Industrial Organization
MECO 6303 – ECON 6430

Simple Monopoly Pricing

Bernhard Ganglmair
University of Texas at Dallas

January 17, 2014

Contents

Chapter 1

Simple Monopoly Pricing

The title of this chapter suggests there is *non-simple* monopoly pricing. *Simple* monopoly pricing means the monopolist charges only a single price for the same good. We look at multiple prices for the same good when we talk about price discrimination in Week 4.

In this chapter we study a monopolist's profit-maximizing price or quantity decision for both single-product and multi-product firms. We first assume that goods are nondurable. We first consider one-period models before looking at inter-temporal pricing behavior in multi-period models. We study a very simple learning model before introducing durable goods.

1.1 Single-Product Monopolist

1.1.1 Monopoly Price and Quantity

We consider a firm that produces a good at cost $c(q)$ with $c'(q) \geq 0$ where q denotes the produced quantity. The demand for this good is $D(p)$ where $D'(p) < 0$. As a regularity condition we want the profit function (objective function) $\Pi(p) = R(p) - c(D(p))$ with $R(p) = pD(p)$ to be concave. Even with convex costs, if the revenue function is not concave then the objective function may not be concave and marginal revenue is not decreasing everywhere. We will typically assume that costs are (weakly) convex. Furthermore, as a regularity condition we require that $D(p)$ must not be too convex. The second-derivative of the revenue function $R(p)$ is

$$R''(p) = 2D'(p) + pD''(p). \tag{1.1}$$

By $D'(p) < 0$ the first term is negative. The second derivative of $R(p)$ is negative (and $R(p)$ concave so that the objective function $\Pi(p)$ is concave and the second-order conditions satisfied) if $D''(p)$ is not too positive, i.e., if demand is not too convex. The inverse demand function is denoted by $P(q) = D^{-1}(q)$ with $P'(q) < 0$. Our regularity assumption also applies to the inverse demand function.

Exercise 1.1. *Exercise 1.4 in Tirole (1988).*

The monopolist maximizes its profits by choosing the profit-maximizing quantity q^m or the profit-maximizing price p^m. The profit-maximizing quantity solves the following problem:

$$\max_{q \geq 0} \{qP(q) - c(q)\} \tag{1.2}$$

The first-order condition is

$$MR(q) = P(q) + qP'(q) \quad \stackrel{!}{=} \quad c'(q) = MC(q) \tag{1.3}$$

that is, marginal revenue is equal to marginal cost. By the regularity condition the second-order condition is satisfied and so that $q = q^m$ that satisfies (1.3) is the profit-maximizing quantity. The profit-maximizing price is then $p^m = P(q^m)$.

Alternatively, instead of finding the quantity q^m that maximizes its profits, the monopolist can find the price p^m that maximizes its profits. This price solves the following problem:

$$\max_{p \geq 0} \{pD(p) - c(D(p))\} \tag{1.4}$$

Rearranging the first-order condition

$$\frac{d}{dp} = D(p) + pD'(p) - c'(D(p))D'(p) \stackrel{!}{=} 0 \tag{1.5}$$

we obtain

$$p - c'(D(p)) = -\frac{D(p)}{D'(p)}. \tag{1.6}$$

Again, for the moment we assume the second-order condition holds so that the price $p = p^m$ that satisfies (1.6) is the profit-maximizing price.

Let the price-elasticity of demand at price p^m be defined as

$$\varepsilon = -\frac{D'(p^m)p^m}{D(p^m)} > 0 \tag{1.7}$$

which is strictly positive because $D' < 0$.[1] Then condition (1.6) can be rewritten as

$$\frac{p^m - c'(D(p^m))}{p^m} = -\frac{D(p^m)}{D'(p^m)p^m} = \frac{1}{\varepsilon}. \tag{1.8}$$

This expression (1.8) is the so-called *Lerner index*. It states that the *relative markup* (difference between marginal costs and price) is inversely proportional to the demand elasticity. This means, the more sensitive consumer react to

[1] We follow Tirole (1988) by defining the price elasticity as a strictly positive number.

prices changes (the higher the price elasticity of demand) the smaller the relative markup a monopolist can charge. In the limit where $\varepsilon \to \infty$ (perfectly elastic demand: the monopolist loses all customers with any arbitrarily small price increase) the relative markup is zero and the profit-maximizing price is equal to marginal cost (cf. perfect competition).[2]

Observe that for non-zero (and non-negative) marginal cost, the monopoly operates in the elastic price region where $\varepsilon > 1$ because

$$\frac{p^m - c'}{p^m} = \frac{1}{\varepsilon}.$$

Suppose $\varepsilon < 1$, then $\frac{p^m - c'}{p^m} > 1$ which implies $c' < 0$. Note that if the price elasticity of demand is less than unity the marginal revenue $D(p) + pD'(p)$ is negative. For strictly positive marginal costs, the monopolist's profits are decreasing in quantity (less revenue and higher cost), i.e., increasing in price.

Exercise 1.2. *Show that if the elasticity of demand is independent of price (the demand function is $q = kp^{-\varepsilon}$, where k is a positive constant), the Lerner index is constant.*

Solution: With the given demand function, we obtain

$$\frac{p - c'(D(p))}{p} = -\frac{kp^{-\varepsilon}}{-\varepsilon kp^{-\varepsilon-1}p} = \frac{kp^{-\varepsilon}}{\varepsilon kp^{-\varepsilon}} = \frac{1}{\varepsilon}.$$

The *Lerner index* does not depend on the price. Tirole (1988, 66) provides an example of a rule-of-thumb pricing rule. Suppose a monopolist's technology exhibits constant returns to scale, then the marginal cost is equal to average cost (or unit cost). If price elasticity is 2 the monopolist charges twice the unit cost. With constant price elasticity (as in this example) such rule-of-thumb pricing may be profit-maximizing. □

A general property of the monopoly pricing is that p^m is a non-decreasing function of marginal costs. We want to show that

$$\frac{dp^m(c')}{dc'} \geq 0 \tag{1.9}$$

where c' denotes marginal costs. For this, consider two alternative cost functions $c_1(q)$ and $c_2(q)$ with $c_1'(q) < c_2'(q)$ for all $q > 0$. The monopolist sets p_1^m and q_1^m if costs are $c_1(q)$ and p_2^m and q_2^m if costs are $c_2(q)$. We therefore want to show that $p_2^m \geq p_1^m$. From p_1^m and q_1^m maximizing the monopolist's profits when $c_1(q)$ we know that

$$p_1^m q_1^m - c_1(q_1^m) \geq p_2^m q_2^m - c_1(q_2^m). \tag{1.10}$$

Likewise for $c_2(q)$:

$$p_2^m q_2^m - c_2(q_2^m) \geq p_1^m q_1^m - c_2(q_1^m). \tag{1.11}$$

[2] For empirical data on the Lerner index see Bresnahan (1989).

Combining (1.10) and (1.11) we get

$$[c_2(q_1^m) - c_2(q_2^m)] - [c_1(q_1^m) - c_1(q_2^m)] \geq 0 \tag{1.12}$$

which holds if and only if

$$\int_{q_2^m}^{q_1^m} [c_2'(x) - c_1'(x)]\, dx \geq 0 \tag{1.13}$$

because

$$\begin{aligned}\int_{q_2^m}^{q_1^m} [c_2'(x) - c_1'(x)]\, dx = [c_2(x) - c_1(x)]\Big|_{q_2^m}^{q_1^m} \quad = \\ [c_2(q_1^m) - c_2(q_2^m)] - [c_1(q_1^m) - c_1(q_2^m)]\end{aligned} \tag{1.14}$$

Further, because (by assumption) $c_2'(q) > c_1'(q)$ for all q we must have $q_2^m \leq q_1^m$ otherwise the expression in (1.13) is negative. Finally, because $D'(p) < 0$, $q_2^m \leq q_1^m$ implies $p_2^m \geq p_1^m$ which is what we wanted to show.

1.1.2 Deadweight Loss and Other Distortions

Monopoly power and monopoly pricing are said to result in welfare distortions when compared to the outcome in a perfectly competitive industry. In perfect competition, the quantity traded is such that the price is equal marginal cost. In Figure 1.1 this price is denoted by p^c (and the quantity q^c; not in the picture). At this price the relative markup is equal to zero. Because $D'(p) < 0$ and by the definition of ε, the Lerner index is always positive implying that the monopoly price is above the perfectly competitive price. This in return implies that the quantity traded in monopoly is less than the quantity traded in perfect competition. This reduction in the trade volume (relative to perfect competition) results in a welfare loss, the so-called *deadweight loss (DWL)*. The deadweight loss of monopoly pricing is represented in Figure 1.1 by the triangle *EFG*.[3] The idea is the following: For all units between q^m and q^c the demand curve lies above the marginal cost curve, i.e., the consumers' marginal valuation for these unit is at least as high as the costs of producing them. Trading these units will result in welfare gains. The deadweight loss is therefore the sum of all these *unconsummated gains from trade.*

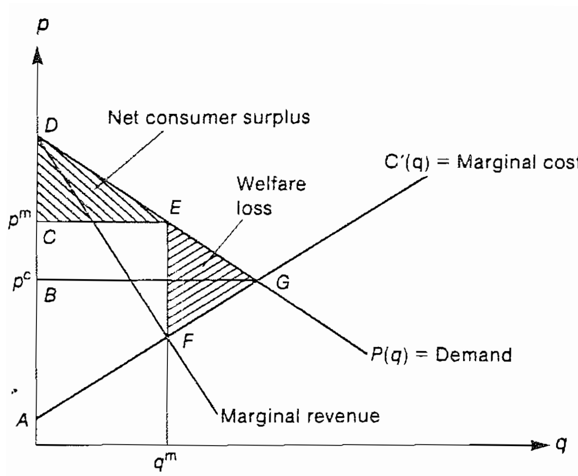
Formally, the deadweight loss is the (unrealized) social surplus for the units between q^m and $q^c = D(p^c)$:

$$DWL = \int_{q^m}^{q^c} [P(x) - c'(x)]\, dx \tag{1.15}$$

In part (c) of the following exercise we look at the relationship between the price elasticity of demand and the DWL. Generally, this relationship is not

[3]The triangular analysis of the deadweight loss goes back to Harberger (1954).

Figure 1.1: Monopoly Pricing and Deadweight Loss Tirole (1988, Fig. 1.1)



monotonic. If ε decreases and the Lerner index increases, then we a higher price which, at first, suggests a larger distortion. However, because for smaller ε the quantity effect of a higher price is also smaller (consumers are less price sensitive), the overall effect is non monotonic.

Exercise 1.3. *Exercise 1.1 in Tirole (1988)*

Solution: 1. We first compute the social welfare in a competitive industry (or social planner), W^c. In perfect competition, $p^c = c$ so that $q^c = D(p^c) = D(c) = c^{-\varepsilon}$. The social surplus W^c is then the "area" between $P(q)$ and c for $q \in [0, q^c]$.

$$\begin{aligned} W^c &= \int_0^{c^{-\varepsilon}} \left[x^{-1/\varepsilon} - c\right] dx = \left[\frac{x^{\frac{\varepsilon-1}{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}} - cx\right]\Bigg|_0^{c^{-\varepsilon}} = \frac{\left(c^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}} - c\left(c^{-\varepsilon}\right) \\ &= \ldots = \frac{c^{1-\varepsilon}}{\varepsilon - 1}. \end{aligned} \tag{1.16}$$

Because $\varepsilon > 1$ we know the competitive social surplus is strictly positive.

2. The DWL is $W^c - W^m$ where W^m is the social surplus under monopoly pricing:

$$W^m = \int_0^{q^m} \left[x^{-1/\varepsilon} - c\right] dx. \tag{1.17}$$

The monopoly quantity q^m is such that marginal revenue is equal to marginal cost. Revenue is $R = qP(q) = q^{\frac{\varepsilon-1}{\varepsilon}}$. For constant marginal cost, we are looking for a $q = q^m$ such that $q^{\frac{\varepsilon-1}{\varepsilon}} = c$. This monopoly quantity is

$$q^m = \left(\frac{c\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon}. \tag{1.18}$$

Then

$$\begin{aligned} W^m &= \int_0^{\left(\frac{c\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}} \left[x^{-1/\varepsilon} - c\right] dx = \frac{x^{\frac{\varepsilon-1}{\varepsilon}}}{\frac{\varepsilon-1}{\varepsilon}} - cx \Bigg|_0^{\left(\frac{c\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}} \\ &= \ldots = \frac{\varepsilon c^{1-\varepsilon\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} - \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-1-\varepsilon}\right]}}{\varepsilon - 1}. \end{aligned} \tag{1.19}$$

We obtain the deadweight loss by subtracting W^m in (1.19) from W^c in (1.16), or by calculating the area between the demand curve and the cost curve for all non-traded units:

$$DWL = \int_{q^m}^{q^c} \left[x^{-1/\varepsilon} - c\right] dx \tag{1.20}$$

with q^m in (1.18) and $q^c = c^{-\varepsilon}$. After some straightforward (but a bit tedious) algebra we obtain

$$DWL = \frac{c^{1-\varepsilon}}{\varepsilon - 1}\left[1 - \frac{2\varepsilon - 1}{\varepsilon - 1}\left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon}\right]. \tag{1.21}$$

3. We skip the non-monotonicity of DWL in ε. We show that the *relative* DWL, DWL/W^c, is increasing in ε. The relative DWL is

$$\frac{DWL}{W^c} = 1 - \frac{2\varepsilon - 1}{\varepsilon - 1}\left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon} = 1 - \kappa(\varepsilon). \tag{1.22}$$

To show that DWL/W^c is increasing in ε we need to show that $\kappa(\varepsilon)$ is decreasing in ε. We first take a monotone transformation (natural log transformation) of $\kappa(\varepsilon)$ so that $\kappa(\varepsilon)$ *is decreasing in* ε *if and only if* $\log\kappa(\varepsilon)$ *is decreasing in* ε:

$$\log\kappa(\varepsilon) = \log(2\varepsilon - 1) - \log(\varepsilon - 1) - \varepsilon\log\varepsilon + \varepsilon\log(\varepsilon - 1) \tag{1.23}$$

and we show that

$$\frac{d\kappa(\varepsilon)}{d\varepsilon} < 0. \tag{1.24}$$

First,

$$\frac{d\kappa(\varepsilon)}{d\varepsilon} = \frac{\kappa'(\varepsilon)}{\kappa(\varepsilon)} = \frac{2}{2\varepsilon - 1} + \log\frac{\varepsilon - 1}{\varepsilon}. \tag{1.25}$$

To show that this expression (1.25) is negative we take its first derivative with respect to ε (second derivative of $\kappa(\varepsilon)$) and obtain

$$\frac{d^2\kappa(\varepsilon)}{d\varepsilon^2} = -\frac{4}{(2\varepsilon-1)^2} + \frac{1}{\varepsilon-1} - \frac{1}{\varepsilon} = \frac{1}{(1-2\varepsilon)^2\,\varepsilon\,(\varepsilon-1)} > 0. \tag{1.26}$$

The inequality holds because (by assumption) $\varepsilon > 1$. Moreover,

$$\left.\frac{d\kappa(\varepsilon)}{d\varepsilon}\right|_{\varepsilon\to\infty} = \left.\frac{\kappa'(\varepsilon)}{\kappa(\varepsilon)}\right|_{\varepsilon\to\infty} = 0 + \log 1 = 0. \tag{1.27}$$

Expression (1.25) is strictly increasing in ε and 0 as $\varepsilon \to \infty$. As a result the expression is negative for all $\varepsilon < \infty$ implying that $\kappa(\varepsilon)$ is decreasing in ε implying that the relative DWL is increasing in ε.

4. We show that the fraction of the potential consumer surplus the monopolist can capture is increasing in ε. First, note that with constant marginal cost and perfect competition, the consumer surplus is equal to the social surplus, W^c. In a monopoly the firm captures some (or all) of this *potential consumer surplus.* The fraction of this potential consumer surplus the monopolist can capture is Π^m/W^c where

$$\Pi^m = q^m P(q^m) - q^m c = \left(\frac{c}{\varepsilon-1}\right)^{1-\varepsilon} e^{-\varepsilon} \tag{1.28}$$

so that

$$\frac{\Pi^m}{W^c} = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} = \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}. \tag{1.29}$$

The fraction Π^m/W^c is increasing in ε if and only if $d\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}/d\varepsilon > 0$. Recall that for $y = (f(x))^{g(x)}$ we get

$$y' = (f(x))^{g(x)}\left(g'(x)\log f(x) + g(x)\frac{f'(x)}{f(x)}\right). \tag{1.30}$$

We have $f(x) = \frac{\varepsilon-1}{\varepsilon}$ and $g(x) = \varepsilon$ and therefore obtain

$$\begin{aligned}\frac{d\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}}{d\varepsilon} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}\left(\log\left(\frac{\varepsilon-1}{\varepsilon}\right) + \varepsilon\frac{\frac{1}{\varepsilon^2}}{\left(\frac{\varepsilon-1}{\varepsilon}\right)}\right)\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}\left(\log\left(\frac{\varepsilon-1}{\varepsilon}\right) + \frac{1}{\varepsilon-1}\right).\end{aligned} \tag{1.31}$$

This expression is positive if $\log\left(\frac{\varepsilon-1}{\varepsilon}\right) > -\frac{1}{\varepsilon-1}$ or

$$e^{\frac{1}{\varepsilon-1}} > \log\frac{\varepsilon}{\varepsilon-1}.$$

After some tedious algebra it can be shown that this holds true.

Is the above result (relative DWL is increasing in the price elasticity of demand) in line with intuition? Increasing price elasticity means that consumers react more to price changes; by the Lerner index we know that the relative markup is smaller, hence the monopoly price lower. We would expect less price distortion and higher quantities traded. As a result, if W^c were fixed, we would expect the opposite result, namely that the DWL is decreasing in ε. Similarly for the fraction of the potential consumer surplus the monopolist can extract. $\square$

The simple DWL analysis works well with nicely downward sloping demand curves. A slightly higher price (above competitive price) implies a lower quantity traded. Moreover, the monopolist sets the profit-maximizing price higher than the perfectly competitive price. However, when consumer demand is not nicely downward sloping with finite price elasticity but homogeneous consumers exhibit unit demand and buy if and only if the price is at or below their reservation price, things change. The following exercise illustrates this point.

Exercise 1.4. *Exercise 1.2 in Tirole (1988)*

Solution: We solve the problem in three steps: We first determine the socially optimal price p^c before we look at the monopolist's profit-maximizing price. We finally argue that monopoly pricing does not result in a deadweight loss. We assume that $c'(n) < \bar{s}$. As we will see, this assumption ensures that it is optimal to serve *all* consumers

1. We have n consumers with unit demand and reservation price $\bar{s}$. The demand function is therefore

$$D(p) = \begin{cases} n & \text{if } p \leq \bar{s} \\ 0 & \text{if otherwise.} \end{cases} \tag{1.32}$$

   Trade is optimal if and only if a consumer's valuation of the good (reservation price) is at least as high as the (marginal) costs of producing that item. All consumers have the same reservation price. Hence, trade is optimal if and only if $\bar{s} \geq c$. If this condition holds, then n units are traded. Then an optimal price must be such that $p \geq c'(n)$ and $p \leq \bar{s}$. Let $\tilde{p}^c = c'(n)$ the lowest such price. If $p < \tilde{p}^c$ then not all n consumers would be served; if $p > \bar{s}$, then consumers should not buy. Hence, $p^c \in [\tilde{p}^c, \bar{s}]$. For any price p^c there are n units traded. By assumption of $c'(n) < \bar{s}$, this set is not empty.

2. The monopolist faces demand $D(p)$ and maximizes its profit function

$$\Pi(p) = pD(p) - c(D(p)) = \begin{cases} pn - c(n) & \text{if } p \leq \bar{s} \\ 0 & \text{if } p > \bar{s} \end{cases} \tag{1.33}$$

   The monopolist's profits are therefore increasing in p as long as $p \leq \bar{s}$ and constant and equal to zero for all $p > \bar{s}$. The profit-maximizing price for the monopolist is then $p^m = \bar{s}$. Again, assume that $c'(n) < \bar{s}$, and the monopolist is willing to serve *all* consumers.

3. The monopoly price p^m is element of the set of optimal prices, $[\tilde{p}^c, \bar{s}]$. Under monopoly pricing, the trade volume is n, just like in the case of perfect competition (or under a social planner/benevolent dictator who sets the price). All wealth-creating transactions are consummated and there is no deadweight loss.

If we relax our assumption of $c'(n) < \bar{s}$ we may have a situation where the marginal costs of serving the nth consumer are higher than the consumer's willingness to pay. Trade for the nth consumer is then suboptimal. In this case, rationing is the optimal solution (assuming that consumers do not waste resources standing in line). The optimal price is $p^c = \bar{s}$ and a quantity $\bar{q}$ such that $c'(\bar{q}) = \bar{s}$ is traded. The monopolist will sell this quantity $q^m = \bar{q}$ for a price $p^m = \bar{s}$. Again, there is no price distortion. □

The discussion of deadweight loss has been one-sided.

1. The deadweight loss is an upper bound on the efficiency gain that can be realized when correcting monopoly pricing. We juxtaposed monopoly pricing with competitive price and an efficient allocation. If the alternative to monopoly pricing is not efficient marginal-cost pricing and this alternative benchmark has lower social surplus, then the deadweight is (mechanically) smaller.

2. The deadweight loss (as a demand side effect) may not be the only distortion of monopoly pricing. Monopoly pricing can also have effects on the supply side when a monopolist produces at a higher cost than a competitive firm otherwise would. The argument is that firms in a monopoly situation pay little attention to cost-cutting strategies or engage in slack. The "quiet-life hypothesis" (Hicks, 1935) or "X-inefficiency" (Leibenstein, 1966) capture this. See the discussion in Tirole (1988, 75f).

3. A possible third distortion (beside deadweight loss and X-inefficiency) from monopoly pricing are wasteful expenses that firms incur to obtain or maintain a monopoly position. As long as positive rents are generated by a monopoly, the monopolist will incur costs and effort to erect barriers that keep entrants out; and potential entrants will incur costs and effort to enter the monopolistic market. To what extent such expenditures (from R&D, barriers to entry, lobbying and advertising campaigns) are wasteful depends on the nature of the expenses. R&D may or may not be wasteful, lobbying or rent-seeking expenditures are typically viewed as wasteful. See Tirole (1988, 76ff) for an extended discussion.

4. The fact that only one firm operates in a market does not automatically imply distortive monopoly pricing. A monopolist can engage in monopoly pricing only if it has the market power to do so. Moreover, the threat of entry (presence of potential entrants) may induce the monopolist to set a price lower than the monopoly price or even equal to the competitive

price. The theory of contestable markets (Baumol, 1982) illustrates this point—we will return to it later.

1.2 Multi-Product Monopolist

We now consider a monopolist producing n goods, indexed $i = 1, \ldots, n$. The monopolist sets prices $\bar{p} = (p_1, \ldots, p_n)$ or quantities $\bar{q} = (q_1, \ldots, q_n)$. Demand for each good i is $D_i(\bar{p})$, total production costs of $\bar{q}$ are $c(\bar{q})$. This notation allows for both dependent and independent demand as well as separable and non-separable costs. The monopolist's profits are

$$\Pi^m(\bar{p}) = \sum_{i=1}^{n} p_i D_i(\bar{p}) - c(D_1(\bar{p}), \ldots, D_n(\bar{p})) \tag{1.34}$$

The monopolist solves $\max_{\bar{p}} \Pi^m(\bar{p})$. We have n first order conditions (one for each i)

$$D_i + p_i \frac{\partial D_i(\bar{p})}{\partial p_i} + \sum_{j \neq i} p_j \frac{\partial D_j(\bar{p})}{\partial p_i} \stackrel{!}{=} \sum_{j=1}^{n} \frac{\partial c(\bar{q})}{\partial q_j} \frac{\partial D_j(\bar{p})}{\partial p_i} \quad \forall i = 1, \ldots, n \tag{1.35}$$

where $q_j = D_j(\bar{p})$.

1.2.1 Dependent Demand and Separable Costs

We assume separable costs:

$$c(\bar{q}) = \sum_{i=1}^{n} c_i(q_i) \tag{1.36}$$

which means the production of good j does not affect the costs of production of good i. Note, however, that because demand is interdependent, the price of good i affects the demand of good j and thus the total production costs for good j. We rewrite our first order conditions as

$$\begin{aligned} D_i + p_i \frac{\partial D_i(\bar{p})}{\partial p_i} + \sum_{j \neq i} p_j \frac{\partial D_j(\bar{p})}{\partial p_i} &\stackrel{!}{=} \frac{dc_i(q_i)}{dq_i} \frac{\partial D_i(\bar{p})}{\partial p_i} + \sum_{j \neq i} \frac{dc_j(q_j)}{dq_j} \frac{\partial D_j(\bar{p})}{\partial p_i} \quad \forall i \\ D_i + p_i \frac{\partial D_i(\bar{p})}{\partial p_i} + \sum_{j \neq i} p_j \frac{\partial D_j(\bar{p})}{\partial p_i} &\stackrel{!}{=} \sum_{j=1}^{n} \frac{dc_j(q_j)}{dq_j} \frac{\partial D_j(\bar{p})}{\partial p_i} \quad \forall i \end{aligned} \tag{1.37}$$

We define the (positive) own-price elasticity as

$$\varepsilon_{ii} = -\frac{\partial D_i(\bar{p})}{\partial p_i} \frac{p_i}{D_i(\bar{p})} \tag{1.38}$$

and the cross-price elasticity as

$$\varepsilon_{ij} = -\frac{\partial D_j(\bar{p})}{\partial p_i}\frac{p_i}{D_j(\bar{p})}. \tag{1.39}$$

With these two expressions and after some algebraic manipulation (*In Tirole we trust.*) we can rewrite (1.37) as

$$\frac{p_i - c_i'}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j\neq i}\frac{\left(p_j - c_j'\right) D_j(\bar{p})\varepsilon_{ij}}{R_i(\bar{p})\varepsilon_{ii}} \tag{1.40}$$

with $R_i(\bar{p}) = p_i D_i(\bar{p})$. This is the Lerner index for the monopoly price for good i when demand is interdependent. First, note that if there are no interdependencies and $\varepsilon_{ij} = 0$ for all j, then (1.40) reduces to (1.8).

With $\varepsilon_{ij} \neq 0$ for some j we consider two polar cases:

1. Suppose *all* goods are substitutes, i.e, for all $j \neq i$ we have $\partial D_j/\partial p_i > 0$ or $\varepsilon_{ij} < 0$. Intuitively, for substitutes an increase in the price for good i increases the demand for (all goods) j as consumer buy less of i and instead more of j. By the definition of the cross-price elasticity, $\varepsilon_{ij} < 0$. (Hint: negative sign means the quantities of i and j move in opposite directions). In this case we have

$$\frac{p_i - c_i'}{p_i} > \frac{1}{\varepsilon_{ii}}, \tag{1.41}$$

   the Lerner index (the relative markup) exceeds the inverse of the own-price elasticity of demand. The substitutability between goods creates an externality. Suppose separate divisions (the monopolist is non-integrated) set prices to maximize their own profits $R_i - c_i$. Each business unit would not take the effect of its price on $D_j(\bar{p})$ into account but price according to the simple Lerner index in (1.8). This effect is negative in the sense that a lower price for good i has a negative effect on the demand for good j. The divisions become *de facto* competitors and the result is *underpricing* by non-integrated divisions; the integrated monopolist sets higher prices.

2. Suppose *all* goods are complements, i.e., i.e, for all $j \neq i$ we have $\partial D_j/\partial p_i < 0$ or $\varepsilon_{ij} > 0$. Intuitively, for complements an increase in the price for good i decreases the demand for (all goods) j as consumer buy less of i and less of j. By the definition of the cross-price elasticity, $\varepsilon_{ij} > 0$. (Hint: positive sign means the quantities of i and j move in the same direction). In this case we have

$$\frac{p_i - c_i'}{p_i} < \frac{1}{\varepsilon_{ii}}, \tag{1.42}$$

   the Lerner index (the relative markup) is less than the inverse of the own-price elasticity of demand. Again, non-integrated divisions would not take the effect of their prices on the other divisions' revenues into account. Unlike in the case of substitutes, the result is now underpricing. In the extreme, a firm may in fact price a good below marginal cost to increase demand for another good. We come back to related issues when we talk about predatory pricing.

Exercise 1.5. *Exercise 1.5 in Tirole (1988)*

1.2.2 Independent Demand and Non-Separable Costs

For this second case we consider an inter-temporal of a multi-product firm: The firm produces goods in two periods, $t = 1, 2$. Costs are non-separable as we assume that the production in period 1 has an effect on the costs in period 2. The goods are nondurable and consumers have demand for the goods in both periods. The demand for goods in period 1 is $D_1(p_1)$, the demand in period 2 is $D_2(p_2)$. Production costs in period 1 are $c_1(q_1)$ and in period 2 are $c_2(q_2, q_1)$. We make the usual assumptions for the demand functions and the following for the cost functions:

1. c_1 is non-decreasing
2. c_2 is decreasing in q_1: $\partial c_2/\partial q_1 < 0$.
3. The marginal costs in period 2 are decreasing in q_1:

$$\frac{\partial^2 c_2(q_2, q_1)}{\partial q_1 \partial q_2} < 0. \tag{1.43}$$

The latter two effects capture the learning effect. The monopolist maximizes its profits[4]

$$\Pi^m(p_1, p_2) = p_1 D_1(p_1) + p_2 D_2(p_2) - c_1(D_1(p_1)) - c_2(D_2(p_2), D_1(p_1)) \tag{1.44}$$

by solving the following problem:

$$\max_{p_1 \geq 0, p_2 \geq 0} \Pi^m(p_1, p_2)$$

It chooses p_1^m and p_2^m such that the first-order conditions are satisfied:

$$p_1: \qquad \left(p_1^m - \left[\frac{dc_1(q_1^m)}{dq_1} + \frac{\partial c_2(q_2^m, q_1^m)}{\partial q_1}\right]\right) D_1'(p_1^m) \stackrel{!}{=} -D_1(p_1^m) \tag{1.45}$$

$$p_2: \qquad \left(p_2^m - \frac{\partial c_2(q_2^m, q_1^m)}{\partial q_2}\right) D_2'(p_2^m) \stackrel{!}{=} -D_2(p_2^m) \tag{1.46}$$

There are two observations:

[4]For simplicity, there is no discounting of future profits.

1. We have *marginal revenue equals marginal cost* only in period $t = 2$. In period $t = 1$, the monopolist's optimal price p_1^m also takes the effect of production in $t = 1$ on the costs of production in $t = 2$ into account. The profit-maximizing p_1^m is lower than the price a myopic monopolist would set; consequently, the quantity is higher. One could interpret $\left[\frac{dc_1(q_1^m)}{dq_1} + \frac{\partial c_2(q_2^m, q_1^m)}{\partial q_1}\right]$ as the *effective marginal costs* (accounting for the inter-temporal effect). Because $\partial c_2/\partial q_1 < 0$, these effective marginal costs are lower. Earlier we showed that the monopoly price is increasing (to be precise: non-decreasing) in marginal costs; lower marginal costs then result in a lower price.

2. If (unlike in the third assumption above), $\frac{\partial^2 c_2(q_2,q_1)}{\partial q_1 \partial q_2} = 0$, then the profit-maximizing price p_2^m and the myopic price are the same because q_1 does not affect the marginal costs of production in period $t = 2$. Under the third assumption this does not hold. Then the profit-maximizing price p_2^m is lower and the profit-maximizing quantity q_2^m higher than in the myopic case because a higher q_1^m (observation 1) reduces c_2' which in return yields a lower price (by the above argument).

1.3 Intertemporal Pricing Behavior

For the above example of learning-by-doing we assumed non-durable goods. The firm produces a good in both periods and consumers have demand for the good in both periods (the goods are "used up" or "perishable"). In the following simple example we now turn to *durable* goods.

Suppose there are seven consumers with reservation prices of $v = 1, \ldots, 7$; no production costs; a monopolist produces goods that are infinitely durable. That means, if a consumer buys the good in period 1 it does not demand the good in period 2. We consider a two-period model with a discount factor δ.

Assume for the moment a single-period model (or a myopic monopolist who maximizes only period-1 profits). This single-period monopoly price is $p_1^m = 4$. Consumers with valuations $v = 4, \ldots, 7$ purchase, and residual demand is $v = 1, 2, 3$. Given this residual demand, a monopoly price of $p_2^m = 2$ maximizes the monopolist's period-2 profits.

But is p_1^m really an optimal price in period 1? We assume rational consumers that are able to anticipate $p_2^m = 2$. That means, a consumer v can purchase the good in period 1 for $p_1^m = 4$ with a surplus of $v - 4$, or the same consumer can wait one period and purchase the good for $p_2^m = 2$ with a discounted surplus of $\delta(v - 2)$. Consequently, consumers buy in period 1 if

$$v - 4 \geq \delta(v - 2) \iff v \geq \frac{4 - 2\delta}{1 - \delta}. \tag{1.47}$$

As we can see, the marginal consumer, $v = 4$, will not buy in period 1 but wait until period 2. This is because the marginal consumer can expect a lower price and is willing to wait one more period. The result is lower demand in period 1 and higher demand is period 2, which affects the optimal prices p_1^m (will be lower) and p_2^m, which affects the consumers' purchasing decisions, which affects the demand in both periods, which affects optimal prices, and so forth. An equilibrium is characterized by a *sequence of prices and expectations such that expectations are rational given the firm's behavior (prices) and the firm's behavior is optimal given the consumers' expectations.* The observation that in such an equilibrium the high valuation customers pay a higher price in period 1 and lower valuation customers pay a lower price in period 2 is an example of *inter-temporal price discrimination.*

In the next section we study a more general form of this game. We will see in equilibrium a *decreasing price sequence* over time where the price (when infinitely long) converges to marginal cost. We will see in the discussion to follow that the monopolist can increase its profits when it is able to commit to not lowering prices. Without commitment, the monopolist in a durable-goods market competes with future incarnations of herself. With commitment the monopolist can soften or eliminate this competition and eventually earn higher profits.

1.3.1 The Coase Conjecture

Coase (1972) discusses the example of landowner who, for the sake of the argument, owns the entire USA. The question he asks is *At which price will the landowner sell the price.* He concludes from our above arguments that the price will be the competitive price. The fact that the monopolist competes with future incarnations of herself drives the first period price down to the perfectly competitive price. He makes the analogous argument for a firm that produces a good at (constant) marginal cost: The price is the perfectly competitive price, all is produced and sold in period one.[5]

Coase's analysis based on the assumption that the monopolist can change prices at an arbitrary rate. A durable-good monopolist who can cut prices sufficiently rapidly charges marginal cost. The so-called *Coase Conjecture* qualifies this a bit by stating that *as price adjustments become more and more frequent, the monopolist's profits converge to zero.*

Bulow (1982) and Stokey (1981) prove the Coase conjecture formally for particular demand functions, Gul, Sonnenschein, and Wilson (1986) prove it for more general demand structures. Below we go through a simple version of this proof; related analysis can be found in Sobel and Takahashi (1983).

Exercise 1.6. *Exercise 1.8 in Tirole (1988)*

Solution: A monopolist sells a perfectly durable good; consumers are infinitely lived. We consider discrete time with a discount factor of δ. The marginal costs of production are equal to zero. Consumers have unit demand for one item, there (per-period) valuation is $v \sim U\,[0, 1]$. Observe, the lowest consumer's valuation

[5]With increasing marginal costs, production of all durable goods at period 1 is more costly than spreading production over multiple periods. The result may change slightly.

is (weakly) less than production costs.[6] Also, buyer's decision on whether to accept a price does not depend on the history of the market.[7] There is no resale market. Once a consumer has purchased, it leaves the market. Consumer's time-0 utility when buying the good in $t = 1$ is $\delta^t(v - p_t)$. The monopolist's profits (at $t = 0$) are

$$\sum_{t=1}^{\infty} \delta^t p_t q_t \tag{1.48}$$

We look for a *linear and stationary equilibrium.*

- *Linear* means that the monopolist's and the consumers' behavior can be described by a simple linear rule.
- *Stationary* means that this rule does not depend on time t.
- *Equilibrium* means that
  - The monopolist's optimization has to be consistent with consumers' rational expectations.
  - Consumers' expectations (about the price) have to be rational given the firms' behavior.

Our approach does not impose linear rules, but maximization is over the price sequence. We take the first-order conditions to verify that a linear and stationary equilibrium exists. Alternatively, we could impose linear rules and maximize over the parameters of these rules.

We will look out for two linear rules:

1. Consumers with valuation v purchase if $v \geq w(p) =: \lambda p$ with $\lambda > 1$.
2. Producer have to decide what price to charge. Consumers with valuations exceeding $\bar{v}$ have already purchased, others have not. Given this $\bar{v}$, the monopolist sets a price $p(\bar{v}) = \mu\bar{v}$ with $\mu < 1$.

Note that $\lambda > 1$, and $\mu < 1$. We are looking for μ^* and λ^*.

Suppose some period t. When do consumers buy? A consumer buys in any period t if $v - p_t > \delta(v - p_{t+1})$ or

$$v > \frac{p_t - \delta p_{t+1}}{1 - \delta} =: v_{t+1}. \tag{1.49}$$

We denote the *state* of the economy in any given t (as a function of the current price and the previous price) as

$$v_t := \frac{p_{t-1} - \delta p_t}{1 - \delta} \tag{1.50}$$

[6] Fudenberg, Levine, and Tirole (1985) provide results for the Coase conjecture when the buyer's valuation is bounded away from production costs. In this case the Coase conjecture states that the monopolist sets the price close to the buyer's lowest valuation.

[7] Ausubel and Deneckere (1989) show that with this assumption relaxed many other outcomes (including something close to the monopoly price) are possible.

Note that *all* $v \geq v_t$ who did not buy in $t-2$ have bough in $t-1$ because in $t-1$ they satisfied (1.49). In any period t the set of consumers who have not yet bought is $[0, v_t]$. The producer knows in t this set of consumers who are still in the market.

With this notation we can reconsider a stationary equilibrium: There is no history in this equilibrium; the firms' behavior is determined by v_t and the consumers' behavior is determined by v_t and p_t.

Find μ. According to the linear rule, $p_t(\bar{v}) = \mu\bar{v}$. In any given t, this $\bar{v}$ is equal to v_t (all consumers $v > v_t$ have purchased). The price in t is then $p_t(v_t) = \mu v_t$. The quantity sold in period t is $q_t = v_t - v_{t+1}$ where v_{t+1} is the lowest consumer who buys in this period (is the highest consumer who buys in the next period) and v_t the highest consumer who buys in this period. This last consumer who buys in t is $v_{t+1} = \lambda p_t(v_t)$; according to the linear purchase rule for the consumers all $v > \alpha p_t$ will have bough in t and v_{t+1} are remaining. Rearranging yields

$$\begin{aligned} q_t &= v_t - \lambda p_t(v_t) \\ &= v_t - \lambda\mu v_t \\ &= (1-\lambda\mu)v_t \end{aligned} \tag{1.51}$$

and $\lambda\mu < 1$. Now we know the demand the firm faces in each period. Let's set up its profits. The monopolist reaches t with v_t consumers. The profits, as of t, are then

$$\begin{aligned} \Pi &= \ldots + \delta^t p_t q_t + \delta^{t+1} p_{t+1} q_{t+1} + \delta^{t+2} p_{t+2} q_{t+2} + \ldots \\ &= \ldots + \delta^t p_t [v_t - \lambda p_t] + \delta^{t+1} p_{t+1} [\lambda p_t - \lambda p_{t+1}] \\ &\quad + \delta^{t+2} p_{t+2} [\lambda p_{t+1} - \lambda p_{t+2}] + \ldots \end{aligned} \tag{1.52}$$

Given the firm's beliefs about what consumers believe about what price will be charged in $t+1$, $t+2$, ..., the firm maximizes over p_t, p_{t+1}, p_{t+2}, ... Let's look at the first three first-order conditions.

$$\begin{aligned} p_t: \quad 0 &= \delta^t \left[v_t - \lambda p_t - \lambda p_t\right] + \delta^{t+1} \lambda p_{t+1} \\ &= \delta^t \left[v_t - 2\lambda p_t + \delta\lambda p_{t+1}\right] && (1.53) \\ p_{t+1}: \quad 0 &= \delta^{t+1} \left[\lambda p_t - 2\lambda p_{t+1} + \delta\lambda p_{t+2}\right] && (1.54) \\ p_{t+2}: \quad 0 &= \delta^{t+2} \left[\lambda p_{t+1} - 2\lambda p_{t+2} + \delta\lambda p_{t+3}\right] && (1.55) \end{aligned}$$

These first order conditions have to hold for $t+i$ for all $i \geq 0$. We find μ such that each row holds. However, we have no μ in this expression. Recall that $p_t = \mu v_t$ and $v_{t+1} = \alpha p_t$. By substituting these expressions we can see a pattern for the first-order conditions. Take for instance the first-order condition for p_{t+1}

$$
\begin{aligned}
\lambda p_t - 2\lambda p_{t+1} + \delta\lambda p_{t+2} &= 0 \qquad (1.56)\\
\lambda\mu v_t - 2\lambda\mu v_{t+1} + \delta\lambda v_{t+2} &= 0 \qquad (1.57)\\
\lambda\mu v_t - 2\lambda\mu\lambda p_t + \delta\lambda v_{t+2} &= 0 \qquad (1.58)\\
\lambda\mu v_t - 2\lambda\mu\lambda\mu v_t + \delta\lambda\mu\lambda\mu\lambda\mu v_t &= 0 \qquad (1.59)\\
\lambda\mu v_t \left[1 - 2\lambda\mu + \delta\lambda^2\mu^2\right] &= 0 \qquad (1.60)
\end{aligned}
$$

We can look at the FOC for p_t as well as for p_{t+2} and we will see the same pattern. In order for (1.60) to hold true for all t we must have

$$
1 - 2\lambda\mu + \delta\lambda^2\mu^2 = 0. \qquad (1.61)
$$

We can take (1.61) and solve for μ. This yields

$$
\mu = \frac{1}{\lambda\left[1 + \sqrt{1-\delta}\right]} \qquad (1.62)
$$

This is still a function of λ, the linear rule on the consumer side.

Find λ. In equilibrium in t, consumers will expect prices

$$
\begin{aligned}
p_{t+1} &= \mu v_{t+1} = \mu\lambda p_t(v_t) = \mu\lambda\mu v_t\\
&= \mu^2\lambda v_t \qquad (1.63)
\end{aligned}
$$

The marginal consumer is

$$
\begin{aligned}
v_{t+1} - p_t &= \delta\left(v_{t+1} - p_{t+1}\right)\\
&= \delta\left(v_{t+1} - \mu v_{t+1}\right) \qquad (1.64)\\
&= \delta\left(1-\mu\right)v_{t+1} \qquad (1.65)
\end{aligned}
$$

We can rearrange to obtain

$$
p_t = \left[1 - \delta(1-\mu)\right] v_{t+1} \qquad (1.66)
$$

Further rearranging yields

$$
v_{t+1} = \frac{p_t}{1 - \delta(1-\mu)} \qquad (1.67)
$$

Because $v_{t+1} = \lambda p_t$ we find

$$
\lambda = \frac{1}{1 - \delta(1-\mu)} \qquad (1.68)
$$

Combining (1.62) and (1.68) we obtain

$$
\begin{aligned}
\mu^* &= \frac{\sqrt{1-\delta}}{1+\sqrt{1-\delta}} \qquad (1.69)\\
\lambda^* &= \frac{1}{\sqrt{1-\delta}} \qquad (1.70)
\end{aligned}
$$

for the monopolist's pricing rule and the consumers' purchasing rule in a linear and stationary equilibrium. Now, if $\delta \to 1$, i.e., if consumers become perfectly patient (which is formally equivalent to saying the monopolist's price changes are arbitrarily frequent), then $\mu \to 0$. This means, the monopolist's markup is equal to zero, $p_t = 0v_t$ for all t. The monopolist sets the price equal to marginal cost. □

Exercise 1.7. *Exercise 1.9 in Tirole (1988) (Advanced)*

The monopolist's problem is that consumers rationally expect the monopolist to flood the market with the durable good after early periods (at high price). Different strategies for the monopolist to avoid the Coase problem are:

1. Credibly commit to a sequence of prices. How?
2. Escrow with third party (an "arbitrator")
3. Reputation can enter these considerations.
4. Destroy factory or production facilities after today's production
5. Increasing marginal cost (decreasing returns to scale) will force the monopolist to spread out production and prevent the monopolist from flooding the market too fast. Increasing costs can be a profitable strategy for the monopolist.
6. Money-back guarantee or most-favored nation clauses.
7. Reduce the durability of the good (planned obsolesce)
8. Leasing instead of selling.

Exercise 1.8. *Read Section 1.5.2.4 in the textbook (Tirole, 1988) and solve Exercise 1.10.*

1.3.2 Leasing vs. Selling

Consider a two-period model in which the seller of a durable good can either lease the good in each period or sell the good. Assume zero marginal costs, a discount factor δ, and linear demand $D(p) = 1 - p$ and $P(q) = 1 - p$. A resale market for the durable good exists; i.e., consumers who purchase the durable good in period 1 can enter as competitors for the monopolist in period 2. We take a very simplistic approach to the resale market.

Leasing. Leasing is a means for the monopolist to transform the durable good into a non-durable good. Its per-period profit function is

$$\pi_t^l = p_t D(p_t) \tag{1.71}$$

for $t = 1, 2$. Given the linear demand function the optimal leasing prices are $p_1^l = p_2^l = 1/2$; the quantities leased in each period are $q_1^l = q_2^l = 1/2$ and the monopolist's total profits are

$$\Pi^l = 1/4 + \delta 1/4 = \frac{1+\delta}{4}. \tag{1.72}$$

Selling. When consumers purchase the good in period $t = 1$ they can resell it on the market in period $t = 2$. This means that in $t = 2$ a quantity of q_1 will be offered by previous buyers. We solve the problem by backward induction.

In period $t = 2$ the total quantity offered is q_1 (by resellers) and q_2 by the monopolist, $q_1 + q_2$. Given q_1 the monopolist solves

$$\max_{q_2} \left[q_2(1 - q_1 - q_2) \right]. \tag{1.73}$$

The optimal quantity is $q_2^s = \frac{1-q_1}{2}$, the optimal price is $p_2^s(q_1) = \frac{1-q_1}{2}$ and the period-2 profits are $\pi_2^s = \frac{(1-q_1)^2}{4}$.

In period $t = 1$ the consumers' willingness to pay is

$$P_1(q) = p_1 = 1 - q_1 + \delta p_2^a \tag{1.74}$$

where p_2^a denotes the expected price in period $t = 2$ at which they can resell the good. The willingness to pay is then according to the demand function, but increased by the discounted price they can achieve from reselling the good. We assume that expectations are correct and $p_2^a = p_2$ where we know from $(t = 2)$ that $p_2 = p_2^s(q_1) = \frac{1-q_1}{2}$. The effective demand in period $t = 1$ is then

$$P_1(q) = 1 - q_1 + \delta \frac{1 - q_1}{2} = (1 - q_1) \left(1 + \frac{\delta}{2} \right) \tag{1.75}$$

The monopolist's period-1 profits are

$$\pi_1^s = q_1(1 - q_1)(1 + \frac{\delta}{2}) \tag{1.76}$$

Because its choice of q_1 does not only affect its period-1 profits but (see above) also affects its period-2 profits, the monopolist solves the following problem:

$$\max_{q_1} \Pi^s = \max_{q_1} (\pi_1^s + \delta\pi_2^s) = \max_{q_1} \left(q_1(1 - q_1)(1 + \frac{\delta}{2}) + \delta \frac{(1 - q_1)^2}{4} \right) \tag{1.77}$$

We obtain

$$q_1^s = \frac{2}{4 + \delta}. \tag{1.78}$$

With this quantity the total profits are

$$\Pi^s = \frac{1+\delta}{4} - \frac{\delta}{4(4+\delta)} < \Pi^l \tag{1.79}$$

which is strictly less than the profits from leasing.

Bibliography

Ausubel, L. M., and R. J. Deneckere (1989): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 57(3), 511–531.

Baumol, W. J. (1982): "Contestable Markets: An Uprising in the Theory of Industry Structure," *American Economic Review*, 72(1), 1–15.

Bresnahan, T. F. (1989): "Empirical Studies of Industries with Market Power," in *Handbook of Industrial Economics*, ed. by R. Schmalensee, and R. Willig, vol. 2, chap. 17, pp. 1011–1057. Elsevier.

Bulow, J. I. (1982): "Durable Goods Monopolists," *90*, 90, 314–332.

Coase, R. (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15, 143–149.

Fudenberg, D., D. Levine, and J. Tirole (1985): "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information," in *Game-Theoretic Modeling of Bargaining*, ed. by A. Roth. Cambridge University Press.

Gul, F., H. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155–190.

Harberger, A. C. (1954): "Monopoly and Resource Allocation," *American Economic Review*, 44(2), 77–87.

Hicks, J. (1935): "Annual Survey of Economic Theory: The Theory of Monopoly," *Econometrica*, 3, 1–20.

Leibenstein, H. (1966): "Allocative Efficiency vs. 'X-Efficiency'," *American Economic Review*, 56, 392–415.

Sobel, J., and I. Takahashi (1983): "A Multistage Model of Bargaining," *Review of Economic Studies*, 50(3), 411–426.

Stokey, N. L. (1981): "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, 12(1), 112–128.

Tirole, J. (1988): *The Theory of Industrial Organization.* MIT Press, Cambridge, Mass.