Pricing Seminar of Professor Ozel @ UTD: first session

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SCM
IT for better decisions, from NPD to distribution

Pricing as a cross functional opertion

Covering the handbook for the course

Cost, market share, game theory

Pricing strategy

Little knowledge about people and firms, and list pricing

Revenue management and obsoletion

Dynamic programming, game theory, and sequential deicison making

Behavioral issue of pricing

Reaction to the pricing decision of multiple firms

Overview of dynamic programming as an important issue

Lot of reading and review them, they are int he form of tutorial, no need for preliminary

Top 5 choices of sessions to be emailed, so that 2 session assigned for every session so that they read the papers

There would be communication between faculty before presentation

See things across differnet perspective

Read chapters very carefully

First level: Abstract-introduction-conclusion

second level: how motivated, how model is built, and how describe, and what are key results

Third level: Take one kewy result, and see how they prove it

Some of papers need to be read third level. Those assigned require a lot more reading

Game theory would be deeper coverage, here is from pricing perspective

Project: pick industry and study actual models, suggest extension, initial idea about next step; analytica, impact, variation

Something like a litrature survey on the industry, and some initial work

Not just litrature, and there are two more things: 1. pick industry and area (the best way is not given in chapters, and they are saying how it is done); there are a lot of research areas, and you identify several and pick one 2. anything that you find interesting nad want to explore.

Project should be started early on

There would be timeline for project that should be followed

Buy dynamic programming book

Send top 5 sessions you want to be involved

Not so much empirical, but more analytical

Sessions 4, 5, 6, 7, 9, 10, 11, 12

Dynamic programming

Make decision on the sequential bases, with or without uncertainty, but mostly in the situation that uncertaitly is involved

static decisions, you make upfront

Dynamic decisions: you make decision right now, for today, and you observe what happens and based on that you find out what happens, and incorporate what you have seen, for future deicison

Decision would be based on the information you have collected until now, woul dbe in the form of dynamic deicisions

Dynamic programming is about single decision makers, and dynamic decisions over time

Situation starts with initial information and you apply some actions; based on information action, and random event, you will encounter a cost, and you transite into new state, and you make another decision

Open loop decision making is about deciding everything upfront.

Close loop is collecting information over time

Close loop is better, and difference is called value of information

We would have random event then we will make the decision. So the random event would have a tree that with probability p the first outcome will come and with probability (1-p) the second outcome will come

Something happens that could be demand, and you make the deicision

If it would be decided up front decision in each situation $3(1-p)+4p$ and for down it would be $6(1-p)+2p$, and you compare them.

But if you do not decide upfront then decide based on the state that identified, and in this case maximum would be chosen in each case, so with probability half if you choose one it would be $(3+6).0.5 = 4.5$

Selecting up or down is an action. If the policy says that if it is head you will choose up, and if tail down, then it would be contingent policy, which would be function rather than choice, and you will optimize them

We will talk about the horizon now (finite and then infinite)

1. Time: $t = 1, 2, n$

2. x_k=state of the system $k = 0, 1, , n-1$

State is the description of information you got.

You would have all information that you have from what happened previously

What is the relevent information for the state to go forward

There are some information that is discarded, and there are some that you need

For example for the inventory, how were the variation previously were is not relevant, but the number that I have in inventory now is important

3. $u_k =$ is the control at time k, and $k = 0, , n-1$

This control means for example ordering.

There would be notion of random event, or noise, or disturbance:

4. $w_k =$ Random variable/ distrubrance at time $k = 0, , n-1$

$w_k \sim p_k(|x_t, u_k)$

This is strong assumption, but by definition you can have different setting applied. This is saying that the random event would be based on information and current controls.

You can now have the minimization of cost problem

$g_k(x_k, u_k, w_k)$ for $k = 0, 1, , n-1$

It could be hold up cost vs. backlog cost.

$g_n(x_n)$ for the last period we have final cost

5. $x_{k+1)=f_k(x_k,u_k,w_k)}$ for $k = 0, , n-1$

This is state evolution.

6. We are interested in minimizing the expectation of cost:
$E[\sum_{k=0}^{n-1} g_k(x_k, u_k, w_k) + g_n(x_k)]$

There are many other explicit assumptions, such as that expectation exists.

They are technical issues and we are not going to talk about it.

Cost is additive over time and separable is another assumption.

You can fit many cases into this, although these are strong assumption for optimization.

Discounting could be incorporated into g_k, and it is not so much relevent for the finite horizon.

7. Definition of states are the following:

$x_k \in X_k$

$u_k \in C_k$

$w_k \in D_k$

8. Let π be the policy, where $\pi = (\mu_0, \mu_1, , \mu_{n-1})$

and $\mu_k : X_k \to C_k$

It is a plan of how you are going to act over the horizon.

9. $J_\pi(x_0) = E[\sum_{k=0}^{N-1} g_k(x_k, \mu_t(x_k), w_k) + g_N(X_N)]$

This is the cost of policy π

We are minimizing this cost $\pi \in \Pi$

10. Π is the set of admissible policies.

A policy is admissible if $\mu_k(x_k) \in U_k(x_k)$

You are making decision based on information given at this point.

11. Optimal cost is $J_\pi(x_0) = J*(x_0)$

$J_{\pi^*}(x_0) = J^*(x_0) \Rightarrow \Pi^*$ is an optimal policy

Policy is function of state and not random variable.

Random variable changes the state.

we say "an optimal policy" since it is not unique, and it could be multiple.

Forecast will affect how it is evolute, and forecast depends on what happens, and the distribution of what the forecast is tommorow is dependent on that, as a result today's forecast is part of the state.

Principle of optimality

You have time zero 0 to N, and intermediate i.

Suppose $\pi^* = (\mu_0, , \mu_{n-1})$ is an optimal policy.

Then $(\mu_1, , \mu_{N-1})$ has to be optimal for the "tail problem"; true for all.

It is proofed by contradiction, since the shortest path if could be found, then that would be optimal.

Suppose that we have the graph of $G = a_1a_2 = 5, a_2a_3 = 1, a_3a_5 = 5, a_1a_6 = 4, a_6a_3 = 3, a_2a_4 = 3, a_4a_5 = 4$

The optimal is $a_1a_2a_3a_5$

Let's start from 2 and try to find the path that helps us to find hte second largest path, then a_2 to a_5 has two path and a_2, a_4, a_5, this is violation, since cost is not additive here, and since they are not additive principle of additivity is violated.

Principle of optimality is important part of machinery of dynamic programming.

When the tail helps, then backward resolution would be good solution.

You go backward up to period zero, and for each iteration you will solve relatively simple problem.

This is notion of backward induction.

So you start from the end and solve it backward.

Life can only be understood backward, but it should be lived forward.

$J_k(x_k)$ is the optimal cost to go from period k to period N. $k = 0, , N$

$J_N(x_N) = g_N(x_N)$, and $J_k(x_k) = min_{u_k \in U_k(x_k)} E[g_k(x_k, u_k, w_k) + J_{k+1}(f_k(x_k, u_k, w_k))$

$k = 0, , N-1$

There is minimization of myopic, and you just save the day and you are not thinking about future.

This is recursion equation, the main that is used in the dynamic programming. It is like greedy, you do what is best for today.

Single stage inventory problem.

Indivisible units.

Supplier with unlimited capacity.

A lead time of 1

i.i.d. demand. $d_k \sim F_k$

F_k would be cdf.

Full backlogging.

Holding and backorders costs.

Salvage value of zero, mean dispose of remaining would be free.

Linear order cost c

Now we try to formulate the problem:

State would be inventory at the beggining of period k

If it took time for the order to reach, you need to take care of time it will take to recieve, and you need to keep track of that as well.

u_k : Amount to order at time k

w_k : Demand in period k $p_k(|x_k, u_k)$

$g_k(x_k, u_k, w_k) = c.u_k + r(x_k)$

You have to be careful about how you define it, and you will have holding cost, and order cost here.

$f_k(x_k, u_k, w_k) = x_k + u_k - w_k$

$g_N(x_N) = r(X_N)$ Whatever we are left that is it

$J_N(x_N) = r(x_N)$

Start with x_0, good thing for dynamic programming is that it is not dependent on the initial state.

$J_k(x_k) = min_{u_k \in Z} E_{w_k}[c.u_k + r(x_k) + J_{k+1}(x_k + u_k - w_k)]$

You will write it in the table for every possible k what would be the J_k, and given J_{k+1} you will calculate for J_k

$X_k \in Z$

All integers could be in state space, and could be positive and negative.

Put every information you have on the paper, and then try to reduce them to state space, you can do it for two state inventory and backlog.

$X_I = \{1, 2, , n\}$ for all k.

The state space for inventory was infinite, since it is for all the integers.

In reality you try to truncate the state space.

Once you do the truncation, you will left with the chunk of state levels, and finite number of states.

$C_k = 1, , m$ for all k's

You will have a markov chain, and tranistion probability matrix, and states.

Normal markove chain we just observe, but here we can traject, and impact how things are evolved.

It is kind of markove decision processes.

We can have some impact on how things are evolving.

$p_{ij}(u_k)$ mean the transition probability is based on what the decision is

$J_N(x_N) = g_N(x_N)$

$J_k(x_k) = min_{u \in \{1,,m\}} \tilde{g}(x_k, x_{k+1}, u_k + sum_{x_{k+1} \in \{1,,n\}} P_{x_k, x_{k+1}}(u_k).J_{k+1}(x_{k+1})$

$\tilde{g_k}(x_k, u_{k+1}, u_k) = \sum_{x_{k+1}} p_{x_k, x_{k+1}}(u_k) \tilde{g}(x_k, x_{k+1}, w_k)$

Cost is depends only on where was I, and where I did end up, and what was the control.

Complexity would be $n^2.m.N$. It is bionomial pretty simple, but there is curse of dimensionality.

As far as state space is assumed, it is finite.

small n is number of states.

Lead time and more dimensions of keeping track up would be called curse of dimensionality.

Computing optimal solution more than four state is usually prohibited.

When you are defining states, you need to reduce states as much as possible, since it can reduce the order of your solution.

Backward induction can help easily, when you reduce the the number of states.

Model of diffusion that you have the earn, and black and white, and at poisson time you take black or white and change the color. Steady state number of white balls? White balls would be that. State transition would be i/n, and you can define transition probability matrix easily.

Using this you can calculate the steady state distribution and you will see that the number is bionomial.

When you have n dimensional state, and each have m possible values, and state space would be much much larger.

Number of balls would be independent of eachother, and it will alternate, but the time between them is identically distributed, and with probability of one half, it would be bernouli which would be bionomial, and it is much easier than putting markove chain. There are states as a result that proof is easy, but those are special cases.

J_k is called value function.

We compute them, and the question is whether are the value function is right. Does this algorithm define the g^* optimal, and indeed it helps to reach the optimal cost.

The policy that is optimal in each step of recursion is also optimal for the whole.

Napsack problem. Napsack with certain capacity, each has the value of v_i and weight w_i, and you want to pick the set that fit, and has the largest value.

You can think about putting things one by one, and then index that as time.

The control is whether to put in or not.

State is left over capacity. You do not need to put what is put until now, but you put how much wieght is remained.

Traveling sales person is about city that you are in, and which you want to select. As a result you need to also have track of the cities that you have visited until now.

Homeworks.

Pricing Seminar of Professor Ozel @ UTD: second session

Meisam Hejazinia

1/24/2013

Read chapter and one of the paper would be the plan for two week from now.

We talked about Ballman equation. Converging to the optimal solution specially for infinite cases are important.

Number of state of n and the number of controls were m and the complexity become $n^2 * m$. Also the number of states had impact on the complexity, and that was called curse of dimentionality.

If you look at all possible number of states, naive methods, give you exponential complexity in term of n. Dynamic method could solve this and give the polynomial complexity, yet has its own limitation.

Today: look how we can use Dynamic programming to look at the structure of policy and problems:

We will look at different setting of marketing and operation management.

The first one is optimall stopping problem, and revenue management problem. We will look at inventory problem as well.

You are talking about system evolving, for example markov chain that you don't control, and is exogeneous, and you decide to stop it, and in previous movement there were cost and revenues. When you stop there would be no cost and revenue and question is when you want to stop.

Example is secretary problem, that you need to either give them a job or reject them so that nobody else comes in. Question is when to hire, and the trade off is that if I hire right now, is there anybody else that is waiting our side. I may be loosing this person that I have right now.

In the context of selling an asset, you have bids, and each time a bid comes in, you have to decide whether to accept or reject offer.

You have to make the decision if you sell the house, it is done, and if you don't sell you will move to next bidder. It is almost like everyday you have the offer, in the form of discrete. You may not recieve it, but you can model it in the form of dummy offer that you recieve everyday.

Asset Selling

N days. There is hard deadline. By day N if you have solved, you will sell it to anyone comes in.

State: if you have already sold the house it is done.

T: termination state. After which you will not incure any cost. This is optimal selling problem.

The control is: accept or reject.

Current offer on the table is state of the system.

Let's assume offers are comming indep. ident. distributed.

How many days remain would be as index of dynamic program.

Past offers are not part of the state and I have not turn them down. I can not go back and sell the house to them. If there was the relationship of offers over time that would be important. If we are thinking about i.i.d. offers, then past offers are irrelevant.

$x_k = (c_k, s_k)$ where c_k is the offer on the table, and $s_k = \{0$: if the house is not sold, 1 : if asset is sold $\}$.

reducing to single dimension could also happen in the following form:

y_k offer on the table if $y_k \geq 0$, and terminatino state if $y_k = -1$

$u_k = \{0$: if we do not sell, 1 if we sell, 1 if we sell$\}$.

w_k is the offer for next period. Suppose that w_k is i.i.d., with cdf F. $y_{k+1} = f_k(y_k, u_k, w_k) = \{w_k$ if $y_k \geq 0$ and $u_k = 0$, -1 if $y_k \geq 0$ and $u_k = 1$, -1 if $y_k = -1\}$

$g_k(y_k, u_k, w_k) = \{y_k.u_k$ if $y_k \geq 0$, 0 if $y_k = -1\}$

$g_N(y_N) = \{y_N$ if $y_k \geq 0$, and 0 if $y_N = -1\}$

$U_k(y_k) = 0, 1$

Discount factor should be $\frac{1}{1+r}$

$J_k(y_k) = max_{u_k \in \{0,1\}} y_k.u_k + \frac{1}{1+r} E_{w_k}[J_{k+1}(f_k(y_k, u_k, w_k)]$ if $y_k \geq 0$

$= max y_k, \frac{1}{1+r} E_{w_k}[J_{k+1}(w_k)]$

$J_k(y_k) = 0$ if $y_k = -1$

$\pi^* = (\mu_0^*, \mu_1^*, , \mu_{N-1}^*)$ be the optimal policy.

$\mu^*(y_k) =$ is one step function. It means if the price is above specific price I will sell it, else I won't.

$\alpha_k = \frac{1}{1+r} E[J_{k+1}(w_k)]$

As you go closer to the end of the horizon you are going to expect lower and lower offers, and at the beggining can be cavelier.

The expectation of w_k would be the same and if you got $\alpha_k \geq \alpha_{k+1}$ then we must have $J_{k+1} \geq J_{k+2}$ for all y_k.

Since you are getting the maximum, it is going to be at least larger than the first, so the offer going to be monotonic, so thereshold are increasing over time.

If you ar going over the infinite horizon then it will converge. For that you need to define what is the setting of the infinite horizon.

Next problem is revenue management type of the problem. We have a situation where we have a flight, which we have to take off, and we are trying to sell the sits in the flight.

Suppose that you can retain the past offers. On that case you need to keep track of the maximum, and the state evolution is that if it is more than maximum then this states, else previous states.

Let say everybody leaves after three days, then the sequence of the offers should be retained and not the maxium. In this one you would have three dimensional state rather than one dimensional state.

Single leg revenue management

The capacity of the flight would be c.

C : capacity on the flight : $C \geq 0$, integer.

n customer classes

r_i : is the revenue from class i customer.

N: periods. Such that in every period you don't expect more then one customer.

Not more than one customer in every period.

p_{ik}: probability of type i customer in period k.

p_{0k}: probability of no customers in period k.

$\sum_i p_{ik} = 1$ for all k.

We may end up selling to business class, or you can end up selling all to economic class.

We basically have to accept the business class offer.

Risk of just accepting business class is that the sits would be empty, and marginal cost would be zero.

Plane leaving empty is big loss.

Typically used to be more static decision.

Bascially as it got closer they cut the price.

In every period either we have a customer or we don't have. We have to decide do we sell the seat or do we not sell.

$x_k = (c_k, s_k)$

c_k : Number of seats sold

s_k : revenue from current customer

u_k : {1 : sell the seats to current customers, 0 : otherwise }

w_k : Next offer or revenue from next customer.

$f_k(c_k, s_k, u_k, w_k) = (c_k + u_k, w_k)$ if $c_k < c$

(c, w_k) if $c_k = c$

$g_k(c_k, s_k, u_k, w_k) = u_k.s_k$ if $c_k < c$
and 0 if $c_k = c$

$g_N(c_N, s_N) = 0$

$J_K(c_k, s_k) = max_{u_k \in \{0,1\}}[s_k.u_k + E_{w_k}[J_{k+1}(c_k + u_k, w_k)]$ if $c_k < c$
$J_k(c_k, s_k) = 0$ if $c_k = c$

Let $g(s) = E_{w_k}[J_{k+1}(s, w_k)]$

$J_k(c_k, s_k) = max_{u_k \in \{0,1\}}[s_k.u_k + g_k(c_k + u_k)]$ if $c_k < c$

$\Delta_k(c_k) = g_k(c_k) - g_k(c_k + 1)$: Marginal revenue of one additional sit

$J_k(c_k, s_k) = \{g_k(c_k)$ if $u_k = 0$, and $s_k, g_k(c_k + 1)$ if $u_k = 1\}$

Sell if $s_k \geq \Delta_k(c_k)$, otherwise don't sell.

The value function $\Delta_k(c_k)$ would be concave, since $J_k(c_k, s_k)$ would be concave.

$b_{ik} = max\{c|r_i \geq \Delta_k(c)\}$ This is Booking limit.

If the offer is more than Δ this will be called booking unit.

I am going to sell if more than b_{ik} is avaiable I would sell.

Intuition for Booking unit is that, if there are still a lot of sit for economy sit I am going to sell, and if there is two sit, I am not going to sell. It is the limite that you are going to sell to thse customers or not.

Booking limit policy, so would be optimal.

Inventory Example

Single stage, single product

Supplier has infinite capacity.

Lead time = 1

i.i.d. demand.

Holding/ Backorder cost $r(c)$ which would be convex

$lim_{s \to |\infty|} r(s) = \infty$

$-c$: order cost (linear)

X_t : inventory on hand, or net inventory, inventory level in period k

Number of unitis if positive

Number of backlogged if negative

$x_k \in X_k = Z$ all integer

u_k : order quantity

$u_k \in U_k(x_k) = Z_0^+$

$x_{k+1} = f_k(x_k, u_k, w_k) = x_k + u_k - w_k$

$g_k(x_k, u_k, w_k) = r(x_k + u_k - w_k)$

$g_N(x_N) = 0$

$J_k(x_k) = min_{u_k \geq 0} E_{u_k}[c.u_k + r(x_k + u_k - w_k) + J_{k+1}(x_k + u_k - w_k)]$

We do not have lost sales, and every customer will eventually be satisfied, so in long run the revenue is fixed.

$k = 0, , N - 1$

The base type policy is optimal, mean if the inventory is below the threshold you try to increase it to that level.

$y_k = x_k + u_k$

Change of variables that appear as a sum, and then we will have:

$J_k(x_k) = min_{y_k} E_{w_k}[c.y_k + r(y_k - w_k) + J_{k+1}(y_k - w_k)] - c.x_k$

Suppose J_{k+1} is convex, and goes to infinity on both sides.

if $f(y)$ would be convex in y then $f(y - \alpha)$ is also convex, since it is just shifting.

Taking the expectation of the convex is also convex since you it is weighted average of convex function that leads to a new convex function.

$J_k(x_k) = min_{y_k \geq x_k} G_k(y_k) - cx_k$

There exists a minimizer, and let that to be s_k

Let $s_k = argmin_{y_k G_k(y_k)}$

If $x_k \leq s_k$ then $y_k = s_k$

If $x_k > s_k$ then $y_k = x_k$

This is called Base stock policy

The assumption here was convexity.

Next step is to show that actual function is convex, and to do this we need to use induction

$J_N(X_N) = 0$

$J_{N-1}(X_{N-1}) = min_{Y_{N-1} \geq X_{N-1}} E(c.y_{N-1} + r(y_{N-1} - w_{N-1})] - cX_{N-1}$

since r goes to infinity on both side and we need to show that $G_{N-1}(y_{N-1}) = E(c.y_{N-1} + r(y_{N-1} - w_{N-1})]$ is convex and goes to infinity on both side.

$J_k(x_k) = min_{y_k \geq x_k} G_k(y_k) - c.X_k$

$= G_k(S_K) - cX_k$ if $X_k \leq S_k$
$= G_k(X_k) - cX_k$ if $X_k > S_k$

$H(x_k) = G_K(s_k)$ if $x_k \leq s_k$
$= G_k(x_k)$ if $x_k > s_k$

$J_k(x_k) = H(x_k) - c.x_k$

The whole cycle is complete.

This concept of proving things using Bellman recursion was the point here, that if the value function has specific form, then using induction you can show that the plicy is optimal.

Choose the structure in value function that shed light on how the optimal policy is.

You can extend it to couple of settings. If lead time is more than one.

Pricing Seminar of Professor Ozel And Profesor Moharram Oghli @ UTD: Third session

Meisam Hejazinia

1/31/2013

Set up the meeting for discussion of the presentation of the session.

Infinite horizon dynamid program. Difference with finite horizon is not conceptually that much.

Taking expectation, and taking limit when we have infinity would be complicated. Whether it is defined or not, would be an issue. We will look at certain type of problems.

Overall outline is to see several versions of dynamic programming. when there is no end to the horizon, and continuous. Minimizing cost, discounted or long run average would be defined.

We will define basic structure, and look at the result we look on. Why we need to be careful. We will talk about computation issues. We have talked about bellman recursion, you start from the end you come back, yet in infinite horizon there is not finite end, so you can not use that.

Set up is similar $x_{k+1)=f(x_k,u_k,w_k)}$, function that maps the control, state and random variable to the next stage. F would not be f_k and dynamic would be constant over time, since you can not define for each period, since storing would be infinite. Costs are stationary, and dynamic function is stationary. You have control state. Control set is also stationary, and state space, control space, and disturbance state would be stationary. We will assume that random variable take values n countlable set. If it wouldn't be countable things would not be simple. You need to discretize. This continueity assumption is not a big deal.

Policies are not necessarily end. It is the set that maps from state to the goal.

$$J_\pi = lim_{N\to\infty} E_{wk} Sum_{k=0}^{N-1} \alpha^k g(x_k, \mu_k(x_k), w_k)$$

We minimum across all the policies and get the optimal value function. $J^*(x) = min_{\pi\in\Pi} J_\pi(x), x \in S$. Cost functions are stationary, in the finite horizon the stationary policy does not work, yet in the infinite it would not work. In the infinite if we are in the period 5 or 6 we have infinite amount of time left, and costs are the same, so stationary policy works $\pi = \mu, \mu, ...$

Here for stationary policy we use J_μ.

Three classes of problems:

1. Discounted problems with bounded cost per stage. The cost is bounded by constant $|g(x, u, w)| < M$

2. When we do not have this bound then discounting does not work, so discounting versus un-discounting does not work. We need to have some conditions for those states that cost per stage is unbounded.

3. Average cost per stage problems. You look at

the long run average over time.

Notation for any function $J : S \to handR$

$(TJ)(x) = min_{x \in U(x)} E_w g(x, u, w) + \alpha J(f(x, u, w)), x \in S$

It converges one function in one state space to the other. It is one recursion in the ballman recursion function. TJ itself is a function defined on S. TJ is optimal cost function for the one stage problem that stage cost g and terminal cost αJ

If you do this mapping recursively we have T^k :

$(T^k J)(x) = (T(T^{k-1}J))(x), (T_\mu^K)J(x) = (T_\mu(T_\mu^{k-1}J))(x)$

If we have policy $\pi = \{\mu_0, .., \mu_{k-1}\}$, then we define $T_{\mu 0}, T_{\mu 1}, ..., T_{\mu k-1}$ as opposed to the stationary one that all are identical.

Monotonicity Lemma: If we have a function $J, J' : S \to R$, if we define the bellman relation after mapping we must also have $(T^k J)(x) \leq (T^k J')(x)$ If k would be one then you are minimizing for two period, and here $T^k J$ you are minimizing the cost over k periods.

You have two problems, two period are the same, and the minimum cost would be lower than all of them, you can apply one policy to the other one and incure the same cost.

The sae applies for the particular policy.

Additivity Lemma says $T^k(J + re))(x) = (T^k J)(x) + \alpha^k r$ for each x you are adding constant r, and you can show that the r could be taken out of minimization, you are taking out the constant out, and it comes out of expectation. You can prove it by induction.

Speculated results: you start from $T^k J$, and you move $k \to \infty$, we hope that it would converge to $J^*(x)$. This should apply to any function J.

Assume you have two different cost, and variance, then discount applies the same behavior for any bounded function.

Bellman equation $J^*(x) = min_{\{} u \in U(x) E_w g(x, u, w) + \alpha J^*(f(x, u, w))\}$, or equivalently $J^* = TJ^*$.

When costs are bounded $(x, u, w) \in S \times C \times D$

We are not going to talk about stochastic shortest path, and we use the assumption of countable set.

$J^*(x) = lim_{N \to \infty}(T^N J)(x)$

You take some k and break into two parts. You can bound the summatoion. You assume that $J(x)$ is bounded. K-period problem with finite cost J. The sandwitch theory helps to show that the infinite horizon problem converges.

Now $J_\mu(x) = lim_{N \to \infty}(T_\mu^N J)(x)$ can be proved, although does not have minimum, by assuming one stage.

J^* should be unique.

$TJ* = J*$, optimal value function satisfies the bellman recursion.

All the result that you got also applies to single policy $J_\mu = T_\mu . J_\mu$.

Stationary policy is optimal if mapping T_μ would be optimal policy, you take the minimum $TJ^* = T_\mu J^*$ μ would be in argmin set, means it minimizes this function. μ attains the minimum in Bellman recursion. Suppose that it attains the minimum then we have $TJ^* = T_\mu J^*$, so $J^* = T_\mu . J^*$, and unique solution is J_μ, and $J^* = J_\mu$. This means the cost that you apply the policy μ would be equal to the cost of optimal cost, means μ would be optimal policy. It is in the form of if and only if, so the reverse also correct. This holds when the cost are bounded. If it is optimal then it obtains minimum, and it is necessary, and if it is minimum it is sufficient to be optimal.

Contraction mapping: suppose we have two bellman functions and we apply it k times. $max_{x\in S}|(T^kJ)(x) - (T^kJ')(x)| \leq max_{x\in S}|J(x) - J'(x)|$

Convergence rate, there would be difference smaller than $\alpha^k max_{x\in S}|J(x) - J^*(x)$, and as k increase the difference will become smaller and smaller.

Contraction mapping fixed point theorem also exists in game theory, for Nash Equlibrium. It can help to show that the solution exists.

Monotonicity, contraction, additivity results are three results we get.

The key assumption was that the cost was bounded in each period.

For the asset selling the offers should be from the limited distribution to satisfy this assumption.

In your paper when you use the poisson distribution, you would have infinite tail, and you need to justify why this assumption is right. Therefore you can not use the result for the optimal stopping rule. Holding and backlog situation again this assumption does not hold for inventory problem.

To shape the problem of inventory if we apply the convex function, we will have infinite from two sides in the integer space. To shape it you need to bound the demand, and if it is lower than certain level you order. Moreover, you never order more than certain amount. State space finite, disturbance and control finite, then finitie number of dimensions then the maximum would be finite as well, mean cost would be bounded.

Computational methods for finite state systems. You need to have finite state system. You can have transition matrix. You write cost as $g(i, u)$, and for finite state you will have $g(i, u)$, α for discount, $p_{ij}(u)$ and $J(j)$. you can write this in the vector form. Given a policy μ, we have cost of $g(i, \mu(i))$, and it would be a vector of length n, and for every state we will have value function $J(j)$, and matrix of transition. Matrix notion $T_\mu J = g_\mu + \alpha P_\mu J$. You can compute of any policy by taking the probability: $J_\mu = (I - \alpha P_\mu)^{-1} g_\mu$. The complexity mainly is due to inverse function of n^3, and you can gaurantee that inverse exist. We know that $T_\mu J = J$ would not have unique solution.

Finite state system with n states, you can evluate cost of any policy.

How to identify the optimal policy. We showed that finite horizon converges to the infinite horizon. You iterate and by applying the bellman recursion, and you show that in limit it converges to the optimal policy. The policy that minimizes the bellman would be optimal policy. Specially when you have finite number of controls, then you can always show that as you get ϵ close to the optimal function, if you look at the minimizing function f and g, for the finite set, you can show that you can minimize it. You can show that for ϵ small enough you can find the policy.

For descrete control you can find that two controls will become close enough. Can we find the optimal value? Taking the expectation of the infinite sum, you would have cost of the optimal policy in the for of $J_\mu = (I - \alpha P_\mu)^{-1} g_\mu$.

For Bellman equation you don't have any bound, you can compute the error bound. If you define $underlinec_k$, and $\bar{c}_k$ then you know that optimal policy is bounded from two sides, and the bounds are getting shrinked over time, and you get closer to the optimal value. When you get upper bound and higher bound, using error bound you can find out that they are converging. In this case minimum and maximum is taken over all states. For this you need to compute value over states and take the minimum. This makes it difficult for doing the parallel competition.

For example for the inventory with space 401, range +200, and -200, you would start with zero, since it does not make any difference. You create a vector of states with 401 items. You need to then calculate TJ, mean cacluating TJ for one time. $J(x) = g(x) = bx^- + hx^+$, v shape of holding cost and backlog cost.

Then this is applied for any value between -200, and +200, and this vector would be called v of size 401. We want to apply the Bellman recursion, $(T^k)J(x) = min_{0 \leq u \leq 200} E[g(x,u,w) + \alpha(T^{k-1}J)(f(x,u,w))]$
$min_{0 \leq u \leq 200-x} E(bx^- + hx^+(T^{k-1})(x+u-w)]$

$(x+u-w)$ is the index of the vector.

for $x = 5, u = 16$ we will have $6 \times 0 + h \times 5 + v(222)$

The state space would be $V(1)...V(401)$ with values of $-200, -199, ..., +200$.

You need to write the statement of 'for' operation and you start for the vector, and you have to do it for every state, and inside it you would have for loop of control, and inside that you need to do computation for every demand value w, and you take the weighted varage, and out of the first you take the minimum, and you do this for everyone, and you use the last vector in this.

For your papers, do the computation of your models you build, and your proof at least gets clear, and you will understand how recursion works. You may wind up with the value function that is not convex when you simulated it, while you were trying to prove that it was convex, and you may find out that you need to add some conditions.

Second optimal policy calculation:

Step1: start from an initial stationary policy μ^0.

Step2: Compute cost of any given policy μ^k. $(I - \alpha^P_{\mu^k})J_{\mu^k} = g_{\mu^k}$ Every iteration you are getting inverse

Step 3: once you compute it, given the cost of that policy try to improve and find the better policy.
If you start fo find out that you start with one policy and wind up with it, then it means that the policy has converged.

If it converges, you have something that solves the Bellman recursion, we know that it is optimal. It finally converges in fewer iterations.

Linear programming. Will lead to problem with lots of constraints, and multiple lines of MATLAB will be required, but for linear program it would be polinomial in term of state and control, and it is the only way for dynamic programming.

Technically we need boundedness, and we need to show that value iteration works, and the result is optimal policy for paper.

Convergence theorem. $J^*(x) = lim_{N \to \infty}(T^N J_0)(x), x \in S$

One way is finite control set. If the state would be unbounded, yet control is bounded, with countable distrubance, still convergene will happen.

In asset selling, we have finite number of control, so even if offers are from unbounded distribution, it is fine, and offers are positive then things work in infinite horizon.

If there would be payment for maintenance, we talk about cost minimization, you should not think about asset selling but buying. Until you buy everyday you will get the interest on that money. If it would not have unbounded negativity, shift will resolve the problem.

One other condition for convergence. You have compactness assumption. Expected value gives the subset of u's, called levo set. For the subset if it is compact then it also works. You can apply it for any k. As long as it applies for some k and above then it is fine.

Optimality equation under assumption that n is inevitable, since it would be recurrent class, then the cost would only depend on the dynamic of the recurrent class, and not on the initial state.

The optimal value is not dependent on the initial state, and the form would be different. In term of value iteration, and policy iteration are kind of the same.

Sennot has a book that describes these conditions. Black well optimality says we look at the optimal policy under discounted policy, under certaint policy sequency of policy converges to optimal policy.

Pricing Seminar of Professor Ozel And Profesor Moharram Oghli @ UTD: Fourth session

Meisam Hejazinia

02/07/2013

We will start from modeling of the demand, and where the pricing should start.

Start from context and history is important. What is the process of setting the prices. Think about what is important and why things are happening.

If you don't understand what the modality of the pricing is. Modality is business process analysis. What are the boundary of the processes, some transformation is in place. What are the rules of the game, and what is the process, what information is there and what would be output. There is a certain business process that gives you output. Rules of the game will tell you what you can do to improve the strategy.

Several reason why certain modalities exist. Why are people setting prices as they are?

Pricing from the economist point of view is equlibrium, and there is no reason why they should deviate. How it is institutionalize, and if one deviates things would change. Path dependance and historic evolution was another definitive factor. The third one is embededness, and social factors. There are other reasons regarding the government who sets price is also definitive.

What are the underlying factors that makes eastern world to shift to different equilibrium than eastern world?

HW:

Pick 4 product categories:

1. Corn flex
2. Iphone 6
3. Rental property
4. Washing machine

Write the commencing, essay describing what kind of model you would use to be able to describe what is happenning in this market. Doesn't have to be demand model, and it could be choice model. You need to describe why you select the model you chosen? You can discuss with your friend. If you have read the chapter carefully you would have the chapter for this.

Modality is the first key thing you need to understand. You need to start in general. You need to look at the binary relationships. From this you create preference. In order to understand order of preference, you create the utility function. From utility function you start to think about as individual how you select between choices. There could be intertermporal choice, decision to buy now or in future. You need to describe from the perspective of consumer how the decision is make. When you say consumers are similar to me, then you will ask who is going to buy in this period, and who would buy next period. In reality there would be heterogeneous group of consumers, so if you understood how you make decision as individual, then you can aggregate, you will understand how the demand is look like. It could be quality, or other things. Once you have that, you will have demand model. Once you have that demand model, then you will tell how someone makes decision. Pretty much most of time they create demand model, but don't know assumption underlying demand model. Don't motivate problems, with certain product or industry, and use demand model that is absolutely opposite you choose. Whatever you are motivating should have proper assumption discussed.

Preference based approach:

alternatives mutually exclusive.

Strict preference

Indifference relation

at least as good as

Rationality of preference properties:

1. completeness

2. transitivty

Deterministic approach versus random utility model

Multinomial Logit:

attribute of decicion maker (observed)
objects of choice and sets of alternatives
the model of individual choice, behavior, and distribution of behavior patterns in population

taste of the population

universe of choices

Set of available choices

Joint distribution of selection probability

If you have the distribution of choices you could use the demand function from them. You know the probability for each one and you want to calculate the number of people by taking expectation. Three axioms show that you can calculate relative probability and you can calculate those PI's. You would be able to find the closed form. It helps to find the parsimoneious model. What are the assumptions, and what are the key trade offs. All models are wrong, but some are useful. You make assumptions that are not true, but you make assumptions that end up creating useful models.

Axiom 1: independence of irrelevant alternatives (IIA).

Axiom 2: positivity

You will do pairwise comparison rather than doing comparison with all population.

Rather than look at entire set, you will compare with reference choice.

You can make probability by comparing every alternative with fixed point. Given that axiom you can calculate the probability.

For example my fix choice is white, and I compare everything with that.

The third axom makes this even simpler.

Axiom 3: irrelevance of alternative set effect.

$V(s,x,z) = v(s,x) - v(s,z)$

Mean I will just look at the distance. X has a value, z has a value and difference between them will give me the value of one relative.

Constant part will cancel each other, so I only need to look at only one choice rather than two and compare, and will give me the main relation.

People are not necessarily rational, and while you introduce a choice the probability, and likelihood of selection changes as a new choice is introduces, vary people's opinion, and make them to be more likely to choose one over another.

We now try to verify, and say what if the error term had double exponential dsitribution. I want to calculate what the probability should be using joint distribution. You take probability and insert joint distribution, and you will see the same result of axioms.

Lemma1: if error term would be i.i.d. with weibull then the joint probability helps me calculate it. Lemma 2 will give me the reverse. If I calculate the probability, and ϵ's are trnslation complete. Then we would have double exponential.

If you know that epsilon's and uncertainty are weibully distributed, then you would be able to use logit model.

Axiom one and two did not talked about distribution, and if another distribution would fit the axioms then you would be able to use for them as well.

If you have the choices and you have utility function for them then you can use this formula to calculate the probability of choices.

Hundred times probability of choosing a model will give you the demand function.

If you want to learn something teach it. This is the reason for presentations.

Mixed Logit

Why we do not have any detereministic random model. If the utilities are dterministic, and we know it is certain what is the demand? It is the maximum one. There are no specific model, yet in reality that is not the case, distribution of the products, and utilities.

$U_j = u_j + \epsilon_j$ is for statistic form. Then the probability of choosing product j $P_j(s) = P(U_j \geq max\{u_i : i \in S\})$

We have rational customer with rational preference, and axioms are for preferences.

We have dterministic and random part and functional form of each of the probabilities.

When we have error terms are normal when we have probit model.

$P(\epsilon_2 - \epsilon_1 \leq u_1 - u_2) = \Phi(\frac{u_1 - u_2}{\sigma})$

Since the difference of normal would be normal. Probit model has the distribution of error terms are i.i.d. and they are normally distributed.

Joint normal distribution, since you are looking at the maxmum of the joints.

Maximum of weibull and exponential distribution.

Binary logit: you have two choices. You have more set of choices. $F(x) = \frac{1}{1+e^{-\mu x}}$ the error would be weibull, and the difference would be logistic distribution.

We would have close form distirbution in this case.

Utility could be obtained from the price and the quality of product as well. The utility could be price minus the cost.

In major scenarios we will have budget constraints as well. Logit models have price and income as well. If you look at u_1 and u_2 they would be much more than price.

Deterministic part comes from somewhere and we are dealing with random part.

We can talk about joint distribution, and integrate over the density function.

If we know error distributions are i.i.d. we will have error distributions.

The aim is not just estimation, and it would be good to work directly with random variable.

Key restrictions of multimedia logit:

1. IIA. Independence of error term.
2. We need all information at once. It is single decision making, versus when the customer is making dynamic decision, choosing alternative over class. Information required for multinomial logit is much more. The decision happens at last stage.

Misxed multinomial logit captures the nature of sequential decision. Nested decision making, and this would be decision of IIA.

It takes the mixture of multinomial logit, and it can approximate any discrete choice from random utility maximization.

MMNL models are random utility maximization (RUM) models, and any discrete choice model derived from a RUM model has choice probabilities that are approximated by MMNL.

$L_C(i; x, \alpha)$ is logit for certain class.

α would be heterogeneiety.

We take the set of logits (for example for each segment), and take the average of them.

You call anything mixed, when you have the weighted average of them. Means every channel has the mixed the probabilities.

Mcfadden got Nobel prize for logit model, and talks about behavior in psychology and how those can be mapped.

They represent choices in reality, and come from reality.

Any discrete choice model can be mimiced by MMNL.

You can use Maximum Simulated Likelihood Estimation (MSLE).

MLE is for multinomial logit. In the Method of Simulated Moment (MSM) we first simulate for each and take likelihood and use it.

If all consumers are the same you can use MMNL model.

Identify customers behavior, parameter (enter price, and find the choice). In actual, yet you will see many customers and each has different parameters.

The rest would be on aggregate demand.

Properties of demand functions.

1. Regularity of demand function.

Contineously differentiable on set of prices

Demand fucntion strictly decreasing.

Demand function bounded.

Demand tend to zero for sufficiently high price namely.

Revenue function is finite for all prices and has finite maximizes that is interior.

You will try to find the maximum price that makes the demand zero, and demand is decreasing, and it is inferimum.

It is minimum since minimum could be infinity, and we want it to reach zero .

2. Reserve price/ willingness to pay and demand functions.

Consumer budget problem

In optimality you should be that the reservation price be equal to the price of the product. If it is not the case I would purchase more, and increase my utility. By contradiction you can claim this.

Reservation price is maximum price I am going to pay for product. Mean that reservation price you are modling is dependent upon time, budget and all related things to utility.

If willingness to pay has a distribution, you can use it to find the demand.

Elasticity is percentage in demand over percentage change in price. $\frac{\partial ln(d)}{\partial ln(p)}$

In long run there is possibility of substitution and postponement and not purchase so elasticity on long run would be greater.

Elasticity depends on current price and time frame.

If marginal revenue is concave then you want to maximize revenue. Marginal revenue should be equal to marginal cost.

Distribution of willingness to pay.

If revenue function is concave we would be able to get maximum revenue.

Objective function is unimodel. When concave it should be unimodel. If elasticity is increasing, at some point it will start from a place and hit a point.

Unimodelity is important to have unique price to charge.

elasticity would be equal to one then you will have maximum revenue.

Markup, contribution margin.

Your markup, and contribution would be equal to the ealsticity.

For multiple product there are same regularities, yet with difference that they may have cross price elasticity. There would be cross price elasticity. It could be positive or negative.

Common demand function:

Linear
Easy to estimate, price constraint, $d(p) = a - bp$, and you can write for multiple product. Decrease and some point hit zero. underlying assumption: 1. At every price point elasticity will look different . 2. willingness to pay uniformly distributed across population. Log linear
exponential. No price constraint. $d(p) = e^{a-bp}$, rather than linear it drops suddenly. You would not have well defined form.
Constant elasticity:
$\epsilon(p) = -b$, and $d(p) = ap^{-b}$. Logit:
Willingness to pay would look like Logistic distribution. $d(p) = N\frac{e^{bp}}{1+e^{bp}}$
It looks like normal at the middle and extreme on the other side. Integration would be the form of S shape, and we can make it either stipper or less stipper. Means if there is competition in the market, it will drastically affect the demand that would be observed. Some customers may not care at all. Those customers are so brand loyal that they don't care, no matter what price is put. Price elasticity at the end would be so much in elastic.
Using this does not give you the analytical flexibility. At the end of the day to decide and select would be: 1. elasticity: are constant, exponential? 2. Willingness to pay: underlying things of it. 3. Whether you can fit it. 4. Analytical tractability. How to optimize.

You must asking these questions when you are reading this.

Draw these functions on excel and check how does it react when you draw, and change the parameter.

Check the infliction points.

second homework For your homework draw demand and supply and elasticity and put some next to it, and draw the objective function, and verify what are the optimal prices corresponding, and also put the objective function.

Stochastic demand functions.

Source of variation: 1. measurement error 2. models are not perfect. 3. heterogeneity across people (unobservable): idiosyncratic stuff.

Anything that is not observable in error term mean zero, and additive, and not systematic form that you did not captured.

Independance of demand error term from price, otherwise there would be something that you did not captured.

Additive uncertainty, when things realize, elasticity would be changing. In the environment things are additive, and you expect elasticities to be changed.

If error term large, and small demand, you will end up in negative demand. For eseasonality you need to keep an eye on it.

Multiplicative uncertainty:

$D(p, \epsilon) = d(p)$.

The variance would not be independent anymore.

Error term here would not affect elasticity.

In mixture model you can introduce two different error terms.

Bernouli models and poisson models are other ways.

They let you to track demand in discrete time units.

Chapter 30.4.1 capture pricing dynamics.

Salvage cost.

$p^* \leq p^0$ in additive demand case comparing stochastic demand than deterministic could be explained by the variation and CV.

On the multiplicative case this relation is reverase and again could be explaine dby variance and coefficient of variation.

Additive observed price elasticity change, variance would be constant, and not depending on the price, so mineral water is part of this. Price does not change the variance.

For the product that the variance is lower you will use additive case. In multiplicative case current state will affect uncertainty. Forecast update in current state generally higher than what you are doing.

Properties of additive versus multiplicative, and how these properties fit to the market you try to consider. The same things applies to demand model.

Why elasticity not equal to 1? Those elasticities could change depending on time. Elasticities are function of many things that could be aggregate.

Von Neuman utilities.

For long run plan the multiplicative form works.

Pricing Seminar of Professor Ozel And Profesor Moharram Oghli @ UTD: Fifth session

Meisam Hejazinia

02/14/2013

Dynamic list pricing.

There would be a homework. Along with the reading from two list from now.

List pricing, prices posted, and consumers make decision about it.

Value they get above the list price they will purchase. Everybody will know there would be single price. Common use pricing modality.

Everybody see's the same price, and it is not most profitable for the company.

Price discremenation will be discussed. If customrs have different willingness to pay, and you charge the same price, you are leaving money on the table.

There is a way to charge some customer higher, and we did not.

Anything above cost we could make money from them, so if you charge more some people do not purchase, so you are leaving money on the table.

You charge epsilon less than customers willingness to pay.

Coupons could be used so that customers decide themselves based on their willingness to pay to purchase. This would be second degree. For the second degree it is critical for the firm to segment their customers in proper way, so that the product fits the customer.

Flexibility could be valuable for example for business class customers.

The third degree is when you have categories. Different price for students vs. general public. Clear lines about segment of population.

Price for students and general public that demand curve of students and general public would be different, and offering different prices will give you different profit.

Dynamic list pricing, varying prices over time.

The capacity would be a constraint. It does not have to be the only reason. Fad and fashion and seasonal product is another reason for dynamic list pricing. Peoples taste change, and something are more fashionable. Demand distributions change over time. Perishability is also another reason for dynamic list pricing.

List pricing would be same price for all.

List pricing is about we give the price and you decide.

Every given period you charge the same prices, and if it is same customer you are charging different price over time, and if not same category then different customers with different prices.

It is self selecting customers. Whether customer is myopic or forward looking is not relevant issue here.

Second part is where we have control over this capacity.

Demand curves different, from the different population people then could help us to charge different prices for different people.

The product is designed in the way that people would self select it.

Dynamic is over time, static is you have same thing and you will not change it.

Myopic versus forward looking are different settings, and we will just work on myopic customers today.

No close solution for contineous time model, so deterministic model is used instead. Deterministic model solution is fixed.

Fiexed-price heuristic

Periodic pricing review policies

Fabric factory US, assemble Asia, Ship back to U.S.

Characteristic of model: Fixed capacity over time, there is a dealdine (finite horizon). Stochastic demand and price senstivie. No backlogging the demand. Salvage value of unsold items. All costs are considered sunk cost.

Assumption: all the customers are myopic.

Homoteneous, time invariant, poisson process, independent increment and stationery increment, means demand of today does not depend on past prices. ($\lambda(p)$)

When there are not lots of customers come in, poisson assumption is shacky assumption, since they would not be independent. Good is not durable, or we do not have only small portion of the market independent increment would not hold anymore so poisson process would not be halpful.

Maximization of profit can be done over demand rather than supply when we have inverse of $\lambda(p)$ in the form of $p(\lambda)$

Revenue function $r(\lambda) = \lambda.p(\lambda)$, continuous, bounded and concave.

Rather than having condition on the $d(p)$ work over $p(d)$ is generalized form of additive and multiplicative, means you put condition on the revenue, and regularity condition holds.

Customers come with rate c, and whether he purchase or not will depend on price $\lambda(p) = c.\bar{F}(p)$

Different acceptance rate, and assuming the entrance of two poisson process, could make a new one in case we have two different customer streams.

Toss a coin to see whether customers that came accept or not. Price will change the result of that coin.

Marginal value of capacity

Equate with marginal revenue will give optimality condition of dynamic programming.

$\frac{dr(\lambda_t)}{/} d\lambda_t = \Delta V_{t+1}(x)$

Heuristic would be asymptotic to optimal as $n \to \infty$

When you have longer priod you do not need to do it dynamically.

Markdown decisions, and clearances.

Inventory commitment: current on hand, plust plant shipment in the reminder, while on hand is different.

Markdown special case of dynamic pricing.

Why multiplicative: 1. analytical tractability 2. concreteness

$k(t)$ seasonal variation.

You price in such a way that inventory will not have effect, you compensate the effect inventory by varrying your price.

Pricing will not have affect until the end of the season.

How the model is shaped. How they motivated it, and how they justified it.

Seasonality factor in the form of multiplicative model.

Expected profit function of demand and function of q. Change variable.

Sometimes papers are written in reverse order, and to understand what is happening you first try to optimize that by yourself, and you will understand why change of variable is happening.

Coefficient of variance $\frac{\sigma}{y(p)+\mu}$ in additive case which has part of price in it, while in the multiplicative we have $\frac{\sigma}{\mu}$ which does not have price effect.

In additive case we can decrease CV withouth adversely affect the variance by changing price, and we can change price in multiplicative case without affecting CV.

Two strategies of inventory building:

Build it and they will come

Let them come and build it later.

Revenue management and Capacity Building litrature combined.

Previously it was saying that we have capacity and now want to maximize revenue, and here we said now we have demand how to build the capacity. In building capacity there would no more be risk, since we have the demand.

Pricing Seminar of Professor Ozel And Profesor Moharram Oghli @ UTD: Fifth session

Meisam Hejazinia

02/21/2013

Non-linear pricing.

Try with simple excercises so that things perticipate in your mind.

Also try to solve the problems by yourself, even when it is not assigned to you.

Last time we had dynamic list pricing. Now we are talking about the way that people could pay different prices, since they have different willingness to pay.

First degree you can charge everybody at different price. Legal issues. Issue about the execution, and it is rare.

Second degree we talk about designing a product in the way that they reveal something about their own price information, and bundling would be in this framework.

Anybody can select any of the offerings.

The segment will select their own product. Charging different price for higher value orders, you have done that since bigger firm has higher or lower willingness to pay.

Third degree you have segment of customer, and you do not allow people have different choices. You have to be careful about legal issue, and transferability.

You try to price to discriminate, but based on what?

If I know your willingness to pay, I will start to charge, and you will get surplus.

First degree is: pricing based on willingness to pay.

Sometimes these things correspond to things you already no, first best, and second best.

First best is based on willingness to pay.

Second I do not know willingness to pay, but I have info about characteristic of demand. for example quantity.

Additional quantity can bring them more value.

Attribute could be used here. Some people like to wear Armani, and you segment them by quantity and third degree. You are discriminating based on observed characteristic of segment and not demand.

$1 - p_i$ would be demand for the first. What would be second degree?

I have some understanding of this person's utility. Utility as a function of q. For some people it would be different than others. Utility for some group of customers. Business versus residentual for example.

The additional quantity has more value to me than others.

I am going to maximize my expected utility.

On the second degree, people self select based on their utility.

This is adverse selection problem.

Third one I do not want to do self selection. Senior versus student.

You are going to maximize objective function which is sum of the two, and one will have surplus, and they should be separable.

In third degree you don't have self select, so it is better, since you have segmented based on the demand characteristics.

We will talk about self selection today.

Ability to sort customers into groups with different reservation prices.

individisibility

Mixed bundle versus pure bundle.

Mixed bundle you not only will provide each product, but their bundle as well.

Reservation price = valuation.

On the pure bundling you force someone to purchase the good that he really does not need.

On the mixed bundling producer surplus will increase, and total welfare increases.

Small variance in valuation of the component.

Negative corrlation between two goods. In this case you extract more consumer suprlus in mixed bundling.

Factors affecting selection of bundling:

1. Type of customers.

2. cost structures.

Some cases we have need to charge monopoly prices for each.

Objective functions and constraints could be written for three products (first, second, bundle), with constraint that utility areg greater than zero, in the case of first degree.

If able to separate these guys, you can charge price equal to reservation price.

Objective function would not be separable for the second degree.

Correlation between willingness to pay (positive, negative, no correlation), may as a rule of tumb require mixed bundling, pure bundling, and pure component respectively.

Quantity Discount and Ramsey Pricing

Two part pricing. $p(q) = F + cq$

Two block pricing $p(q) = p_h.q,\ p_h q_l + (q - q_l)p_l$

Two block weakly dominates two part. If objective function is profit then two block pricing weakly doinates two part pricing, and all weakly dominate uniform pricing.

consumer suprluse lowers when you change from uniform to two part, but this happens if you move fro uniform to two block.

Separable objective function means you can maximize for each of the prices, and then sum the result up as the maximum of the objective function.

Uniform price comparison with quantity discount based on price elasticity parameter (rate of elasticity over the quantity), in the form of convex function. As quantity increases you become more elastic or less elastic as this rate of elasticity over quantity.

Regulated monopoly, government says you can only earn specific amount of revenue. You can maximize your profit, subject to the constraint that your profit does not reach specific threshold.

You discriminate based on characteristic of customer, which is either residential or commercial. The optimal condition is still separable, due to the inequality. Mainly can be seen in the Lagrangian.

priority pricing

Gaurantee and quality of service.

According to reliability for example.

Indogenizing the choice, you give the people the choice what type of quality they want.

Unbundling quality of service; you maximize your utility, and one parameter is quality.

Priority pricing scheme is optimal among all the possible price schemes whenever the potential demand exceeds the capacity limiation of the firm.

Possible schemese can inclue: 1. one stage mechanism: sealed auction, single price, pay what you want, 2. sequantial mechanism: english auction, two part tarrifs.

Multistage game, with finite set of actions, for example bids.

Revalation. Nash equlibrium in revealing true reservation price. Equivalent scheme for any price schime that is both direct and truthful.

Allocation by rank with a cut-off rank is optimal.

All are second degree price discrimination.

$max_p p_1 - c_1$

Constraint: want to ensure that when $w_1 - p_1 \geq 0$

The customer has willingness to pay.

For the second one you should have:

$max_{p1,p2}(p_1 - c_1) + (p_2 - c_2)$

$w_1 - p_1 \geq 0$

$w_2 - p_2 \geq 0$

$p_1 \geq 0$

$p_2 \geq 0$

The result would be $p_1 = w_1$ and $p_2 = w_2$

Second degree price discrimination says.

I know something about utility distribution.

$u_1(q)$ q could be quality level of first product.

For customer 2 $u_2(q)$

There are different customers in term of utility.

The addition of utility from addition of utility is $u_1'(q) > u_2'(q)$

Cost of providing the quality is positive, and is increasing with quality level mean $c'(q) > 0$ and $c(q) > 0 \nearrow q$

We fix quality now

$max_{p_1,p_2}[p_1 - c_1(q_1)] + [p_2 - c_2(q_2)]$

$u_q(q_1) - p_1 \geq 0$ mean they should have minimum reservation profit, otherwise they will not purchase.

$u_2(q_2) - p_2 \geq 0$

It is still separable, and we can solve for each. We can not solve it, since by choosing price and separating them will be back to the first problem.

The first homework was to solve the first degree state, and second homework is to solve this second one.

See what is p_1* and p_2*

I would like to give two prices p_1 and p_2 and let anybody to select.

If customer 1 gets the price that is designed for him, his utility is larger than the second one $u_1(q_1) - p_1 \geq u_1(q_2) - p_2$

The same is true for second customer $u_2(q_2) - p_2 \geq u_2(q_1) - p_1$, as a third homework solve with these conditions and tell us p_1*, p_2*

Compare these result of second and third problem.

Write customer surpluses for each of the customers for each of the cases as well.

The idea is you are announcing (p_1, q_1), and (p_2, q_2), and customer are selecting.

q_1 could be greater than q_2, try to make your own assumptions to solve problems.

In the third form, we need to be able to identify two segments. We can correct based on customer. Not the characteristic, but based on customer. Male, female, identify, and ...

Since I can identify I would be able to solve the same problem, and I would not encounter the problem that customers can select package of eachother.

There is observable attribute of consumer:

1. Residential
2. Commercial

This price is only for residential (the first one), and second one for the commertial (second price).

I can not observe valuation, but I know the distribution of them.

Come up with price for these possible customers, and allow them to self select. Take first problem, and I have the distribution, since I don't know who will show up, there would be probability that each of the customers could show up, subject to the condition that I know he is either type one or type two. I will include constraints that high type select high price, or low type select low price. It is individual rationality constraints, and I select by adverse selection, which is like second degree price discrimination.

Extra assumption: there is w_h and w_l, and as producer I have distribution for them.

I can come up with p_l^*, p_h^*, so that I maximize profit, subject to the constraint that both low and high type buy from me, and I want high type to choose p_h, and low type to choose p_l^*. This is as as a frounth homework, to formulate the question.

This is participation constraint (to buy from me), in such a way that the customer self select, and each select the price that is for them.

Pricing Seminar of Professor Ozel And Profesor Moharram Oghli @ UTD: Seventh session

Meisam Hejazinia

02/28/2013

Revenue Management

Relatively new topic. Around thirdy years. In compared to other fields of inventory management.

Linked to dynamic pricing. Not really 100% clear distinction. Make decision based on capacity avaialability. Make pricing decision based on availability.

Inventory, and available sits change dynamically. Optimize revenue. Powerful tool for airline industry. One type of revenue management. It is more about managing capacity, than managing prices.

If you just think about airline industry. Midsize carrier. They have thousands of flight everyday. 200 sit per flight. 200,000 sits per day. You can purchase up to the year in advance. 365 days. Around 7 million sits management everyday. Very important for revenue.

Airlines used to be regulated until 1978. Price control lifted as deregulated. People express started in 1981. Rise of low carriers. American airlines. Do things more statically. Bob Crandall recognized one problem. Many flights departing with empty sits. At any price they could make money. You are flying those sits. They introduced discounted fares ('Super Saver').

Initially, once they announced it. Analyst thought it would not be good. There could have been price wars. American airlines started to work better.

They went from \$60 profit to \$160 loss, and bankrupt in next year.

Yield management was origin of the point. Their big chunk of profit was revenue management.

Classes that create differentiation. Variation in terms and conditions.

Categories is opened and closed every second, but prices for categories is not changed so frequently.

In airline we mostly talk about opening and closing the categories.

Revenue Management

Structural decisions: which selling format to use?

Price decision: how to price across categories?

Quantity decisions: how to allocated capacity to diff segments?

Single resources:

Two class model

Multi class model

Multiple resources

Consumer Choice Models

Fixed number of sit. Capacity are homogeneous, and perishable.

n product classes with associated prices

Different discount levels with restrictions

Demand for different classes are independent.

Demands are independent from capacity controls

No group booking, cancellation, no shows, and thus no overbooking

Low fare demand arrives before high fare demand

If they were in reverse it would be first come first serve

Denied customer is lost

Fixed cost is high, and the variable cost is very low.

Booking limits: The number that could be sold to the particular class.

Protection levels: the number of booking that should be protected for the class.

$p_2 = p_1.P(D_1 \geq y_1^*)$ or equivalently $y_1^* = F_1^{-1}(1 - \frac{p_2}{p_1})$. The comparison with the expected revenue of the other class.

Newsvendor model. Loos profit, or '$cost-$ salvage value'.

Assumption was that Low fare demand books first.

In reality they may not come in this order, but this analysis is more of the big picture things. Business traveler purchase late most of the time. For individual customers, although they may be variations.

Heuristic Method: expected marginal set revenue. (EMSR)

Create artificial demand combining two classes, and then solve problem based on little rule for two class, and find the little result for the virtual class. $\frac{p_1 E[d_1]+p_2.E[d_2]}{E[d_1]+E[d_2]}$

Heuristic is intuitive, although it is not optimal. It is still used in many cases.

Airline Seat Allocation with Multiple Nested Fare Classes

Assumptions:

1. Single flight leg

2. Independent demand for different fare classes

3. Low before high demands

4. No cancellations

5. Limited information: Not make decision based on time departure, but based on observed demand and left over capacity.

6. Nested classes

Demand for the lowest fare class arrive first.

Two criteria. Booking to the class has booking limit. Sales to the lowest fare class are then closed. Demand for the next lowest fare class arrive.

At each stage you sell ticket to certain fare classes.

Booking policy: whether a fare class should be available for booking at a certain point of time. A general policy may depend on prior demands.

Simplere booking policy: A vector of fixed protection level $y = (y_1, y_2,)$

Closted loop versus open loop.

price for each fare class: $(p_1, p_2, p_3) = (10, 9, 8)$, and fixed protection level $(y_1, y_2) = (20, 70)$.

Total capacity is 100.

Marginal revenue of now versus expected value of future.

bid price control. Local loss versus total gain. Virtual nesting control. Partition booking limit.

Keep the balance, and take into account the network effect problem.

Marginal value of resource.

Bid price control is optimal asymptotically.

Bid price control: optimal when linearly. Not optimal generally. Asymptotic properties. Approximate schme: decomposition, simplify the static problems.

Protection level difinition is not the same as quantity revenue management. This is network case, and has different legs.

Instead of putting all combination in, you assign dollar value to each.

Revenue management under choice model of consumer behavior

Assumptions of earlier models:

consumer demand complete independent of control from sellers. Here different. Demand dependance on current price, only one product sold at one price; binary choice: buynobuy.

Reality:

Many offers, consumers chooose based on price and preferences.

Problem: Single leg yield management: model consumer choice behavior, specify probability of purchase for each fare product as a function of set of available fare products.

Propose an optimal policy.

At some point open two fare classes, but at other point not. Which subset to make available is the problem. Revenue for each class is fixed. Consumer choice is these are available choices, and which one I select.

Demand based on offer to select which one?

How optimal policy can be implemented. nested by fare order. Nested allocation policy.

Fixed probability of arrival. At most one arrival in each period.

n fare products, set N. Fixed revenue from each product. a subset of fare products to offer. A choice probability.

Willingness to pay for each class is available. (Yes vs. No).

Given availablity of each classes what would be the probability of purchase of each of the products.

We have two cases: purchase, or not purchase.

On purchase we will have one extra capacity to fill.

For not purchase: Arrival that did not selected, or there was not arrival.

Preferences are revealed in some way.

Maximum over efficient sets.

For given subset total expected revenue, denote by $R(S)$. For each subset getting total probability of purchase, denote by $Q(S)$.

Be on the frontire rather than frontier.

Efficient sets.

At each capacity level you will calculate which efficient set is optimal.

Demand exogeneously than consumer choice, still we are using the same policy, but how you reach to those numbers are different.

In this case we do not know which demand is coming, and the assumption of reverse demand coming does not hold, so it is more broader framework.
Set up would be different in this case.

This segementation does not come from fare classes, but from maximization problem.

Here we tried to learn about customer segments.

Pricing Seminar of Professor Ozel and Profesor Moharram Oghli @ UTD: Eighth session

Meisam Hejazinia

03/21/2013

Game Theory and the models of pricing

On research?

What are the optimal decisions.

What pulses could be used to make the decision optimal.

How can we optimize, and then check what decisions will be affected by actors that have not been anticipated.

The same methodology of Game theory. How capture on pricing decision.

Interaction between two firms. Implication of contracts. Whether save the information or different, and how the impact is on my decision.

Individual when take decision, there hare behavioral matters. Does not always mean soft, but there are some aspects that are not captured by classical econs, and game theoric, and we will think about them too.

Firm level, individual decision making, how decision is made, and how to optimized decision from the decision maker aspects, and how the equilibrium will be constructed, and then check the behavioral and see what would be their role.

Regulator perspective, customer perspective, and supply chain perspective.

Another thing is within the firm perspective. There are things that need to be done within the firm. There are multiple decision making.

When pricing decision, it is not that one person makes the decision, but there would be multiple decision makers that will involve, and you need to see how they are fit within the firm.

Need of game theory pricing:

Single agent has been discussed. Why we need game theory, cournot bertrand, stackleberg.

Single period and multiple period pricing games will be discussed as well.

Objective functions of the firm, and some constraints to maximize it.

Objective function example: 1. consumer surplus 2. financial goals 3. market share and so on.

Pricing policies:

1. single price or multiple price

2. price discrimination or yield management

3. Quantity discount, bundling

4. Customer price sensitivity

Complication resulting from taking into account the objective function of the other actor, makes firm to like to do the single.

Need for guess, conflicting objective, and key to success: practice hard, and predict the other players move.

How other player will react to your decision.

Bertrand competition:

$Q(p) = \alpha - \beta.p$ for the demand function.

Assumption:

homogeneous product.

Same market power

There is no loyalty, and split market equally when same price

FOC: $\pi_i = c.q_i(p_i, p_j)$

There would be tie breaking rule, due to the discontinuity of demand

Reaction function

Converging, and then check whether it exists or not.

Start from one point and then check whether you converge or not, and check it for multiple points.

In Bertrand show that it is unique.

Feasible sets of equilibrium could exist, and it is also a reduction of the space, and that would also be helpful.

$q_i = \frac{\alpha-\beta.C}{2}$ with zero profit since $p = c$.

Bertrand	Cournot
(C,C)	$(\frac{A+2C}{3}, \frac{A+2C}{3})$
$(\frac{\alpha-\beta.C}{2}, \frac{\alpha-\beta.C}{2})$	$(\frac{A-c}{3B}, \frac{A-c}{3B})$
(0,0)	$(\frac{(A-c)^2}{9B}, \frac{(A-c)^2}{9B}$

On the stackelberg, when leader selects High or Low, second firm observs that and then follower selects his strategy based on the signal recieved. To solve you use backward induction. We try to get subgame equilibrium.

The dot line of information set reduces the number of subgames since it shows that subgame could not be separated.

Subgame Perfect Equilibrium (SPE).

Two stage: 1. decide booking limit 2. Demand arrives.

Demand for low fair demand arrives first, and then demand for high fair demand arrives.

Assumption: D_{ki} is exogeneous and not affected by booking limit.

Fix point theory, and that mixed strategy existance when not strictly concave.

Multiperiod use dynamic programming.

Single period versus multi period.

Single decision maker versus revenue game.

Multiperiod Revenue game: use both tools of game theory and dynamic programming.

If they let the customer go, they can save their inventory for future customer.

Unit sold today may mean less revenue for tommorow, but we may end up with less revenue.

Backward induction, and decide price based on the the capacity left.

In the last period no future value, so it makes both of them to engage in perfect Bertrand competition.

valuation times the probability that the other player has unit of inventory at the last period would make second last period with limited capacity.

In this case we will have asymetric bertrand game.

The person with lower capacity wants to exit first.

Whenever there are the couple of people with the same capcity, and one with more. The firms with highest capacity engage in a price war and we can see negative prices.

In the case of monopoly the person charges reservation price, and the profit of industry would be higher.

For each presentation at the beggining or at the end try to point out key take aways.

problem in chapter 29, section 5, is the problem, right before implementation issues there is a problem about information assymetry, and try to solve the problem as homework.

Pricing Seminar of Professor Ozel and Profesor Moharram Oghli @ UTD: Ninth session

Meisam Hejazinia

03/28/2013

Up to now we looked at the single decision making perspective.

This chapter looks at the SCM perspective, and looks at the business to business interaction.

When the supply chain is owned by single decision maker, I own SCM and everything and how would I decide orders. When you have multiple decision makers, we will have multiple objectives, then what would be different. Due to different marginalization. Profit margin of both would be different from each of the firms. If I make less profit, then you want to care more inventory or higher? It is downside risk versus upside risk.

Upside risk is when the demand is low, there is potential of excess inventory, and downside is there is potential of lack of inventory.

If risks are not balanced in supply chain, then we will have different incentive. That would not be align with different actor's solution.

When you decentralized things:

1. lack of information, local information.

2. No more single objective, but multiple objective

Efficiency is your distance from the benchmark.

The benchmark here is centralized firm.

To become close efficient you either dicate or you put the incentive.

Dictating things when it is difficult to convince people to do something. As a downstream firm convince them that how they should replenish their inventory. There is not necessarily contract terms here. Other discritionary things that make them agree with that.

Garaunteed profit, and improvement from what you are earning, or pareto optimal so that people would be willing to go to the next level.

Decentralization will lead to:

1. Profit margin change

If I need to solve my problem, I need to know her cost strucutre, her revenue structure, response function, payoff of the other firm, and this is local information. The question is is it right incentive for her to share information with him. In this case local information will become global information. Once the information is global you need to find the objectives only. Once I got information I will know his entry, exit strategy, and my entry, exit strategy. How could I used certain contract to get that. The seocnd part tries to find out what will happen if there would be information assymetry.

Coordination means the decisions at the end would be equal to the profit of centrally integrated firm. Some of the contracts in this case coordinate. We will have the same set up and they all do the same thing. Why we have all source of contracts that coordinate channel? Why those contracts would be different? The objective would be different. They might minimize lost.

Elements of information at each node:
Cost structure
profit margin
forecast

If demand turns out to be significantly low then the supplier would have excess capacity. If you do not align supplier will have excess capacity.

If the demand would be significantly higher then both will loose.

Firm tries to maximize the expected profit; this is the reason for balance the trade off of downside risk and upside risk.

It is part of small business, and it is smaller part of the firm that does not affect the firm to go bankrupt. By selling stock and other things they may balance the risk.

$c_u = p - c$ wp $1 - F(Q)$

$c_o = c - v$ wp $F(Q)$

$c_u.(1 - F(Q)) = C_o.F(Q)$

Decision degree of freedom: Affecting buy backs and decrease the cost of underage.

Arbitrary buy back helps to create pareto improvement.

Equivalence of buy back contract and revenue sharing. If you create the contract that creates channel efficiency, and one allows arbitrary sharing, and the other is profit sharing, we can find parameter

integrated firm	$p - c$	$c - v$
whole sale price	$p - w$	$c - v$
buyback	$p - c$	$c - b$
revnue sharing	$fp - c$	$c - v$
renate	$p - w + r$	$w - v$
penalty	$p - w + z$	$w - v$
quantity discount	p-w(Q)	w(Q)-v
Two tarriff	$p - (w + F/Q)$	$(W + F/Q) - v$

that both would be equivalent.

We can take one contract from the other contract by changing the parameters.

Rebate contracract: I will give you extra if you sell. Affecting cost of overage.

Buy back will affect cost of underage.

The contracts affect different part of supply chain. They affect risk in different way. Even though they are equivalent in term of coordinating channel, but they are not. If people are responsive to negative and positive in different way, people may react differently to that.

Pay back for the capacity, or reserve capacity, if I you did not used it I will pay you back.

Stackelberg game, backward induction could be used. Bertrand Nash equilibrium, cornot game was when simultaneous, by mean of they could not see the result of each other movement.

Erricson, hitachi and intel writing contracts, when they are exagerating.

Supply uncertainty

$D = \mu + \xi + \epsilon$ where ϵ is market uncertainty, and ξ would be private information (deterministic) S has a believ random variable. Should these two be independent?

If you are good forecaster, the forecasts should be independent.

When reporting forecast is it incentive into it. It is not verifiable. We can not say whether it is true.

Talk is cheap, and it is cheap talk. This is called dynamic game incomplete information.

We can credibly share information, but the question is whether it is possible.

$\phi(\hat{\xi}|\xi)$

He can not communicate his strategy.

Supply decision: take this information to set capacity. $K(\hat{\xi})$ he does not communicate, but his objective function is clear.

$\phi(\hat{\xi}|\xi)$ and $K(\hat{\xi})$ are response function to eachother.

Any strategy ends up to be uninformative. Posterior belief and prior belief would be the same.

r revnue w whole sales price

$(r-w)E[min(K, \mu+\xi+\epsilon)]$ would be profit of the manufacturer

Supplier problem would be $max_{K\geq 0}(w-c)E[min(K, \mu+\xi+\epsilon)] - c_k.K$

Optimal capacity:

$K^{ws} = \mu + \xi + G^{-1}(\frac{w-c-c_k}{w-c})$

The supplier knows this deviation incentive of you to reveal the truth, so he will as a result he uses his belief about what the forecast of the capacity should be, and optimizes his objective function using that.

Cause:

1. Double marginalization
wholesale price less than sale price

2. No information sharing
M has incentive to inflate the forecast
S cannot verify ex-post

Solution:

1. Risk sharing:
Pay back contract

2. Credible information sharing:
Capacity reservation
Advance purchase

S has a capacity strategy $K(\hat{\xi})$
Does not communicate and commit to an action rule

Supplier communicates and commits to the mechanism. If you agree you will play the game by the rules that I say.

Given the message space M, action rule based on m would be there, so $\{K(m), P(m)\}$.

This would be mechanism design.

The question is what should be the message space?

All message space should not be considered. We can constraint ourselves to the message space which is between lower and higher bound. Restricting to direct mechanism will not limit us.

I can only restrict myself to forecast. It would take low value and high value. You decide which one to use. You will select the one that may be truth telling or not, and then I build and take the price.

Supplier optimization problem then will be solved.

$E\pi^s(K(\xi), p(\xi)|\xi)$, and conditioning on ξ means conditioning on all the sample space.

Participation constraint would be $\pi^m(K(\xi), p(\xi)|\xi) \geq \pi^m_{min}$ which means this participation should provide higher profit than previous non participation profit.

We also need to have incentive compatibility here by mean of $\pi^m(K(\xi), p(\xi)|\xi) \geq \pi^m(k(\hat{\xi}), p(\hat{\xi})|\xi)$

It is difficult because:

1. it is function optimization (in optimal control you will learn that it is very difficult problem)

2. we need to have all constraints of participation for all the possible forecasts.

The very first thing to solve is to simplify space, so you try to simplify the constraint.

If I can not solve it globally what does it mean locally. Can I find properties of optimal solution and use them?

They replaced incentive compatibility constraint, saying that optimial solution satisfies that, and by looking at that you find characteristics of it.

Second I prove that participation constraint binds at one extreme.

In this case all the other constraints would be redundant.

They first forget about the constraint, since if the solution turned out to be increasing they they feel lucky, then if not it would be problem.

Solution that maximizes the integration that maximizes that.

The non linear pricing, showing that the larger the capacity lower per unit price.

Because of the information assymetric, manufacturer ends up paying higher. This is strategic commitment. We used non linear information for segmentation. Manufacturer has a forecast that I don't know, and I give incentive to people who have higher forecast than those who have lower.

Commit an action strategy and act on that, means buy in advance before you build your capacity, and then after you build I will purchase some other amount.

This is principle agent problem, means I change the rule of the game.

Advance purchase it he price that you will pay before supplier purchases.

Signaling game: principle commits an action, and acts based on that.

Given that strategy you decide how to take an action.

Separating equilibrium that gives you perfect information sharing.

Signalling, screening, two type of contract, which one to use?

Compare signalling and screening, and compare and decide which one gives you better?

Two sources of inefficiency:

1. Classical: double marginalization.

2. Assymetric assymetry.

Double marginalization is bad in term of efficiency. $w - c$ and $p - w$. If I know exact demand is, and need to decide? I would order the demand. It does not affect anything on that case.

You need to also capture uncertainty. We need to adjust according to it.

What we need to deal with is risk adjusted double marginalization.

Check the ration of variances is measure of information assymetry.

Screening works better in case of low information assymetry. When the cost of signaling is lower than cost of screening then we have assymetric information.

Before 2000, channel role of contract and 'risk adjusted profit margin' was analyzed prviously. Around and after 2000 people start to see what happens in information space.

Non pecuniry issues such as risk, check how the other things change. Question all source of stuff on that dimension.

Assymetry is mostly higher in high tech per simple empirical.

On high asymmetric we and low risk we will have advance purchase contract with payback contract.

On the high risk adjusted profit and high assymetric we will have advance purchase contract. On symmetric information and low risk adjusted profit we will have payback contract or linear capacity reservation. On the high risk adjusted profit and symmetric information we will have whole sales price contract.

On mediam assymetric and medium risk we will have capacity reservation contract.

There are many questions in the dynamic case.

Pricing Seminar of Professor Ozel and Profesor Moharram Oghli @ UTD: tenth session

Meisam Hejazinia

04/04/2013

Sequential decision making taking into account what has currently happen, and what would be future.

React to the deicsion and the effect of it over others.

These are IQ. The third one is social intelligence. What are the behavioral things that I would react to them.

Well established an well known behavior that effect them.

The first one is individual preference. I make the decision individually different from someone else making decision. Risk aversion, loss aversion, regret. People have shown that there are persistent things happening in that dimensions.

Social preference are second one. Not objective function optimization, but non pecunarity issues. Feel bad when treat you. I expect you to act fair, and if not retalliate. Betrayal and reputation. Persist in the way we make decision.

Last one is bounded rationality. I may be more cognitively limited that you are. I don't optimize things, but do simple heuristic rather than finding the complete way.

What makes them important is that every time you find behavior, does not make them behavioral issue. It has to come from first principle, and first axioms. Foundation of why it is happening. Fairness in apes.

Percieve as fair person, since it is important.

Monkey's unequally treated, when grape to one the other one who used to eat cucumbar, would not eat cucumber anymore.

Behavioral field a very good field to harvest. Be good both at behavioral and quantitative person.

Phases of decision making:

Editing phase of information.

References in the mind of people.

Weight function as percieved probability.

Make up mind phase for making choice.

Theories of fairness:

1. reciprocity: tit for tat

2. inequality aversion: equitable share of wealth

3. Altruism: give up own welfare to increase welfare of others.

Fariness reciprocity models:

$u_i = x_i + f_j^p[1 + f_i]$

f_j : player i's kindness to player j

f_j^p : player i's percieved kindness

Multiplicative form works.

Inequality model: utility maximization, but contributing these to the utility function of individual.

$U_i(x) = U(x - i, \frac{x_i}{\sum_{k \neq i} x_k})$

Fahr and schmidt model.

Evolutionary game theory.

Controlled labratory experiment, rather than not controlling things in the labratory.

Dictator bargaining game, to check the altruism.

Sequential offer, if you do not accept, the amount offered to the second. The amount significant, because of no option to punish.

4 person public good game.

Effort level, free riding.

Expectation frame the behavior.

Affect of how we observe things.

simultaneous game

1. max min rule

2. maximizing expected payoff rule

3. certainty rule

sequential decisions

1. search rules

2. for a set of possible outcomes such that the pay-off is satisfactory for all s in S'.

Quantal response equilibrium.

Logistic quantal response, using choice model.

Multiple actions, and one of them optimal, but solution around it. Some people giving probability assignmen to choices. The likelihood you choose one outcome out of other outcomes. The more error prone you are giving more weight to them rather than yours, and you randomize. Using this you will find the equilibrium.

Pricing Seminar of Professor Ozel and Profesor Moharram Oghli @ UTD: Eleventh session

Meisam Hejazinia

04/11/2013

Tuesday afternoon next week we will have the class.

Try to cluster and see what are the key things that affect people's decision.

Certain things that seem to be persistent. Individual preferences could be different from others preferences. Loss aversion, fairness, endowment effect, status quo.

Trusthworthiness, and finally bounded rationality is the last affecting factor. People do not usually do bayesian updating. This is about bounded rationality.

Today we will give example about pricing. Example would be from consumer ricing and business to business. This is firm to consumer. Mjority of consumer research is in domain of consumer pricing. Even in the B2B decision making, when you set price, you always assume rationality. These are not irrational, but different kind of rationality. Fairness, trust, reputation and framing are different kind of things that B2B firms take into consideration.

People do not usually behave according to the assumption of rational choice theory. Aggregate demadn that assumed rationality. In heterogeneity the aggregate demand will change.

Framing:

Economist subscription:

1. print subscription \$59.00
2. print web subscription \$125.00

When third choice is introduced 3. web subscription \$125. People flow from the selection of first option to the the second option.

Going out with someone who is relatively less attractive to find a date.

\$175 cross on amazon anchors you as a list price, and they put another price as current price. Your perception is that they are giving you a discount.

MSRP: manufacturer suggested price.

Choose the anchor price, and negotiate on.

If you go first you try to anchor him to the price that you want him to get to.

Regret and avilability. It says only two is left. If you do not get this product, there is the likelihood that you will not get it, since it is sold out. They try to make you more myopic customer rather than strategic forward looking customer.

Once I purchased it and am out of the market, I may not regret when the price goes down, since probably I would not search for its cost anymore.

Regret the sold out is salient.

When you can observe the outcome of the alternative choice the regret would exist.

How do you know how many customers come to buy? There is the concept of misperception of probability. Two reason:
1. Information is not fully given to you.

2. Even if I give you what exactly these probabilities are, you will underestimate high probability, and overestimate the lower probability. The main problem is on high probability that percepved probability is usually underestimation. Even this shape curve is challanged right now.

If valuation greater than the price the consumer will purchase.

Majority of consumers are forward looking, strategic consumers, that I will wait and purchase in future. $u(v1 - p_1) > q.u(v_2 - p_2)$ where q is the probability that I purchase the product.

Everyday low price, and no mark down, or rationing the quantity to counter balance the forward looking behavior.

JC penny came with everyday low price, rather than mark down.

On the other hand consumers are irrational. They do not percieve probabilities as they really are. $u(v_1-p_1) > q.u(v_2-p_2)-\beta(1-q)(max\{u(v-p_1), p\}$

Rationing is not good idea, and mark down turns out to be good strategy. Regret of availability is more beneficial, and makes mark down a good strategy.

Limited offer has its root in here.

For something everyday lower price works, but for some it does not.

Empirical research on this could be conducted.

Business to business

Moving to business to business context.

Local information and local control, and come with contracting mechanism that try to coordinate their action so that they behave like centralized, single firm.

The controls and profit margins are different here. Firm could have different information, and downstream has more information than upstream. This is conflicting incentive to collect, process and share information. Exponential growth in this. All this assumes firm only maximize pecuniary incentives. How do these results change? Do we start to observe the same thing?

There is a fairness effect that firms could make the decision. Wholesale price contract at quantity 'Q'. Demand is deterministic. Given the price we will have the demand that is realized.

If the supplier offers high wholesale price, retailer responses with higher 'p'. In this way he is incuring the price, but he is penalizing the supplier. People dislike outcome that are percieved as unfair. This is why whole sale contracts don't work to act like single firm, and be efficient. Theoretically, even whole sale contract can coordinate the entire channel.

Frame contract as fixed plus variable fee, or just as variable fee. There is no uncertainty here. Theoretically those two should result in the same outcome. When actual people are involved they tend to pic quantity discount $F/Q + W$. People prefere quantity discount rather than $F + w.Q$, since fixed fee feels like unfair, and penalty. Neither these contracts coordinate channel, and they are no longer equivalent. Why?

Loss aversion? Then you need to check whether loss aversion addresses this problem.

You were designing this contract, and we were helping them. We realized that with some of them

phenomenon	reference point
markup vs. markdown misperception	original price
fairness	other's price

there is no problem of forecast exageration. Why some companies like best buy exagerate? They use the same contract. Maybe trust and trustworthiness is the key. They end up playing golf with eachother and affect on the information that is being shared. Reputation may be effecting. If you have penalties or not. For reputation to have impact on decision.

Reputation will have effect on decision? Repeated interact, what has to be there for reputation? Only in case there would be reward and punishment. Reputation is when you talk about trigger that I am not going to buy from you anymore. If there is that kind of mechanism, in dynamic game you can occasionaly find that it affects your decision of credibly sharing. Majority of companies do not end up using penalty, yet you still observe good or bad behavior. Trust and trustworthiness affects the way you are going to take action.

We can quantify trustworthiness. Bayesian updating and simple updates. It turns out that trust and trust worthiness affects, and you can quantify them. We can change fundamental axioms, and then outcome of that new model explains what you have observed. Chinese less trustworthy, and trusting in information sharing? Culture affect?

You can start to capture some of thos analytically as well. Lot of things could be done here.

Everyday low pricing can dominate dynamic pricing.

Bargain seeking behavior.

Long run when converging happens.

Type of regret (Two different parameters):

1. Stock out regret
2. High price regret

People inclined to underweight things.

Hetrogeneous reservation price

Replenishment period longer

Decision:

Stage 1: Mark down or Everyday low pricing
Stage 2: Choose price and inventory level

Consumer decision:

1. Whether they want to buy?
2. When to buy?

Availability function

Understanding about the future.

Regret for stock out is more salient than regret for paying high price.

Inaction regret is greater long term impact than action regret.

$q(r) = r^\theta$, where r is actual value.

$\bar{v}$ is maximum valuation

$\pi = (p - c)N(\bar{v} - p)/\bar{v}$

Three affecting factors:

1. Misperception
2. availability
3. high price risk

The existance of strong stock out regret and availability misperception encourages the seller to price discriminate high and low value consumers inter-termporally, as well as reduces the extent of inventory rationing.

	rationality	
sunk cost	sunk cost do not matter	
irrelative alternatives does not matter	if A is chosen of the set $\{A, B\}$, B cannot be chosen from the set $\{A, B, C\}$ singnificantly influence consumers choice	introducing ne
probability	customer esteem expected values	
Caring about other utilities		

If people have problem in calculating behavioral prameters then pricing could be good modifier.

Mark down pricing dominates EDLP in the face of regularities.

Two different way of framing fixed fee:

1. two part tariff
2. Quantity discount

Whether the impact of fixed fee is sensitive to framing.

Two part tariff restores channel efficiency, and quantity discount channel also restores channel inefficiency the same way TPT does.

TPT and QD not better than Linear pricing. On both cases retailer will reject the offer.

TPT and QD are not also equivalent.

Behavioral regularities.

Reference dependent utility function.

Quantile response, decision makers select better options, but not always best option, so prone to errors.

As retailer becomes more loss aversion increases the channel efficiency decreases, and as nash rtionality increases the channel efficiency increases.

Contract complexity same as nash rationality.

Assuming same β using β which is result of pooling rather than fixed effect.

Their model worked better than perfect rationality model.

Presentation:

1. Key message. Paper writing and presentation different things. In writing you have to be very rigrous, but in presentation highlight key things, rather than going through all the assumptions and equations. Once messsage is out the goal is to make the person curious so that they go and read the paper, so that in future they learn about it.

Remember that person for something, or go on the paper and build on it.

Not the next tueseday but the next one.

Pricing Seminar of Professor Ozel and Profesor Moharram Oghli @ UTD: Twlefth session

Meisam Hejazinia

04/23/2013

1 Analytical and Behavioral Approaches: The Case of Forecast Information Sharing

Deterministic vs. Stochastic

Approach: Analytica vs. Behavioral

Foundation at least in one, and solve problem.

Not constraint by toolset, and not narrow minded look at the problem.

Analytical approah vs. Simulation as approaching a problem.

Things will evolve over time. Takes time to look at multiple angle. Research, reviewing and so on will be required.

Even full professor and think you are done and don't study anymore your skill begin to decade.

A complementary approach:

(i) Define problem. Why? Research question. Being curious about something. Important business relevant and pervasive. Is it relevant. Different industries interested in same question.

(ii) Analytical Approaches: Build mathematical descriptive models, stakeholders. Pecuniary objectives, payoffs, key tradeoffs. Decisions/actions, outcomes.
How to build analytica: (1) Identify stake holder. (2) Identify trade offs. (3) Identify most critical trade offs. Also you look at payoffs, objectives, and decision variables. There is a loop in this process.
Behavioral approach: (1)Bring the human perspective. Does it play a role. (2) What are non-pecuniary things that could affect. (3) When, how, and why?. Whether those affect decision. (4) Field study. Try the idea. How to change gift or donation policy. Buy back contracts. Are the effective or efficient. (5) Controlled lab experiment. Control is an important keyword here. Does not mean replicating exact business environment, but set up things in the way that you control everything. Change one thing at the time. Two by two design to understand impact of two variables. H: High, L: Low. (H,H), (H,L), (L,H), (L,L). (6) Theoretical underpinnings. Two ways: (a) Exploratory: too dark, and I start shooting. Shooting starts. Now maybe its going to give me some understanding of what may be happening. Today everything is not dark. You need to be exploring outer space. Today it does not exist. People are exploring and look at certaing aspect of problem, and know what is happening. Smack data to find some result out of. You don't want to be like that. (b) Theoretical underpining: You should have theoritical reason to know what is happening. Certain things is known that are well established to be certain theory. Individual (*)perference(prospect theory, risk aversion), (**)social perference (trust, fairness), (***)bounded rationality: complexity. You need to make sure that your result is robust

to another experiment. Certain things in analytical model that does not capture and you use them to update your model. Don't shoot in the dark.

If the loop is for empirical and you start from analytical, and then behavioral approach, it will be called structural model estimation.

(iii) Goal: To understand, to build predictive model. To prescribe effective policies/strategies optimize (min/max) pecuniary payoffs. Decision support tools.

Don't be just analytica, empirical or behavioral, but try to do all of them.

Use research question to learn.

Co-author is like playing tennis, if they would be good, you will be good, if not, you would not be good either.

Sequence of reading papers.

The more fundamental courses you take, the easier for you to learn in future.

This is marathon, and not 100 meter run. You will run it for your whole time. This is life long. Don't be afraid to invest your time on learning.

During time things come back to you and you deeply understand it on the next time. You will suddenly understand that something is missing.

Discrete simulation versus Monte Carlo simulation.

In empirical approach you need to have theoretical reason to include variables that may affect the outcome.

PhD candidate appeal: creativity, research papers, recommendation that shows person can stand on his feet.

Person should also know area, and be able to say what happens if model elements modified, added, or removed.

Copy style of good papers, and then you will start to develop your sense of writing.

Any paper, even most technical, good presenter is able to present what is going on in there. Those who present and can not clarify it, shows the person does not know what they studied.

Trust and trustworthiness research just had its root in curiosity. They took the risk. Nothing has been done before.

If you are excited then who cares what would be end product. Product will become byproduct of your main product which is learning.

If you put your effort when your heart is not, when you don't get the job you will get misrable, but if you put your effort in where your heart is, at the end you will at least learn something, and would be happy at the end.

Funding as incentive for Health Care.

Research improvement is in the form of spike.

Credible forecast information sharing.

Assymetry of information.

Firms incentive to exagerate forecasts.

Principle agent model

Dynamic game of incomplete information

Heterogeneity in the distribution of sharing information.

You can close loop of analytical and behavioral model in one paper, or do it in multiple papers. You can do multiple iterations on each of the steps for different papers.

2 Experimental Study

Step 1: Set up your research question derieved from previous models. Theoretical guidance, set up. (Do not shoot in the dark and use current theories)

Step 2: Develop testable hypothesis based on well-established theories

Setep 3: Examine if, when, how and why actual behavior deviates?

Carefully controlled lab experiments

Step 4: Go to setp 2:

Validation is needed: Whether or not what I observe in the lab is replicatable. Control for culture for example.

Or test new hypothesis

Step5: Use the observations to build better predictive model.

Closing the model-exp-model loop

How to incorporate exp observation into the model?

(1) Start from original math model and add in behaviors observed in
Labratory experiment
studies from, soc., psyc., and decision theories.

(2) Use a minimalistic approach
Consider existing explanations and models
Add one piece at a time

(3) Check whether the resulting optimal decisions predicted by the new model

Have similar properties as those observed in the lab
Fits the data and makes accurate prediction?
Results in similar comparative statistics
Test for goodness of fit (such as, low AIC info criteria)

Does it match with the properties I observe in the data.
You want to make sure that properties of the model is similar to those visible in data.

Fit first half of data. Fit the model, and then use second half to check how your prediction compared to them.

If all are on 45 degree line shows that you have cheated.

Read the paper, and use the result to give managerial implication in MBA class.