Dynamic Structural modeling Estimation | Dube SU 2011 | 1

1. NFP: nested fix point
2. loosen outer loop stopping criterion
3. GMM objective func: mathematical program with equilibrium Constraints (MPEC) algorithm
4. Jacobian, and Hessian of the Lagrangian are sparse (advantage of MPEC diminishes for large-dim. problems with very few mkts & large # prod)
5. Bellman equation
6. $S_t = (S_{j,t})_{j=1}^{J}$: system of mkt share

defined as mapping b/w the vector of demand shocks and the observed mkt shares: $S_t = s(x_t, p_t, \xi_t | \theta)$

$S_t = s(\xi_t | \theta) \Rightarrow \xi_t = s^{-1}(S_t | \theta) \Rightarrow$ Contraction mapping

7. improving GMM estimator by:

(1) using more functions of Z_{jt}, more instrument, optimal weighting matrix in a second step

(2) using an efficient one-step method such as Continuously updated GMM or empirical likelihood

8. M of potential Consumers at date $t=1$

Cons. drop out of mkt once they make a purchase

prices evolve over time as function of the lagged prices of both firms:

$$p_{j,t} = \rho_{j,0} + \rho_{j,1} p_{j,t-1} + \rho_{j,2} p_{-j,t-1} + \psi_{j,t} = p'_{t-1}\rho_j + \psi_{j,t} \quad j=1,2 \quad (10)$$

$\psi_{j,t}$ → random supply shock

jointly dist. with demand shock: $(\xi_{j,t}, \psi_{j,t}) \sim N(0, \Omega)$

indep. across time periods, firms, and markets

Cons. tastes indexed by $r=1,\dots,R$, prod $j=1,\dots,J$

$$u_{j,t}(p_t, |\theta) = \beta_j^r - \alpha^r p_{j,t} + \xi_{j,t} + \varepsilon^r_{j,t}$$

Cons. forward looking, rational Expectations.

Cons. Expected val. delaying adoption at time t:

$$v_0^r(p_t, \theta^r) = \delta \int \Big(\log\big(\exp(v_0^r(p'_t\rho_j + \psi | \theta^r)) + \textstyle\sum_j \exp(\beta_j^r - \alpha^r(p'_t\rho_j + \psi) + \xi_j)\big)\Big) dF_{\psi,\xi}(\psi, \xi) \quad (11)$$

$\delta \in (0,1)$ discount factor

discr. dist with R mass points to charact. Cons. pop. type at date $t=1$:

$$\theta = \begin{cases} \theta^1 & Pr(1) = \lambda_1 \\ \vdots & \vdots \\ \theta^R & Pr(R) = 1 - \sum_{r=1}^{R-1} \lambda_r \end{cases}$$

$\theta^r = (\alpha^r, \beta^r)$

heterogeneity: Certain type of Cons. to systematically purchase earlier than others

remaining mass of Cons. of type r, who have not yet adopted at the beggining of period t: M_t^r

$$M_t^r = \begin{cases} M\lambda_r & t=1 \\ M^r_{t-1} S_0^r(x_{t-1}|\theta^r) & t>1 \end{cases}$$

Fractions of Cons. of type r who purchase the outside good

9. in a given period t, mkt share of product j: (12)

$$S_j(p_t, \theta) = \sum_{r=1}^{R} \lambda_{t,r} \frac{\exp(\beta_j^r - \alpha^r p_{j,t} + \xi_{j,t})}{\exp(v_0^r(p_t|\theta^r)) + \sum_{k=1}^{J} \exp(\beta_k^r - \alpha^r p_{k,t} + \xi_{k,t})}$$

$\lambda_{t,r}$ = remaining prob mass assoc. type r Cons. at date t

$$\lambda_{t,r} = \begin{cases} \lambda_r & t=1 \\ \dfrac{M_t^r}{\sum_r M_t^r} & t>1 \end{cases}$$

empirical model: (10), (12);

$$u_t = \begin{bmatrix} \psi_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} p_t - p'_{t-1}\rho \\ s^{-1}(p_t, S_t | \theta) \end{bmatrix}$$

$$f_Y(Y_t | \theta, \rho, \Omega) = \frac{1}{(2\pi)^J |\Omega|^{1/2}} \exp(-\tfrac{1}{2} u'_t \Omega^{-1} u_t) \underbrace{|J_{t,u \to Y}|}_{2J \times 2J}$$

Jacobian matrix

max likelihood est. of params:

$$\max_{\{\theta, \rho, \Omega\}} \prod_{t=1}^{T} f_Y(Y_t | \theta, \rho, \Omega)$$

two loops: [max likelihood [Compute the fix point of Bellman equations

$$J_{t,u \to Y} = \begin{bmatrix} \dfrac{\partial \psi_t}{\partial p_t} & \dfrac{\partial \psi_t}{\partial S_t} \\ \dfrac{\partial \xi_t}{\partial p_t} & \dfrac{\partial \xi_t}{\partial S_t} \end{bmatrix} \qquad \frac{\partial \psi_t}{\partial \log(p_t)} = I_J \qquad \frac{\partial \psi_t}{\partial \log(p_t)} = 0_J$$

$$G(S_t, \xi_t) = s(p, \xi_t | \theta) - S_t = 0$$

$$J_{t, \xi_t \to S} = -\left[\frac{\partial G}{\partial \xi_t}\right]^{-1} \left[\frac{\partial G}{\partial S_t}\right] = \left[\frac{\partial s}{\partial \xi_t}\right]^{-1}$$

elements of $\dfrac{\partial S_{j,t}}{\partial \xi_{k,t}} = \begin{cases} \sum_r \lambda_{r,t} S_j(p_t, \xi_t | \theta^r)(1 - S_j(p_t, \xi_t | \theta^r)) & \text{if } j=k \\ -\sum_r \lambda_{r,t} S_j(p, \xi_t | \theta^r) S_k(\xi_t, p | \theta^r) & \text{otherwise} \end{cases}$

chebyshev approx of Expected val. of waiting:

Bellman equation:

$$\gamma^{r\prime} \Lambda(p) = \delta \int \log\big(\exp(\gamma^{r\prime} \Lambda(p\rho + \psi)) + \textstyle\sum_j \exp(\beta_j^r - \alpha^r(p'\rho_j + \psi) + \xi_j)\big) dF_{\psi,\xi}(\psi, \xi)$$

residual func:

$$R(p|\gamma) = \gamma^{r\prime}\Lambda(p) - \delta \int \log\big(\exp(\gamma^{r\prime}\Lambda(p\rho + \psi)) + \textstyle\sum_j \exp(\beta_j^r - \alpha^r(p'\rho_j + \psi) + \xi_j)\big) dF_{\psi,\xi}(\psi, \xi)$$

X: matrix of K chebyshev polynomial at each G grid points.

search param γ: $X'R(p|\gamma) = 0$

Iterated least square approach for NFP:

1. Pick starting value: $\gamma^{r,0}$, $v_0^{r,0}(p|\theta^r) = \gamma^{r,0\prime}\Lambda(p)$
2. use quadrature to compute:
$$Y(p|\gamma^{r,0}) = \delta \int \log\big(\exp(\gamma^{r,0\prime}\Lambda(p\rho + \psi)) + \textstyle\sum_j \exp(\beta_j^r - \alpha^r(p'\rho_j + \psi) + \xi_j)\big) dF_{\psi,\xi}(\psi, \xi)$$
3. Solve the least square problem: $\min R(p|\gamma^r)' R(p|\gamma^r)$

or: $\min_{\gamma^r} (X\gamma^r - Y(p|\gamma^{r,0}))'(X\gamma^r - Y(p|\gamma^{r,0}))$

Solution is: $\gamma^{r,1} = (X'X)^{-1} X' Y(p|\gamma^{r,0})$

4. Compute $v_0^{r,1}(p|\theta^r) = \gamma^{r,1\prime}\Lambda(p)$
5. Repeat steps 2 and 3 until Convergence

Behavioral Pricing project | Estimation ①

105 product codes (sku)

$D_{jt} = (i_{j1t}, i_{j2t}, \dots, i_{jSt})$ inventory level $i_{jst} \in \{0,1\}$

$\lambda_{jt} = \sum_{s=1}^{S} \frac{I_{jst}}{S}$ retail dist

$\lambda_{jt} = a_1 \ln(Inv_{jt}) + a_2 Inv_{jt} + a_3 Inv_{jt} + \varepsilon_{jt}$

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discount factor monthly: 0,99 , weekly: $\boxed{0.9975 = \gamma}$

model params: fixed effect $\alpha = (\alpha_1, \dots, \alpha_J)$

per period consumption: c

segment specific param. $\beta = (\beta_1, \dots, \beta_K)$

$\beta_K = (\beta_{Kp}, \beta_{Km}, \beta_{Ks})$

demand shock std: $\pi = (\pi_2, \dots, \pi_K)$

set of all params: $\Omega = (\alpha, c, \beta, \pi, \sigma_\xi)$

$X_{jt} = (P_{jt}, d_{jt}, S_t)$

$\delta_{jt} = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_1 + \xi_{jt}$

$\bar{\beta}_K = (\beta_K - \beta_1)$ relative to seg 1 $\quad k = 2-K$

market share: $MS_{jt} = \theta_{1jt} \dfrac{\exp(\delta_{jt})}{\exp(W_{1jot}) + \exp(\delta_{jt})} +$

$\sum_{k=2}^{K} \theta_{Kjt} \cdot \dfrac{\exp(X_{jt}\bar{\beta}_K + \delta_{jt})}{\exp(W_{Kjot}) + \exp(X_{jt}\bar{\beta}_K + \delta_{jt})}$

W_{Kjot} : observable value of waiting $\rightarrow$ $t = T_j - 1, T_j - 2, \dots, 1$

$W_{Kjot} = \gamma \int \lambda_{j,t+1} \ln\{\exp[\delta_{j,t+1} + X_{j,t+1}\bar{\beta}_K] + \exp[W_{Kj0,t+1}(Z_{t+1})]\} dF(Z_{t+1} | Z_t)$

expected markdown path: $\Delta_{jt} = (\Upsilon_{t+1}, \Upsilon_{t+2}, \dots, \Upsilon_T)$ $(1, 0, \dots)$ permat(1,0)

Simulate availability path | current state

$\forall$ time period t, product j : draw L rand vect. of $(T_j - t)$ error terms

each corresponds to avail. vector

$\Psi_2 = (e^2_{j,t}, e^2_{j,t+1}, \dots, e^2_{j,T_j-1}) \sim iid\ N(0, \sigma^2_\lambda)$

$\Lambda^2_{jt}(Z_t) = (\lambda^2_{j,t+1}, \lambda^2_{j,t+2}, \dots, \lambda^2_{j,T})$

$W_{Kjot} \rightarrow$ integr. over $\begin{cases} N & \text{rand avail. vect.} \\ L & \text{rand mrk down depth vect} \\ R & \text{rand demand shock vect} \\ M & \text{mark down path} \end{cases}$

First step: depth, avail vect, shock

$W^{lnr}_{Kjot}(Z_t) = \gamma \sum_{m=1}^{M} P(\Delta^m_{jt})[\lambda^2_{j,t+1} \ln\{\exp[\delta^{mn}_{j,t+1} + X^{mn}_{j,t+1}\bar{\beta}_K]$

$+ \exp[W^{lmnr}_{Kjo}]\}\} | \Lambda^2_{jt}(Z_t), MD^n_{jt}(Z_t), \xi^r_{jt}, Z_t]$

Last period waiting normalize zero: $W^{lmnr}_{KjoT_j} = 0$

Second step: Calc. wait. for each period & product by AVG val. 1st step $\forall$ mkdwn depth (L), Avail. (N), Rand shock (R)

$W_{Kjot}(S_t) = \frac{1}{L}\frac{1}{N}\frac{1}{R} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{r=1}^{R} W^{lnr}_{Kjot}(Z_t)$

Berry fixed point $\begin{cases} \text{finite horiz. dyn. prog set.} \\ \text{(optimal stopping prob.)} \end{cases}$

②

- search nonlin. param $\bar{\beta}_K$, π_K (seg size)
  - $\delta_{jt}(S_{jt}, \hat{S}_{jt}) | \bar{\beta}_K, \pi_K$ (inversion contraction mapping)
    - ① $\beta_1, \alpha_j, c \,|\, \delta_{jt}$ ② ξ_{jt} using $\delta_{jt} = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_1 + \xi_{jt}$
    - max likelihood
      - $\hat{S}_{jt}$ by solving dynamic prog. $| \delta_{jt}$
      - Calc. waiting for each seg. (last period bkwrd)

GMM Nevo $\begin{cases} \text{① mkt size (non zero outside good)} \\ \text{② sensitivity} \end{cases}$

- proportional factor $\Rightarrow$ order size (1.25)

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value func. spec.

$V_{ijt}(S_t) = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + \beta_{ip} P_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt} + \varepsilon_{ijt}$: purchase

non purchase $V_{ijot}(S_t) = \gamma E[\lambda_{j,t+1} \max[V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})] | S_t] + \varepsilon_{ijot}$

$V_{ijt} = W_{ijt} + \varepsilon_{ijt}$ $\rightarrow$ observable part of value func. by retailer

$V_{ijot} = W_{ijot} + \varepsilon_{ijot}$

$W_{ijt}(S_t) = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + \beta_{ip} P_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt}$

$W_{ijot}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1}) | S_t]$

dist known $\Rightarrow$ $W_{ijot}(Z_t) = \gamma \int \lambda_{j,t+1} \ln\{\exp[W_{ij,t+1}(Z_{t+1})] + \exp[W_{ij0,t+1}(Z_{t+1})]\} dF(Z_{t+1} || Z_t)$

indiv. uncond. purchase prob: $P_{ijt} = \dfrac{\exp(W_{ijt})}{\exp(W_{ijot}) + \exp(W_{ijt})}$ Comm

- latent class (Kamakura & Russel 1989) K segment in pop $\Rightarrow \beta_K$
- M_{j0}: mkt size for product j $\quad N_{Kjt}$: # remaining cons.
- $N_{Kjt} = N_{Kj,t-1}(1 - P_{Kj,t-1})$ or $N_{Kjt} = M_{j0}\pi_K \prod_{l=1}^{t-1}(1 - P_{Kjl})$
- Size of seg. K : θ_{Kjt}

$\theta_{Kjt} = \dfrac{N_{Kjt}}{\sum_{m=1}^{K} N_{mjt}} = \dfrac{\pi_K \prod_{l=1}^{t-1}(1 - P_{Kjl})}{\sum_{m=1}^{K} \pi_m \prod_{l=1}^{t-1}(1 - P_{mjl})}$ ⊛

$MS_{jt} = \sum_{k=1}^{K} \theta_{Kjt} P_{Kjt} = \sum_{k=1}^{K} \theta_{Kjt} \dfrac{\exp(W_{Kjt})}{\exp(W_{Kjot} + W_{Kjt})}$

Regret in pricing project ①

proposed model:

$$u_{ij1} = \alpha_j + c + \gamma c + \beta_{ip} \cdot p_{j1} + \underbrace{\alpha_{ip} \lambda_{i2}(p_{j1} - p_{j2})}_{\text{Disutility of high price regret}}$$

utility of purchase

$$u_{ij2} = \underbrace{\gamma(\lambda_{i2}(c + \beta_{ip} p_{j2})}_{\substack{\text{Expected utility}\\ \text{of purchasing}\\ \text{and using}}} + \underbrace{\beta_{ir}(1-\lambda_{i2})\max\{(\gamma c + \beta_{ip} p_{j1}), 0\}}_{\substack{\text{Expected disutility of}\\ \text{stock out regret}}}$$

Tasks

① Data cleaning (@periods ② # periods (c/period) e.g. 15, 10)

② run Bayesian BLP model (How Dynamic?)

③ Build theory for hypoth, competing hypothesis

④ what others have done, litrature, writing

ideas

① different categories (fashion vs. basic)

How to estimate?

① in outer loop rather than β_k use β_k for each segment an π_k on outher loop (π_k: segment size)

② what is objective function?

$$s_j(p_t, \theta) = \sum_{r=1}^{R} \lambda_{t,r} \frac{\exp(\beta_j^r - \alpha^r p_{j,t} + \xi_{jt})}{\underbrace{\underbrace{\exp(v_0^r(p_t|\theta^r))}_{\text{Not purchase}} + \underbrace{\sum_{k=1}^{J} \exp(\beta_k^r - \alpha^r p_{kt} + \xi_{kt})}_{\text{purchase}}}_{\text{For each taste } r}}$$

$$\underbrace{\frac{M_t^r}{\sum_r M_t^r}}_{} \equiv \theta_{kjt}$$

→ probability of purchase of a segment r

$$M_t^r = \begin{cases} M\lambda_r & t=1 \\ M_{t-1}^r \, s_0^r(x_{t-1}|\theta^r) & t>1 \end{cases}$$

$$\delta_{jt} = \alpha_{jt} + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_i + \xi_{jt}$$

inner loop $\underline{\xi_{jt}}$ by solve dynamic problem $| \delta_{jt}$

Calc: No purchase prob
purchase prob

Dynamic prog problem:

① Control: Purchase or not (u_k)

② $t=2$, plan to how act in horizon: $\pi = (\mu_0, \mu_1, \dots, \mu_{n-1})$

③ $S_t = \{$Price, wait, markdown status, seasonality, demand shocks, error terms$\}$

$$P(S_{t+1}|S_t) = p(Z_{t+1}, \varepsilon_{t+1}|Z_t, \varepsilon_t) = p(Z_{t+1}|Z_t) \cdot p(\varepsilon_{t+1}|\varepsilon_t)$$

all state variables except the unobservable error terms

optimal cost $J_\pi(x_0) = J(x_0)$
⇑
$\min E[\sum_{k=0}^{n-1} g_k(x_k, u_k, w_k) + g_n(x_n)]$

④ w_k: noise or disturbance

⑤ Cost minim: $g_k(x_k, u_k, w_k)$ $k = 0, 1, \dots, n-1$

⑥ State evolution: $x_{k+1} = f_k(x_k, u_k, w_k)$

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②

non lin param β_k, π_k (seg size) (metropolis hasting)

$\delta_{jt}(s_{jt}, \hat{s}_{jt}) | \beta_k, \pi_k$ (Inversion) Cont map

$\beta_i, \alpha_j, c | \delta_{jt}$, $\xi_{jt} = \delta_{jt} - \alpha_i - \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_i$ (Gibbs sampling)

$\hat{s}_{jt} | \delta_{jt}$ (Dynamic prog) each seg.

① no need for Instr.

Role of Dynamic programming:

- Calculate not purchase utility $\underline{w_{kjot}}$ for each period ⇒

Pricing Project

Ozlap & Gonca

Scratch

(1) Gonca

Mkdown + Seasons (dum)

$$\begin{cases} u_{ijt} = \alpha_j + \sum_{\tau=t}^{\tau_j} \gamma^{\tau-t} c + \beta_{ip} p_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt} + \varepsilon_{ijt} \\ V_{ij0t}(S_t) = \gamma E(\underbrace{\lambda_{j,t+1}}_{\text{avail.}} \max\{v_{ij,t+1}(S_{t+1}), V_{ij0}(S_{t+1})\} + (1-\lambda_{j,t+1}) v_0(S_{t+1}) \end{cases}$$

$+ V_{ij0t}(S_t)$: no purchase

pred dist. $\lambda_{jt} = a_1 \ln(Inv_{jt}) + a_2 Inv_{jt} + a_3 Inv_{jt} + \varepsilon_{jt}$

prob mkdown: $p_{jt} = \begin{cases} \underbrace{p_{j,t-1}}_{\text{price}} - \underbrace{MP_{jt}}_{\text{Depth}} & \text{prob } \varphi_{jt} \\ p_{jt-1} & \text{prob } \varphi_{0jt} = (1-\varphi_{jt}) \end{cases}$

$$\Rightarrow \begin{cases} (\varphi_{jt} \mid p_{j,t-1} = p_{j1}) = \dfrac{\exp(a_0 + a_1 p_{j1} + a_2 time_{jt})}{1+\exp(a_0 + a_1 p_{j1} + a_2 time_{jt})} & \text{Given No prev MKDn} \\ (\varphi_{jt} \mid p_{j,t-1} \neq p_{jt}) = \dfrac{\exp(b_0 + b_1 p_{j1} + b_2 time_{jt})}{1+\exp(\ldots)} \end{cases}$$

Depth: $\ln(\underbrace{MP_{jt}}_{\text{Depth}}) = \theta_0 + \theta_1 \ln(p_{j1}) + \theta_2 mdum_{jt} + \varepsilon_{jt} \sim N(0, \sigma^2_{MD})$ Price shock

avail; $\lambda_{j,t+1} = \gamma_0 \lambda_{jt} + \gamma_1 p_{j1} + \gamma_2 time_{jt} + e_{jt} \sim N(\mu_\lambda, \sigma_\lambda^2)$

segmnts: $N_{kjt} = N_{kj,t-1}(1 - P_{kj,t-1})$ # rem. Cons seg k

$N_{kjt} = \underbrace{M_{j0}}_{\text{MKT size}} \underbrace{\pi_k}_{\text{seg size}} \prod_{l=1}^{t-1} (1 - P_{kjl})$ → prop. Cons. not bought before

Size seg avail j →

$$\theta_{kjt} = \frac{N_{kjt}}{\sum_{m=1}^{K} N_{mjt}} = \frac{\pi_k \prod_{l=1}^{t-1}(1-P_{kjl})}{\sum_{m=1}^{K} \pi_m \prod_{l=1}^{t-1}(1-P_{mjl})}$$

$$MS_{jt} = \sum_{k=1}^{K} \theta_{kjt} P_{kjt} = \sum_{k=1}^{K} \theta_{kjt} \frac{\exp(u_{kjt})}{\exp(V_{kj0t} + u_{kjt})}$$

$$\delta_{jt} = \alpha_i + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_1 + \xi_{jt} \quad X_{jt} = (p_{jt}, d_{jt}, S_t)$$

(2) Ozalp: $u_1(v, p_1) = (v - p_1) - \alpha q (p_1 - \delta p_1)$

$u_2(v, p_1) = q(v - \delta p_1) - \beta(1-q) \max\{(v - p_1), 0\}$

Gonca meeting $\boxed{\text{may 28th}}$

① define two segment → latent class
& solve for each of segments

② outside goods → probability of not making any purchase

③ search on regret whether empirical exist?

④ time discount of future utility β

⑤ write utility functions, and write down what each part captures $\boxed{\text{Deliverable}}$

⑥ things to worry about: what if I don't purchase when total market size is Estimated from somewhere else: e.g usage of americans

⑦ discount factor of utility to be taken into account

⑧ data cleaning
assumption about when t_2 since People may purchase after second mark down

⑨ Mom → ask for

Estimation algorithm

* fixed point estimation

Steps

[Outer loop]

S1 ① $\bar{\beta}_k$ param diff. seg one

② π_k relative segment size $k=(2,\dots,K)$

S2 δ_{it} : mean valuation | Given $\bar{\beta}_k$, π_k

↳ observed mkt share ⊖ predicted mkt share — inverting (14)

↦ Contraction mapping

S3 ① $(\beta_1, \alpha_j; c)$ → response of first segment

② ξ_{jt} calculated using $\delta_{jt} = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{T-\tau} c + X_{jt}\beta_1 + \xi_{jt}$

S4 [Fourth loop] Calculate predicted market share |

- Given a value of nonliner params, mean valuation δ_{jt}

(Solve consumers' dynamic optimization problem)

↦ Calc waiting for each segment [Backward]

* discrete heterogeneity (GMM)

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two considerations

① allow for nonzero share of outside goods

② check sensitivity of results to market definition

↳ high level estimation (e.g. total # of office based employees) proportionality factor 1.25

- two segment spec fits data best ⇒ 70 params

① 61 fix effect (α_j)

② per period consumption param (c)

③ β
- Price sensitivity x2 seg
- markdown sensitivity x2 seg
- seasonality x2 seg

④ segment size params
- relative size of first seg (initial) π
- Standard deviation mean zero normal dist. demand shock : σ_ξ

→ Endogeneity to future

approach

Erdem 1996

① what observable?

② what unobservable? ① parameter ② Hidden markov mixture ③ likelihood
④ fixed & random effect

Assymetry

③ Theoretical underpinning → Connect to his theory paper

④ relation → ① signaling (advertising) ② learning

⑤ Find instrument to measure (1) to indirectly estimate (2) or break down to another mixture

⑥ assymetric model

⑦ Forward looking vs. myopic

⑧ Error term of utility

⑨ parameter list: ① q: fill rate probability
② θ: distorted percived q → other probability model?
mean, variance } ③ α: marginal value high price regret
mean, variance } ④ β: stock out regret
mean, variance } ⑤ v: Valuation distribution → (life style, demographic, estimate?)

⑩ observable: ① mark down rate δ
② r
③ p

⑪ what hypothesis? just test theory?

⑫ type of data { ① aggregate → random coefficient
② panel data

⑬ problem: lack data → ① need scrap internet
② multiple src

⑭ solution: Instrument? measurement effect

⑮ logit & log normal?

⑯ multi level mixture model ① signals company
② Consumer Charact.
③ Brand characteristics

⑰ problem: ① not observe not purchase
② do not know first examination of product

⑱ is multibrand better

1

Erdem 2003 | model - summary

- dynamic model : ① learn about ② choose b/w brands

data
① src visited each period
② search duration
③ price expectations measure
④ attitude toward the alternatives during the search process

① Bayesian learning model of consumer choice behavior (uncertainty: active learning model - optimal sequential decision)

→ Result: better estimation of price elasticity (mean reversion of expectations) → incentive to buy now

hightech, high involvement, durable goods

$$u_{ijrt} = \beta_i (1 - e^{-\alpha G_{ijrt}}) + \gamma C_{ijrt} + \varepsilon_{ijrt}$$

Consumer, Brand; discrete $ to spend; indiv spec; $\sim$ iid idiosyncratic taste

$\beta_i = \beta_0 + \beta Q_i$

$G_{ijrt} = \pi_{jt} P_r Q_j$: index of effic capab/$; spending on P.C.; Efficiency unit = Quality of tech / $; cte over time

$C_{ir} = I_i - P_r$: outside good consumption; income

price reduce ≠ grow over time

Substitute: $\boxed{V_{ijrt} = \beta_i (1 - e^{-\alpha \pi_{jt} P_r Q_j}) - \gamma P_r + \varepsilon_{ijrt}}$

Consumer Perception: $E[Q_j | I_{it}] = Q_j + Z_{ijt}$, $Z_{ijt} \sim N(0, \sigma^2_{ijt})$

Consumer Quality Expectation; info set (signal set); prior $Q_j \sim N(Q_{j0}, \sigma^2_{j0})$

$$E(u_{ijt} | I_{it}) = \beta_i \{1 - e^{[-\alpha \pi_{jt} P_r E(G_j | I_{it})] + (\alpha \pi_{jt} P_r)^2 \frac{\sigma^2_{ijt}}{2}}\}$$
$$- \gamma P_r + \varepsilon_{ijt}$$

: log normal dist → Risk aversion degree

dynamic: time = ↓ percieved variance

assump: liner utility in outside good consumption

Forces:
① ↓ price durin time (delay) → π_{ij} ↑
② during time: uncertaint ↓ → utilit ↑ (variance) (root: risk aversion)
③ delay opportunity cost

not purchase utility: $u_{i0}(\alpha)$ ~~not~~ depend on { Comput, demographi }

Src of learning: ① article ② wom ③ TV ...

Percieved Quality Signal (Noisy)

$S_{jkt} = Q_j + x_{ijkt}$, $x_{ijkt} \sim N(0, \sigma^2_{kt})$

$\hookrightarrow \sim N(Q_j, \sigma^2_k)$

Source: k

threshold (ordinal)
$q_{ijt} = L$ if $E[Q_j | I_{it}] < \mu_{iL}$
$q_{ijt} = M$ if $E[Q_j | I_{it}] < \mu_{iM}$
$q_{ijt} = H$ if $E[G_j | I_{it}] = \mu_{iH}$

$$Pr(D_{ijrt} = 1 | \theta, Z_{it}, \tau) = \frac{e^{E(U_{ijrt} | I_i)}}{e^{\bar{V}_{it}^N} + \sum_{k=1}^{2} \sum_{q=1}^{R} e^{E[\bar{V}_{ipqt} | I_{it}]}}$$

Perception Error Vector; Consumer type latent

j = IBM, apple   r = 1, R

Gozer 20 | Zheng

- optimal pricing
- inventory strategy
  - ① mark down (II)
  - ② every day low pricing (I)

- affect of non pecunary on consumer purchase decision

MKT: 2 stage selling season

Competing force: (FW looking not relevent)
① anticipated regret
② misperception of product availability

{ (i) high price (ii) markdown; (ii) wait (ii) stock out }

percieve likelihood of product availability

- measure: seller profit: 6% loss
- Control: inventory rationing
- Calibrate based on behavioral parameter

↓ disclose low inventory level ← ★ adv content

monopolistic seller: (I) optimize p^b on two period
(II) P_1: first period, $P_2 = \delta P_1$ 2nd period, $\delta \in (0,1)$

inventory level: K
unit cost: c, beggining period
long replenishment lead: stock out ⚠

Seller only knows this

Consumer reservation price: $v \sim Unif[0, \bar{v}]$ → { $f(\cdot)$ pdf, $F(\cdot)$ cdf }

market size: N

Consumer decision: which period to purchase?

q: percieved probability period 2 product avail

Expected utility purchase in each period:

$U_1(v, P_1) = (v - P_1) - \alpha q (P_1 - \delta P_1)$ (marginal value high price regret, $\in [0,1]$)

$U_2(v, P_1) = q(v - \delta P_1) - \beta (1 - q) \max\{(v - P_1), 0\}$

Consumption utility; utility loss: high price regret, stock out regret

- Linear benouli utility function; marginal value of stock out regret

Consumer underweight period 2

Fill rate: $\boxed{q(r) = r^\theta \quad \theta \geq 1}$ → strength of deviation

$\theta = 1$ no distortion

- buy if: $U_1(v, P_1) \geq \max\{U_2(v, P_1), 0\}$ (period 1)

→ Threshold policy & Cutoff value v_1: (indifference deviation)

$$Q = \frac{(1+\beta)(\bar{v} - P_1)}{(1+\beta)\bar{v} + (\alpha - \beta - (1+\alpha)\delta) P_1} < 1 \qquad v_1 = \frac{(1 + \beta + (\alpha - \beta - (1+\delta) q)}{(1-q)(1+\beta)} \geq$$

Period 1 if: $v \in [v_1, \bar{v}]$ period 2 if: $v \in [\delta P_1, v_1)$: $q \in [0, Q$
not buy if: $v \in [0, \delta P_1)$

Period 2 if $v \in [\delta P_1, \bar{v}]$
not buy if $v \in [0, \delta P_1)$: $q \in (Q, 1]$

δ: mark down ratio

everyday low price optimal price: $P^b = \frac{\bar{v} + c}{2}$ | inventory level $K^b = \frac{N(\bar{v} - c)}{2\bar{v}}$ | optimal profit $\pi^b = \frac{N(\bar{v} - c)}{4\bar{v}}$

Corollary 1: when the optimal markdown solution achieved (boundary), markdown strategy dominated by everyday-low-price.

Prop 4 (i) if $\alpha \geq \beta$ & $\theta = 1$ everyday low price always optimal
(ii) if $\alpha < \beta$ & $\theta \geq 1 \Rightarrow \exists$ a $\bar{v}^*$ such that ⓐ $\forall \bar{v} > \bar{v}^*$ markdown strategy is optimal, ⓑ $\forall \bar{v} \leq \bar{v}^*$ everyday low price optimal

Prop 5 under markdown P_1^s, K^s, π^s ↗ β, θ ↘ α (optimal, inventory, profit)

Gonca 2012, krishna murthi
Soysal

①

- expectation of price & expectation of availability
- early markdown more favorable (sales, revenue)
  ① timing ② depth of markdown
  trade off
- decreasing product availability reduces consumer's incentive to wait

- intertemporal price discrimination
- key: supply side of limited availability
- strategic consumer behavior
- heterogeneity in willingness to pay, change in consumer utility over time
- create rationing risk
- firm would have increased its revenue if it had smaller markdowns earlier in the season
- multiple stages
- independent discrete choices (only purchase once)
- { future price expectation / stockout expectation (Risk)
- durable product: MKT shrinks when exits
- heterogeneity allows different adjustment (segments)
- MKT composition change: proportion of price sensitive increases
- sales ↓ markdown after

$$U_{ijt} = \alpha_j + \sum_{\tau=t}^{T} \gamma_c + \beta_{ip} P_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt} + \epsilon_{ijt}$$

Control taste; intrinsic preference; length of season; Consum ↓ in time; Product; Relative to outside options; Price sensitivity; discount factor; Postpurchase utility; ≡ later purchase = lower benefit in season (finite < ∞); Price; markdown dummy; markdown sensitivity; hetrogeneity in willingness to pay; seasonal dummy; Product time specific demand shock; unobservable characteristics (e.g. TV show) (P2); mean zero stochastic term

Limitation: α_j, c_j not identifiable separately
(not double duty ϵ_j only)
⊛ Common across segments
↳ { γ_i, c, γ }
based on calculation
c_i: (only hedonic) not utilitarian
c: utilitarian (1) warmth (2) durability

key of model advantage: less bias of estimation

d_{it} → shock of whether marked down or not affect utility (noticeability is also captured)
(0, shock)

P_{it} → depth of markdown [actual mark down]

S_t → lower or higher utility from product during peak or low seasonal period (e.g. holiday shopping, thanks giving, cermonies, holiday's) → 1: holiday, 0: other period

$U_{ij0t} = \epsilon_{ij0t}$ not buying

Availability
① probability of finding in the store
② total sales and opening inventory level for each (active) SKU reported for 109 weeks (supplementary $\frac{57}{105}$)
dist $\frac{\text{\# avail stock outlets}}{\text{Total stores}}$

dist. level: $D_{jt} = (i_{j1t}, i_{j2t}, \ldots, i_{jSt})$ S: store
↳ inventory level
$I_{jst} \in [0,1]$ indicator of existance of item j
$\lambda_{jt} = \sum_{s=1}^{S} \frac{I_{jst}}{S}$
for 48: observed: $Inv_{jt} = \sum_{s=1}^{S} i_{jst}$

②

Predict retail dist. level: $\lambda_{jt} = a_1 \ln(Inv_{jt}) + a_2 Inv_{jt} + a_3 Inv_{jt}$ + ϵ_{jt}
↓ observed in 57
$R^2 = .98$
⇒ param estimate a_1, a_2, a_3 ⇒ Predict λ_{jt}

Expectation: rational about future of state variable (consumers)
- observe for period and predict for future

price, p, ↕ depth

prob (price markdown) = f (retail price, time in season, ? mkdown)

↑ price ⇒ ↑ deeper mkdown & ↓t earlier mkdown

p, t_1, t_2, d_1, d_2, t
$t_1 > t_2 >$
$d_1 > d_2 > \ldots$

multivariate jump

φ_{jt} prob. of markdown ⇒ stay constant in period (t_j, t_0) with prob: $\varphi_{0jt} = (1 - \varphi_{jt})$

$$P_{jt} = \begin{cases} P_{jt-1} - MD_{jt} & \text{prob } \varphi_{jt} \\ P_{jt-1} & \text{prob } \varphi_{0jt} \end{cases}$$
P_{j1}: initial price

$time_{jt}$: # weeks since intro of prod.

Decision process

no previous mkdown (seperate process): $t = 2, \ldots, T$
$$(\varphi_{jt} \mid P_{j,t-1} = P_{j1}) = \frac{\exp(a_0 + a_1 P_{j1} + a_2 time_{jt})}{1 + \exp(a_0 + a_1 P_{j1} + a_2 time_{jt})} \quad ②$$

③
mkdown prob | previous mkdown: $t = 3, \ldots, T$
$$(\varphi_{jt} \mid P_{j,t-k} P_{jt}) = \frac{\exp(b_0 + b_1 P_{j1} + b_2 time_{jt})}{1 + \exp(b_0 + b_1 P_{j1} + b_2 time_{jt})}$$

mkdown depth
↑ price shock ~ iid $N(0, \sigma^2_{MD})$
$$\ln(MD_{jt}) = \theta_0 + \theta_1 \ln(P_{j1}) + \theta_2 mdum_{jt} + \epsilon_{jt} \quad ④$$
↳ markdowndummy { 1 previous mk down, 0 otherwise }

- predictive power of multiple hypoth

⑤
$$\lambda_{jt+1} = \gamma_0 \lambda_{jt} + \gamma_1 P_{j1} + \gamma_2 time_{jt} + e_{it}$$
(Consumer)
$e_{it} \sim N(\mu_\lambda, \sigma_\lambda^2)$

optimal purchase: finite horizon dynamic programming
Z_t: all state var except unobs. error term ($\epsilon_{ijt}, \epsilon_{ij0t}$) retailer transition prob.

state var { ① time ② price ③ availability ④ markdown status ⑤ demand shock ⑥ Error terms }

$$P(S_{t+1} \mid S_t) = P(Z_{t+1}, \epsilon_{t-1} \mid Z_t, \epsilon_t) = P(Z_{t+1} \mid Z_t) P(\epsilon_{t+1} \mid \epsilon_t)$$

$\epsilon_{ijt} \sim$ iid extreme value $\xi_{jt} \sim N(0, \sigma_\xi^2)$

Consumers observe: (1) time (2) seasonality (3) current period demand shock (4) error terms

Retailer observes: (1) time (2) price (3) availability (4) markdown status (5) seasonality (6) prod. time spec. demand shock (ξ_{jt}) but not ϵ_{ijt}

$V_{ijt}(S_t) = U_{ijt}(S_t)$
↳ value of consumption (strategic consum.)

$V_{ijt}(S_t)$, $U_{ij0}(S_t)$

$$V_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0}(S_{t+1})\} + (1 - \lambda_{j,t+1})^{*}$$
$$O[S_t] + U_{ij0t}[S_t])$$
↳ no purchase utility

Gonca 2012 Soysal krishnamurthi (3)

normalize $U_{ij0t}(S_t)$ to ε_{ij0t}

$$\Rightarrow V_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\} | S_t] + \varepsilon_{ij0t}$$

$$\begin{cases} V_{ijt} \geq V_{ij0t} \text{ and } V_{ij\tau} < V_{ij0\tau} \quad \forall \tau < t \Rightarrow \boxed{\text{purchase period } t} \\ V_{ijt} < V_{ij0t} \text{ and } V_{ij\tau} < V_{ij0\tau} \quad \forall \tau < t \end{cases}$$

Estimation

$$V_{ijt}(S_t) = \alpha_i + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + \beta_{ip} P_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt} + \varepsilon_{ijt}$$

$$V_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\} | S_t] + \varepsilon_{ij0t}$$

- Expectation w.r.t dist. future var. unknown | Conditional current info
- $\varepsilon_{ijt}, \varepsilon_{ij0t}$ evolve indep from other state var's

$$\begin{cases} V_{ijt} = W_{ijt} + \varepsilon_{ijt} \\ V_{ij0t} = W_{ij0t} + \varepsilon_{ij0t} \end{cases}$$

W_{ijt} → observed by retailer

$$W_{ijt}(S_t) = \alpha_i + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + \beta_{ip} P_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt}$$

$$W_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0t+1}(S_{t+1})\} | S_t]$$

$$\Rightarrow W_{ij0t}(Z_t) = \gamma \int \lambda_{j,t+1} \ln\{\exp[W_{ij,t+1}(Z_{t+1})] + \exp[W_{ij0,t+1}(Z_{t+1})]\} \times dF(Z_{t+1} | Z_t)$$

unconditional purchase prob. $P_{ijt} = \dfrac{\exp(W_{ijt})}{\exp(W_{ij0t}) + \exp(W_{ijt})}$

- discrete approx. — aggregate analog latent class
- K segments → β_K shared $\quad \beta_K = (\beta_{Kp}, \beta_{Km}, \beta_{Ks})$
- Consumers belong to segment k
- π_K prop. Consumers — segment size changes (adoption)
- $\sum_{k=1}^{K} \pi_K = 1$

$N_{kjt} = N_{kj,t-1}(1 - P_{kj,t-1})$ # remaining Cons. { seg k, Product j, Period t }

$N_{kjt} = M_{j0}\, \pi_K \prod_{l=1}^{t-1}(1 - P_{kjl})$ → prop. Con who have not bought before t

π_K → segment size; M_{j0} → total market size

θ_{kjt} : size segment k in market for prod. j at t

$$\theta_{kjt} = \frac{N_{kjt}}{\sum_{m=1}^{K} N_{mjt}} = \frac{\pi_K \prod_{l=1}^{t-1}(1 - P_{kjl})}{\sum_{m=1}^{K} \pi_m \prod_{l=1}^{t-1}(1 - P_{mjl})}$$

(14)

$$MS_{jt} = \sum_{k=1}^{K} \theta_{kjt} P_{kjt} = \sum_{k=1}^{K} \theta_{kjt} \frac{\exp(W_{kjt})}{\exp(W_{kj0t} + W_{kjt})}$$

Parameters

$\alpha = (\alpha_1, \dots, \alpha_J)$ Product fixed effect

c Consumption Param

$\beta = (\beta_1, \dots, \beta_K)$ segment specific param $\beta_K = (\beta_{Kp}, \beta_{Km}, \beta_{Ks})$

σ_ξ

$\pi = (\pi_2, \dots, \pi_K)$ segment size (initial)

$\Omega = (\alpha, c, \beta, \pi, \sigma_\xi)$ set of model param

γ → set to pre specific determined $\quad$ 0.99 monthly, 0.9975 weekly

(4)

$X_{jt} = (P_{jt}, d_{jt}, S_t)$ Covar's

segment 1: $\delta_{jt} = \alpha_i + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_1 + \xi_{jt}$ mean utility

$\tilde{\beta}_K = \beta_K - \beta_1$ param diff of seg k

$$MS_{jt} = \theta_{1jt} \frac{\exp(\delta_{jt})}{\exp(W_{1j0t}) + \exp(\delta_{jt})} + \sum_{k=2}^{K} \theta_{kjt} \frac{\exp(X_{jt}\tilde{\beta}_K + \delta_{jt})}{\exp(W_{kj0t}) + \exp(X_{jt}\tilde{\beta} + \delta_{jt})}$$

Value from waiting Calculation

$$\begin{cases} t = T_j - 1, T_j - 2, \dots, 1 \\ k = 2, \dots, K \end{cases}$$

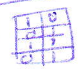
$$W_{kj0t} = \gamma \int \lambda_{j,t+1} \ln\{\exp[\delta_{j,t+1} + X_{j,t+1}\tilde{\beta}_K] + \exp[W_{kj0,t+1}(Z_{t+1})]\} dF(Z_{t+1} | Z_t) \quad \text{simulate}$$

(*) assumed consumer knows distribution of demand shock

- Price path conditional on current state
- expected markdown path $\Delta_{jt} = (\gamma_{t+1}, \gamma_{t+2}, \dots, \gamma_T)$

if new markdown place in period t $\quad 1 = \dots$

$\Delta_{jt}^m \quad m = 1, \dots, M$ markdown path

two period e.g. MkDown path 

$P(\Delta_{jt}^m | S_t)$ from (2), (3)

$T_j - t$ Error term from (4) ⇒ Mk down depth

$E_n = (\varepsilon_{j,t+1}^n, \varepsilon_{j,t+2}^n, \dots, \varepsilon_{jT}^n) \sim N(0, \sigma_{MD}^2)$

$MD_{jt}^n(S_t) = (MD_{j,t+1}^n, MD_{j,t+2}^n, \dots, MD_{j,T_j}^n)$

(1) Simulated price path { possible markdown path, simulated markdown depth } Conditional on current state

(2) simulate availability path | Conditional on current state

→ draw L random vectors $(T_j - t)$ Error terms: $\varphi_l = (e_j^l, e_{j,t+2}^l, \dots, e_{jT_j-1}^l) \sim N(0, \sigma_\lambda^2)$ → estimate from (5)

↳ has corresponding future availability

$\Lambda_{jt}^J(Z_t) = (\lambda_{j,t+1}^l, \lambda_{j,t+2}^l, \dots, \lambda_{jT}^l)$

(3) use σ_ξ^2 to integrate over time and product-specific demand shock $\xi_{jt} \sim N(0, \sigma_\xi^2)$

(4) W_{kj0t} → AVG (integrand) on (2 steps)

{ (1) N rand Avail. vector; (2) L rand markdown depth vector; (3) R random demand shock vector; (4) M markdown path }

(*) steps: (1) value from waiting corres. (I) avail vector (II) markdown depth vector (III) demand shock vector

Backward recursive finite horizon

$$W_{kj0t}^{lnr}(Z_t) = \gamma \sum_{m=1}^{M} P(\Delta_{jt}^m)[\lambda_{j,t+1}^l \ln\{\exp[\delta_{j,t+1}^{mn} + X_{j,t+1}^{mn}\tilde{\beta}_K] + \exp[W_{kj0,t+1}^{lmnr}]\} | \Lambda_{jt}^l(Z_t), MD_{jt}^n(Z_t), \xi_{jt}^r, Z_t]$$

(2) value from waiting for each product and time (averaging)

$$W_{kj0t}(S_t) = \frac{1}{L}\frac{1}{N}\frac{1}{R}\sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{r=1}^{R} W_{kj0t}^{lnr}(Z_t)$$

Pricing project

Ozlap & Gonca

Scratch:

First Draft

$$\begin{cases} u_{ij_1} = \overbrace{a_j + \Sigma_{\tau=1}^{2} \gamma_c^{\tau-t} + \beta_{ip} P_{j1}}^{u_1} + a_{ip} \cdot \lambda_{i2} (P_{j1} - P_{j2}) \\ u_{ij2} = \underbrace{\lambda_{i2} (\Sigma_{t=2}^{2} \gamma_c^{\tau-t} + \beta_{ip} P_{j2})}_{u_2} - \beta_{ir} (1-\lambda_{i2}) (\gamma_c^{i} + \beta_{ip} P_{j1}) \end{cases}$$

(*)

Br

1. $v_1, v_2 \rightarrow$ 10c, 15c..
2. regret should also be discounted
3.
4. categories of Fashion & Basic classification (20-30 weeks)
5. Then extend dynamic multiperiod

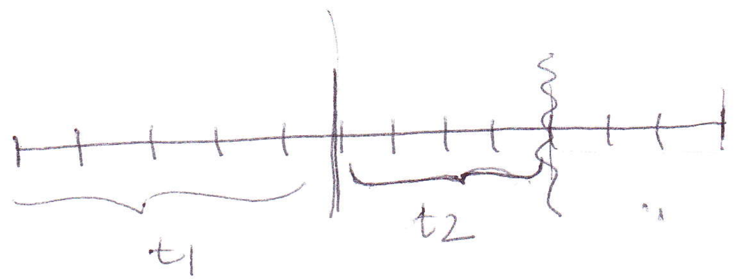
t1 — t2

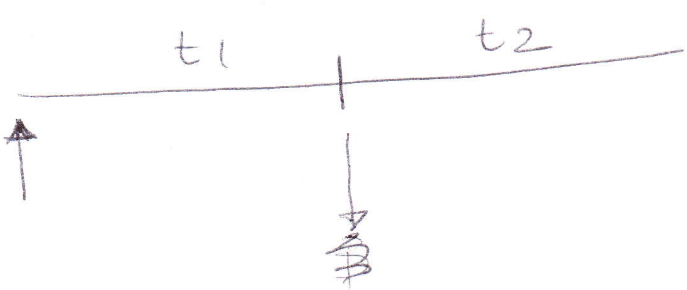
Purchase

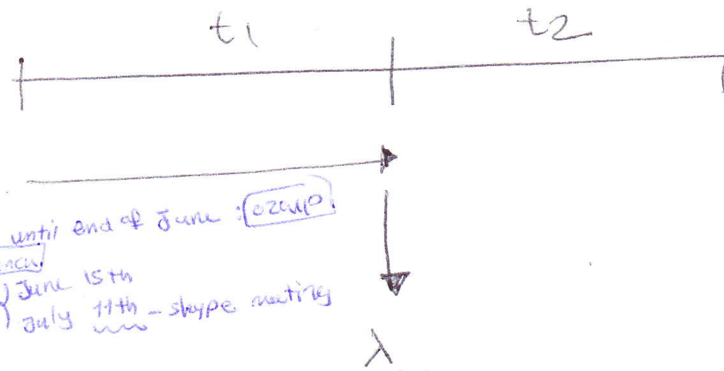
Wait — Purchase — available — $\lambda \max\{u_2, 0\}$

not — $(1-\lambda) \max\{u_1, 0\}$

~~want No purchase~~ $\rightarrow$







until end of June: Ozlap

Gonca
- June 15th
- July 11th - skype meeting

M $\rightarrow$ m_1 $\rightarrow$ period 1

m_2 $\rightarrow$ period 2

m_3 $\rightarrow$ not available

didn't want to buy it?

ZSLS

- intertemporal price skimming
- high valuation but they are strategic
- while waiting their valuation also drop
- sooner markdown makes you winner

$\Rightarrow$ (new): when regret you need to ration for some category you don't ration

$\Rightarrow$ not only strategic but also Behavioral (e.g. regret

$\Rightarrow$ ① Get data & see how clean & estimate

② build theory for hypoth. (what people have done? what hypoth. (competing hypothesis)] $\Rightarrow$ take notes of literature

③ next meeting $\Rightarrow$ Calender invitation

June 06

Regret in Pricing Project

/① Drop Box

② maximum likelihood rather than GMM

③ cleaning the data → would you need anything else

④ try to use the GMM, likelihood method

⑤ need more categories? ⟹ only one category right now

⑥ man's coat prices as instrum for women

(without

at product level

Hypothesis?

Fashion vs. basic

mkdown vs. non markdown

women's coat currently availab

⑦ availability is not concern

creativity to define periods

full avail until first mkdown

① what are actual data and what calc

② inventory is at time or after period

explicit assumptions

③ data Explanation post in Explanation

④ Literature → availability regret

⇒ you are in market

⑤ masis → everyday low price

for certain products

⑥ ~~[illegible]~~

ask question about how

aggregated (outlet with...)?

across regions?

population changes?

zip code information?

⑦ ask header and 5 data points

⑧ granular

⑨ cc preferer a2alp

⑩ summary of emails in excel sheets

⑪ write assumptions

✓ [HW0] email Ozlap for appointment.

✓ [HW1] Read and understand job market paper of Gonca

[HW2] sales, period, availability ⇒ likelihood?

[HW3] How measure regret?

[HW4] what else if new data?

[HW5] Can we compare two models? merge them?

[HW6] Think about other projects

[Prof Ozalp]

① it is there effect Purchase decision

② Something more than probability

③ Company affects α, β (regret)

→ regret hetrogeneity (soft & Behavioral) estimate

④ or signaling and find result

⑤ don't aggregate markdown

⑥ whether markdown only on christmas & remove the probability (& take majority 20% - 80% only) [20-80]

⑦ Only on markdown

⑧ Choice
Consistent markdown
Dynamic program of two Period

⑨ Forget about misperception of availability
later on we can add it

$u_1(\vartheta, p) = (\vartheta_1 - p_1) - \alpha q (p_1 - \delta p_1)$

$u_2(\vartheta, p) = q(\vartheta_2 - \delta p_1) - \beta(1-q$

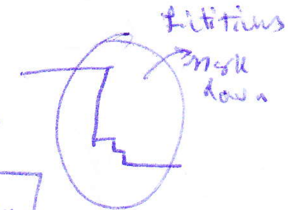
⑩ update u $u_{ijt} \to u_2(\vartheta, p)$

$u_{ijt} \to v_1(\vartheta, p)$

⑪ Start simple & during time Enrich Consumer's decision (hetrogeneous First) → No time varying currently

⑫ For different product category different

⑬ do the aggregation



⑭ Deadline → [September]

[Deliverable]

① write model & say how to Calc utility

② after meeting with Gonca (3 people meeting & my presentation of what I plan to do)

Q how mk Down sensitivity

season

↳ purchase time

ξ_{jt} ϵ_{ijt} → Mean ter stock

↳ product oriented

$$u_{ijt} = \alpha_i + \sum_{\tau=t}^{T_j} \gamma_c^{\tau-t} + \beta_{ip} P_{jt} + \beta_{im} d_{jt} + \beta_{is} S_t + \xi_{jt} + \epsilon_{ijt}$$

Length T_j of sale season

JTG → TTpremium

Q How marry two models?

Brand utilities → in Garcia paper → No bias in people perception of availability

Period utility in Ozalp paper

Signaling

problem: how observe?

instrument?

is there by β_{is}, β_{im}

heterogeneity

Seasonality or markdown

where store availability is captured?

→ idea Assymmetry should be point of marry

really enjoyed reading her paper

- if exists in next period definitely lower price Ozalp

d_{ijt} → availability Dynamic Programming Garcia

β_{it} → Captures high price regret

Q What theory predicts?

Configure by perception change

Q How Garcia Captures updating perception?

Current state → effects are captured with different name in Garcia's paper

$$\begin{cases} y_{it} = a_i + u_{it} \\ u_{it} = \rho u_{it-1} + u_{it} \end{cases}$$

$$\Rightarrow y_{it} = \underbrace{(1-\rho) a_i}_{0} + y_{it-1} + u_{it}$$

$$\Rightarrow y_{it} = \cancel{\rho}\, y_{it-1} + u_{it}$$

$$\Rightarrow y_{it} = \sum_{i=1}^{T} u_{it}$$

$$\Rightarrow \hat{\rho} - 1 = \frac{\sum y_{it-1} u_{it}}{\sum y_{it-1}^2}$$

$$y_{it}^2 = \left(\sum_{i=1}^{t-1} u_{it}\right)^2 = (T-1)\sigma_u^2$$

$$\text{plim} \sum_{t=1}^{T} (t-1)\sigma_u^2 = \boxed{\frac{T^2}{2}\sigma_u^2}$$

Var

σ_u^2

$$\left(\sum u_i \sum u_{it}\right)^2 = \text{square} + \text{cross}$$

$$\sum^{T} (t-1)\sigma_u^4 = \frac{T^2}{2}\sigma_u^4$$

$$\boxed{\sum u_i y_{it-1}} \neq E\left(\sum u_i y_{it-1}\right)^2 = \text{squar} + \text{cross}$$

1. $\text{plim}\,(u_i y_{it-1}) = (u_{it} \sum u_{it-1}) = 0$

   $\sqrt{N}\, T(\hat{\rho}-1) \to^{d} N($

2. Var