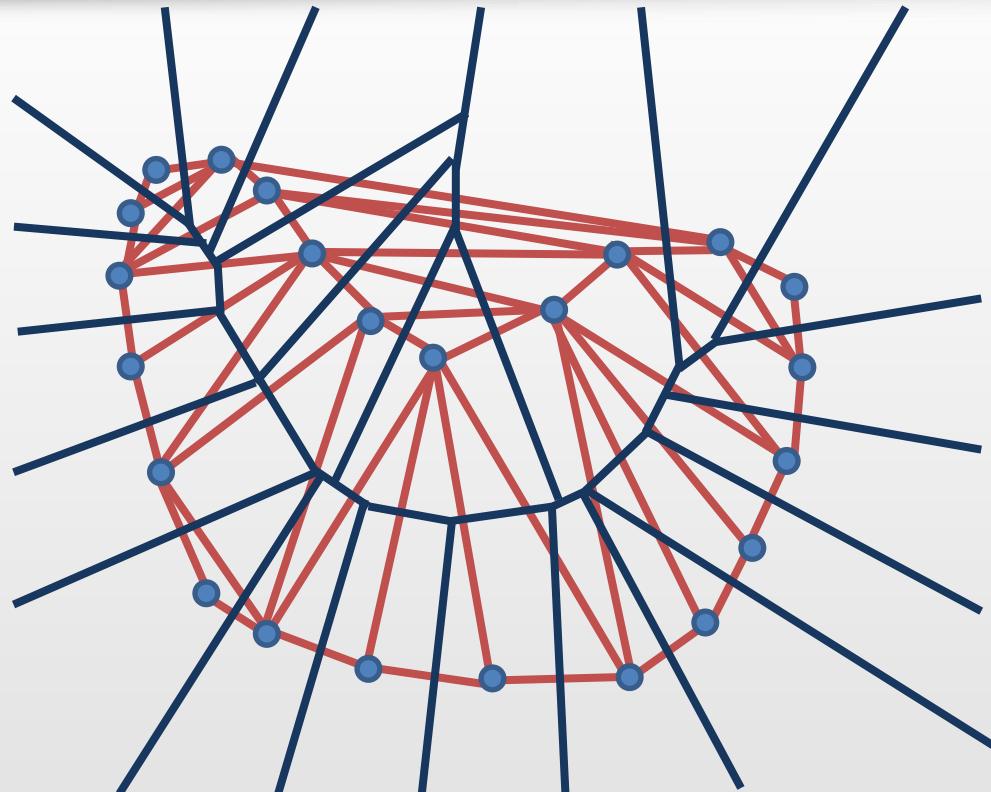
Duality



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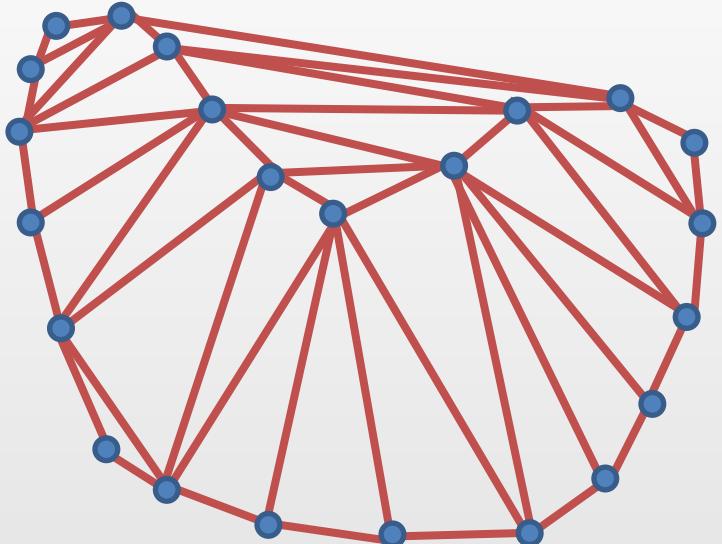
Inner/Mixed/Outer Edges



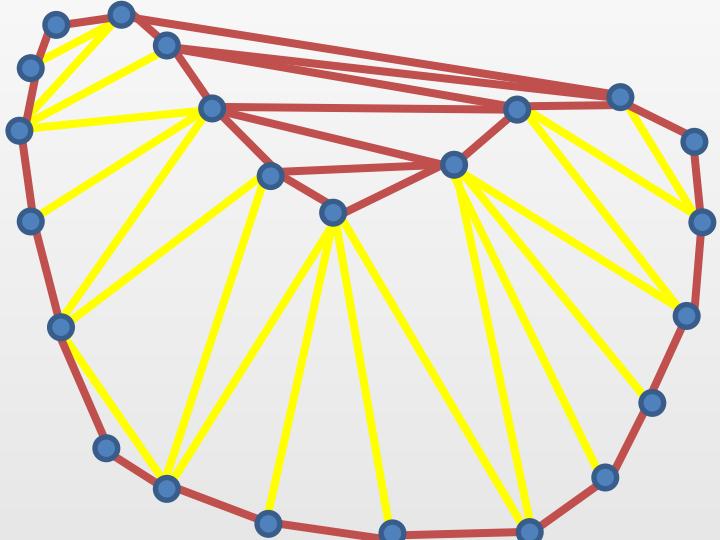
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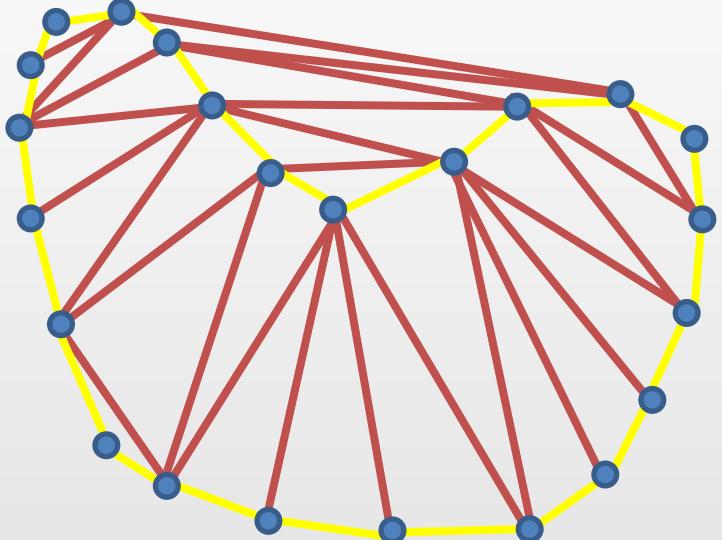
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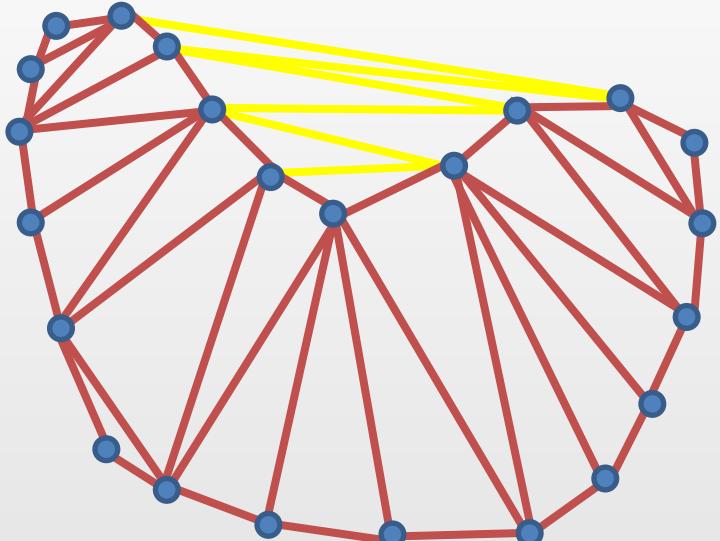
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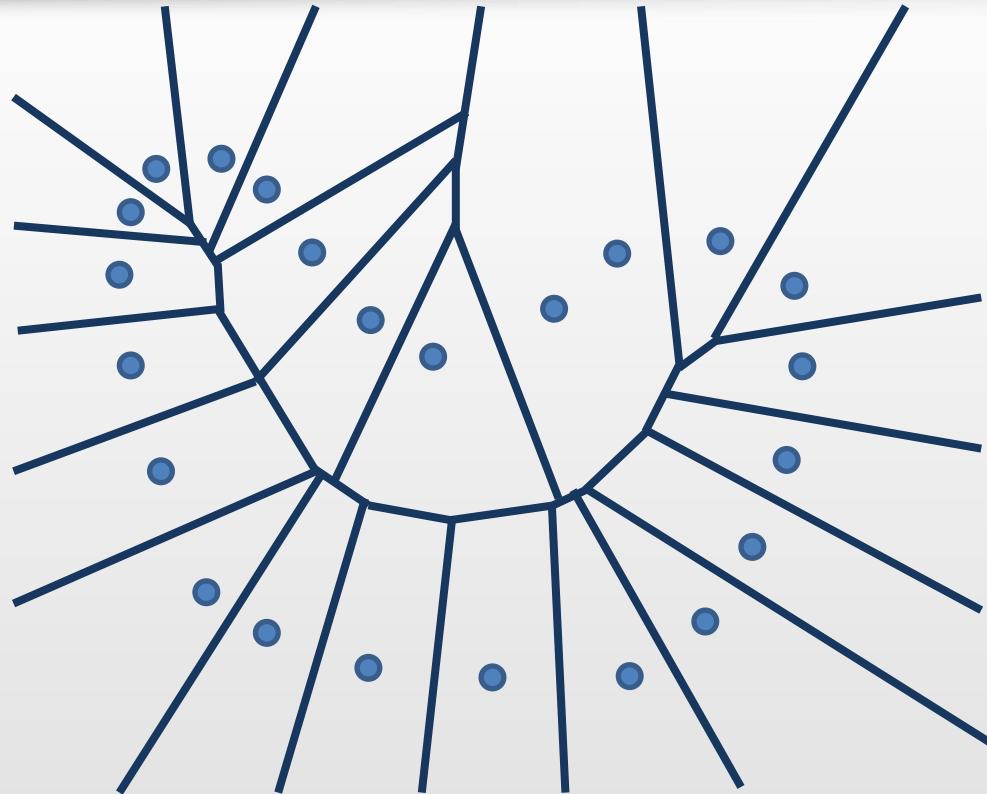
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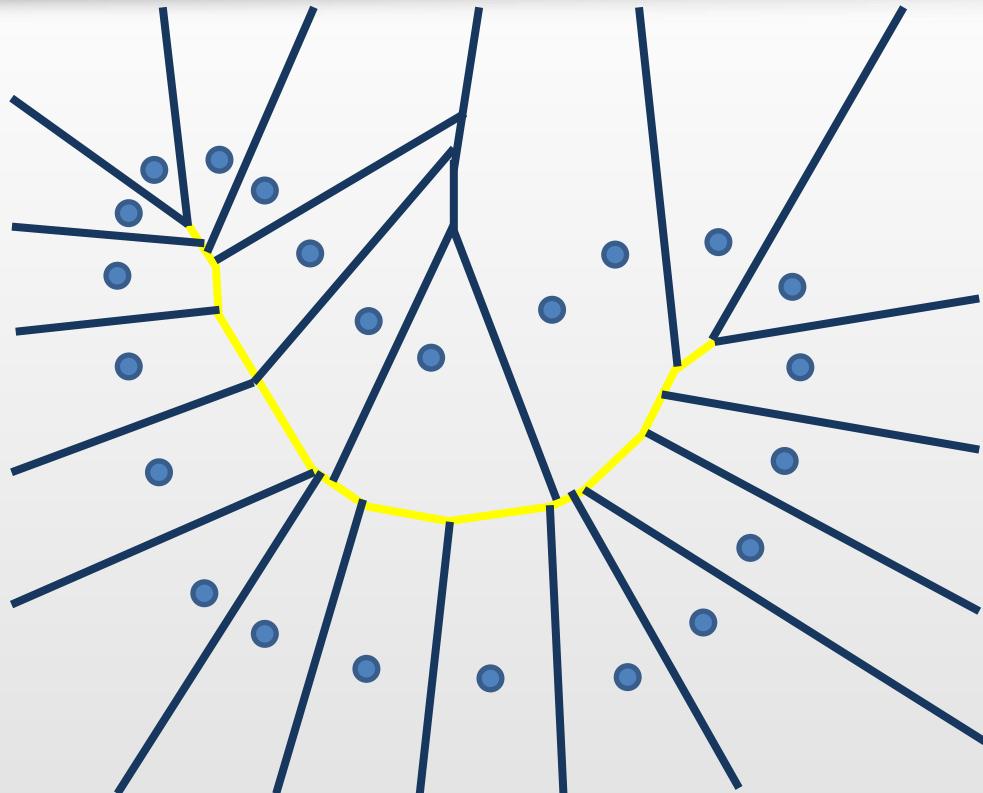
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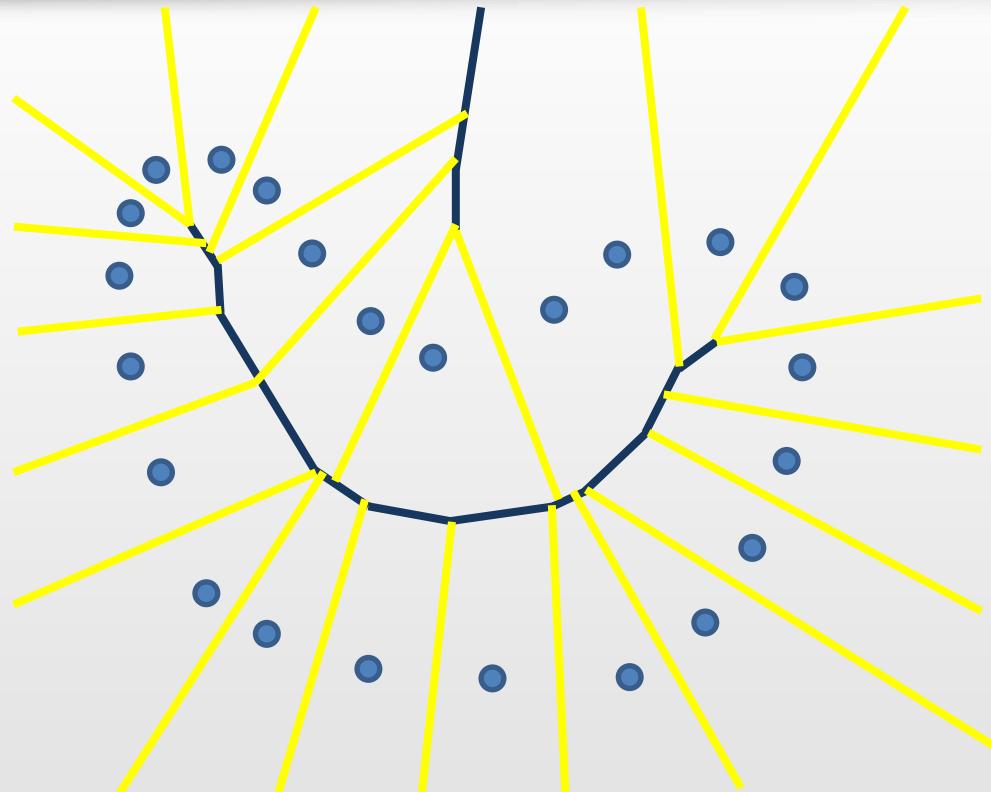
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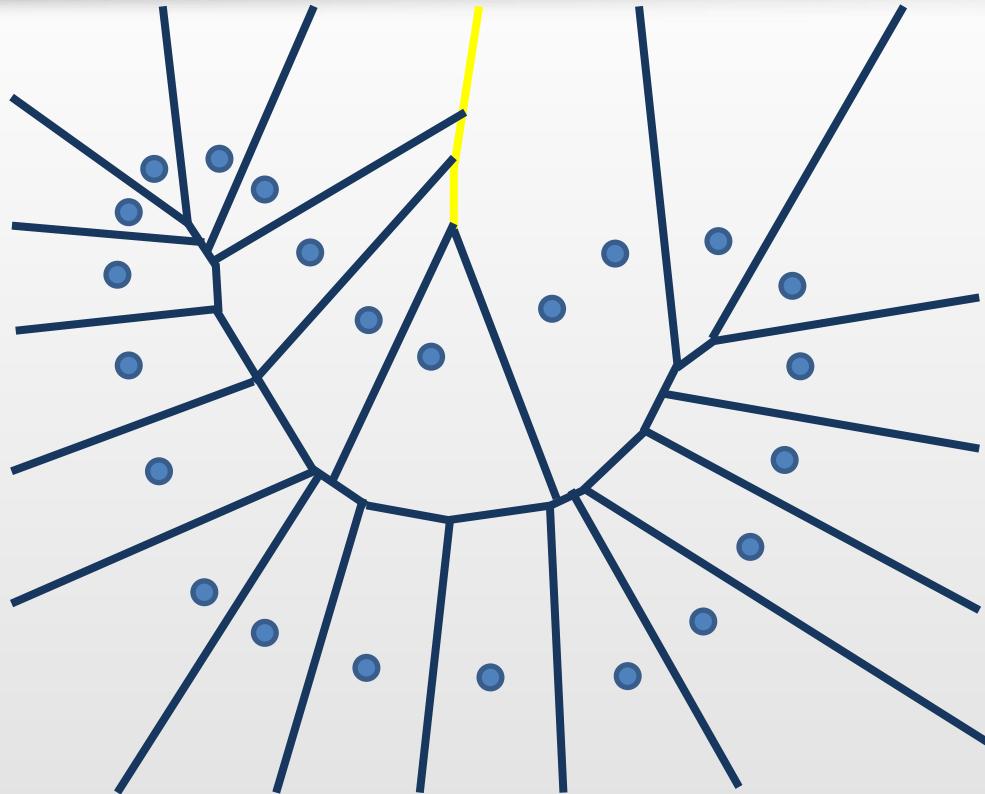
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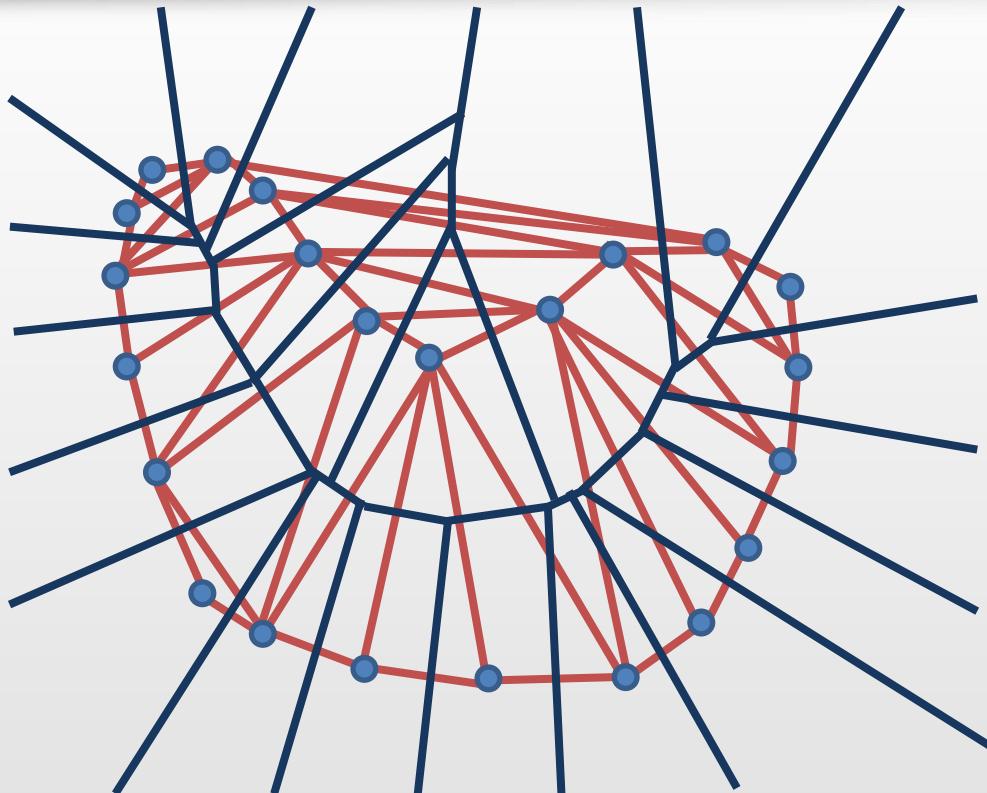
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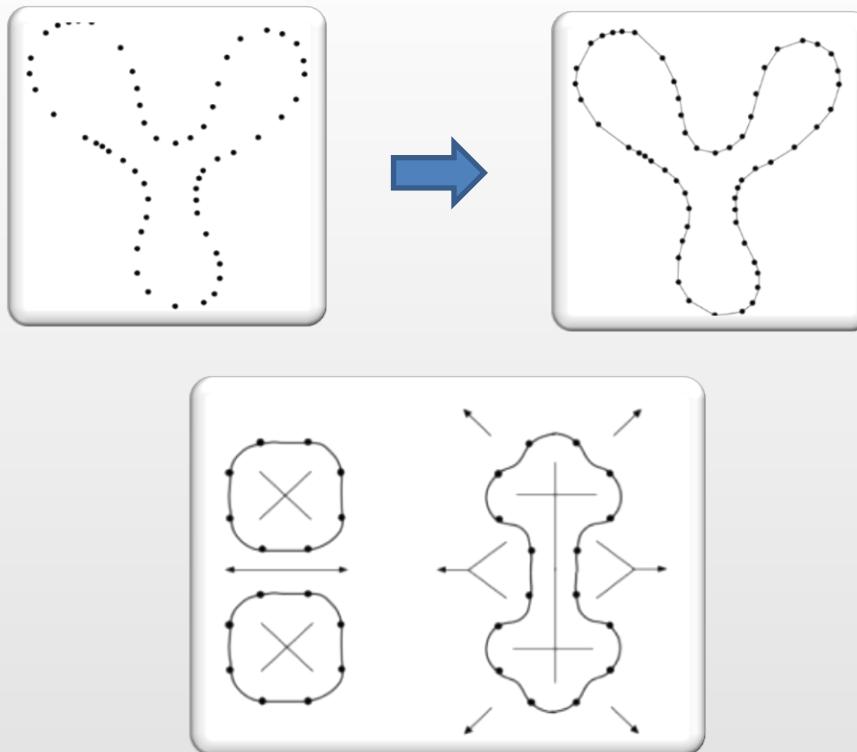
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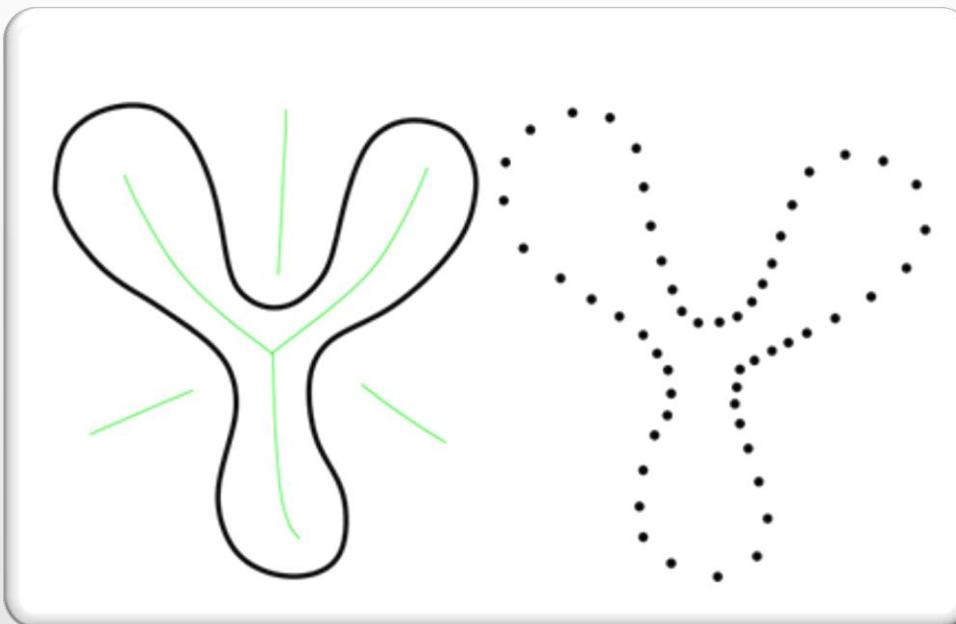
Curve Reconstruction



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Sampling



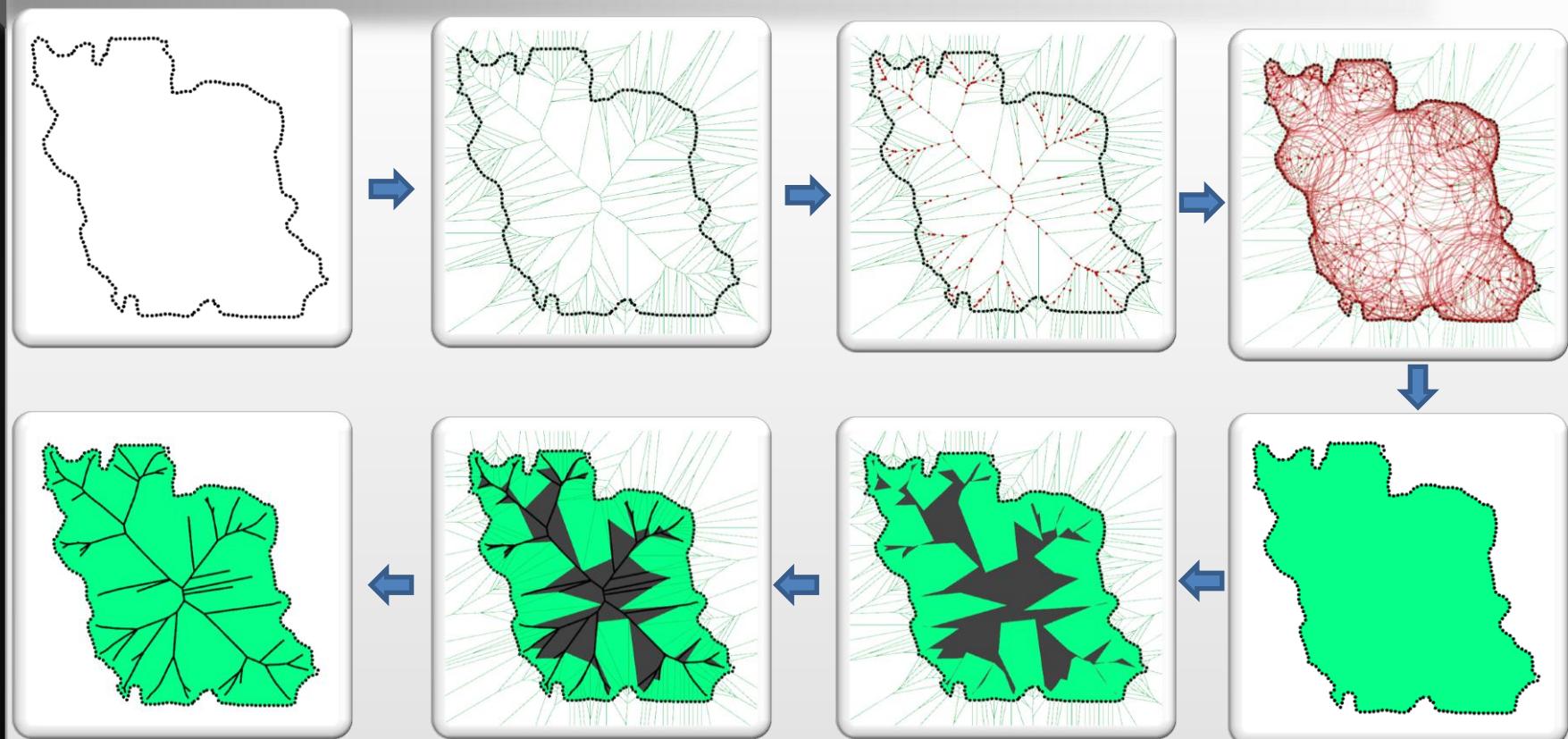
A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



- 1 •Introduction
- 2 •Geometric Definitions
- 3 •Voronoi-based Algorithms

Voronoi Ball Algorithm

Mark Pauly et al. (2008)

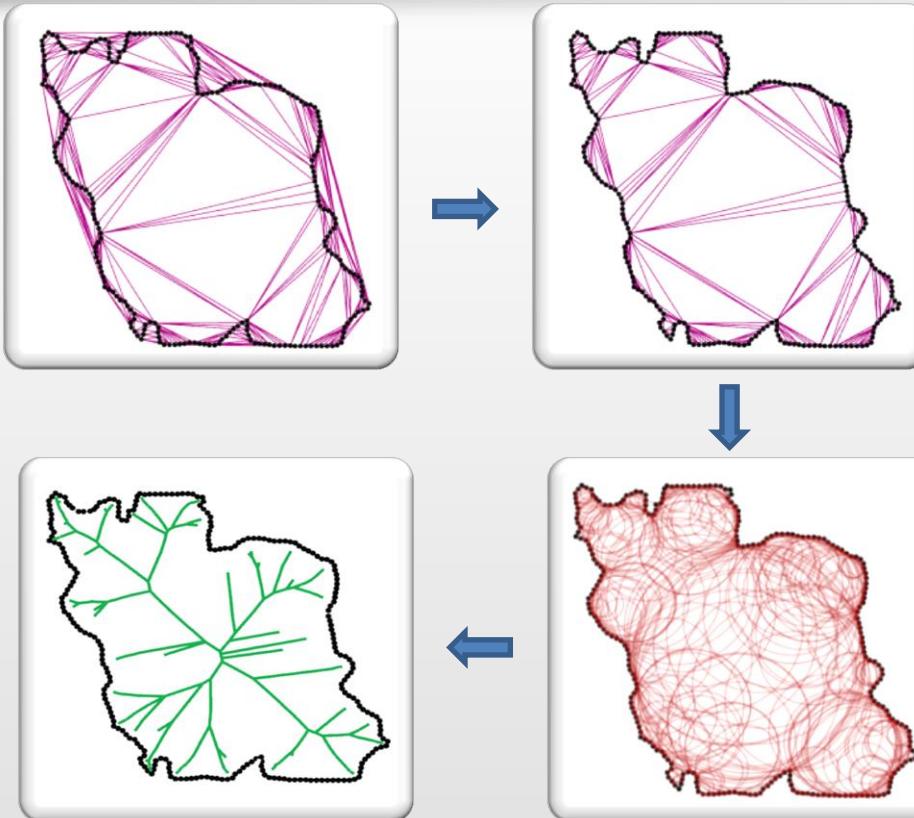


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Voronoi Edge Algorithm

Joachim Giesen et al.(2009)

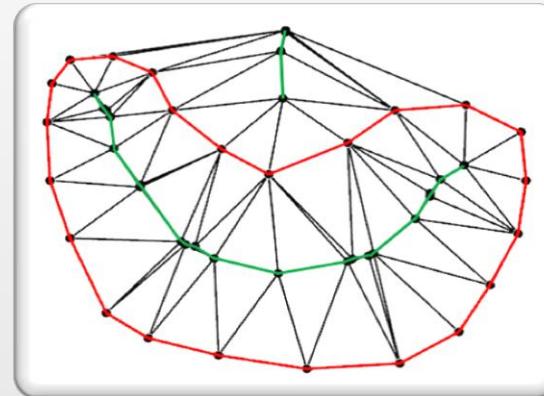
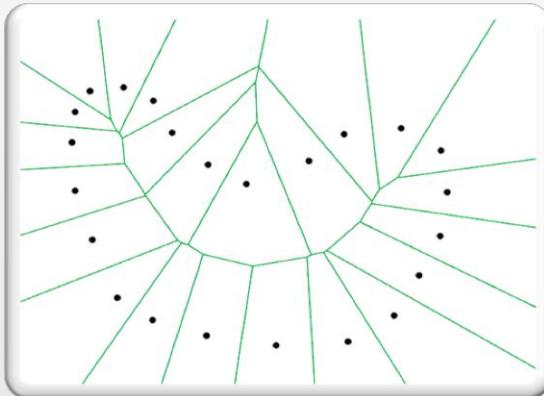


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Crust Algorithm

Amenta et al.(1998)

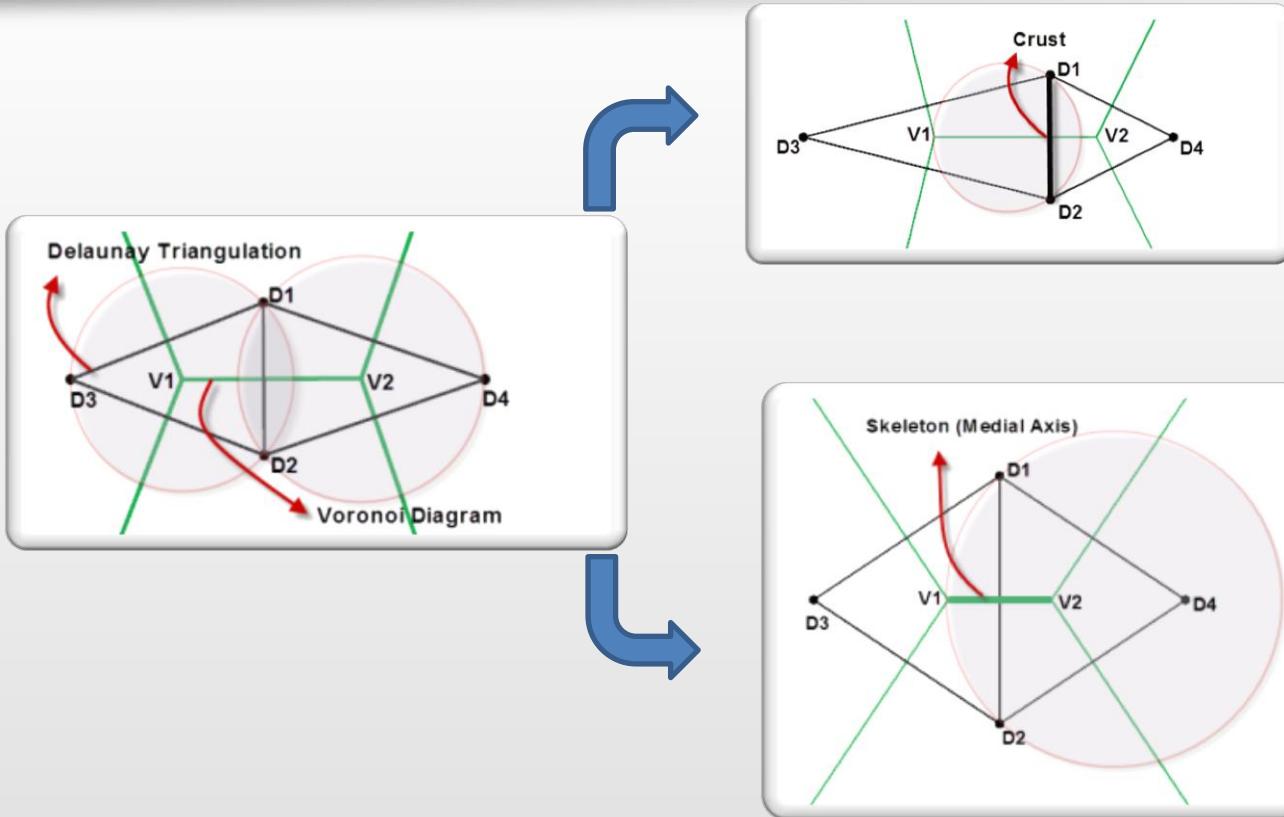


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



One Step Crust and Skeleton Algorithm

Christopher Gold and Maciek Dakowicz (2000)

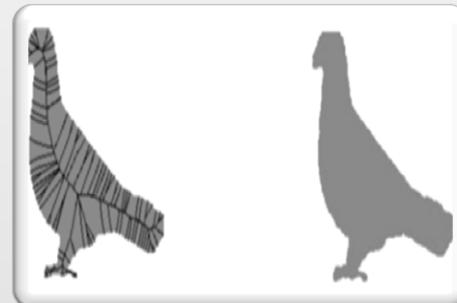
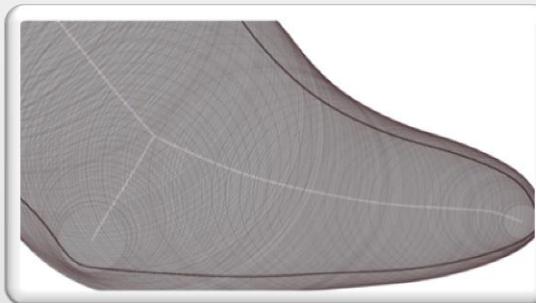
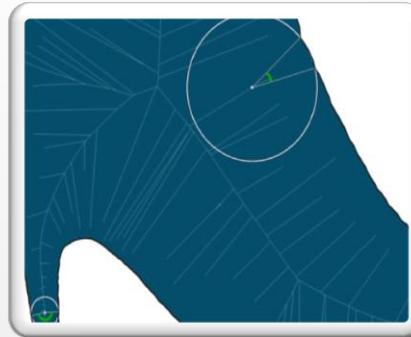
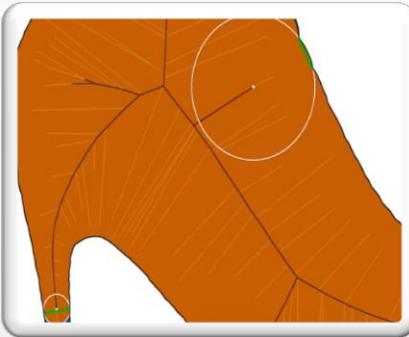


A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Pruning Methods

- Distance
- Angle
- Area
- Scale



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



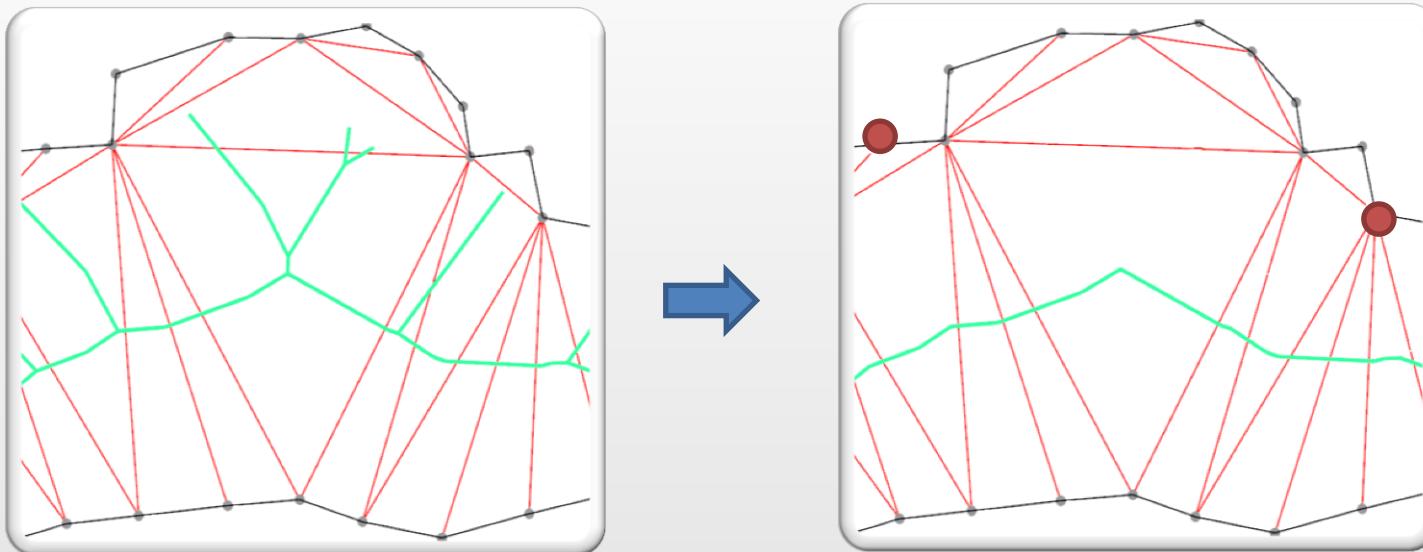
Pruning Methods (Shortcomings)

- Irrelevant branches
- Shortening the main branches
- Disconnection in the main structure
- Cannot preserve the topology
- Multiple parameters
- Do not work automatically

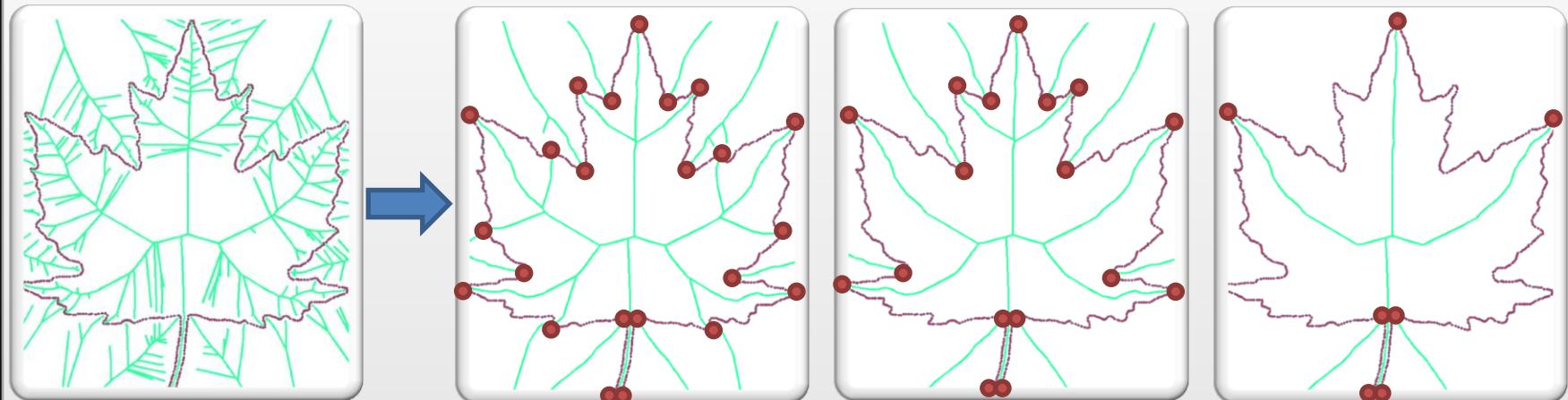


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Main Idea (Labeling the Sample Points)



An Example



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



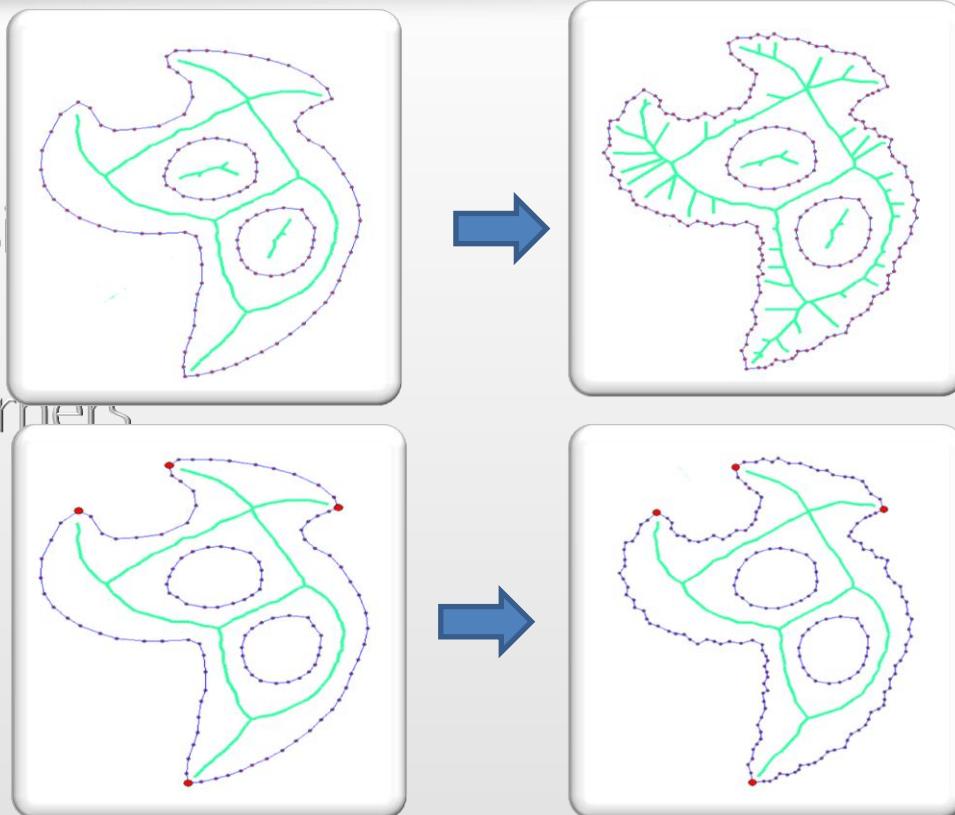
Advantages

- Stability
- Flexibility
- Accuracy & precision
- Complexity
- Handling sharp corners
- Open curves



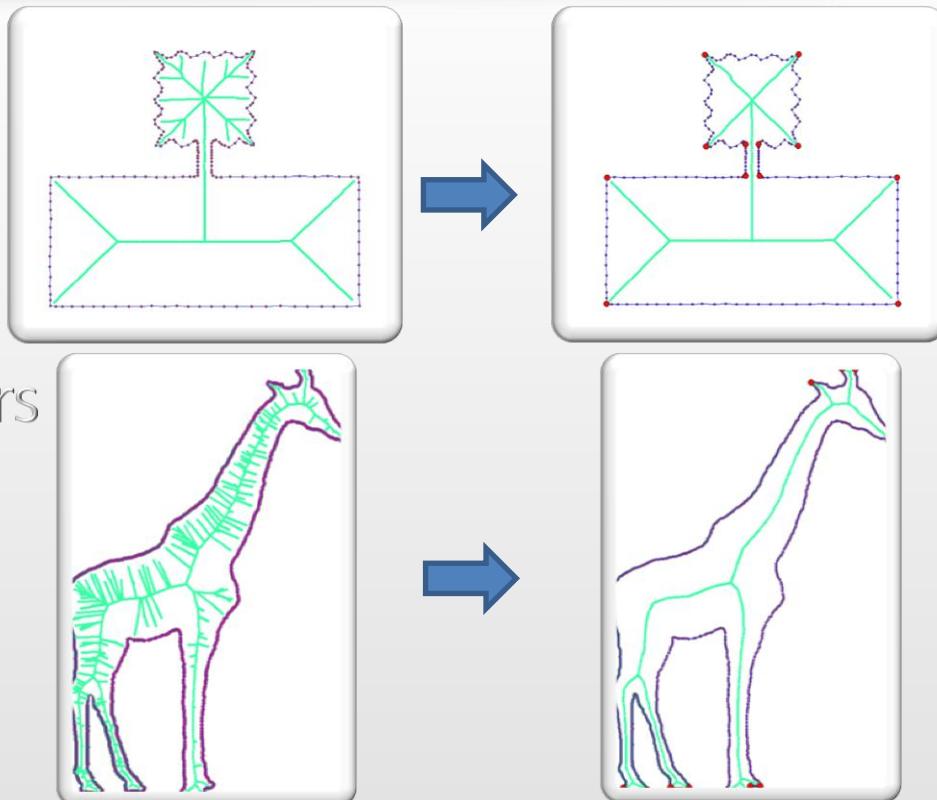
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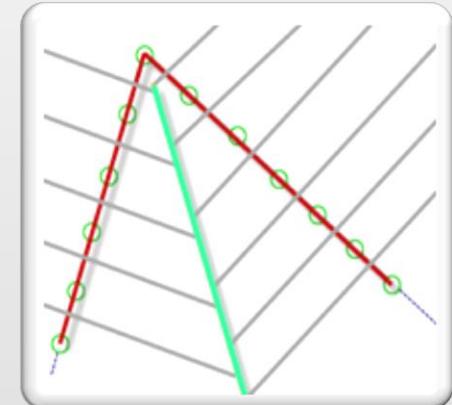
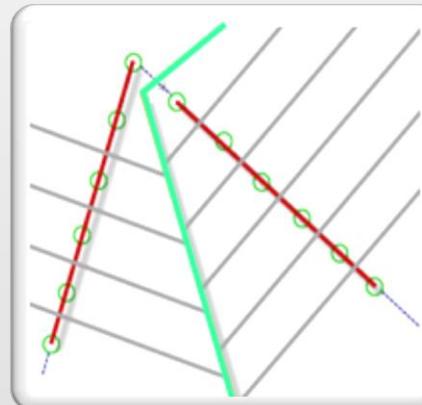
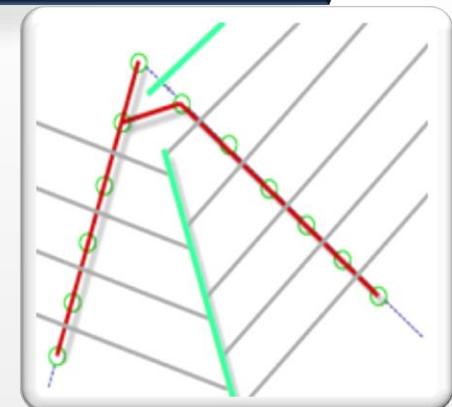
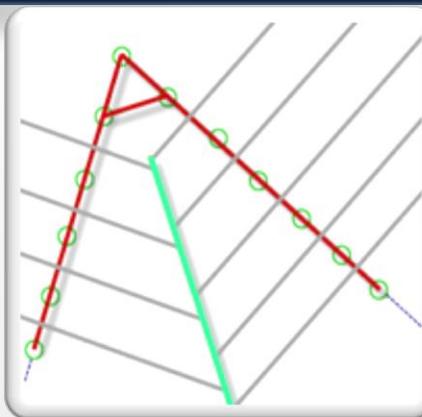
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$O(n \log n)$



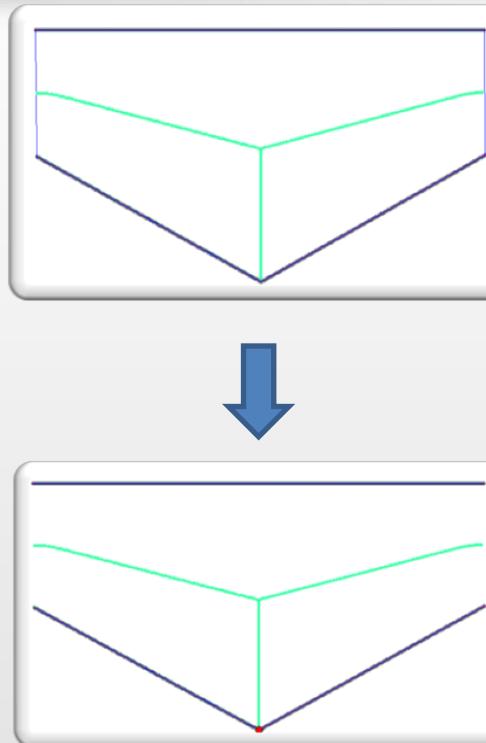
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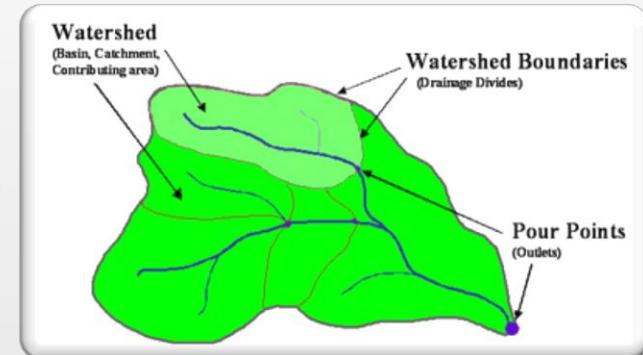
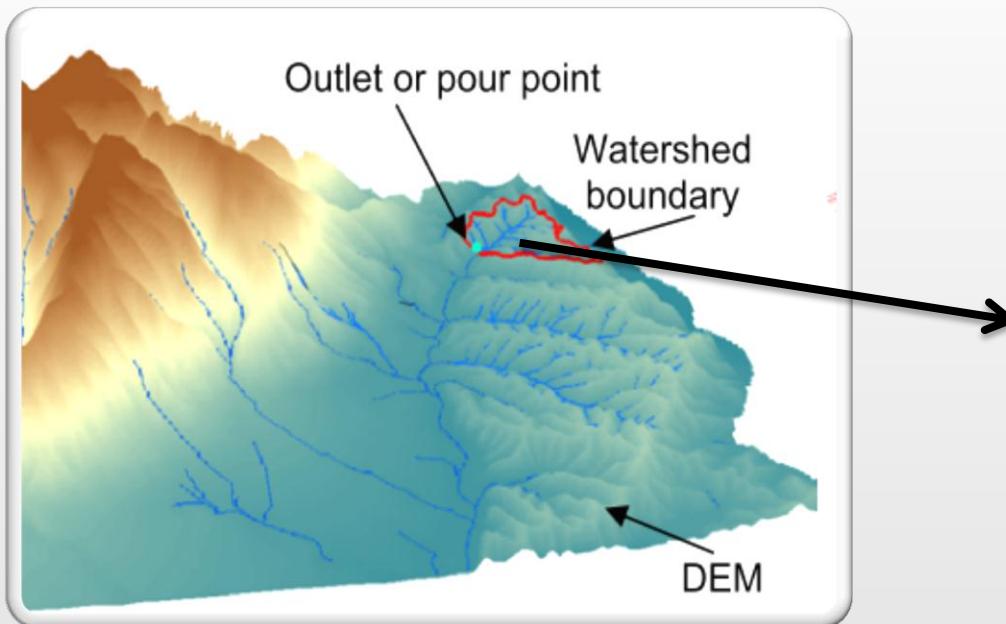
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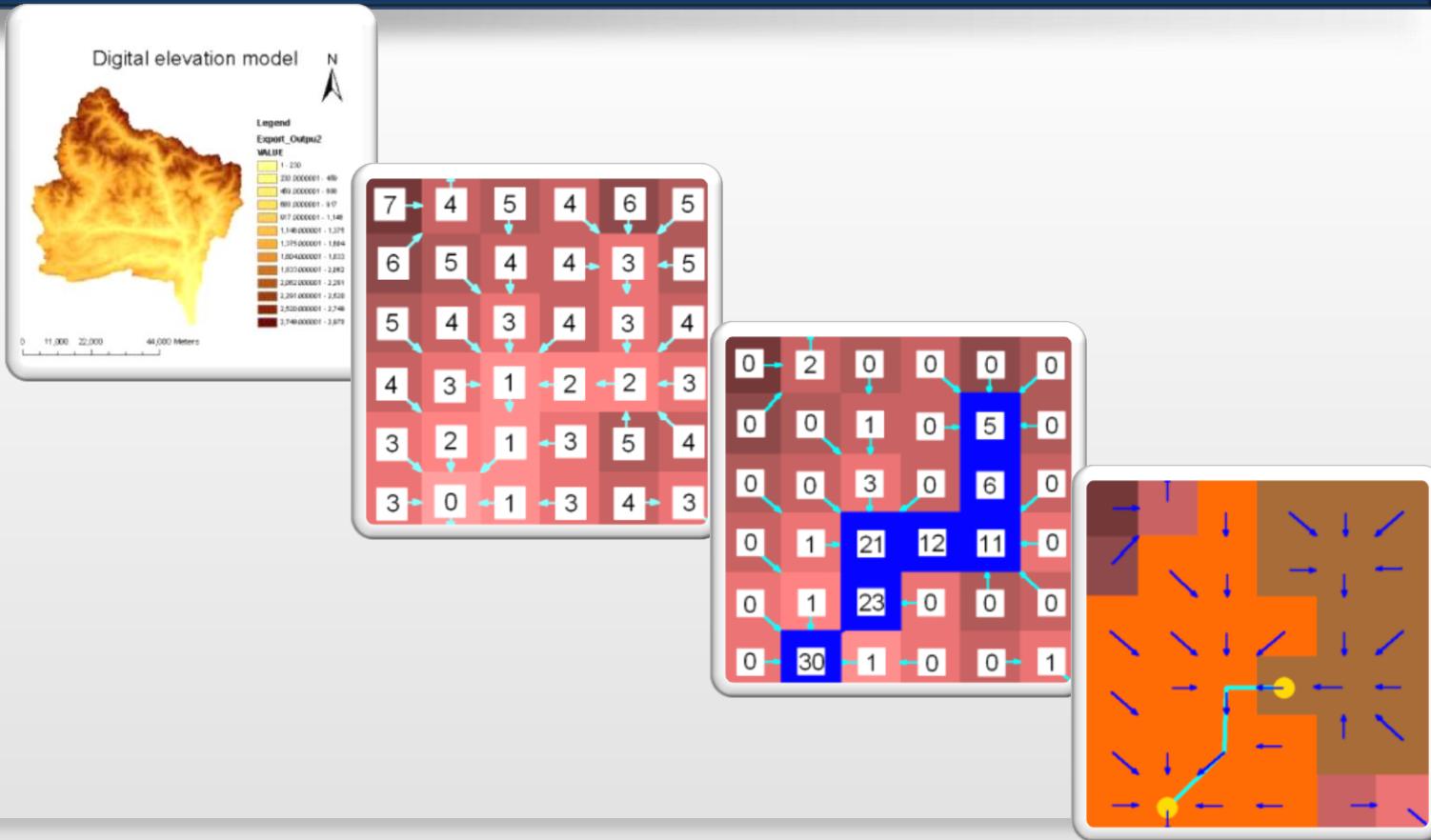


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- 4 •Proposed Algorithm
- 5 •Case Study

Watershed



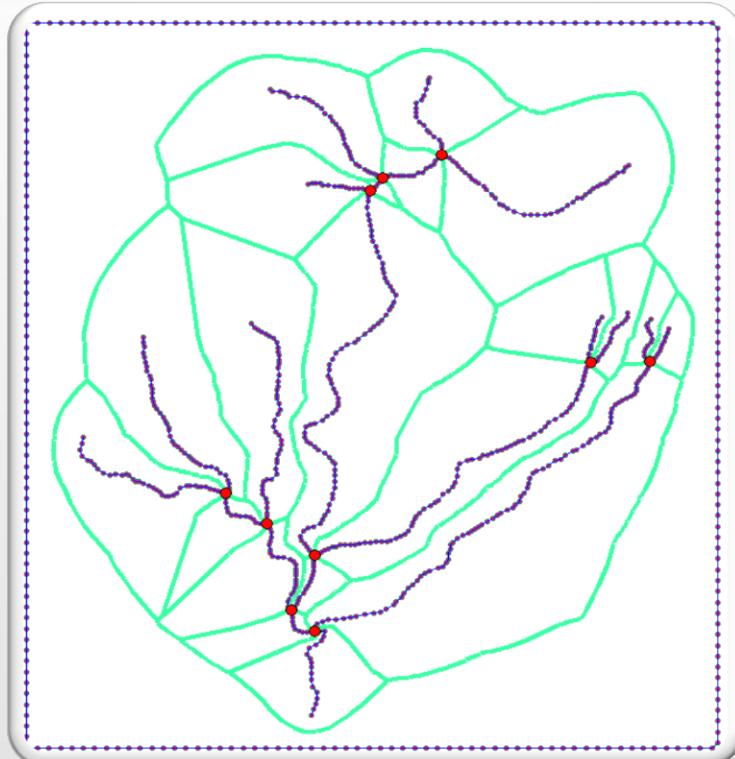
Watershed Delineation using DEM



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



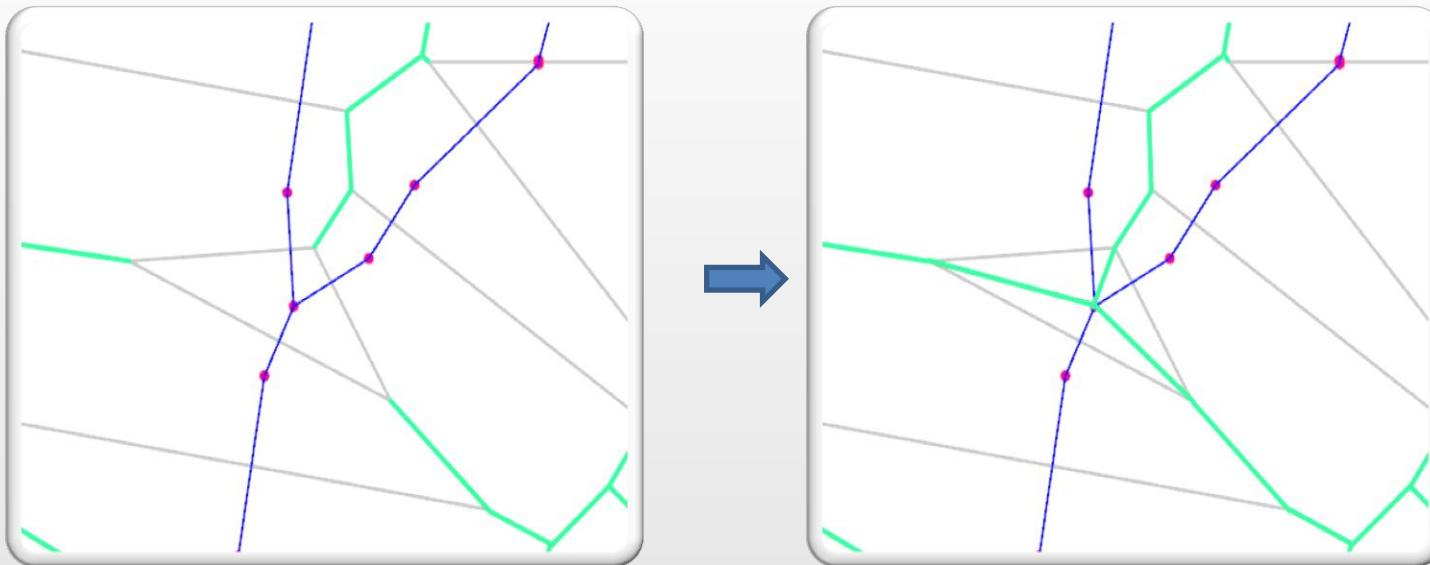
Watershed Delineation using the Proposed Method



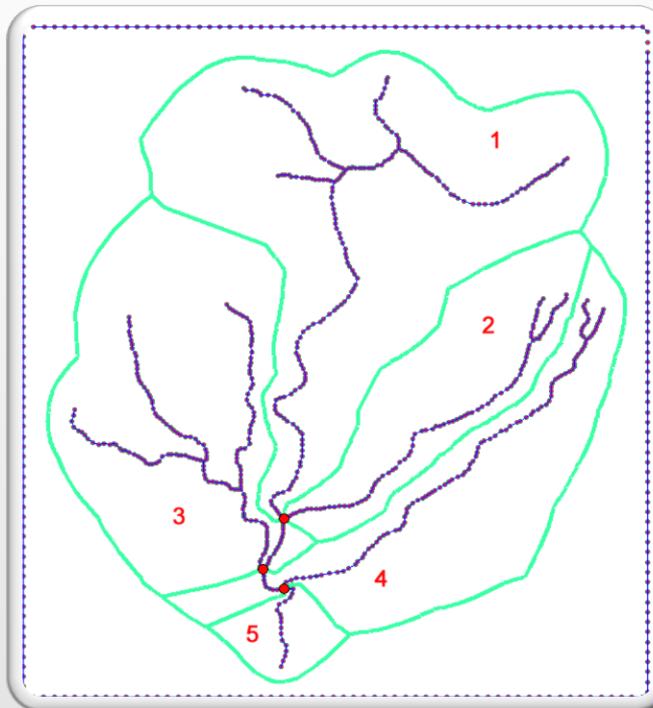
A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Closing the Polygons



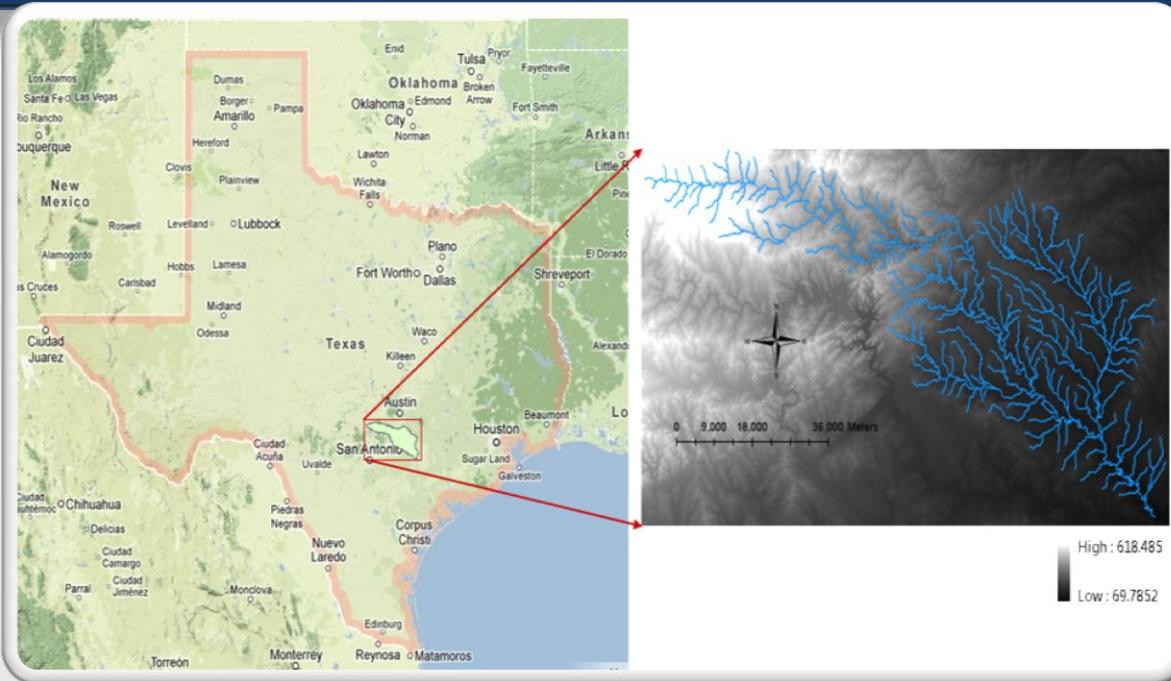
Hierarchical Delineation



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Results (Study Area & Data Set)



River data: National Hydrography Dataset (NHD)

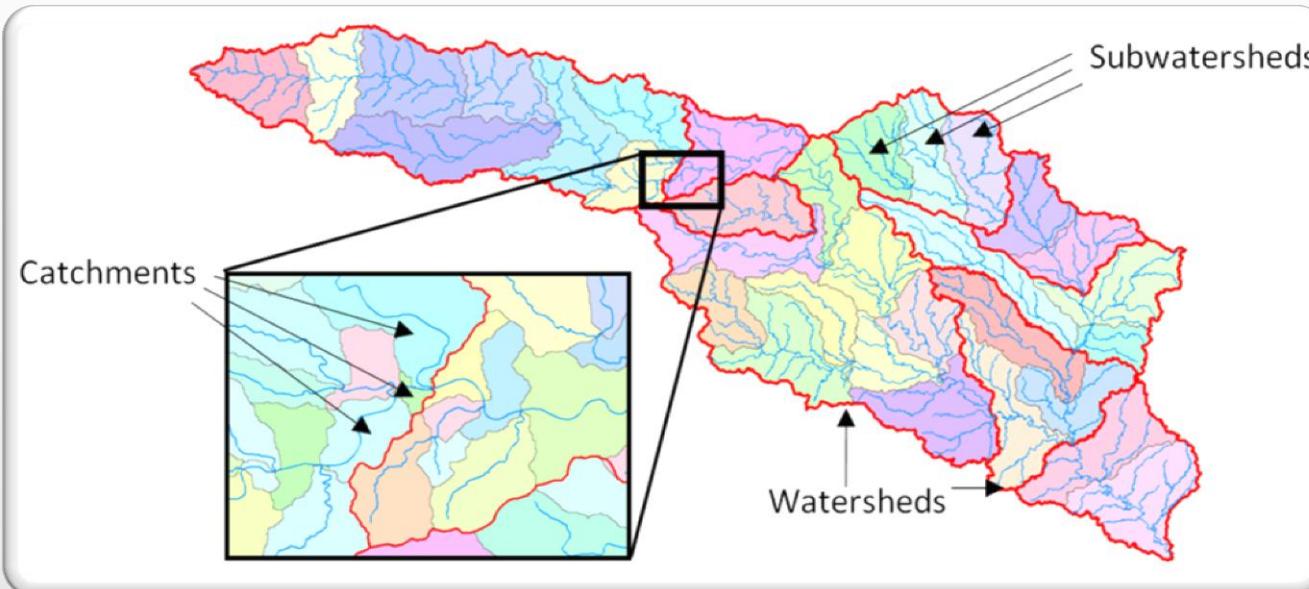
Raster DEM: National Elevation Dataset (NED) with the spatial resolutions of 1 arc-second
(approximately 30m)



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



Results (Study Area & Data Set)

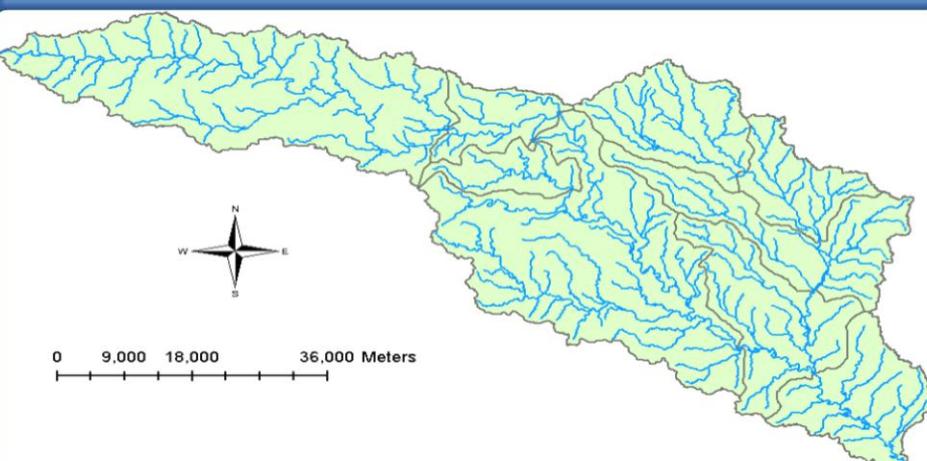
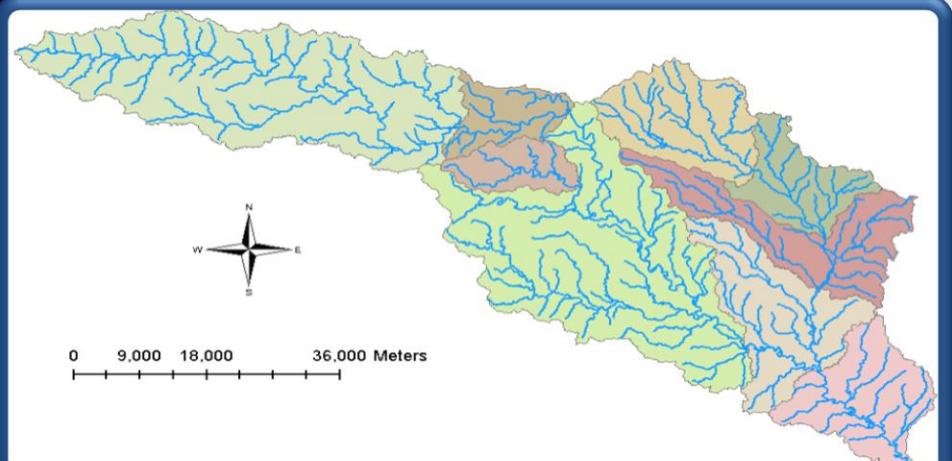
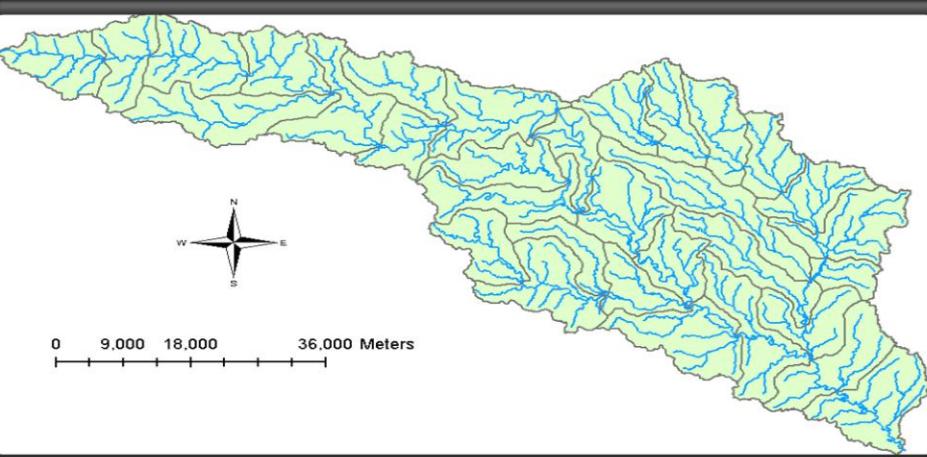
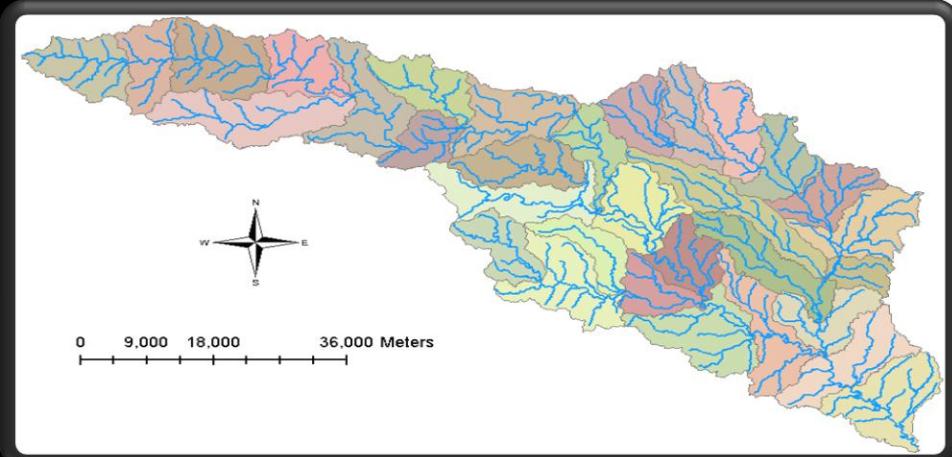
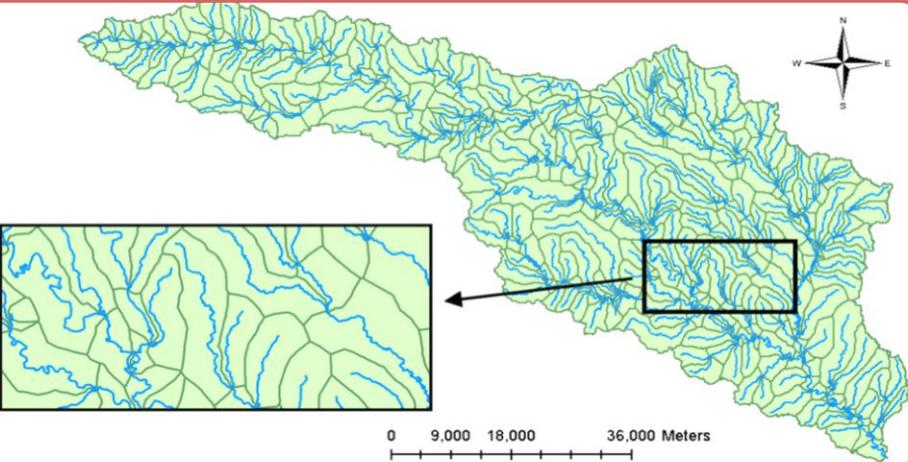
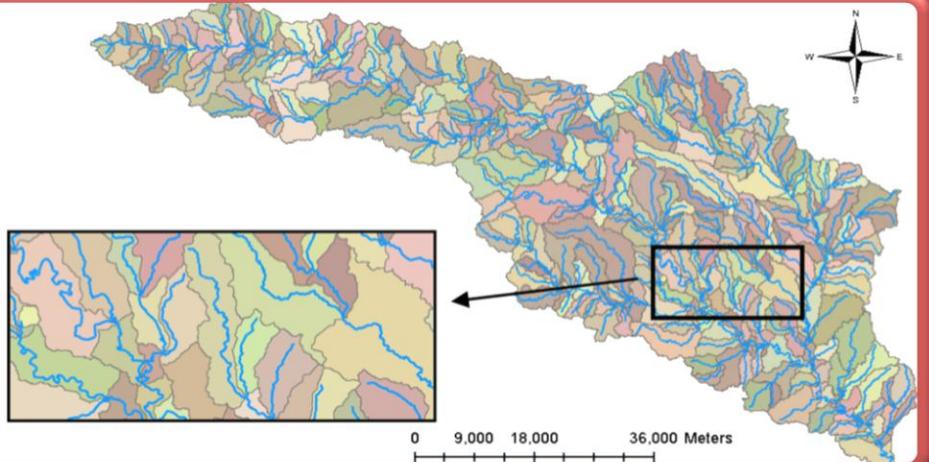


Hydrologic units from the National Hydrography Dataset



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)





Results (Speed Assessment)

- Watershed delineation is a time-consuming analysis
- The complexity of proposed method is $O(n \log n)$
- In a data set with 30000 sample points, the catchment area was extracted in 93 seconds



Results (Accuracy Assessment)

		Accuracy Assessment																																
																																		
		Error Matrix <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="3" style="text-align: center;">$j = \text{Columns}$ (Reference Data)</th><th style="text-align: center;">Row Total n_{i+}</th></tr> <tr> <th style="text-align: center;">$i = \text{Rows}$ (Results)</th><th style="text-align: center;">1</th><th style="text-align: center;">2</th><th style="text-align: center;">k</th><th style="text-align: center;">n_{i+}</th></tr> </thead> <tbody> <tr> <td style="text-align: center;">1</td><td style="text-align: center;">n_{11}</td><td style="text-align: center;">n_{12}</td><td style="text-align: center;">n_{1k}</td><td style="text-align: center;">n_{1+}</td></tr> <tr> <td style="text-align: center;">2</td><td style="text-align: center;">n_{21}</td><td style="text-align: center;">n_{22}</td><td style="text-align: center;">n_{2k}</td><td style="text-align: center;">n_{2+}</td></tr> <tr> <td style="text-align: center;">k</td><td style="text-align: center;">n_{k1}</td><td style="text-align: center;">n_{k2}</td><td style="text-align: center;">n_{kk}</td><td style="text-align: center;">n_{k+}</td></tr> <tr> <td style="text-align: center;">Column Total n_{+j}</td><td style="text-align: center;">n_{+1}</td><td style="text-align: center;">n_{+2}</td><td style="text-align: center;">n_{+k}</td><td style="text-align: center;">n</td></tr> </tbody> </table>				$j = \text{Columns}$ (Reference Data)			Row Total n_{i+}	$i = \text{Rows}$ (Results)	1	2	k	n_{i+}	1	n_{11}	n_{12}	n_{1k}	n_{1+}	2	n_{21}	n_{22}	n_{2k}	n_{2+}	k	n_{k1}	n_{k2}	n_{kk}	n_{k+}	Column Total n_{+j}	n_{+1}	n_{+2}	n_{+k}	n
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		overall accuracy = $\frac{\sum_{i=1}^k n_{ii}}{n}$																																
		$\hat{K} = \frac{n \sum_{i=1}^k n_{ii} - \sum_{i=1}^k n_{i+} n_{+i}}{n^2 - \sum_{i=1}^k n_{i+} n_{+i}}$																																



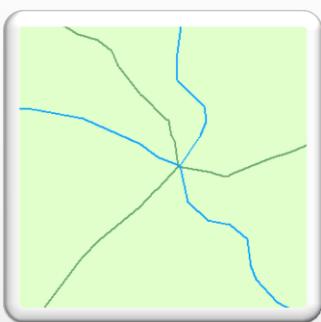
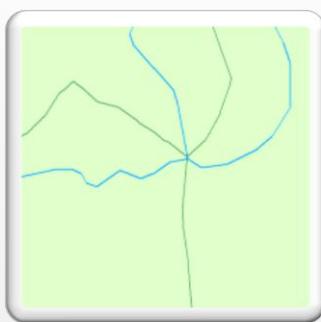
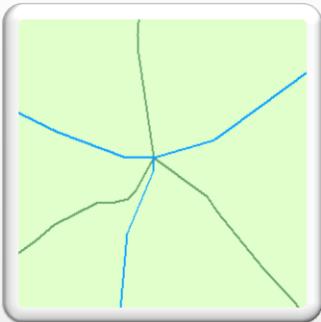
Results (Accuracy Assessment)

	Proposed Method		DEM-based Method	
	Overall accuracy (%)	Kappa (%)	Overall accuracy (%)	Kappa (%)
Catchment area	77.46	77.34	93.25	92.14
Subwatershed	92.00	91.72	97.73	97.23
Watershed	96.43	95.67	99.20	98.76

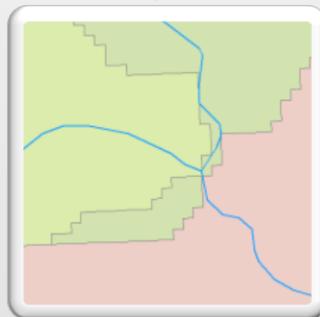
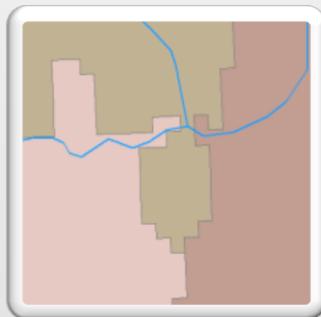
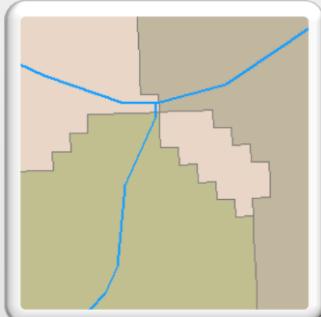
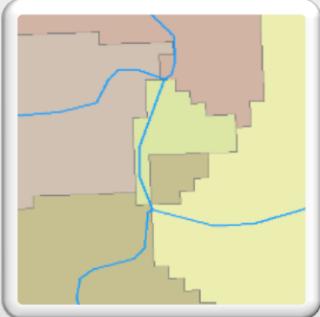


Results (Accuracy Assessment)

Proposed Method



DEM-Based Method



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



1

•Introduction

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•Geometric Definitions

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•Voronoi-based Algorithms

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•Proposed Algorithm

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•Case Study

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•Conclusions & Future Work

Summary

- Medial axis
 - Theory
 - Algorithms
 - Applications in GIS
- Proposed algorithm
- Watershed delineation form river networks



Conclusions

- Presenting a Voronoi-based algorithm to extract the medial axis through labeling sample points.
- Our proposed approach can deal elegantly with different cases of sample points and solves the problems that may occur in other algorithms.
- The conceptual structure and the results illustrate that our method are stable, easy to implement, robust and able to handle sharp corners, even in the presence of significant noise and perturbations.



Conclusions

- A fast and automatic watershed delineation algorithm is presented that uses only the vector-based river network and does not use any elevation data.
- The results indicate that the medial axis of river network is a reasonable approximation of watershed.
- The method can handle very large areas and produce catchment polygons quickly.



Contributions

- Proposing a novel Voronoi-based algorithm to extract the medial axis through labeling sample points
- Creating a rational relation between labeling the sample points and extraneous edges in the medial axis structure
- Reducing the time of medial axis extraction by simultaneous filtering process
- Improving the stability, accuracy and precision of medial axis in comparison with other voronoi-based algorithm
- Proposing a fast and automatic watershed delineation algorithm that uses only the vector-based river network



Recommendations & Future Work

- Surface reconstruction and 3D medial axis extraction
- Medial axis for other applications in GIS
- Computational geometry & GIS



Publications (Conferences)

- Ghandehari, M. and Karimipour, F. (2012). 'Voronoi-based Curve Reconstruction: Issues and Solutions', (ICCSA 2012)
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A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)



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- Karimipour, F. and **Ghandehari, M.** (2012). '*Voronoi-based Medial Axis Approximation from Samples: Issues and Solutions*', Submitted to the Graphical Models Journal.
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Acknowledgments

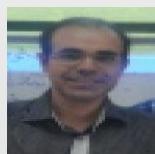
Advisors



Examination Committee



Friends



A Voronoi-based Algorithm for Medial Axis Extraction (Case Study: Watershed Delineation from River Networks)





Thanks for Your Attention

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