IO @ UTD: Third session

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Couse conjecture: price of monopolist will reach to marginal cost, as price adjustment become more frequent (in small period), when the product would be durable good.

Monopolist make sense to lease the good. The monopolist has option to either lease, and then get everything back, and then lease again.

Gul paper:

We have an infinite number of periods. It is monopoly of durable goods. Consumers have unit demand for one item.

Resell market in the previous examle exist, but here that does not exist, and anyone purchased would be out of market.

Consumer has valuation v, and consumers with $v > \lambda.p; \lambda > 1$ will purchase.

Producer have to decide what price to charge. Producers with $\bar{V}$ have already purchase, and all consumers lower than this will pruchase. The producer rule therefore would be $p = \mu.\bar{v}, \mu < 1$

so > 1, and $\mu < 1$

We want to find μ^*, λ^*

In some period we must have $v - p_t \geq \delta(v - p_{t+1})$

δ would be the discount factor.

p_{t+1} would be expectation of the next period price.

In this period consumers would purchase that have the relation of $v > \frac{p_t - \delta_{p_{t+1}}}{v_{t+1}}$

State of economy would be: $v_t = \frac{p_{t-1} - \delta p_t}{1 - \delta}$

the producer knows that $v \in [0, v_t]$

According to linear rule, $p_t(v_t) = \mu.v_t$

Quantity sold in period t: $q_t = v_t - v_{t+1}$

last consumer who would buy would be: $v_{t+1} = \lambda p_t(v_t)$ if my valuation is more than $\lambda.p$ I am going to purchase.

$q_t = v_t - \lambda.p_t(v_t) = v_t - \lambda.\mu.v_t$

$q_t = (1 - \lambda.\mu).v_t$

We must have had the assumption that $\lambda.\mu < 1$

$\pi = ... + \delta^t.p_t.q_t + \delta^{t+1}.p_{t+1} + \delta^{t+2}.p_{t+1}.q_{t+1} = ... + \delta^t.p_t[v_t - \lambda.p_t] + \delta^{t+1}p_{t+1}[v_{t+1} - \lambda.p_{t+1}] + \delta^{t+2}.p_{t+2}[v_{t+2} - \lambda.p_{t+2}] + ...$

v_t was assumed to be uniformly distributed between zero and one. We want to miximize profit with respect to p.

We want to find μ, and we want to maximize over all p's.

FOC with respect to p_t: $= \delta^t[v_t - \lambda.p_t - \lambda p_t] +$

$\delta^{t+1}.\lambda.p_{t+1} = \delta^t v_t - 2\lambda p_t + \delta\lambda p_{t+1}$

FOC: $\Leftrightarrow = \delta^{t+1}\lambda.p_t - 2\lambda p_{t+1} + \delta\lambda p_{t+2}$

FOC: $\Leftrightarrow = \delta^{t+2}\lambda.p_{t+1} - 2\lambda p_{t+2} + \delta\lambda p_{t+3}$

We start to see patterns here.

$\lambda.p_t - 2\lambda.p_{t+1} + \delta.\lambda.p_{t+2}$

$\lambda.\mu.v_t - 2.\lambda.\mu.v_{t+1} + \delta.\lambda.v_{t+2}$

$\lambda.\mu.v_t - 2.\mu.\lambda.\mu.v_t + \delta.\lambda.\mu.\lambda.\mu.\lambda.\mu.v_t$

$\lambda.\mu.v_t[1 - 2\lambda.\mu + \delta.\lambda^2.\mu^2] = 0$

Let's assume that $\lambda.\mu < 1$

$\mu = \frac{1}{\lambda[1+\sqrt{1-\delta}]}$

Equilibrium at period t: $p_{t+1} = \mu.v_{t+1} = \mu.\lambda.p_t(v_t) = \mu.\lambda.\mu.v_t$

$p_{t+1} = \mu^2.\lambda.v_t$

Marginal consumer will be: $v_{t+1} - p_t = \delta(v_{t+1} - p_{t+1}) = \delta(v_{t+1} - \mu.v_{t+1})$

$v_{t+1} - p_t = \delta(1-\mu).v_{t+1}$

$p_t = [1 - \delta(1-\mu)]v_{t+1}$

$v_{t+1} = \frac{p_t}{1-\delta(1-\mu)}$

$\lambda.p_t = \frac{p_t}{1-\delta(1-\mu)}$

$\delta = \frac{1}{1-\delta(1-\mu)}$

$\mu^* =$
$\lambda^* = \frac{1}{\sqrt{1-\delta}}$

Linear non stationary equlibrium so would be in this form.

Quose conjecture says that the price will go down, and monopoly then will charge marginal cost, and zero profit.

$\delta \to 1 \Rightarrow u^* = 0$

$v_1 = 1$ means $p_1 = \mu^*.v_1 = 0.1 = 0.MC$

Intuitively story makes sense, and we showed theoritically.

We tried to keep things simple here.

Rubenstein has infinite horizon game, and players play until infinity, and he introduces discount factor. Stahl has different approach, and assumes barganining lasts one pariod, but offers stand form limited time. The number of offers with this approach goes to infinity.

Shapiro 1983 paper:

Experience goods, consumers ex ante is not much, and only by using it they will learn about it. Search good you don't know what the quality is and by inspection without using it you wil learn about it. Experience goods you have to purchase and use them to find out about them.

Experience good introduced by Nelson, and consumer have imperfect info. until they purchase and use it. The sales is the dynamic one. If the buyer learns, the demand function shifts. The seller is bundling information with actual product. With the product the information about product and attribute is selled. With purchasing favorability of information will be understood.

Consumers are pessimistic $q > R$ R would be expectation. Redbull coke for example. Prior is pessimistic. Second case is when consumers are optimistic. The true quality would be smaller than expected quality $q < R$. For the first case result is that there would be introductory offer to inform consumers. It would be two stage introductory scheme. Monopoly charges high prices at the beggining, and then they realize that the good is good one. Once the introductory expires, the monopolist will charge higher price. Pricing scheme is milking reputation.

When consumers are optimistic, everybody think that it is better than it is, so you charge a lot, and once they bought they will more or less out of market. Then you skimm. You slowly skim the market, and you start with high price and slowly the price will go on, and you jump back up.

Here assumption is that everybody is optimistic or pessimistic, and it means everybody are homogeneous.

All expectations are homogneous, and consumers have point expectation, mean there would be no uncertainty.

The product is non durable. Jump back would be to monopoly price.

Consumers are myopic.

Monopolist who sells the product of quality q and produces quantity x. There is a cost function $c(x, q)$. There are standard assumption; it is either u shape, average cost curve, or u shape constant average cost curve, means decreasing marginal cost curve.

The choice variable is the p, and it is discrete time model, so it would be discrete time $r > 0$ would be the interest rate.

It is discrete time model, infinite horizon. Consumers have unit demand. In each t, and they are indexed by their taste of quality. $\theta \in [0, 1]$.

There is density function $f(\theta)$.

Consumer surplus is $\theta.q - p$.

$R > 0$, if $R = q$ then it would be no learning, and we are back into static monopoly.

In updating, it is instantaneously and perfect. Consumers with information purchase if and only if $iff \theta \geq \frac{p}{1}$

uninformed: purchase $iff \theta \geq \frac{p}{R}$

The cumulative distribution is left hand usually and here it is not so, and we have right hand. $F(\theta) = \int_{\theta}^{1} f(t)dt$. This is for simplicity.

$F(bar$
$theta)$ is mass of consumer with valuation of $\bar{\theta}$ or higher $\theta \geq \bar{\theta}$

Demand function for each price will gie us quantity demanded. Each consumer will need one unit. $S(p) = F(\frac{p}{R})$ gives us consumers whose valuation is higher than p. Mean consumers who would purchase.

This is for no information case, and for full information we will have $Z(p) = F(p/q)$

If price would be 2 in one period, and consumers will find out, and if price change the demand will not change, and everybody has bought, and know the true quality. If the price increases the structure of the economy will not change.

Lemma 1. if $p_T > p_{T-1}$ then, $p_t = P_T t >> T$

The economy will not change, since there would be no additional consumer that learned. If it was optimal in P_T, then it will still be optimal for all periods.

The implication is that we only need to consider the declining path of price. If it was optimal for the consumer to charge the lower price in the next period, and I did not change the market today, and helped consumer learning, for the next one it will not change anymore.

Learning is trhough experiment, and is complete.

The pessimistic consumers are not necessarily low type. They are distributed between zero and one. Pessimistic consumers expect quality R, and the real quality would be q, they expect $R < q$.

If monopolist informs a lot of consumers, then they will have higher willingness to pay, and they can charge higher price in the later periods.

We have to consider only declining price path. The Lemma applies for everything that happens before, and it applies to both optimistic, and pessimistic.

$R < q$, it is not informing all of consumers, but part of consumers. It is two stage pricing policy. Introductory price is low, and high in the second.

Price path is kind of clear. When there are fully informed demand. Suppose that all consumers are fully informed. Then $\pi(x, q) = z^{-1}.x - c(x, q)$. It is for fully informed case.

For incomplete informed $\pi(x, R) = S^{-1}(x)x - c(x, q)$

$\phi(x, \hat{x}) = \pi(x, q)$ when $x \leq \hat{x}$, and $\pi(x, R)$ when $x > \hat{x}$

$S(p) = F(\frac{p}{R})$

$S^{-1}(x) = R.F^{-1}(x)$

$Z^{-1}(x) = q.F^{-1}(x)$

Marginal revenue of the informed lies above the uninformed one.

If people are uninformed the quantity would be much less than the informed.

For uninformed the profit would be lower. Both in the form of reverse U.

The right leg of the informed, and left leg of the uninformed would be monopolist profit curve.

$P_L : X = S(P_L)$ Monopolist chooses quantity and not the price, and this quantitiy will determine the price.

$P_H = Z^{-1}(x) = Z^{-1}(S(P_L))$

Z is the informed demand function. X consumer will know what is the true quality, and the second price is that all this consumers who are informed are willing to the purchase. All this consumers will have second demand function.

There is a trade off for the producer. Producer sees profit in two stages, or periods. By producing the amount between the uninformed, and informed, you would sacrifice for later greater profit.

The profit for this two stage policy would be $v(x) = \pi(x, R) + \pi(x, q)/r$

It is infinite horizon game, so the future profit is discounted.

The first derivitive would be $\frac{dV(x)}{dx} = 0 \Leftrightarrow X^*(R) < X < X^*(q)$

If all consumers are pessimistic, you want to make sure that they become informed quickly.

The curve is because if you increase the price the people who are uninformed will not purchase.

The strategy is dynamic pricing without letting information out too quickly when consumers are optimistic.

$R > q$

There are two types: The first one is declining price path: $P_1 \geq P_2 \geq P_3 \geq ...$

The second one is $P_1 > P_2 > ... > P_{T-1} < P_T = P_{T+1}$

We would have two uninformed would be higher profit revers U, and informed would be lower. You skimm uninformed. The price and quantity will go to perfect information case. The price is lower when perfect information price.

At some point you jump up. Optimal strategy is to jump back up for perfect information cost, and it would be discrete jump.

You skimm uninformed, you slowly lower the price until it is not profitable anymore, and you have enough consumer. The heterogeneity is on marginal value of quality, and how much they value quality.

Now will go over price discremination, and continue next session.

Problem set with questions will include one price discrimination would be in two weeks.

When we say coke today is different from coke tommorow, mean same price to different consumers, or different price to same consumer.

Different prices to different consumers due to different costs.

The possibility of arbitrage hinders the price discrimination. If the good is transferable, then price discrimination does not work. Other consumers will not purchase from monopolist, since he could purchase from another.

Services is hard to transfer, e.g. counselling, movie, financial assistance, medical treatment. If consumers are offered two different packages, or contents, then we will have transferability, and price discrimination will not work. Certain price quanity, and quality menues to different consumers. If all consumers could purchase the preferable ones, then discrimination work. Consumers will self select. i will target to two different, so that there would be no transferability.

Incentive compatibility constraint would be an important issue here. There are trhee degree of price descrimination:

1. perfect price discrimination: monopolist knows valuation. It is unlikely, and not that likely. Usually have incomplete information. The monopolist should have perfect information of valuations. Arbitrage would be easy. If you can not prevent first degree will not work.

2. Second degree would be the signal that we can not observe it. How we put incentive so that each group choose the packages that belongs to their group. There should be compatible designs for this.

3. Third degree: Direct and observable signals could be used for discrimination. Getting the hair cut. There is some direct signals that are observable. Signal is directly observable, so it is controlable. You don't give your student ID, you would not be chared. This was the third type.

First Degree

Determine willingness to pay. If the monopolist has all information, he can charge prices for different units. There is different case that we have identical downward slopping demand curves. Different willingness to pay for different units. $q = \frac{D(p)}{n}$, where n is number of consumers.

Aggregate demand is $Q = D(p)$. Monopolist comes with pricing schedule, but some transfer $T(q)$ is the total amount that consumer pays.

It could be linear $T(q) = p.q$

It could be affine linear: linear with intercept: fixed fee for each consumer $T(q) = A + pq$. Now what we want to show is that with appropriate pricing scheme, monopolist can increase profit with discrimination.

If monopoly can charge two prices, then consumer would be better off.

We are thinking about stealing consumer surplus. Monopolist wants to steal it, and needs to find how big it is and charge it in the form of fixed fee.

The best way is to charge the marginal cost. If he charges price that equals to marginal cost then he charges fixed fee. Fixed fee of consumer surplus, and variable cost that maximizes the consumer surplus. This would be Aggregate demand, and for each it should be A/n. Due to dead weightloss the in one part tariff can not take suprlus, but in two part monopolist can do this.

Paper of shmalinze, and the related papers in sylabus.

IO @ UTD: Fourth session

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Linear pricing scheme: $T(q) = p.q$ price would be monopoly price.

Two part tarriff: fix fee plus the variable fee: $T(q) = A + p.q$

The firm has too choose price, and once price is selected. With appropriate pricing scheme the consumer will extract the consumer surplus.

How to maximize social suprlus:

1. Price equal to competitive price p^c. If we maximize the social suprlus, there would be no consumer surplus. $T(q) = p^c.q + A$. The consumer surplus would be $S^c = \int_0^{q^c}[p(q) - p^c]dq$. License fee in the form of fixed premium. A would be individual fee and it should be lower than each individual share $A \leq \frac{s^c}{n}$.

Total two part tarriff would be $T(q) = s^c/n + p^c.q$ if $q > 0$, and zero if $q = 0$. Each consumer consumes $q = \frac{q^c}{n}$. Each consumer consumes the share.

The monopolist profit is higher than uniform monopoly pricing:

$\pi^{FD} = \frac{s^c}{n}.n + q^c.p^c - C(q^c)$ profit with first degree price descrimination.

$= s^c + [q^c.p^c - c(q^c)]$

$\pi^m = p^m.D(p^m) - C(D(p^m))$

Does $\pi^{FD} > \pi^m$. social surplus would be $ss^m = \pi^m + s^m + (DWL)$

Therefore, $S^{SM} < SS^{FD}$

excercise 3.1 is streight forward. Competitive fringe: means there are samll number of firms that are price taking, and are competing with the monopolist. $p^c < p^0 < p^m$. If the conopolist charges lower than their price which is p^0 then they would go out of market. If monopolist charges more than this amount, monopolist will not get any maret share.

$\tilde{p}(q)$ monopolist decides to charge price less than this price then the monopolist price would be lower, and if monopolist decides to produce more, then the price would be lower. For any quantitiy, so less than q^0 the price of the monopolist will be $\tilde{p}(q) = p^0$. if $q \geq D(p^0)$ the price of monopolist would be $p(q)$.

If MC is equal to zero we will have $p = 0 = p^c$, and $A = S^c(p^c)$. $p^c = 0$ then $A = \int_0^{q^c}[\tilde{p}(q)]dq/n$.

It means that from the triangle created from the two axis and the demand function, now the top triangle created with (p^0, q^0) can be removed, since consumer will go to the cometitor if the monopolist charges the price higher than them.

Here we assumed that consumers are identical. What if the consumers are not identical. In this case consumer must charge A_i for each of the consumers.

You have to find a way to remove the arbitrage possibility. Most of the time it would be intermarket arbitrage, or personal arbitrage. Intra personal personal arbitrage is that I claim to be low type and then I pay the low price, but I am really high type.

Third degree price discrimination. Suppose we are single product, and the cost of production is $c(q)$. We divide the market by m groups. This is based on some exogeneous information: gender age, status. This is exogeneous information or signal.
This correlated with price elasticity or dmeand elasticity.

Your signal is correlated with their characteristics. m groups, would be m distinct demand.

We assume there is no arbitrage between group. Mean they can not sell to other group.

Also there is no discrimination within the group. You know their characteristics. You don't have any detail about consumer in each group. If you know you can split it into other groups.

Linear tarriff for each group, t_i for each group, where $T_i(q) = p_i.q$. Old and young people for example will pay different prices p_i.

We have to find price vector $\bar{p} = (p_1, , p_m)$, $\bar{q} = (D_1(p_1), D_2(p_2), , D_m(p_m))$.

Total quantity would be $q = \sum_{i=q}^{m} D_i(p_i)$. Monopolist will sell one product. Therefore there is only one cost function $c(q)$. If there is different type of product for different group, we also would have product differentiation problem.

We have to set up monopolist profit $max[\sum_{i=1}^{n} p_i.D_i(p_i) - c(\sum_{i=1}^{m} D_i(p_i)]$

This is like multiple product with independent demand, but the cost is not independent. We assume that regularity condition holds, and everything behave nicely, and second order condition holds.

First order condition would be $D_i(p_i)+p_i.D_i'(p_i) - c'(q).D_i'(p_i) = 0$

$\frac{p_i - c'(q)}{p_i} = -\frac{D_i(p_i)}{D_i(p_i).p_i} = \frac{1}{\epsilon_i}$

$\epsilon_i = -\frac{D_i'(p_i).p_i)}{D_i(p_i)}$

Group with smaller elasticity have higher markup, and pay more. Rich people pay more. They don't care about price. Addiction, and less sensitivity is intuition. Means these consumers have less alternative and they can not avoid the prices. Consumers in third degree price elasticity excercise with less elasticity play more.

Higher elasticity means you have more alternative, and they pay lower price.

With linear demand the total quantity does not change. $\Delta q_i = q_i - q^m$ then some group consume more and some groups consume less. As a result for high elasticity group it will have one direction and for low elasticity it would go another direction. With linear demand curve we will have $\sum \delta q_i = 0$ means there would be no welfare change.

We did not made an ssumption about what actual demand will look like. An increase in total quantity would be necessary, but not sufficient. Even if we have increase it is no gaurantee that we will see an improvement.

What there is intermediate good and final goods the structure would change, and here all discussion were about final prices.

From first degree , you had i consumers, and in third group you had groups. In the first you recieve signal about each consumer, but in the third you do not know anything about individual within group, you recieve signals about each of the groups only. Second degree you know there are group, and the structure of demand for each group, but you don't know who is who. Also first degree can move to second if you know the demand distribution, yet don't know who is who you will move to the second degree.
second degree there is private information. First

degree are direct price discrimination, and second and third are indirect.

We will compare third degree with uniform pricing. First degree in terms of welfare improves, unless all are identical.

What we know is that the smallest inverse dmeand elasticity $min_i \frac{1}{\epsilon_i} = \leq \frac{\bar{p}-c}{\bar{p}} \leq max \frac{1}{\epsilon_i}$. This means all the consumers will buy. For third degree linear demand will not change total quantity, assuming that everybody is willing to buy. If one group is not buying then total quantity changes, since you increase the quantity by introducing one group to the market.

This does not mean that everybody buys, but we assume that everybody buys under uniform pricing.

John robinson (193?) denoted that strong markets are the ones that pay more under the third degree price discrimination, than under uniform pricing, and there is at least one group that pays less, called weak markets, than under uniform pricing.

We give one group more social surplus, and the other group pays less. Some changes in monopolist pricing, and some change in consumer suprplus. It depends on the value and distribution.

Constant marginal cost: $MC = C(\sum q_i) = c.(\sum q_i)$

To get whole consumer suprlus we add up to have $CS = \sum_{i=1}^{m} S_i(p_i)$, and $\pi = \sum_{i=1}^{m}(p_i - c)D_i(p_i) = \sum(p_i - c)q_i$. Constant marginal cost we had here.

Under unifor price we have $\bar{c}s = \sum S_i(\bar{p})$

Profit would be $\bar{\pi} = \sum(\bar{p} - c)\bar{q}_i$

$\Delta q_i = q_i - \bar{q}_i$

The change in welfare would be $\bar{W} = S_i(p_i) + \sum(p_i - c)q_i - [\sum S_i(\bar{p}) + \sum(\bar{p} - c)\bar{q}_i]$

We try to say something about the shape of the function: $S_i(p) = \int_0^{D_i(p)} [p_i(q) - p]dq$

The distance between cost curve and the demand curve would be consumer surplus. The cost for consumer is price.

We apply the Libnitz rule: $V = \int_{a(x)}^{b(x)} f(x,t)dt$ then $\frac{\partial V}{\partial x} = int_{a(x)}^{b(x)} \frac{\partial f(x,t)}{\partial x} dt - \frac{\partial a(x)}{\partial x} f(a(x),t) + \frac{\partial b(x)}{\partial x} f(b(x),t)$

$\frac{\partial s_i(p)}{\partial p} = \frac{\partial D_i(p)}{\partial p}[p_i(D_i(p)) - p] - \int_0^{D_i(p)} dp = -D_i(p)$

$S_i'(p) = -D_i(p) < 0$

$S_i'(p) = -D_i'(p) > 0$

Figures for this are in the book.

We know how the slopes are. $\frac{S_i(\bar{p}) - S_i(p_i)}{\bar{p} - p_i} \geq S_i'(\bar{p})$

$S_i(p_i) - S_i(\bar{p}) \geq S_i'(\bar{p})[p_i - \bar{p}]$

$S_i(p_i) - S_i(\bar{p}) \geq -D_i(\bar{p})[p_i - \bar{p}]$

$S_i(p_i) - S_i(\bar{p}) \geq -\bar{q}_i(p_i - \bar{p})$

$S_i(p_i) - S_i(\bar{p}) - \bar{q}_i.\bar{p} \geq -\bar{q}_i.p_i$

$S_i(p_i) - S_i(\bar{p}) + q_i.p_i - \bar{q}_i.\bar{p} \geq (q_i - \bar{q}_i)p_i$

$S_i(p_i) - S_i(\bar{p}) +$
$S_i(p_i) - S_i(\bar{p}) + q_i(p_i - c) - \bar{q}_i(\bar{p} - c) \geq \Delta q_i(p_i - c)$

If this inequality holds for all i, it will also hold for sum of them.

$\sum S_i(p_i) - \sum s_i(\bar{p}) + \sum q_i(p_i - c) - \sum \bar{q}_i(\bar{p} - c) \geq \sum \Delta q_i(p_i - c)$

$\Delta w \geq \sum \Delta q_i(p_i - c)$ Lower bound.

$\Delta w \leq (\bar{p} - c). \sum \Delta q_i$ Upper bound.

Two part tarriffs

Instead of offering the fixed quanity or the price, the manufacturer will define price and quanity slabs. There is personal arbitrage, since you can not identify who the person is. Different menue to different people will be offered. As a result there is self selection problem. The manufacturer is interested to design the incentive. We have to design incentive compatible constraints.

Potential to continium of contracts quanity and quality will be in the form of $\{T, q\}$, $T(q) = A + p.q$. $A > 0$.

A is not consumer suplus since we don't know who you are, it is fixed fee, e.g. just a licence fee.

The total tarriff is decreasing. For consumers utility would be in the form of :

$u = \theta v(q) - T$ if the consumer accepts it.

and zero if the consumer does not buy.

We assume that $v(0) = 0$, and $v'(.) > 0, v''(.) < 0$. θ represents consumer valuation of quality. θ_1 is probability λ, and θ_2 with probability $1-\lambda$. $\theta_2 > \theta_1$. $\frac{1}{\theta} = \frac{\lambda}{\theta_1} + \frac{1-\lambda}{\theta_2}$

$v(q) = \frac{1-(1-q)^2}{2}$

$v(q) = 1 - q$

We want to derive the mand function for the consumer:

Consumer will $max[\theta_i.v(q) - [A + pq]]$ so that $u(q) \geq 0$

$\theta_i v'(q) - p = 0$

$\theta_i(1 - q) = p = p(q)$ inverse demand function.

$D_i(p) = q_i - 1 - \frac{p}{\theta_i}$

$U(D_i(p)) = S_i(p) = \theta_i.v(D_i(p)) - pD_i(p)$

$S_i(p) = \frac{(\theta_i - p)^2}{2\theta_i}$

v would be valuation, and θ would be marginal valuation.

$s_1(p) < s_2(p)$

Always know that there is some type two that will have higher surplus if sees this price.

For monopolist to choose optimal price we have to determine aggregate demand.

There is $D(p) = \lambda D_1(p) + (1-\lambda)D_2(p) = 1 - \frac{p}{\theta}$

If perfect discrimination (First order Discrimination) would be possible. $p^{FD} = c, A_i = S_i(c) = S_i(P^{FD})$

Problem of personal arbitrage or full arbitrage is that you can not target one price to one specific group. What is the optimal pricing to the monopolist. $mx_p(p - c)D(p)$ then the price of monopolist should be $p^m = \frac{c+\theta}{2}$

$\pi^m_R = \frac{(\theta - c)^2}{4\theta}$

$S_1(p^m) \geq 0$ and $\theta_1 \geq p^m = \frac{c+\theta_2}{2}$

$\pi^m_R \geq \pi^m_2$

Two part tarrif:

I can not identify consumers with fixed fee, and I can only offer one fixed fee. There is some price fee, and monopolist should find it. What is the highest level of fixed fee?

$A \leq S_1(p)$

$A = S_1(p)$

The monopolist profit would be $max_p[(\lambda S_1(p) + (1-\lambda)S_1(p) = S_1(p)) + (p - c)D(p)]$

To maximize you must get first order condition:

$p^{tp} =$

The first comparison is profit comparison:

$\pi^{FD} \geq \pi^{TP} \geq \pi^{m}$

We allo the monopolist to choose the price and fixed fee in two part tarrifs, so we are back to uniform pricing.

$p^{FD} = c < p^{TP} < p^{m}$

age 146 you can read the intuition behanid this.

FOC m: $D(p) + (p - c)D'(p) = 0$

FOC p: $D(p) + (p - c)D'(p) = D_1(p) > 0$

We know that $S_1'(p) = -D_1(p)$

$S'1(p) = -D_1(p)$

Price in two part tarriff is strictly less than monopoly pricing.

$D(p) - D_1(p) + (p - c)D'(p) = 0$

$\lambda D_1(p) + (1 -)D_2(p) - D_1(P) = (1 - \lambda)[D_2(p) - D_1(p)]$

We have something positive plus $(p - c)D'(p) = 0$, and this means $(p - c)$ should be greater than zero since $D'(p) < 0$.

Two part tarriff is not optimal for monopolist, and non-linear is much better.

For non-linear prices. I charge you much much more for larger quantity than smaller quantity.

We have two values, we offer one menue of contract to first type, and one menue to the second.

$\{T_1, q_1\}$, $\{T_2, q_2\}$

$\theta_1, \lambda\ , \theta_2, (1 - \lambda)$

$\pi^{m} = \lambda[T_1 - c.q_1] + (1 - \lambda)[T_2 - c.q_2]$

You can select from the menues.

We have two rationality constraints. We design the contract so that we make sure that group 1 will not go for group 2 contract.

$\theta_1 v(q_1) - T_1 \geq \theta_1 v(q_2) - T_2$ (IC1)

If this holds true the contracts are incentive compatible. The same should hold for group 2. Compatibility constraint should also hold for group 2as following:

$\theta_2 v(q_2) - T_2 \geq \theta - 2v(q) - T_1$ (IC2)

We assume that individual rationality also holds:

$\theta_1 v(q_1) - T_1 \geq 0$ (IR1)

$\theta_2 v(1_2) - T_2 \geq 0$ (IR2)

if IC2, IR1 $\rightarrow$ IR2 not binding

if IC1, IR1 $\rightarrow$ IC2 not binding

$\theta_2 v(q_2) - T_2 \geq \theta_2 v(q_1) - T_1 > \theta_1 v(q_1) - T_1 \geq 0$

$\theta_2 v(q_2) - T_2 > 0$ IR2 not binding

IR1

$\theta_1 v(q_1) = T_1$

$\theta_1 v(q_1) - T_1 = 0 \geq \theta_1 v(q_2) - T_2$

$T_2 > \theta_1 v(q_2)$

Conclusions:

1. The low demand consumers, low types, will go home with zero surplus.

high types will go with positive surplus (information effect).

They know they are good, and this helps them to go home with good contract.

2. Relevant IC prevents high demand from buying low demand.

You have to offer the high demand guys sufficient contract, so that they do not go for the low demand.

3. There is efficiency at the top: High demand by socially efficient quantity.

Low demand guy pay cost more than marginal cost.

Read chapter on vertical integration: chapter 4 that we will start next week. Problem set 1 is due next week, and next one will be posted next week, and you have 2 weeks for.

IO @ UTD: Fifth session

Meisam Hejazinia

02/12/2013

New problem set is posted for price discrimination that we will finish today, and the vertical integration that we will discuss.

There is guide to say how to solve. Non linear optimal pricing or contract, and assumption of downward sloping demand curve is also there.

We looked at two part pricing, and said that two part pricing is good since it allows monopolist to extract more welfare. It is still not optimal.

$A = S_1(p) < S_2(p)$

First type of consumersurplus. Second type consumer go home with something positive.

Tarriff is function of quantity $T(q) = A + pq$ for linear tarrif. For non linear tarrif we do not have this anymore. You have n group, but you can not identify them.

In second degree prevention of arbitrage not possible. You know that this consumer type are there, and you have to offer a contract to them so that they choose based on fit.

You extract everything from low type, then high type are happy since they go home with positive suprlus.

$(T_1, q_1), (T_2, q_2), (T_3, q_3), (T_4, q_4)$, you offer different set of contracts and let them self select for non linear pricing.

High value consumers pick the one targetted for them, and Low type also select one targetted to them. High valuation buy one that targetted to them.

This was part of incentive compatibility constraint.

$\theta = \theta_1, \theta_2$ are valuation of consumers either high value or low value. $\theta_2 > \theta_1$. share are per following:

θ_2 is $1 - \lambda$

θ_1 is λ

(T_1, q_1)

(T_2, q_2)

The monopolist has interest and they make sure everybody buys.

There is share that consumer one purchases.

Maximize over bundels that stated

$\pi^m = \lambda[T_1 - cq_1] + (1 - \lambda)[T_2 - c.q_2]$

Maximization problem comes with constraing, incentive compatibility, and rationality constraint.

$\theta_1 V(q_1) - T_1 > 0$ Individual Rationality 1
$\theta_2 V(q_2) - T_2 \geq 0$ Individual Rationality 2 (IR2)

$\theta_1 V(q_1) - T_1 \geq \theta_1 V(q_2) - T_2$ Incentive compatibility 1

$\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1$ Incentive compatibility 2 (IC2)

1) IR1 & IC2 $\rightarrow$ IR2 is not binding
$\rightarrow IC1$ is not binding.

$\theta_2 V(q_2) - T_2 = \theta_2 V(q_1) - T_1 > \theta v(q_1) - T_1 = 0$
$\rightarrow IC2 > IR1$

$\rightarrow \theta_2 V(q_2) - T_2 > 0$

2) $IR_1 \rightarrow \theta v(q_1) = T1$

$\theta_1 v(q_1) - T_1) = 0 > \theta_1 V(q_2) - T_2$

$T_2 > \theta_1 V(q_2)$

$\theta_2 V(q_2) - T_2 = \theta_2 V(q_1) = T_1$

$\theta_2 V(q_2) - T_2 = \theta_2 V(Q_1) - \theta_1 V(q_1)$

$\theta_2 V(q_2) - T_2 = (\theta_2 - \theta_1) V(q_1)$

$\theta_2 V(q_2) - [\theta_2 - \theta_1] v(q_1) = T_2 > \theta_1 v(q_2)$

$\theta_2 v(q_2) - \theta_1 v(q_2) - (\theta_2 - \theta_1) v(q_1) > 0$

$(\theta_2 - \theta_1)(v(q_2) - v(q_1)) > 0$

$\theta_2 > \theta_1 > 0$ if $q_2 > q_1$

IR1 binding: $T_1 = \theta_1 v(q_1)$

IC2 is binding: $T_2 = \theta_2 V(q_2) - (\theta_2 -_1) V(q_1)$
Monopolist will choose $\pi_m(T_1, T_2)$

$max_{q_1,q_2} \pi^m = \lambda[\theta_1 . v(q_1) - cq_1] + (1-\lambda)[\theta_2 v(q_2) - (\theta_2 - \theta_1) v(q_1) - cq_2]$

We assumed IC1 is binding by not adding it to this problem.

FOC 1 : $\lambda \theta_1 V(q_1) - \lambda c - (1-\lambda)(\theta_2 - \theta_1) V(q_1) = 0$
$\rightarrow \theta_1 V'(q_1) = \frac{c}{1 - \frac{1-\lambda}{\lambda} \frac{\theta_2 - \theta_1}{\theta_1}}$

FOC 2.: $(-\lambda)[\theta_2 V(q_2) - c] = 0$
$\theta_2 V(q_2) = c$

There is optimal time condition by consumer.

It is socially efficient or optimal by type two consumer.

Socially optimal q_2

What do we know about denominator of q_1 is lower than one, so $\theta_1 V'(q_1) > c$

$V' > 0$ yet, $V'' < 0$, since q_1 is small, this ends up to be quality. $\rightarrow q_1 < q_1*$, means distortion for the low types, and efficiency on the high type.

We were not able to extract all the benefit from high type.

$q_2^* > q_q^*$

q_2^* such that $\theta_2 V(q_2^*) = c$

q_1^* such that $\theta_1 V(1_2^*) = c$

θ identified consumer types.

Two incentive compatibiltiy constraint, and two individual rationality constraints. The high demand consumers go home with strictly positive surplus, due to information rent.

Incentive compatibility binding does not allow incentive compatibility to be binding.

$\theta \in [\theta_l, \bar{\theta}]$

$f(\theta)$ density

$F(\theta)$ cdf

nonlinear tarriff $T(q)$

consumers θ buys $q(\theta)$
$T(q(\theta))$

We assume that $q(\theta) \nearrow \theta$
$T(.) \nearrow \theta$

max $\int_{\theta_l}^{\bar{\theta}}$[T(q($\theta$)) $- c.q(\theta)]f(\theta)d\theta$

so that (IR), (IC)

(IR) $\theta V(q(\theta)) - T(q(\theta)) \geq 0\theta$

$\theta_l : \theta_l v(q(\theta_l)) - T(q(\theta_l)) = 0$

(IC): $\theta = \tilde{\theta}$

$\theta v(q(t)) - T(q(\theta)) \geq \theta v(q(\tilde{\theta})) - T(q(\tilde{\theta}))$ for $\theta_1, \tilde{}$

Lowest type: $\tilde{\theta} = \theta - d\theta$

$\frac{\theta V(q(\theta)) - T(q(\theta))}{d} - \theta \frac{v(q(\theta - d\theta)) - T(q(\theta - d\theta)))}{d} \geq 0$

$lim_{d \to 0} \theta [\frac{V(q(\theta)) - V(q(\theta - d\theta))}{d} - lim_{d \to 0} \frac{T(q(\theta)) - T(q(\theta - d\theta))}{d} \geq 0$

$\theta . V'(q(\theta)) - T'(q(\theta)) \geq 0$

$\tilde{\theta} = \theta + d\theta$

$0 \geq \theta v'(q(\theta)) - T'(q(\theta))$

$\theta v'(q(\theta)) - T'(q(t)) =)$ for θ IC.

Means that imitation would not be possible.

Merlis 1971

If monopolist were to maximize their utility by choosing the type that they mimic $max_{\tilde{\theta}} \theta V(q(\tilde{\theta})) - T(q(\tilde{\theta}))$ would be suprlus.

It is optimal for me that I don't deviate and the θ that I select would be my own.

$U(\theta) = \theta V(q(\theta)) - T(q(\theta)) =$ the one above means the maximized $\tilde{\theta}$

Monopolist wants each to purchase based on it's type mean if it is θ_i we want him to select q_i, T_i. By mimicing I would choose the quantity that is not meant to be to me. By incentive compatibility we will try to make self selection to be truth revealing.

$\frac{dU(\theta)}{d\theta} = U'(\theta) = V(q(\theta)) + [\theta \frac{dv(.)}{dq} - \frac{dT(.)}{dq}] \frac{dq(\theta)}{d\theta}$

$[\theta \frac{dv(.)}{dq} - \frac{dT(.)}{dq}] = 0$

$\to U'(\theta) = u(q(\theta))$

$U(\theta) = \theta V(q(\theta)) - T(q(\theta))$

$T(q(\theta)) = \theta V(q(t)) - U(\theta)$

$\pi = \int_{\theta_l^{\bar{\theta}} [T(q(\theta)) - c(q(theta))] d\theta}$

$U(\theta) = \int_{\theta_l}^{\theta} V(q(u))du + U(\theta_l)$

By individuality rationality constraint $U(\theta_l) = 0$

So: $= \int_{\theta_l}^{\bar{\theta}} V(q(u))du$

FOC $_\theta \theta v'(q'(\theta)) = c + \frac{1-F(\theta)}{f(\theta)} . V'(q(\theta))$

For all theta, other than the highest type, we have marginal cost on the other side. Quantity on the other part of tarriff is not socially optimal.

I don't know who you are, but I have to offer you a contract. Best quantity or quality would be derieved by something like this.

Chapter 4

Go through book, and make sure you understand things in the book.

Plug function in and go through them by yourself.

Go through those, and try to change things to find out how things would change.

Upstream monopolist

Intermediary product : Retailer

Retailer sells to consumer.

We have upstream market and then we have downstream market.

Upstream is between manufacturing and retailer

Downstream is between retailer and consumers.

Verticaly integration means, the upstream firm or downstream firm integrates.

If upstream firm has control directly or indirectly through the whole chane.

This could be by deligation or by contract.

The aggregate profit of the upstream and downstream is the profit.

Whatever retailer does will be internalized by the manufacturer, and vice versa if they are integrated.

Production $MC = c$.

Whole sales price p_w

Retailer sells good to downstream, and the quantity sold is whatever demand function is $q = D(pr)$

The demand from manufacturer is $D(pr)$

The quantity demanded in the upstream market would be whatever it is needed in the downstream.

There are number of different contract and technologies. One is linear pricing.

$T(p_w) = p_w.q$

Two part tarriff would be $T(.) = A + p_w.q$

There would be contractually retail price controls/ retail price maintenance.

Firms could fix quality/quantity

There would be some contractual solutions.

Whether or not feasible depend on the environment. You might not know enough about retailer to fix quantity or quality.

The vertical constraint also depends on what is feasible.

You might select territory to allow them to have profit in downstream, and extract as much as possible.

Another way is tie in supplier. If they would need many input factors and other firms provide those, it would be constraint.

You get good from me, and then I would be your exclusive contractor.

Decision variables: whole sales price, then francise fee A, another is quantity purchased by retailer (another decision variable), consumer price, retail location are all decision variables.

Instruments, targets, control problem, and sufficient examples.

Instrument: something the retailer can use, try to implement vertically integrated market.

Target directly affect aggregate problem.

Retail quantity, amount of service is a target.

Whatever retailer does affects the profit.

Any of these instruments is taken from retailer and given to manufac.

How to use instrument to reach desired value of target.

Sufficient instrument is the one that maximizes integrated profit.

Retailer monopolist, manufacturer is monopolist.

The benchmark is vertically integrated profit, profit of entire structure.

Profit for upstream is: $c.q + T - T + q.p$
T what recieves from the retailer.

Total revenue of vertically integrated would be :

$\pi = p.q - c.q = (p - c)D(p)$

If we want to maximize the entire profit we have to mazimize this.

$p^m = argmax_p(p - c)D(p)$

$q^m = D(p^m)$

The upstream monopolist does not have full control on retailer.

What if it is decentralized, and each make their own decision?

First manufacturer and then retailer, using backward induction:

Retailer has to set price $p_r = argmax(p - p_m)D(p)$

FOC: $D'(p)(p - p_w) + D(p) = 0$

Price is not decreasing in marginal cost. The higher marginal cost the higher marginal cost.

If $c < p_w$, the whole sales price larger than the actual cost of producing. $p_r > p^m$ and manufacturer does not want it.

The target is not sufficient, the target is not equal to vertical integration price.

$D(p_r) = D(p_r(p_w)) = D(p_w)$

The monopolist will $p_w = argmax_{p_w} = (p_w - c)D(p_w) \rightarrow p_w > c \rightarrow$ first marginalisation

$p_r > p_w \rightarrow$ second marginalisation

Problem of marginalisation. The downstream starts with markup.

The first marinalization enforces second marginalisation.

$D(p) = 1 - p$

$p^r = argmax_p(p - p_w)(w - p)$
$p_r = \frac{1+p_2}{2}$

$q^r = \frac{1-p^r}{2}$

$\pi = (\frac{1-p_w}{2})^2$

$p_w = argmax_p(p - c)(\frac{1-p}{2})$

$p_w = \frac{1+c}{2}$

$\pi^m = \frac{1-c}{2}^2$

$\pi^r + \pi^m < \pi^I = \frac{(1-c)^2}{4}$

non integrated profit:

$\pi^{ni} = \frac{3}{16}(1 - c)^2$ which is lower than the integrated one

If upstream is monopolist and downstream firm is competitor then $p_r = p_w$

p_w is marginal cost in downstream market.

In this case monopolist $max_p(p_w - c).D(p_w)$

$p_w = p_m$ means equal to benchamrk price

There is no markup in the downstream, and we are still not in deadweight loss. We maximize decentralize profit.

Vertical integration does not increase profit.

Upstream is perfect competitor, and downstream competitor.

Then we will have $p_w = c$

$p_r = argmax(p - c)D(p)$

First order condition for the monopolist:

For manufacturer: $D'(p)(p - c) + D(p_w) = 0$

For retailer FOC:

$D'(p)(p - p_w) + D(p) = 0$
Marginalization has an effect on monopolist. The monopolist will make lower profit.

Horizontal externality external 4.2 in the book.

Briefly we will talk about this set up.

Two firms each produce one good. Goods are perfect complement. One firms sells to the downstream market.

One decision maker, what happens if both firms sell the complementary product when there is only one customer in the market. Combine profits from two, and you will see decision variable.

Horizontal structure, but firm 1 makes decision first, and firm 2 makes his decision based on the price firm 1 decides. This could be solved by backward induction.

Two firms choose prices simultaneously. Show that the price is equal to some of these experessions. In this excercise you will have sequential decision making.

This was very basic, now lets talk about vertical instruments.

Retailer sets price greater than marginal cost p_w. Manufacturer also does this, so would be double marginalization. Target is price and we want to implement, and charge this price $p^m = argmax_p(p - c)D(p)$

First one is franchise fee.

The two part tarrif would be $T(q) = A + p_w.q$

$p_w = c$

$\rightarrow p_r = p^m$

$\pi^r = \pi^m - A$

A $= \pi^m$ Franchise fee, which would be equal to consumer surplus.

Manufacturer profits are $\pi_w = A + (p_w - c)D(p_2)$

$A = \pi^m$, so every one would be happy.

Retailer gets everything, and is claimer of what is left.

Another draw back is if there is private information, we may not know what A should be. Two part tarriff could be used as screening price.

$p_r = p_w$, if you have many retailer, the will charge whole sale price.

Manufacturer could require retailer to charge the target price.

$p^T = p^m$

How can we use retailer price maintenance, and whole sales pricing that include all vertically integrated profit.

We could have $p_w = p_m$ for the input. Our manufacturer charges target price. Then manufacturer require retailer to charge $p_r = MC_r = p_w = p_m$

That implements target price, and it insures that manufacturer has all profit, and retailer walks home with nothing.

This is vertical integration by contractual control, and not by ownership.

Retailer exerts some promotional effort.

Manufacturer has incentive not just price but service. Service is not contractable. Either because it is not observable, or costly to control. (customer service, promotion, and advertising are sample of these services).

Retailer don't want to, since they don't acquire the service and should send to manufacturer.

This is model of vertical integration. Higher service results in higher quality of product.

$q = D(p, s)$

$q \searrow p, q \nearrow s$

Cost of service is $\Phi(s) \nearrow s$

$Phi(s)$ per unit of the good

$max_p(p - c - \Phi(s))D(p_s)s)$

$\pi^m = (p^m - c - \Phi(s^m))D(p^m, s^m)$

FOC with respect to s would be $(p - c - \Phi(s)).\frac{2D(p,s)}{s} - \Phi(s) - D(q, s) = 0$

with p: $(p - c - \Phi(s))\frac{2D(p,s)}{2p} + D(p, s) = c$

Manufacturer charges the price:

Manufacturer:

FOC: $D(p_w) + (p - c)D'(p, s) = 0$

Retailer's: $max_{p,s}(p - p_w - \Phi(s))D(p, w)$

FOC, p $D(p, s) + (p - p_w - \Phi(s))\frac{\partial D(p,s)}{\partial p} = 0$

FOC, s $(p - p_w - \Phi(s))\frac{\partial(p,s)}{\partial s} - \Phi'(s).D(p, s)$

The retailer of service would be lower than benchmark case. There are so two distortion. Second one would be double marginalization.

IO @ UTD: Sixth session

Meisam Hejazinia

02/19/2013

The instruments that monopolist can use.

Sufficient vertial restraints.

Downstream moral hazard.

Franchise fee as an instrument.

$T(q) = A + p_w.q$

For franchise fee to work, the proposed instrument is as stated. Monopolist will charge $p_w = c$

Double marginalization is problem since does not allow the vertically integrated profit.

The retailer first order condition are now

First order condition for the price.

$D(p,s) + (p - c - \Phi(s))\frac{D(p,s)}{\partial p} = 0$

c is marginal cost for the retailer.

First order condition for the service:

$-\Phi'(s)D(p,s) + (p - c - \Phi(s)\frac{\partial D(p,s)}{\partial s} = 0$

The retailer has efficient incentive to provide the service.

p^m is the price retailer pays for the input.

p^r would be the price that retailer charges.

Monopolist wants the retailer to charge the price that maximizes the vertically integrated means $p^r = p^m$.

$s = s^m$

The retailer profit $\pi^m = p^{in} - A$

For manufacturer we have $\pi^{in} = A$

Retailer can not set p^r, but we wanted to still provide efficient level of service. We knew that the price that retailer will charge is p^{in}

The input price would be p^w in this case.

RPM

Retailer price maintenance is that retailer sticks to that price.

$\bar{p}_r = p_{in}$

The monopolist makes profit by charging whole sales price $p^m > c$

Monopolist wants to engage in markup pricing.

Retailer will pay something more mean $p^w > 0$ and retail price maintenance would not be proper, since retailer would not provide enough level of service.

Quantitative forcing

sufficient vertical restraint ex. 4.3.

The moral hazard problem is that the retailer provides service, which is not observable by the manufacturer, and manufacturer want to extract maximum profit.

There is double side mean there is state when manufacturer also wants to provide the service. In this case in excercise 4.4 $D(p, S, \sigma)$

σ could be advertising expenditure of mcdonalds.

Demand function is increasing in both σ and s

It is called, bilateral Moral hazard.

We have upstream manufacturer who produced two input factors. Upstream manufactuerer produces two goods x and $\tilde{x}$, and there is second manufacturer produced substitute good in the production process $\tilde{x}$.

The retailer in downstream needs x or $\tilde{x}$ for output process. The output function would be $f(x, \tilde{x})$. You can used more of x, and less of $\tilde{x}$, and this is called substitute in the production process.

$f(0, \tilde{x}) = 0$ and $f(x, 0) = 0$

Also the marginal cost for the first manufacturer is c and for the second one is $\tilde{c}$.

The first intermediate good has marginal cost of c and if it is producing $\tilde{x}$ then the marginal cost would be $\tilde{c}$.

The retailer set price p, and the produce output level of $f(x, \tilde{x})$

Inverse demand would be $p(.) = D^{-1}(.)$ resulting from the demand of $q = D(p)$. Then the price in the donstream market would be $p(.) = D^{-1}(f(x, \tilde{x}))$

What would be the benchmark?

The joint profit of manufacture one and the retailer. For vertically integrated is manufacture one and whatever retailer produces. We are interested in the optimal input factor. If the optimal input factor would be there the incentive would be to charge p.

$(x, \tilde{x}) = argmax_{x,\tilde{x}}[p(f(x, q))f(x, \tilde{x}) - cx - \tilde{c}\tilde{x}]$

First order condition on x would be $f'_x[p'(x, \tilde{x}) + p] = c$ marginal benefit factor would be equal to the the marginal cost, and for the second one we will have $f'_{\tilde{x}}[p'.f(x, \tilde{x}) + p] = \tilde{c}$

Marginal rate of technical substitution would be $MRTS = -\frac{c}{\tilde{c}}$

$|MRTS| = \frac{MP_x}{MP_{\tilde{x}}} = \frac{f'_x}{f'_{\tilde{x}}} = \frac{c}{\tilde{x}}$

p_w price for input factor x

$p_w > c$

actual price ration $= \frac{p_w}{\tilde{p}_w} = \frac{p_w}{\tilde{c}} > \frac{c}{\tilde{c}}$

Since there would be competition for the second factor, the price would be equal to marginal cost, but for the first input factor we have monopolist, so p_w would be greater than its cost.

We want the retailer to use xx^{in}, and $\tilde{x} \to \tilde{x}^{in}$

$p_w = c$

$\tilde{p}_w = \tilde{c}$

A franchise fee that extracts profit.

Frachise fee is sufficient vertical restraints.

The second one is tie in with resale price maintencance

The downstream unit, purchases both of the factors. Monopolist of first factor, says if you are not going to purchase second factor, I will not give you first input factor.

The upstream manufacturer should decide p_w and p_w.

The upstream manufacturer will select $\frac{p_w}{\tilde{p}_w} = \frac{c}{\tilde{c}}$ to force the retailer to manufacture the amount required $x = x^{in}$, and $\tilde{x} = \tilde{x}^{in}$

The retailer has to charge p^{in}, and then upstream monopolist will go home with zero profit.

We use retail price maintenance, to make sure retailer charges this price, so that monopolist extracts profit.

Tie in would be in the form of exclusive contract.

The next section is interbrand compeition that we will not consider here.

If you don't use instrument, you will have double marginalization problem.

Through franchise fee you solve this problem.

The tarriff would be in the form of three part tarriff $T(x, \tilde{x}) = A + p_w.x + \tilde{p}_w.\tilde{x}$, which will give you the same condition as price maintenance.

Set the retail price may not be problem due to the anti trust law.

We leave the real of monopolist, and now we look at the strategic interaction between two firms. First wee look at the static form, one shot. In three weeks after spring break, we will look at the dynamic form.

We start with the cournot model, then bertrand competition of prices, and then we look at the capacity constraints. The problem with bertrand is that we have imperfect competition. Both firm will earn zero profit in Bertrand. Bertrand paradox in Nash equlibrium will tell us that this is not case. Once product are differentiated, both firms will earn positive benefits. The third solution is two stage problem. The first is what is the production capacity, and second they decide about the price. This is on the day to day, firms decide to how set their prices. On the daily basis they set prices. This will back to cournot equlibrium. Firms compete in capacity and then on prices. Then we look at quantity and price leadership, and finanlly on conjecure of prices.

Cournot

Firms compete in quantities of non differentiated products (identical). Goods are perfect substitutes.

$\pi^i(q_i, q_j)$

$P(Q)$ would be demand function

$Q = q_i' + q_j$

$\pi^i(q_i, q_j) = p(q_i + q_j)q_i - c_i(q_i)$

π is concave, and we want it to be twice differentiable in q_i

We look at the best response or reaction function.

We denote by $R_i(q_j)$.

The quantity that firm i will produce would be, $q_i = R_i(q_j)$ so that $\pi_i'(R(q_j), q_j) = 0$

$\frac{\pi^i(q_i,q_j)}{\partial q_j} = 0$ $R_i(q_j)$

$R_i(q_j)$ is unique, and single valued.

Slope of the reaction function $R_i'(q_j) = \frac{\partial R_i(q_j)}{\partial q_j} = \frac{\pi^i_{ij}(R_j(q_j),q_j)}{-\pi^i_{ij}(R_i(q_j),q_j)}$

By cancavity assumption the denominator is positive.

$sgn(R_i'(q_j)) = sgn(\pi^i_{ij})$

If the cross derivative is negative, this means the marginal profit for firm i is decreasing the more j

produces, and this is defintion of strategic substitute $\pi_i' j < 0$

We can draw reaction functions of each quantity based on other quantity, and when two reaction functions intersect, we would have equlibrium.

Quantity competition is not always strategic substitute, but most of the time.

Nash equlibrium $q_i = R_i(q_j)$ and $q_j = R_j(q_i)$, and we want these two to be mutual best responses. $q_i^* = R_i(R_j(q_i^*))$ and $q_j^* = R_j(R_i(q_j^*))$.

$\pi^i(q_i, q_j) = p(q_i + q_j)q_i - c_i(q_i)$

$\pi_o^i(q_i, q_j) = q^i \frac{dP}{dQ} \frac{\partial Q}{\partial q_i} + p(q_i + q_j) - C_i'(q_j) = 0$

$p(q_i + q_j) - c'(q_j) + q_i.p'(q_i + q_j)$
Where $p(q_i + q_j) - c'(q_j)$ is profitability of extra unit.

$q_i.p'(q_i + q_j)$: losess of all individual marginal units.

$p(q_i + q_j) - c_i'(q_j) + (q_i + q_j)p'(q_i + q_j)$

'social costs > 'individual costs'

The total quantity in cournot model would be greater than monopoly.

We can compare three different structure, competitive, cournot, and monopoly.

comp $q >$ Cournot $Q >$ monopoly Q

comp $p <$ cournot $P <$ monpoly p.

First order condition $p(Q) - c_i'(q_i) = -q_i.p'(Q)$

$-\frac{p(Q) - c_i(q_i)}{p(Q)} = -\frac{q_i.p'(Q)}{p(Q)} = -\frac{q_i}{Q}\frac{p'(Q).Q}{P(Q)}$

$= -p(Q)p'(Q).Q$

$\frac{q_i}{Q} = \alpha_i$

$L_i = \frac{\alpha_i}{\epsilon}$

Lerner index would be the following. The more firm you have α_i would be smaller.

$L_i^{courn} < L_i^{monop}$

$L_i^{courn} > L_i^{comp} = 0$

In most IO papers, people have assumed linear demand.

$P(Q) = 1 - Q = 1 - q_i - q_j$
$C_i(q_i) = c_i.q_i$

For nation equlibrium we must find (q_i^*, q_j^*).

Then we must find $\pi^i(q_i^*, q_j^*)$

$\pi^i(q_i, q_j) = q^i(1 - q_i - q_j) - c_i.q_i$

FOC: $1 - q_i - q_j - q_i - c_i = 0$

$Q_i = \frac{1 - q_j - c_i}{2} = R_i(q_j)$

The same symmetric things happen for j, by replacement. As a result:

$q_i^* = \frac{1 - 2c_i + c_j}{3}$

$q_j^* = \frac{1 - 2c_j + c_i}{3}$

$p = \frac{1 + c_i + c_j}{3}$

The profits are $\pi^i = (\frac{1 - 2c_i + c_j}{3})^2$

For linear demand we usually have this quadratic form of quantity as the profit.

What if we have Cournot with n firms? identical firms

$Q = \sum_i^n q_i$

$Q_{-i} = \sum_{j \neq i} q_j$

$\pi^i = p(Q).q_i - c_i.q_i$

Assuming different marginal costs. Assuming identical we can get rid of i here.

$\pi^i = p(Q) + q_i.p'(Q) = c$

We assume linear demand so:

$\pi^i = 1 - Q - q_i = c$

$= 1 - Q_i - q_i - q_i = c$

Therefore, best response function, given what everybody else produces would be:

$q_i = \frac{1-Q_{-i}-c}{2} = R_i(Q_i)$

Symmetric equlibrium, so would be:

$q_i = qi$

$Q_{-i} = (n-1)q$

$\Rightarrow = \frac{1-(n-1)q-c}{2} = q$

$q^* = \frac{1-c}{n+1}$

The equlibrium price would be $p^* = 1 - nq^* = c + \frac{1-c}{n+1} > c$

$\pi^* = (\frac{1-c}{n+1})^2$

We have lot of firms, then $lim_{n\to\infty} q^* = 0$ and $lim_{n\to\infty} p^* = c$, and $lim_{n\to\infty} \pi^* = 0$

In long run, each firm will produce at minimum efficient scale.

We skip the existance of equilibria, and uniqueness.

The reaction function was downward sloping, and here we had strategic substitute.

Inverse demand function give us strategic complements.

1. $D(Q) = \frac{a}{Q}^{\frac{1}{\epsilon}} = \frac{a}{q_i+q_j}^{\frac{1}{\epsilon}}$

We take the first derivative, and we get $\frac{1}{q_i+q_j}^{\frac{1}{\epsilon}}(1 - \frac{1}{\epsilon}\frac{q_1}{q_1+q_2}) = \frac{c}{1\epsilon}$

We take the cross derivative:

$\frac{\partial^2 \pi^i}{q_1 \partial q_2} = (\frac{\alpha}{q_1+q_2})^{\frac{1}{\epsilon}} \frac{q_1 - \epsilon q_2}{\epsilon^2 (q_1+q_2)^2}$

There would be two sectors cutting each of the axis, and their center on each axis, and their intersection will give us the equlibrium.

ϵ is constant elasticity.

Stackleberg equlibrium (1934)

$P(Q) = 100e^{1/10)\sqrt{Q}}$

$R_1(q_2) = 200 + 20\sqrt{100 + q_2}$

$R_1(q_2) \nearrow q_2$

U shape Average cost curve.

Convergence example.

$-\frac{c(q)}{q}$

Minimum efficient scale at 1.

All firm produce at minimum efficient scale which we assume 1.

Marginal cost at $q = 1$ is c.

The minimum efficient scale is 1 and $MC = c$.

$p = D(Q)$

We allow them to enter at minimum efficient scale α.

Then we reduce α. All of them who enter should produce α, and as we decrease it, the number of firms will go to infinity, and this is convergence.

$c_\alpha(q) = \alpha.c(\frac{q}{\alpha})$. for this minimum efficient scale would be α.

We minimize $min_q \frac{c_\alpha(q)}{q} = min \frac{c(q/\alpha)}{q/\alpha}$

$\frac{q}{\alpha} = 1$

$q = \alpha$

$Mc = c$

The smaller α, more firms would be in this market.

The number of firms would be $\frac{Q}{\alpha}$

It depends on how you can squeez them, for example $\frac{9}{2}$ would be 4.

We fix α, and then we change the number of firms.

We are going to show that $Q \in [Q^* - \alpha, Q^*]$.

Q^* = perfectly competitive quantity.

$Q^* = D(c)$.

$Q > Q^*$:

$P(Q) < c \leq \frac{c_\alpha(q_i}{q_i}$, mean not greater than average cost of each of the firm.

Negative profit for operating firm.

This will not work.

$Q < Q^* - \alpha$

The firm that is not yet in the industry makes zero profit. If $Q < Q^* - \alpha$, then no firms wants to enter, since any additional firm will not get profit, otherwise there would be no equilibrium. We want to take contradiction.

$Q < Q^* - \alpha$. If this was an equilibrium, an extra firm will get the negative profit. If one firm enters $Q + \alpha < Q^* \rightarrow Q < Q^* - \alpha$. Means, one firm can enter the market, so that everyone makes positive profit.

The total amount that is produced has to be somewhere in $[Q^* - \alpha, Q^*]$.

As we take $lim_{\alpha \rightarrow 0}[Q^* - \alpha, Q^*] = Q^*$

This would be competitive equlibrium.

Raffen (1971) in review of economics and statistics:

$c(q) = c - d.q^2 + e.q^3$

$a - c > 0, d - b > 0$

$p(Q) = a - bQ$

The long run equlibrium would be where price= marginal cost = Average cost. $p = MC = AC$.

$q = c$, and $q = \frac{d}{2.e}$.

$q_c^{LR} = q = \frac{d}{2e}$

$p_c^{LR} = AC(q_c^{LR} = c - \frac{d^2}{4e}$

$Q_c^{LR} = \frac{Q-c}{b} + \frac{d^2}{4bc}$

What's the total quantity of all the other firms given what I produced.

$Q_{-i}(q_i) = \frac{Q-c+2q(d-b)-3eq^2}{b}$

We have many firms here. $\pi^i(q_i, Q_{-i}) \rightarrow \pi^i(q_i, Q_{-i}(q_i))$

$\pi^i(q_i) = 2eq_i^2(q_i - \frac{d-b}{2e})$

we want q_i so that $\pi_i(q_i) =^? 0$

$\pi^i(0) = \pi^i(\frac{d-e}{2e}) = 0$

$\frac{\partial \pi^i}{\partial q_i}_{q_i=0} > 0$

$q_{cor}^{LR} = \frac{d-e}{2e} <$
$p_{cor}^{LR} = c - \frac{d^2-b^2}{4e} > p_c^{LR}$

$Q_{cor}^{LR} = \frac{a-c}{b} + \frac{d^2-b^2}{4be} < Q_c^{LR}$

$N_{comp}^{LR} = \frac{Q_c^{LR}}{}$
$N_{cor}^{LR} = \frac{Q_{cor}^{LR}}{q_{cor}^{LR}}$

$N_{cor}^{LR} > N_{comp}^{LR}$

In this equlibrium of cournot, we have more firms, and less is produced in long term than competitive market.

In second case, we had zero profit, in long run, and everybody only produced minimum efficient scale.

The number of firms in all are indogeneous. The cost structure would be different.

Bertrand

Firms make simultaneous pricing decision.

Identical products. Everybody produces the same.

$\pi^i(p_i, p_j) = (\pi_i - c)D_i(p_i, p_j)$

Question is what is demand function: $D_i(p_i, p_j) = D(p_i)$ $p_i < p_j$

$\frac{D(p_i)}{2} = \frac{D(p_j)}{2}$ $p_i = p_j$

0 $p_i > p_j$

Nash equlibrium would be $\pi^i(p_1^*, p_j^*) \geq \pi^i(p_i, p_j^*)p_i$

if $c_1 = c_2 = c \rightarrow p_1 = p_2 = c$

Bertrand Paradox.

$c_1 < c_2$

$p_1 = c_2 - \epsilon$

$p_2 = c_2$; p_1^m if $p_1^m < c_2 - \epsilon$

$p_1^m < c_2 - \epsilon$

$\pi_1 = (c_2 - c_1)D(c_2) = (p_1^m - c_1)D(p_1^m)$ Monopoly

Look at 5.3, 5.4, and 5.5. The solutions are at the back of the book.

Three solutions for Bertrand Paradox.

1. repeated interaction

2. product differentiation (not identical products, gives market power to them, leading positive profit).

3. Capacity constraints.

Cournot and bertrand are simultaneous, with price or quantity leadership, we will have leader and follower model, and we will look at subgame equilibrium. Variation in supply function, and conjectural variation will be considered last.

IO @ UTD: Sixth session

Meisam Hejazinia

02/26/2013

Familiarize yoursef with mathematica or maple to simplify the problem, if calculation becomes cumbersome.

$D[f[x], x] = f'[x]$

You can put assumptions also there.

Example solution is online. It is with the same name.

We went over static oligopoly, and we went through cournot analysis.

Law for repeated interaction, in the market situation when we have two firms, oligopoly.

Two firm that have market power yet the result is price equal to marginal cost.

Two solution to bertrand paradox:

1. Product differentiation (imperfect substitute): a week after spring break.

2. dynamic game: in multiple periods.

Final solution is capacity constraing, and then decide on prices. The result is Bertrand, but the capacity constraints equals the equlibrium to cournot.

Kreps-Shenkman in 1983 worked over this model.

In Tirole 5.7.2 explains this.

Equlibrium in two stage game is cournot equlibrium.

Suppose we have a situation of decreasing return to scale, increasing average cost. The firm will not want to produce more.

Suppose $\bar{q}_i = S_i(p) = D(p)$ is firm supply that the firm is willing to supply at price p. Firm will face residual demand in this case. $D(p) - S_i(p)$. Sometimes we denote this by $D_j^r(p) = D(p) - S_i(p)$ residual demand of firm j.

The result is that firms have capacity constraing, so we have **firm Rationing**

$\Pi = p(q_1 + q_2).q_1$

What if $p = 1 - Q$, and $p = 1 - q_1 - q_2$. The price would be to the demand I face which is not full demand but residual demand.

Firm 2 supplies certain side of the market. Who are my customers? What is the rationing rule? Efficient rationing and proportional rationing are two types.

Efficient rationing You have consumers with lower marginal valuation and so on, and the customer who is the most eager will buy, and whoever is left firm j will face. Mean most eager consumer will buy from i, and that shifts q_j.

Proportional Rationing Rule: Probability of not having bought earlier is $(D(p) - S_i(p))/D(p)$.

The probability of being part of the residual demand is $(D(p) - S_i(p))/D(p)$.

The large part of discussion is on rationing rule.

At stage one firm choose capacity, and that capacity is fixed.

On stage 1 set the capacity, and on stage 2 set prices, and all firms do that simultaneously.

It is not first cournot then bertrand, but it is about production capacity. I choose my storage or production capacity, and once the pre stage is over, I then compete with someone else on quantity. You can do that as inventory. In cournot that was not the case, since we through in the market and let the market decide the price.

We try to solve this by backward induction.

First stage: find Nash equlibrium in pricing game.

Second stage: Nash equilibrium in capacity, giving, or anticipating price equlibrium.

First we think ahead, and then think about the capacity.

Pricing game:

Both firms have some capacity constraints. There are capacity constraints $\bar{q}_i$, $\bar{q}_j$

We assume $c = 0$, mean marginal cost equal to zero, $p" < 0$

Efficient rationing, the highest willingness to pay buys first, and the low willingness to pay will purchase then. This is efficient, with those with higher valuation to purchase, if it was with limited supply.

$p_1 = p_2 = p(\bar{q}_1 + \bar{q}_2)$

We can read the price of the demand function. Lemma 1 is that both firms select from this price.

Lemma 2: $p_i \geq p(\bar{q}_j + R_i(\bar{q}_i))$

From these two lemma $\bar{q}_i \leq R_i(\bar{q}_j)$ impure strategies.

mixed strategies: Tirole

We will have interest of little capacity. We have a join interest. If we hit the price, it would be lower than capacity. The competition for lower price would not be there. Jointly have interest to keep the capacity lower.

We look at the capacity game stages. There are cost of producing capacity. The cost for selling later on will be equal to zero $c = 0$.

Nash equlibrium is such that $\bar{q}_i = q_j^{**}$

$\bar{q}_j = q_j^{**}$

$q_i^{**} = argmax_{q_i} q_i[p(q_i + q_j^{**}) - c_o - c]$

This happens to be cournot equlibrium of quantity.
I maximize my cost margin, given your best quanity response, and vise versa.

At first stage firms will select cournot quantities as their capacities. q_i^{**}, q_j^{**} and on the second stage of pricing they will have $p_i = p_j = p(q_i^{**} + q_j^{**})$

The basic intuition is that both firms want to constraint themselves at lower capacity. In equlibrium, capacity is simultaneously non-cooperatively selected.

The outcome of two stage game boils down to one stage cournot game.

The quantity decision is long run, and price is day to day decision. Capacity could not be configured on daily basis, but price could. Cournot is simple way of representing this two stage game.

We will look at the product differentiation in more detail in one week after spring break.

For cournot model, the demand function is $p(q) = 1 - q_1 - q_2$ for homogeneious goods.

We can add one parameter for differentiation. $p(q) = 1 - q_1 - \gamma q_2$, if competitor increases his price by one unit, the price will go down by less than one unit $\gamma < 1$. This is for non homogeneous case, or product differentiation. When goods are not perfectly substitutable.

$\gamma < 0$ complement

$\gamma = 1$ homogeneous

$\gamma = 0$ monopoly

This would be differentiated cournot model.

You can write for differentiated Bertrand model in the form of $D(p) = 1 - p_1 + c.p_2$ The only difference is that we have positive sign for the bertrand model, yet we had negative sign in cournot model.

If $c > 0$ we will have substitute, and if $c < 0$ we will have complements.

Leviton (1920s), and Boley (1924)

You can have the cournot one in the form of $p(q) = \alpha - \beta q_1 - \gamma q_2$. For the Bertrand we will have $D(p) = a - b.p_1 + c.p_2$

There is inverse demand function $p_2(Q) = \alpha_2 - \beta_2.q_2 - p_2.q_1$

Q would be function of q_1, q_2, but not summ of them. $p_2(q_1, q_2) = p_2(Q)$

$D_1(p) = a - b.p_1 + c.p_2$

$D_2(p) = a + c.p_1 - b.p_2$

Imact on both are the same.

After spring break we will think about the decision to differentiate the product.

Make your life simple and use this structure rather than decision to differentiate, only with one parameter, to make your life simple.

Suply function competition

Firm compete in supply function. They do not produce certain price or quantity, but the choice variable is supply function. It was introduce first by Hert (1982), and Grossman (1981). Whey should competition be in price or quanity. How firm decide to choose in price or quanity. Given the market price I will decide how much I will produce.

Firms produce homogeneous goods. Firm do legally binding contracts, saying that if market price this, I will produce this much, if another I will produce another quantity.

The assumption is that firms are able to commit.

We have n-firm oligopoly, and we have standard downward slopping demand curve.

We have a collection of supply functions $(S_i(.), i = 1, n)$

There is a demand function and the market clears at the price that demand will be $D(p) = \sum_{i=1}^{n} S_i(p)$

This is the market clearing condition, and there would be such a price.

The firm's profit is $_i = p.S_i(p) - C(S_i(p))$

These may not necessarily be identical firms, the supply are different, but the costs are identical.

The firms are identical in term of marginal cost, but they do not supply the same quantity.

You want to play your best response to whatever everybody else does.

If marginal cost would be different, the equlibrium would not be symmetric anymore, and the model would not be tractable anymore.

Firm choose supply function simultaneously.

Given supply function we will find out the market price, and given market price we will find out equlibrium quantities.

There is multiplicity of equlibrium. I see your supply function, and given the anticipation of your supply function, I will decide my supply function.

Klemperes, eyer (1989): added uncertain demand.

Strategy is supply function, and you can have mixture of supply function.

$D(p, \theta) = D(p) + \theta \ b > 0 \ \theta[0, \infty)$

If we do this we will get unique equlibrium in linear supply function, if demand is linear.

If $D(p) = -b.p, D(p, \theta) = \theta - b.p$ there would be unique supply function in the form of linear. So you don't look for any other form.

The paper is on the sylabus and you can check it out there.

The firm commits to twice differentiable function. They select quantity price competition.

The firm commite to supply function. Commitment before θ is observed. This helps us to boil down to unique equlibrium.

Once θ is realized there would be $p(\theta)$ such that $D(p, \theta) = \sum_{i=1}^{n} S_i(p)$, since theta will shift the demand.

In equlibrium θ will drop out. The profit will be $\pi_i = p(\theta)S_i(p(\theta)) - c(S_i(p(\theta)))$

The choice variable, and action set is, the set of all linear supply functions. They make choices simultaneously, we look for equlibrium supply function. We know this is going to be unique supply function.

b would be as parameter in supply function $S_i(p)$, since $p(\theta)$ you supply function will have θ inside. Those prices depend on θ, the quantity will depend on it, but indirectly through price.

$D(p) = \theta - b.p$, and $S(p) = l.p$ linear supply function in the form of aggregate. $p = \frac{\theta}{l-b}$

With different realization of θ we will have different price.

How do we choose the supply function?

$S_j(.)$

Residual demand is $D_i^r(p, \theta) = D(p, \theta) - \sum_{j \neq i} S_j(p)$

$max_p[p.D_i(p, \theta) - C(D_i^r(p, \theta))]$

Now we assume that for given θ, there would be a price. We have to consider all possible θ.

We assume that a unique invertible $p(\theta)$ exists.

The supply function is the collection of all these prices $(p_i(\theta), q_i(\theta))$ where $q_i(\theta) = D_j^r(p, \theta)$

Instead of writing residual demand you can write your supply $S_i(p)$

First order condition would be $p - c = -\frac{S_i}{\frac{\partial D_i^r(p,\theta)}{\partial p}}$

$\frac{\partial D_i^r(p,\theta)}{\partial P} = D' - \sum_{j \neq i} S'_j$

$(n-1)S'(p) = \frac{s}{p-c(s)} + D'(p)$

For two firms:

$max_p[p.[D(p)+\theta) - q_2(p)] - C(D(p)+\theta - q_2(p))]$

First order condition is $D(p)+\theta-q_2(p)+\{p-c'(D(p)+\theta-q_2(p))\}[D'(p)-q_2'(p)] = 0$

Where $D(p)+\theta-q_2(p) = q_1(p)$ which is residual.

Rewriting this will result in $q_i(p) + \{p - c'(q_1(p))\}[D'(p) - q_2'(p)] = 0$

In the original paper θ was not there so we removed.

$q'(p) = \frac{q(p)}{p-c'(q(p))} + D'(p)$

Show (Problem set 3):

$D(p) = -bp$

$D(p,\theta) = \theta - bp; \theta[0,\infty)$

$c(q) = \frac{c.q^2}{2}; c > 0$

$S(2) = d.p$

$d = \frac{1}{2}[\frac{1}{c} - b + \sqrt{(\frac{1}{c}-b)^2 + \frac{4b}{c}})$

Compare with cournot and Bertrand.

Conjectual variation

Our assumption in cournot was that I choose my quantity given my rivals quantity. My best response is q_1. In bertrand, yet we did not have any quantity effect.

The cournot was static game, simultaneous select. Static game does not allow us how does my quantity affect the quantity my rival selects. My choice will not affect my rival's choice. My choice should not fit back.

The total revenue is $TR(q_1,q_2) = p(q_1+q_2).q_1$, and marginal revenue is $MR_1 = p(q_1+q_2) + q_1.\frac{dP}{dQ}\overbrace{[\frac{dq_1}{dq_1} + \partial dq_2 dq_1 = 1 + \frac{dq2}{dq_1}]}^{\frac{\partial Q}{\partial q_1}}$

I form belief about how my quantity will affect my rivals quantity. It is still static. This is the place that classic IO theory crashes with game theory. Hicks (1936), and Bowley (1924).

$\frac{dq_2}{dq_1} = \lambda_1$

If $\lambda_1 = 0$ we will have cournot.

If $\lambda_1 < 0 \rightarrow q_1 \searrow \Rightarrow q_2 \nearrow$

If $\lambda_1 > 0 \rightarrow q_1 \searrow \Rightarrow q_2 \searrow$

FOC:

$q_1 = q_2 = q$

$\lambda_1 = \lambda_2 = \lambda$

$\frac{p-c'(q)}{p} = \frac{1+\lambda}{2\epsilon}$

This is Lerner index.

$\lambda = -1 \rightarrow$ competitive

$\lambda = 0 \rightarrow$ cournot

$= 1 \rightarrow$ monopoly . collosive

Conjectural elasticity.

$\alpha_1 = \frac{q_1}{q+2}\frac{dq_2}{dq_1} = \frac{dlogq_2}{dlogq_1}$

$\frac{dq_2}{dq_1} = \frac{q_2}{q_1}.\alpha_1$

$\frac{p-c'(q)}{p} = \frac{\alpha+(1-\alpha)\frac{1}{2}}{\epsilon}$

$\alpha = 1$ monopoly case

$\alpha = 0$ cournet

$\alpha = -1$ competition

Elasticity here is quantity elasticity

Frish (1933)

$R_2(q_1)$

$R_2'(q_1) = \frac{dq_2}{dq_1}$

Numerical example:

Leontiff $p(q) = b - q_1 - q_2$

$c(q_1) = \frac{q_1^2}{4}$

$\frac{\partial \pi_i}{\partial q_1} = b - (5/2 + \lambda_1)q_1 - q_2 = 0$

$\frac{\partial \pi_i}{\partial q_1^2} = -(5/2 + \lambda_1) < 0$

$q_1 = \frac{6-q_2}{5/2+\lambda_1}$

$R_2(q_1) = q_2 = \frac{6-q_1}{5/2+\lambda_2}$

$R_2'(q_1) = \frac{-1}{5/2+\lambda_2}$

$R_1'(q+2) = \frac{-1}{5/2+\lambda_1}$

$\lambda_1 = R_2'(q_1)$

$\lambda_2 = R_1'(q_2)$

$\lambda = -2$

$\lambda = -\frac{1}{2}$

This gaves the same result without going through full fledged dynamic game.

IO @ UTD: Eighth session

Meisam Hejazinia

03/05/2013

	player 2 coop	player 2 defect
player 1 coop	3,3	$-1, 4$
player 1 defect	$4, -1$	$1, 1$

Asset colussion/ static colusion

How many firm expected

Selthon

Dynamic colusion

repeated game

super games

Payoff matrix of normal form game

cooperation $q^m/2$, each will produce. When defection we will have couple of production quantity $(q^m/2, q^m/2)$.

In one shot game equilibrium tells us there is no cooperation. Cooperation means firms will collude, since they would be better off jointly.

Today we are interested in condition in which collusive equiliubrium is sustainable.

Then we will look at the super game, repeated interaction game, what is the parameter δ that we must have to make sure that we have collusive equilibrium.

Static colusion is not that interesting from game theory point of view, since we need enforcable curtail agreement, which is usually illegal. It is not really interesting to look at the institution economist point of view. We look at the overt colusion.

It is an open agreement. They have quantity setting, and they decide what is the optimal quantity that they should produce.

q is quantity per firm

n is the number of firms

Q is the total quantity

Different size firms may end up with different quantities. They sit together and try to maximize profit. They maximize total profit minus cost of production:

$max_{\bar{q},Q} p(Q).Q - \sum_{i=1}^{n} c_i(q_i)$

All the individual quantities should be less than total quantity: $\sum_{i=1}^{n} q_i - Q \geq 0$. It is an agreement, and this is to have it make sense.

You also assume that everybody produces non negative quantity $q_i \geq 0$.

First order condition with respect to individual quantity q_i: $p'(Q).Q + p(q) - c_i'(q_i) = 0 \ \forall i$

We run this true. There is proposition. Going through this analysis we find out that they produce less than cournot, so Cortel: higher profit, of non cooperative equlibrium. Also Coutel produces lower quanity.

The probalem is that without external enforcement the Cortel is not sustainable. If you do not get punished, you will deviate.

Selten, (1973), IJGT (international Journal of Game Theory). Something was previously in german. 'Four or two few, six or too many' was the name.

n firms in the market

k firms in the Cortel

$k \leq n$

It does not say anything about size of Cortel. It is about size of the market.

$n \geq 2$ at least two firms.

$MC = 0$

Linear demand: $p(Q) = 1 - Q$ $Q \equiv \sum q_i$

three stage

1. joint or not

2. The curtail writes an agreement: Q_k

3. Firms play a Cournot game

$\rightarrow$ Cortel firms: q_k stick to what is agreed.

Al other firms $q_i = R_i(q_j, Q_k)$ where Q_k is the amount that Cortel produces.

There are k Cortel members, they range from $1 \ldots k$ $\{1, \ldots, k\}$

non members would be $\{k+1, \ldots, n\}$

Cortel would be leader, and the rest are followers. Stackleberg view that first Q_k amount of Cortel will be set, and then the rest will decide how much they want to produce.

STACKELBERG

The price would be function of the amount the Cortel would produce and the rest of members produce. The quantity would be fixed. $P(Q_k, \sum_{i=k+1}^{n} q_i)$

When the firm want to enter, and when the firm wants to exit, sit the thing that we are searching for.

Stage 3 we get the Q_k given, and then each members profit N : non member, of firm i:

$\pi_{iN} = q_i(1 - Q_k - \sum j \neq i q_j - q_i)$

We got the first order condition :

$max_{q_i} \pi_{iN} : (1 - Q_k - (n-k-1)q_N) - 2q_i = 0$

$q_j = q_N$

$q_N = \frac{1-Q_k}{n-k+1}$

$Q_N = \frac{(n-k)(1-Q_k)}{n-k+1}$

Stage 2:

$M_k = (1 - Q_N(Q_k) - Q_k).Q_k$

FOC: $Q_k = \frac{1}{2}$

This is general result of stackelberg. $\pi_k = \frac{1}{4(n-k+1)}$

$\pi_k(k,n) = \frac{1}{4k(n-k+1)}$

$\pi_N(k,n) = \frac{1}{4(n-k+1)^2}$

Stage 1: All inclusive: $k = n$

$pi_k(k,n) \geq \pi_N(k-1,n)$

$n > 4 : k < n$

Internally stable. When there is no incentive to quit. $pi_k(k, n) \geq \pi_N(k-1, n)$ Do not quit

The external stability is do not joint the Cortel: $\pi_N(k, n) \geq \pi_k(k+1, n)$

$n = 5/n = 6 \Rightarrow k = 4$

$n = 9/n = 10 \Rightarrow k = 6$

Cortail all inclusive agreement works only when market is small.

Super Games

Repeated Games

We have a stage game: In each stage firm have price or quantity decision

Stage game is played T times

$T < \infty$

$T = \infty$

Super game is stage game that is played T times. Payoff in each stage game is time invariant.

We need to specify what to do in each of the priod, on each of the contingency.

Let (a_{it}, a_{jt}) are firms actions in the stage game. π_i is the firm's profit in the stage game.

Action is going to be quantity.

The profit is the profit from stage cournot game or Bertrand game.

The profit function in itself, is real life and is dependent upon action which is dependent upon t. $\pi_{it} = \pi_i(a_{it}, a_{jt})$.

In each period the firm makes the profit $\sum_{t=0}^{T} \pi_i(a_{it}, a_{jt}$

$\delta < 1$

Low delta means high impatience, or low patience

Cournot game $a_{it} = q_{it}$

In bertrand would be price: $ait = p_{it}$

The profit function does not depend on t. We allow the actions to be function of the history

$H_t = (a_{i0}, a_{j0}, a_{i1}, a_{j1}, , a_{it-1}, a_{jt-1})$

For bertrand we will have:

$T < \infty$

$a_{it} = p_{it}$

$p_{it} = c$

$c_i = c_j = c$

We do backward induction.

Read 6.5.(.2) section Kleps, Milgram, Roberts, Wilson (1982), Gang of 4. They considered the simple stage game. Super game, and they played T periods. There is a share α of crazy people. Not mean completely rational, but they have strong preference for cooperation. There is considerable amount of people who will cooperate if you cooperate.

With small fraction of players who cooperate, will be sufficient for the collusive equilibrium to be sustainable, even in the finite game.

If there is this type of uncertainty, then even in the finite game there would be sustainable equilibrium.

When we say collusion it means tacit collusion. They do not talk with each other. This is 'coopera-

	a_{Ncc}	a_{Nash}
a_{Ncc}	3,3	-1,4
a_{Nash}	4,-1	1,1

tive' equilibrium in the non-cooperative game.

Tricky thing is that they do not talk to eachother.

If $T = \infty$ playing the Nash equilibrium is subgame Nash equilibrium, but it is not the only equilibrium. There are many equilibriums.

There are many subgame Nash equilibrium. Play that in all equilibrium is Nash equlibrium. Then we say Tacit collusion is sustainable equilibrium. (Friedman, 1971), Fold theorem.

Always cooperating is jointly Nash equilibria. In any stage game, there is strategy a_{NCC} mean non cooperative collusion, since we did not talked to each other, but it is tacit collusion.

Trigger strategy:

1. Start with place a_{NCC}, which is good strategy. Continue as long as all other player play the same.

2. If somebody deviates from the strategy that does not play a_{NCC} revert back to a_{Nash}.

I observer whether someone defects at the end of period, and I will shift to a_{Nash} as soon as I observe that.

The profits for Nash, or profits if we both play Nash equilibrium strategy: $\pi_{i,Nash} = \pi_i(a_{nash}, a_{nash})$

$pi_{i,Def} = \pi_i(a_{Def}, a_{Ncc})$

$\pi_{i,Def} > \pi_{i,Ncc} > \pi_{i,Nash}$

In the Cooperation case:

$PDV = pi_{iNcc} + \delta\pi_{iNcc} + \delta^2\pi_{iNcc} + \ldots = \frac{1}{1-\delta}\pi_{i,Ncc}$

Defect:

$\pi_{i,Def} + \delta.\pi_{i,Nash} + \delta^2\pi_{i,Nash} + = \pi_{iDef} + \frac{\delta}{1-\delta}\pi_{i,Nash}$

$\frac{1}{1-\delta}\pi_{iNcc} \geq \pi_{i,Def} + \frac{\delta}{1-\delta}\pi_{i,Nash}$

$\delta \geq$
$\pi_{i,NCC} = \pi_m/2$

$\pi_{i,Def} = \pi^m$

$\pi_{i,Nash} = 0$

$\delta \geq \frac{1}{2}$

Couple of applications:

1. with n firms

2. information lag (not immediate punishment): you do not find out quickly

3. Price wars during doom: low demand state. Lower competition results in collusion, but in boom the collusion collapsed.

1. Tacit collusion is easier, smaller number of firms

We assume Bertrand with n firms. $\pi_{i,Ncc} = \frac{\pi^m}{n}$

$\pi_{i,DEF} = \pi^m$

$pi_{i,NASH} = 0$

$\delta \geq \frac{\pi_{i,DEF} - \pi_{i,NCC}}{\pi_{i,DEF} - \pi_{i,NASH}}$

Tacit collusion would be easier smaller the number of firms.

2. Long information Lag.

Presented discounted value:

$PDV_{NCC} = \frac{1}{1-\delta}\pi_{i,NCC}$

$PDV_{DEF} = pi_{i,DEF} + \delta.\pi_{i,DEF} + \delta^2.\pi_{i,Nash} + \delta^3 \ldots = (1+\delta)\pi_{i,DEF} + \frac{\delta^2}{1-\delta}\pi_{i,Nash}$

The longer the lag, the harder the sustainability of equilibrium.

$PDV_{NCC} \geq PDV_{DEF}$

$\delta \geq \sqrt{\frac{\pi_{i,DEF}-\pi_{i,NCC}}{\pi_{i,DEF}-\pi_{i,NASH}}} = \sqrt{\frac{1}{2}} > \frac{1}{2}$

Demand can be high or low in period t.

$Pr(highDemand) = \frac{1}{2}$

Demand is iid.

$D_2(p) > D_1(p)$

$s = 1, 2$

There would be three phases in each stage game. At the beggining of each stage demand is observed. Then given that observation price is set. In the third period profits are realized. At the end of the period profit are realized and actions are observed.

What do we want to find? We are interested in (p_1, p_2) price in low demand state and high demand state.

1. Both firms set price p_s, $s = 1, 2$, when demand is s.

2. Sustainable in equilibrium.

3. PDV may not be pareto-dominated by the other equilibrium payoffs.

What is the price that we want?

Each of this two p_s will maximize the profit.

$p_s \equiv argmax_p p.D_s(p)$

$\pi_{i,NCC} = \frac{\pi_s^m}{2}$

Present discounted value if I always collude would be:

$\sum_{t=0}^{\infty} \delta^t [\frac{1}{2}\frac{\pi_1^m}{2} + \frac{1}{2}\frac{\pi_2^m}{2}]$

$PDV_{NCC} = \frac{\pi_1^m + \pi_2^m}{4(1-\delta)}$

The cost of defection $\delta\frac{\pi_1^m + \pi_2^m}{4(1-\delta)}$

Gains of defection undercutting.

If I defect I make the full monopoly profit, so my gain from defection are $\pi_s^m - \frac{\pi_s^m}{2} = \frac{\pi_s^m}{2}$

Equlibrium would be sustainable:

$\frac{pi_s^m}{2} \leq \delta\frac{\pi_1^m + \pi_2^m}{4(1-\delta)}$

$\frac{\pi_1^m}{2} < \frac{pi_2^m}{2}$

$\Rightarrow \frac{\pi_2^m}{2} \leq \delta\frac{\pi_1^m + \pi_2^m}{4(1-\delta)}$

$\delta > \underline{\delta} = \frac{2\pi_2^m}{3.\pi_2^m + \pi_1^m}$

$\frac{1}{2} < \underline{\delta} < \frac{2}{3}$

What happens if $\delta < \underline{\delta}$?

Then π_1^m, π_2^m would not be sustainable

This collusive agreement would not be sustainable since the condition does not hold, but this does not mean that there would not ever be collusive equlibrium.

We have to find p_1, p_2 such that the firms profit are maximized:

We have to find collusive prices that maximize the firm's profit.

$max[\frac{1}{2}.\pi_1(p_1)/2 + \frac{1}{2}\frac{\pi_2(p_2)}{2}]/(1-\delta)$

What are the prices that maximize each firms profit, subject to not deviation?

S.t $\frac{\pi_1(p_1)}{2} \leq \delta[\frac{1}{2}\frac{\pi_1(p_1)}{2} + \frac{1}{2}\frac{\pi_2(p_2)}{2}]/(1-\delta)$ (Not Binding)

$\frac{\pi_2(p_2)}{2} \leq \delta[\frac{1}{2}\frac{\pi_1(p_1)}{2} + \frac{1}{2}\frac{\pi_2(p_2)}{2}]/(1-\delta)$ (Binding)

$max[\pi_1(p_1) + \pi_2(p_2)]$

So that: $\pi_1(p_1) \leq k.\pi_2(p_2)$

$\pi_2(p_2) \leq k.\pi_1(p_1)$

$k = \frac{\delta}{2-3\delta} \geq 1$

$p_1 = p_1^*$

Set $p_2' < p_2^m$ so that $\pi_2(p_2') < \pi_1(p_1^m)$

The last thing we will do today is look at the price versus quantity:

So far we have looked at the Bertrand. Now we will look at the Bertrand when the product is differentiated. The products are not perfect substitute.

Now our profit from Nash equilibrium would be different.

We will compare the delta we found from Bertrand and Cournot, and show that collusion will be easier to sustain when it is quantity competition.

$p_i = 1 - q_i - \theta\sum_{j+i} q_j$

Symetric game. Assume n firms. Take first order condition.

You can assume different level of substitutability between different products, but once you go more than two firm it will become intractable.

$FOC_{q_i} = R_i(q_j)$

$q_i = q_j$ (impose symmetry)

$q_c = \frac{1-c}{2+\theta(n-1)}$

$\pi_c = (\frac{1-c}{2+\theta(n-1)})^2 \Rightarrow \pi_{i,NASH}$

π_{iNCC} :

$(p(.)-c)q_i + \sum_{i\neq j}(p_j(.)-c)q_j$

FOC $\Rightarrow R_i(q_j)$

$q_i = q_j = q_{NCC}$

$q_{NCC} = \frac{1-c}{2[1+\theta(n-1)]}$

$\pi_{NCC} = (\frac{1-c}{2[1+\theta(n-1)]})^2$

$max_{q_i}[(1-q_i-\theta(n-1).q_{NCC}-c).q_i]$

In Bertrand we said best response is ϵ below the collusive price. In cournot I need to look at the best response of others.

What is my defection strategy? q_{DEF}

$q_{DEF} = \frac{1-c-\theta(n-1)q_{NCC}}{2}$

$\pi_{DEF} = [\frac{2+(n-1)\theta}{1+(n-1)\theta}.\frac{1-c}{4}]^2$

$\delta = \frac{1}{1+r}$

$\frac{1}{r} \geq \frac{\pi_{iDEF}-\pi_{iNCC}}{\pi_{iNCC}-\pi_{iNASH}} = \frac{1}{4}[\frac{(2+(n-1)\theta)^2}{1+(n-1)\theta}]$

For price competition, we must have:

$\frac{1}{4} \geq \frac{1}{4}\frac{1}{1-\theta}\frac{(2+(n-3)\theta)^2}{1+(n-2)\theta}$

$4(1-\theta)\frac{1+(n-2)\theta}{[2+(n-3)\theta]^2} < r' < 4\frac{1+(n-1)\theta}{(2+(n-1)\theta)^2}$

It satisfied for quantity case, but not satisfied for price.

IO @ UTD: Ninth session

Meisam Hejazinia

03/19/2013

Today we will talk about product differentiation. Chapter 3.6 − 4.2. It is two stage game, first price contract or quantity, and then they compete over the type of contract they have selected. Linear demand function with differentiated product.

We refer to ancient paper that discussed demand function by different product.

We start with the representative model that is aggregate model, although it has some product differentiation.

Some degree of off differentiation, and how the market works. The product differentiation is part of the firm choices, and the other half is it is not part of choice.

We have some degree of market power, but it does not seem to be that firms use market power excessively. What type of decision the firm is used is important. If instead of looking at the perfectly differentiated product to imperfect substitute the conclusion of bertrand paradox will collapse, and we will see similar pattern. The price would be above the marginal cost in cournot. With imperfect substitute we will see that two models are close.

Representative model of product differentiation:

Bowley 1924, singh-Viven (1984)

Consumer utility would be: $U(q_1, q_2) = a(q_1, q_2) - \frac{b[q_1^2 + 2\theta q_1 q_2 + q_2^2]}{2}$

Inverse dmeand function would be $p_1 = a - b(q_1 + \theta q_2)$ a is shift parameter, and b is slope. If θ is equal to zero then independent demand, if $\theta \equiv 1$ then we will have perfect susbstitute, and if $\theta < 0$ shows complement.

$p_2 = a - b(q_2 + \theta.q_1)$

Inverse linear demand function.

$q_1 = \frac{(1-\theta)a - p_1 + \theta.p_2}{b(1-\theta^2)}$

$q_2 = \frac{(1-\theta)a - p_2 + \theta.p_1}{b(1-\theta^2)}$

Demand function with respect to other quantity the slope is positive, with respect to mine it is negative.

Profits are:

$\pi_1 = q_1(a - b(q_1 + \theta.q_2) - c)$

$\pi_2 = q_2(a - b(q_2 + \theta.q_1) - c)$

First order condition with respect to good one:

$2q_1 + \theta.q_2 - \frac{a-c}{b}$

First order with respect to second good is:

$\theta.q_1 + 2.q_2 = \frac{a-c}{b}$

This is good with strategic substitute.

The game is symmetric:

$q_1 = q_2 = q = \frac{1}{2+c} . \frac{a-c}{b}$

$p_1 = p_2 = p = c + \frac{1}{2+\theta}(a-c)$

If we have n firms we can do the same thing, we get the FOC for each, and then we impose symmetry, saying that $q_i = q_j = q$, then the result would be:

$q_c = \frac{1}{2+(n-1)\theta} . \frac{a-c}{b}$

$p_c = c + \frac{1}{2+(n-1)\theta} .(a-c)$

The less product differentiation the lower the price, means less product differentiation we will have more competition. Higher product differentiation means I will have more markup pricing, and I will have more monopoly in the market.

Qualitatively we will have the same outcome here. We talked about cournot to this point, which was quantity competition. Now we will discuss Bertrand about the price competition.

$\pi_1 = (p_1 - c).q_1(p_1, p_2)$

$\pi_2 = (p_2 - c).q_2(p_1, p_2)$

We take First order condition with respect to price:

FOC1: $\frac{(1-\theta)a+c+\theta.p_2}{2} = p_1$

FOC2: $\frac{(1-\theta)a+c+\theta.p_1}{2} = p_2$

This is reaction function since form is $p_1 = R_1(p_2)$

The goods are substitute but prices are strategic complements for this.

$p_1 = p_2 = c + \frac{1+\theta}{2-\theta}(a-c)$

$p_b = c + \frac{1+\theta}{2+(n-3)\theta}(a-c)$

Once the firm choose prices, if goods are differentiated, firms can walk home with strictly positive prices.

As n increases profit for firm will also go down. We same the same qualitative result as a result.

Qualitatively we get the same prediction from this model.

$R'_i = \frac{\pi_i"}{\pi_{ij}"}$

$n = 2 : p \nearrow \theta$

$n > 5 : p \searrow \theta$

In cournot model higher theta gets lower price, and lower theta givs higher price, but here in bertrand more differentiation gives lower price at the $n = 2$.

Here the only decision variable were quantity or price, in the next stage we want to give decision variable of product differentiation.

We will look at the horizontal product differentiation and vertical product differentiation.

The goods are differentiated, since consumers have different reservation value, and you add another level of complexity, and then the argument could be if some consumers have higher value for some, and other lower for others, probably it is vertical differentation. As a result horizontal means consumers have same reservation value for all the goods. Some consumers have higher value for some and some lower, it means it would be same quality. In vertical form it would be two dimensional, due to consumer income we will have different form.

horizontal differentiation

Consumers are different in their preferences.

Spacial approach: taste adjustment cost comes from spacial adjustment cost. The good that one consumer supplies is within the same category, but is different from what I am looking form. If it is not equal to my taste, I have to adjust a little bit. WE can use hotteling model for both of these forms.

You have two streets, and one end of town is 0, and the other on 1. All consumers live in the main street. One firm is at one end of main street, and hte other at the other end. I am consumer X, and it is my location. I can not decide whether I want to purchase from A or B. Both firms produce different brand characteristic. The only differentiation is the distance, and products are the same.

Supplying good at two different location differentiates goods. Not only you pay P_A or P_B, but you also incure transportation cost $P_A + t(x)$.

transportation cost could be linear or quandratic. The distance would be for good t included so $P_B + t(1 - x)$. By locating at different points firms differentiate themselvs. This was special differentiation. Firm A and Firm B is not located geographically, but they locate themslves in term of preference. You like Vanila ice-cream, and then there is no adjustment. If you like choclate then you have to adjust your taste. This was the distance interpretation. Why it is maximum product differentiated when K-mart and wallmart are at different side of town but mcdonald and berger king are close to each other.

Cosumers: uniformly distributed on the line with length l.

Two firms: A, B. Firm A locates at $a > 0$, unit from the left end of the line, and then there is firm B locates at $b > 0$ which is at the right end of the line.

Eventually these two would be decision variable for the firm.

$a + b \leq l$ $(*)$.

Firm A will charge mill price P_A. The alternative would be delivered price. Firm A will set mill price P_A, and firm B will set the mill price P_B. We have a consumer which is x unit away from firm A, and y unit away from firm B. If line has unit length, then consumer is defined as x, and then y would be $1 - x = y$.

The consumer will incure transportation cost $t(x)$ to go to firm A or $t(y)$ to go to firm B. We assume linear transportation costs. If consumer from A wants to by from the transportation would be $c.x$ and from B $\rightarrow c.y$.

Total costs:

From A: $P_A + c.x$
From B: $P_B + c.y$

$a + b + x + y = l$, also from $(*)$.

$y = l - a - b - x$.

The indifferent consumer $\tilde{x}(P_A, P_B)$: $P_A + c.\tilde{x} - p_B + c.(l - a - b - \tilde{x})$

The indifferent consumer is: $\tilde{x} = \frac{1}{2}[l - a - b + \frac{P_B - P_A}{c}]$

As a result demand for firm A would be $q_A = a + \tilde{x} = \frac{1}{2}(l + a - b + \frac{P_B - P_A}{c})$

$q_A(p_A, p_B) = a + \tilde{X}(p_A, p_B)$

$q_B = l - q_A(p_A, p_B)$

Full market coverage

$pi_A = p_A.q_A(p_A, p_B)$

$pi_B = p_B.q_B(p_A, p_B)$

mill price means the firm offers the price and you get the good, and alternative is delivered price which means the firm encounters the transportation cost.

We basically get the demand function, and we have profit function, and then we have price competition for bertrand.

$P_A^* = (b + \frac{a-b}{3}).c$
$p_B^* = (b + \frac{b-a}{3}).c$

$\pi_A^* = \frac{1}{2c}.{P_A^*}^2$

$pi_B^* = \frac{1}{2c}.{p_B^*}^2$

$pi_A^*(a,b) = \frac{1}{2c}[(b + \frac{a-b}{3}).c]^2$

$pi_B^*(a,b) = \frac{1}{2c}[(b + \frac{b-a}{3}).c]^2$

We have restriction that $a + b \leq l$, since the role will flip.

We will have minimal differentiation.

$a + b = l$

A and B will sit over eachother, means minimal differentiation.

Gabtzweicz, thisse (1979) showed that for large a and b the pure nash equilibrium in second stage pricing game may not exist. This means we will not have reduced form for profit. We need to have that so that the firm know where to locate, but if it does not exist then the whole will fall apart. This would be discontinuity in profit function. Pure strategy does not exist, although mixed strategy may exist.

As a result linear transportation cost does not work, so they showed that if we assume quandratic transportation cost everything will work. The costs are quandratic in the distance. As a result we assume $c.x^2$ and $c.y^2$ for the firms.

From A: $P_A + c.x^2$
From B: $P_B + c.y^2$

$\tilde{x} : P_A + c.\tilde{x}^2 - P_B + c(l - a - b - \tilde{x})^2$

$\tilde{x} = \frac{1}{2}[l - a - b + \frac{p_B - p_A}{c(b-a-b)}]$

We take a and b is given and we derieve equilibrium prices P_A and P_B.

$P_A^* = \frac{c(l-a-b)}{3}(3b + a - b)$

$P_B^* = \frac{c(l-a-b)}{3}(3l + b - a)$

Profits at equilibrium:

$\pi_A^* = \frac{c}{18}(l - a - b)(3l + a - b)^2$

$\frac{\partial \pi_A^*}{\partial Q} = -\frac{c}{18}(l + 3a + b)(3l + a - b) < 0$

if $a \geq 0$ then $a^* = 0$

if $a \in R$ then $a^* < 0$ (to the left of zero)

if $b > 0$ then $b^* = 0$

if $b \in R$ then $b^* < 0$ (to the the right of l)

Use hotelling mode only when you are really interested in location, else use the normal aggregate form.

With quadratic transportation cost, we will have maximum differentiation.

Take the book and go through the examples of hotelling of the problem. Slightly different different, and make sure you understand them.

There are two effect:

1. strategic effect: Firm want to locate furthure away from the competitor. We sell the same good to the same market, we will have fierece price competition. Firm want to locate as far away from the other firm. This alleviates market competition.

2. Market share effect: firm want to locate near the center of the market since you will have more consumers, since there would be more demand.

interaction of these two effect will define the equilibrium.

These were equilibrium results. Socially optimal location will be $a^* = \frac{l}{4}$ $b^* = \frac{3.l}{4}$ that minimizes the transportation cost. This is under quandratic cost.

Location that not only reduces the transportation cost, but also minimizes the prices, will give us the socially optimal location.

On linear case, we have multiple equilibria, but the dominant one is when $a = b$, mean they sit on the top of each other, but on that case the price is not defined.

If full coverage they will always have same profit. When they sit on top of each other they equally compete over all the market.

Salop 1979 is about circular city. The problem with the hotelling is when you sit at one side, there would be no consumer and no competitor. Salope solved the problem with circular city. The consumers are all located on the circular line. Each firm will have competitor on either side, the circumfrence is equal to one.

The question of interest is not location of the firm, but how many firm we will have in this market. hotelling allows for any given number of firms.

It is symetric game, and consumers buy one unit of the firm. There is linear transportation cost. Firms will have fixed cost of entry. Fixed cost would be f, and firm will have marginal cost of product which would be c.

$\pi_i = (p_i - c).D_i - f$

The Salope game is two stage game. Firms simultaneously decide to enter the market, and once they decided then they locate equally distanced on the circle.

Stage 1: firm decide to enter simultaneously

Stage 2: Pricing stage, mean competition

Maximaly differentiated is when they are euqlly distanced on the circle.

The maximal differentiation is not derived. As long as entry has positive profit a firm will want to decide to enter.

In the free entry equilibrium we will have zero profit.

We derive Nash equlibrium for pricing game, and then at the second stage we drive nash equilibrium in the entry stage. This is backward induction.

So first: Pricing:

Everybody price at p. We assumed symmetry $p_j = p$. I want to know my optimal price, given everybody else selects equilibrium price.

The share of the market between two firms would be $\frac{1}{n}$.

The question is what is the price p_i. The indifferent consumer will sit in $\tilde{x} \in (0, \frac{1}{n})$

indifferent consumer will pay either pay $p_i + t\tilde{x} = p + t(\frac{1}{n} - \tilde{x})$

$\tilde{x} = \frac{p - p_i + \frac{t}{n}}{2t}$

$D_i(p_i, p) = 2.\tilde{x} = \frac{p - p_i + \frac{t}{n}}{t}$

FOC:

$p_i = D_i(p)$

$p_i = p$

$p = c + \frac{t}{n}$

The price so should be above marginal cost.

What is number of firms? Now we look at the Entry stage:

$D_i(p_i, p) = \frac{1}{n}$

$\pi_i = (p - c).\frac{1}{n} - f$

$\pi = (c - \frac{t}{n} - c)\frac{1}{n} - f$

$\pi = 0$

$n^* = \sqrt{\frac{t}{f}}$

In the entry stage all firms will make zero profit, and we determine the number of firms that enter the market.

If transportation cost is zero nobody will enter. If cost of entery is low many firms enter.

$p^* = c + \sqrt{t.f}$

This is price of free entry equilibrium.

Firm still have market power, with strictly positive transportation cost, firm will have at least some market power.

The more firm we will have higher total cost, although the less would be transportation cost. The solution is at the end of the book for this socially optimal and calculate for yourself.

Vertical differentiation

Firm do not choose their horizontal location, and adjustment, but firm decide the quality of their offer.

Tirol 75

Shaked sutton (1982)

We have oligopolistic competition and quality differentiation. We have consumers and consumers have following utilities.

$U = \theta.s - p$ for one unit of the good. For quality s and price p.

With vertical differentiation, consumer are different in marginal valuation of the good. They attach higher value to quality, and lower value to additional quality. If the consumer buys something that would be utility, and if not buy:

$U = 0$ otherwise ($q = 0$)

$\theta \sim U[\underline{\theta}, \underline{\theta} + 1]$

$\theta \leq 1, \bar{\theta} = \underline{\theta} + 1$

2 firms

quality S_i, $S_s > S_1$

unit cost of production $= c$

Assumption 1: $\bar{\theta} \geq 2\underline{\theta}$
Assumption 2: Market is covered: means each consumers buys at least one of the goods.

$\Delta S = S_2 - S_1$

There is unit demand.

high θ will buy high quality

low θ will buy the low quality

This is like the price differentiation.

There is indifferent consumer.

$\theta.s_i - p_1 = \theta.s_2 - p_2$.

$\theta = \frac{p_2 - p_1}{\Delta s}$

All consumers that sit in $\theta \in [\underline{\theta}, \tilde{\theta}] \rightarrow s_1$

$\theta \in [\tilde{\theta}, \bar{\theta}] \rightarrow$ buy s_2

$D_1(p_1, p_2) = \frac{p_2 - p_1}{\Delta S} - \underline{\theta}$

$D_2(p_1, p_2) = \bar{\theta} - \frac{p_2 - p_1}{\Delta S}$

$\pi_i(p_i, p_j) = (p_i - c).D_i(p_i, p_j)$

1. $max_{p_i} \pi_i$

2. $p_i = R_i(p_j)$

3. $p_1 = c + \frac{\bar{\theta} - 2\_\theta}{3}.\Delta S$
$p_2 = C + \frac{2\bar{\theta} - \_\theta}{3}\Delta S > p$

$pi_1(s_1, s_2) = \frac{(\bar{\theta} - 2.\_\theta).\Delta S}{q}$

$pi_2(s_1, s_2) = \frac{(2.\bar{\theta} - \_\theta).\Delta S}{q}$

$s \in [\_s, \bar{s}]$

For firm 1:

$pi_1(s_1, s_2) = \frac{\bar{\theta} - 2\_\theta}{q}(s_2 - s_1)$

1. $max_{s_1}\pi_1(s_1, s_2)$

2. $s_1 = R_1(s_2)$

Suppose $s_1 < s_2$

$s_1^* = \_s, s_2 = \bar{s}$

$s_1 > s_2$

$s_1^* = \bar{s}, s_2^* = \_s$

high quality firm makes higher profit.

Monopolistic competition is not location decision, but is one where firm supply differentiated product. They are able to engage monopolistic pricing, but they engage in markup pricing. elements:
1. markup pricing

2. zero profit

IO @ UTD: Ninth session

Meisam Hejazinia

03/19/2013

1 continue on product differentiation

We talked about horizontal differentiation. We could have companies competed in three dimension in cube. Three parameters θ and α, and you can extend them. You can have multiple number of firms. Horizontal and vertical differentiation was discussed, you can have combination. Consumer differentiation from horizontal, and vertical differentiation in term of quality input of firms.

Fixit, spense, stiglitz have couple of papers. They come with idea of monopolistic competition. Firms compete with all firms, and not just their neighbors. The one we usually solve comes with two firms. Two products are substitute, but they are imperfect substitutes. In hotelling you just compete with the firm that is next to you. Circle model also does not let us to have competition in cloud, and competition with neighbors would only be possible.

Tirol has different approach. We have representative consumer that has utility over different brands. We have utility function over n brands. Each firm $i = 2, \ldots, n$ produces his hown brand of product. Monopolistic competitive equilibrium comes in 3 ways:

1. Goods are imperfect substitutes. Try to combine monopoly into competitive market. Firm behave as their are monopolistic as their own brand. So first is to set monopolistic price.

2. Consumer choose whichever product maximizes their utility, given price.

3. Free entry. Means competition aspect of the model.

In long run we will have zero profit condition. Each firm makes zero profit. Price would be above marginal cost. At the end each firm makes zero profit. We have to assume entry cost, otherwised infinite firm. Questions: What are entry costs? How many firms in equilibrium?

Unlike monopoly that entry was impossible, here is possible, but with cost.

Assume inverse demand function: $p_i = A(n)q_i^{\beta-1}$ $A(n) < 0$, and $0 < \beta < 1$. So there is some constant $A(n)$ times quantity.

Production cost $c.q_i + f$.

1. interested in price and quantity?

2. At the end we want to say something about the number of firms in equilibrium?

We go from inverse demand function to direct demand function:

$q_i = (\frac{p_i}{A(n)})^{-\frac{1}{1-\beta}}$

Price elasticity in this example would be $-\frac{1}{1-\beta} < 0$

We know what the markups will be.

Learner index is going to $\frac{p_m - c}{p_m} = \frac{1}{|c|} = \frac{1-\beta}{1}$

$max[(p_i - c)q_i(p_i)] = max_{p_i}[(p_i - c)(\frac{p_i}{A(n)})^{\frac{1}{1-\beta}}]$

$p_i^m = \frac{c}{\beta}$

Means there is some market power, since the price is not equal to marginal cost.

Zero profit condition for entry:

Each firm has to make zero profit: $p_i^m.q_i^m - c.q_i^m - f = 0$

As long as strictly positive profit, firms will enter. As the number of firm increases n will increase, and this will affect inverse demand function.

$p_i^m.q_i^m = c.q_i^m + f$

We want to solve for quantity q_i^m that each firm will sell in zero profit condition equilibrium.

$q_i^m = \frac{\beta}{1-\beta}.\frac{f}{c}$

If fix costs increase the quantity of each firm in the market will increase. Price is markup pricing, and fixed cost does not depend on it. The higher the fixed cost, the lower your profit after your revenue. Small number of firms will survive. Then we will have higher value of firms in the industry.

$q_i^m \nearrow f$
$q_i^m \searrow f$

What can we say about n?

We have inverse demand function we have: $p_i = A(n).q_i^{\beta-1}$

$\frac{c}{\beta} = A(n).q_i^{\beta-1}$

$\frac{c}{\beta} = A(n)[\frac{\beta}{1-\beta}.\frac{f}{c}]^{\beta-1}$

$A(n) = \frac{1}{\beta}(\frac{\beta}{1-\beta})^{1-\beta}.c^{\beta}.f^{1-\beta}$

$A(n) \nearrow c \to n \searrow c$

$A(n) \nearrow f \to n \searrow f$

Two effect that counter each other for c since increases the price, at the same time the number of firms decreases, so these would be competing factors.

The fix cost will not affect the price.

2 entry, accomodation and exit condition

One firm that has entered due to the technology advantage or chance, and this incumbant faces entry. What happens to some of the previous structures that we considered.

We look at three types of interaction:

1. Accomodation: accepts the fact that entrant arrives, and tries to make more. Adjust behavior after entry.

2. Blockade: Entry will not happen. The entrant has the production cost that incumbant does not have to change his behavior.

3. Deter the entry: Change the behavior so that it deters the entry.

Bormel(1982), contestable market

Contestable market

n firms

Perfect substitutes

Production cost $c(q)$, $c(0) = 0$

fixed cost if $q > 0$

m incumbands

$n - m$ potential entrants

What is the industry configuration which is feasable?

$q_i, \dots, q_m$ that are produced, each with price p

Feasible if market clears $\sum_{i=1}^{n} q_i = D(p)$

If the firm makes no profit, it will exit the market.

Firms make non negative provit

Each firms revenue not smaller than cost of production $p.q_i \geq c(q_i) \forall i = 1, \dots, m$

Second sustainability: If they are strictly positive then firm will enter, then the industry is not sustainable. It is sustainable mean nobody out would b e better off.
$q^e \leq p, q^e \leq D(p)$, such that $p^e.q^e > c(q^e)$.

If cross subsidization will jeopardize the sustainablity condition.

A contestable market is the one that entry comes with zero cost.

As soon as the revenue is slightly higher than cost of production, an entrant could enter the market. They will exit the market as the soon as profit becomes negative.

Under single product case, where production cost is $c(q) = f + c.q$ $\tilde{\pi} m \equiv max_q[(p(q) - c)q] - f$

unique industry configuration:

$\{p^c, q^c\}$ such that $AC = p$

identical firms

Contestable market is one that the intersection of demand curve and the average cost curve is where the average cost would be decreasing.

If $p < AC$ everybody will make negative profit, violation of feasibility

If $p > AC$ then we will see entrants, violation of sustainability

The only way that the firm would want to enter, is to charge the price less than market, means less than average cost, so get negative profit.

There is only one firm that is able to survive in this market. It is technological efficiency.

We have seen lot of things but not average cost pricing. It is socially efficient given that there would be no subsidy. Nobody wants to produce could be alternative, since we will still have consumer surplus. Allocation here is constraint efficient, and not overall efficiency.

Marginal cost price will not work, since the products are not substitute.

Excercise 8.1

No allocation will exist if $D(p^c)$ would be larger than minimum efficient scale

industry $\{q^c, p^c\}$

Demand is lower than minimum efficient scale: mean the quantity that minimizes firms average cost. $MES \equiv argmin_q AC(q)$

Price is equal to average cost. $P = AC$

Demand given this price would be higher than minimum efficient scale. Such industry configuration would not be sustainable.

Entrant could charge $p^e = p^c - \epsilon$ $q^e = MES$

Natural monopoly characterized with high fixed cost.

WAR OF ATTRITION

Once the second firm leaves the remain firm will be monopoly. We will have mixed strategy that each firm has strictly positive probability of winning.

Tirol p.311

It is natural monopoly, since there is no enough room for the firms to survive.

We assume time is continuous.

interest rate r.

Two firms

Cost of productions are $c(f) = f + c.q$ if $q > 0$, and $c(0) = 0$

If Bertrand $p = c$ profit $-f$

If one firm leaves the market monpoly condition will be built p^m

$\tilde{\pi}^m - f$

Both firms at the market at the $t = 0$

If the probability was strictly positive they will not start again.

At any time t a firm can stay or leave. The firm should be indifferent between staying and leaving.

At each time you need to be indifferent between dropping out, or staying in.

If t were drop out the profit would be equal to zero.

If I stay in then I would make duopoly losses of $f.dt$

dt is the time that elapses from now to the next.

t is now

$t + dt$ is the last instance in time

If firm i stays then firm j will exit with probability

Both firms have to leave with strictly positive probability.

j drops out with probability pr

It will also drop out from now t to $t + dt$.

Once it dropped out, I will make monopoly profit.

$\frac{\tilde{\pi}^m - f}{r}$

$\int_p^\infty (\pi - f)e^{rt}dt = \frac{\tilde{\pi}^m - f}{r}$

If I am staying out (condition for firm i) $0 = f.dt + pr_j.\frac{\pi^m - f}{r}$

$p_{ij} = \frac{f.r}{\tilde{\pi}^m - f}.dt$

FOC (j) $0 = -f.dt + pr_j \frac{\tilde{pi}^m - f}{r}$

We have random length of time which is technologically inefficient. Means we have encountered fixed cost once.

$pr_j = \frac{f.r}{\tilde{\pi}^m - f}.dt$

First price is competitive, and once one firm leaves the price will go to the monopoly price.

Excercise 8.2 in the book, look at it. Look at the contestable market, and then compare the result with result of contestable market. The welfare of the contestable market would be higher.

First mover, and Leader follower model

Stackelberg. See some capacity or some costs to commit certain actions.

You can affect entry by deterring entry, or accomodate entry and make best of it.

Interpretation of capital. How much capital is produced to make best out of entry.

How the idea is different from quantity interpretation. First I decide how much quantity to produce then why don't I adjust quantity. The leader can again adjust capacity. The capacity can be adjusted, and is not fixed.

Two firms:

Firm1: level of capital K_1, fixed

Firm 2: level of capital K_2.

$\pi'(k_1, k_2) = k_1(1 - k_1 - k_2)$

$\pi^2(k_1, k_2) = k_2(1 - k_1 - k_2)$

The higher quantity firm 2 produces smaller would be firm 1:
1. $\pi^i_j < 0$

2. $\pi^i_{ij} < 0$

$K^s_1 = \frac{1}{2}$, $k^s_2 = \frac{1}{4}$, $\pi^1 = \frac{1}{8}$ $\pi^2 = \frac{1}{16}$

$k^N_1 = \frac{1}{3}$, $k^N_2 = \frac{1}{3}$, $\pi^1 = \pi^2 = \frac{1}{9}$

Reaction function:

$R_1 = \frac{1-k_2}{2} = \frac{1-k^s_2}{2} = \frac{3}{8} \neq k^S_1$

Entry cost or set up cost is strictly positive, since then we can look at the entry blockade.

If firm one knows he can not deter the entry, it reduces the scale of firm 2 production, and maximizes the production.

Entry accomodition, then firm one will produce $k^S_1 = \frac{1}{2}$

Entry happens no matter what the fixed cost would be. We want to find the critical point of fix cost that can deter entry.

Entry Deterrence: $f > 0$ $\pi^2 = k^S_2(1 - k^S_1 - k^S_2) - f$

$\frac{1}{16} - f$

Suppose f is close to $\frac{1}{16}$, is k^S_1 optimal for firm 1?

If firm one increases its capacity so much, then that could be optimal. If I increase capacity then the other firm might decide to not enter.

The quantity is not always optimal. We have to find K^b_1 that discourages entry by firm 2.

Given this k^b_1 firm two wants to maximzie $max_{k_2} K_2(1 - k^b_1 - k_2) - f$

$R_2 = \frac{1-k^b_1}{2}$

$\pi^2 = \frac{1-k^b_1}{2}(1 - k^b_1 - \frac{1-k^b_1}{2}) - f$

$pi^2 = (\frac{1-k^b_1}{2})^2 - f = 0 - \epsilon$

$k^b_1 = 1 - 2\sqrt{f} > \frac{1}{2} = K^S_1$

We start the game is from stage 1 that incumband produces an amount, and then on the second stage the entrant decides to enter.

$\pi' = (1 - 2\sqrt{f})(1 - 1 + 2.\sqrt{(f)}) \geq \frac{1}{8}$

True as long as $f > \frac{3-2\sqrt{2}}{32} \propto 0.005 = \frac{1}{16}$

In section 8.2. stigletz approach to this, which is quantity rather than capacity would be revised. Entry deterance as public good and what are other forms of capital.

Stackeleberg: Some times we interpret it as over investment. I produce too much to deter entry. It is always this type of over investment. We have to model it in the form of two firms a leader and the follower. We have the technology leader K_1, and then both firms compete in quantities.

1. investment K_1

2. on second stage quantities is selected simultaneously.

Profit for firm one would be $\pi^m(k_1, X^m(K_1))$ on case of No Entry

Entry:

$\pi^1(K_1, X_1, X_2)$

$\pi^2(K_1, X_1, X_2)$

$X_1^*(K_1), X_2^*(K_1)$

We have Deterence if firm one chooses K_1 such that $\pi^2(K_1, X_1^*(K_1), X_2^*(K_1)) < 0$
Accomodition if $\pi^2(K_1, X_1^*(K_1), X_2^*(K_2)) > 0$

K_1 such that $\pi^2(K_1, X_1^*(K_1), X_2^*(K_1)) = 0$

$$\frac{\partial \pi^2}{dK_1} = \underbrace{\frac{\partial \pi^2}{\partial K_1}} + \underbrace{\frac{\partial \pi^2}{\partial X_1}\frac{dX_1^*}{\partial K_1}} + \underbrace{\frac{\partial \pi^2}{\partial X_2}.\frac{dX_2^*}{dK_1}}$$

The first term is demand effect, the second one would be strategic effect, and the third one would be equal to zero based on envelop theory.

Inverted firm 1: "Taugh" if $\frac{d\pi^2}{dK_1} < 0$
"Soft" if $\frac{d\pi^2}{dK_1} > 0$

Accomodation:

$\pi^1(K_1, X_1^*(K_1), X_2^k(K_1))$

$$\frac{d\pi^1}{dk_1} = \underbrace{\frac{\partial \pi^1}{\partial K_1}} + \underbrace{\frac{\partial pi^1}{\partial X_1}\frac{\partial X_1}{dK_1}} + \underbrace{\frac{\partial \pi^1}{.}\frac{\partial X_1}{K_1}}$$

The first one is direct effect, the second one would be equal to zero, and the third one would be equal to strategic effect.

$\frac{dX_2}{dK_1} = (\frac{dX_2}{dX_1})(\frac{dX_1}{dK_1}) = R_2^1.\frac{dX_1}{dK_1}$

$Sign(\frac{\partial pi^1}{\partial X_2}\frac{dX_2}{dk_1} = Sign(\frac{\partial \pi^2}{\partial X_1}\frac{dX_1}{dK_1}).SignR_2$

Business strategies, and top dog, puppy dog, lean and hungry cat, and fat cad explanation in Tirol book.

Bundeling and entery deterant

Reduce residual demand, and limite competitor business opportunity, and with fix cost means you can deter entry by over investing.

Two papers bundling as an entry barrier, and at the end we look at bundling as entry deterrant.

Bundling in chapter 8 example 8 in Tirol

Rulebuff (2004)

Two goods A, B

Consumers with unit demand for good A and good B (for both)

The valuation of the consumers are $\alpha_A, \alpha_B \in [0, 1]$, independently uniformly disributed

Suppose consumers always want to purchase full coverage.

Incumbent produces A and B. Could have been market leader. $MC = 0$. Entrant enters the market. The challanger enters either market 'A' or market 'B'.

Entrant will produce either A or B.

Entry cost is common knowledge.

Timing:

First incumbant sets prices (pure bundle, or two different cost for them)

The prices would be fixed for the rest of the game.

Second, the entrant decides to enter or not.

Three, the entrant sets the prices.

We look at different situations.

(i) First we look at independent pricing.

(ii) pure bundling

When incumbent alone. Independent prices:

In no entry:

$P_A = P_B = \frac{1}{2}$

$\pi_A = \frac{1}{4}$ $\pi_B = \frac{1}{4}$

$\pi_A + \pi_B = \frac{1}{2}$

Entry

$P_A^e = P_A - \epsilon$

$P_B^e = P_B - \epsilon$

$\pi^e = \frac{1}{4}$

$\pi^{inc} = \frac{1}{4}$

Limit pricing

What are optimal prices?

$P_A = \frac{1}{4}$

$\pi^A = \frac{1}{4}\frac{3}{4} = \frac{3}{16}$

The reduced decision variable for the incumbent are the profit that are made in this market.

Set P_A and P_B such that profit would be equal to $Z(P_A, P_B)$ then the entrant can make half of that $\frac{Z(P_A,P_B)}{2} = f$

$Z(P_A, P_B) = 2f - \epsilon$, so that you limit entry. If your profit would be higher you have given the entrant the chance to enter.

IO @ UTD: Eleventh session

Meisam Hejazinia

04/02/2013

1 continue on entry, and exit

In pure bundling case we have consumers who are uniformly distributed b/w zero and one, and two dimensions independent (in hoteling).

$\bar{p} = 1$. Consumers are willing to buy bundle if their valuation is at least one.

The says down triangle will not buy, and upper triangle will buy in hotelling square.

In order to figure out what is optimal price, we need to figure out what is the demand.

If price is larger than one then we will have the intersection of the new line which is higher than $y = -x$ will intersect the square of hottelling, and would be in the form of $x = 2 - \bar{p}$.

Demand if $\bar{p} > 1$ would be in the form of $D(\bar{p}) = \frac{(2-\bar{p})^2}{2}$, and if $\bar{p} \leq 1$ then $D(\bar{p}) = 1 - \frac{\bar{p}^2}{2}$. In this case we calculate $max_{\bar{p}}[\bar{p}.D(\bar{p})]$, so $\bar{p}^* = \sqrt{2}3 < 1$.

Uniform pricing will result in $p_A = p_B = \frac{1}{2}$.

We will have pure bundling effect. Bundling discount will be seen here. This will deter incumbent of entry, but incumbent still allows entry.

Incumbent benefits from bundling here, and if there was no such benefit then incumbent would charge $\bar{p} = 1$.

In the uniform pricing case then the entrant needed to slightly undercut the entrant, say $p_e^* = p_I - \epsilon$. Suppose we have $\bar{p} = 1$. Who will buy from entrant?

Whose surplus is $\alpha_A + \alpha_B - 1 \geq \alpha_B - \frac{1}{2}$ consumer will buy from entrant. This holds true for $\alpha_A \geq \frac{1}{2}$.

For $\beta > \frac{1}{2}$ will buy from incumbent. Buy bundling incumbent could keep the first quadratic, and this deters some entry. Now the entrant profit is only half of the profit we had in the previous case. Incumbent has to bundle two goods, and this profits on two quadratic. The axis of this system of corrdinate that make the quadratic is center of line $y = -x$.

Suppose we have price $\bar{p}$ as a line that intersects both α_A and α_B axis. When entrants entrs with the price p_e for α_B, all the area under this line which is greater than p_e for α_B will buy from entrant.

$\alpha_A + \alpha_B - \bar{p} \geq \alpha_B - p_e$, so $\alpha_A \geq \bar{p} - p_e$ This means the demand for entrant would be $D^e(p_e) = (1 - p_e)(\bar{p} - p_e)$, since we have pure bundling for the second, so incumbent will not satisfy the need for $\bar{p} - p_e$.

Proposition 1 is $p_e^* = \frac{1+\bar{p}}{3} - \frac{\sqrt{1-\bar{p}+\bar{p}^2}}{3}$

Mixed bundling case as an extension would be in the form of p_A, p_B, $\bar{p}$ mean different price for each of the goods, and $\bar{p}$ for the bundle. Until now we just discussed the pure bundling.

Now the common theme is what we can use to deter the entry.

p. Aghion- p. Bolton (1987)

downstream buyers $v = 1$.

Downstream sellers who produce/ incumbent $cost = \frac{1}{2}$.

We have supplier who produces with $c_e \sim U[0, 1]$ (private information).

We have three stage game.

at first stage $t = 1$ the buyer will write the contract. At second stage an entrant arrives $t = 2$ it observses cost c_e, and at third stage $t = 3$ the entrant makes price offer to the buyer.

The initial contract will specify $\{p, p_0\}$. p is the main price of purchase, and the price $_0$ is the payment penalty for bridging the contract.

The good comes with some quality and some quantity, and it is called WIDGET.

The benchmark is that we have integrated structure: Incumbent/ buyer.

Incumbent wants to minimize the production cost.

Trade is better than not, but want to minimize price.

Internally cost would be $\frac{1}{2}$.

If they outsource the cost would be T, mean the price to give the other supplier, mean buying from entrant.

The benchmark is hypothetical situation.

If they were integrated we would have no contract. We bechmark with the first institution, and it maximizes everything, since it has no purturbation.

Etnrant will accept price if $c_e \leq T$. This is how entrant decides.

In order for integrated structure to make the offer is $p(c_e \leq T) = T$

$T.Pr(c_e \leq T) + \frac{1}{2}(1 - pr(c_e \leq T)) = min_T[T.T + \frac{1}{2}(1 - T)] \Rightarrow T^* = \frac{1}{4}$.

What does this mean for efficiency?

The entrant will supply product if its cost are less than T.

Incumbent produces at the cost of $\frac{1}{2}$.

The incumbent supplying is inefficient. The incumbent could produce with the lower cost.

We compare no contract situation with contract situation.

When we have no contract:

Incumbent makes take it or leave it offers.

It is like monopoly pricing.

Incumbent could extract full surplus from the buyer, because the incumbent will set the price equal to one.

When will we see entry?

We assume there would be bertrand condition.

$c_E > \frac{1}{2}$ the entrant will not enter.

As a result the only case is $c_E \leq \frac{1}{2}$.

$\phi = pr(c_E \leq \frac{1}{2}) = \frac{1}{2}$.

Price of bertrand would be $p = \frac{1}{2}$.

The profit for the buyer would be:

He knows with probability $(1-\phi)$ there would be entry, surplus would be zero.

With probability ϕ we will have entry. As a result:

$(1-\phi).0 + \phi.(1-\frac{1}{2}) = \phi.\frac{1}{2} = \frac{1}{4}$.

Profit for incumbent would be $(1-\phi)\frac{1}{2} = \frac{1}{4}$.

Incumbent has two options: either not offer contract, or offer.

The contract will look as follows:

p is payment if there is trade.

p_0 if there is no trade. penalty.

If there is no entry the buyer surplus would be $1-p$.

If there is entry, then the buyer will switch if the entrant offers surplus of at least $1-p$.

At stage 3 the entrant makes price offer.

if entry: buywer switches if $\tilde{p}$ is at least the surplus is $\geq 1-p$.

STages would be the following:

$t=1$ accept p, p_0 if surplus $\geq \frac{1}{4}$.

$t=3$ buyer will accept the price offer if $\tilde{p}+p_0 \leq p$, since p is the price of staying with the incumbent.

$\tilde{p} \leq p - p_0$.

The entrant so first should make this offer, and the entrant should make strictly non negative profit mean $p \geq C_E \rightarrow p - p_0 \geq C_e$.

$\{p, p_0\}$.

Probability of entry would be $\phi = p(c_e \leq p - p_0)$ $\phi = p - p_0$.

Incumbant price problem would be:

$max_{p_0,p}\phi.p_0 + (1-\phi)(p-\frac{1}{2})$ so that $1-p \geq \frac{1}{4}$.

This is as a result pricing problem for the incumbent. $p=\frac{3}{4}$ and $p_0 = \frac{1}{2}$.

What are the expected profit for this contract for incumbent?

$\frac{1}{2}\frac{1}{4} + \frac{3}{4}\frac{1}{4} = \frac{1}{8} + \frac{3}{16} = \frac{5}{16} > \frac{1}{4}$.

This contract allows for entry. The penalty is less than price, so there is entry with probability of one quarter. In term of efficiency, entrant should enter. This contract gives us inefficient entry.

How would entry blockade look like? $p=\frac{3}{4}$, and $p_0 = \frac{3}{4}$.

The price p could not be higher than $\frac{3}{4}$, since it violates the condition of $1-p \geq \frac{1}{4}$.

In this case buyer and incumbent will form a coalition that entrant compensates not only the buyer, but also the incumbent as well. They find a way to extract additional rent from the contract. Contracts here is used as entery deterance. You basically create a contract as barrier to entry.

This is all about entry, exit and incumbant.

Because there is entry with no cost, the incumbent has not extracted all he could from the buyer.

If they could sit together and negotiate for collusion surplus of $\frac{5}{16}$ would also be shared.

Spier, whinston Rano 1995, extended this paper, and discussed investment part of it.

2 pricing under asymetric information

Static game with asymetric information.

One player knows something that the other player does not.

$t = 1$ there is information.

$t = 2$ there is pricing.

Differentiated duopoly.

Two firm setting prices.

One firm has incomplete information about the cost of rival.

It is symmetric in term of demand $D(p_i, p_j) = a - b.p_i + d.p_j$. Goods are substitute but strategy complement. We have upward sloping reaction function.

Firms are risk neutral, since we are dealing with expectation we need to form some assumption. Constant return to scale production. (CSR). Marginal cost is going to be constant.

$d \leq b$, and $d > 0$.

Firm 2 we have c_2 is common knowledge.

Firm 1 would have $c_1 \in \{c_1^L, c_1^H\}$.

$C_q^L < C_1^H$

$pr(c_1 = c_1^L) = x$

$pr(c_1 = c_1^H) = 1 - x$

$c_1^e := x.c_1^L + (1-x)c_1^H$, would be expected marginal cost.

Ex-post: would be profit function of the firm: $\pi^i(p_i, p_j) = (p_i - c_i)(a - bp_i + dp_j)$

Firm i knows its type and depend on type maximizes profit.

Firm 2 does not know firm 1 cost, and has to maximize profit given what it expects the cost of firm 1 would be.

Bayesian Nash equlibrium should be calculated.

Action set is strategy set, and you either go left or right.

If you think there is type then strategy would be different.

The action set contains two actions, but with different type combination you will have four actions.

What are the action sets?

$A_i \subset R_+$ means any amount could be the action.

$S_2 \subset R_+$

$S_1 \subset R_+ * R_+$

Suppose 2:

p_2^* firm suppose $k = L, H$.

$\pi_i^*(p_1^k, p_2^*) = (p_1^k - c_1^k)(a - bp_1^k + dp_2^*)$

$j = 2, i = 1$

$p_1^k = \frac{+dp_2^* + b.c_1^k}{2b}$ for $k \in \{L, H\}$

$c_1^L < c_1^H$, we know that $p_1^L < p_1^H$

$p_1^e = x.p_1^L + (1-x)p_1^H$.

Reaction function is linear in the cost.

We can take expectation of the price, or take the reaction function, since it is linear in the cost we can use $p_1^e = \frac{a+d.p_2^*+b.c_1^e}{2b}$

Mean the expectation would take this direct form.

c_1^e is expected marginal cost.

Firm 2 needs to know the reaction function of firm 1. We will work with expected reaction function.

We will have to look for the reaction function of each, and how the firms will react to that.

We are looking for the price the firm is going to charge.

The full expected payoff would be $E_{c_1}\pi_2 = x[(p_x - c_2)(a - bp_x + dp_1^L)] + (1-x)[(p_2 - c_2)(a - bp_2 + d.p_1^H)] = (p_2 - c_2)(a - bp_2 + d.p_1^e)$

What would be the expected price of firm 1 and then react to it would be done by firm 2.

$p_2 p_1^c = \frac{a+dp_1^c+bc_2}{2b}$

$p_1^c(p_2) = \frac{a_d p_2 + bc_1^e}{2b}$

$p_2^{**} = \frac{3+c_1^e+2c_2}{3}$

$p_1^k = \frac{3+2c_1^k+c_2}{3}$ would be for $k \in \{L, H\}$

There is no signaling power here. This is static game. You have interest to signal, but here we do not allow that.

Let say the type could be verified. The higher the price firm two charges the higher the profit for firm one.

Low type will not send the message that I am low type since this will reduce its profit. Highest type wants to reveal the type since the second firm will then set the price higher and it will capture more of the market.

We restrict to what does pricing say about firms price structure. We have price competition over two periods. On the first period they will signal type, and then set the actual value.

There are two cases:

1. If you can not deter entry. You would like to signal high cost. If you signal high cost, the other firm will set the high cost optimally. If you happen to be someone of low type you would be better off.

2. If entry can be detered you will signal low type, since you want the firm to behave less agressively.

The other firm does not know your actual cost, by setting the price you will signal the the cost.

Two firms both in the market with two periods.

Marginal cost would be zero.

Demand would be $q_i = a - p_i + p_j$. a is unknown, but distributed on real line. Does not really matter what it is.

a^e is expected value (mean).

Firm one will $max_{p_i} E_a[(a - p_i + p_j).p_i] - max_{pi}(a^e - p_i + p_j).p_j$

$p_i = \frac{a^e+p_j}{2}$

Equilibrium would be $p_i^+ = p_j^+ = a^e$

Means what we expect the demand to be.

Suppose we have two period model, where a does not change and is same for both period.

They only see the realization of their demand q_i, they know their own price, but they do not know they rival price.

We can not allow firms to observe price, since we want to keep that uncertainty.

a is common for both firms.

Suppose firms price symmetrically.

$p_i^A = \alpha$ and it is symmetric.

If that is the case $D_i^A = a - \alpha + \alpha = a$, means on the equilibrium we can infer what a is.

What if the firm deviates?

i deviates. Then $p_i^A \neq \alpha$.

What is that firm j observes? $D_j^A = a - \alpha + p_i^A = \tilde{a} = a + (p_1^A - \alpha)$

In the symmetric equilibrium it would be correct. Means deviation would be the same. Now we assume that one firm deviates and the other things what he sees is correct value.

Firm j assumes complete information, and is on the equilibrium path.

$D_i^A = a - p_i^A + \alpha$ means firm i knows true value of a.

What would be the second stage?

in this case B on the second period while A first period: $p_j^B = \tilde{a}(p_i^A)$

$max_{p_i}(a - p_i^B + \tilde{a}(p_i^A)).p_i^B$

As a result $p_i^B = a + \frac{p_1^A - \alpha}{2}$

In equilibrium j sets $p_j^A = \alpha$ and $p_j^B = \tilde{a}(p_i^A)$

$p_i^B = a + \frac{p_i^A - \alpha}{2}$

What is the deviation?

$\pi_i = E_a[(a - p_i^A + \alpha).p_i^A + \delta(a - (a + \frac{p_i^a - \alpha}{2}) + \tilde{a}(p_i^A)).p_i^B] = max_{p_i^A} E_a[(a - p_i^A + \alpha).p_i^A + \delta.\pi_i^B(p_i^B(p_i^A), p_i^A)]$

Expectation since firm A does not have any knowledge.

$\frac{d\pi_i^3}{d_i^A} = \frac{\partial \pi_i^B}{\partial p_i^B} . \frac{\partial p_i^B}{2.p_i^A} + \frac{\partial \pi_i^B}{\partial \pi_i^A}$

$\frac{\partial \pi_i^B}{\partial \pi_i^A} = \pi_i^B \frac{\partial \tilde{a}}{\partial p_i^a} = p_i^B$

Envelop theorem, or from maximization the first term would be equal to zero

$E_a[a - 2p_i^A + \alpha + \delta.p_i^B(p_i^A)] = 0$

$E_a[a - 2.p_i^A + \alpha + \delta[a + \frac{p_i^A - \alpha}{2}] = 0$

$a^e - 2.p_i^A + \alpha + \delta[a^e + \frac{p_i^A - \alpha}{2}] = 0$

Symmetry , both firm set price α $p_i^A = \alpha$

$\alpha = a^e(1 + \delta)$

IO @ UTD: Twelfth session

Meisam Hejazinia

04/09/2013

1 Limit pricing

Last example we went through was one with stochastic demand.

Two firms (incumbent, and entrant).

Incumbent sets price p. Incumbent knows its cost.

$c_1 \in \{c_1^L, c_1^H\}$, $c_1^L < c_1^H$

Firm 2 does not know firm ones marginal cost.

Firm 2 knows c_1^k after entry. When firm two wants to enter does not know cost of entering, but as it enters it knows it.

We have assymetric of information in duopoly.

Firm two will learn firm one's cost.

$k \in \{L, H\}$

Firm 1:

$\pi_1^k(p_1) = (p_1 - c_1^k).Q_1^m(p_1)$

Same demand. Firm 1's monopoly profit will be in this form.

Concave with regularity conditioning.

p_m^k monopoly price for type k

Because $c_1^L < c_1^H$ then $p_m^L < p_m^H$

For simplicity $\pi_1^k = \pi_1^k(p_m^k)$

After entery: $D_1^k \& D_2^k$ is firm 1's and firm 2's duopoly profit given type k.

Assumption:

$D_2^H > 0 > D_2^L$ The entrants dupoly is smaller than zero if it faces low type firm 1. If firm 1 is high cost type only firm 2, entrant, will have positive profit.

The problem is that at the time of entery firm 2 does not know cost of firm 1, mean low type or high type.

Firm 1 wants to signal firm 2 that it is low cost type.

$\pi_1^k > D_1^k$

Firm 1 wants to signal low type by setting its price p_1^L.

Setting this price p_1^l will result in the negative profit for the first period for firm 1.

If the incumbent sets the price p_1^L that is $\neq p_m^k$ then losses in stage 1 are offset by game in stage 2 with no entery.

There is some price that should convince the entrant that you are in fact low type.

Perfect bayesian equilibrium:

1. Separating equilibria, where we have two different types with different prices.

2. Pooling equilibria. Two types set the same price.

Separating equilibrium the high type price induces entery and low type price does not induce the entery.

Incumbent knows that there is potential enterant.

In the separating equilibrium we have some type k, high type price will induce entery.

For high type we will have:

p_m^H will induce entery.

The low type does not want to signal that it is high type.

The payoff for high type is $\pi_1^H + \delta D_1^H$

The low types price p_1^H. There is some price that if the high type wants to mimic the low type it will set this price.

$\pi_1^H(p_1^L) + \delta.\pi_1^H$

High type charges high price, and low type will set low price p^L.

We do not want high type to set low price so, $\pi_1^h + \delta D_1^H \geq \pi_1^H(p_1^L) + \delta\pi_1^H$ Incentive compatibility Condition High (ICH)

$\pi_1^L(p_1^L) + \delta\pi_1^L$ p_1^L is equilibrium price, and any deviation from this means it is not low type. These are equlibrium payoffs.

Now the low type deviates $\rightarrow$ will have entery.

Now we want to find off equilibrium payoffs. I need to deviate to have entery. What is the best price that low type could charge that is not equal to p_1^L

$\pi_1^L(p_m^H)$

$\pi_1^L(p_m^L)$

$p_m^L \neq p_1^L$

p_m^L is maximized monopoly price for low type

$\pi_1^L(p_m^L) > \pi_1^L(p_m^H)$ $p_m^H \neq p_m^L$

$M_1^L|\delta D_1^L$

Condition 2 is : $\pi_1^L(p_1^L) + \delta\pi_1^L \geq \pi_1^L + \delta D_1^L$

$\pi_1^L - \pi_1^L(p_1^L) \leq \delta(\pi_1^L - D_1^L)$ Incentive compatibility for Low (ICL)

The paper shows that under reasonable condition we will have :

$[\tilde{\tilde{p}}_1 \ \tilde{p}_1 \ ]$ where $\tilde{p}_1 < p_m^L$

$p_1^L \in [\tilde{\tilde{p}}_1, \tilde{p}_1] < p_m^L$

Equilibrium prices $p_m^H, p_m^L \in [\tilde{\tilde{p}}_1, \tilde{p}_1]$

$\rightarrow p_1^L = \tilde{p}_1$

Suppose there is $p \notin \{p_1^L, p_m^H\}$ unexpected event

Arbitrary belief $x = 1$ $pr(H|p) = 1$ We will not have bayesian here.

We needed to provide some believes other than incentive compatibility that supports separating equilibrium. Here we founded it.

Social welfare is higher than under symetric information. The second stage is always identical, for social we look at the first stage. We have lower monopoly price for the low type. The lower price here is good.

The entrant will not enter if the condition $xD_2^H + (1-x)D_x^L \geq 0$.

Results are sensitive to assumption. We have abundance of equilibria. We have abundance of multiplicity of equilibria.

Once you take all these equilibria in standard signaling game there is separating equilibria that survives the standard refinement.

Arbitrary believes help the refinements. The typical signaling game has one equilibria that survives that the high selects high education.

You need to motivate something, boil it down to simple signaling game, and then find equlibria, and go to the refinement and try to convince that it makes sense.

Don't stop at perfect nash equilibria; you need to go through some refinements.

We skip 9.5 and 9.6 section. We will look at 9.7.

Lemon pricing, you choose the action in early stage to convince other party that you are of certain type. Now we affect the actual pay off of the entery. We will affect the actual pay off of entery.

We assume now that there is entery, and we engage in predatory pricing to push the rival out of market. We try to get of entrant once entrant entered the market.

You better able to sustain the losses. You have deeper market. Bearing losses hurts both of you, but you have deeper pocket and can endure it more.

Rival could go to the bank and tries to convince that he will run out and then I will be able to repay. Pay me now and I will recover. There should be some constraint. There should be some imperfection in the capital market.

Tirol direction of deep pocket.

Firm 2: financing project to debth.

The cost of the project is k which is total investment.

E is the firm's equity. The amount that firm can finance by itself.

$D = K - E$ is the amount of finance by the bank.

There is a random profit. Profits are $\tilde{\pi} \in [\underline{\pi}, \bar{\pi}]$ F would be CDF and f would be the pdf.

r would be interest rate

The repayment would be $D(1+r)$

if $\tilde{\pi} > D(1+r)$ then $\tilde{\pi} - D(1+r)$ as profit

if $\tilde{\pi} < D(1+r)$ then the bank will keep $\tilde{\pi}$

Bankrupcy will cost the bank $\tilde{\pi} -^-$

Firm expected profit would be $U(D,r) = \int_{D(1+r)}^{\tilde{\pi}} [\tilde{\pi} - D(1+r)] f(\tilde{\pi}) d\tilde{p}$ there was second term which was equal to zero.

Bank's profit:

$V(D,r) = \int_{\underline{\pi}}^{D(1+r)} [\tilde{-}B] f(\tilde{\pi}) d\pi + D(1+r)[1 - F(D(1+r))]$

Realization less that what the firm will repay

Additional assumptions:

Banks are competitive: so $(1+r_0).D$

r_0 would be the banks interest rate

Zero profit condition:

$V(D,r) = (1+r_0).D$

$r(D) \nearrow D$ exists

Higher lowen will come with higher cost of bankrupcy.

The firm will invest in the project only if
Opportunity cost for the firm is that:

The firm holds the equity. The firm could become a lender, and lend money to other bank.

Firm 2 will hold on the project so if $E \ldots (1+r_0)E$

if profit would be greater than opportunity cost the firm only will go for the project. $U(D,r) \geq (1+r_0)E$

$w = U(D,r) - (1+r_0)E \geq 0$

Firm one has effect on E the cash holding of firm two is lower. Firm one makes sure that it competes very aggressively drives the profit down so that the firm two exits profit.

$W = E\tilde{\pi} - (1+r_0).k - B.F((1+r)(K-E)) \geq 0$

The expected value of cash flow minus total investment cost. The joint opportunity cost the net value of project would be positive.

The expected bankrupcy cost will affect the interest rate.

Higher E will give you higher expected payoff for the project.

If in the first stage very aggressive firm's equity will be driven down, and we will have lower w, and this increases the chance that firm 2 would not be able to finance the project in the second stage.

The higher E the smaller expected bankrupcy cost, so smaller rate.

This is pricing strategy without assymetry information.

The firm one strategy could be pricing strategy or quantity strategy all leading to reducing the firms equity.

Now we want to talk about assymetric information in advertising.

Next week we will tak about search.

Alkerof (1970). Lemon. We have sellers that know the quality of the good, buyers that do not.

quality q: the characteristic of the good.

Seller valuation is $\theta_s.q$ if it keeps it, and p if it is sold.

Buyers payoff is $\theta_b.q - p$ if bought, and 0 if not bought.

$\theta_B > \theta_s$ mean the buying is efficient.

$q \sim U[0, q^{max}]$

Buyer $\theta_b.q^e - p \geq 0$

$q^e \equiv E[q|sold]$

The buyer will buy if the price is $p \in [0, \theta_B.q^e]$

Seller will sell if $q \in [0, \frac{p}{\theta_s}]$ and $p \geq \theta_s.q$

The expected quality of the good would be $E[q|sold]$ $E[q|q < \frac{p}{\theta_s}]$

$q^e(p) = \frac{p}{2\theta_s}$

$p \leq \theta_B.\frac{p}{2\theta_s}$

$\theta_B \geq 2\theta_s$

The assumption here is that advertising reveals information of the good in verifiable manner.

Advertising for search goods: goods that quality is verifiable before purchase. Hard facts. ascertain the quality of the good before purchase.

Experience good advertising would be about soft facts.

The quality can be observed only after consumption.

The firm could only fool you once, and if fool you twice means you are fool.

Dorfiman Steiner condition (1954).

Firm sells bundle of two thing the good itself and information about it.

If the information and physical data could be complementary.

Higher advertising implies higher demand. Mean hard fact reduce uncertainty, and come iwth higher demand.

Provide degree of product information by releasing more information. Release some information about your good. You differentiate your good from others. You update your quality belief, and by releasing information you can control how different your product is from the othe product.

$max_{p,A}\pi = max_{p,A}[(p-c)Q(p,A) - F - p^A.A]$
A is advertising.

P^A unit price of advertising.

$Q_p < 0, Q_H > 0$

We maximize over price and advertising.

FOC of p:

Learner index would be $\frac{p-c}{p} = \frac{1}{\epsilon_p}$

FOC of A:

$(p-c).Q_A - p^A = 0$

What is advertising elasticity of demand?

$\epsilon_A = \frac{Q_A.A}{Q}$

$\frac{p-c}{p} = \frac{p^A}{Q_A.p}$

$\epsilon_A \frac{p-c}{p} = \frac{p^A}{-Q_A.p}\frac{Q_A.A}{Q} = \frac{P^A.A}{p.Q}$

$\frac{\epsilon_A}{\epsilon_p} = \frac{p^A.A}{p.Q}$

A, p are choice

If learner index would be equal to zero then advertising would be equal to zero. Consequently, when we have no markup, we will have no advertising.

We had bundling of service and product in previous sessions. $Q(s,p)$ some very simple form of investment cost.

No markup pricing leads to no advertising. It is necessary condition. If there is no perfect condition there would be no advertising. If advertising elasticity would be zero, we will have no advertising.

The higher profit associated with new product the higher your advertising. The more you benefit from attracting people to buy your good you want to advertise more.

In static game you will advertising in every period.

Advertising would be stock of good will. Could be brand. That consists of all advertising in the past.

A_t is advertising in t.

a_t is the amount of advertising in t

$a_t = A_t + (1-\gamma)a_{t-1}$ This is deprication of advertising.

$a_t = A_t = (1-\gamma)[A_{t-a} + (1-\gamma)a_{t-2}]\dots$

$a_t = \sum_{t=0}^{t}(1-\gamma)^{t-\tau}A_t + (1-\gamma)^{t+1}a_{-1}$

The monopolist cash flow in t is $(p_t - c)Q(p_t, a_t) - p_t^A{}_t$ which depends on stock of good will.

PDV (presented discounted value) of cost follows:

$max_{p_t,a_t} \sum_{t=0}^{\infty} \delta^t[(p_t - c).Q(p_t, a_t) - p_t^A[a_t - (1 - \gamma).a_{t-1}]$

Here p_t, a_t are choice variables. We select this to maximize above.

Here the assumption is that the product is in-durable good.

FOC w.r.t p_t learner index, and we get also w.r.t. a_t

If I invest today it will effect on stock today, tommorow, the day after tommorow and ....

Dorfman stiener condition tells us that:

$p_t^A = \sum_{t=0}^{\infty}(\frac{1-\gamma}{1-r})^t(p_{t+\tau}\frac{\partial Q_{t+\tau}}{\partial a_{t+\tau}}$

Consumers have different tastes. Some uncertainty about the characteristics.

Nelson $1970 - 1974$

Non informative advertising campaign. Just say your product is around. Invest in non-informative, since you want consumer that your product exists.

Every informative would be cheap talk, unless you as the high quality firm find it cheaper to advertise.

If it were to be informative, it has to be truthful.

The argument here is about repeat purchase.

Milgram robert (1986)

Signaling model of advertising. Signal the quality. Low quality means it is satisfactory with probability L.

High quality with probability H, where $L < H < 1$.

$\pi(p, H, H)$, $price, type$, consumer perception.

Consumer are willing to pay for high quality good. The higher their perception the more they are willing to pay.

The low quality guys want to signal high quality, and high quality firm want to offer in separate equilibrium high advertising that is difficult for the low quality to mimic.

Again here we have incentive compatibility constraints.

Incentive compatibility constriant for high quality firm. In equlibrium suppose that we have (p, A) combination. This is in separating equilibrium.

$\pi(p, H, H) - P_A.A$ in equilibrium

In the off equilibrium high quality firm knows that he could not convince that it is of high quality, then why should I advertise.

$\pi(p_{HL}, H, L)$ In this case off equilibrium the consumer will believe that I am a low type

ICH: $\pi(p, H, H) - p_A.A \geq \pi(p_{HL}, H, L)$

incentive compatibility in low would be (ICL)

off equilibrium would be $\pi(p, L, H) - p_A.A$

in separating equilibrium: $\pi(p_{LL}, L, L) - 0$

$p_{LL} \neq p$

$p_{HL} \neq p$

Incentive compatibility would be now : (ICL) $\pi(p_{LL}, L, L) \geq \pi(p, L, H) - p_A.A$.

Any deviation from this would not make sense for the low type either.

IO @ UTD: Thirteen session

Meisam Hejazinia

04/16/2013

1 Economics of search

Emanuele Tarantino

Empirical evidance of price dispersion.

Theoritical foundation of search

1. Sequential search and the Diamond paradox

2. Sequential search with heterogeneous agents Reinganum, Wolinskey

3. Alternative search. Non sequential search. Fixed sample size. Varian 1980.

4. Empirical test of search models

Consistency with behavioral consumers in the market.

Motivation. Started back in the 60's stigler. Standard models of competition with homogeneous products, like cournot and bertrand. There is one price in the market. "'law of one price"', yet you observe dispersion in the market.

Empirical litrature on search and price dispersion, between 5% to 30%. For books like software, book, gasoline and so on. There is in the online marketplace as well.

Goods are different, but you do not observe these differences.

Theoritical foundation for price dispersion. In the standard framework, like bertrand, introducing search cost, and assymetry of inormation results in different result.

Consumers have imperfect information about the prices. The incure cost to get this information.

Recall gives you the option to go back over the previous options and search again, or not.

These models could be applied to labor economics.

Object could be good or job offer. Recieve prize y_n, if stop at period n. Search with recall $y_n = max\{v_1, \ldots, v_n\}$, search without recall $y_n = v_n$. The utility you attach to job will be job offer that you recieved.

Stopping rule, describes after which sequence you should stop.

Stopping time N is integer N after which you should stop.

The rule is to stop after n if an only if $y_n \geq y$, where level of y is the **reservation price**.

Stopping rule is recursive. Environmet is stationary, and at any point the decision is whether to stop or continue the search.

Expected benefit, using marginal conditional. The additional benefit of one more search. $\int_y^b [v - y] f(v) dv$. truncated at y. b is maximum value. y is reservation price. v is the value touched to the given price. Uncertainty is captured by density function $f(v)$.

Additional cost to visit next store is s. Optimality requires that $\int_y^b [v-y]f(v)dv = s$.

This will tell us the optimal time to stop search.

Stopping time $N(y)$ is geometrically distributed: 1 with $F(y)[1-F(y)]$ and for n with $F(y)^n[1-F(y)]$.

Diamond showed that under certain condition, all firms will set monopoly prices.

Large number of identical firms. Many identical consumers. Inelastic demand. Each consumers before starting the process recieves a price quota. If you want to know more you undertake search process. Finally the consumer decides whether and where to buy. Homogeneity of the firms and consumers are key assumptions here.

If increase the price slightly below the search cost, noone will search.

Under diamond equilibrium despite no search price bertrand that we had $p_i = 0$ we have monopoly price.

How to solve this?

1. Introduce heterogeneity on consumers' and/ or firm's side.

2. Consider alternative search rules: Fixed size search

Until now we had hetrogeneity on the firm side. Now we have Wolinskey 1986 which has heterogeneity on the consumer side.

The decision to search again depends on the outcome of the previous search.

Varian 1980. Show price dispersion by assuming that $1-\lambda$ of consumers remain uninformed (pick at a firm at random), and other λ consumers are informed (purchase from the lowest price firm).

$((1-\lambda)/n+\lambda)$ will of uninformed purchase from lower price firm, and $(1-\lambda)/n$ of uninformed will buy from greater price firm. No pure strategy.

$\pi_i(p_i, F(p)) = p_i[\frac{1-\lambda}{n} + \lambda(1-F(p_i))^{n-1}] = \frac{\bar{v}(1-\lambda)}{n}$ is the expected profit. where the second term is the probability that you would be the lowest firm.

In online market, do we have precommitment to fixed sample size, or we have sequential search?

IO @ UTD: Thirteen session

Meisam Hejazinia

04/16/2013

1 Principle Agent problem

I missed first part

q contineous

$q = a + \epsilon \ \epsilon \sim N(0, \sigma^2)$

principle risk rental

constant absolute risk aversion

$u(w, a) = -e^{-y[w-\psi(a)]}$

$\eta = -\frac{u''}{u'}$

$\psi(a) = \frac{c.a^2}{2}$

Linear contracts $w - t + s.q$

principle maximizes expected payoffs:

$max_{a,t,s} E(q - t - s.q)$

Subject to (participation constratints) $E(-e^{-y[t+s.q-\psi(a)]}) \geq 0$

$a \in argmaxE(-e^{-\eta[t+sq-\frac{c.a^2}{2}]})$

$E(-e^{-\eta[t+s(a+\epsilon)-\frac{c.a^2}{2}})$

$E(-e^{-\eta[t+sa-\frac{ca^2}{2}]}).E(e^{-\eta.s\epsilon})$

$(-e^{-\eta[t+sa-\frac{ca^2}{2}]}).E(e^{-\eta.s\epsilon})$

For normally distributed variable $E(e^{\gamma.\epsilon}) = e^{\gamma^2 \frac{\sigma^2}{2}}$

$\gamma = -\eta.s$

$E(e^{-\eta.s.\epsilon}) = e^{\eta^2.s^2.\sigma^2/2}$

$\rightarrow -e^{-\eta[t+sa-\frac{c.a^2}{2}-\frac{\eta.s^2.\sigma^2}{2}]} = -e^{-\eta.\hat{2}(a)}$

The first term of exponent of left side is ..., second term is cost, and third term is risk preimum, or volatility..

$w\hat{(a)} = t + sa - \frac{1}{2}c.a^2 - \frac{\eta.s^2.\sigma^2}{2}$

$max - e^{-\eta(\hat{w}(a))} = max\hat{w}(a)$

The optimal would be $a^0 \frac{s}{c}$. Agent will put this much effort.

Principle maximizes this: $E(q - w)$

$E[a + \epsilon - \omega] = a - \omega + E[\epsilon]$

where $E(\epsilon) = 0$

$E[a + \epsilon - t - s(a + \epsilon)] = (1 - s)a - t$

Principle tries to extract everything from the agent. Suppose there is outside option which comes with $\bar{w}$, means outside option wage, so $\hat{\omega}(s/c) = \bar{w}$.

$max_{s,t}(1 - s)\frac{s}{c} - t$

Subject to $\hat{w}(s/c) = \bar{w}$

$a^0 = s/c$

$s = \frac{1}{1+\eta.c\sigma^2}$

Fixed fee t will depend on outside auction.

The principle could only offer outside contingent contract. Participation agent (aget not negative profit, or not less than outside option), and incentive compatibility(agent will produce optimal level of effort) to agent.

Principle maximizes profit conditioned on participation and incentive compatibility constraints. More effort will increase q. I will earn more on higher q. If the costs are higher I want to exert less effort. Principle can use fixed fee t to exert entier surplus.

Linear contracts are nice and simple, and you just need two number. However, they are not optimal. Optimal are non linear contracts.

Inefficiency of contract now will be discussed.

What is non linear contract?

There is some output q which is function of effort level and state of nature. $q = u(\theta, a)$

$\theta \in \omega$ Natural disaster that affect outcome. The good state of nature bad state of nature, or anything that will codetermine tha output.

Principle payoff: $-v(q - w)$

Agent payoff: $u(w) - \psi(a)$

$q \in [\hat{q}, \bar{q}]$

CDF $F(q|a)$ The higher effort level the higher outcome of the process (first order dominance)

PDF: $f(q|a)$

Contraction contract $w(q)$ non linear tariff some function of outcome.

The principle interested to maximize payoff:

$max \int_{\bar{q}}^{\hat{q}} V(q - w(q)).f(q|a).dq$

$\psi(q)$ is cost of exerting effort

Given the materialization of q, agents problem would be $\int u(w(q))f(q|a)dq - \psi(a) \geq 0 (IR)$ Individual rationality

$a \in argmax_{\hat{a}}\{\int u(w(q))f(q|\hat{a})dq - \psi(\hat{a})\}(IC)$ Incentive compatibility constraint

$IC \rightarrow$ FOC: (ICa)
SOC (Second order constraint): (ICb)

Set up lagrangian.

$\frac{V'(q-w(q))}{u'(w(q))} = \lambda + \mu.\frac{f_a(q|a)}{f(q|a)}$ This is for optimal risk sharing (co insurance)

If μ is larger than zero we have deviation from co-insurance. Effort of agent will not be verifiable. Principle is interested in maximizing its expected payoff.

The principle could ignore incentive compatibility constraint. Principle offers $w(q)$ that leads to co-insurance.

$\mu > 0$ typically

We want to see higher wage for higher outcome. t was fixed fee. s was positive. We want to see something similar for non-linear contract, but the problem is that it is not automatically given. We need the assumption of cdf satisfying first order stochastic dominance. Intuitively the probability of higher outcome is increasing in 'a'.

If $\mu > 0$ the agent would be risk averse.

We also need something that puts some structure on $\frac{f_a(q|a)}{f(q|a)}$.

Suppose principle is risk neutral. We have $\frac{1}{u'(w(q))} = \lambda + \mu$, and suppose we have two levels of effort: a_H, a_L.

$f_a(q|a) = f(q|a_H) - f(a|q_L)$

$f(q|a) > f(a|a_H)$

Optimal risk sharing gives us
$\frac{1}{u'(w(q))} = \lambda + \mu.[1 - \frac{f(q|a_L)}{f(q|a_H)}]$

If $f(q|a_L) < f(q|a_H)$ then I will get more if I put more effort.

This is monotone likelihood ratio. Any contract is possible, but we need first order stochastic dominance as a structure we put to make sure that aget gets higher as it puts higher effort.

First order approach is not always valide. We only consider first order, we can not make sure it is optimal level of effort, we can pin point local maximum or minimum and not maximum.

Mirrless(1975) investigates it.

First order is necessary but not sufficient.

Rogoser (1985) added one condition to this which was convexity of this condition. Maximum Likelihood Ration Property (MLRP). It comes with convex CDFC (Distribution Function Condition). In this case first order stochastic condition works.

Grossman-Hart(1983). Why we started with first order? Since we had continium of types, and we had incentive compatibility, and we do not wanted types to mimic eachother. We had huge amount of incentive compatibility constraints. They tried to use discrete type space. Discrete performance space. $q_1 < q_2 < \ldots < q_n$. It might not be too tractable, but it is still better than infinite number.

It looked at two stages: first stage, implemenation and, second stage, optimization.

The effort levels you can put it could be finite. How much do I have to pay? For each of effort level he knows cost, and had cost function and he tries to maximize this.

IO @ UTD: Fifteen session

Meisam Hejazinia

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Two stage game. At the second stage you solve for signaling game. In the signaling game you will have information sets. The incumbant in the second stage does not know whether it is new entrant or it is the old one. Prey and accomodate strategies on each of stage. Prior is given in the form of common knowledge. What changes is posterior that the entrant may be able to update or not. α is common knowledge, and both player know it. The incumbant knows his type when he makes the move. α is entrant beleif about the type. The entrant starts the game, and decides whether to enter or not. There is also a second round. Nature draws a type. Enterant knows the payoff structure for each one. The only uncertainty is α, whether it is old or new. α is for entrant and non incumbant. One sided assymetry of information.

Here we look at the perfect bayesian equilibrium.

What are they player, what they can do. What do they know. Strategic aspect. Nash equilibrium? Perfect Bayesian Equlibrium, since players move sequentially under assymetric information. What is the objective function.

Perfect equlibrium, add probability of epsilon of fault, trembeling hand, will equilibrium be equilibrium again.

Topics covered:
Monopoly
Price discrimination
Second degree price discrimination
Third degree price discrimination
Hotelling model
Little bit of product differentiation
Vertical control
Strategic interaciton (Static dynamic duopoly)
Limit pricing
Predetary pricing
Entry barriers
Search
Strategic decision making

Bertrand paradox: homogeneous goods. Price equal to marginal cost
Solution: (1) differentiated product, (2) repeated interaction, (3) search cost

Linear demand functions