Micro Economics Summary | intution | ① Shapes & ② vector & matrix ③

① preference relation : at least as good as (strict vs normal: indifference set exists) $x \succeq y$ | $y \sim x$ (indifference)

② Rationality relation ① Completeness $x, y \in X \Rightarrow x \succeq y$ or $y \succeq x$ ② Transitivity $x, y \in X \Rightarrow x \succeq y, y \succeq z \Rightarrow x \succeq z$ → violate by Change of pref.

③ Choice Preference & Budget set

④ Rational choice to get utility Function $x \succeq y \Rightarrow u(x) \geq u(y)$

utility func : summerizes effect of all attributes

(Resource limitation balances this out)

⑤ choice structure $(B, C(\cdot))$ $\mathcal{B} \subset X$ → every element in B | B: Budget set

$C(B)$: Choice rule non empty set of choice | list of all feasible choice experiment

e.g. $x = \{x, y, z\}$ $B1 = \{x, y\}$ $B2 = \{x, y, z\}$ $C(B1) = \{x\}$ $C2(B2) = \{y\}$

⑥ weak Axiom of revealed Preference (WARP): Consistency

$x, y \in B$, $x \in C(B)$; $x, y \in B'$, $y \in C(B') \Rightarrow x \in C(B')$

⑦ revealed prefered: $x \succeq^* y$ $x, y \in B$, $x \in C(B)$ (WARP ✓)

⑧ choice for rational pref at least as good as: $C^*(B, \succeq) = \{x \in B, x \succeq y \; y \in B\}$

$\Rightarrow x \in C^*(B, \succeq) \Rightarrow x \succeq y$

⑨ choice satisfy y may not mean Rationality: $C(\{x,y\}) = \{x\}$, $C(\{y,z\}) = \{y\}$, $C(\{x,z\}) = \{z\}$ But transitivity ⊗

⑩ All three budget sets & WARP ⇒ Rationality

⑪ Consumption bundle → vector

⑫ walrasian Demand Function (WDF), walras Law (WL): $xp = w$

⑬ Homogeneity of Degree Zero (HDZ) : $x(\alpha p, \alpha w) = x(p, w)$

⑭ wealth Expansion: One variable fix ; $D_w x(p,w) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial w} \\ \frac{\partial x_2(p,w)}{\partial w} \\ \vdots \\ \frac{\partial x_L(p,w)}{\partial w} \end{bmatrix}$ $1 \times n$

(wealth effect)

$D_p x(p,w) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} & \cdots & \frac{\partial x_1(p,w)}{\partial p_L} \\ \frac{\partial x_L(p,w)}{\partial p_1} & \cdots & \frac{\partial x_L(p,w)}{\partial p_L} \end{bmatrix}$ $n \times n$

⑭ WDF & HDZ (differentiate w.r.t α): Elasticity Conversion

$$\sum_{k=1}^{L} \frac{\partial x_\ell(p,w)}{\partial p_k} p_k + \frac{\partial x_\ell(p,w)}{\partial w} w = 0$$ Property of Cournot

matrix: $D_p x(p,w) \cdot p + D_w x(p,w) \cdot w = 0$

$\varepsilon_{\ell k} = \frac{\partial x_\ell(p,w)}{\partial p_k} \cdot \frac{p_k}{x_\ell(p,w)}$ elasticity of demand w.r.t price

$\varepsilon_{\ell w} = \frac{\partial x_\ell(p,w)}{\partial w} \cdot \frac{w}{x_\ell(p,w)}$ elasticity of demand w.r.t. wealth

$\Rightarrow \sum_{k=1}^{\ell} \varepsilon_{\ell k}(p,w) + \varepsilon_{\ell w}(p,w) = 0$ $\ell = 1, \dots, L$ | Equal change in price & wealth ⇒ no change in demand

property of Engel

⑮ WL ⇒ $xp = w \Rightarrow \sum_{\ell=1}^{L} p_\ell \frac{\partial x_\ell(p,w)}{\partial p_k} + x_k(p,w) = 0$ $k = 1, \dots, L$ ≡ Consumption in proportion

differentiate w.r.t p_k ⇒ $p D_p(p,w) + x(p,w)^T = 0$

⑯ WL ⇒ $xp = w \Rightarrow \sum \frac{\partial x_\ell(p,w)}{\partial w} p_\ell = 1 \Rightarrow p \cdot D_w x(p,w) = 0$

⑰ WDF satisfies WARP : $p x(p', w') \leq w$, $x(p', w') \neq x(p,w)$ $\Rightarrow w' < p \cdot x(p,w)$

⑱ slutsky wealth Compensation (SWC) : $\Delta w = \Delta p \cdot x$

Compensated Law of demand (CLD) | Change in budget set

⑲ $x(p,w)$: WDF, HDZ, WL ⇒ WARP ✓ if

$(p' - p)(x(p', w') - x(p,w)) \leq 0$

$\boxed{SWC}$ $dx = [D_p x(p,w) + D_w x(p,w) \cdot x(p,w)^T] \, dp$

interaction of change to wealth & price is chang in demand

Slutsky Matrix (SM) : Substitution Effect :

$[D_p x(p,w) + D_w x(p,w) \cdot x(p,w)^T] \Rightarrow s_{\ell k} = \frac{\partial x_\ell(p,w)}{\partial p_k} + \frac{\partial x_\ell(p,w)}{\partial w} x_k(p,w)$ ②

⑳ Differentiable WDF / WL, HDZ, WA, with SM : $S(p,w)$

satisfies: $v \cdot S(p,w) \cdot v \leq 0$ $\forall v \in \mathbb{R}^L$

→ negative semidefinite (NS)

≡ Diagonal of zero or negative (Neg Sensitivity)

(inferior = Giffen analysis) CLS ⇒ NS

㉑ WDF | differentiable, HDZ, WL ⇒ $p \cdot S(p,w) = S(p,w) \cdot p$ $\forall p, w$

㉒ Rationality needs Symmetry as well ~~transitivity ⇒ Desirability~~

$L = 2$ symmetry Exists

㉓ Non Satiation : ∃ neighbor of y, called x that $y \succ x$

㉔ monotonicity : $u(x) > u(y)$ if $x \gg y$ increasing func

if monotone ⇒ locally non satiated ⇒ Desirability

㉕ Upper/Lower Counter ($\succeq$) : UC / LC

㉖ Convexity assumption

$\succeq$ convex if UC convex : $x, y \succeq x \Rightarrow \alpha y + (1-\alpha) z \succeq x$

Diminishing marginal rate of substitution (DMRS):

Diversification

㉗ Homothetic if indifference set proportional expansion

$x \sim y \Rightarrow \alpha x \sim \alpha y$

㉘ Quasilinear ① parallel displacement $x \sim y \Rightarrow (x + \alpha e_1) \sim (y + \alpha e_1)$

$e_1 = (1, 0, \dots, 0)$

② Good 1 desirable: $x + \alpha e_1 \succ x$ ($\forall x, \alpha > 0$)

㉙ Continuous if preserves under limits

$\{(x^n, y^n)\}_{n=1}^{\infty}$, $x^n \succeq y^n$ $x = \lim_{n \to \infty} x^n$ $y = \lim_{n \to \infty} y^n$

$\Rightarrow x \succeq y$ (closed UC/LC, diagonal ray)

㉚ Continuity of $\leq$ ⇒ utility func exists (Restriction duality)

㉛ Differentiability (Leontiff func: problem of non differentiability)

㉜ strict / Convexity of preference = strict / Quasiconcavity of utility func: $u(\alpha x + (1-\alpha) y) \geq \min\{u(x), u(y)\}$

㉝ $\succeq$ homothetic ⟺ utility: homogeneous of degree one $\$ u(\alpha x) = \alpha u(x)$

㉞ $\succeq$ quasilinear if admit utility func of form:

$u(x) = x_1 + \phi(x_2, \dots, x_L)$

㉟ Ordinal properties of $u(\cdot)$: ① Increasingness ② Quasiconcavity

㊱ ① Rational ② Continuous ③ locally non satiation preference

⇒ $u(\cdot)$ Continuous

㊲ utility maximization problem (UMP): $\max_{x \geq 0} u(x)$ s.t. $p \cdot x \leq w$ (Compact set)

Continuous $u(\cdot)$ ⇒ solution

㊳ $u(\cdot)$ continuous, local non satiated $\succeq$ ⇒ W.D.C : (Correspondence)

HD(Z), W.L, Convexity / uniqueness ⇒ $u(\cdot)$ Quasiconcave

Intuition: ① Reveal preference ② Consistency (WARP)

③ utility func Exist result of rational choice

④ utility maximization problem & Demand

⑤ Demand & Competition (all about shapes)

㊴ Kuhn Tucker Condition: solution to (UMP). Lagrange multiplier

$Du(x^*(p,w)) = \lambda p$

$Du(x) = \left[\frac{\partial u(x)}{\partial x_1}, \frac{\partial u(x)}{\partial x_2}, \dots, \frac{\partial u(x)}{\partial x_L} \right]$

Micro Economics Summary

(40) marginal rate of substitution of two goods (MRS)

$$\frac{\frac{\partial u(x^*)}{\partial x_\ell}}{\frac{\partial u(x^*)}{\partial x_k}} = \frac{\partial P_\ell}{\partial P_k} \Rightarrow \nabla u(x(P,w))\, D_w x(P,w) = \lambda P\, D_w x(P,w) = \lambda$$

(41) Indifference curve characteristics:

① negative slope (substitution effect) + Diminishing marginal utility

② Linear: perfect substitute 

③ L shape: perfect complement 

(42) Cobb Douglas utility function (Intution: Decomposition to subelements)

$$u(x_1,x_2) = k\, x_1^{\alpha} x_2^{1-\alpha}$$

(43) Constant elasticity of substitution utility (CES):

$$u(x) = (\alpha_1 x_1^{\alpha} + \alpha_2 x_2^{\alpha})^{1/\alpha}$$

$\alpha \to 0$ Cobb douglas simulator

$\alpha \to -\infty$ Leontef simulator

$\alpha \to 1$ Linear simulation

(44) Demand func. properties: ① w^L ② HDZ ③ uniqueness ④ Continuity ⑤ monotonicity ⑥ Quasiconvexity

(45) Homogeneity of Degree 1 (HDO) on w: $\varepsilon_{\ell w} = 1 \Rightarrow a = \frac{1}{w} \Rightarrow D_w x$ = func of P only

$\Rightarrow$ wealth expansion passing $x(p,1)$

(46) Revealed preferred $\underline{a}$ over $\underline{b}$:

① wealth of an option greater than wealth of better modified demand a

② modified price, has greater wealth $\underline{a}$ compared to wealth $\underline{b}$

Summary: under current price I went for x, since based on cur. it uses most of my money; therefore under new price since the other demand is selected, this demand should have required more wealth, hinder me to select it, and select b instead.

(47) (HDF) is HDZ) & (EF) (Expenditure function) (HDF) Hicksian Demand Function: minimize expenditure

$$e(p,u) = p.x^* = p.h(p,u) \Rightarrow h(p,u) = \nabla_p e(p,u)$$

(Conshots) $h(p,\bar u) = \arg\min_{x} \sum P_i x_i$ subject to $u(x) \geq \bar u$

$$S(p,u) = D_p h(p,u) = D_p^2 e(p,u)$$

$D_p h(p,u)$: Symmetric, NSD; since $\underline{e}$ is Concave

utility max $\Rightarrow$ WDF

(48) Cross price effect:
$\frac{\partial h_\ell(p,u)}{\partial p_k} > 0$ substitution
$\frac{\partial h_\ell(p,u)}{\partial p_k} < 0$ Complementary

(49) $h_\ell(p,u) = x_\ell(p, e(p,w))$

$$\frac{\partial x_\ell(p,e(p,w))}{\partial p_k} = \frac{\partial h_\ell(p,u)}{\partial p_k} + \left[\frac{\partial x_\ell(p,e(p,w))}{\partial w}\right]\left[-x_k(\bar p,\bar w)\right]$$

Price effect = Substitution effect + wealth effect

(50) (IDF) & WDF: $\bar u = v(\bar p, \bar w) \Rightarrow x_\ell(\bar p, \bar w) = -\dfrac{\frac{\partial v(\bar p,\bar w)}{\partial p_\ell}}{\frac{\partial v(\bar p,\bar w)}{\partial w}}$ Roy's Identity

↳ Indirect demand function: gain utility from Price & wealth

? IUF

welfare change: ~~$e(p, v(p,w))$~~ $= v(p',w) - v(p^2,w)$

EF is IUF (strictly increasing)

(51) $e(\bar p, v(p,w))$ money metric indirect utility function

$P_0 \to P_1$: Equivalent variation (E.V): $e(p^0,u') - w$

(52) Compensation Variation (C.V) = $w - e(p', u^0)$

Parallel Streight wealth Expansion path Condition (Gorman form) $v_i(p, w_i) = a_i(p) + b(p) w_i$

(53) Gorman function: $v(p,w) = a(p) + b(p)w$: IUF given:

① $a(p)$ Constant
② Hom. Deg. 1
③ $b(p) \geq 0$
④ $\nabla b(p) \leq 0 \quad \forall p > 0$

(54) production plan/vector: $y = (y_1, y_2, \ldots, y_L) \in R^L$

(55) Transformation function: $F(\cdot)$: $Y = \{y \in R^L : F(y) \leq 0\}$

(56) Transformation Frontier: $Y = \{y \in R^L : F(y) = 0\}$

(57) free disposal: $y \in Y,\ y' \leq y \Rightarrow y' \in Y$

(58) Additivity (Free entry): $y \in Y,\ y' \in Y \Rightarrow y + y' \in Y$

(59) profit function: $\pi(p) = p \cdot y(p)$

(60) Single output $f(z)$: P – output price, w – input price vector $\Rightarrow$ Profit = $p.f(z) - wz$

(61) marginal Rate of transformation:

$$MRT_{\ell k} = \frac{\frac{\partial F(y)}{\partial y_\ell}}{\frac{\partial F(y)}{\partial y_k}} = \frac{\partial y_k}{\partial y_\ell}$$ Diminishing Product Production

(62) irreversibility: $y \in Y,\ y \neq 0 \Rightarrow -y \notin Y$

intuition: Math is tool & intuition is more important

(63) Convex Cone: $y \in Y,\ y' \in Y,\ \alpha, \beta > 0 \Rightarrow \alpha y + \beta y' \in Y$

(64) $\frac{\partial f(y^*)}{\partial z_\ell} p = w_\ell$ & $MRT_{\ell k} = \dfrac{\frac{\partial f(y^*)}{\partial z_\ell}}{\frac{\partial f(y^*)}{\partial z_k}} = \dfrac{w_\ell}{w_k}$

(65) $q = (q_1, q_2, \ldots, q_m) \in R_+^m$ output

$z = (z_1, z_2, \ldots, z_{L-m}) \in R_+^{L-M}$: inputs

$Y = \{(z,q) : F(-z,q) \leq 0$ for $z \in R_+^{L-M},\ q \in R_+^L\}$

Production function $f(z)$: maximum output for given input

(66) Non increasing return to scale: $y \in Y, \alpha \in [0,1] \Rightarrow \alpha y \in Y$

(67) Profit Maximization Problem (PMP)

① price vector $(p_1, p_2, \ldots, p_\ell) \gg 0$

② Firms are price takers

③ Firm objective: $\max_y p.y = \sum_{i=1}^{L} p_\ell y_\ell$ s.t $y \in Y$

$L = py - \lambda F(y)$, $F(y) \leq 0 \Rightarrow p_\ell = \lambda \frac{\partial F(y^*)}{\partial y_\ell}$

(68) $$\frac{\frac{\partial f(y^*)}{\partial z_\ell}}{w_\ell} = \frac{\frac{\partial f(y^*)}{\partial z_k}}{w_k}$$

(69) $MRT_{\ell k} = \dfrac{\frac{\partial F(\bar z)}{\partial z_\ell}}{\frac{\partial F(\bar z)}{\partial z_k}}$, $q = f(\bar z)$

marginal rate of technical substitution

(70) non decreasing return to scale: $y \in Y, \alpha \in [1,\infty) \Rightarrow \alpha y \in Y$

(71) Profit function properties:

① $\pi(\cdot)$ – HD.1

② $\pi(\cdot)$ Convex in p

③ If production function is Convex:
$Y = \{y \in R^L, p.y \leq \pi(p)\ \forall p \gg 0\}$

④ $y(p)$ H.D.Z in p

(72) production function properties:

① Y is non empty

② Y is Closed (boundary inclusion)

③ No free lunch: if $y \in Y$, $y \geq 0 \Rightarrow y = 0$

④ 0 is part of Y (inaction)

Micro Economics Summary

(73) Constant return to Scale: $y \in Y \quad \alpha \in [0,\infty) \rightarrow \alpha y \in Y$

(74) $MRT = \dfrac{\frac{\partial F(y^*)}{\partial y_\ell}}{\frac{\partial F(y^*)}{\partial y_k}} = \dfrac{P_\ell}{P_k}$ PMP: profit maximization problem

(75) y: Strictly/Convex $\Rightarrow y(P)$: single value/Convex

(76) $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$ Convexity

(77) (Hotelling Lemma) if $y(P)$: Single value $\Rightarrow$ Production function $\pi(P)$ is defferentiable and $\nabla\pi(\bar{p}) = y(\bar{p})$

(78) If $y(p)$ is differentiable, then $D_y(\bar{p}) = D^2\pi(\bar{p})$ is ① symmetric ② positive semidefinite (diagonal $\in \mathbb{Z}^+$)

with $D_y(\bar{p}) \cdot \bar{p} = 0$, $dP\, D^2\pi(\bar{p}) dP \geq 0$, $(p'-p)(y(p')-y(p)) \geq 0$

(79) $\nabla\pi(p) = y(p) + p \cdot D_y(p)$ HDZ in P: $y(\alpha p) = y(p)$

(80) Cost minimization problem (CMP)

z: input, w: input price $\mapsto \begin{cases} \min & w \cdot z \\ \text{s.t.} & f(z) \geq P \end{cases}$ $c(w,q)$: Cost func; $z(w,q)$: Conditional factor demand

$\Rightarrow$ ① $w_\ell = \lambda \dfrac{\partial F(z^*)}{\partial z_\ell}$ ② $\dfrac{\partial z(w,q)}{\partial q} \cdot w = \dfrac{\partial c(w,q)}{\partial q}$

(81) $f(z(w,q)) = q \Rightarrow \dfrac{\partial c(w,q)}{\partial q} = \lambda$

(82) properties of $C(\cdot)$:
① $c(w,q)$: H.D.1 & non dec. in q
② $c(w,q)$: Concave (w)
③ $\{z \geq 0 \mid f(z) \geq q\}$: Convex $\forall q$
$\Rightarrow Y = \{(-z, q): w z \geq c(w,q) \ \forall w \gg 0\}$
④ $z(w,q) = H.D.Z(w)$
⑤ $\{z \geq 0 : f(z) \geq q\}$: Strictly Convex
$\Rightarrow z(w,q)$: single value/Convex
⑥ (Shepard's Lemma) $z(\bar{w},q)$: Single value
$\Rightarrow c(\cdot)$ differentiable & $\nabla_w c(\bar{w},q) = z(\bar{w},q)$
⑦ $z(\cdot)$ differentiable at $\bar{w} \Rightarrow$
$D^*_m z(\bar{w},q)$: asymmetric neg. semidef.
⑧ $f(\cdot)$: H.D.1 & $z(\bar{w},q) \Rightarrow c(\cdot)$ & $z(\cdot)$: HD1 (q)
⑨ $f(\cdot)$: Concave $\Rightarrow c(\cdot)$: Convex (q)

(83) Coup Douglas prod. Func: $F(z_1, z_2) = z_1^\alpha z_2^\beta$

(84) $p = \dfrac{\partial c(w,q^*)}{\partial q}$: $MC = MR$

(85) Proc. of Profit maximization & Cost minimization:
(1) $F(\cdot)$ (2) $\dfrac{\frac{\partial f(y^*)}{\partial z_\ell}}{w_\ell} = \dfrac{\frac{\partial f(y^*)}{\partial z_k}}{w_k}$ (3) $z(w_i's, q)$
(4) $c' = zw$, $c' = p$ (5) analysis

intuition: ① First procurement department tells the ~~point~~ cost of each good production
② then marketing department or production planning dept. says how much to produce

(86) Average Cost (AC) $= C(\bar{w}, q)/q$

(87) Max (AC): 1st order Cond: $AC = MC$

(88) Shortrun: always something is fixed

(89) Long run: Reshuffling

(90) profit maximization: efficient $\neg\exists y' > y \in Y$

(91) Efficiency | Convex $\rightarrow$ profit max

(92) economic allocation: $(x_1, x_2, \dots, x_I, y_1, y_2, \dots, y_J)$ $x_i \in X_i$ $y_j \in Y_j$

(93) $(x_1, x_2, \dots, x_I, y_1, y_2, \dots, y_J)$ is feasible if: $\sum_{i=1}^{I} x_i \leq w_j + \sum_{j=1}^{J} y_j$ intuition: usage of good L < production of good L

(94) $(x_1, x_2, \dots, x_I, y_1, y_2, \dots, y_J)$ pareto optimal if $(x_1', x_2', \dots, x_I', y_1', y_2', \dots, y_J')$:
$\nexists (x_1', x_2', \dots, x_I', y_1', y_2', \dots, y_J')$ $\forall i = 1 \dots I$ $u_i(x_i') \geq u_i(x_i)$ $\exists i: u_i(x_i') > u_i(x_i)$

(95) I: Consumer $i = 1, \dots, I$ $x_i(x_{1i}, \dots, x_{Li}) \in R^L$ Consumer I's bundle
J: Firms $j = 1, \dots, J$ $Y_j = (y_{1j}, \dots, y_{Lj}) \in Y_j$
L: goods $\ell = 1, 2, \dots, L$ Production plan of firm j
U_i - Consumer utility
$(y_1', y_2', \dots, y_J') \in R^{LJ}$: production plan of all J firms
$w_\ell \geq 0$, $\ell = 1, 2, \dots, L$: Initial endowment of good ℓ
$w_\ell + \sum_{j=1}^{t} y_{\ell j}$: total amount of good L

(96) Equilibrium Condition:
(1) profit maximization: for each firm y_j^* solves:
$\max p^* \cdot y_j \quad y_j \in Y_j$
(2) utility maximization for each Consumer
I, x^* solves: $\max u_i(x_i) \quad x_i \in X_i$
s.t. $p^* \cdot x_i \leq p^* w_i + \sum_{j=1}^{J} \theta_{ij}(p^* \cdot y_j^*)$
③ market clearing for each good: $\ell = 1, \dots, L$
$\sum_{i=1}^{I} x_{\ell i}^* = w_\ell + \sum_{j=1}^{J} y_{\ell j}^*$

(97) First order Condition (O.C):
$p'(q^m) \cdot q^m + p(q^m) \leq c'(q^m) = c'(q^m)$ if $q^m > 0$
$p'(q^m) \cdot q^m + p(q^m) = c'(q^m)$: $MC = MR$

(98) Bertrand model: $P_1 = P_2 = C$

(99) Cournot model: $p'(q_1^* + q_2^*) q_2^* + p(q_1^* + q_2^*) \leq c$ $= c$ if $q_2^* > 0$

(100) monopoly: $p'(q^m) q^m + p(q^m) = c$

(101) product differentiation: $x_1(P_1, P_2) = \begin{cases} 0 & \text{if } P_1 > P_2 + t \\ (t + P_2 - P_1)M & \text{if } P_1 \in (P_2 - t, P_2 + t) \\ M & \text{if } P_2 > P_1 + t \Rightarrow P_1 < P_2 - t \end{cases}$

$\bar{P}_{-j} = c + 3t$

$b(\bar{P}_{-j}) = \begin{cases} \bar{P}_{-j} + t & \text{if } \bar{P}_{-j} \leq c - t \\ \dfrac{t + \bar{P}_{-j} + c}{2} & \text{if } \bar{P}_{-j} \in [c - t, c + 3t] \\ \bar{P}_{-j} - t & \text{if } \bar{P}_{-j} \geq c + 3t \end{cases}$ $P_j = \dfrac{t + P_{-j} + c}{2}$
$P^* = P_j = P_{-j} = \dfrac{t + P^* + c}{2}$
$P^* = c + t$

(102) $r_A(x) = -\dfrac{u''(x)}{u'(x)}$ (absolute risk aversion) $r_R(x) = -\dfrac{x u''(x)}{u'(x)} = x \cdot r_A(x)$ Relative Risk aversion

(103) $U(F) = \int u(x) dF$
$V(C(F,u)) = \int u(x) dF(x)$
$\begin{cases} u(\int x \, dF(x)) = \int u(x) dF(x) & \text{risk neutral} \\ u(\int x \, dF(x)) \geq \int u(x) dF(x) & \text{risk averse} \\ u(\int x \, dF(x)) > \int u(x) dF(x) & \text{Strictly risk averse} \end{cases}$

MicroEconomics summary

(109) Firm's profit : $\pi(q,\alpha)=p(\alpha)\cdot q-C(q,\alpha)$

FOC (profit max): $p(\alpha)=C_q(q,\alpha)$

$q_*(\alpha)$: Firm's reduced-form profits : $\pi_*(\alpha)=\pi(q_*(\alpha),\alpha)$

Envelope theorem : $\left.\frac{\partial \pi(q,\alpha)}{\partial q}\right|_{q=q_*(\alpha)}=0$:

$\pi_*'(\alpha)=\partial\pi(q_*(\alpha),\alpha)/\partial\alpha=p'(\alpha)\cdot q_*(\alpha)-C_\alpha(q_*(\alpha),\alpha)$

By assumption $C_q(\cdot)\le 0$

(110) lottery $\begin{cases} A & q \\ D & 1-q \end{cases}$ (prob) unnecessary evacuation: 15%

ⓐ assign utility: $u_A=1$ $\quad u_B=p\cdot 1+(1-p)\cdot 0=p$

$u_D=0$ $\quad u_C=q\cdot 1+(1-q)\cdot 0=q$

ⓑ prob dist. (Criterion 1): $(p_A,p_B,p_C,p_D)=(0.891,0.098,0.009,0.001)$

(111) ⓐ $\sqrt{z_1+z_2}$

ⓑ $\sqrt{\min(z_1,z_2)}$

ⓒ $(z_1^\rho+z_2^\rho)^{1/\rho}$

$\boxed{\text{intuition}}$ double story : recall ① intuitive story ② math story & curve story

$c(w,q)=\begin{cases} q^2 w_1 & \text{if } w_1<w_2 \\ z_1w_1+z_2w_1 & \text{if } w_1=w_2 \text{ when } q=\sqrt{z_1+z_2} \\ q^2 w_2 & \text{if } w_1>w_2 \end{cases}$

$y(p)=\begin{cases} (\frac{1}{2w_1}{}^2, 0, \frac{1}{2w_1}) & \text{if } w_1<w_2 \\ (-z_1,-z_2,\frac{1}{2w_2}) & \text{if } w_1=w_2 \quad q=\sqrt{z_1+z_2} \\ (-\frac{1}{2w_2}{}^2, 0, \frac{1}{2w_2}) & \text{if } w_1>w_2 \end{cases}$

$c(w,q)=(w_1+w_2)q^2$ $\quad AC_j(q_j)=\alpha+\beta_j q_j \quad MC_j(q_j)=\alpha+2\beta_j q_j$

$q^q=\frac{1}{2(w_1+w_2)}$

(112) $\frac{\partial z}{\partial w}(p,w)=\frac{1}{p}[D^2 f(z(p,w))]^{-1}$

$\frac{\partial z}{\partial p}(p,w)=-\frac{1}{p}[D^2 f(z(p,w))]^{-1}\nabla f(z(p,w))$

(113) $\Sigma_j q_j=q$ and $MC_j(q_j)=MC_{j'}(q_{j'})$

$q_j=(q/\beta_j)/(\Sigma_h 1/\beta_h)$

$\boxed{\text{Sample Exam}}$ ① $q=\sqrt{z_1+z_2}$ ⇒ substitution of input z_1, z_2

Ⓘ ⇒ procurement Dept select based on price of input w_1, w_2 one with q^2 amount (given output level) ⇒ $\boxed{\text{Cost minimizat.}}$

Ⓘⓘ now given input marketing Dept maximizes profit

$\boxed{\text{Profit maximizat.}}$ ⇒ F.OC π : $q-q^2w_1=0$ $\quad w_1<w_2$ ⇒ $q=\frac{1}{2w_1}$, $z_1=\frac{1}{4w_1^2}$

⇒ production frontier: $(-\frac{1}{4w_1^2}, 0, \frac{1}{2w_1})$ = (input, output)

Ⓘⓘⓘ normal solution: lagrange multiplier: $\frac{\partial f}{\partial z_1}/\frac{\partial f}{\partial z_2}=\frac{w_1}{w_2}$

$\boxed{\text{optimization problem}}$

ⓒ $q=(z_1^\rho+z_2^\rho)^{1/\rho}$ ⇒ $\begin{cases} q^\rho - z_1^\rho - z_2^\rho=0 \\ \max_{w_1,w_2,q} pq-z_1w_1-z_2w_2 \end{cases}$ $\boxed{\text{Optimization problem}}$

⇒

⇒ write lagrangian for differentiation with r.t w_1, w_2, q, then find λ, and put back

$\boxed{\text{intuition:}}$ translation b/w math, intuitiation, and what people say

③ $\frac{\partial z_\ell(w,q)}{\partial q}>0 \Leftrightarrow \frac{\partial(\frac{\partial c(q)}{\partial q})}{\partial w_\ell}>0$

integration by parts of right side : $\frac{\partial(\frac{\partial w_\ell}{\partial q} z_\ell)}{\partial w_\ell}=\frac{\partial^2 w_\ell}{\partial w_\ell \partial q}z_\ell+\frac{\partial z_\ell}{\partial w_\ell}\cdot\frac{\partial w_\ell}{\partial q}$

$\frac{\frac{\partial z_\ell}{\partial q}w_\ell}{\partial w_\ell}=\frac{\partial^2 z_\ell}{\partial q\partial w}+\frac{\partial w_\ell}{\partial w_\ell}\cdot\frac{\partial z_\ell}{\partial q}$ all terms except left side zero

④ Demand functions: $\begin{cases} h(p,u) & \text{hicsian demand func} : \nabla_p e(p,u) \text{ expenditure} \\ x(p,w) & \text{walrasiand} \\ v(p,w) & \text{indirect utility func} \end{cases}$

$h(p,u)$: solution to cost minimization (given utility)

$x(p,w)$: solution to utility maximization (given wealth)

e.g $\boxed{u=2x_1^{1/2}+4x_2^{1/2}}$ Ⓘ

$\begin{cases} \frac{\partial u}{\partial x_1}=\lambda p_1 \\ \frac{\partial u}{\partial x_2}=\lambda p_2 \end{cases}$ ⇒ $\begin{cases} x_1^{-1/2}=\lambda p_1 \\ 2x_2^{-1/2}=\lambda p_2 \\ p_1x_1+p_2x_2=w \end{cases}$ ⇒ $x_1=(\frac{p_2}{2p_1})^2 x_2$ ①

$\xrightarrow{+①}$ $p_1(\frac{p_2}{2p_1})^2 x_2+p_2x_2=w$ ⇒ x_2, x_1 ✓

ⓐ ⇒ $x_1(p,w)$, $x_2(p,w)$ ≡ walrasian

ⓑ to find hicksian put ① into Ⓘ Equation ⇒ $h_1(p,u)$ ✓ $h_2(p,u)$ ✓

ⓒ $e(p,u)=p_1h_1(p,u)+p_2h(p,u)$

ⓓ $v(p,w)=p_1x(p,w)+p_2x(p,w)$ $\boxed{\text{Roy's identity}}$: $x(p,w)=\frac{\frac{\partial v(p,w)}{\partial p}}{\frac{\partial v(p,w)}{\partial w}}$

⑤ slutsky matrix symmetric ⇒ when two demand x_1, x_2 available to find params:

$\frac{\partial D x_1}{\partial p_1}+\frac{\partial D x_1}{\partial w}x_1=\frac{\partial D x_2}{\partial p_2}+\frac{\partial D x_2}{\partial w_2}x_2$

⑥ $q=\min(x_1,x_2)$: Leontif func ⇒ $x_1=x_2$ indiff.

⑦ 2 Firms Cournot $c_1>c_2$

$P(q)=a-bq \quad a>c_1$

Nash) $\pi_1=(a-b(q_1+q_2))q_1-q_1c$

$\pi_2=(a-b(q_1+q_2))q_2-q_2c$

⇒ FOC : $\frac{\partial\pi}{\partial q_1}=a-2bq_1-bq_2-c_1=0$

$\frac{\partial\pi_2}{\partial q_2}=a-2bq_2-bq_1-c_2=0$

Solu: $q_1=\frac{a-2c_1+c_2}{3b}$, $q_2=\frac{a-2c_2+c_1}{3b}$

Firm 2 remains, firm 1 higher cost ⇒ out

$a-2c_1+c_2>0$ ⇒ $a>2c_1-c_2$

sensitivity to cost: $\frac{\partial q_1}{\partial c_1}=-2/3b$ $\quad \frac{\partial q_2}{\partial c_1}=1/3b$

evaluate $\frac{\partial\pi_1}{\partial c_1}<0$ $\quad \frac{\partial\pi_2}{\partial c_1}>0$

⑧ when investment in cost reduction : I

⇒ objective function (optimization) : $\pi=p(q)\cdot q-c(I)q-I$

F.O.C $\frac{\partial\pi}{\partial q}=p'(q)\cdot q-c(I)+p(q)=0$, $\frac{\partial\pi}{\partial I}=c'(I)\cdot q=-1$

Socially optimal (sell to everybody with price: cover underline)

obj fun : $\int_0^q p(x)\,dx-c(I)\cdot q-I$ ⇒ $\frac{\partial obj}{\partial q}=0 \Rightarrow p(q^*)=c(I)$

$\frac{\partial obj}{\partial I}=0 \Rightarrow c'(I)\cdot q=-1$

MicroEconomics summary

$\Rightarrow$ ① $q^m < q^*$: since monopoly: $p'(q)\cdot q + p(q^*) = c(I)$

neg price sensitivity $p'(q) < 0$, $p'(q)\cdot q < 0$

$\Rightarrow$ Social optimal: $p(q^*) = c(I)$

② Given $q^m < q^* \Rightarrow c'(I) = -\frac{1}{q}$ Both $\Rightarrow -c'_m(I_m) > -c'_{so}(I_{so})$

$\Rightarrow c'_{so}(I_m) < c'_{so}(I_{so})$

③ $c''(I) > 0 \Rightarrow I_m < I_{so}$

---

9 two period. q c_1 $c_2 = c_1 - mq_1$ $a > c$ $b > m$ (12.B.8)

$\pi = (a - bq_1)\cdot q_1 - c_1 q_1 + (a - bq_2)\cdot q_2 - (c_1 - mq_1) q_2$

$\frac{\partial \pi}{\partial q_1} = a - 2bq_1 - c_1 + mq_2 = 0 \qquad \frac{\partial \pi}{\partial q_2} = a - 2bq_2 - c_1 + mq_1 = 0$

intuition: when π symmetric w.r.t q_1 and q_2 then just put $q_1 = q_2$ in one eq. and calculate

$\Rightarrow q_1 = q_2 = \frac{c_1 - a}{m - 2b}$

ⓑ Social Benevolent agent:

$\int_0^{q_1} (a - bq_1) dq - c_1 q_1 + \int_0^{q_2} (a^2 - bq) dq - (c_1 - mq_1) q_2$

$= a(q_1 + q_2) + b/_2 (q_1^2 + q_2^2) - c_1 q_1 - (c_1 - mq_1) q_2$

$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow \boxed{q_1 = \frac{c_1 - a}{m - b}}$

MC = MR
if we assume price = surplus so mq_1 is saving of company in first period

ⓒ if only first period effect: $\pi = \int_0^{q_1} (a - bq) dq - c_1 q_1$

$\Rightarrow q_1 = \frac{a - c_1}{b} > q_1^m = \frac{a - c}{2b - m}$ since $b > m$

---

10 $x_1 = a - \theta_1 p$ Group 1 $\quad \theta_2 < \theta_1 \quad$ C: prod. cost

$x_2 = a - \theta_2 p$ Group 2

ⓐ MC = MR (for Competition) : $c = p_1 = p_2$

$\Rightarrow \begin{cases} x_1 = a - \theta_1 c \\ x_2 = a - \theta_2 c \end{cases}$

ⓑ monopolist with no price discrim.

$p_1 = p_2 \Rightarrow \pi = (2a - (\theta_1 + \theta_2)p)(p - c)$

$(2a - (\theta_1 + \theta_2)p)(p - c) \Rightarrow$ FOC: $p = \frac{a}{\theta_1 + \theta_2} + \frac{c}{2}$

discrim better since lower price for price sensitive

ⓒ discrimination $\max_{p_m, p_w} (a - \theta_m p_m)(p_m - c) + (a - \theta_w p_w)(p_w - c)$

$p_m = \frac{a + c\theta_m}{2\theta_m} \qquad p_w = \frac{a + c\theta_w}{2\theta_w} \Rightarrow q_m = (a - c\theta_m)/2$

$q_w = (a - c\theta_w)/2$ welfare in price disc. higher

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11 $u(x) = \varphi(x) + m \quad P_m = 1 \quad P_\ell = P \quad \frac{\partial u}{\partial m} = 1$ (two goods)

ⓐ $\Rightarrow \frac{\frac{\partial u}{\partial \ell}}{\frac{\partial u}{\partial m}} = \frac{P_\ell}{P_m} \Rightarrow \boxed{x = \frac{\beta}{P}}$

ⓑ Firm: $c(q) = \sigma q$ Competition: $MC = MR \Rightarrow p = c$

$\Rightarrow \sigma = p$

ⓒ Equilibrium $x = \frac{\beta}{\sigma}$, $p = \sigma$

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12 Stackleberg: (quantity leadership) $\rightarrow$ know reaction of F2

$P(Q) = a - b(q_1 + q_2)$ Backward induction: reaction of firm 2

assume $c = 0$

① $\max_{q_2} (a - b(q_1 + q_2)) q_2 \Rightarrow$ reaction function (given q_1) = FOC

FOC: $-bq_2 + (a - b(q_1 + q_2)) = 0 \Rightarrow R_2(q_1) = q_2 = \frac{a - bq_1}{2b}$

② $\max_{q_1} (a - b(q_1 + q_2)) q_1 \Rightarrow \max_{q_1} (a - b(q_1 + \frac{a - bq_1}{2b})) q_1$

$\Rightarrow \frac{aq_1}{2} - \frac{bq_1^2}{2} \Rightarrow q_1^s = \frac{a}{2b} \Rightarrow$ substitute back to firm 2

reaction function: $q_2^s = \frac{a}{4b}$, $q = q_1^s + q_2^s = \frac{3a}{4b}$

Price leadership: like bertrand $P_1 = P_2 = C$

intuition:
① all is about information set, what I know and what I don't!
② at each level assume one value known $\equiv$ Serializing & Conditioning & solve for it
③ optimization problem: just take first order condition, plug in prob for Dynamic programming
④ for Game theory forms do Backward induction (recursive thinking)

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13 Product differentiation: (Hoteling, linear city model)

$c > 0$, Cost of Purchase: $p_j + t/_2 (2d) = p_j + td$

travel cost 4 distance round trip

indifference $\hat{z}$: $p_1 + t\hat{z} = p_2 + (t - \hat{z}) \Rightarrow \hat{z} = \frac{t + p_2 - p_1}{2t}$

m: market size $\Rightarrow x_1(p_1, p_2) = \begin{cases} 0 & \hat{z} < 0 \\ \hat{z}m & \hat{z} \in [0, 1] \\ M & \hat{z} > 1 \end{cases}$

intuition: ① Condition on distance ② Calc Quant based on distance

$x_1(p_1, p_2) = \begin{cases} 0 & p_1 > p_2 + t \\ \frac{(t + p_2 - p_1)m}{2t} & p_1 \in [p_2 - t, p_2 + t] \\ M & p_2 > p_1 + t \Rightarrow p_1 < p_2 - t \end{cases}$

Firm j's best respons: $\max_{p_j} (p_j - c)(t + \bar{p}_{-j} - p_j)\frac{m}{2t}$

$p_j = [\bar{p}_j - t, \bar{p}_j + t]$

$t + \bar{p}_{-j} + c - 2p_j \begin{cases} \leq 0 & p_j > \bar{p}_j - t \\ = 0 & p_1 \in [p_2 - t, p_2 + t] \\ \geq 0 & p_j > \bar{p}_j + t \Rightarrow p_1 < p_2 - t \end{cases}$

intuition $\Rightarrow$ Solve for Boundary

$t + \bar{p}_{-j} + c - 2(\bar{p}_{-j} + t) = 0 \Rightarrow \bar{p}_{-j} = c - t$

$t + \bar{p}_{-j} + c - 2(\bar{p}_{-j} - t) = 0 \Rightarrow \bar{p}_j = c + 3t$

$b(\bar{p}_j) = \begin{cases} \bar{p}_{-j} + t & \bar{p}_j > \bar{p}_j \leq c - t \\ \frac{t + \bar{p}_j + c}{2} & \bar{p}_{-j} \in [c - t, c + 3t] \\ \bar{p}_{-j} - t & \bar{p}_{-j} \geq c + 3t \end{cases}$

$\Rightarrow p_j = \frac{t + p_j + c}{2}$

$\Rightarrow p^* = p_j = p_{-j} = \frac{t + p^* + c}{2}$

$\Rightarrow p^* = c + t$

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14 Compound lottery: $L_k = (p_1^k, p_2^k, \dots, p_N^k)$ $k = 1, 2, \dots, K$

$\alpha_k \geq 0$, $\sum_k \alpha_k = 1$: $(L_1, L_2, \dots, L_k; \alpha_1, \alpha_2, \dots, \alpha_k)$

Yields simple lottery L_k with Prob. α_k

$L = (p_1, p_2, \dots, p_N) \Rightarrow p_n = \alpha_1 p_n^1 + \alpha_2 p_n^2 + \dots + \alpha_k p_n^k$

Consider $u(\int x\, dF(x))$, $\underbrace{\int u(x) dF(x)}_{\text{uncertainty}}$

(15) Insurance Problem:

initial wealth: w — possibility of loss: D (amount)

probability of loss: π — ~~percentage~~ amount of coverage: α (e.g. \$10)

Cost/Coverage unit: q

obj func $\Rightarrow \max_{\alpha\geq 0} (1-\pi)\,u(w-\alpha q) + \pi\, u(w-\alpha q - D + \alpha)$

$\Rightarrow FOC(\alpha^*)$: $q(1-\pi)u'(w-\alpha^* q) + \pi(1-q)u'(w-\alpha q - D + \alpha^*) = 0$

Fairness Condition: $q = \pi \times \$1 + (1-\pi)\times \$0 = \pi$

$\boxed{\text{intuition}}$: $\boxed{\text{amount I pay = Expected amount I get}}$

$\Rightarrow u'(w-D+\alpha^*(1-\pi)) = u'(w-\alpha^*\pi) \Rightarrow D+\alpha^*(1-\pi) = \alpha^* \Rightarrow \alpha^* = D$

(16) demand func.: $x(p) = A - Bp$

(a) Cost func.: $c(q) = k + \alpha q + \beta q^2$ $\quad \alpha,\beta > 0$

J firms $\boxed{\text{Competition}}$

Profit max ① $p = c'(q) = 2\beta q + \alpha$

market clearing ② $A - Bp = Jq$

Free entry ③ $pq - c(q) = pq - k - \alpha q - \beta q^2 = 0$

$\Rightarrow q^* = \sqrt{k/\beta}$ $\quad p^* = \alpha + 2\sqrt{k\beta}$ $\quad J^* = (A - \alpha B)\sqrt{\beta/k} - 2\alpha\beta$

aggreg output: $Q^* = J^* q^* = A - \alpha\beta - 2\alpha\beta\sqrt{k/\beta}$

(b) ignore $J \in \mathbb{Z}^+$ $\Rightarrow$ How change vary with A $\Rightarrow$ diffr w.r.t A

treat func(A) $\Rightarrow$ ① $p'(A) = 2\beta q'(A)$ ② $1 - Bp'(A) = Jq'(A)$

$\Rightarrow p'(A) = \dfrac{1}{J^* + 2\beta}$ $\qquad q'(A) = \dfrac{2\beta}{J^* + 2\beta}$ $\qquad J^* \nearrow_A \Rightarrow p'(A) \searrow A$, $q'(A) \searrow A$

large market equil. # firm = large

each firm's prod. needs to change slightly to accomodate

short run shift in demand

market price insensitive to short run demand shift

(17) $u(x_1, x_2) = -\frac{1}{x_1} - \frac{1}{x_2}$ $\quad$ indirect utility func.? Expenditure?

$u(x_1,x_2) = -\frac{1}{x_1} - \frac{1}{x_2}$ $\quad$ UMP: $\max\, u(x_1,x_2) = \frac{-1}{x_1} - \frac{1}{x_2}$ s.t. $p_1x_1 + p_2x_2$ $\Rightarrow \frac{x_2^2}{x_1^2} = \frac{p_1}{p_2}$

$\Rightarrow x_2 = x_1\sqrt{\frac{p_1}{p_2}}$ $\Rightarrow$ Put in: $p_1x_1 + p_2x_2 = w \Rightarrow x_1 = \dfrac{w}{p_1 + \sqrt{p_1p_2}}$, $\quad x_2 = \dfrac{w}{\sqrt{p_2p_1} + p_2}$

$\Rightarrow x(p,w) = \left(\dfrac{w}{p_1 + \sqrt{p_1p_2}}, \dfrac{w}{p_2 + \sqrt{p_1p_2}}\right)$

Indirect util. func: $u(x(p,w)) = v(p,w)$

$\Rightarrow v(p,w) = \dfrac{-(p_1+\sqrt{p_1p_2})}{w} + \dfrac{-(p_2+\sqrt{p_1p_2})}{w} \Rightarrow v(p,w) = \dfrac{-(\sqrt{p_1}+\sqrt{p_2})^2}{w}$

$\boxed{EMP}$ $\quad p_1x_1 + p_2x_2$ s.t. $u(x) \geq v$ $\qquad \frac{p_1}{p_2} = \frac{x_2^2}{x_1^2} \Rightarrow x_2 = x_1\frac{\sqrt{p_1}}{\sqrt{p_2}}$

$\Rightarrow$ put back in $u = \dfrac{-\sqrt{p_2} - \sqrt{p_1}}{x_1\sqrt{p_2}} \Rightarrow x_1 = \dfrac{-\sqrt{p_2} - \sqrt{p_1}}{u\sqrt{p_1}}$

$h(p,u) = \left(\dfrac{-\sqrt{p_1} - \sqrt{p_2}}{u\sqrt{p_1}}, \dfrac{-\sqrt{p_1} - \sqrt{p_2}}{u\sqrt{p_2}}\right)$ $\quad e(p,u) = p\cdot h(p,u)$

$e(p,u) = \dfrac{-(\sqrt{p_1}+\sqrt{p_2})(\sqrt{p_1}+\sqrt{p_2})}{u} \Rightarrow e(p,u) = \dfrac{-(\sqrt{p_1}+\sqrt{p_2})^2}{u}$

$\boxed{\text{intuition}}$ $\boxed{\text{to get elasticity of any type, or change just differentiate w.r.t. the variable}}$

(18) (x,y) goods $\quad$ price (p,q)

utility $u(x,y)$ $\quad$ wealth: $w > 0$

$q = \alpha q_0$ $\quad$ price change in prop to another

$\tilde{u}(x,z) = \max_y u(x,y)$ s.t. $q_0 y \leq z$

good in econ: x, composite z $\quad \alpha$: price of composite

$\max_{x,z} \tilde{u}(x,z)$ s.t. $p\cdot x + \alpha z \leq w$ $\Rightarrow z = q_0 y$

(19) $u(x) = (x_1-b_1)^\alpha (x_2-b_2)^\beta (x_3-b_3)^\gamma \quad \alpha+\beta+\gamma = 1$

$\tilde{u}(x) = \ln u(x)$

$\Rightarrow \tilde{u}(x) = \alpha\ln(x_1-b_1) + \beta\ln(x_2-b_2) + \gamma\ln(x_3-b_3)$

s.t. $p_1x_1 + p_2x_2 + p_3x_3 \leq w$

$\Rightarrow x_1 = \dfrac{\alpha}{\lambda p_1} + b_1, \ldots$ $\qquad \sum b_i = b$

put in budget constraint:

$p_1\left(\frac{\alpha}{\lambda p_1} + b_1\right) + p_2\left(\frac{\beta}{\lambda p_2} + b_2\right) + p_3\left(\frac{\gamma}{\lambda p_3} + b_3\right) = w$

$\Rightarrow \lambda = \dfrac{1}{w - pb} \Rightarrow x_1 = \dfrac{\alpha}{p_1}(w - pb) + b_1, \ldots$

$v(p,w) = u(x(p,w)) = \left[\frac{\alpha}{p_1}(w-pb)\right]^\alpha \cdots = \left(\frac{\alpha}{p_1}\right)^\alpha \cdots (w-pb)$

$v(p, e(p,u)) = u \Rightarrow e(p,u) - pb = u\cdot\left(\frac{p_1}{\alpha}\right)^\alpha \left(\frac{p_2}{\beta}\right)^\beta \left(\frac{p_3}{\gamma}\right)^\gamma$

$\Rightarrow e(p,u) = u\cdot\left(\frac{p_1}{\alpha}\right)^\alpha\left(\frac{p_2}{\beta}\right)^\beta\left(\frac{p_3}{\gamma}\right)^\gamma + p\cdot b$

$h_1(p,u) = \dfrac{\partial e(p,u)}{\partial p_1} = u\cdot\left(\frac{p_1}{\alpha}\right)^{\alpha-1}\left(\frac{p_2}{\beta}\right)^\beta\left(\frac{p_3}{\gamma}\right)^\gamma + b_1$

...

(20) $q = (z_1^\rho + z_2^\rho)^{1/\rho}$ ①

λ will cancel out but

$\begin{cases} \rho q^{\rho-1} = \lambda p \\ \rho z_1^{\rho-1} = \lambda w_1 \\ \rho z_2^{\rho-1} = \lambda w_2 \end{cases} \Rightarrow \begin{cases} z_1 = \left(\frac{w_1}{p}\right)^{\frac{1}{\rho-1}} \\ z_2 = \left(\frac{w_2}{p}\right)^{\frac{1}{\rho-1}} \\ q = \left(\frac{1}{p}\right)^{\frac{1}{\rho-1}} \end{cases}$ Normalize price to 1

$\rho < 1$

$\Rightarrow$ ①: $w_1^{\frac{\rho}{1-\rho}} + w_2^{\frac{\rho}{1-\rho}} = 1 \Rightarrow (\alpha(-w_1^{\frac{1}{\rho-1}}, -w_2^{\frac{1}{1-\rho}}, \text{sum}^{1/\rho}))$ Production frontier

② when $w_1^{\frac{\rho}{1-\rho}} + w_2^{\frac{\rho}{1-\rho}} \underset{\text{Price}}{<1}$ $\quad \phi$ $\quad$ you can produce huge amount

③ when $w_1^{\frac{\rho}{1-\rho}} + w_2^{\frac{\rho}{1-\rho}} \underset{\text{Price}}{>1}$ $\quad \{0\}$ under loss

$\rho = 1$: $q = z_1 + z_2$ $\quad \Rightarrow \min(z_1, z_2) < 1 \quad \infty$

$\min(z_1, z_2) > 1$

(21) $v(p,w) = -\exp\left(-b\frac{p_1}{p_2}\right)\left[\frac{w}{p_2} + \frac{1}{b}\left[a\frac{p_1}{p_2} + \frac{a}{b} + c\right]\right]$

① $x(p,w) = \dfrac{\frac{\partial v(p,w)}{\partial p}}{\frac{\partial v(p,w)}{\partial w}}$ Roy's identity

② to get $e(p,u)$: $w = e(p,u)$, $v(p,w) = u$ and use main equation

③ $h(p,u) = ?$ $\quad h(p,u) = \dfrac{\partial e(p,u)}{\partial p}$