In[1432]:=
```
Clear["Global`*"]
Expand[FullSimplify[(θ * s11 * (1 - s11) + (1 - θ) * (1 - s21) * s21) *
     (θ * s12 * (1 - s12) + (1 - θ) * s22 * (1 - s22)) -
   (θ * s11 * s21 + (1 - θ) * s21 * s22) * (θ * s11 * s21 + (1 - θ) * s21 * s22)]]
```

Out[1433]=
$$s21\ s22 - s21^2\ s22 - s21\ s22^2 + s12\ s21\ \theta - s12^2\ s21\ \theta - s12\ s21^2\ \theta + s12^2\ s21^2\ \theta + s11\ s22\ \theta - s11^2\ s22\ \theta - 2\ s21\ s22\ \theta + 2\ s21^2\ s22\ \theta - 2\ s11\ s21^2\ s22\ \theta - s11\ s22^2\ \theta + s11^2\ s22^2\ \theta + 2\ s21\ s22^2\ \theta + s11\ s12\ \theta^2 - s11^2\ s12\ \theta^2 - s11\ s12^2\ \theta^2 + s11^2\ s12^2\ \theta^2 - s12\ s21\ \theta^2 + s12^2\ s21\ \theta^2 - s11^2\ s21^2\ \theta^2 + s12\ s21^2\ \theta^2 - s12^2\ s21^2\ \theta^2 - s11\ s22\ \theta^2 + s11^2\ s22\ \theta^2 + s21\ s22\ \theta^2 - s21^2\ s22\ \theta^2 + 2\ s11\ s21^2\ s22\ \theta^2 + s11\ s22^2\ \theta^2 - s11^2\ s22^2\ \theta^2 - s21\ s22^2\ \theta^2$$

In[1434]:=
```
(* Setting negative jacobean that is used in the likelihood function:*)
LogJacob :=
 -FullSimplify[Log[s21 s22 - s21^2 s22 - s21 s22^2 + s12 s21 θ - s12^2 s21 θ - s12 s21^2 θ +
      s12^2 s21^2 θ + s11 s22 θ - s11^2 s22 θ - 2 s21 s22 θ + 2 s21^2 s22 θ -
      2 s11 s21^2 s22 θ - s11 s22^2 θ + s11^2 s22^2 θ + 2 s21 s22^2 θ + s11 s12 θ^2 -
      s11^2 s12 θ^2 - s11 s12^2 θ^2 + s11^2 s12^2 θ^2 - s12 s21 θ^2 + s12^2 s21 θ^2 -
      s11^2 s21^2 θ^2 + s12 s21^2 θ^2 - s12^2 s21^2 θ^2 - s11 s22 θ^2 + s11^2 s22 θ^2 + s21 s22 θ^2 -
      s21^2 s22 θ^2 + 2 s11 s21^2 s22 θ^2 + s11 s22^2 θ^2 - s11^2 s22^2 θ^2 - s21 s22^2 θ^2]]
(*Gradient Element: first derivative w.r.t θ, this is okay as here
 we do not have density of unobserved demand shocks in effect*)
Dθ = D[LogJacob, θ]
(*Hessian Element: second derivative,
again unobserved demand shock is not in effect *)
D[Dθ, θ]
```

Out[1435]=
$$-\Big(-s21\,(-(-1+s12)\,s12+(-1+s22)\,s22)\,(-1+\theta)+(-1+s11)\,s11\,(s12-s22)\,(-1+s12+s22)\,\theta-s21\,((-1+s22)\,s22\,(-1+\theta)-(-1+s12)\,s12\,\theta)+(-1+s11)\,s11\,((-1+s22)\,s22+(s12-s22)\,(-1+s12+s22)\,\theta)+s21^2\Big((-1+2\,s11)\,s22\,(-1+\theta)-s11^2\,\theta+\Big(-s11^2+(1-s12)\,s12\Big)\,\theta+s22\,(1-\theta+2\,s11\,\theta)+s12\,(-1+s12+\theta-s12\,\theta)\Big)\Big)\Big/\Big(-s21\,(-1+\theta)\,((-1+s22)\,s22\,(-1+\theta)-(-1+s12)\,s12\,\theta)+(-1+s11)\,s11\,\theta\,((-1+s22)\,s22+(s12-s22)\,(-1+s12+s22)\,\theta)+s21^2\Big(s22\,(-1+\theta)\,(1-\theta+2\,s11\,\theta)+\theta\Big(-s11^2\,\theta+s12\,(-1+s12+\theta-s12\,\theta)\Big)\Big)\Big)$$

Out[1436]=
$$\Big(-s21\,(-(-1+s12)\,s12+(-1+s22)\,s22)\,(-1+\theta)+(-1+s11)\,s11\,(s12-s22)\,(-1+s12+s22)\,\theta-s21\,((-1+s22)\,s22\,(-1+\theta)-(-1+s12)\,s12\,\theta)+(-1+s11)\,s11\,((-1+s22)\,s22+(s12-s22)\,(-1+s12+s22)\,\theta)+s21^2\Big((-1+2\,s11)\,s22\,(-1+\theta)-s11^2\,\theta+\Big(-s11^2+(1-s12)\,s12\Big)\,\theta+s22\,(1-\theta+2\,s11\,\theta)+s12\,(-1+s12+\theta-s12\,\theta)\Big)\Big)^2\Big/\Big(-s21\,(-1+\theta)\,((-1+s22)\,s22\,(-1+\theta)-(-1+s12)\,s12\,\theta)+(-1+s11)\,s11\,\theta\,((-1+s22)\,s22+(s12-s22)\,(-1+s12+s22)\,\theta)+s21^2\Big(s22\,(-1+\theta)\,(1-\theta+2\,s11\,\theta)+\theta\Big(-s11^2\,\theta+s12\,(-1+s12+\theta-s12\,\theta)\Big)\Big)\Big)^2-\Big(2\,(-1+s11)\,s11\,(s12-s22)\,(-1+s12+s22)+s21^2\Big(-2\,s11^2+2\,(1-s12)\,s12+2\,(-1+2\,s11)\,s22\Big)-2\,s21\,(-(-1+s12)\,s12+(-1+s22)\,s22)\Big)\Big/\Big(-s21\,(-1+\theta)\,((-1+s22)\,s22\,(-1+\theta)-(-1+s12)\,s12\,\theta)+(-1+s11)\,s11\,\theta\,((-1+s22)\,s22+(s12-s22)\,(-1+s12+s22)\,\theta)+s21^2\Big(s22\,(-1+\theta)\,(1-\theta+2\,s11\,\theta)+\theta\Big(-s11^2\,\theta+s12\,(-1+s12+\theta-s12\,\theta)\Big)\Big)\Big)$$

In[1703]:=
```
(* Define Likelihood term, in code sum up across all J items*)
(* the part of -0.5/var*(δ1-Z1*ϵ)^2,
is removed as variance is simply mean(δ1-Z1*ϵ)^2,
so it will cancel out, and the part that remains is Log[2*π*σ^2],
also T=2 multiplication is removed as it is not function of these variables*)
(*-------------------------------------------------Calculating
 Gradient w.r.t βpd---------------------------------------*)
(*Definition of s1=f(δ1), s2=f(δ2), δ1=f^(-1)(s1), δ2=f^(-1)(s2)*)
```

```
Sp1s1δ1'[δ1_] := s11[βpd] * (1 - s11[βpd])
Sp1s1δ2'[δ2_] := -s11[βpd] * s21[βpd]
Sp1s2δ1'[δ1_] := s12[βpd] * (1 - s12[βpd])
Sp1s2δ2'[δ2_] := -s12[βpd] * s22[βpd]
Sp2s1δ1'[δ1_] := -s11[βpd] * s21[βpd]
Sp2s1δ2'[δ2_] := (1 - s21[βpd]) * s21[βpd]
Sp2s2δ1'[δ1_] := -s12[βpd] * s22[βpd]
Sp2s2δ2'[δ2_] := (1 - s22[βpd]) * s22[βpd]
Sp1δ1[δ1_] := θ * Sp1s1δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp1δ2[δ2_] := θ * Sp1s1δ2[δ2] + (1 - θ) * Sp1s2δ2[δ2]
Sp2δ1[δ1_] := θ * Sp1s2δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp2δ2[δ2_] := θ * Sp2s1δ2[δ2] + (1 - θ) * Sp2s2δ2[δ2]
Sp1[δ1_, δ2_] := Sp1δ1[δ1] + Sp1δ2[δ2]
Sp2[δ1_, δ2_] := Sp2δ1[δ1] + Sp2δ2[δ2]
(*Definitions: derivative with respect to δ with parameter βpd,:*)
(*Detail: when we have s1=e^(ax1)/(1+e^(x1*a)+e^(x2*a)),
and s2=e^(ax2)/(1+e^(x1*a)+e^(x2*a)) then ds1/da=
  x1s1s3+(x1-x2)s1s2 and ds2/d=x2s2s3+(x2-x1)s1s2*)
s12'[βpd_] := P1 * s12[βpd] * (1 - s12[βpd] - s22[βpd]) +
  (P1 - γ * λ * P2) * s12[βpd] * s22[βpd]
s22'[βpd_] := (γ * λ * P2) * s22[βpd] * (1 - s12[βpd] - s22[βpd]) +
  (γ * λ * P2 - P1) * s12[βpd] * s22[βpd]
s11'[βpd_] := 0
s21'[βpd_] := 0
δ1'[βpd_] := Dδ1βpd[βpd]
δ2'[βpd_] := Dδ2βpd[βpd]
Ds1βpd[βpd_] :=
 (1 - θ) * P1 * s12[βpd] * (1 - s12[βpd] - s22[βpd]) + (P1 - γ * λ * P2) * s12[βpd] * s22[βpd]
Ds2βpd[βpd_] := (1 - θ) * (γ * λ * P2) * s22[βpd] * (1 - s12[βpd] - s22[βpd]) +
  (γ * λ * P2 - P1) * s12[βpd] * s22[βpd]
Dδ1βpd[βpd_] := FullSimplify[
  (D[Sp2δ2[δ2], δ2] * Ds1βpd[βpd] - D[Sp1δ2[δ2], δ2] * Ds2βpd[βpd]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
Dδ2βpd[βpd_] := FullSimplify[
  (D[Sp2δ1[δ1], δ1] * Ds1βpd[βpd] - D[Sp1δ1[δ1], δ1] * Ds2βpd[βpd]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
(*definitions of the gradient-------------------------*)
(*use DNErrDens[βpd_]=Expand[D[Log[2*π*(Φ1[βpd]-Z1*ϵ)*(Φ1[βpd]-Z1*ϵ)]+
      Log[2*π*(Φ2[βpd]-Z2*ϵ)*(Φ2[βpd]-Z2*ϵ)],βpd]]= 2 Φ1'[βpd]/(-Z1 ϵ+Φ1[βpd]) + 2 Φ2'[βpd]/(-Z2 ϵ+Φ2[βpd]) *)
(*Manually put the relevant replacement of δ with Φ*)
DNErrDens[βpd_] := 2 Dδ1βpd[βpd] / (-Z1 ϵ + δ1[βpd]) + 2 Dδ2βpd[βpd] / (-Z2 ϵ + δ2[βpd])
NJβpd[βpd_] :=
 FullSimplify[Log[s21 s22[βpd] - s21^2 s22[βpd] - s21 s22[βpd]^2 + s12[βpd] * s21 θ -
    s12[βpd]^2 * s21 θ - s12[βpd] * s21^2 θ + s12[βpd]^2 s21^2 θ + s11 s22[βpd] θ -
    s11^2 s22[βpd] θ - 2 s21 s22[βpd] θ + 2 s21^2 s22[βpd] θ - 2 s11 s21^2 s22 [βpd] θ -
    s11 s22[βpd]^2 θ + s11^2 s22[βpd]^2 θ + 2 s21 s22[βpd]^2 θ + s11 * s12[βpd] * θ^2 -
```

```
    s11^2 * s12[βpd] * θ^2 - s11 * s12[βpd]^2 θ^2 + s11^2 * s12[βpd]^2 θ^2 - s12[βpd] * s21 θ^2 +
    s12[βpd]^2 * s21 θ^2 - s11^2 s21^2 θ^2 + s12[βpd] * s21^2 θ^2 - s12[βpd]^2 * s21^2 θ^2 -
    s11 s22[βpd] θ^2 + s11^2 s22[βpd] θ^2 + s21 s22[βpd] θ^2 - s21^2 s22[βpd] θ^2 +
    2 s11 s21^2 s22[βpd] θ^2 + s11 s22[βpd]^2 θ^2 - s11^2 s22[βpd]^2 θ^2 - s21 s22[βpd]^2 θ^2]]
DNJβpd[βpd_] := D[NJβpd[βpd], βpd]
DNLLβpd[βpd_] := DNJβpd[βpd] + DNErrDens[βpd]
(*================================Gradient with respect to βpd becomes:*)
DNLLβpd[βpd]
(* Hessian w.r.t. βpd*)
DDβpd = D[DNLLβpd[βpd], βpd]
(* Hessian w.r.t θβpd:*)
DθH1[βpd_] := -(-s21 (-(-1 + s12[βpd]) s12[βpd] + (-1 + s22[βpd]) s22[βpd]) (-1 + θ) +
      (-1 + s11) s11 (s12[βpd] - s22[βpd]) (-1 + s12[βpd] + s22[βpd]) θ -
      s21 ((-1 + s22[βpd]) s22[βpd] (-1 + θ) - (-1 + s12[βpd]) s12[βpd] θ) + (-1 + s11) s11
       ((-1 + s22[βpd]) s22[βpd] + (s12[βpd] - s22[βpd]) (-1 + s12[βpd] + s22[βpd]) θ) +
      s21^2 ((-1 + 2 s11) s22[βpd] (-1 + θ) - s11^2 θ + (-s11^2 + (1 - s12[βpd]) s12[βpd]) θ +
         s22[βpd] (1 - θ + 2 s11 θ) + s12 [βpd] (-1 + s12[βpd] + θ - s12[βpd] θ))) /
   (-s21 (-1 + θ) ((-1 + s22[βpd]) s22[βpd] (-1 + θ) - (-1 + s12[βpd]) s12[βpd] θ) +
     (-1 + s11) s11 θ
      ((-1 + s22[βpd]) s22[βpd] + (s12[βpd] - s22[βpd]) (-1 + s12[βpd] + s22[βpd]) θ) +
     s21^2 (s22[βpd] (-1 + θ) (1 - θ + 2 s11 θ) +
        θ (-s11^2 θ + s12[βpd] (-1 + s12[βpd] + θ - s12[βpd] θ))))
H1DDθβpd = D[DθH1[βpd], βpd]
```

Out[1731]=

```
(-P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[βpd]) (-1 + s12[βpd]) s12[βpd] -
    P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[βpd]) s12[βpd]^2 +
    P2 (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) (1 - s22[βpd])
     s22[βpd] - 2 P2 (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) (1 - s22[βpd]) s22[βpd]^2) /
  (-s11^2 s21^2 θ^2 - θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (-1 + s12[βpd]) s12[βpd] +
    (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) s22[βpd] -
    (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) s22[βpd]^2) +
 (2 (-((-P1 + P2 γ θ λ) s12[βpd] + P2 γ (-1 + θ) λ (-1 + s22[βpd])) s22[βpd]
       (-θ s11[βpd] s21[βpd] + (-1 + θ) s12[βpd] s22[βpd]) +
      s12[βpd] (P1 (-1 + θ) (-1 + s12[βpd]) + (P1 θ - P2 γ λ) s22[βpd])
       (-θ (-1 + s21[βpd]) s21[βpd] + (-1 + θ) (-1 + s22[βpd]) s22[βpd]))) /
  (((-1 + s12[βpd]) s12[βpd] (-θ s11[βpd] s21[βpd] + (-1 + θ) s12[βpd] s22[βpd]) +
      (-θ (-1 + s11[βpd]) s11[βpd] + (-1 + θ) (-1 + s12[βpd]) s12[βpd])
       (-θ (-1 + s21[βpd]) s21[βpd] + (-1 + θ) (-1 + s22[βpd]) s22[βpd])) (-Z1
      ϵ + δ1[βpd])) -
 (2 ((-θ (-1 + s11[βpd]) s11[βpd] + (-1 + θ) (-1 + s12[βpd]) s12[βpd])
       ((-P1 + P2 γ θ λ) s12[βpd] + P2 γ (-1 + θ) λ (-1 + s22[βpd])) s22[βpd] +
      (-1 + s12[βpd]) s12[βpd]^2 (P1 (-1 + θ) (-1 + s12[βpd]) + (P1 θ - P2 γ λ) s22[βpd]))) /
  (((-1 + s12[βpd]) s12[βpd] (-θ s11[βpd] s21[βpd] + (-1 + θ) s12[βpd] s22[βpd]) +
      (-θ (-1 + s11[βpd]) s11[βpd] + (-1 + θ) (-1 + s12[βpd]) s12[βpd])
       (-θ (-1 + s21[βpd]) s21[βpd] + (-1 + θ) (-1 + s22[βpd]) s22[βpd])) (-Z2
      ϵ + δ2[βpd]))
```

Out[1732]=

```
-((-P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[βpd]) (-1 + s12[βpd]) s12[
      βpd] - P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[βpd]) s12[βpd]^2 +
```

$$
\begin{aligned}
&\mathrm{P2}\,(-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\,(1-\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad \mathrm{s22}[\beta\mathrm{pd}]-2\,\mathrm{P2}\,(-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(1-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]^2\big)\\
&(-\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])-\\
&\quad \theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s12}[\beta\mathrm{pd}]\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad (-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\\
&\quad ((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,\lambda\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}])-\\
&\quad 2\,(-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s22}[\beta\mathrm{pd}]\\
&\quad ((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,\lambda\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]))\big)\Big/\\
&\big(-\mathrm{s11}^2\,\mathrm{s21}^2\,\theta^2-\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\,\mathrm{s12}[\beta\mathrm{pd}]+\\
&\quad (-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\,\mathrm{s22}[\beta\mathrm{pd}]-\\
&\quad (-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s22}[\beta\mathrm{pd}]^2\big)^2+\\
&\big(-\mathrm{P1}\,\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(1-\mathrm{s12}[\beta\mathrm{pd}])\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])-\\
&\quad 3\,\mathrm{P1}\,\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(1-\mathrm{s12}[\beta\mathrm{pd}])\,\mathrm{s12}[\beta\mathrm{pd}]\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad \mathrm{P1}\,\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\,\mathrm{s12}[\beta\mathrm{pd}]\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad \mathrm{P1}\,\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s12}[\beta\mathrm{pd}]^2\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad \mathrm{P2}\,(-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\,(1-\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad ((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,\lambda\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}])-\\
&\quad \mathrm{P2}\,(-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\,\mathrm{s22}[\beta\mathrm{pd}]\\
&\quad ((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,\lambda\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}])-\\
&\quad 4\,\mathrm{P2}\,(-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(1-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]\\
&\quad ((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,\lambda\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad 2\,\mathrm{P2}\,(-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s22}[\beta\mathrm{pd}]^2\\
&\quad ((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,\lambda\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]))\big)\Big/\\
&\big(-\mathrm{s11}^2\,\mathrm{s21}^2\,\theta^2-\theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\,\mathrm{s12}[\beta\mathrm{pd}]+\\
&\quad (-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\,\mathrm{s22}[\beta\mathrm{pd}]-\\
&\quad (-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s22}[\beta\mathrm{pd}]^2\big)-\\
&\big(2\,(-((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\theta\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\mathrm{s22}[\beta\mathrm{pd}]))\,\mathrm{s22}[\beta\mathrm{pd}]\\
&\quad (-\theta\,\mathrm{s11}[\beta\mathrm{pd}]\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad \mathrm{s12}[\beta\mathrm{pd}]\,(\mathrm{P1}\,(-1+\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])+(\mathrm{P1}\,\theta-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad (-\theta\,(-1+\mathrm{s21}[\beta\mathrm{pd}])\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]))^2\big)\Big/\\
&\big((-1+\mathrm{s12}[\beta\mathrm{pd}])\,\mathrm{s12}[\beta\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\beta\mathrm{pd}]\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad (-\theta\,(-1+\mathrm{s11}[\beta\mathrm{pd}])\,\mathrm{s11}[\beta\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\,\mathrm{s12}[\beta\mathrm{pd}])\\
&\quad (-\theta\,(-1+\mathrm{s21}[\beta\mathrm{pd}])\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]))^2\\
&\quad (-\mathrm{Z1}\,\epsilon+\delta 1[\beta\mathrm{pd}])^2\big)-(2\,(-((-\mathrm{P1}+\mathrm{P2}\,\gamma\,\theta\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]+\mathrm{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\mathrm{s22}[\beta\mathrm{pd}]))\\
&\quad \mathrm{s22}[\beta\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\beta\mathrm{pd}]\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad \mathrm{s12}[\beta\mathrm{pd}]\,(\mathrm{P1}\,(-1+\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])+(\mathrm{P1}\,\theta-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad (-\theta\,(-1+\mathrm{s21}[\beta\mathrm{pd}])\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}]))\\
&\quad ((-1+\mathrm{s12}[\beta\mathrm{pd}])\,(-\theta\,\mathrm{s11}[\beta\mathrm{pd}]\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad \mathrm{s12}[\beta\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\beta\mathrm{pd}]\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad (\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2}\,\gamma\,\lambda)\,\mathrm{s12}[\beta\mathrm{pd}]\,\mathrm{s22}[\beta\mathrm{pd}])+\\
&\quad (-\theta\,(-1+\mathrm{s21}[\beta\mathrm{pd}])\,\mathrm{s21}[\beta\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\beta\mathrm{pd}])\,\mathrm{s22}[\beta\mathrm{pd}])\\
&\quad ((-1+\theta)\,(-1+\mathrm{s12}[\beta\mathrm{pd}])\,(\mathrm{P1}\,\mathrm{s12}[\beta\mathrm{pd}]\,(1-\mathrm{s12}[\beta\mathrm{pd}]-\mathrm{s22}[\beta\mathrm{pd}])+
\end{aligned}
$$

```
          (P1 - P2 γ λ) s12[βpd] s22[βpd]) + (-1 + Θ) s12[βpd]
         (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) + (P1 - P2 γ λ) s12[βpd] s22[βpd])) +
      (-1 + s12[βpd]) s12[βpd] ((-1 + Θ) s22[βpd] (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) +
          (P1 - P2 γ λ) s12[βpd] s22[βpd]) + (-1 + Θ) s12[βpd]
         ((-P1 + P2 γ λ) s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd])) +
      (-Θ (-1 + s11[βpd]) s11[βpd] + (-1 + Θ) (-1 + s12[βpd]) s12[βpd])
       ((-1 + Θ) (-1 + s22[βpd]) ((-P1 + P2 γ λ) s12[βpd] s22[βpd] +
          P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd]) + (-1 + Θ) s22[βpd] ((-P1 + P2 γ λ)
          s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd])))) /
   (((-1 + s12[βpd]) s12[βpd] (-Θ s11[βpd] s21[βpd] + (-1 + Θ) s12[βpd] s22[βpd]) +
      (-Θ (-1 + s11[βpd]) s11[βpd] + (-1 + Θ) (-1 + s12[βpd]) s12[βpd])
       (-Θ (-1 + s21[βpd]) s21[βpd] + (-1 + Θ) (-1 + s22[βpd]) s22[βpd]))^2
     (-Z1 ϵ + δ1[βpd])) + (2 (-((-P1 + P2 γ Θ λ) s12[βpd] + P2 γ (-1 + Θ) λ (-1 + s22[βpd]))
      (-Θ s11[βpd] s21[βpd] + (-1 + Θ) s12[βpd] s22[βpd])
      ((-P1 + P2 γ λ) s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd]) +
     (P1 (-1 + Θ) (-1 + s12[βpd]) + (P1 Θ - P2 γ λ) s22[βpd])
      (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) + (P1 - P2 γ λ) s12[βpd] s22[βpd])
      (-Θ (-1 + s21[βpd]) s21[βpd] + (-1 + Θ) (-1 + s22[βpd]) s22[βpd]) -
     s22[βpd] (-Θ s11[βpd] s21[βpd] + (-1 + Θ) s12[βpd] s22[βpd])
      ((-P1 + P2 γ Θ λ) (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) +
          (P1 - P2 γ λ) s12[βpd] s22[βpd]) + P2 γ (-1 + Θ) λ
         ((-P1 + P2 γ λ) s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd])) +
     s12[βpd] (-Θ (-1 + s21[βpd]) s21[βpd] + (-1 + Θ) (-1 + s22[βpd]) s22[βpd])
      (P1 (-1 + Θ) (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) +
          (P1 - P2 γ λ) s12[βpd] s22[βpd]) + (P1 Θ - P2 γ λ)
         ((-P1 + P2 γ λ) s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd])) -
     ((-P1 + P2 γ Θ λ) s12[βpd] + P2 γ (-1 + Θ) λ (-1 + s22[βpd])) s22[βpd]
      ((-1 + Θ) s22[βpd] (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) +
          (P1 - P2 γ λ) s12[βpd] s22[βpd]) + (-1 + Θ) s12[βpd]
         ((-P1 + P2 γ λ) s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd])) +
     s12[βpd] (P1 (-1 + Θ) (-1 + s12[βpd]) + (P1 Θ - P2 γ λ) s22[βpd])
      ((-1 + Θ) (-1 + s22[βpd]) ((-P1 + P2 γ λ) s12[βpd] s22[βpd] +
          P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd]) + (-1 + Θ) s22[βpd] ((-P1 + P2 γ λ)
          s12[βpd] s22[βpd] + P2 γ λ (1 - s12[βpd] - s22[βpd]) s22[βpd])))) /
   (((-1 + s12[βpd]) s12[βpd] (-Θ s11[βpd] s21[βpd] + (-1 + Θ) s12[βpd] s22[βpd]) +
      (-Θ (-1 + s11[βpd]) s11[βpd] + (-1 + Θ) (-1 + s12[βpd]) s12[βpd])
       (-Θ (-1 + s21[βpd]) s21[βpd] + (-1 + Θ) (-1 + s22[βpd]) s22[βpd])) (-Z1
     ϵ + δ1[βpd])) -
  (2 ((-Θ (-1 + s11[βpd]) s11[βpd] + (-1 + Θ) (-1 + s12[βpd]) s12[βpd]) ((-P1 + P2 γ Θ λ)
        s12[βpd] + P2 γ (-1 + Θ) λ (-1 + s22[βpd])) s22[βpd] + (-1 + s12[βpd])
       s12[βpd]^2 (P1 (-1 + Θ) (-1 + s12[βpd]) + (P1 Θ - P2 γ λ) s22[βpd]))^2) /
   (((-1 + s12[βpd]) s12[βpd] (-Θ s11[βpd] s21[βpd] + (-1 + Θ) s12[βpd] s22[βpd]) +
      (-Θ (-1 + s11[βpd]) s11[βpd] + (-1 + Θ) (-1 + s12[βpd]) s12[βpd])
       (-Θ (-1 + s21[βpd]) s21[βpd] + (-1 + Θ) (-1 + s22[βpd]) s22[βpd]))^2 (-Z2
     ϵ + δ2[βpd])^2) -
  (2 (2 (-1 + s12[βpd]) s12[βpd] (P1 (-1 + Θ) (-1 + s12[βpd]) + (P1 Θ - P2 γ λ) s22[βpd])
      (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) + (P1 - P2 γ λ) s12[βpd] s22[βpd]) +
     s12[βpd]^2 (P1 (-1 + Θ) (-1 + s12[βpd]) + (P1 Θ - P2 γ λ) s22[βpd])
      (P1 s12[βpd] (1 - s12[βpd] - s22[βpd]) + (P1 - P2 γ λ) s12[βpd] s22[βpd]) +
```

$$
\begin{aligned}
&(-\theta\,(-1+\text{s11}[\beta\text{pd}])\,\text{s11}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}])\\
&\quad((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\beta\text{pd}]+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\beta\text{pd}]))\\
&\quad((-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])+\\
&\quad((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\beta\text{pd}]+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\beta\text{pd}]))\,\text{s22}[\beta\text{pd}]\\
&\quad((-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+\\
&\qquad(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+(-1+\theta)\,\text{s12}[\beta\text{pd}]\\
&\qquad(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]))+\\
&\quad(-\theta\,(-1+\text{s11}[\beta\text{pd}])\,\text{s11}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}])\\
&\quad\text{s22}[\beta\text{pd}]\,((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+\\
&\qquad(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\\
&\qquad((-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}]))+\\
&\quad(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}]^2\,(\text{P1}\,(-1+\theta)\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+\\
&\qquad(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,((-\text{P1}+\text{P2}\,\gamma\,\lambda)\\
&\qquad\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])))\Big)\Big/\\
&((((-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}]\,(-\theta\,\text{s11}[\beta\text{pd}]\,\text{s21}[\beta\text{pd}]+(-1+\theta)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+\\
&\quad(-\theta\,(-1+\text{s11}[\beta\text{pd}])\,\text{s11}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}])\\
&\quad(-\theta\,(-1+\text{s21}[\beta\text{pd}])\,\text{s21}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}]))\,(-\text{Z2}\\
&\quad\epsilon+\delta 2[\beta\text{pd}]))+\\
&\Big(2\,\Big((-\theta\,(-1+\text{s11}[\beta\text{pd}])\,\text{s11}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}])\\
&\quad((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\beta\text{pd}]+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\beta\text{pd}]))\,\text{s22}[\beta\text{pd}]+\\
&\quad(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}]^2\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\beta\text{pd}])\Big)\\
&\quad((-1+\text{s12}[\beta\text{pd}])\,(-\theta\,\text{s11}[\beta\text{pd}]\,\text{s21}[\beta\text{pd}]+(-1+\theta)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])\\
&\qquad(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+\\
&\qquad\text{s12}[\beta\text{pd}]\,(-\theta\,\text{s11}[\beta\text{pd}]\,\text{s21}[\beta\text{pd}]+(-1+\theta)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])\\
&\qquad(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+\\
&\quad(-\theta\,(-1+\text{s21}[\beta\text{pd}])\,\text{s21}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])\\
&\quad((-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+\\
&\qquad(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+(-1+\theta)\,\text{s12}[\beta\text{pd}]\\
&\qquad(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]))+\\
&\quad(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}]\,((-1+\theta)\,\text{s22}[\beta\text{pd}]\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+\\
&\qquad(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+(-1+\theta)\,\text{s12}[\beta\text{pd}]\\
&\qquad((-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}]))+\\
&\quad(-\theta\,(-1+\text{s11}[\beta\text{pd}])\,\text{s11}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}])\\
&\quad((-1+\theta)\,(-1+\text{s22}[\beta\text{pd}])\,((-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\\
&\qquad\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])+(-1+\theta)\,\text{s22}[\beta\text{pd}]\,((-\text{P1}+\text{P2}\,\gamma\,\lambda)\\
&\qquad\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}]))))\Big)\Big/\\
&\Big(((-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}]\,(-\theta\,\text{s11}[\beta\text{pd}]\,\text{s21}[\beta\text{pd}]+(-1+\theta)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+\\
&\quad(-\theta\,(-1+\text{s11}[\beta\text{pd}])\,\text{s11}[\beta\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\beta\text{pd}])\,\text{s12}[\beta\text{pd}])\\
&\quad(-\theta\,(-1+\text{s21}[\beta\text{pd}])\,\text{s21}[\beta\text{pd}]+\\
&\qquad(-1+\theta)\,(-1+\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])^2\,(-\text{Z2}\,\epsilon+\delta 2[\beta\text{pd}])\Big)
\end{aligned}
$$

Out[1734]=

$$
\begin{aligned}
&\Big(-(-1+\text{s11})\,\text{s11}\,\theta\,(-1+\text{s12}[\beta\text{pd}]+\text{s22}[\beta\text{pd}])\\
&\quad(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]-\\
&\qquad(-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]-\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])-\\
&\quad(-1+\text{s11})\,\text{s11}\,\theta\,(\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+\\
&\qquad(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+(-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+\\
&\qquad\text{P2}\,\gamma\,\lambda\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])\,\text{s22}[\beta\text{pd}])+\text{s21}\,(-1+\theta)\,((1-\text{s12}[\beta\text{pd}])\\
&\qquad(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])-\\
&\qquad\text{s12}[\beta\text{pd}]\,(\text{P1}\,\text{s12}[\beta\text{pd}]\,(1-\text{s12}[\beta\text{pd}]-\text{s22}[\beta\text{pd}])+(\text{P1}-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}])+\\
&\qquad(-1+\text{s22}[\beta\text{pd}])\,((-\text{P1}+\text{P2}\,\gamma\,\lambda)\,\text{s12}[\beta\text{pd}]\,\text{s22}[\beta\text{pd}]+
\end{aligned}
$$

$$
\begin{aligned}
&\quad P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+s22[\beta pd]\\
&\quad((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd]))+\\
&s21\,(-\theta\,(-1+s12[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+\\
&\quad(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])-\theta\,s12[\beta pd]\\
&\quad(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])+\\
&\quad(-1+\theta)\,(-1+s22[\beta pd])\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+\\
&\quad P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+(-1+\theta)\,s22[\beta pd]\\
&\quad((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd]))-\\
&(-1+s11)\,s11\,(\theta\,(-1+s12[\beta pd]+s22[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+\\
&\quad(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]-(-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]-\\
&\quad P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+(-1+s22[\beta pd])\\
&\quad((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\\
&\quad s22[\beta pd]\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+\\
&\quad P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\theta\,(s12[\beta pd]-s22[\beta pd])\\
&\quad(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+\\
&\quad(-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd]))-\\
&s21^2\,((-1+\theta+s12[\beta pd]-\theta\,s12[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+\\
&\quad(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])+(-1+2\,s11)\,(-1+\theta)\\
&\quad((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\\
&\quad(1-\theta+2\,s11\,\theta)\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+\\
&\quad P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\\
&\quad s12[\beta pd]\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]-\\
&\quad\theta\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]))+\\
&\quad\theta\,((1-s12[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+\\
&\quad(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])-s12[\beta pd]\\
&\quad(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])))\Big)\Big/\\
&\Big(s21^2\,\Big(\theta\,\Big(-s11^2\,\theta+s12[\beta pd]\,(-1+\theta+s12[\beta pd]-\theta\,s12[\beta pd])\Big)+\\
&\quad(-1+\theta)\,(1-\theta+2\,s11\,\theta)\,s22[\beta pd]\Big)-\\
&\quad s21\,(-1+\theta)\,(-\theta\,(-1+s12[\beta pd])\,s12[\beta pd]+(-1+\theta)\,(-1+s22[\beta pd])\,s22[\beta pd])+\\
&\quad(-1+s11)\,s11\,\theta\\
&\quad((-1+s22[\beta pd])\,s22[\beta pd]+\theta\,(s12[\beta pd]-s22[\beta pd])\,(-1+s12[\beta pd]+s22[\beta pd]))\Big)-\\
&\Big(\Big(-(-1+s11)\,s11\,\theta\,(s12[\beta pd]-s22[\beta pd])\,(-1+s12[\beta pd]+s22[\beta pd])-\\
&\quad s21^2\,\Big(-s11^2\,\theta+s12[\beta pd]\,(-1+\theta+s12[\beta pd]-\theta\,s12[\beta pd])+\\
&\quad\theta\,\Big(-s11^2+(1-s12[\beta pd])\,s12[\beta pd]\Big)+\\
&\quad(-1+2\,s11)\,(-1+\theta)\,s22[\beta pd]+(1-\theta+2\,s11\,\theta)\,s22[\beta pd]\Big)+\\
&\quad s21\,(-1+\theta)\,((1-s12[\beta pd])\,s12[\beta pd]+(-1+s22[\beta pd])\,s22[\beta pd])+\\
&\quad s21\,(-\theta\,(-1+s12[\beta pd])\,s12[\beta pd]+(-1+\theta)\,(-1+s22[\beta pd])\,s22[\beta pd])-\\
&\quad(-1+s11)\,s11\\
&\quad((-1+s22[\beta pd])\,s22[\beta pd]+\theta\,(s12[\beta pd]-s22[\beta pd])\,(-1+s12[\beta pd]+s22[\beta pd]))\Big)\\
&\Big(-s21\,(-1+\theta)\,(-\theta\,(-1+s12[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+\\
&\quad(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])-\theta\,s12[\beta pd]\\
&\quad(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])+\\
&\quad(-1+\theta)\,(-1+s22[\beta pd])\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+\\
&\quad P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+(-1+\theta)\,s22[\beta pd]\\
&\quad((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd]))+\\
&\quad(-1+s11)\,s11\,\theta\,(\theta\,(-1+s12[\beta pd]+s22[\beta pd])\\
&\quad(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]-\\
&\quad(-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]-P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\\
&\quad(-1+s22[\beta pd])\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+
\end{aligned}
$$

$$P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+s22[\beta pd]\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\theta\,(s12[\beta pd]-s22[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+(-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd]))+s21^2\,((-1+\theta)\,(1-\theta+2\,s11\,\theta)\,((-P1+P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]+P2\,\gamma\,\lambda\,(1-s12[\beta pd]-s22[\beta pd])\,s22[\beta pd])+\theta\,((-1+\theta+s12[\beta pd]-\theta\,s12[\beta pd])\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])+s12[\beta pd]\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd]-\theta\,(P1\,s12[\beta pd]\,(1-s12[\beta pd]-s22[\beta pd])+(P1-P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])))))\Big)\Big/\Big(s21^2\,\Big(\theta\,\Big(-s11^2\,\theta+s12[\beta pd]\,(-1+\theta+s12[\beta pd]-\theta\,s12[\beta pd])\Big)+(-1+\theta)\,(1-\theta+2\,s11\,\theta)\,s22[\beta pd]\Big)-s21\,(-1+\theta)\,(-\theta\,(-1+s12[\beta pd])\,s12[\beta pd]+(-1+\theta)\,(-1+s22[\beta pd])\,s22[\beta pd])+(-1+s11)\,s11\,\theta\,((-1+s22[\beta pd])\,s22[\beta pd]+\theta\,(s12[\beta pd]-s22[\beta pd])\,(-1+s12[\beta pd]+s22[\beta pd]))\Big)^2$$

In[1874]:=
```
(*-------------------------------------------------Calculating
 Gradient w.r.t αpd----------------------------------------*)
(*Main definition correction*)
Sp1s1δ1'[δ1_] := s11[αpd] * (1 - s11[αpd])
Sp1s1δ2'[δ2_] := -s11[αpd] * s21[αpd]
Sp1s2δ1'[δ1_] := s12[αpd] * (1 - s12[αpd])
Sp1s2δ2'[δ2_] := -s12[αpd] * s22[αpd]
Sp2s1δ1'[δ1_] := -s11[αpd] * s21[αpd]
Sp2s1δ2'[δ2_] := (1 - s21[αpd]) * s21[αpd]
Sp2s2δ1'[δ1_] := -s12[αpd] * s22[αpd]
Sp2s2δ2'[δ2_] := (1 - s22[αpd]) * s22[αpd]
Sp1δ1[δ1_] := θ * Sp1s1δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp1δ2[δ2_] := θ * Sp1s1δ2[δ2] + (1 - θ) * Sp1s2δ2[δ2]
Sp2δ1[δ1_] := θ * Sp1s2δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp2δ2[δ2_] := θ * Sp2s1δ2[δ2] + (1 - θ) * Sp2s2δ2[δ2]
Sp1[δ1_, δ2_] := Sp1δ1[δ1] + Sp1δ2[δ2]
Sp2[δ1_, δ2_] := Sp2δ1[δ1] + Sp2δ2[δ2]
(*Definitions: derivative with respect to parameter βpd,
for the first period:*)
Ds1βpd[αpd_] := (1 - θ) * s12[αpd] * (1 - s12[αpd]) * λ * (P1 - P2)
Ds2βpd[αpd_] := -(1 - θ) * s12[αpd] * s22[αpd] * λ * (P1 - P2)
Dδ1βpd[αpd_] :=
 FullSimplify[(D[Sp2δ2[δ2], δ2] * Ds1βpd[αpd] - D[Sp1δ2[δ2], δ2] * Ds2βpd[αpd]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
Dδ2βpd[αpd_] := FullSimplify[
  (D[Sp2δ1[δ1], δ1] * Ds1βpd[αpd] - D[Sp1δ1[δ1], δ1] * Ds2βpd[αpd]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
s12'[αpd_] := s12[αpd] * (1 - s12[αpd]) * λ * (P1 - P2)
s22'[αpd_] := -s12[αpd] * s22[αpd] * λ * (P1 - P2)
s11'[αpd_] := 0
s21'[αpd_] := 0
δ1'[αpd_] := Dδ1βpd[αpd]
```

```
δ2'[αpd_] := Dδ2βpd[αpd]

DNErrDens[αpd_] := 2 Dδ1βpd[αpd] / (-Z1 ϵ + δ1[αpd]) + 2 Dδ2βpd[αpd] / (-Z2 ϵ + δ2[αpd])

NJβpd[αpd_] :=
 FullSimplify[Log[s21 s22[αpd] - s21^2 s22[αpd] - s21 s22[αpd]^2 + s12[αpd] * s21 θ -
    s12[αpd]^2 * s21 θ - s12[αpd] * s21^2 θ + s12[αpd]^2 s21^2 θ + s11 s22[αpd] θ -
    s11^2 s22[αpd] θ - 2 s21 s22[αpd] θ + 2 s21^2 s22[αpd] θ - 2 s11 s21^2 s22 [αpd] θ -
    s11 s22[αpd]^2 θ + s11^2 s22[αpd]^2 θ + 2 s21 s22[αpd]^2 θ + s11 * s12[αpd] * θ^2 -
    s11^2 * s12[αpd] * θ^2 - s11 * s12[αpd]^2 θ^2 + s11^2 * s12[αpd]^2 θ^2 - s12[αpd] * s21 θ^2 +
    s12[αpd]^2 * s21 θ^2 - s11^2 s21^2 θ^2 + s12[αpd] * s21^2 θ^2 - s12[αpd]^2 * s21^2 θ^2 -
    s11 s22[αpd] θ^2 + s11^2 s22[αpd] θ^2 + s21 s22[αpd] θ^2 - s21^2 s22[αpd] θ^2 +
    2 s11 s21^2 s22[αpd] θ^2 + s11 s22[αpd]^2 θ^2 - s11^2 s22[αpd]^2 θ^2 - s21 s22[αpd]^2 θ^2]]

DNJβpd[αpd_] := D[NJβpd[αpd], αpd]

DNLLβpd[αpd_] := DNJβpd[αpd] + DNErrDens[αpd]

DNLLβpd[αpd]

DDβpd = D[DNLLβpd[αpd], αpd]

DθH2[αpd_] := -(-s21 (-(-1 + s12[αpd]) s12[αpd] + (-1 + s22[αpd]) s22[αpd]) (-1 + θ) +
     (-1 + s11) s11 (s12[αpd] - s22[αpd]) (-1 + s12[αpd] + s22[αpd]) θ -
     s21 ((-1 + s22[αpd]) s22[αpd] (-1 + θ) - (-1 + s12[αpd]) s12[αpd] θ) + (-1 + s11) s11
      ((-1 + s22[αpd]) s22[αpd] + (s12[αpd] - s22[αpd]) (-1 + s12[αpd] + s22[αpd]) θ) +
     s21^2 ((-1 + 2 s11) s22[αpd] (-1 + θ) - s11^2 θ + (-s11^2 + (1 - s12[αpd]) s12[αpd]) θ +
        s22[αpd] (1 - θ + 2 s11 θ) + s12 [αpd] (-1 + s12[αpd] + θ - s12[αpd] θ))) /
  (-s21 (-1 + θ) ((-1 + s22[αpd]) s22[αpd] (-1 + θ) - (-1 + s12[αpd]) s12[αpd] θ) +
    (-1 + s11) s11 θ
     ((-1 + s22[αpd]) s22[αpd] + (s12[αpd] - s22[αpd]) (-1 + s12[αpd] + s22[αpd]) θ) +
    s21^2 (s22[αpd] (-1 + θ) (1 - θ + 2 s11 θ) +
       θ (-s11^2 θ + s12[αpd] (-1 + s12[αpd] + θ - s12[αpd] θ))))

H2DDθαpd = D[DθH2[αpd], αpd]

DβpdH4[αpd_] :=
 (-P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[αpd]) (-1 + s12[αpd]) s12[αpd] -
     P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[αpd]) s12[αpd]^2 +
     P2 (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) (1 - s22[βpd])
      s22[αpd] - 2 P2 (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) (1 - s22[αpd]) s22[αpd]^2) /
   (-s11^2 s21^2 θ^2 - θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (-1 + s12[αpd]) s12[αpd] +
     (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) s22[αpd] -
     (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) s22[αpd]^2) +
  (2 (-((-P1 + P2 γ θ λ) s12[αpd] + P2 γ (-1 + θ) λ (-1 + s22[αpd])) s22[αpd]
       (-θ s11[αpd] s21[αpd] + (-1 + θ) s12[αpd] s22[αpd]) +
      s12[αpd] (P1 (-1 + θ) (-1 + s12[αpd]) + (P1 θ - P2 γ λ) s22[αpd])
       (-θ (-1 + s21[αpd]) s21[αpd] + (-1 + θ) (-1 + s22[αpd]) s22[αpd]))) /
   (((-1 + s12[αpd]) s12[αpd] (-θ s11[αpd] s21[αpd] + (-1 + θ) s12[αpd] s22[αpd]) +
      (-θ (-1 + s11[αpd]) s11[αpd] + (-1 + θ) (-1 + s12[αpd]) s12[αpd])
       (-θ (-1 + s21[αpd]) s21[αpd] + (-1 + θ) (-1 + s22[αpd]) s22[αpd])) (-Z1 ϵ +
      δ1[αpd])) - (2 ((-θ (-1 + s11[αpd]) s11[αpd] + (-1 + θ) (-1 + s12[αpd]) s12[αpd])
       ((-P1 + P2 γ θ λ) s12[αpd] + P2 γ (-1 + θ) λ (-1 + s22[αpd])) s22[αpd] + (-1 +
         s12[αpd]) s12[αpd]^2 (P1 (-1 + θ) (-1 + s12[αpd]) + (P1 θ - P2 γ λ) s22[αpd])) /
```

```
(((-1 + s12[αpd]) s12[αpd] (-θ s11[αpd] s21[αpd] + (-1 + θ) s12[αpd] s22[αpd]) +
     (-θ (-1 + s11[αpd]) s11[αpd] + (-1 + θ) (-1 + s12[αpd]) s12[αpd])
      (-θ (-1 + s21[αpd]) s21[αpd] + (-1 + θ) (-1 + s22[αpd]) s22[αpd])) (-Z2
      ϵ + δ2[αpd]))
H4DDβpdαpd = D[DβpdH4[αpd], αpd]
```

Out[1902]=

$$\Big(-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2+(P1-P2)\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]-2\,(P1-P2)\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)\Big/\Big(-s11^2\,s21^2\,\theta^2-\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd]+(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,s22[\alpha pd]-(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,s22[\alpha pd]^2\Big)+\Big(2\,(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,\big(\theta-\theta\,s12[\alpha pd])\,s21[\alpha pd]^2-(-1+\theta)\,s22[\alpha pd]\,(-1+s12[\alpha pd]+s22[\alpha pd])+\theta\,s21[\alpha pd]\,(-1+s12[\alpha pd]+s11[\alpha pd]\,s22[\alpha pd])\big)\Big)\Big/\Big(((-1+s12[\alpha pd])\,s12[\alpha pd]\,(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+(-\theta\,(-1+s11[\alpha pd])\,s11[\alpha pd]+(-1+\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd])\,(-\theta\,(-1+s21[\alpha pd])\,s21[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd]))\,(-Z1\,\epsilon+\delta1[\alpha pd])\Big)-\Big(2\,(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,(-\theta\,(-1+s11[\alpha pd])\,s11[\alpha pd]\,s22[\alpha pd]+(-1+s12[\alpha pd])\,s12[\alpha pd]\,(-1+s12[\alpha pd]+(-1+\theta)\,s22[\alpha pd])\Big)\Big/\Big(((-1+s12[\alpha pd])\,s12[\alpha pd]\,(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+(-\theta\,(-1+s11[\alpha pd])\,s11[\alpha pd]+(-1+\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd])\,(-\theta\,(-1+s21[\alpha pd])\,s21[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd]))\,(-Z2\,\epsilon+\delta2[\alpha pd])\Big)$$

Out[1903]=

$$-\Big(\Big(-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2+(P1-P2)\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]-2\,(P1-P2)\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)\Big(-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2-(P1-P2)\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]+2\,(P1-P2)\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)\Big)\Big/\Big(-s11^2\,s21^2\,\theta^2-\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd]+(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,s22[\alpha pd]-(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,s22[\alpha pd]^2\Big)^2+\Big(-(P1-P2)^2\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda^2\,(1-s12[\alpha pd])^2\,(-1+s12[\alpha pd])\,s12[\alpha pd]-3\,(P1-P2)^2\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda^2\,(1-s12[\alpha pd])^2\,s12[\alpha pd]^2+(P1-P2)^2\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda^2\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]^2+(P1-P2)^2\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda^2\,(1-s12[\alpha pd])\,s12[\alpha pd]^3+(P1-P2)^2\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,\lambda^2\,(1-s12[\alpha pd])\,s12[\alpha pd]\,s22[\alpha pd]-(P1-P2)^2\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))$$

$$\lambda^2\,\mathrm{s12}[\alpha\mathrm{pd}]^2\,\mathrm{s22}[\alpha\mathrm{pd}] - 2\,(\mathrm{P1}-\mathrm{P2})^2\,(-1+\theta)$$
$$(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\lambda^2\,(1-\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}]^2 +$$
$$4\,(\mathrm{P1}-\mathrm{P2})^2\,(-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\lambda^2\,\mathrm{s12}[\alpha\mathrm{pd}]^2\,\mathrm{s22}[\alpha\mathrm{pd}]^2\Big) \Big/$$
$$\Big(-\mathrm{s11}^2\,\mathrm{s21}^2\,\theta^2 - \theta\,((-1+\mathrm{s21})\,\mathrm{s21}\,(-1+\theta)-(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}] +$$
$$(-1+\theta)\,((-1+\mathrm{s11})\,\mathrm{s11}\,\theta+\mathrm{s21}\,(-1+\mathrm{s21}+\theta+(-1+2\,\mathrm{s11})\,\mathrm{s21}\,\theta))\,\mathrm{s22}[\alpha\mathrm{pd}] -$$
$$(-1+\theta)\,(\mathrm{s21}\,(-1+\theta)+(-1+\mathrm{s11})\,\mathrm{s11}\,\theta)\,\mathrm{s22}[\alpha\mathrm{pd}]^2\Big) -$$
$$\Big(2\,(\mathrm{P1}-\mathrm{P2})^2\,(-1+\theta)^2\,\lambda^2\,\mathrm{s12}[\alpha\mathrm{pd}]^2\,\big((\theta-\theta\,\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]^2-(-1+\theta)\,\mathrm{s22}[\alpha\mathrm{pd}]$$
$$(-1+\mathrm{s12}[\alpha\mathrm{pd}]+\mathrm{s22}[\alpha\mathrm{pd}])+\theta\,\mathrm{s21}[\alpha\mathrm{pd}]\,(-1+\mathrm{s12}[\alpha\mathrm{pd}]+\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])\big)^2\Big) \Big/$$
$$\Big(((-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$(-\theta\,(-1+\mathrm{s11}[\alpha\mathrm{pd}])\,\mathrm{s11}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}])$$
$$(-\theta\,(-1+\mathrm{s21}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\alpha\mathrm{pd}])\,\mathrm{s22}[\alpha\mathrm{pd}]))^2$$
$$(-\mathrm{Z1}\,\epsilon+\delta 1[\alpha\mathrm{pd}])^2\Big) + \Big(2\,(\mathrm{P1}-\mathrm{P2})^2\,(-1+\theta)\,\lambda^2\,(1-\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]$$
$$\Big((\theta-\theta\,\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]^2-(-1+\theta)\,\mathrm{s22}[\alpha\mathrm{pd}]\,(-1+\mathrm{s12}[\alpha\mathrm{pd}]+\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$\theta\,\mathrm{s21}[\alpha\mathrm{pd}]\,(-1+\mathrm{s12}[\alpha\mathrm{pd}]+\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])\Big)\Big) \Big/$$
$$(((-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$(-\theta\,(-1+\mathrm{s11}[\alpha\mathrm{pd}])\,\mathrm{s11}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}])$$
$$(-\theta\,(-1+\mathrm{s21}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\alpha\mathrm{pd}])\,\mathrm{s22}[\alpha\mathrm{pd}]))$$
$$(-\mathrm{Z1}\,\epsilon+\delta 1[\alpha\mathrm{pd}]))) - \Big(2\,(\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,\mathrm{s12}[\alpha\mathrm{pd}]$$
$$\Big((\theta-\theta\,\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]^2-(-1+\theta)\,\mathrm{s22}[\alpha\mathrm{pd}]\,(-1+\mathrm{s12}[\alpha\mathrm{pd}]+\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$\theta\,\mathrm{s21}[\alpha\mathrm{pd}]\,(-1+\mathrm{s12}[\alpha\mathrm{pd}]+\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])\Big)$$
$$\Big((\mathrm{P1}-\mathrm{P2})\,\lambda\,(1-\mathrm{s12}[\alpha\mathrm{pd}])\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]$$
$$(-\theta\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])+(\mathrm{P1}-\mathrm{P2})\,\lambda$$
$$(1-\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]^2\,(-\theta\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,\Big((\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,(1-\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}]-$$
$$(\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,\mathrm{s12}[\alpha\mathrm{pd}]^2\,\mathrm{s22}[\alpha\mathrm{pd}]\Big)+\Big((\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,(1-\mathrm{s12}[\alpha\mathrm{pd}])$$
$$(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]+(\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,(1-\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]^2\Big)$$
$$(-\theta\,(-1+\mathrm{s21}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\alpha\mathrm{pd}])\,\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$(-\theta\,(-1+\mathrm{s11}[\alpha\mathrm{pd}])\,\mathrm{s11}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}])$$
$$\Big(-(\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,\mathrm{s12}[\alpha\mathrm{pd}]\,(-1+\mathrm{s22}[\alpha\mathrm{pd}])\,\mathrm{s22}[\alpha\mathrm{pd}]-$$
$$(\mathrm{P1}-\mathrm{P2})\,(-1+\theta)\,\lambda\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}]^2\Big)\Big)\Big) \Big/$$
$$\Big(((-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$(-\theta\,(-1+\mathrm{s11}[\alpha\mathrm{pd}])\,\mathrm{s11}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}])$$
$$(-\theta\,(-1+\mathrm{s21}[\alpha\mathrm{pd}])\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s22}[\alpha\mathrm{pd}])\,\mathrm{s22}[\alpha\mathrm{pd}]))^2$$
$$(-\mathrm{Z1}\,\epsilon+\delta 1[\alpha\mathrm{pd}]))\Big) - \Big(2\,(\mathrm{P1}-\mathrm{P2})^2\,(-1+\theta)^2\,\lambda^2\,\mathrm{s12}[\alpha\mathrm{pd}]^2$$
$$(-\theta\,(-1+\mathrm{s11}[\alpha\mathrm{pd}])\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}]+$$
$$(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,(-1+\mathrm{s12}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s22}[\alpha\mathrm{pd}]))^2\Big) \Big/$$
$$\Big(((-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}]\,(-\theta\,\mathrm{s11}[\alpha\mathrm{pd}]\,\mathrm{s21}[\alpha\mathrm{pd}]+(-1+\theta)\,\mathrm{s12}[\alpha\mathrm{pd}]\,\mathrm{s22}[\alpha\mathrm{pd}])+$$
$$(-\theta\,(-1+\mathrm{s11}[\alpha\mathrm{pd}])\,\mathrm{s11}[\alpha\mathrm{pd}]+(-1+\theta)\,(-1+\mathrm{s12}[\alpha\mathrm{pd}])\,\mathrm{s12}[\alpha\mathrm{pd}])$$

$$
\begin{aligned}
&(-\Theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])^2\\
&(-\text{Z2}\,\epsilon+\delta 2[\alpha\text{pd}])^2\Big)-\Big(2\,(\text{P1}-\text{P2})^2\,(-1+\Theta)\,\lambda^2\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\\
&(-\Theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]+\\
&(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-1+\text{s12}[\alpha\text{pd}]+(-1+\Theta)\,\text{s22}[\alpha\text{pd}]))\Big)\Big/\\
&(((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\Theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+\\
&(-\Theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\\
&(-\Theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}]))\\
&(-\text{Z2}\,\epsilon+\delta 2[\alpha\text{pd}]))-\Big(2\,(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\\
&\Big((\text{P1}-\text{P2})\,\Theta\,\lambda\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]+(\text{P1}-\text{P2})\,\lambda\\
&(1-\text{s12}[\alpha\text{pd}])\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-1+\text{s12}[\alpha\text{pd}]+(-1+\Theta)\,\text{s22}[\alpha\text{pd}])+\\
&(\text{P1}-\text{P2})\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\,(-1+\text{s12}[\alpha\text{pd}]+(-1+\Theta)\,\text{s22}[\alpha\text{pd}])+\\
&(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\\
&((\text{P1}-\text{P2})\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]-(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])\Big)\Big)\Big/\\
&(((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\Theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+\\
&(-\Theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\\
&(-\Theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}]))\\
&(-\text{Z2}\,\epsilon+\delta 2[\alpha\text{pd}]))+\Big(2\,(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\\
&(-\Theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]+\\
&(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-1+\text{s12}[\alpha\text{pd}]+(-1+\Theta)\,\text{s22}[\alpha\text{pd}]))\\
&\Big((\text{P1}-\text{P2})\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\\
&(-\Theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+(\text{P1}-\text{P2})\,\lambda\\
&(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\,(-\Theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+\\
&(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,\Big((\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]-\\
&(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,\text{s12}[\alpha\text{pd}]^2\,\text{s22}[\alpha\text{pd}]\Big)+\Big((\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\\
&(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]+(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\Big)\\
&(-\Theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])+\\
&(-\Theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\\
&\Big(-(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}]-\\
&(\text{P1}-\text{P2})\,(-1+\Theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]^2\Big)\Big)\Big)\Big/\\
&\Big(((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\Theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+\\
&(-\Theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,(-\Theta\,(-1+\text{s21}[\\
&\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\Theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}]))^2\,(-\text{Z2}\,\epsilon+\delta 2[\alpha\text{pd}])\Big)
\end{aligned}
$$

Out[1905]=
$$\Big(-(-1+s11)\,s11\,\theta\,(s12[\alpha pd]-s22[\alpha pd])\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])-(-1+s11)\,s11\,\theta\,(-1+s12[\alpha pd]+s22[\alpha pd])\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]+(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])-s21^2\Big((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]\,(-1+\theta+s12[\alpha pd]-\theta\,s12[\alpha pd])+s12[\alpha pd]\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd])+\theta\Big((P1-P2)\,\lambda\,(1-s12[\alpha pd])^2\,s12[\alpha pd]-(P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2\Big)-(P1-P2)\,(-1+2\,s11)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]-(P1-P2)\,(1-\theta+2\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]\Big)+s21\,(-1+\theta)\Big((P1-P2)\,\lambda\,(1-s12[\alpha pd])^2\,s12[\alpha pd]-(P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2-(P1-P2)\,\lambda\,s12[\alpha pd]\,(-1+s22[\alpha pd])\,s22[\alpha pd]-(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)+s21\Big(-(P1-P2)\,\theta\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2-(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,(-1+s22[\alpha pd])\,s22[\alpha pd]-(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)-(-1+s11)\,s11\Big(-(P1-P2)\,\lambda\,s12[\alpha pd]\,(-1+s22[\alpha pd])\,s22[\alpha pd]-(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2+\theta\,(s12[\alpha pd]-s22[\alpha pd])\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])+\theta\,(-1+s12[\alpha pd]+s22[\alpha pd])\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]+(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])\Big)\Big)\Big/\Big(s21^2\Big(\theta\Big(-s11^2\,\theta+s12[\alpha pd]\,(-1+\theta+s12[\alpha pd]-\theta\,s12[\alpha pd])\Big)+(-1+\theta)\,(1-\theta+2\,s11\,\theta)\,s22[\alpha pd]\Big)-s21\,(-1+\theta)\,(-\theta\,(-1+s12[\alpha pd])\,s12[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd])+(-1+s11)\,s11\,\theta\,((-1+s22[\alpha pd])\,s22[\alpha pd]+\theta\,(s12[\alpha pd]-s22[\alpha pd])\,(-1+s12[\alpha pd]+s22[\alpha pd]))\Big)-\Big((-(-1+s11)\,s11\,\theta\,(s12[\alpha pd]-s22[\alpha pd])\,(-1+s12[\alpha pd]+s22[\alpha pd])-s21^2\Big(-s11^2\,\theta+s12[\alpha pd]\,(-1+\theta+s12[\alpha pd]-\theta\,s12[\alpha pd])+\theta\Big(-s11^2+(1-s12[\alpha pd])\,s12[\alpha pd]\Big)+(-1+2\,s11)\,(-1+\theta)\,s22[\alpha pd]+(1-\theta+2\,s11\,\theta)\,s22[\alpha pd]\Big)+s21\,(-1+\theta)\,((1-s12[\alpha pd])\,s12[\alpha pd]+(-1+s22[\alpha pd])\,s22[\alpha pd])+s21\,(-\theta\,(-1+s12[\alpha pd])\,s12[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd])-(-1+s11)\,s11\,((-1+s22[\alpha pd])\,s22[\alpha pd]+\theta\,(s12[\alpha pd]-s22[\alpha pd])\,(-1+s12[\alpha pd]+s22[\alpha pd]))\Big)\Big(s21^2\,(\theta\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]\,(-1+\theta+s12[\alpha pd]-\theta\,s12[\alpha pd])+s12[\alpha pd]\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]))-(P1-P2)\,(-1+\theta)\,(1-\theta+2\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])-s21\,(-1+\theta)\Big(-(P1-P2)\,\theta\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2-(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,(-1+s22[\alpha pd])\,s22[\alpha pd]-(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)+(-1+s11)\,s11\,\theta\Big(-(P1-P2)\,\lambda\,s12[\alpha pd]\,(-1+s22[\alpha pd])\,s22[\alpha pd]-(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2+\theta\,(s12[\alpha pd]-s22[\alpha pd])\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])+\theta\,(-1+s12[\alpha pd]+s22[\alpha pd])\,((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]+(P1-P2)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd])\Big)\Big)\Big)\Big/\Big(s21^2\Big(\theta\Big(-s11^2\,\theta+s12[\alpha pd]\,(-1+\theta+s12[\alpha pd]-\theta\,s12[\alpha pd])\Big)+(-1+\theta)\,(1-\theta+2\,s11\,\theta)\,s22[\alpha pd]\Big)-s21\,(-1+\theta)\,(-\theta\,(-1+s12[\alpha pd])\,s12[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd])+(-1+s11)\,s11\,\theta\,((-1+s22[\alpha pd])\,s22[\alpha pd]+\theta\,(s12[\alpha pd]-s22[\alpha pd])\,(-1+s12[\alpha pd]+s22[\alpha pd]))\Big)^2$$

Out[1907]=
$$-\Big(\Big(-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]-(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2-(P1-P2)\,(-1+\theta)$$

$$
\begin{aligned}
&((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]+\\
&2\,(P1-P2)\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)\\
&\Big(-P1\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\\
&s12[\alpha pd]-P1\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(1-s12[\alpha pd])\\
&s12[\alpha pd]^2-2\,P2\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,(1-s22[\alpha pd])\,s22[\alpha pd]^2+\\
&P2\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\\
&s22[\alpha pd]\,(1-s22[\beta pd])\Big)\Big)\Big/\\
&\Big(-s11^2\,s21^2\,\theta^2-\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd]+\\
&(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,s22[\alpha pd]-\\
&(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,s22[\alpha pd]^2\Big)^2+\\
&\Big(-P1\,(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\\
&(1-s12[\alpha pd])^2\,(-1+s12[\alpha pd])\,s12[\alpha pd]-\\
&3\,P1\,(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])^2\,s12[\alpha pd]^2+\\
&P1\,(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\\
&\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]^2+\\
&P1\,(P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^3+\\
&4\,(P1-P2)\,P2\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\\
&\lambda\,s12[\alpha pd]\,(1-s22[\alpha pd])\,s22[\alpha pd]^2-\\
&2\,(P1-P2)\,P2\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^3-\\
&(P1-P2)\,P2\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\\
&\lambda\,s12[\alpha pd]\,s22[\alpha pd]\,(1-s22[\beta pd])\Big)\Big/\\
&\Big(-s11^2\,s21^2\,\theta^2-\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd]+\\
&(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,s22[\alpha pd]-\\
&(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,s22[\alpha pd]^2\Big)-\\
&\Big(2\,(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,\Big((\theta-\theta\,s12[\alpha pd])\,s21[\alpha pd]^2-(-1+\theta)\,s22[\alpha pd]\\
&(-1+s12[\alpha pd]+s22[\alpha pd])+\theta\,s21[\alpha pd]\,(-1+s12[\alpha pd]+s11[\alpha pd]\,s22[\alpha pd])\Big)\\
&((-(-P1+P2\,\gamma\,\theta\,\lambda)\,s12[\alpha pd]-P2\,\gamma\,(-1+\theta)\,\lambda\,(-1+s22[\alpha pd]))\,s22[\alpha pd]\\
&(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+\\
&s12[\alpha pd]\,(P1\,(-1+\theta)\,(-1+s12[\alpha pd])+(P1\,\theta-P2\,\gamma\,\lambda)\,s22[\alpha pd])\\
&(-\theta\,(-1+s21[\alpha pd])\,s21[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd]))\Big)\Big/\\
&\Big(((-1+s12[\alpha pd])\,s12[\alpha pd]\,(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+\\
&(-\theta\,(-1+s11[\alpha pd])\,s11[\alpha pd]+(-1+\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd])\\
&(-\theta\,(-1+s21[\alpha pd])\,s21[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd]))^2\\
&(-Z1\,\epsilon+\delta1[\alpha pd])^2\Big)-\Big(2\,((-(-P1+P2\,\gamma\,\theta\,\lambda)\,s12[\alpha pd]-P2\,\gamma\,(-1+\theta)\,\lambda\,(-1+s22[\alpha pd]))\\
&s22[\alpha pd]\,(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+\\
&s12[\alpha pd]\,(P1\,(-1+\theta)\,(-1+s12[\alpha pd])+(P1\,\theta-P2\,\gamma\,\lambda)\,s22[\alpha pd])\\
&(-\theta\,(-1+s21[\alpha pd])\,s21[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd]))\\
&\Big((P1-P2)\,\lambda\,(1-s12[\alpha pd])\,(-1+s12[\alpha pd])\,s12[\alpha pd]\\
&(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+(P1-P2)\,\lambda\\
&(1-s12[\alpha pd])\,s12[\alpha pd]^2\,(-\theta\,s11[\alpha pd]\,s21[\alpha pd]+(-1+\theta)\,s12[\alpha pd]\,s22[\alpha pd])+\\
&(-1+s12[\alpha pd])\,s12[\alpha pd]\,\Big((P1-P2)\,(-1+\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]\,s22[\alpha pd]-\\
&(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]^2\,s22[\alpha pd]\Big)+\Big((P1-P2)\,(-1+\theta)\,\lambda\,(1-s12[\alpha pd])\\
&(-1+s12[\alpha pd])\,s12[\alpha pd]+(P1-P2)\,(-1+\theta)\,\lambda\,(1-s12[\alpha pd])\,s12[\alpha pd]^2\Big)\\
&(-\theta\,(-1+s21[\alpha pd])\,s21[\alpha pd]+(-1+\theta)\,(-1+s22[\alpha pd])\,s22[\alpha pd])+\\
&(-\theta\,(-1+s11[\alpha pd])\,s11[\alpha pd]+(-1+\theta)\,(-1+s12[\alpha pd])\,s12[\alpha pd])\\
&\Big(-(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,(-1+s22[\alpha pd])\,s22[\alpha pd]-\\
&(P1-P2)\,(-1+\theta)\,\lambda\,s12[\alpha pd]\,s22[\alpha pd]^2\Big)\Big)\Big)\Big/
\end{aligned}
$$

$$\Big(\big((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,(-\theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])\big)^2\,(-\text{Z1}\,\epsilon+\delta\text{1}[\alpha\text{pd}])\Big)+\Big(2\Big(-(\text{P1}-\text{P2})\,\lambda\,\text{s12}[\alpha\text{pd}]\,(-(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\alpha\text{pd}]-\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\alpha\text{pd}]))\,\text{s22}[\alpha\text{pd}]\,(-\theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+\text{s22}[\alpha\text{pd}]\,(-\theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])\Big(-(\text{P1}-\text{P2})\,\lambda\,(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]+(\text{P1}-\text{P2})\,\text{P2}\,\gamma\,(-1+\theta)\,\lambda^2\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]\Big)+(-(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\alpha\text{pd}]-\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\alpha\text{pd}]))\,\text{s22}[\alpha\text{pd}]\Big((\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]-(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,\text{s12}[\alpha\text{pd}]^2\,\text{s22}[\alpha\text{pd}]\Big)+(\text{P1}-\text{P2})\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\alpha\text{pd}])\,(-\theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])+\text{s12}[\alpha\text{pd}]\,(\text{P1}\,(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]-(\text{P1}-\text{P2})\,\lambda\,(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])\,(-\theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])+\text{s12}[\alpha\text{pd}]\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\alpha\text{pd}])\Big(-(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}]-(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]^2\Big)\Big)\Big)\Big/\Big((\big((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,(-\theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])\big)\,(-\text{Z1}\,\epsilon+\delta\text{1}[\alpha\text{pd}]))-\Big(2\,(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,\text{s12}[\alpha\text{pd}]\,(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]+(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-1+\text{s12}[\alpha\text{pd}]+(-1+\theta)\,\text{s22}[\alpha\text{pd}]))\,\Big((-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\alpha\text{pd}]+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\alpha\text{pd}]))\,\text{s22}[\alpha\text{pd}]+(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\alpha\text{pd}])\Big)\Big)\Big/\Big(\big((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,(-\theta\,(-1+\text{s21}[\alpha\text{pd}])\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s22}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}])\big)^2\,(-\text{Z2}\,\epsilon+\delta\text{2}[\alpha\text{pd}])^2\Big)-\Big(2\Big(-(\text{P1}-\text{P2})\,\lambda\,\text{s12}[\alpha\text{pd}]\,(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\alpha\text{pd}]+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\alpha\text{pd}]))\,\text{s22}[\alpha\text{pd}]+\Big((\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]+(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\Big)\,((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\alpha\text{pd}]+\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\alpha\text{pd}]))\,\text{s22}[\alpha\text{pd}]+2\,(\text{P1}-\text{P2})\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\alpha\text{pd}])+(\text{P1}-\text{P2})\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^3\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\alpha\text{pd}])+(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])\,\text{s22}[\alpha\text{pd}]\,\Big((\text{P1}-\text{P2})\,\lambda\,(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]-(\text{P1}-\text{P2})\,\text{P2}\,\gamma\,(-1+\theta)\,\lambda^2\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}]\Big)+(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]^2\,(\text{P1}\,(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,(1-\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]-(\text{P1}-\text{P2})\,\lambda\,(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])\Big)\Big)\Big/\Big(\big((-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}]\,(-\theta\,\text{s11}[\alpha\text{pd}]\,\text{s21}[\alpha\text{pd}]+(-1+\theta)\,\text{s12}[\alpha\text{pd}]\,\text{s22}[\alpha\text{pd}])+(-\theta\,(-1+\text{s11}[\alpha\text{pd}])\,\text{s11}[\alpha\text{pd}]+(-1+\theta)\,(-1+\text{s12}[\alpha\text{pd}])\,\text{s12}[\alpha\text{pd}])$$

```
        (-θ (-1 + s21[αpd]) s21[αpd] + (-1 + θ) (-1 + s22[αpd]) s22[αpd])) (-Z2
        ϵ + δ2[αpd])) +
    (2 ((-θ (-1 + s11[αpd]) s11[αpd] + (-1 + θ) (-1 + s12[αpd]) s12[αpd])
        ((-P1 + P2 γ θ λ) s12[αpd] + P2 γ (-1 + θ) λ (-1 + s22[αpd])) s22[αpd] +
       (-1 + s12[αpd]) s12[αpd]^2 (P1 (-1 + θ) (-1 + s12[αpd]) + (P1 θ - P2 γ λ) s22[αpd]))
      ((P1 - P2) λ (1 - s12[αpd]) (-1 + s12[αpd]) s12[αpd]
        (-θ s11[αpd] s21[αpd] + (-1 + θ) s12[αpd] s22[αpd]) + (P1 - P2) λ
        (1 - s12[αpd]) s12[αpd]^2 (-θ s11[αpd] s21[αpd] + (-1 + θ) s12[αpd] s22[αpd]) +
       (-1 + s12[αpd]) s12[αpd] ((P1 - P2) (-1 + θ) λ (1 - s12[αpd]) s12[αpd] s22[αpd] -
          (P1 - P2) (-1 + θ) λ s12[αpd]^2 s22[αpd]) +
       ((P1 - P2) (-1 + θ) λ (1 - s12[αpd]) (-1 + s12[αpd]) s12[αpd] +
          (P1 - P2) (-1 + θ) λ (1 - s12[αpd]) s12[αpd]^2)
        (-θ (-1 + s21[αpd]) s21[αpd] + (-1 + θ) (-1 + s22[αpd]) s22[αpd]) +
       (-θ (-1 + s11[αpd]) s11[αpd] + (-1 + θ) (-1 + s12[αpd]) s12[αpd])
        (-(P1 - P2) (-1 + θ) λ s12[αpd] (-1 + s22[αpd]) s22[αpd] -
          (P1 - P2) (-1 + θ) λ s12[αpd] s22[αpd]^2))) /
     (((-1 + s12[αpd]) s12[αpd] (-θ s11[αpd] s21[αpd] + (-1 + θ) s12[αpd] s22[αpd]) +
        (-θ (-1 + s11[αpd]) s11[αpd] + (-1 + θ) (-1 + s12[αpd]) s12[αpd]) (-θ (-1 + s21[
            αpd]) s21[αpd] + (-1 + θ) (-1 + s22[αpd]) s22[αpd]))^2 (-Z2 ϵ + δ2[αpd]))
```

In[1908]:=
```
(*----------------------------------------------------Calculating
 Gradient w.r.t θrd-----------------------------------------*)
(*Main definition correction*)
Sp1s1δ1'[δ1_] := s11[θrd] * (1 - s11[θrd])
Sp1s1δ2'[δ2_] := -s11[θrd] * s21[θrd]
Sp1s2δ1'[δ1_] := s12[θrd] * (1 - s12[θrd])
Sp1s2δ2'[δ2_] := -s12[θrd] * s22[θrd]
Sp2s1δ1'[δ1_] := -s11[θrd] * s21[θrd]
Sp2s1δ2'[δ2_] := (1 - s21[θrd]) * s21[θrd]
Sp2s2δ1'[δ1_] := -s12[θrd] * s22[θrd]
Sp2s2δ2'[δ2_] := (1 - s22[θrd]) * s22[θrd]
Sp1δ1[δ1_] := θ * Sp1s1δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp1δ2[δ2_] := θ * Sp1s1δ2[δ2] + (1 - θ) * Sp1s2δ2[δ2]
Sp2δ1[δ1_] := θ * Sp1s2δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp2δ2[δ2_] := θ * Sp2s1δ2[δ2] + (1 - θ) * Sp2s2δ2[δ2]
Sp1[δ1_, δ2_] := Sp1δ1[δ1] + Sp1δ2[δ2]
Sp2[δ1_, δ2_] := Sp2δ1[δ1] + Sp2δ2[δ2]
(*Definitions: derivative with respect to parameter βpd,
for the first period:*)
Ds1βpd[θrd_] := - (1 - θ) * s12[θrd] * s22[θrd] * (1 - λ) * γ * (Dur1 / 2 + γ * Dur2)
Ds2βpd[θrd_] := (1 - θ) (1 - s22[θrd]) * s22[θrd] * (1 - λ) * γ * (Dur1 / 2 + γ * Dur2)
Dδ1βpd[θrd_] :=
 FullSimplify[(D[Sp2δ2[δ2], δ2] * Ds1βpd[θrd] - D[Sp1δ2[δ2], δ2] * Ds2βpd[θrd]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
Dδ2βpd[θrd_] := FullSimplify[
  (D[Sp2δ1[δ1], δ1] * Ds1βpd[θrd] - D[Sp1δ1[δ1], δ1] * Ds2βpd[θrd]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
```

```
s12'[θrd_] := -s12[θrd] * s22[θrd] * (1 - λ) * γ * (Dur1 / 2 + γ * Dur2)
s22'[θrd_] := (1 - s22[θrd]) * s22[θrd] * (1 - λ) * γ * (Dur1 / 2 + γ * Dur2)
s11'[θrd_] := 0
s21'[θrd_] := 0
δ1'[θrd_] := Dδ1βpd[θrd]
δ2'[θrd_] := Dδ2βpd[θrd]
DNErrDens[θrd_] := 2 Dδ1βpd[θrd] / (-Z1 ϵ + δ1[θrd]) + 2 Dδ2βpd[θrd] / (-Z2 ϵ + δ2[θrd])
NJβpd[θrd_] :=
 FullSimplify[Log[s21 s22[θrd] - s21^2 s22[θrd] - s21 s22[θrd]^2 + s12[θrd] * s21 θ -
    s12[θrd]^2 * s21 θ - s12[θrd] * s21^2 θ + s12[θrd]^2 s21^2 θ + s11 s22[θrd] θ -
    s11^2 s22[θrd] θ - 2 s21 s22[θrd] θ + 2 s21^2 s22[θrd] θ - 2 s11 s21^2 s22 [θrd] θ -
    s11 s22[θrd]^2 θ + s11^2 s22[θrd]^2 θ + 2 s21 s22[θrd]^2 θ + s11 * s12[θrd] * θ^2 -
    s11^2 * s12[θrd] * θ^2 - s11 * s12[θrd]^2 θ^2 + s11^2 * s12[θrd]^2 θ^2 - s12[θrd] * s21 θ^2 +
    s12[θrd]^2 * s21 θ^2 - s11^2 s21^2 θ^2 + s12[θrd] * s21^2 θ^2 - s12[θrd]^2 * s21^2 θ^2 -
    s11 s22[θrd] θ^2 + s11^2 s22[θrd] θ^2 + s21 s22[θrd] θ^2 - s21^2 s22[θrd] θ^2 +
    2 s11 s21^2 s22[θrd] θ^2 + s11 s22[θrd]^2 θ^2 - s11^2 s22[θrd]^2 θ^2 - s21 s22[θrd]^2 θ^2]]
DNJβpd[θrd_] := D[NJβpd[θrd], θrd]
DNLLβpd[θrd_] := DNJβpd[θrd] + DNErrDens[θrd]
DNLLβpd[θrd]
DDβpd = D[DNLLβpd[θrd], θrd]
DθH3[θrd_] := -(-s21 (-(-1 + s12[θrd]) s12[θrd] + (-1 + s22[θrd]) s22[θrd]) (-1 + θ) +
     (-1 + s11) s11 (s12[θrd] - s22[θrd]) (-1 + s12[θrd] + s22[θrd]) θ -
     s21 ((-1 + s22[θrd]) s22[θrd] (-1 + θ) - (-1 + s12[θrd]) s12[θrd] θ) + (-1 + s11) s11
      ((-1 + s22[θrd]) s22[θrd] + (s12[θrd] - s22[θrd]) (-1 + s12[θrd] + s22[θrd]) θ) +
     s21^2 ((-1 + 2 s11) s22[θrd] (-1 + θ) - s11^2 θ + (-s11^2 + (1 - s12[θrd]) s12[θrd]) θ +
        s22[θrd] (1 - θ + 2 s11 θ) + s12 [θrd] (-1 + s12[θrd] + θ - s12[θrd] θ))) /
  (-s21 (-1 + θ) ((-1 + s22[θrd]) s22[θrd] (-1 + θ) - (-1 + s12[θrd]) s12[θrd] θ) +
    (-1 + s11) s11 θ
     ((-1 + s22[θrd]) s22[θrd] + (s12[θrd] - s22[θrd]) (-1 + s12[θrd] + s22[θrd]) θ) +
    s21^2 (s22[θrd] (-1 + θ) (1 - θ + 2 s11 θ) +
       θ (-s11^2 θ + s12[θrd] (-1 + s12[θrd] + θ - s12[θrd] θ))))
H5DDθθrd = D[DθH3[θrd], θrd]
DβpdH5[θrd_] :=
 (-P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[θrd]) (-1 + s12[θrd]) s12[θrd] -
     P1 θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (1 - s12[θrd]) s12[θrd]^2 +
     P2 (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) (1 - s22[θrd])
      s22[θrd] - 2 P2 (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) (1 - s22[θrd]) s22[θrd]^2) /
   (-s11^2 s21^2 θ^2 - θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (-1 + s12[θrd]) s12[θrd] +
     (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) s22[θrd] -
     (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) s22[θrd]^2) +
  (2 (-((-P1 + P2 γ θ λ) s12[θrd] + P2 γ (-1 + θ) λ (-1 + s22[θrd])) s22[θrd]
        (-θ s11[θrd] s21[θrd] + (-1 + θ) s12[θrd] s22[θrd]) +
       s12[θrd] (P1 (-1 + θ) (-1 + s12[θrd]) + (P1 θ - P2 γ λ) s22[θrd])
        (-θ (-1 + s21[θrd]) s21[θrd] + (-1 + θ) (-1 + s22[θrd]) s22[θrd]))) /
   (((-1 + s12[θrd]) s12[θrd] (-θ s11[θrd] s21[θrd] + (-1 + θ) s12[θrd] s22[θrd]) +
```

```
        (-θ (-1 + s11[θrd]) s11[θrd] + (-1 + θ) (-1 + s12[θrd]) s12[θrd])
         (-θ (-1 + s21[θrd]) s21[θrd] + (-1 + θ) (-1 + s22[θrd]) s22[θrd])) (-Z1 ϵ +
        δ1[θrd])) - (2 ((-θ (-1 + s11[θrd]) s11[θrd] + (-1 + θ) (-1 + s12[θrd]) s12[θrd])
         ((-P1 + P2 γ θ λ) s12[θrd] + P2 γ (-1 + θ) λ (-1 + s22[θrd])) s22[θrd] + (-1 +
           s12[θrd]) s12[θrd]^2 (P1 (-1 + θ) (-1 + s12[θrd]) + (P1 θ - P2 γ λ) s22[θrd]))) /
   (((-1 + s12[θrd]) s12[θrd] (-θ s11[θrd] s21[θrd] + (-1 + θ) s12[θrd] s22[θrd]) +
        (-θ (-1 + s11[θrd]) s11[θrd] + (-1 + θ) (-1 + s12[θrd]) s12[θrd])
         (-θ (-1 + s21[θrd]) s21[θrd] + (-1 + θ) (-1 + s22[θrd]) s22[θrd])) (-Z2
         ϵ + δ2[θrd]))
H5DDβpdθrd = D[Dβpdh5[θrd], θrd]
Dαpdh6[θrd_] :=
 (-(P1 - P2) θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) λ (1 - s12[θrd]) (-1 + s12[θrd])
      s12[θrd] - (P1 - P2) θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ)
      λ (1 - s12[θrd]) s12[θrd]^2 + (P1 - P2) (-1 + θ)
      ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) λ s12[θrd] s22[θrd] -
     2 (P1 - P2) (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) λ s12[θrd] s22[θrd]^2) /
   (-s11^2 s21^2 θ^2 - θ ((-1 + s21) s21 (-1 + θ) - (-1 + s11) s11 θ) (-1 + s12[θrd]) s12[θrd] +
     (-1 + θ) ((-1 + s11) s11 θ + s21 (-1 + s21 + θ + (-1 + 2 s11) s21 θ)) s22[θrd] -
     (-1 + θ) (s21 (-1 + θ) + (-1 + s11) s11 θ) s22[θrd]^2) +
  (2 (P1 - P2) (-1 + θ) λ s12[θrd] ((θ - θ s12[θrd]) s21[θrd]^2 - (-1 + θ) s22[θrd]
        (-1 + s12[θrd] + s22[θrd]) + θ s21[θrd] (-1 + s12[θrd] + s11[θrd] s22[θrd]))) /
   (((-1 + s12[θrd]) s12[θrd] (-θ s11[θrd] s21[θrd] + (-1 + θ) s12[θrd] s22[θrd]) +
        (-θ (-1 + s11[θrd]) s11[θrd] + (-1 + θ) (-1 + s12[θrd]) s12[θrd])
         (-θ (-1 + s21[θrd]) s21[θrd] + (-1 + θ) (-1 + s22[θrd]) s22[θrd]))
     (-Z1 ϵ + δ1[θrd])) - (2 (P1 - P2) (-1 + θ) λ s12[θrd] (-θ (-1 + s11[θrd]) s11[θrd]
        s22[θrd] + (-1 + s12[θrd]) s12[θrd] (-1 + s12[θrd] + (-1 + θ) s22[θrd])))) /
   (((-1 + s12[θrd]) s12[θrd] (-θ s11[θrd] s21[θrd] + (-1 + θ) s12[θrd] s22[θrd]) +
        (-θ (-1 + s11[θrd]) s11[θrd] + (-1 + θ) (-1 + s12[θrd]) s12[θrd])
         (-θ (-1 + s21[θrd]) s21[θrd] + (-1 + θ) (-1 + s22[θrd]) s22[θrd])) (-Z2
         ϵ + δ2[θrd]))
H6DDαpdθrd = D[Dαpdh6[θrd], θrd]
```

Out[1936]=
$$\Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta$$
$$((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]+$$
$$\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-2\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)$$
$$(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\Big)\Big/$$
$$\big(-\text{s11}^2\,\text{s21}^2\,\theta^2-\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]+$$
$$(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))\,\text{s22}[\theta\text{rd}]-$$
$$(-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,\text{s22}[\theta\text{rd}]^2\big)+$$
$$(\gamma\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)\,(-1+\theta)\,\theta\,(-1+\lambda)\,\text{s21}[\theta\text{rd}]$$
$$(\text{s12}[\theta\text{rd}]\,(-1+\text{s21}[\theta\text{rd}])-\text{s11}[\theta\text{rd}]\,(-1+\text{s22}[\theta\text{rd}]))\,\text{s22}[\theta\text{rd}])\,/$$
$$(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+$$
$$(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])$$
$$(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))$$
$$(-\text{Z1}\,\epsilon+\delta1[\theta\text{rd}]))+\big(\gamma\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)\,(-1+\theta)\,(-1+\lambda)\,\text{s22}[\theta\text{rd}]$$
$$\big((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(\text{s12}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}]))+$$
$$\theta\,\text{s11}[\theta\text{rd}]\,(-1+\text{s22}[\theta\text{rd}])+\text{s11}[\theta\text{rd}]^2\,(\theta-\theta\,\text{s22}[\theta\text{rd}])\big)\big)\,/$$
$$(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+$$
$$(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])$$
$$(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))\,(-\text{Z2}$$
$$\epsilon+\delta2[\theta\text{rd}]))$$

Out[1937]=
$$-\Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta$$
$$((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]+$$
$$\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-2\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)$$
$$(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\Big)^2\Big/$$
$$\big(-\text{s11}^2\,\text{s21}^2\,\theta^2-\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]+$$
$$(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))\,\text{s22}[\theta\text{rd}]-$$
$$(-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,\text{s22}[\theta\text{rd}]^2\big)^2+$$
$$\Big(\gamma^2\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)^2$$
$$(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+$$
$$\gamma^2\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)^2\,\text{s12}[\theta\text{rd}]^2\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+$$

$$\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2 (-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)^2\,(1-\text{s22}[\theta\text{rd}])^2\,\text{s22}[\theta\text{rd}]-$$
$$\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2 \theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)^2$$
$$(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]^2-3\,\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2 \theta$$
$$((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)^2\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]^2-$$
$$\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2 (-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)^2\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2-4\,\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2 (-1+\theta)$$
$$(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)^2\,(1-\text{s22}[\theta\text{rd}])^2\,\text{s22}[\theta\text{rd}]^2+$$
$$2\,\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)^2 (-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)^2\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^3\Big)\Big/$$
$$\Big(-\text{s11}^2\,\text{s21}^2\,\theta^2-\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]+$$
$$(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))\,\text{s22}[\theta\text{rd}]-$$
$$(-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,\text{s22}[\theta\text{rd}]^2\Big)-$$
$$\Big(\gamma^2\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)^2\,(-1+\theta)^2\,\theta^2\,(-1+\lambda)^2\,\text{s21}[\theta\text{rd}]^2$$
$$(\text{s12}[\theta\text{rd}]\,(-1+\text{s21}[\theta\text{rd}])-\text{s11}[\theta\text{rd}]\,(-1+\text{s22}[\theta\text{rd}]))^2\,\text{s22}[\theta\text{rd}]^2\Big)\Big/$$
$$\Big(2\,((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+$$
$$(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])$$
$$(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))^2$$
$$(-\text{Z1}\,\epsilon+\delta\text{1}[\theta\text{rd}])^2\Big)+\Big(\gamma^2 \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(\text{Dur1}+2\,\text{Dur2}\,\gamma)\,(-1+\theta)\,\theta\,(1-\lambda)$$
$$(-1+\lambda)\,\text{s21}[\theta\text{rd}]\,(\text{s12}[\theta\text{rd}]\,(-1+\text{s21}[\theta\text{rd}])-\text{s11}[\theta\text{rd}]\,(-1+\text{s22}[\theta\text{rd}]))$$
$$(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\Big)\Big/$$
$$(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+$$
$$(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])$$
$$(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))$$
$$(-\text{Z1}\,\epsilon+\delta\text{1}[\theta\text{rd}]))+\Big(\gamma\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)\,(-1+\theta)\,\theta\,(-1+\lambda)\,\text{s21}[\theta\text{rd}]$$
$$\text{s22}[\theta\text{rd}]\left(-\gamma \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,(-1+\text{s21}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-\right.$$
$$\left.\gamma \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s11}[\theta\text{rd}]\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\right)\Big)\Big/$$
$$(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+$$
$$(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])$$
$$(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))$$
$$(-\text{Z1}\,\epsilon+\delta\text{1}[\theta\text{rd}]))-\Big(\gamma\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)\,(-1+\theta)\,\theta\,(-1+\lambda)\,\text{s21}[\theta\text{rd}]$$
$$(\text{s12}[\theta\text{rd}]\,(-1+\text{s21}[\theta\text{rd}])-\text{s11}[\theta\text{rd}]\,(-1+\text{s22}[\theta\text{rd}]))\,\text{s22}[\theta\text{rd}]$$
$$\left(-\gamma \left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]\right.$$

$$
\begin{aligned}
&(-\theta\, s11[\theta rd]\, s21[\theta rd] + (-1+\theta)\, s12[\theta rd]\, s22[\theta rd]) - \gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(1-\lambda)\\
&s12[\theta rd]^2\, s22[\theta rd]\,(-\theta\, s11[\theta rd]\, s21[\theta rd] + (-1+\theta)\, s12[\theta rd]\, s22[\theta rd]) +\\
&\left(-\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)(1-\lambda)(-1+s12[\theta rd])\, s12[\theta rd]\, s22[\theta rd] -\right.\\
&\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)(1-\lambda)\, s12[\theta rd]^2\, s22[\theta rd]\right)\\
&(-\theta\,(-1+s21[\theta rd]))\, s21[\theta rd] + (-1+\theta)(-1+s22[\theta rd])\, s22[\theta rd]) +\\
&(-1+s12[\theta rd])\, s12[\theta rd]\left(\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)(1-\lambda)\, s12[\theta rd]\,(1-s22[\theta rd])\right.\\
&\left.s22[\theta rd] - \gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)(1-\lambda)\, s12[\theta rd]\, s22[\theta rd]^2\right) +\\
&(-\theta\,(-1+s11[\theta rd]))\, s11[\theta rd] + (-1+\theta)(-1+s12[\theta rd])\, s12[\theta rd])\\
&\left(\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)(1-\lambda)(1-s22[\theta rd])(-1+s22[\theta rd])\, s22[\theta rd] +\right.\\
&\left.\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)(1-\lambda)(1-s22[\theta rd])\, s22[\theta rd]^2\right)\right)\Big/\\
&\Big(((-1+s12[\theta rd])\, s12[\theta rd]\,(-\theta\, s11[\theta rd]\, s21[\theta rd] + (-1+\theta)\, s12[\theta rd]\, s22[\theta rd]) +\\
&(-\theta\,(-1+s11[\theta rd]))\, s11[\theta rd] + (-1+\theta)(-1+s12[\theta rd])\, s12[\theta rd])\\
&(-\theta\,(-1+s21[\theta rd]))\, s21[\theta rd] + (-1+\theta)(-1+s22[\theta rd])\, s22[\theta rd]))^2\\
&(-Z1\,\epsilon + \delta1[\theta rd])\Big) - \Big(\gamma^2\,(Dur1 + 2\,Dur2\,\gamma)^2\,(-1+\theta)^2\,(-1+\lambda)^2\, s22[\theta rd]^2\\
&\Big((-1+s12[\theta rd])\, s12[\theta rd]\,(s12[\theta rd] + (-1+\theta)(-1+s22[\theta rd])) +\\
&\theta\, s11[\theta rd]\,(-1+s22[\theta rd]) + s11[\theta rd]^2\,(\theta - \theta\, s22[\theta rd]))^2\Big)\Big/\\
&\Big(2\,((-1+s12[\theta rd])\, s12[\theta rd]\,(-\theta\, s11[\theta rd]\, s21[\theta rd] + (-1+\theta)\, s12[\theta rd]\, s22[\theta rd]) +\\
&(-\theta\,(-1+s11[\theta rd]))\, s11[\theta rd] + (-1+\theta)(-1+s12[\theta rd])\, s12[\theta rd])\\
&(-\theta\,(-1+s21[\theta rd]))\, s21[\theta rd] + (-1+\theta)(-1+s22[\theta rd])\, s22[\theta rd]))^2\\
&(-Z2\,\epsilon + \delta2[\theta rd])^2\Big) + \Big(\gamma^2\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(Dur1 + 2\,Dur2\,\gamma)(-1+\theta)\\
&(1-\lambda)(-1+\lambda)(1-s22[\theta rd])\, s22[\theta rd]\\
&\Big((-1+s12[\theta rd])\, s12[\theta rd]\,(s12[\theta rd] + (-1+\theta)(-1+s22[\theta rd])) +\\
&\theta\, s11[\theta rd]\,(-1+s22[\theta rd]) + s11[\theta rd]^2\,(\theta - \theta\, s22[\theta rd]))\Big)\Big/\\
&(((-1+s12[\theta rd])\, s12[\theta rd]\,(-\theta\, s11[\theta rd]\, s21[\theta rd] + (-1+\theta)\, s12[\theta rd]\, s22[\theta rd]) +\\
&(-\theta\,(-1+s11[\theta rd]))\, s11[\theta rd] + (-1+\theta)(-1+s12[\theta rd])\, s12[\theta rd])\\
&(-\theta\,(-1+s21[\theta rd]))\, s21[\theta rd] + (-1+\theta)(-1+s22[\theta rd])\, s22[\theta rd]))\\
&(-Z2\,\epsilon + \delta2[\theta rd]))) + \Big(\gamma\,(Dur1 + 2\,Dur2\,\gamma)(-1+\theta)(-1+\lambda)\, s22[\theta rd]\\
&\left(-\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(1-\lambda)(-1+s12[\theta rd])\, s12[\theta rd]\right.\\
&\left.(s12[\theta rd] + (-1+\theta)(-1+s22[\theta rd]))\, s22[\theta rd] -\right.\\
&\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(1-\lambda)\, s12[\theta rd]^2\,(s12[\theta rd] + (-1+\theta)(-1+s22[\theta rd]))\, s22[\theta rd] +\right.\\
&\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)\theta\,(1-\lambda)\, s11[\theta rd]\,(1-s22[\theta rd])\, s22[\theta rd] -\right.\\
&\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)\theta\,(1-\lambda)\, s11[\theta rd]^2\,(1-s22[\theta rd])\, s22[\theta rd] +\right.
\end{aligned}
$$

$$(-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]\right)\Big)\Big/$$
$$(((-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+(-\theta\ (-1+\text{s11}[\theta\text{rd}])\ \text{s11}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}])\ (-\theta\ (-1+\text{s21}[\theta\text{rd}])\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]))\ (-\text{Z2}\,\epsilon+\delta 2[\theta\text{rd}]))-\Big(\gamma\ (\text{Dur1}+2\,\text{Dur2}\,\gamma)\ (-1+\theta)\ (-1+\lambda)\ \text{s22}[\theta\text{rd}]$$
$$\big((-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ (\text{s12}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}]))+\theta\ \text{s11}[\theta\text{rd}]\ (-1+\text{s22}[\theta\text{rd}])+\text{s11}[\theta\text{rd}]^2\ (\theta-\theta\ \text{s22}[\theta\text{rd}])\big)$$
$$\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]\right.$$
$$\left.(-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\right.$$
$$\left.\text{s12}[\theta\text{rd}]^2\ \text{s22}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+\right.$$
$$\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]-\right.$$
$$\left.\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]^2\ \text{s22}[\theta\text{rd}]\right)$$
$$(-\theta\ (-1+\text{s21}[\theta\text{rd}])\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])+$$
$$(-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]\ (1-\text{s22}[\theta\text{rd}])\right.$$
$$\left.\text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]^2\right)+$$
$$(-\theta\ (-1+\text{s11}[\theta\text{rd}])\ \text{s11}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}])$$
$$\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]+\right.$$
$$\left.\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\ (1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]^2\right)\Big)\Big)\Big/$$
$$\big(((-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+(-\theta\ (-1+\text{s11}[\theta\text{rd}])\ \text{s11}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}])\ (-\theta\ (-1+\text{s21}[\theta\text{rd}])\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]))^2\ (-\text{Z2}\,\epsilon+\delta 2[\theta\text{rd}])\big)$$

Out[1939]=
$$\Big(-(-1+\text{s11})\ \text{s11}\,\theta\ (-1+\text{s12}[\theta\text{rd}]+\text{s22}[\theta\text{rd}])$$
$$\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]-\right.$$
$$\left.\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]\right)-$$
$$(-1+\text{s11})\ \text{s11}\,\theta\ (\text{s12}[\theta\text{rd}]-\text{s22}[\theta\text{rd}])\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]+\right.$$
$$\left.\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]\right)+$$
$$\text{s21}\ (-1+\theta)\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\ (1-\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]+\right.$$

$$
\begin{aligned}
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\Big)+\\
&\text{s21}\Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\Big)-\\
&\text{s21}^2\Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,(-1+\theta+\text{s12}[\theta\text{rd}]-\theta\,\text{s12}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\\
&(-1+2\,\text{s11})\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\theta+2\,\text{s11}\,\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\\
&\text{s12}[\theta\text{rd}]\Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]\Big)+\\
&\theta\Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]\Big)\Big)-\\
&(-1+\text{s11})\,\text{s11}\Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2+\\
&\theta\,(-1+\text{s12}[\theta\text{rd}]+\text{s22}[\theta\text{rd}])\Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]-\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\Big)+\\
&\theta\,(\text{s12}[\theta\text{rd}]-\text{s22}[\theta\text{rd}])\Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\\
&\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\Big)\Big)\Big)\Big/\\
&\Big(\text{s21}^2\left(\theta\left(-\text{s11}^2\,\theta+\text{s12}[\theta\text{rd}]\,(-1+\theta+\text{s12}[\theta\text{rd}]-\theta\,\text{s12}[\theta\text{rd}])\right)+\right.\\
&\left.(-1+\theta)\,(1-\theta+2\,\text{s11}\,\theta)\,\text{s22}[\theta\text{rd}]\right)-\\
&\text{s21}\,(-1+\theta)\,(-\theta\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}])+\\
&(-1+\text{s11})\,\text{s11}\,\theta\\
&\left((-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\theta\,(\text{s12}[\theta\text{rd}]-\text{s22}[\theta\text{rd}])\,(-1+\text{s12}[\theta\text{rd}]+\text{s22}[\theta\text{rd}])\right)\Big)-
\end{aligned}
$$

$$\Big(\Big(-(-1+s11)\,s11\,\theta\,(s12[\theta rd]-s22[\theta rd])\,(-1+s12[\theta rd]+s22[\theta rd])-s21^2\,\Big(-s11^2\,\theta+ s12[\theta rd]\,(-1+\theta+s12[\theta rd]-\theta\,s12[\theta rd])+\theta\,\Big(-s11^2+(1-s12[\theta rd])\,s12[\theta rd]\Big)+ (-1+2\,s11)\,(-1+\theta)\,s22[\theta rd]+(1-\theta+2\,s11\,\theta)\,s22[\theta rd]\Big)+ s21\,(-1+\theta)\,((1-s12[\theta rd])\,s12[\theta rd]+(-1+s22[\theta rd])\,s22[\theta rd])+s21 (-\theta\,(-1+s12[\theta rd])\,s12[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd])-(-1+s11)\,s11 ((-1+s22[\theta rd])\,s22[\theta rd]+\theta\,(s12[\theta rd]-s22[\theta rd])\,(-1+s12[\theta rd]+s22[\theta rd]))\Big)$$

$$\Big(-s21\,(-1+\theta)\,\Big(\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,\theta\,(1-\lambda)\,(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd]+ \gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,\theta\,(1-\lambda)\,s12[\theta rd]^2\,s22[\theta rd]+ \gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,(-1+s22[\theta rd])\,s22[\theta rd]+ \gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]^2\Big)+ (-1+s11)\,s11\,\theta\,\Big(\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,(1-s22[\theta rd])\,(-1+s22[\theta rd])\,s22[\theta rd]+ \gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]^2+ \theta\,(-1+s12[\theta rd]+s22[\theta rd])\,\Big(-\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]- \gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]\Big)+ \theta\,(s12[\theta rd]-s22[\theta rd])\,\Big(-\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]+ \gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]\Big)\Big)+ s21^2\,\Big(\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(-1+\theta)\,(1-\theta+2\,s11\,\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]+ \theta\,\Big(-\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,s12[\theta rd]\,(-1+\theta+s12[\theta rd]-\theta\,s12[\theta rd])\,s22[\theta rd]+ s12[\theta rd]\,\Big(-\gamma\,\Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]+\gamma \Big(\frac{Dur1}{2}+Dur2\,\gamma\Big)\,\theta\,(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]\Big)\Big)\Big)\Big)\Big/$$

$$\Big(s21^2\,\Big(\theta\,\Big(-s11^2\,\theta+s12[\theta rd]\,(-1+\theta+s12[\theta rd]-\theta\,s12[\theta rd])\Big)+ (-1+\theta)\,(1-\theta+2\,s11\,\theta)\,s22[\theta rd]\Big)-s21\,(-1+\theta) (-\theta\,(-1+s12[\theta rd])\,s12[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd])+(-1+s11)\,s11\,\theta ((-1+s22[\theta rd])\,s22[\theta rd]+\theta\,(s12[\theta rd]-s22[\theta rd])\,(-1+s12[\theta rd]+s22[\theta rd]))\Big)^2$$

Out[1941]=

$$-\Big(\Big(-P1\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(1-s12[\theta rd])\,(-1+s12[\theta rd])\,s12[\theta rd]-P1\,\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(1-s12[\theta rd])\,s12[\theta rd]^2+ P2\,(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,(1-s22[\theta rd])\,s22[\theta rd]-2\,P2\,(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,(1-s22[\theta rd])\,s22[\theta rd]^2\Big)$$

$$\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\right.$$
$$(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta$$
$$((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]+$$
$$\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-2\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)$$
$$\left.(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\right)\Big)\Big/$$
$$\left(-\text{s11}^2\,\text{s21}^2\,\theta^2-\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]+\right.$$
$$(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))\,\text{s22}[\theta\text{rd}]-$$
$$\left.(-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,\text{s22}[\theta\text{rd}]^2\right)^2+$$
$$\left(\text{P1}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\right.$$
$$(1-\text{s12}[\theta\text{rd}])\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+$$
$$3\,\text{P1}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)\,(1-\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]-$$
$$\text{P1}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]-$$
$$\text{P1}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)$$
$$(1-\lambda)\,\text{s12}[\theta\text{rd}]^3\,\text{s22}[\theta\text{rd}]+$$
$$\text{P2}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])^2\,\text{s22}[\theta\text{rd}]-$$
$$\text{P2}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))$$
$$(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2-4\,\text{P2}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)$$
$$(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])^2\,\text{s22}[\theta\text{rd}]^2+$$
$$2\,\text{P2}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)$$
$$\left.(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^3\right)\Big/$$
$$\left(-\text{s11}^2\,\text{s21}^2\,\theta^2-\theta\,((-1+\text{s21})\,\text{s21}\,(-1+\theta)-(-1+\text{s11})\,\text{s11}\,\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]+\right.$$
$$(-1+\theta)\,((-1+\text{s11})\,\text{s11}\,\theta+\text{s21}\,(-1+\text{s21}+\theta+(-1+2\,\text{s11})\,\text{s21}\,\theta))\,\text{s22}[\theta\text{rd}]-$$
$$\left.(-1+\theta)\,(\text{s21}\,(-1+\theta)+(-1+\text{s11})\,\text{s11}\,\theta)\,\text{s22}[\theta\text{rd}]^2\right)-$$
$$(\gamma\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)\,(-1+\theta)\,\theta\,(-1+\lambda)\,\text{s21}[\theta\text{rd}]$$
$$(\text{s12}[\theta\text{rd}]\,(-1+\text{s21}[\theta\text{rd}])-\text{s11}[\theta\text{rd}]\,(-1+\text{s22}[\theta\text{rd}]))\,\text{s22}[\theta\text{rd}]$$
$$((-(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\theta\text{rd}]-\text{P2}\,\gamma\,(-1+\theta)\,\lambda\,(-1+\text{s22}[\theta\text{rd}]))$$
$$\text{s22}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+$$
$$\text{s12}[\theta\text{rd}]\,(\text{P1}\,(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\theta\text{rd}])$$
$$(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))\Big)\Big/$$

$$\Big(\big((-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+(-\theta\ (-1+\text{s11}[\theta\text{rd}]))\ \text{s11}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}])\ (-\theta\ (-1+\text{s21}[\theta\text{rd}]))\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])\big)^2\ (-\text{Z1}\,\epsilon+\delta\text{1}[\theta\text{rd}])^2\Big)-\Big(2\ \big((-(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\ \text{s12}[\theta\text{rd}]-\text{P2}\,\gamma\ (-1+\theta)\ \lambda\ (-1+\text{s22}[\theta\text{rd}]))\ \text{s22}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+\text{s12}[\theta\text{rd}]\ (\text{P1}\ (-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\ \text{s22}[\theta\text{rd}])\ (-\theta\ (-1+\text{s21}[\theta\text{rd}]))\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])\big)\ \Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (1-\lambda)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]^2\ \text{s22}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+\Big(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]^2\ \text{s22}[\theta\text{rd}]\Big)\ (-\theta\ (-1+\text{s21}[\theta\text{rd}]))\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])+(-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ \Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]^2\Big)+(-\theta\ (-1+\text{s11}[\theta\text{rd}]))\ \text{s11}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}])\ \Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]^2\Big)\Big)\Big)\Big/\Big(\big((-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+(-\theta\ (-1+\text{s11}[\theta\text{rd}]))\ \text{s11}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])\ \text{s12}[\theta\text{rd}])\ (-\theta\ (-1+\text{s21}[\theta\text{rd}]))\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])\big)^2\ (-\text{Z1}\,\epsilon+\delta\text{1}[\theta\text{rd}])\Big)+\Big(2\ \Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (1-\lambda)\ (-(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\ \text{s12}[\theta\text{rd}]-\text{P2}\,\gamma\ (-1+\theta)\ \lambda\ (-1+\text{s22}[\theta\text{rd}]))\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])+\text{s22}[\theta\text{rd}]\ (-\theta\ \text{s11}[\theta\text{rd}]\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}])\ \Big(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (1-\lambda)\ (-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]-\text{P2}\,\gamma^2\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ \lambda\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]\Big)-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]\ (\text{P1}\ (-1+\theta)\ (-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\ \text{s22}[\theta\text{rd}])\ (-\theta\ (-1+\text{s21}[\theta\text{rd}]))\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])+\text{s12}[\theta\text{rd}]\ \Big(-\text{P1}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (-1+\theta)\ (1-\lambda)\ \text{s12}[\theta\text{rd}]\ \text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\ (1-\lambda)\ (\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\ (1-\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}]\Big)\ (-\theta\ (-1+\text{s21}[\theta\text{rd}]))\ \text{s21}[\theta\text{rd}]+(-1+\theta)\ (-1+\text{s22}[\theta\text{rd}])\ \text{s22}[\theta\text{rd}])+(-(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\ \text{s12}[\theta\text{rd}]-\text{P2}\,\gamma\ (-1+\theta)\ \lambda\ (-1+\text{s22}[\theta\text{rd}]))\ \text{s22}[\theta\text{rd}]$$

$$\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)\,\text{s12}[\theta\text{rd}](1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]^2\right)+\text{s12}[\theta\text{rd}]\,(\text{P1}(-1+\theta)(-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\theta\text{rd}])\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)(1-\text{s22}[\theta\text{rd}])(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\right)\right)\Big/\Big(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}](-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))(-\text{Z1}\,\epsilon+\delta1[\theta\text{rd}]))+\Big(\gamma\,(\text{Dur1}+2\,\text{Dur2}\,\gamma)(-1+\theta)(-1+\lambda)\,\text{s22}[\theta\text{rd}]\left((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}](\text{s12}[\theta\text{rd}]+(-1+\theta)(-1+\text{s22}[\theta\text{rd}]))+\theta\,\text{s11}[\theta\text{rd}](-1+\text{s22}[\theta\text{rd}])+\text{s11}[\theta\text{rd}]^2(\theta-\theta\,\text{s22}[\theta\text{rd}])\right)\left((-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\theta\text{rd}]+\text{P2}\,\gamma(-1+\theta)\,\lambda(-1+\text{s22}[\theta\text{rd}]))\,\text{s22}[\theta\text{rd}]+(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]^2(\text{P1}(-1+\theta)(-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\theta\text{rd}])\right)\Big)\Big/\Big(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}](-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))^2(-\text{Z2}\,\epsilon+\delta2[\theta\text{rd}])^2\Big)-\Big(2\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)(-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\theta\text{rd}]+\text{P2}\,\gamma(-1+\theta)\,\lambda(-1+\text{s22}[\theta\text{rd}]))(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-2\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}](\text{P1}(-1+\theta)(-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\theta\text{rd}])-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]^3\,\text{s22}[\theta\text{rd}](\text{P1}(-1+\theta)(-1+\text{s12}[\theta\text{rd}])+(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)\,\text{s22}[\theta\text{rd}])+((-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\theta\text{rd}]+\text{P2}\,\gamma(-1+\theta)\,\lambda(-1+\text{s22}[\theta\text{rd}]))\,\text{s22}[\theta\text{rd}]\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]\right)+(-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)(-\text{P1}+\text{P2}\,\gamma\,\theta\,\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\text{P2}\,\gamma^2\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)\,\lambda(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\right)+(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]^2\left(-\text{P1}\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)(\text{P1}\,\theta-\text{P2}\,\gamma\,\lambda)(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\right)\right)\Big)\Big/\Big(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}](-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))(-\text{Z2}$$

$$\epsilon + \delta 2[\theta rd])) +$$

$$\left(2\left((-\theta\,(-1+s11[\theta rd])\,s11[\theta rd] + (-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\right.\right.$$
$$((-P1+P2\,\gamma\,\theta\,\lambda)\,s12[\theta rd] + P2\,\gamma\,(-1+\theta)\,\lambda\,(-1+s22[\theta rd]))\,s22[\theta rd] +$$
$$(-1+s12[\theta rd])\,s12[\theta rd]^2\,(P1\,(-1+\theta)\,(-1+s12[\theta rd]) + (P1\,\theta - P2\,\gamma\,\lambda)\,s22[\theta rd]))$$
$$\left(-\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(1-\lambda)\,(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd]\right.$$
$$(-\theta\,s11[\theta rd]\,s21[\theta rd] + (-1+\theta)\,s12[\theta rd]\,s22[\theta rd]) - \gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(1-\lambda)$$
$$s12[\theta rd]^2\,s22[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd] + (-1+\theta)\,s12[\theta rd]\,s22[\theta rd]) +$$
$$\left(-\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd] -\right.$$
$$\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,s12[\theta rd]^2\,s22[\theta rd]\right)$$
$$(-\theta\,(-1+s21[\theta rd])\,s21[\theta rd] + (-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd]) +$$
$$(-1+s12[\theta rd])\,s12[\theta rd]\left(\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,s12[\theta rd]\,(1-s22[\theta rd])\right.$$
$$\left.s22[\theta rd] - \gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]^2\right) +$$
$$(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd] + (-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])$$
$$\left(\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,(-1+s22[\theta rd])\,s22[\theta rd] +\right.$$
$$\left.\left.\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]^2\right)\right)\Big/$$
$$\left((((-1+s12[\theta rd])\,s12[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd] + (-1+\theta)\,s12[\theta rd]\,s22[\theta rd]) +\right.$$
$$(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd] + (-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\,(-\theta\,(-1+s21[$$
$$\left.\theta rd])\,s21[\theta rd] + (-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd]))^2\,(-Z2\,\epsilon + \delta 2[\theta rd])\right)$$

Out[1943]=
$$-\left(\left((-P1+P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta) - (-1+s11)\,s11\,\theta)\,\lambda\,(1-s12[\theta rd])\,(-1+s12[\theta rd])\right.\right.$$
$$s12[\theta rd] - (P1-P2)\,\theta\,((-1+s21)\,s21\,(-1+\theta) - (-1+s11)\,s11\,\theta)$$
$$\lambda\,(1-s12[\theta rd])\,s12[\theta rd]^2 + (P1-P2)\,(-1+\theta)$$
$$((-1+s11)\,s11\,\theta + s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,\lambda\,s12[\theta rd]\,s22[\theta rd] -$$
$$2\,(P1-P2)\,(-1+\theta)\,(s21\,(-1+\theta) + (-1+s11)\,s11\,\theta)\,\lambda\,s12[\theta rd]\,s22[\theta rd]^2\Big)$$
$$\left(\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)\theta\,((-1+s21)\,s21\,(-1+\theta) - (-1+s11)\,s11\,\theta)\,(1-\lambda)\right.$$
$$(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd] + \gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)\theta$$
$$((-1+s21)\,s21\,(-1+\theta) - (-1+s11)\,s11\,\theta)\,(1-\lambda)\,s12[\theta rd]^2\,s22[\theta rd] +$$
$$\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)\,((-1+s11)\,s11\,\theta + s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))$$
$$(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd] - 2\,\gamma\left(\frac{Dur1}{2} + Dur2\,\gamma\right)(-1+\theta)$$
$$\left.(s21\,(-1+\theta) + (-1+s11)\,s11\,\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]^2\right)\Big)\Big/$$
$$\left(-s11^2\,s21^2\,\theta^2 - \theta\,((-1+s21)\,s21\,(-1+\theta) - (-1+s11)\,s11\,\theta)\,(-1+s12[\theta rd])\,s12[\theta rd] +\right.$$
$$(-1+\theta)\,((-1+s11)\,s11\,\theta + s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,s22[\theta rd] -$$

$$
\begin{aligned}
&(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,s22[\theta rd]^2\Big)^2+\\
&\Big(-(-P1+P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,(1-s12[\theta rd])\,(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd]+\\
&\quad 2\,(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,(1-s12[\theta rd])\,s12[\theta rd]^2\,s22[\theta rd]-\\
&\quad(-P1+P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,(1-s12[\theta rd])\,s12[\theta rd]^2\,s22[\theta rd]+\\
&\quad(-P1+P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,(-1+s12[\theta rd])\,s12[\theta rd]^2\,s22[\theta rd]-\\
&\quad(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,s12[\theta rd]^3\,s22[\theta rd]+(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\\
&\quad((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,(1-\lambda)\,\lambda\\
&\quad s12[\theta rd]\,(1-s22[\theta rd])\,s22[\theta rd]-(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\\
&\quad((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,(1-\lambda)\,\lambda\,s12[\theta rd]\,s22[\theta rd]^2-\\
&\quad 4\,(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,s12[\theta rd]\,(1-s22[\theta rd])\,s22[\theta rd]^2+\\
&\quad 2\,(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\\
&\quad(1-\lambda)\,\lambda\,s12[\theta rd]\,s22[\theta rd]^3\Big)\Big/\\
&\Big(-s11^2\,s21^2\,\theta^2-\theta\,((-1+s21)\,s21\,(-1+\theta)-(-1+s11)\,s11\,\theta)\,(-1+s12[\theta rd])\,s12[\theta rd]+\\
&\quad(-1+\theta)\,((-1+s11)\,s11\,\theta+s21\,(-1+s21+\theta+(-1+2\,s11)\,s21\,\theta))\,s22[\theta rd]-\\
&\quad(-1+\theta)\,(s21\,(-1+\theta)+(-1+s11)\,s11\,\theta)\,s22[\theta rd]^2\Big)-\\
&\Big((P1-P2)\,\gamma\,(Dur1+2\,Dur2\,\gamma)\,(-1+\theta)^2\,\theta\,(-1+\lambda)\,\lambda\,s12[\theta rd]\,s21[\theta rd]\\
&\quad(s12[\theta rd]\,(-1+s21[\theta rd])-s11[\theta rd]\,(-1+s22[\theta rd]))\,s22[\theta rd]\\
&\quad\Big((\theta-\theta\,s12[\theta rd])\,s21[\theta rd]^2-(-1+\theta)\,s22[\theta rd]\,(-1+s12[\theta rd]+s22[\theta rd])+\\
&\qquad\theta\,s21[\theta rd]\,(-1+s12[\theta rd]+s11[\theta rd]\,s22[\theta rd])\Big)\Big)\Big/\\
&\Big(((-1+s12[\theta rd])\,s12[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd]+(-1+\theta)\,s12[\theta rd]\,s22[\theta rd])+\\
&\qquad(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd]+(-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\\
&\qquad(-\theta\,(-1+s21[\theta rd])\,s21[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd]))^2\\
&\quad(-Z1\,\epsilon+\delta1[\theta rd])^2\Big)-\Big(2\,(P1-P2)\,\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,\lambda\,s12[\theta rd]\\
&\quad s22[\theta rd]\,\Big((\theta-\theta\,s12[\theta rd])\,s21[\theta rd]^2-(-1+\theta)\,s22[\theta rd]\,(-1+s12[\theta rd]+s22[\theta rd])+\\
&\qquad\theta\,s21[\theta rd]\,(-1+s12[\theta rd]+s11[\theta rd]\,s22[\theta rd])\Big)\Big)\Big/\\
&(((-1+s12[\theta rd])\,s12[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd]+(-1+\theta)\,s12[\theta rd]\,s22[\theta rd])+\\
&\quad(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd]+(-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\\
&\quad(-\theta\,(-1+s21[\theta rd])\,s21[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd]))
\end{aligned}
$$

$$
\begin{aligned}
&(-Z1\,\epsilon+\delta1[\theta rd]))+\Big(2\,(P1-P2)\,(-1+\theta)\,\lambda\,s12[\theta rd]\\
&\Big(\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\theta\,(1-\lambda)\,s12[\theta rd]\,s21[\theta rd]^2\,s22[\theta rd]-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)\\
&\quad(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]\,(-1+s12[\theta rd]+s22[\theta rd])-\\
&\quad(-1+\theta)\,s22[\theta rd]\,\Big(-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]+\\
&\qquad\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]\Big)+\\
&\quad\theta\,s21[\theta rd]\,\Big(-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]+\\
&\qquad\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(1-\lambda)\,s11[\theta rd]\,(1-s22[\theta rd])\,s22[\theta rd]\Big)\Big)\Big)\Big/\\
&(((-1+s12[\theta rd])\,s12[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd]+(-1+\theta)\,s12[\theta rd]\,s22[\theta rd])+\\
&\quad(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd]+(-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\\
&\quad(-\theta\,(-1+s21[\theta rd])\,s21[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd]))\\
&(-Z1\,\epsilon+\delta1[\theta rd]))-\Big(2\,(P1-P2)\,(-1+\theta)\,\lambda\,s12[\theta rd]\\
&\Big((\theta-\theta\,s12[\theta rd])\,s21[\theta rd]^2-(-1+\theta)\,s22[\theta rd]\,(-1+s12[\theta rd]+s22[\theta rd])+\\
&\quad\theta\,s21[\theta rd]\,(-1+s12[\theta rd]+s11[\theta rd]\,s22[\theta rd])\Big)\\
&\Big(-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(1-\lambda)\,(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd]\\
&\quad(-\theta\,s11[\theta rd]\,s21[\theta rd]+(-1+\theta)\,s12[\theta rd]\,s22[\theta rd])-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(1-\lambda)\\
&\quad s12[\theta rd]^2\,s22[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd]+(-1+\theta)\,s12[\theta rd]\,s22[\theta rd])+\\
&\quad\Big(-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(-1+s12[\theta rd])\,s12[\theta rd]\,s22[\theta rd]-\\
&\qquad\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,s12[\theta rd]^2\,s22[\theta rd]\Big)\\
&\quad(-\theta\,(-1+s21[\theta rd])\,s21[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd])+\\
&\quad(-1+s12[\theta rd])\,s12[\theta rd]\,\Big(\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,s12[\theta rd]\,(1-s22[\theta rd])\\
&\qquad s22[\theta rd]-\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,s12[\theta rd]\,s22[\theta rd]^2\Big)+\\
&\quad(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd]+(-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\\
&\quad\Big(\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,(-1+s22[\theta rd])\,s22[\theta rd]+\\
&\qquad\gamma\left(\frac{Dur1}{2}+Dur2\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-s22[\theta rd])\,s22[\theta rd]^2\Big)\Big)\Big)\Big/\\
&\Big(((-1+s12[\theta rd])\,s12[\theta rd]\,(-\theta\,s11[\theta rd]\,s21[\theta rd]+(-1+\theta)\,s12[\theta rd]\,s22[\theta rd])+\\
&\quad(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd]+(-1+\theta)\,(-1+s12[\theta rd])\,s12[\theta rd])\\
&\quad(-\theta\,(-1+s21[\theta rd])\,s21[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd])\,s22[\theta rd]))^2\\
&(-Z1\,\epsilon+\delta1[\theta rd]))\Big)+\Big((P1-P2)\,\gamma\,(Dur1+2\,Dur2\,\gamma)\,(-1+\theta)^2\,(-1+\lambda)\,\lambda\\
&s12[\theta rd]\,s22[\theta rd]\,(-\theta\,(-1+s11[\theta rd])\,s11[\theta rd]\,s22[\theta rd]+\\
&\quad(-1+s12[\theta rd])\,s12[\theta rd]\,(-1+s12[\theta rd]+(-1+\theta)\,s22[\theta rd]))\\
&\Big((-1+s12[\theta rd])\,s12[\theta rd]\,(s12[\theta rd]+(-1+\theta)\,(-1+s22[\theta rd]))+\\
&\quad\theta\,s11[\theta rd]\,(-1+s22[\theta rd])+s11[\theta rd]^2\,(\theta-\theta\,s22[\theta rd])\Big)\Big)\Big/
\end{aligned}
$$

$$\Big(\big((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}])\big)^2(-\text{Z2}\,\epsilon+\delta2[\theta\text{rd}])^2\Big)+\Big(2\,(\text{P1}-\text{P2})\,\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,\lambda\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]\,(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-1+\text{s12}[\theta\text{rd}]+(-1+\theta)\,\text{s22}[\theta\text{rd}]))\Big)\Big/\Big(\big((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}])\big)(-\text{Z2}\,\epsilon+\delta2[\theta\text{rd}])\Big)-\Big(2\,(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,\text{s12}[\theta\text{rd}]\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)\theta\,(1-\lambda)\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]\,(-1+\text{s12}[\theta\text{rd}]+(-1+\theta)\,\text{s22}[\theta\text{rd}])-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]\,(-1+\text{s12}[\theta\text{rd}]+(-1+\theta)\,\text{s22}[\theta\text{rd}])+(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]\right)\right)\Big)\Big/\Big(\big((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}])\big)(-\text{Z2}\,\epsilon+\delta2[\theta\text{rd}])\Big)+\Big(2\,(\text{P1}-\text{P2})\,(-1+\theta)\,\lambda\,\text{s12}[\theta\text{rd}]\,(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]+(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,(-1+\text{s12}[\theta\text{rd}]+(-1+\theta)\,\text{s22}[\theta\text{rd}]))\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]\,(-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+\left(-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,\text{s12}[\theta\text{rd}]^2\,\text{s22}[\theta\text{rd}]\right)(-\theta\,(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}])+(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}]\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,\text{s12}[\theta\text{rd}]\,(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]-\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)\,(1-\lambda)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}]^2\right)+(-\theta\,(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)\,(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])$$

$$\left(\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)(1-\text{s22}[\theta\text{rd}])(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]+\gamma\left(\frac{\text{Dur1}}{2}+\text{Dur2}\,\gamma\right)(-1+\theta)(1-\lambda)(1-\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]^2\right)\Big)\Big/\Big(((-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}](-\theta\,\text{s11}[\theta\text{rd}]\,\text{s21}[\theta\text{rd}]+(-1+\theta)\,\text{s12}[\theta\text{rd}]\,\text{s22}[\theta\text{rd}])+(-\theta(-1+\text{s11}[\theta\text{rd}])\,\text{s11}[\theta\text{rd}]+(-1+\theta)(-1+\text{s12}[\theta\text{rd}])\,\text{s12}[\theta\text{rd}])(-\theta(-1+\text{s21}[\theta\text{rd}])\,\text{s21}[\theta\text{rd}]+(-1+\theta)(-1+\text{s22}[\theta\text{rd}])\,\text{s22}[\theta\text{rd}]))^2(-\text{Z2}\,\epsilon+\delta2[\theta\text{rd}])\Big)$$

In[1944]:=
```
(*-------------------------------------------------Calculating
 Gradient w.r.t δ1----------------------------------------*)
(*Main definition correction*)
Sp1s1δ1'[δ1_] := s11[δ1] * (1 - s11[δ1])
Sp1s1δ2'[δ2_] := -s11[δ1] * s21[δ1]
Sp1s2δ1'[δ1_] := s12[δ1] * (1 - s12[δ1])
Sp1s2δ2'[δ2_] := -s12[δ1] * s22[δ1]
Sp2s1δ1'[δ1_] := -s11[δ1] * s21[δ1]
Sp2s1δ2'[δ2_] := (1 - s21[δ1]) * s21[δ1]
Sp2s2δ1'[δ1_] := -s12[δ1] * s22[δ1]
Sp2s2δ2'[δ2_] := (1 - s22[δ1]) * s22[δ1]
Sp1δ1[δ1_] := θ * Sp1s1δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp1δ2[δ2_] := θ * Sp1s1δ2[δ2] + (1 - θ) * Sp1s2δ2[δ2]
Sp2δ1[δ1_] := θ * Sp1s2δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp2δ2[δ2_] := θ * Sp2s1δ2[δ2] + (1 - θ) * Sp2s2δ2[δ2]
Sp1[δ1_, δ2_] := Sp1δ1[δ1] + Sp1δ2[δ2]
Sp2[δ1_, δ2_] := Sp2δ1[δ1] + Sp2δ2[δ2]
(*Definitions: derivative with respect to parameter βpd,
for the first period:*)
Ds1βpd[δ1_] := θ * s11[δ1] * (1 - s11[δ1]) + (1 - θ) * s12[δ1] * (1 - s12[δ1])
Ds2βpd[δ1_] := -(θ * s11[δ1] * s21[δ1] + (1 - θ) * s12[δ1] * s22[δ1])
Dδ1βpd[δ1_] :=
 FullSimplify[(D[Sp2δ2[δ2], δ2] * Ds1βpd[δ1] - D[Sp1δ2[δ2], δ2] * Ds2βpd[δ1]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
Dδ2βpd[δ1_] := FullSimplify[
  (D[Sp2δ1[δ1], δ1] * Ds1βpd[δ1] - D[Sp1δ1[δ1], δ1] * Ds2βpd[δ1]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
s12'[δ1_] := s12[δ1] * (1 - s12[δ1])
s22'[δ1_] := -s12[δ1] * s22[δ1]
s11'[δ1_] := s11[δ1] * (1 - s11[δ1])
s21'[δ1_] := -s11[δ1] * s21[δ1]
δ1'[δ1_] := Dδ1βpd[δ1]
δ2'[δ1_] := Dδ2βpd[δ1]
DNErrDens[δ1_] := 2 Dδ1βpd[δ1] / (-Z1 ϵ + δ1[δ1]) + 2 Dδ2βpd[δ1] / (-Z2 ϵ + δ2[δ1])
NJβpd[δ1_] := FullSimplify[
  Log[s21[δ1] s22[δ1] - s21[δ1]^2 s22[δ1] - s21[δ1] s22[δ1]^2 + s12[δ1] * s21[δ1] θ -
    s12[δ1]^2 * s21[δ1] θ - s12[δ1] * s21[δ1]^2 θ + s12[δ1]^2 s21[δ1]^2 θ +
    s11[δ1] s22[δ1] θ - s11[δ1]^2 s22[δ1] θ - 2 s21[δ1] s22[δ1] θ + 2 s21[δ1]^2 s22[δ1] θ -
```

```
    2 s11[δ1] s21[δ1]^2 s22 [δ1] θ - s11[δ1] s22[δ1]^2 θ + s11[δ1]^2 s22[δ1]^2 θ +
    2 s21[δ1] s22[δ1]^2 θ + s11[δ1] * s12[δ1] * θ^2 - s11[δ1]^2 * s12[δ1] * θ^2 -
    s11[δ1] * s12[δ1]^2 θ^2 + s11[δ1]^2 * s12[δ1]^2 θ^2 - s12[δ1] * s21[δ1] θ^2 +
    s12[δ1]^2 * s21[δ1] θ^2 - s11[δ1]^2 s21[δ1]^2 θ^2 + s12[δ1] * s21[δ1]^2 θ^2 -
    s12[δ1]^2 * s21[δ1]^2 θ^2 - s11[δ1] s22[δ1] θ^2 + s11[δ1]^2 s22[δ1] θ^2 +
    s21[δ1] s22[δ1] θ^2 - s21[δ1]^2 s22[δ1] θ^2 + 2 s11[δ1] s21[δ1]^2 s22[δ1] θ^2 +
    s11[δ1] s22[δ1]^2 θ^2 - s11[δ1]^2 s22[δ1]^2 θ^2 - s21[δ1] s22[δ1]^2 θ^2]]
DNJβpd[δ1_] := D[NJβpd[δ1], δ1]
DNLLβpd[δ1_] := DNJβpd[δ1] + DNErrDens[δ1]
DNLLβpd[δ1]
```

Out[1972]=

$$
\begin{aligned}
&\big(2\theta\,(1-s11[\delta1])\,s11[\delta1]^2\\
&\quad\big(-\theta\,s12[\delta1]+\theta\,s12[\delta1]^2-\theta\,s21[\delta1]^2-(-1+\theta)\,(-1+s22[\delta1])\,s22[\delta1]\big)+\theta\,s11[\delta1]^2\\
&\quad\big(-\theta\,(1-s12[\delta1])\,s12[\delta1]+2\theta\,(1-s12[\delta1])\,s12[\delta1]^2+2\theta^2\,s11[\delta1]\,s21[\delta1]^2+\\
&\qquad(-1+\theta)\,s12[\delta1]\,(-1+s22[\delta1])\,s22[\delta1]+(-1+\theta)\,s12[\delta1]\,s22[\delta1]^2\big)+\\
&\quad(-1+\theta)\,\theta\,s11[\delta1]\,s21[\delta1]\,\big(\theta\,s12[\delta1]^2\,(-1+s21[\delta1])+s12[\delta1]\,(\theta-\theta\,s21[\delta1])+\\
&\qquad(-1+\theta)\,s22[\delta1]\,(-1+s21[\delta1]+s22[\delta1])\big)+\theta\,(1-s11[\delta1])\,s11[\delta1]\\
&\quad\big(\theta\,s12[\delta1]-\theta\,s12[\delta1]^2+(-1+\theta)\,s22[\delta1]\,\big(-1+2\,s21[\delta1]^2+s22[\delta1]\big)\big)-\\
&\quad(-1+\theta)\,s21[\delta1]\,\big(2\theta\,(1-s12[\delta1])\,s12[\delta1]^2\,(-1+s21[\delta1])+\\
&\qquad\theta^2\,s11[\delta1]\,s12[\delta1]\,s21[\delta1]-\theta^2\,s11[\delta1]\,s12[\delta1]^2\,s21[\delta1]+(1-s12[\delta1])\\
&\qquad\,s12[\delta1]\,(\theta-\theta\,s21[\delta1])-(-1+\theta)\,s12[\delta1]\,s22[\delta1]\,(-1+s21[\delta1]+s22[\delta1])+\\
&\qquad(-1+\theta)\,s22[\delta1]\,(-\theta\,s11[\delta1]\,s21[\delta1]-s12[\delta1]\,s22[\delta1])\big)+\\
&\quad\theta\,s11[\delta1]\,\big(\theta\,(1-s12[\delta1])\,s12[\delta1]-2\theta\,(1-s12[\delta1])\,s12[\delta1]^2-\\
&\qquad(-1+\theta)\,s12[\delta1]\,s22[\delta1]\,\big(-1+2\,s21[\delta1]^2+s22[\delta1]\big)+\\
&\qquad(-1+\theta)\,s22[\delta1]\,\big(-4\theta\,s11[\delta1]\,s21[\delta1]^2-s12[\delta1]\,s22[\delta1]\big)\big)\big)\big/\\
&\quad\big(\theta\,s11[\delta1]^2\,\big(-\theta\,s12[\delta1]+\theta\,s12[\delta1]^2-\theta\,s21[\delta1]^2-(-1+\theta)\,(-1+s22[\delta1])\,s22[\delta1]\big)-\\
&\qquad(-1+\theta)\,s21[\delta1]\,\big(\theta\,s12[\delta1]^2\,(-1+s21[\delta1])+\\
&\qquad\,s12[\delta1]\,(\theta-\theta\,s21[\delta1])+(-1+\theta)\,s22[\delta1]\,(-1+s21[\delta1]+s22[\delta1])\big)+\\
&\qquad\theta\,s11[\delta1]\,\big(\theta\,s12[\delta1]-\theta\,s12[\delta1]^2+(-1+\theta)\,s22[\delta1]\,\big(-1+2\,s21[\delta1]^2+s22[\delta1]\big)\big)\big)+\\
&\big(2\,\big(-(\theta\,s11[\delta1]\,s21[\delta1]-(-1+\theta)\,s12[\delta1]\,s22[\delta1])^2+\\
&\qquad(-\theta\,(-1+s11[\delta1])\,s11[\delta1]+(-1+\theta)\,(-1+s12[\delta1])\,s12[\delta1])\\
&\qquad(-\theta\,(-1+s21[\delta1])\,s21[\delta1]+(-1+\theta)\,(-1+s22[\delta1])\,s22[\delta1])\big)\big)\big/\\
&\quad\big(((-1+s12[\delta1])\,s12[\delta1]\,(-\theta\,s11[\delta1]\,s21[\delta1]+(-1+\theta)\,s12[\delta1]\,s22[\delta1])+\\
&\qquad(-\theta\,(-1+s11[\delta1])\,s11[\delta1]+(-1+\theta)\,(-1+s12[\delta1])\,s12[\delta1])\\
&\qquad(-\theta\,(-1+s21[\delta1])\,s21[\delta1]+(-1+\theta)\,(-1+s22[\delta1])\,s22[\delta1]))\,(-Z1\,\epsilon+\delta1[\delta1]))-\\
&\big(2\,\big(-\theta\,s11[\delta1]+\theta\,s11[\delta1]^2-(-1+\theta)\,(-1+s12[\delta1])\,s12[\delta1]\big)\\
&\qquad(\theta\,s11[\delta1]\,s21[\delta1]+s12[\delta1]\,(1-s12[\delta1]+s22[\delta1]-\theta\,s22[\delta1]))\big)\big/\\
&\quad\big(((-1+s12[\delta1])\,s12[\delta1]\,(-\theta\,s11[\delta1]\,s21[\delta1]+(-1+\theta)\,s12[\delta1]\,s22[\delta1])+\\
&\qquad(-\theta\,(-1+s11[\delta1])\,s11[\delta1]+(-1+\theta)\,(-1+s12[\delta1])\,s12[\delta1])\\
&\qquad(-\theta\,(-1+s21[\delta1])\,s21[\delta1]+(-1+\theta)\,(-1+s22[\delta1])\,s22[\delta1]))\,(-Z2\,\epsilon+\delta2[\delta1]))
\end{aligned}
$$

In[2007]:=

```
(*-------------------------------------------------Calculating
 Gradient w.r.t δ2----------------------------------------*)
(*Main definition correction*)
Sp1s1δ1'[δ1_] := s11[δ2] * (1 - s11[δ2])
Sp1s1δ2'[δ2_] := -s11[δ2] * s21[δ2]
Sp1s2δ1'[δ1_] := s12[δ2] * (1 - s12[δ2])
```

```
Sp1s2δ2'[δ2_] := -s12[δ2] * s22[δ2]
Sp2s1δ1'[δ1_] := -s11[δ2] * s21[δ2]
Sp2s1δ2'[δ2_] := (1 - s21[δ2]) * s21[δ2]
Sp2s2δ1'[δ1_] := -s12[δ2] * s22[δ2]
Sp2s2δ2'[δ2_] := (1 - s22[δ2]) * s22[δ2]
Sp1δ1[δ1_] := θ * Sp1s1δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp1δ2[δ2_] := θ * Sp1s1δ2[δ2] + (1 - θ) * Sp1s2δ2[δ2]
Sp2δ1[δ1_] := θ * Sp1s2δ1[δ1] + (1 - θ) * Sp1s2δ1[δ1]
Sp2δ2[δ2_] := θ * Sp2s1δ2[δ2] + (1 - θ) * Sp2s2δ2[δ2]
Sp1[δ1_, δ2_] := Sp1δ1[δ1] + Sp1δ2[δ2]
Sp2[δ1_, δ2_] := Sp2δ1[δ1] + Sp2δ2[δ2]
(*Definitions: derivative with respect to parameter βpd,
for the first period:*)
Ds1βpd[δ2_] := -(θ * s11[δ2] * s21[δ2] + (1 - θ) * s12[δ2] * s22[δ2])
Ds2βpd[δ2_] := θ * s21[δ2] * (1 - s21[δ2]) + (1 - θ) * s22[δ2] * (1 - s22[δ2])
Dδ1βpd[δ2_] :=
 FullSimplify[(D[Sp2δ2[δ2], δ2] * Ds1βpd[δ2] - D[Sp1δ2[δ2], δ2] * Ds2βpd[δ2]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
Dδ2βpd[δ2_] := FullSimplify[
  (D[Sp2δ1[δ1], δ1] * Ds1βpd[δ2] - D[Sp1δ1[δ1], δ1] * Ds2βpd[δ2]) /
   (D[Sp1δ1[δ1], δ1] * D[Sp2δ2[δ2], δ2] - D[Sp1δ2[δ2], δ2] * D[Sp2δ1[δ1], δ1])]
s12'[δ2_] := -s12[δ2] * s22[δ2]
s22'[δ2_] := s22[δ2] * (1 - s22[δ2])
s11'[δ2_] := -s11[δ2] * s21[δ2]
s21'[δ2_] := s21[δ2] * (1 - s21[δ2])
δ1'[δ2_] := Dδ1βpd[δ2]
δ2'[δ2_] := Dδ2βpd[δ2]
DNErrDens[δ1_] := 2 Dδ1βpd[δ2] / (-Z1 ϵ + δ1[δ2]) + 2 Dδ2βpd[δ2] / (-Z2 ϵ + δ2[δ2])
NJβpd[δ2_] := FullSimplify[
  Log[s21[δ2] s22[δ2] - s21[δ2]^2 s22[δ2] - s21[δ2] s22[δ2]^2 + s12[δ2] * s21[δ2] θ -
    s12[δ2]^2 * s21[δ2] θ - s12[δ2] * s21[δ2]^2 θ + s12[δ2]^2 s21[δ2]^2 θ +
    s11[δ2] s22[δ2] θ - s11[δ2]^2 s22[δ2] θ - 2 s21[δ2] s22[δ2] θ + 2 s21[δ2]^2 s22[δ2] θ -
    2 s11[δ2] s21[δ2]^2 s22 [δ2] θ - s11[δ2] s22[δ2]^2 θ + s11[δ2]^2 s22[δ2]^2 θ +
    2 s21[δ2] s22[δ2]^2 θ + s11[δ2] * s12[δ2] * θ^2 - s11[δ2]^2 * s12[δ2] * θ^2 -
    s11[δ2] * s12[δ2]^2 θ^2 + s11[δ2]^2 * s12[δ2]^2 θ^2 - s12[δ2] * s21[δ2] θ^2 +
    s12[δ2]^2 * s21[δ2] θ^2 - s11[δ2]^2 s21[δ2]^2 θ^2 + s12[δ2] * s21[δ2]^2 θ^2 -
    s12[δ2]^2 * s21[δ2]^2 θ^2 - s11[δ2] s22[δ2] θ^2 + s11[δ2]^2 s22[δ2] θ^2 +
    s21[δ2] s22[δ2] θ^2 - s21[δ2]^2 s22[δ2] θ^2 + 2 s11[δ2] s21[δ2]^2 s22[δ2] θ^2 +
    s11[δ2] s22[δ2]^2 θ^2 - s11[δ2]^2 s22[δ2]^2 θ^2 - s21[δ2] s22[δ2]^2 θ^2]]
DNJβpd[δ2_] := D[NJβpd[δ2], δ2]
DNLLβpd[δ2_] := DNJβpd[δ2] + DNErrDens[δ2]
DNLLβpd[δ2]
```

Out[2035]=
$$\frac{\begin{aligned}&\Big(-2\,\theta\,s11[\delta 2]^2\,s21[\delta 2]\\&\quad\Big(-\theta\,s12[\delta 2]+\theta\,s12[\delta 2]^2-\theta\,s21[\delta 2]^2-(-1+\theta)\,(-1+s22[\delta 2])\,s22[\delta 2]\Big)+\\&\quad\theta\,s11[\delta 2]^2\Big(-2\,\theta\,(1-s21[\delta 2])\,s21[\delta 2]^2+\theta\,s12[\delta 2]\,s22[\delta 2]-2\,\theta\,s12[\delta 2]^2\,s22[\delta 2]-\\&\qquad(-1+\theta)\,(1-s22[\delta 2])\,(-1+s22[\delta 2])\,s22[\delta 2]-(-1+\theta)\,(1-s22[\delta 2])\,s22[\delta 2]^2\Big)-\\&\quad(-1+\theta)\,(1-s21[\delta 2])\,s21[\delta 2]\Big(\theta\,s12[\delta 2]^2\,(-1+s21[\delta 2])+s12[\delta 2]\,(\theta-\theta\,s21[\delta 2])+\\&\qquad(-1+\theta)\,s22[\delta 2]\,(-1+s21[\delta 2]+s22[\delta 2])\Big)-\theta\,s11[\delta 2]\,s21[\delta 2]\\&\quad\Big(\theta\,s12[\delta 2]-\theta\,s12[\delta 2]^2+(-1+\theta)\,s22[\delta 2]\,\Big(-1+2\,s21[\delta 2]^2+s22[\delta 2]\Big)\Big)-\\&\quad(-1+\theta)\,s21[\delta 2]\Big(-\theta\,s12[\delta 2]\,(1-s21[\delta 2])\,s21[\delta 2]+\theta\,s12[\delta 2]^2\,(1-s21[\delta 2])\,s21[\delta 2]-\\&\qquad2\,\theta\,s12[\delta 2]^2\,(-1+s21[\delta 2])\,s22[\delta 2]-s12[\delta 2]\,(\theta-\theta\,s21[\delta 2])\,s22[\delta 2]+\\&\qquad(-1+\theta)\,(1-s22[\delta 2])\,s22[\delta 2]\,(-1+s21[\delta 2]+s22[\delta 2])+\\&\qquad(-1+\theta)\,s22[\delta 2]\,((1-s21[\delta 2])\,s21[\delta 2]+(1-s22[\delta 2])\,s22[\delta 2])\Big)+\\&\quad\theta\,s11[\delta 2]\Big(-\theta\,s12[\delta 2]\,s22[\delta 2]+2\,\theta\,s12[\delta 2]^2\,s22[\delta 2]+\\&\qquad(-1+\theta)\,(1-s22[\delta 2])\,s22[\delta 2]\,\Big(-1+2\,s21[\delta 2]^2+s22[\delta 2]\Big)+\\&\qquad(-1+\theta)\,s22[\delta 2]\,\Big(4\,(1-s21[\delta 2])\,s21[\delta 2]^2+(1-s22[\delta 2])\,s22[\delta 2]\Big)\Big)\Big)\Big/\\&\Big(\theta\,s11[\delta 2]^2\Big(-\theta\,s12[\delta 2]+\theta\,s12[\delta 2]^2-\theta\,s21[\delta 2]^2-(-1+\theta)\,(-1+s22[\delta 2])\,s22[\delta 2]\Big)-\\&\quad(-1+\theta)\,s21[\delta 2]\Big(\theta\,s12[\delta 2]^2\,(-1+s21[\delta 2])+\\&\qquad s12[\delta 2]\,(\theta-\theta\,s21[\delta 2])+(-1+\theta)\,s22[\delta 2]\,(-1+s21[\delta 2]+s22[\delta 2])\Big)+\\&\quad\theta\,s11[\delta 2]\Big(\theta\,s12[\delta 2]-\theta\,s12[\delta 2]^2+(-1+\theta)\,s22[\delta 2]\,\Big(-1+2\,s21[\delta 2]^2+s22[\delta 2]\Big)\Big)\Big)-\end{aligned}}{-Z2\,\epsilon+\delta 2[\delta 2]}\;\frac{2}{}$$

In[2100]:=
```
(*Calculate the gradient of constraint s^(it)=s(it), t=1,2*)
(*first element w.r.t θ*)
DS1θ = D[θ * s11 + (1 - θ) * s12, θ]
Ds2θ = D[θ * s21 + (1 - θ) * s22, θ]
(*second element w.r.t βpd*)
s12'[βpd_] :=
 P1 * s12[βpd] * (1 - s12[βpd] - s22[βpd]) + (P1 - γ * λ * P2) * s12[βpd] * s22[βpd]
s22'[βpd_] := (γ * λ * P2) * s22[βpd] * (1 - s12[βpd] - s22[βpd]) +
   (γ * λ * P2 - P1) * s12[βpd] * s22[βpd]
s11'[βpd_] := 0
s21'[βpd_] := 0
DS1βpd = D[θ * s11[βpd] + (1 - θ) * s12[βpd], βpd]
DS2βpd = D[θ * s21[βpd] + (1 - θ) * s22[βpd], βpd]
(*Third element w.r.t αpd*)
s12'[αpd_] := s12[αpd] * (1 - s12[αpd]) * λ * (P1 - P2)
s22'[αpd_] := -s12[αpd] * s22[αpd] * λ * (P1 - P2)
s11'[αpd_] := 0
s21'[αpd_] := 0
DS1αpd = D[θ * s11[αpd] + (1 - θ) * s12[αpd], αpd]
DS2αpd = D[θ * s21[αpd] + (1 - θ) * s22[αpd], αpd]
(*Fourth element w.r.t θrd*)
s12'[θrd_] := -s12[θrd] * s22[θrd] * (1 - λ) * γ * (Dur1 / 2 + γ * Dur2)
s22'[θrd_] := (1 - s22[θrd]) * s22[θrd] * (1 - λ) * γ * (Dur1 / 2 + γ * Dur2)
s11'[θrd_] := 0
s21'[θrd_] := 0
DS1θrd = D[θ * s11[θrd] + (1 - θ) * s12[θrd], θrd]
DS2θrd = D[θ * s21[θrd] + (1 - θ) * s22[θrd], θrd]
(*Fourth element w.r.t δ1*)
s12'[δ1_] := s12[δ1] * (1 - s12[δ1])
s22'[δ1_] := -s12[δ1] * s22[δ1]
s11'[δ1_] := s11[δ1] * (1 - s11[δ1])
s21'[δ1_] := -s11[δ1] * s21[δ1]
DS1δ1 = D[θ * s11[δ1] + (1 - θ) * s12[δ1], δ1]
DS2δ1 = D[θ * s21[δ1] + (1 - θ) * s22[δ1], δ1]
(*Fifth element w.r.t δ2*)
s12'[δ2_] := -s12[δ2] * s22[δ2]
s22'[δ2_] := s22[δ2] * (1 - s22[δ2])
s11'[δ2_] := -s11[δ2] * s21[δ2]
s21'[δ2_] := s21[δ2] * (1 - s21[δ2])
DS1δ2 = D[θ * s11[δ2] + (1 - θ) * s12[δ2], δ2]
DS2δ2 = D[θ * s21[δ2] + (1 - θ) * s22[δ2], δ2]
```

Out[2100]= $s11 - s12$

Out[2101]= $s21 - s22$

Out[2106]= $(1-\theta)\,(P1\,s12[\beta pd]\,(1 - s12[\beta pd] - s22[\beta pd]) + (P1 - P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd])$

Out[2107]= $(1-\theta)\,((-P1 + P2\,\gamma\,\lambda)\,s12[\beta pd]\,s22[\beta pd] + P2\,\gamma\,\lambda\,(1 - s12[\beta pd] - s22[\beta pd])\,s22[\beta pd])$

Out[2112]= $(P1 - P2)\ (1 - \theta)\ \lambda\ (1 - s12[\alpha pd])\ s12[\alpha pd]$

Out[2113]= $-(P1 - P2)\ (1 - \theta)\ \lambda\ s12[\alpha pd]\ s22[\alpha pd]$

Out[2118]= $-\gamma \left(\frac{Dur1}{2} + Dur2\ \gamma\right)\ (1 - \theta)\ (1 - \lambda)\ s12[\theta rd]\ s22[\theta rd]$

Out[2119]= $\gamma \left(\frac{Dur1}{2} + Dur2\ \gamma\right)\ (1 - \theta)\ (1 - \lambda)\ (1 - s22[\theta rd])\ s22[\theta rd]$

Out[2124]= $\theta\ (1 - s11[\delta 1])\ s11[\delta 1] + (1 - \theta)\ (1 - s12[\delta 1])\ s12[\delta 1]$

Out[2125]= $-\theta\ s11[\delta 1]\ s21[\delta 1] - (1 - \theta)\ s12[\delta 1]\ s22[\delta 1]$

Out[2130]= $-\theta\ s11[\delta 2]\ s21[\delta 2] - (1 - \theta)\ s12[\delta 2]\ s22[\delta 2]$

Out[2131]= $\theta\ (1 - s21[\delta 2])\ s21[\delta 2] + (1 - \theta)\ (1 - s22[\delta 2])\ s22[\delta 2]$