STATISTICS THEORY ASSESSMENT

W4 assessment

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1 Problem 2

Question 1. Is Θ^n an unbiased estimator of θ?

Explanation 1. In order to answer this, we need to use mathematical expectations. If the results equals to θ, we prove that Θ^n an unbiased estimator of θ.
based on the question we have:

$$E[\Theta^n] = E[12\bar{X} - 6]$$

Explanation 2. First, we need to calculate the mathematical expectation of the X_i:

$$E[X_i] = \int_0^1 [\theta(x - \frac{1}{2}) + 1]x\,dx$$

Explanation 3. Split the Integral:

$$E[X_i] = \int_0^1 [\theta(x - \frac{1}{2}) + 1]x\,dx = \int_0^1 \theta(x - \frac{1}{2})x\,dx + \int_0^1 x\,dx$$

Explanation 4. Simplify the first one:

$$E[X_i] = \theta \int_0^1 (x^2 - \frac{1}{2}x)\,dx + \int_0^1 x\,dx$$

Explanation 5. Calculate each integral:

$$\theta \int_0^1 (x^2 - \frac{1}{2}x)\,dx = \theta \left(\frac{x^3}{3} - \frac{x^2}{4}\right)\Bigg|_0^1 = \frac{\theta}{12}$$

$$\int_0^1 x\,dx = \frac{x^2}{2}\Bigg|_0^1 = \frac{1}{2}$$

Explanation 6. then we combine these results:

$$E[X_i] = \frac{\theta}{12} + \frac{1}{2} = \frac{\theta + 6}{12}$$

Explanation 7. Now, we just need to replace it in the first equation:

$$E[\Theta^n] = E[12\bar{X} - 6]$$

$$E[\Theta^n] = 12E[\bar{X}] - 6$$

$$E[\Theta^n] = 12 \cdot \frac{\theta + 6}{12} - 6$$

$$E[\Theta^n] = \theta$$

So, we can conclude that Θ^n is an unbiased estimator of θ.

2 Problem 3

Explanation 8. For a 95% confidence interval and a sample size > 30, we typically use a z-score of 1.96. The formula for a confidence interval is:

$$\bar{x} \pm (z \times (\frac{std}{\sqrt{n}}))$$

From the question we have:

- Sample size = 100
- Sample mean = 50.1
- Population variance = 81

Explanation 9. The standard deviation of the population can be calculated as:

$$STD = \sqrt{Var(X_i)} = \sqrt{81} = 9$$

Explanation 10. Then we have:

$$50.1 \pm (1.96 \times (\frac{9}{\sqrt{100}})) = 50.1 \pm (1.96 \times 0.9)$$

Explanation 11. so:

$$50.1 - (1.96 \times 0.9) = 48.336$$

$$50.1 + (1.96 \times 0.9) = 51.864$$

Therefore, 95% confidence interval for $\theta = EX_i$ are $\boxed{48.336}$ and $\boxed{51.864}$

3 Problem 4

Explanation 12. Formula:

$$n = (\frac{z_{\frac{\alpha}{2}}}{2 \cdot \text{margin of error}})^2$$

- n: necessary sample size
- z: z-score or critical value

This formula calculates the sample size needed to estimate the population proportion with a specified level of confidence.

Question 2. How large does n need to be so that we can obtain a 90% confidence interval with 3% margin of error?

Explanation 13. z-score for 90% confidence interval = 1.645

$$(\frac{1.645}{2 \cdot 0.03})^2 = \boxed{751.673}$$

Question 3. How large does n need to be so that we can obtain a 99% confidence interval with 3% margin of error?

Explanation 14. z-score for 99% confidence interval = 2.576

$$(\frac{2.576}{2 \cdot 0.03})^2 = \boxed{1843.271}$$