Chapter 1	Chapter 2	Chapter 3	Chapter 3 (cont.)	Chapter 3
Preference Relation: at least as good as (Strict vs. normal: indifference set existence)	consumption bundle as vector(multidimensional)	Desirability (following 2) Non-satiation: existence of x neighborhood of y such that y $\succ$ x	Utility maximization problem: $\max_{x>=0} u(x)$ s.t. p. x <=w (compact set) Continuous u(.) => solution	HDF (is HDZ)& EF (Expenditure): e(p,u) = p.x* = p.h(p,u); $h(p,u) = \nabla_p e(p,u)$ S(p,u)= $D_p h(p,u) = D_p^2 e(p,u)$ $D_p h(p,u)$: Symmetric ,NSDsince e:Concave
Rationality Relation (Completeness, transitivity)	Walrasian Demand Function (WDF), Walras Law(WL) x.p=w	Monotonicity: u(x)>u(y) if x>>y: increasing func If monotone => locally non satiated	u(.) continuous, local non satiated $\geq$ => W.D.C (correspondence): HDZ, W.L. , convexity/uniqueness => u(.) quasiconcave	Cross price effect $\frac{\partial h_l(p,u)}{\partial p_k} > 0$ substitution $\frac{\partial h_l(p,u)}{\partial p_k} < 0$ complementary
Choice preference & Budget set	Homogeneity of degree zero (HDZ) x(αp,αw)=x(p,w)	Upper /Lower counter ($\geq$): UC/LC	Kuhn Tucker condition: Solution to UMP: Lagrange multiplier: $\nabla u(x^*(p,w)) = \lambda p$ $\nabla u(x) = [\frac{\partial u(x)}{\partial x_1}, \frac{\partial u(x)}{\partial x_2}, \ldots, \frac{\partial u(x)}{\partial x_1}]$	$h_l(p,u) = x_l(p, e(p,w))$ $\frac{\partial x_l(p,e(p,w))}{\partial p_k} = \frac{\partial h_l(p,u)}{\partial p_k} + [\frac{\partial x_l(p,e(p,w))}{\partial w}(-x_k(\bar{p},\bar{w}))]$ Price effect= Substitution effect+wealth effect
Weak Axiom of Revealed preference (WARP)	Wealth expansion: one variable fix; Dw x(p,w):1*n, Dp:x(p,w)n*n	Convexity assumption $\geq$ convex if UC convex: y,z $\geq$ x=>αy+(1-α)z $\geq$ x : Diminishing Marginal Rates of substitution (DMRS): Diversification	Marginal rate of substitution of two goods (MRS): $\frac{\partial u(x^*)}{\partial x_l} \Big/ \frac{\partial u(x^*)}{\partial x_k} = \frac{p_l}{p_k}$ $\nabla u(x(p,w)) D_w x(p,w) = \lambda p. D_w x(p,w) = \lambda$: W.L.	IDF & WDF: $\bar{u} = v(\bar{p}, \bar{w})$ $x_l(\bar{p}, \bar{u}) = \frac{\partial v(\bar{p}, \bar{w})}{\partial p_l} \Big/ \frac{\partial v(\bar{p}, \bar{w})}{\partial w}$ Roy's identity Welfare change: = v(p^1,w)-v(p^I,w) EF is IUF (strictly incr.)
Weak Axiom of Revealed preference & Rationality of choice	WDF & HDZ (differentiate w.r.t. α): $\sum \frac{\partial x_l(p,w)}{\partial p_k} p_k + \frac{\partial x_l(p,w)}{\partial w} w = 0$ Elasticity conversion	Homothetic: if indifference set proportional expansion x~y => αx~αy	Indifference curve characteristics: 1. Negative slope (substitution effect) + Diminishing marginal utility 2. Linear: perfect substitute 3. L shape: perfect complement	e($\bar{p}$, v(p, w)) money metric indirect utility function; p0 to p^1 :**Equivalent variation**(E.V.): $e(p^0, u^1)$ -w
All three budget sets & WARP => Rationality	Two properties of Cournot & Engel aggregation WL: differentiate w.r.t. w $\sum \frac{\partial x_l(p,w)}{\partial p_k} p_k + \partial x_l(p,w) = 0$ $p.D_p x(p,w) + x(p,w)^T = 0^T$	Quasilinear: 1. Parallel displacement: x~y => (x+αe$_1$)~ (y+αe$_1$) e$_1$=(1,0,…,0) 2. Good 1 desirable: x+αe$_1$>x (all x & α>0)	Cobb-Douglas utility function: u(x_1, x_2)= k $x_1^{\alpha} x_2^{1-\alpha}$ Constant elasticity of substitution (CES): u(x)=$[\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}]^{1/\rho}$ Cobb-Douglas simulator p→0 Leontief simulator p→-∞ Linear simulator p→1	**Compensating variation** (C.V.): = w- $e(p^1, u^0)$ Parallel straight wealth expansion path condition (Gorman form): $v_i(p, w_i) = a_i(p) + b(p) w_i$
	WL: $\sum \frac{\partial x_l(p,w)}{\partial w} p_k = 1$ $p.D_w x(p,w) = 0$	Continues: If preserved underlimits $\{(x^n, y^n)\}_{n=1}^{\infty}, x^n \geq y^n$ $x = \lim_{n\to\infty} x^n, y = \lim_{n\to\infty} y^n$ => $x \geq y$ Closed UC/LC; diagonal ray	Demand function properties: 1. WL 2. HDZ 3. Unique 4. Continuity 5. Monotonicity 6. Quasiconvexity	Gorman function v(p,u)=a(p)+b(p)w; IUF | 1. a(p) constant 2. Hom. Degr. 1 3. Quasiconcave 4. b(p)>=0 5. $\nabla b(p) \leq 0$ for every p>>)
	WDF satisfies WARP if: p.x(p',w')<=w & x(p',w')!=x(p,w) => p.x(p,w)>w'	Continuity of $\leq$ =>utility fun exists (Restriction duality)	HDO(deg. 1) on w=> ε_{lw}=1 => α=1/w => $D_w x$= function of p only=> wealth expansion=ray passing x(p,1)	
	Slutsky wealth compensation (SWC): $\Delta w = \Delta p.x$ Compensated Law of demand (CLD)	Differentiability (Loentiff function having problem of non-differentiability)		
	CLD ~WT satisfies HDZ, WL: (dp. dx) $\Delta p..\Delta x$ <=0 =>WARP SWC: dx= $[D_p x(p,w) + D_w x(p,w) x(p,w)^T] dp$	Strict/Convexity of preference = strict/quasiconcavity of utiliy func: u(αx+ (1-α)y) >=Min {u(x), u(y)}	Revealed preferred a over b: 1. wealth of a option greater than wealth of better modified demand a; 2. Modified pricewealth a greater than wealth b	
	Slutsky matrix(SM): substitution effect $[D_p x(p,w) + D_w x(p,w).x(p,w)^T]$	$\geq$ homothethic ⇔utility:homogeneios of degree one = u(αx)=αu(x)	Summer of previous cell: under this price I went for x, since based on WL it uses most of my money; so under new price since the other demand is chosen, this demand should have required more wealth that lead me not to choose that, and go for option b.	
	Differentiable WDF| WL,HDZ,WA SM v.S(p,w).v<=0 for any v S called Negative Semidefinite (NS) (inferior = Giffen analysis) CLD => NS	$\geq$ quasilinear if admist utility function of form: u(x)=x1+ϕ (x$_2$,…,x$_L$)		
	WDF| differentiable, HDZ, WL => p.S(p,w)=S(p,w).p for any p,w	Ordinal properties of u(.): 1. Increasingness 2.Quasiconcavity		
	Rationality needs Symmetry as well L=2 however symmetry exists	Rational, continues, locally non satiation preference → u(.) contineous		

Production plan/vector: y =(y_1,y_2,…,yl)=R^L Transformation function: F(.): $Y = \{y \in R^L : F(y) \le 0\}$ Transformation frontier: $Y = \{y \in R^L : F(y) = 0\}$	Marginal Rate of Transformation $MRT_{lk} = \dfrac{\partial F(\bar{y})}{\partial y_l} \Big/ \dfrac{\partial F(\bar{y})}{\partial y_k} = \dfrac{\partial y_k}{\partial y_l}$ Diminishing product productivity	$q = (q_1, q_2, \dots, q_m) \in R_+^{\ m}$:outputs $z = (z_1, z_2, \dots, z_{l-m}) \in R_+^{\ L-M}$: would be inputs $Y=\{(z,q): F(-z,q) \le 0 for z \in R_+^{L-M} \& q \in R_+^L\}$ Production function f(z): maximum output for given input	$MRT_{lk} = \dfrac{\partial F(\bar{z}) / \partial z_l}{\partial F(\bar{z}) / \partial z_k}, \bar{q} = f(\bar{z})$: Marginal rate of technical substitution	Production function properties: 1. Y is non-empty 2. Y is closed (boundary inclusion) 3. No free lunch: if $y \in Y, y \ge 0$ then y=0 4. 0 is part of Y (inaction)
5. free disposal: $y \in Y, y' \le y \to y' \in Y$	6. irreversibility: $y \in Y, y \ne 0$ then $-y \notin Y$	7. Non increasing return to scale: $y \in Y, \alpha \in [0,1] \to \alpha y \in Y$	8.Nondecreasing return to scale $y \in Y, \alpha \in [1,\infty] \to \alpha y \in Y$	9 constant return to scale $y \in Y, \alpha \in [0,\infty] \to \alpha y \in Y$
10. Additivity (Free entry) $y \in Y, y' \in Y \to y + y' \in Y$ Profit function: π(p)=p.y(p)	11. convex cone: $y \in Y$ $y' \in Y$, α, β>0 => $\alpha y + \beta y' \in Y$	Profit Maximization Problem (PMP):1. Price vector (p1,p2,…,pl)>>0 , Firms are price takers 2. Firm's objective: $\boxed{L=p.y-\lambda F(y),}$ $\boxed{F(y)<=0}$ $\max_y p.y = \sum_{l=1}^{L} p_l y_l, s.t. y \in Y$		PMP: $\boxed{p_l = \lambda \dfrac{\partial F(y^*)}{\partial y_l}}$ $\boxed{MRT_{lk} = \dfrac{\partial F(y^*)/\partial y_l}{\partial F(y^*)/\partial y_k} = \dfrac{p_l}{p_k}}$
Single output f(z): p- output price w- input price vector profit = p.f(z)-w.z	$\dfrac{\partial f(y^*)}{\partial z_l} p = w_l$ & $MRT_{lk} = \dfrac{\partial f(y^*)/\partial z_l}{\partial f(y^*)/\partial z_k} = \dfrac{w_l}{w_k}$	$\dfrac{\partial f(y^*)/\partial z_l}{w_l} = \dfrac{\partial f(y^*)/\partial z_k}{w_k}$	Profit function properties: 1. π(.) : H.D.1 2. π(.) convex in p 3. If production function is convex: $Y = \{y \in R^L, p.y \le \pi(p) for \quad all \quad p >> 0\}$ 4. y(p) H.D.Z in p	5. y: str/convex => y(p) : single value/convex. f(αx+(1-α)y)<=αf(x)+(1-α)f(y): convexity

6. (Hotelling's Lemma) if y(p) is single value then the production function π(p) is differentiable and $\nabla \pi(\bar{p}) = y(\bar{y})$	If y(p) id differentiable, then $D_y(\bar{y}) = D^2\pi(\bar{p})$ is symmetric and positive semidefinite with $D_y(\bar{p}).\bar{p} = 0$ $dpD^2\pi(\bar{p})dp \ge 0$;(p'-p)(y(p')-y(p))>=0	$\nabla \pi(p) = y(p) + p.Dy(p)$ $HDZ \quad in\ p: y(\alpha p) = y(p)$
Cost minimization Problem (CMP): z: input; w: input price: $\min_z$ w.z,s.t. f(z) >= q ; c(w,q): cost func.; z(w,q): conditional factor demand		$w_l = \lambda \dfrac{\partial f(z^*)}{\partial z_l}; w.\dfrac{\partial z(w,q)}{\partial q} = \dfrac{\partial c(w,q)}{\partial q}$

f(z(w,q))=q => $\dfrac{\partial c(w,q)}{\partial q} = \lambda$	Properties of c(.) (i) c(w,q): H.D.Q & non-decr. in q (ii) c(w,q): concave(w) (iii) $\{z \ge 0: f(z) \ge q\}: convex \forall q \Rightarrow Y = \{(-z,q): wz \ge C(w,q) \forall w >> 0\}$ (w) (iv) z(w,q): H.D.Z	(v) $\{z \ge 0: f(z) \ge q\}$: convex /strictly=> z(w,q): convex/single val.

(vi) (shepend's Lemma) $z(\bar{w},q)$: single valued => c(.): differentiable & $\nabla_w C(\bar{w},q) = z(\bar{w},q)$	(vii) z(.) differentiable @ $\bar{w}$ => $D^*_m z(\bar{w},q)$: asymmetric neg. semidef.

(vii) f(.): H.D.1& $z(\bar{w},q)$ => c(.) & z(.) : H.D.1. (q)	(ix) f(.): concave => c(.) convex(q) Coup Douglas: $F(z_1,z_2)=z_1^{\alpha} z_2^{\beta}$	$p = \dfrac{\partial c(w,q^*)}{\partial q}$: MC=MR	**Doing firm Profit maximization and cost minimization: 1.F(.)2.** $\dfrac{\partial f(y^*)/\partial z_l}{w_l} = \dfrac{\partial f(y^*)/\partial z_l}{w_k}$ **3.z(w_i's,q)5.c=zw6.c'=p7. analysis**

Average cost (AC)= $C(\bar{w},q)/q$	Max(AC): 1st order=> AC=MC	Short run: always something is fixed.	Long run: reshuffling.	Prof max → efficient $\neg\exists y' > y \in Y$

Effic.|convex→ prof max economic allocation $(x_1, x_2, \dots, x_I, y_1, y_2, \dots, y_J)$ $x_i \in X_i$ $y_j \in Y_j$ $(x_1, x_2, \dots, x_I, y_1, y_2, \dots, y_J)$ is feasible if $\sum_{u-1}^{I} x_{li} \le w_l + \sum_{j=1}^{J} y_{lj}$ $(x_1, x_2, \dots, x_I, y_1, y_2, \dots, y_J)$ pareto optimal if $\neg\exists(x'_1, x'_2, \dots, x'_I, y'_1, y'_2, \dots, y'_J)$ $\forall i = 1..I, u_i(x'_i) \ge u_i(x_i)$ & $\exists i: u_i(x'_i) > u_i(x_i)$	I: consumers i=1,…, I J- firms, j=1,….,J l-goods l=1,2,…, L Ui – consumer utility Xi (x1i,…,xsi) $\in R^L$ consumer I's consumption bundle Yj=(y1i,…,yli) $\in Y_j$ production plan of firm j (y'1,y'2,…, y'j) $\in R^{Lj}$ production plan of all J firms Wl ≥ 0, l=1,2,…, L Initial endowment of good l $w_l + \sum_{j=1}^{J} y_{lj}$: total amount of good l	Equilibrium conditions: (i)Profit maximization: for each firm y*j solves, Max p*.yi , $y_j \in Y_j$ (ii) Utility maximization for each consumer I, x* solves: Max ui(xi) $x_i \in X_i$ so that. P*.xi $\le p^* w_i + \sum_{j=1}^{J} \theta_{ij}(p^*.y^*_j)$ (iii) Market clearing for each good l=1,…,L $\sum_{i=1}^{I} x^*_{lj} = w_l + \sum_{j=1}^{J} y^*_{lj}$		