A Decision Support System for Auction Platforms:

An empirical study of emotionally rational bidders on eBay's Big Data

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1. Abstract

Bidders do not have only the rational side, but also they have the emotional side. They account for their emotions in their decision making, by their counterfactual thinking. More precisely, the bidders may become regretful for the money they spend, or for the purchase decisions they forgone. We call the first type of regret the action, and the second one, the inaction regrets. In this study, we empirically test the theories of bidders' regret in the auction platforms for the self-centric luxury good categories, on the eBay's big data. More particularly, we develop a decision support system (DSS) called AucDSS that allows the auction platform to run the counterfactual analysis on the policies that affect the bidders' regret. Our DSS estimates the parameters of the dynamic structural model of the auctions, the auctioneers' valuation, and the number of auction participants. Our DSS uses various methods appropriate for big data to enable the counterfactual analysis that is both scalable and efficient. In particular, to estimate the discrete state space structural model, our DSS uses a stochastic variational Bayesian expectation maximization methods, with a latent Dirichlet allocation (LDA), and a Dirichlet Process (DP) priors to optimize the estimate of the maximum a posteriori (MAP). To allow our DSS to estimate parameters promptly on a big data, we had to use the simulated annealing, as a heuristic method rather than an exhaustive search of model's parameter space. We estimated our model over a sample data set of around a thousand and six hundred eBay's auctions with around twelve thousand bidders. We find that incongruent with the theoretical predictions, action regret is significant in many bidders' segments for eBay's second price auction, yet our results do not confirm the significance of the bidders' inaction regret. We run a counterfactual on the effect of auction platforms' policies that affect the bidder's action regret. We find that five percent decrease in the action regret of the emotionally rational bidders can increase the highest bid of

some of the auctions from two to four folds, yet it may not touch the highest bid in the other auctions. We believe our DSS can be of interest of both the practitioners and the scholars in academia.

Key Words: Emotionally rational bidders, Auction Platform, Bidder's Regret, Big Data analytics, Machine Learning, Dynamic Structural Model, Variational Bayesian, Probabilistic Graphical model, Dirichlet Process, Kalman Filter, Parallel High Performance Computing, Expectation Maximization, Simulated Annealing

2. Introduction

Bidders do not have only the rational side, but also they have the emotional side as well. Practitioners in the industry have recognized this emotional side to the point that they have designed the bidding systems in which the bidder's bid by their emotion indirectly and not their money directly[1]. In the other hand, the auction systems have a lot of data, e.g. tens of petabytes of data on eBay[2], which can be useful to extract some insights, if it is considered as a whole. The practitioners already know the best way to run a descriptive and a predictive analysis over a huge amount of data, big data, yet the prescriptive analytics of a big data requires further studies[3]. The methods in the prescriptive analytics field require the approaches that trade off the best possible answer with the acceptable good one, which is generalizable on unseen data.

Prescriptive analytics requires knowledge of the domain to model the dynamics structurally; however, the structural models are notorious for the long running times, the Time after which the results may not be relevant for the industry anymore, because then the state of the system is

[1] http://www.cnet.com/news/artwork-auctioned-to-the-highest-emotional-bidder/
[2] http://www.zdnet.com/article/how-ebay-uses-big-data-to-make-you-buy-more/
[3] http://www.informationweek.com/big-data/big-data-analytics/big-data-analytics-descriptive-vs-predictive-vs-prescriptive/d/d-id/1113279

already changed, so the previous recommendation may not be relevant anymore. Big Data comes with four main features: volume, variety, velocity, and veracity. On the variety side, we have a lot of individuals with the different response parameters. An appropriate application for a big data may need to account for this heterogeneity to allow the decision makers to target the different customers, or the bidders separately. On the volume side, a designed system for a big data may not only account for the examples seen, but also for new examples that might not have been seen before. And finally on the velocity and the variety sides, the system should not only be able to produce results dynamically within the planning horizon, but also it should account for the possible uncertainty and the bounded rationality embedded in the consumers' decisions.

Given these requirements of for a viable solution that is useful for industry, we asked: can we design a computationally tractable system to allow an auction platform to estimate how the bidders are sensitive to the action or the inaction regret and to compare them at the individual level over a big data? Can we design a system to allow the auction platforms to know how the bidders update their beliefs about the value of an auction item, and how their hands tremble in the bidding process? More importantly, can we design a system that allows the auction platform to analyze the counterfactuals on how the changes in the consumers' regret can affect the bidding policies? Can this system be scalable, efficient, and robust to the unseen examples?

To answer these questions we developed a structural model of the dynamic evolution of the consumers' bids, the evolution of the number of auction participants, and the evolution of the individual consumers' affiliated valuations. We casted these models into the discrete state space models, and we estimated them with Kalman Filter Forward Filtering Backward Smoothing (KFFBS). As one of the features of a big data is the sparsity, we had to use the regularization terms to prevent the models' over-fittings. As a result we used a Latent Dirichlet Process (LDA) model as a prior on the auctions' parameters, and the Dirichlet Process (DP) as a prior on the bidders' parameters. We used the maximum a posteriori (MAP) variational Bayesian expectation maximization(VEM) optimization approximated methods rather than the sampling methods, such as a Gibbs Sampler, a Metrapolis Hasting or a hybrid of both to estimate the model parameters. Sampling methods may have the problem of mixing, the long run time convergence, and the inefficiency problems, and these features may make them inappropriate for big data applications. In addition, to optimize the MAP of our model, we used a global optimization simulated annealing method rather than a gradient descend, a quasi Newton, or a conjugate gradient methods, because the number of the parameters we want to estimate is very large, and on a big data, we had to trade off an exhaustive search for the efficient computationally tractable procedure.

We used the LDA to cluster the parameters of the auctions in the auctions' hierarchical model, because each auction consists of a vector of 963 keywords in their title, and their category name, the number of bidders, the number of bids, and the auction duration. The LDA is a model of a collection of items' topics. It assumes that each feature of the data (i.e. column of the data) has a topic with a probability distribution, and based on this prior and the likelihood of the model of the probability distribution of topics over each of the data point (i.e. rows of the data), it estimates the joint distribution of the topics over the features and over the data points. The main advantage of this model is that it is a generative model, which means for the new data we can use the estimated model to find an appropriate prior probability of an unseen data based on the estimated parameters of the model. We used DP over the number of the bidders as a prior, as it is a model for the infinite mixture of parameters. Anticipating that an online auction system may face a new unseen bidder at each point in time, we used an infinite mixture model, rather than a finite mixture one to allow for generalizability.

We tested our AucDSS on a data set of 1,646 auctions for luxury goods categories, such as the jewelries' (rings, bracelets, body, and anklets), the watches', the potteries' and glasses', the crafts', the toys' and hobbies', the stamps', the accessories' and collectibles' categories. In our data set that we crawled and scraped from web, we observe 58,285 bids of 12,247 bidders. For each auction, we had the auction description, the category name, the number of bidders, the number of bids, and the auction duration. For each bidder, we have the feedback score, the number of bids on this item, the total number of bids within the last thirty days, the total number of items bids, within the thirty days; the number of bid activity with the current seller, and the number of categories bided on within the last 30 days. We find that incongruent with the theoretical discussions (Ozbay and Ozbay 2007), the action regret is significant in many bidders' segments at the eBay second price auction, yet our results do not confirm the significance of the bidders' inaction regret. We run the counterfactual analysis on the effect of the auction platforms' policies that affect the bidder's action regret. We find that a five percent decrease in the action regret of the emotionally rational bidders can increase the highest bid of some auctions from two to four folds, yet this modification may not affect the highest bid in the other auctions. We believe our DSS can be of interest of both the practitioners and the scholars in academia.

Our study extends the literature on the luxury auctions, and the regretful bidders by characterizing the heterogeneity in the regret, the consumers' valuation, and the consumers' Bayesian updating parameters, and with externally validating the consumers' regret theories. In addition, our study extends research on the applied work to use big data for prescriptive analytics.

This study resides in the intersection of four streams of literature: (1) the emotionally rational consumer, or so called the regretful consumers (2) the bounded rational consumers and bidders (3) the empirical and the experimental studies of auctions and (4) the big data prescriptive analytics. On the first dimension, the main relevant works are the works by Engelbrecht-Wiggans and Kotak (2008) and Filiz et al. (2007), Ding et al. (2005), which build models of regret and experimentally validate the existence of bidders' action and inaction regrets. In modeling literature, Bell (1982) models regret in the decision science literature, and Zeelenberg et al. (2001) validates it in the psychology literature. A recent work by Ozer and Zhang (2013), study the effect of the consumers' regret on the firm's selection of the everyday low price, or the promotion policy. Parallel with this literature on the regretful agents, a stream of work emerged by Simon (1972), Selten, (1975), and Tversky and Kahneman (1992) argue for the consumers' misperception and the bounded rationality. Our study builds over these two streams of literature to expose them to the field of big data, and by extending them to account for the affiliated value, the uncertainty in the private values, the heterogeneity, the dynamics, and the big data methods, which according to Bajari and Hortascu (2003) and Aierly et al. (2005) are important factors in explaining the bidders behavior, to build a DSS that allows an auction platform to run the counterfactual analysis.

Another stream of literature by Laffont et al. (1995) and Guerre et al. (2000), Li et al. (2002), focuses on the non-parametric identification, the theories of auction, the bidder's valuation estimation, and the common value models. On a big data prescriptive dimension, Little (1979) discusses the necessity of decision support systems for managers. A relevant work by Jap and Naik (2008), proposes Kalman filter method for estimating and selecting the dynamic bid models, but not much other work has been done on a big data prescriptive analytics on the auction data. We are inspired by these works, so we built our AucDSS as a prescriptive analytics system for a big auction data, over the concept proposed by Jap and Naik (2008) to allow the auction platform managers to run the counterfactuals on the impact of the bidders' regret on the revenue of the sellers and the auction platform.

In the next section we first plot out our structural model of the optimal bid, the evolution of auction bids, the evolution of the consumers' affiliated valuation, and the evolution of the number of bidders. Then we plot out our estimation strategy, and refer you to the appendix A for more details. Next we explain our results, and our counterfactual analysis, and finally we conclude.

3. Model

We start our model by specifying the utility of an emotionally rational consumer. Similar to Engelbrecht-Wiggans and Kotak (2008) and Filiz et al. (2007) we define the bidder i's utility of, with the valuation v_{it}, and the bid b_{it}, formally as follows:

$$u_{it} = \underbrace{(v_{it} - b_{it})F(b_{it})}_{\text{Rational gain from winning the auction}} - \underbrace{\int_{z_t \le b_{it}} \alpha_i (b_{it} - z_t) dG(z_t)}_{\text{Action regret}} - \underbrace{\int_{b_{it} \le z_t \le v_{it}} \beta_i (v_{it} - z_t) dG(z_t)}_{\text{Inaction regret}} \tag{1}$$

The first component is the utility of the bidder from winning the auction, the second component action regret (paying higher price), and the third component inaction regret (not bidding higher). z_{it} is the highest bid among N-1 bids, and $G(z_t)$ denotes its probability. The

$F(b_{it})$ denotes the probability that b_{it} is the highest bid among N bids. Finally, α_i and β_i are individual specific action and inaction regret respectively.

We can rewrite this utility function in the following form:

$$u_{it} = (v_{it} - b_{it})F(b_{it}) - \alpha_i b_{it} G(b_{it}) + \int_{z_t \le b_{it}} \alpha_i z_t dG(z_t) - \int_{b_{it} \le z_t \le v_{it}} \beta_i (v_{it} - z_t) dG(z_t) + \varepsilon_{it} \quad (2)$$

An optimal bid for the consumer i at time t satisfies the following first order condition:

$$\frac{\partial u_{it}}{\partial b_{it}} = -F(b_{it}) + (v_{it} - b_{it}) f(b_{it}) - \alpha G(b_{it}) - \alpha b_{it} g(b_{it}) + \alpha b_{it} g(b_{it}) + \beta (v_{it} - b_{it}) g(b_{it}) = 0 \quad (3)$$

As a result we can have the following inversion to recover the valuation of consumer i, from her bid and the bids distribution:

$$v_{it} = \frac{F(b_{it}) + b_{it} f(b_{it}) + \alpha_i G(b_{it}) - \alpha_i b_{it} g(b_{it}) + (\alpha_i + \beta_i) b_{it} g(b_{it})}{f(b_{it}) + \beta_i g(b_{it})} \quad (4)$$

Both the trembling hand theory (Selten1975) and the bounded rationality theory (Kahneman 2003, Simon 1972), suggest that consumers may not perfectly measure their valuation and they may not perfectly put their bids. To model this uncertainty, we use Kalman Filter theory (Kalman and Bucy 1961) to model this imperfect measurement and action of the bidders, so we define the latent actual valuations υ_{it} and the latent target actual bids θ_{it} of the bidders in the state space form. For latent actual valuations υ_{it} , we have the following systems of equations:

$$\begin{aligned} &E\left(\frac{F_{t-1}(\theta_{it}) + b_{it} f_{t-1}(\theta_{it}) + \alpha_i G_{t-1}(\theta_{it}) - \alpha_i z_t g_{t-1}(\theta_{it}) + (\alpha_i + \beta_i)\theta_{it} g_{t-1}(\theta_{it})}{f_{t-1}(\theta_{it}) + \beta_i g_{t-1}(\theta_{it})}\right) \\ &\quad = \vartheta_{it} + \varepsilon_{it}, \vartheta_{it} \sim LN(0, \sigma_\vartheta) \end{aligned} \quad (5)$$

$$\vartheta_{it} = \delta_i \vartheta_{it-1} + \rho_i b_{-it} + \mu_{it}, \mu_{it} \sim LN(0, \sigma_\mu) \tag{6}$$

The first equation is the observation equation for the latent valuation υ_{it}, the valuation of consumer i at time t (event of new bid), and the second one is it's the state equation. The learning or the signaling theory in marketing suggests that individuals' update their valuation by observing the market signals (Ariely et al. 2005, Hossain 2008, and Wilcox 2000). In addition our notion of learning is consistent with the experimental studies' findings such as Greenleaf (2004). The signals in bidding context are the bids of other bidders. Our specification of the state equation imposes first order Markov process to the evolution of the valuations. In short, bidders start with an ex-ante valuation and they update their valuation to ex-post, based on the signals or bids they observe.

The ϑ_{it} and μ_{it} are orthogonal white noises with log normal distribution, because valuation is a positive number and it is supposed to increase with larger incoming bids, according to the theory. The simultaneous estimation of the latent valuation state equations allow us to model interdependency of consumers' affiliated valuations as well. Notice that as we do not have the actual value of latent bid that the bidder wanted to issue, and we only observe the bid after bidders' trembled hand, we have to take expectation of the latent bid, to build our observed valuation of bidder. Furthermore, in order to identify the model, we assume that the valuation across the population has a normal distribution, and the expected valuation of a bidder is a draw from the normal distribution of valuation of the bidder population that evolves during time. As a result, we assume that all the bidders share the same distribution of valuation that evolves during time.

This modeling approach is consistent with what auction literature calls affiliated values. Kagel et. al. (1987) defines affiliated private value function an auction in which each bidder has perfect information regarding his/her own value for the object at auction, but higher value of the item for one bidder make higher values for other bidders more likely. Formally, we define the valuation observation and state equations as follows:

$$E\left(\frac{F_{t-1}(\theta_{it})+b_{it}f_{t-1}(\theta_{it})+\alpha_i G_{t-1}(\theta_{it})-\alpha_i z_t g_{t-1}(\theta_{it})+(\alpha_i+\beta_i)\theta_{it}g_{t-1}(\theta_{it})}{f_{t-1}(\theta_{it})+\beta_i g_{t-1}(\theta_{it})}\right)$$
$$=\vartheta_{jt}+\varepsilon_{jt},\vartheta_{jt}\sim LN(0,\sigma_{\vartheta})$$

$$\vartheta_{jt}=\delta_i\vartheta_{jt-1}+\rho_i b_{-it}+\mu_{jt},\mu_{jt}\sim LN(0,\sigma_{\mu})$$

Notice that this action reduces our state space of valuation of consumers from the size of the total bidders' population, i.e. I, to the size of total auctions, i.e. J.

The ρ_i and δ_i are parameters to estimate. We allow the parameter vector of the valuations' states' equations' $\Theta_i=(\alpha_i,\beta_i,\delta_i,\rho_i)$ to come from an infinite mixture normal distribution. This is an applied choice, because we observe sparse data, so estimating each bidders' sensitivity separately on each auction results in over fitting our model, so our Dirichlet process (DP) prior acts as regularization term from machine learning perspective, so called shrinkage in Bayesian estimation. Formally we have the following.

$$G\sim DP(\alpha,G0)$$
$$\eta_i\sim G$$
$$\Theta_i\sim MVN(.\,|\,\eta_i)$$

Where G0 is the prior on the Dirichlet process. DP is the Dirichlet process, a measure on measure, which models Chinese restaurant or Polya urn process. α is parameter to estimate, and

η_j is latent cluster component. Since we draw parameters from G, the data themselves will cluster according to those values drawn from the same parameters. A stick-breaking construction scheme may give a better interpretation of Dirichlet Process (Blei and Jordan 2005), in which we successively break pieces off a unit length stick with size proportion to random draw from a Beta distribution. We consider two collection of independent random variable $V_c \sim Beta(1,\alpha)$ and $\eta_c{}^* \sim G0$ for components $c = \{1,..\infty\}$. We can therefore write G as:

$$\theta_c = V_c \prod_{j=1}^{i-1}(1-V_i)$$
$$G(\eta) = \sum_{c=1}^{\infty}\theta_c \delta(\eta, \eta_c^*)$$

In this scheme θ_c denotes proportions of each of the infinite pieces of stick relative to its original size. It may be useful to consider the variable Z_n as a mixture component to which Θ_j is associated. We should note that in estimation we augmented our data of characteristic of each individual to Θ_j, to use more data for our soft clustering DP approach. Therefore, we have the following generative process for bidder specific parameters:

$$\begin{aligned}
&V_c \sim Beta(1,\alpha), c = \{1,..\infty\}\\
&\eta_c \sim G0, c = \{1,..\infty\}\\
&Z_i \sim Mult(\theta)\\
&\Theta_i \sim MVN(.\,|\,\eta_{z_i})
\end{aligned}$$

We indexed the density functions of F and G in, to mention that the bidders update their expectation of the mean and the variance of bids from observing any new incoming bid, the same approach Jap and Naik (2008) suggest. However, as a new bidder puts his bid in, other bidders will change their perception of the distribution of the bids, and they take into account the updated distribution of bids in their next bidding action. It may be relevant to not that, a sufficient statistic that bidders' use for this update is the maximum of bids that are issued until now. Also note that as in each auction we only observe couple of bids from each bidder, we assume the same variance for both observation and state equation of bidders' valuation update.

For the latent bids θ_{jt} in auction j, at time t, we have the following systems of equations:

$$b_{jt} = \theta_{jt} + v_{jt}, v_{jt} \sim N(0, \sigma_{jv}) \tag{7}$$

$$\theta_{jt} = \tau_j \theta_{jt-1} + \gamma_j + \omega_{jt}, \omega_{jt} \sim N(0, \sigma_{jw}) \tag{8}$$

The first equation is the observation equation for the latent actual bid b_{jt}, and the second one is it's the state equation. The state equation assumes that the bidders generate any new bid by simply increasing the previous bid with a drift γ_j, but they have errors to do so. Both τ_j and γ_j are auction specific parameters.

We assume that bidders use Bayesian updating theory to update their prior on the distribution of the bids. Any time a new bid comes the Kalman Filter theory allows us to update the actual distribution of the underlying actual bids, as suggested by Jap and Naik (2008). The Kalman Filter procedure starts with a prior on the distribution of state parameters, and it updates this prior with filtering errors by the Kalman gain (fraction of the variance of observation equation with state equation) to build the posterior, based on the multivariate normal theory. In this sense the stochastic time varying parameters of the state space process informs both the customers and our estimation procedure. Kalman filter assumes normal distribution for latent state variable, to get a nice closed form, so we adopted the same assumption. In addition to mathematical convenience, we find this assumption innocuous, because Engelbrecht-Wiggans and Katok (2008) suggest a linear mapping between the reservation prices and the bids.

We also have to specify the distribution of maximum bids to complete definition of auction specific bid evolution, so we derive the distribution of maximum of N bidders as follows with simple probability theory:

$$\begin{aligned} G(\theta_t) &= F_{B_{\max}}(\theta_t) = P(B_{\max} \le \theta_t) = P(B_1 \le \theta_t, ..., B_n \le \theta_t) \\ &= P(B_1 \le \theta_t)...P(B_n \le \theta_t) = F(\theta_t)...F(\theta_t) \end{aligned} \tag{9}$$

$$g(\theta_t) = f_{B_{\max}}(\theta_t) = nF(\theta_t)^{n-1} f(\theta_t) \tag{10}$$

The last element of our model includes the number of the bidders. We assume that bidders also update their expectation about the actual number of bidders at each point in time by observing the number of distinct bidders, who have bid up until the current moment. As a result, again we allow the number of bidders who have bid until a given time n_{jt} to be a noisy measure of the actual number of bidders κ_{jt}, so we adopt the state space approach for the evolution of the number of bidders as follows:

$$n_{jt} = \kappa_{jt} + \zeta_{jt}, \zeta_{jt} \sim N(0, \sigma_{j\zeta}) \tag{11}$$

$$\kappa_{jt} = \kappa_{jt-1} + \iota_j + \eta_j \tau_{jt} + \xi_{jt}, \xi_{jt} \sim N(0, \sigma_{j\xi}) \tag{12}$$

We use first order Markov process to specify the evolution of the actual number of bidders, with drift that varies with time. The sort feature of eBay that allows the potential bidder to sort the bids according to the time to deadline, and the Roth and Ockenfels (2000) addressing of snipping behavior bidders guided our specification. We use structural restriction on the number of bidders to be an integer number greater than zero.

. Similar to the approach for the individual specific parameters of valuation state space model, we allow the parameter vector of the auction states' equations' $\psi_j = (\gamma_j, \tau_j, \iota_j, \eta_j)$ to come from a two level hierarchical mixture normal distribution. Formally we have:

$$
\begin{aligned}
&\psi_j \sim Dir(\alpha') \\
&z_n \sim Multinomial(\theta) \\
&f^j{}_n \mid z_n, \beta \sim Multinomial(\beta)
\end{aligned}
$$

where $f^j{}_n$ is actualization of the features n (i.e. columns) in auction j of our data. Where θ' is latent prior on the cluster of a given auction. z_j latent is the cluster index of feature j, and $f^j{}_n$ observed features value n in auction j. α' and β are parameters of the model to estimate. Needless to say, the main benefit that both LDA and DP prior bring to our model is flexibility.

In summery our model uses the hierarchical prior of LDA and DP for the individual bidders, the auction categories, to account for heterogeneity in these entities, and to prevent over fitting of our model. In figure 1, we show the probabilistic graphical model that we used. This model further allows inference after estimation for various probabilistic queries that an auction platform may want to answer on its big data. In addition probabilistic graphical model allows identification of conditional independence much easier, so it may be useful to marketing as much as Feynman is to physics. Each rectangle is a plate to show replication of portion of the model with the number at right south corner of it. Each circle is a random variable, and it is filled, if it is observed. Microsoft Xbox live currently uses probabilistic graphical models to update

Microsoft's information about the preferences of consumers in a collaborative filtering framework[4].

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4. Estimation

As the number of our parameters is huge, 4 for each of the 12,603 bidders, and 4 for each of the 1646 auctions, with total of 55,572, we had to use regularization to avoid model over fitting. As a result, we use two level hierarchical model, i.e. LDA for the auction fixed parameters, and Dirichlet Process for individual fixed parameters. Further, we put structural restriction on the growth and drift parameters of incoming bid value, the valuation of consumers, and the number of bidders, to be non-negative. To estimate our model we use maximum a posterior (MAP) method, because sampling methods such as Gibbs and Metrapolis Hasting sampling methods are computationally intractable over our big data. To optimize the MAP of our data, we used MCVEM, a combination of MCMC method, and adaptive quadrature methods for integrations, and Variational Bayesian Expectation Maximization methods.

The prior choice makes calculation of gradient of MAP function computationally intractable. This suggests that methods such as gradient descend, quasi Newton, and conjugate gradient are computationally intractable for our optimization problem. Calculating the gradient computationally also will increase the run-time of the algorithm exponential in the number of the parameters, as each iteration of MAP calculation takes around 20 minutes over our big data after parallelization. Therefore, we used simulated annealing method, as a heuristic global optimization method. Simulated annealing is a generic probabilistic heuristic method for global optimization. As our use of expectation maximization methods exposes us to a range of local

[4] http://research.microsoft.com/pubs/208585/fose-icse2014.pdf

modes, this choice may be more useful than methods such as quasi Newton that suffer from the problem of local optimum convergence. This method starts with an initial values, and it continues until it reaches the maximum of the iterations, or an energy level is less than the tolerance. It randomly generates neighbors of each state at the point in time as the next searching point. It starts with a reasonably high probability of accepting worse solution, as it explores the solution space, and then it gradually decreases this probability (Belisle 1992).

Further, again to allow our model to work as a part of DSS of an auction platform owner, we parallelized estimation sub-processes. We parallelized our code at two stages: first for DP estimation, and second in separate integration of observed expected valuation of consumer. Next we explain stages of our estimation method. Our estimation method consists of six stages. We ran Kalman filter forward filtering backward smoothing to recover latent state mean and variance for auction bids evolution, auction number of bidders evolution, and individual valuation evolution in the first two stage and in the fourth stage. We used dlm package in R to do so. We used adaptive quadrature to find the expected value of consumer’s valuation in the third stage. We then run LDA method which is available in R in the fifth stage, and in the sixth stage we ran DP estimation code we have implemented in R. We explain the detail of LDA estimation and DP estimation in appendix A. Our method is referred to as a form of stochastic optimization as a whole. The MCEM portion of it to estimate Kalman filter is suggested by de Valpine, P. (2012), as one classical approach to estimate Kalman filter. As we are searching a stochastic surface, our simulated annealing approach may be a good match. Murphy (2012) refers to our version of MCEM method as a form of stochastic approximation EM, in which we perform brief sampling in E-step, followed by partial parameter update. This approach is more efficient for big data prescriptive analytics.

5. Literature review

This study resides in the intersection of three streams of literature: (1) the emotionally rational consumer, or so called the regretful consumers (2) the bounded rational consumers and (3) the empirical and the experimental studies of auctions (4) Big data prescriptive analytics. Next we explore the relevant studies.

5.1. Emotionally Rational Consumers (Regret)

Regret construct has been subject of many studies ranging from the consumer behavior, psychology, and decision science to the behavioral economics and marketing. Peluso (2011) distinguishes between the consumers' satisfaction and the consumers' regret constructs by defining regret as a negative psychological response of an individual to his/her perception of a present situation that would have been better if only she had decided differently (Gilovich and Medvec 1995, Van Dijk and Zeelenberg 2005). Given the regret's negative valence, people tend to be averse to regret, and are motivated to avoid making regrettable decisions (Peluso 2011, Pieters and Zeelenberg 2007, Zeelenberg and Pieters 2007) When consumers feel regret, they tend to engage in the amelioration behavior afterward, so called the regret regulation (Peluso 2011, Zeelenberg and Pieters 2007).

In the consumption context, the consumers tend to feel regretful when they perceive that the performance of a purchased product is worse than that of the forgone alternative (Peluso 2011, Boles and Messik 1995, Tsiros and Mittaal 2000). Both consumer regret and consumer satisfaction are outcomes of comparison process between the performance of a chosen good and a reference point. While consumer satisfactions is the result of a comparison between the perceived performance of the purchased product and an internal standard (e.g. expectation), the regret is the result of comparison between the perceived performance and an external standard (i.e. the performance of forgone alternative), as discussed by Peluso (2011). Tsiros and Mittal (2000) examine the consumer satisfaction and the consumer regret, by arguing that they are separate constructs with the different antecedents and consequences. They find that both consumer satisfaction and regret affect both the re-purchasing decisions and the complaining intentions.

The regret is more likely to be felt under the three circumstances: (1) the perceived performance of the purchased products fall below the relevant expectations (2) The resulting negative disconfirmation is irreversible (3) The choice of the chosen product represents a departure from the status quo (such as in the purchase of a branded good for the first time (Peluso 2011). Feeling of rejoice and regret can be treated as two opposites of a broader one-dimensional construct, both concern with the emotional response of a consumer to the performance of the purchased product in relation to that of the forgone alternatives (Peluso 2011). Consumers may change their decision ex-ante, as a result of feeling regret in a form of a close loop feedback, as many studies suggest (e.g., Simonson 1992, Zeelenberg etal. 2000, Inman and Zeelenberg 2002).

In summary, we may explain regret as a form of a counterfactual thinking ("might-have-been", reconstruction of the past outcomes), which may serve as an affective function, feeling better, and a preparative function, future improvement (Roes 1994). Bell (1982) argues that after making a decision under uncertainty, the decision maker may discover the relevant outcomes, by learning that another alternative would have been preferable. This learning creates a sense of loss, or regret that if incorporated explicitly into the expected utility framework predicts individuals' decision better. Loomes and Sugden (1986) argue that violation of the conventional expected utility suggests that important influential factors in many people's choices are overlooked, perhaps because of the misspecification of the conventional theory. This evidence, proposes an alternative theory that accounts for individual's capacity to anticipate feelings of regret and rejoice. Guided by the same argument, Keinan and Kivetz (2008) find that long-term regret makes consumers to buy pleasurable goods rather than practical necessities, so to spend more on shopping, and this study argues that the long-term regret relaxes self-control and motivates consumers to counteract their righteousness.

Parallel with the consumer behavior and the psychology studies on consumer regret, the emotionally rational consumers' issues such as loss aversion, hyperbolic discounting, and anticipated regret has been subject of many theoretical studies that seek to characterize optimum pricing behavior (Popescu and Wu 2007, Nasiry and Popescu 2011, Heidhues and Koszegi 2008, Su and Zhang 2009, Diecidue et al. 2012,Nasiry and Popescu 2012, Ozer and Zheng 2012. For example, in a recent study, Ozer and Zhang (2013) drive optimal pricing and inventory strategies for the firm with consumers who experience both the high price regret and the stock out regret in a two period model. This theoretical study also considers the consumers' misperceptions of product availability, to suggest merits of markdown policy to everyday low price policy. Next we will discuss related literature on the bounded rationality and the trembling hand theory.

5.2. Bounded Rationality and trembling hand (Behavioral Economics, Psychology)

Many studies concern bounded rationality of consumers, a stream of research on this issue started with seminal work of Simon (1972), and Tversky and Kahneman (1992). This stream of literature discusses that consumers may have misperception about the state of the world, and they may also have error in their action Selten, (1975) at theoretical level. In addition, it argues that consumers' have nebulous idea about the distribution of occurrences of the events (Kahneman 2003, Camerer and Weber 1992). Deviation between the actual probability of an outcome and an individual's perceived probability is another robust behavioral phenomenon addressed by these studies (Kahneman and Tversky 1979, Camerer and Ho 1994,Hey and Orme 1994).

Kaufman (1999) suggests emotional arousal as an alternative to the Simon (1972) cognitive explanation of bounded rationality. Salant (2011) proves that choice rules that result from an optimal tradeoff between maximizing utility and minimizing procedural complexity are history dependent satisficing procedures that exhibit primacy and recency effects, as well as a default tendency. Ellison (2006) discusses bounded rationality in industrial organization, and it explains three main approaches discussed in literature: the rule-of-thumb approaches that specify simple rules for behavior; the explicit bounds approaches that consider agents who maximize payoffs net of cognitive costs; the psychology and economics approaches that cite experimental evidence to motivate utility-like frameworks.

In particular in the auction context, Ely and Hossain (2009) compare late bidding (sniping) to early bidding (squatting) in auctions for newly-released DVDs on eBay, using a field experiment to test the benefit from sniping. This study defines naive bidder as the bidder who acts as if the amount she pays, conditional on winning, equals to her bid. Ely and Hossain (2009) posits that the key contrast with sophisticated bidders is that when a naive bidder becomes the high bidder in an auction, she does not submit a proxy bid, but rather remains inactive until another competitor arrives and competes. Wilcox (2000) finds that experience lead to behavior which is more consistent with theory although the proportion of experienced bidders who behave in a manner inconsistent with theory is quite large. In other word, more experienced bidders are more likely than less experienced bidders to follow Nash equilibrium bidding strategies, and most nonprofessional bidders do not bid in a manner consistent with game-theorists' predictions.

Hossain (2008) suggests that people do not always know their exact private valuation for a good, so the bidders learn spontaneously at a posted price. This study defines uninformed bidder a type of bidder that upon seeing a posted price learns whether his valuation is above that price. The main idea is that people get new information about their own preferences in addition to information about other player's preferences as they participate in the game. Ariely et al. (2005) finds considerable incremental bidding that is reduced but not eliminated with experience. This study discusses how behavior is shaped by different opportunities for learning provided in the auction conditions. Ockenfels and Roth (2002) discusses the possible explanation of multiple-bid phenomenon without positing inexperience or irrational bidders is that bidders sometimes can get information from others' bid that cause them to revise their willingness to pay in auction with interdependent values.

Okenfels and Roth (2002) also discusses the possible existence of naive, inexperienced bidders, who mistakenly treat the eBay auction like English first-price auctions in which the high bidder pays his/her maximum bid.. Kagel et al. (1987) posits that the dominant strategy equilibrium does not organize second-price auction outcomes, as bids consistently exceed private values. This study argues that ad hoc reasoning in the second-price and English auctions call for overbidding, or underbidding, relative to the dominant strategy, and that second-price errors are not robust. These studies are relevant to our study in that they also emphasize the role of naïve bidder, who mistake second price auction with English auction, in explaining results that might not be congruent with the theory.

5.3.The empirical and the experimental studies of auctions

Many studies on auctions concern about theories of auctions such as Laffont et al. (1995) structural estimation of first price auction, Guerre et al. (2000) non parametric identification and estimation in IPV first price auction model, Li et al. (2002) and Campo et al. (2003) studies of nonparametric identification and estimation of the affiliated private values, Haile et al. (2003) study of common value models existence, and Haile and Tamer (2003)'s study of "incomplete" model of English auction. There are also studies that concern about sniping. While theoretically savvy, these studies leave out the emotional aspect of the bidders. Parallel with a stream of literature on the empirical study of the rational behavior of consumers, another stream of literature studies the regret and the rejoice of consumers with experimental studies, studies such as Greenleaf (2004) study of reservation price, regret and rejoice in an open English auction, Engelbrecht-Wiggans and Katock (2008) study of the regret and the feedback information in the first price sealed-bid auctions, the study by Filiz-Ozbay and Ozbay (2007) on the auction with anticipated regret, and the study by Astor et al. (2011) on regret measurement. While these studies have lots of merits, and they give us good evidence of internal validity of the regret concept, they do not prove the external validity and they do not identify the distribution of heterogeneity of consumers to give an appropriate guideline for auction platform of various luxury goods.

This study is different from above study in that it uses a big data on bidders' bid in various categories, and it focuses on estimating regret parameters distribution, the consumers' valuation and bid actual distribution, the consumers' learning behavior and the bidders' entrance processes' parameters across various luxury product categories.

6. Data

We acquired our data by scraping a sample of 1646 auctions from eBay. We observe sequence of bids of bidders in twenty luxury category auctions. These categories include: (1) the jewelries (rings, bracelets, body, and anklets), (2) the watches, (3) the potteries and glasses, (4) the crafts, (5) the toys and hobbies, (6) the stamps, (7) the clothing and the accessories (8) the collectibles (9) the tickets and the experiences (10) The antiques (11) the art works (12) the music record (13) the gift cards and coupons (14) the DVD and movies (15) The books (16) . the consumer electronics (17) the dolls and bears (18) the entertainment goods (19) the health and beauty products (20) the video game and consoles. We collected the data of around a hundred auctions in each luxury good category.

These categories include high involvement ego-involved goods, which makes them appropriate to capture emotional response of consumers. We selected this context and this data, as luxury auctions are usually high involvement, susceptible to the consumers' regret. In addition, as the experimental studies corroborated the internal validity of bidder regret theories (Filiz-Ozbay and Ozbay 2007, Engelbrecht-Wiggana and Katok 2008), we expect the external validity of the theory to be relevant. We note that external validity that we look for is related to the discussion of the mentioned papers, which suggest that regret might not be a relevant factor on the second price auctions, such as eBay auction.

----------------Insert figure 2 around here -----------------

Our data set consists of 58,285 bids of 12,247 bidders over these luxury category goods. We observe the amount that each bidder puts in as its bid. In eBay auction, there is a proxy engine that puts increment on the current bids automatically given that the bid proxy amount is less than the threshold of the bid until the proxy bid is the highest one. This process turns eBay into a second price auction. We filtered such automatic bids, as we are interested to model how the consumers react to each other's bid, rather than how the system actually shows the bids. This system structure is important for bidders, because if one does not understand it well, she might act as if eBay is a first price auction, so she might experience regret.

For each auction item, we know the number of bidders, the number of bids, the duration and its category, and title. For each bidder, we have feedback score, number of bids on this item, total number of bids within the last thirty days, total number of items bids on within thirty days; number of bid activity with the current seller, and number of categories bided on with last 30 days. Figure 2 and figure 3 present the evolution of observed bids and the number of bidders for a sample of six auctions in the different auction categories. As we can observe we see exponential growth of bids as new bids enter the auction. This figure may suggest that most of the biddings are incremental bids, which according to (Hossain 2008, Ariely et al. 2002, and Ockenfels and Roth 2002), may suggest that the bidders under our study might have mistaken eBay's second price auction for the English auction.

----------------Insert figure 3 around here -----------------

We cluster bidders and auctions based on their observable characteristics in the first stage, to account for heterogeneity in our estimation. For segmenting bidders we use the following bidders' characteristics: the feedback score, which is a proxy for bidder's reputation, the number of bids the bidder issued on the auctions under our study, which represents the importance of the current item that we studied for the bidder, the total number of bids within thirty day, which represents how active the bidder has been recently, the total number of items the bidder has bided on within thirty days, which represents whether the bidder diversifies her bids, the number of bidders bid activity with the current seller, which shows how experienced the bidder is with the current seller, and the number of different auction categories the bidder has bided on, which shows whether the bidder has focused on the current auction category. Table 1 presents basic statistic of the profile of the bidders' segments.

----------------Insert table 1 around here -----------------

In order to capture heterogeneity in the auctions, we cluster the auctions based on the word vector of the auction description and auction category, the number of bidders participating the auction, the number of bids issued in the auction, and the duration of the auction. Table 2 presents the basic statistics of the profile of each auction cluster. The histogram of the durations and number of bidders visually has Pareto distribution, with lots of auction clusters with duration of five days, and around nine to ten bidders. In addition on average we observe around forty bids for each auction.

----------------Insert table 2 around here -----------------

7. Results

Table 3 presents the log likelihood of different portions of our model, i.e. the number of bidders' evolution, the bids evolution and the evolution of the valuations. Given forty days average number of bids and that we have around a thousand six hundred auctions in our sample, our data consists of around forty thousand panel observations, so the relatively large log likelihood may suggest that our model captures the bidding behavior with a reasonable performance.

------------insert table 3 around here--------------

Table 4 presents the estimates of the action regret parameters across different bidders' segments. Six segments have significant negative regret coefficient with two tail 5% confidence interval, and thirty six segments out of forty seven segments have significant negative regret coefficients with one tail 5% confidence interval. To better visualize the action regret coefficients figure 4 presents the histogram of the regret coefficients across different segments. The histogram indicates that there are at least two segments of consumers with respect to the sensitivity to action regret. This might be a surprise to see that there are segments of bidders who are regretful for the relatively high amount that they might need pay for an item if they win, in the eBay second price auction.

According to Ozbay and Ozbay (2007), the bidders should not experience regret on the English or second price auctions. The possible explanation for this finding is bidders' uncertainty on the value of the product and learning. In other word, the bidder does not know the value of the rating, so she updates her valuation on the item by observing other bids that its competitors raise. In addition, the bidder might have mistaken the eBay's second price auction with English auction. Therefore, the second highest bid might become an important reference of regret for the decision of the bidder who is cautious of the winners' curse, a curse that might be either due to artificial (shill) bidding (Boze and Daripa 2011) or false learning (Wilcox 2000, Hossain 2008, Ariely et al. 2005).

-------------insert table 4 around here--------------

Table 5 presents the inaction regret parameter estimates across 47 segments. Although the inaction regret parameters are left skewed, however, none of the inaction regret coefficients are significant with 5% either one or two tail test. This finding may arise from the learning behavior of the bidders or the type of the auction item the bidders bid for. In other word, the bidders may avoid thinking about the missed opportunity of winning in a given auction, by considering her other options that may not be far from the auction items that they bid on. The importance of search aspect of bidding, and other comparable options is emphasized by Wilcox (2000). All in all, this result is consistent with Ozbay and Ozbay (2007) theoretical discussion that in the second price auction, the bidder inaction regret is insignificant. Again to better visualize the inaction regret coefficients figure 4 presents the histogram of the regret coefficients across different segments. The histogram indicates that there are at least two segments of consumers with respect to the sensitivity to inaction regret.

-------------insert table 5 around here--------------

Table 6 presents the parameters of the growth and the drift of the valuation of bidders' population. The growth rate is greater than one, which suggests an exponential growth. To explain this high growth we refer to figure 2 of sample evolution of bids with semi exponential growth. Our model assumed that each bidder will issue a bid given her regret parameters, and her perception of the distribution of the bids based on the number of bidders. As time passes more bidders enter the bid, and this translates to higher bids. In parallel, each bidder that enters the auction brings her valuation to the auction, which by translating into bids updates the valuation of the population. In addition, this high growth in our model may be explained by signaling theory. In other word, the bidders deliberately bid less to signal that this product may not worth too much, as they know other bidders infer their valuation from each bids. As time passes the bidders reveal their true valuation, so the bidders' valuation distribution evolves and its mean grows over time. Again to better visualize the population valuation evolution parameters figure 4 presents the histogram of the evolution parameters across different segments. The histogram indicates that there are at least two segments of bidders with respect to the updating behavior of their valuation.

-------------insert table 6 around here--------------

-------------insert figure 4 around here--------------

Table 7 presents the estimates of growth and drift parameters of the bids and the number of bidders' evolution across different auction clusters. We explain high growth rate of bids again with referring to figure 2 of sample bid evolutions, which show the exponential growth of the bids. The estimates of the bidders' mean entrance rate suggest that on average in each auction cluster three bidders enter any time a new bid is issued. Also it suggests that this rate approximately doubles as we close to the deadline for the auction. Figure 5 presents the histogram of these auction cluster specific parameters. Here, we observe a long tail distribution of auction specific parameters, which may suggest the auction platform to customize the auction policies differently for different auctions. Next we will discuss our counterfactual analysis.

-------------insert table 7 around here--------------

-------------insert figure 5 around here--------------

8. Counterfactual analysis

To show the capability of our suggested model for counterfactual analysis we run an analysis for six auctions as a toy example. In our counterfactual analysis we seek to answer how 0.05 decrease in the action regret can affect the seller's revenue. If the auction platform gets revenue share from the buyers, then such an objective function can also be relevant to the auction platform. Generally, the auction platform can reduce consumers' regret by sharing more information that reduces the consumers' uncertainty, or by designing regretless specific mechanisms. To run our counterfactual analysis, giving all the estimated parameters we reduced the action regret of each auction participant by multiplying it to 0.95. Then we start from the beginning, and at a given point in time we find a bid value that maximizes the bidders' utility according to equation (2), by calling an optimization algorithm that uses BFGS algorithm, with initial value of the actual bid that we observed. At each point in time, we use a history of optimal bids that we have found in a Kalman Filter Forward Filtering Backward Smoothing to estimate the mean and variance of the bids.

Figure 6 presents the result of our analysis. As we can see 5% decrease in the action regret can increase the winning bid from 2 to 4 times higher. We can see an step function in this case, because as a result of this decrease some bidders bid so much higher than others that the other bidder's bid becomes irrelevant, as they are prone to raise a bid with lower value. However, we observe such an increase only in watch, pottery and glasses, toy and hobby and stamp auctions. In other word, we observe a sample of jewelry and craft auction that 5% decrease in action regret only raises the bids at initial stages and not the last stages. This may be because the bidders in these two types of auctions are more a dealers, who are more rational. However, further study is needed to make an appropriate conclusion out of this finding.

-------------insert figure 6 around here--------------

9. Conclusion

In this study, we model bidder's decision under regret, and uncertainty, when bidders are rational and update their affiliated valuations. Our main target was to develop an estimation approach that is both scalable and efficient for big data prescriptive analytics. We estimate action and inaction regret parameters, bidder's update valuation parameters, which allow the auction platform to run counterfactual analysis based on. We used stochastic optimization approaches such as VEM, MCEM, and simulated annealing to find the parameters that maximize the posterior distribution of our model. We used LDA and DP priors to explain the variation in the bidders 'and auction specific parameters. Both LDA and DP bring both flexibility and generalizability to our model, to allow our model to part of an auction platform's DSS. To show applicability of our model we run it over a large set of 1646 auctions, with 58,285 bids by 12,247 bidders. We find that incongruent with prior theoretical discussions (Ozbay and Ozbay 2007) action regret is significant in many bidders' segments for eBay's second price auction, yet our results do not confirm the significance of bidders' inaction regret. We run counterfactual on the effect of auction platforms' policies that affect bidder's action regret. We find that five percent decrease in action regret of emotionally rational bidders can increase the highest bid of some auctions from two to four folds, yet it may not touch the highest bid in other auctions. We believe our DSS can be of interest of both practitioners and scholars in academia.

References

Ariely, D., Ockenfels, A., & Roth, A. E. (2005). An experimental analysis of ending rules in internet auctions. RAND Journal of Economics, 890-907.

Astor, P., Adam, M., Jähnig, C., & Seifert, S. (2011). Measuring regret: Emotional aspects of auction design.

Bajari, P., & Hortacsu, A. (2003). Economic insights from internet auctions: A survey (No. w10076). National Bureau of Economic Research.

Bell, D. E. (1982). Regret in decision making under uncertainty. *Operations research*, *30*(5), 961-981.

Belisle, C. J. P. (1992) Convergence theorems for a class of simulated annealing algorithms on Rd. J. Applied Probability, 29, 885–895.

Bierens, H. J., & Song, H. (2011). Nonparametric identification of the first-price auction model. Working paper.(http://econ. la. psu. edu/~ hbierens/auctions_ident. pdf).

Bishop, C. M. (2006). Pattern recognition and machine learning (Vol. 1, p. 740). New York: springer.

Blei, D. M., & Jordan, M. I. (2006). Variational inference for Dirichlet process mixtures. *Bayesian analysis*, *1*(1), 121-143.

Blei, D. M., Ng, A. Y., & Jordan, M. I. (2003). Latent dirichlet allocation. the Journal of machine Learning research, 3, 993-1022.

Boles, T. L., & Messick, D. M. (1995). A reverse outcome bias: The influence of multiple reference points on the evaluation of outcomes and decisions.Organizational Behavior and Human Decision Processes, 61(3), 262-275.

Bose, S., & Daripa, A. (2011). Shills and Snipes.

Camerer, C. F., & Ho, T. H. (1994). Violations of the betweenness axiom and nonlinearity in probability. *Journal of risk and uncertainty*, *8*(2), 167-196.

Camerer, C., & Weber, M. (1992). Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of risk and uncertainty*, *5*(4), 325-370.

Campo, S., Perrigne, I., & Vuong, Q. (2003). Asymmetry in first□price auctions with affiliated private values. Journal of Applied Econometrics, 18(2), 179-207.

de Valpine, P. (2012). Frequentist analysis of hierarchical models for population dynamics and demographic data. Journal of Ornithology, 152(2), 393-408.

Diecidue, E., Rudi, N., & Tang, W. (2012). Dynamic purchase decisions under regret: Price and availability. *Decision Analysis*, *9*(1), 22-30.

Ding, M., Eliashberg, J., Huber, J., & Saini, R. (2005). Emotional bidders—An analytical and experimental examination of consumers' behavior in a priceline-like reverse auction. *Management Science*, *51*(3), 352-364.

Ely, J. C., & Hossain, T. (2009). Sniping and squatting in auction markets. *American Economic Journal: Microeconomics*, 68-94.

Engelbrecht-Wiggans, R., & Katok, E. (2008). Regret and feedback information in first-price sealed-bid auctions. Management Science, 54(4), 808-819.

Ellison, G. (2006). Bounded rationality in industrial organization. *Econometric Society Monographs*, *42*, 142.

Erdem, T., & Keane, M. P. (1996). Decision-making under uncertainty: Capturing dynamic brand choice processes in turbulent consumer goods markets. *Marketing science*, *15*(1), 1-20.

Filiz-Ozbay, E., & Ozbay, E. Y. (2007). Auctions with anticipated regret: Theory and experiment. The American Economic Review, 1407-1418.

Ghahramani, Z., & Beal, M. J. (2001). Propagation algorithms for variational Bayesian learning. *Advances in neural information processing systems*, 507-513.

Gilovich, T., Medvec, V. H., & Kahneman, D. (1998). Varieties of regret: A debate and partial resolution. *Psychological Review*, *105*(3), 602.

Greenleaf, E. A. (2004). Reserves, regret, and rejoicing in open English auctions. *Journal of Consumer Research*, *31*(2), 264-273.

Haile, P. A., Hong, H., & Shum, M. (2003). Nonparametric tests for common values at first-price sealed-bid auctions (No. w10105). National Bureau of Economic Research.

Haile, P. A., & Tamer, E. (2003). Inference with an incomplete model of English auctions. Journal of Political Economy, 111(1), 1-51.

Hey, J. D., & Orme, C. (1994). Investigating generalizations of expected utility theory using experimental data. *Econometrica: Journal of the Econometric Society*, 1291-1326.

Heidhues, P., & Kőszegi, B. (2008). Competition and price variation when consumers are loss averse. *The American Economic Review*, 1245-1268.

Hossain, T. (2008). Learning by bidding. *The RAND Journal of Economics*, 39(2), 509-529.

Inman, J. J., Dyer, J. S., & Jia, J. (1997). A generalized utility model of disappointment and regret effects on post-choice valuation. *Marketing Science*,*16*(2), 97-111.

Inman, J. J., & Zeelenberg, M. (2002). Regret in repeat purchase versus switching decisions: The attenuating role of decision justifiability. *Journal of Consumer Research*, *29*(1), 116-128.

Jap, S. D., & Naik, P. A. (2008). Bidanalyzer: A method for estimation and selection of dynamic bidding models. *Marketing Science*, *27*(6), 949-960.

Kahneman, D. (2003). A perspective on judgment and choice: mapping bounded rationality. *American psychologist*, *58*(9), 697.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 263-291.

Kagel, J. H., Harstad, R. M., & Levin, D. (1987). Information impact and allocation rules in auctions with affiliated private values: A laboratory study. *Econometrica: Journal of the Econometric Society*, 1275-1304.

Kalman, R. E., & Bucy, R. S. (1961). New results in linear filtering and prediction theory. *Journal of Fluids Engineering*, *83*(1), 95-108.

Kaufman, B. E. (1999). Emotional arousal as a source of bounded rationality.*Journal of Economic Behavior & Organization*, *38*(2), 135-144.

Keinan, A., & Kivetz, R. (2008). Remedying hyperopia: The effects of self-control regret on consumer behavior. *Journal of Marketing Research*, *45*(6), 676-689.

Laffont, J. J., Ossard, H., & Vuong, Q. (1995). Econometrics of first-price auctions. Econometrica: Journal of the Econometric Society, 953-980.

Li, T., Perrigne, I., & Vuong, Q. (1998). Structural Estimation of the Affiliated Private Value Model with an Application to OCS Auctions. Mimeo, USC.

Little, J. D. (1979). Decision support systems for marketing managers. *The Journal of Marketing*, 9-26.

Loomes, G., & Sugden, R. (1986). Disappointment and dynamic consistency in choice under uncertainty. *The Review of Economic Studies*, *53*(2), 271-282.

Murphy, K. P. (2012). *Machine learning: a probabilistic perspective*. MIT press.

Nasiry, J., & Popescu, I. (2012). Advance selling when consumers regret.*Management Science*, *58*(6), 1160-1177.

Ockenfels, A., & Roth, A. E. (2002). The timing of bids in internet auctions: Market design, *bidder behavior, and artificial agents*. AI magazine, 23(3), 79.

Peluso, A. M. (2011). Consumer satisfaction: Advancements in theory, modeling, and empirical findings. Peter Lang.

Penny, W. (2002). An EM algorithm for Gaussian Markov Random Fields.

Penny, W. (2001). KL-Divergence of Normal, Gamma, Dirichlet and Wishart densities.

Popescu, I., & Wu, Y. (2007). Dynamic pricing strategies with reference effects. *Operations Research*, *55*(3), 413-429.

Roese, N. J. (1994). The functional basis of counterfactual thinking. *Journal of personality and Social Psychology*, *66*(5), 805.

Roth, A. E., Ockenfels, A. (2000). *Last minute bidding and the rules for ending second-price auctions: Theory and evidence from a natural experiment on the Internet* (No. w7729). National bureau of economic research.

Selten, R. (1975). Reexamination of the perfectness concept for equilibrium points in extensive games. *International journal of game theory*, *4*(1), 25-55.

Simon, H. A. (1972). Theories of bounded rationality. *Decision and organization*, *1*, 161-176.

Simonson, I. (1992). The influence of anticipating regret and responsibility on purchase decisions. *Journal of Consumer Research*, 105-118.

Salant, Y. (2011). Procedural analysis of choice rules with applications to bounded rationality. *The American Economic Review*, 724-748.

Su, X., & Zhang, F. (2009). On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science*, *55*(5), 713-726.

Tsiros, M., & Mittal, V. (2000). Regret: A model of its antecedents and consequences in consumer decision making. *Journal of Consumer Research*,*26*(4), 401-417.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, *5*(4), 297-323.

Van Dijk, E., & Zeelenberg, M. (2005). On the psychology of 'if only': Regret and the comparison between factual and counterfactual outcomes.*Organizational Behavior and Human Decision Processes*, *97*(2), 152-160.

Wilcox, R. T. (2000). Experts and amateurs: The role of experience in Internet auctions. *Marketing Letters*, 11(4), 363-374.

Zeelenberg, M., & Pieters, R. (2004). Beyond valence in customer dissatisfaction: a review and new findings on behavioral responses to regret and disappointment in failed services. *Journal of business Research*, *57*(4), 445-455.

Figure 1: The probabilistic graphical model of the model we estimate in this paper

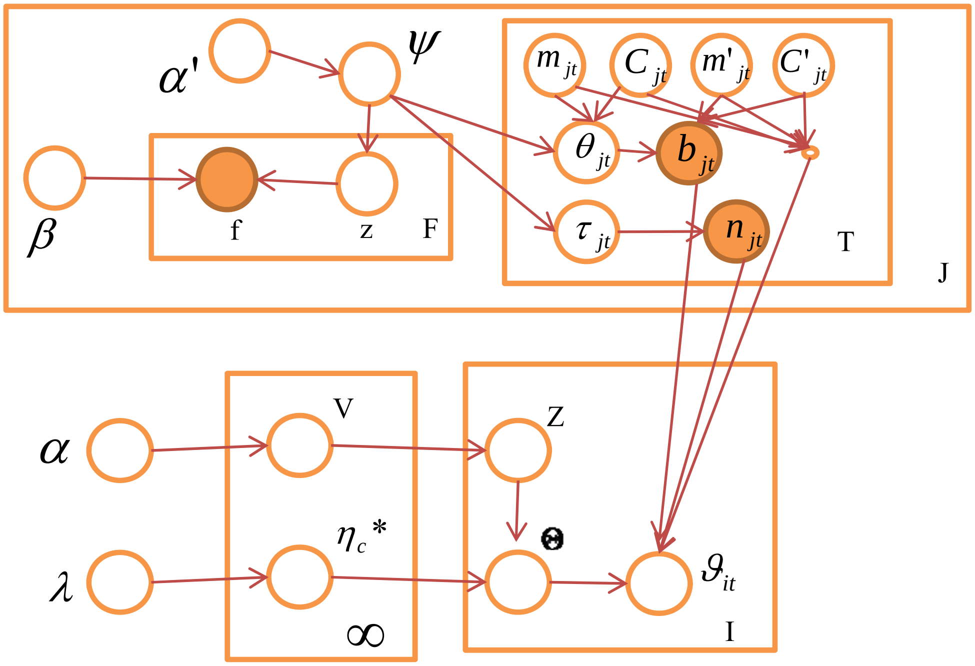


Figure 2: Evolution of Bids in six sample auctions

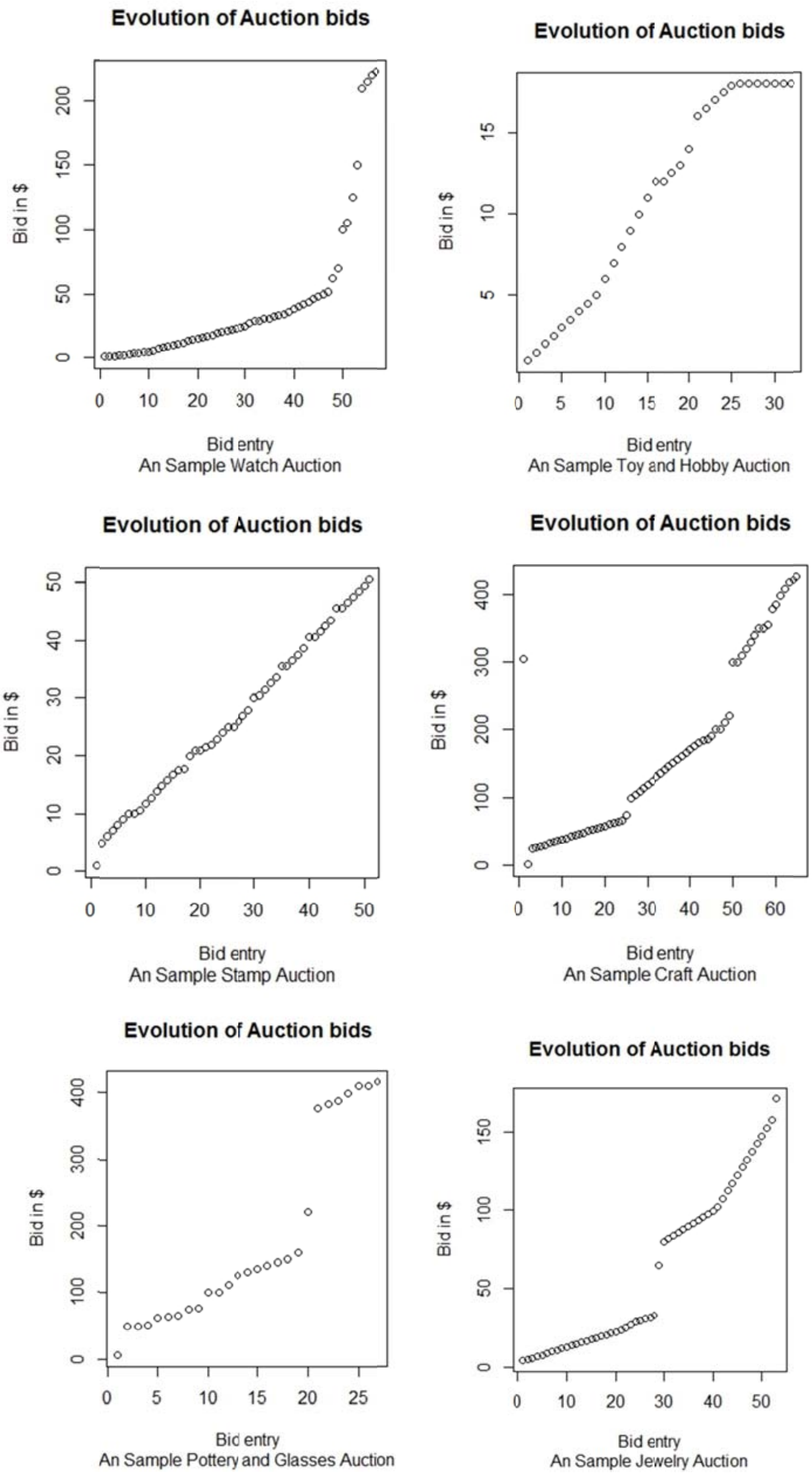


Figure 3: Evolution of number of participating bidders in six sample auctions

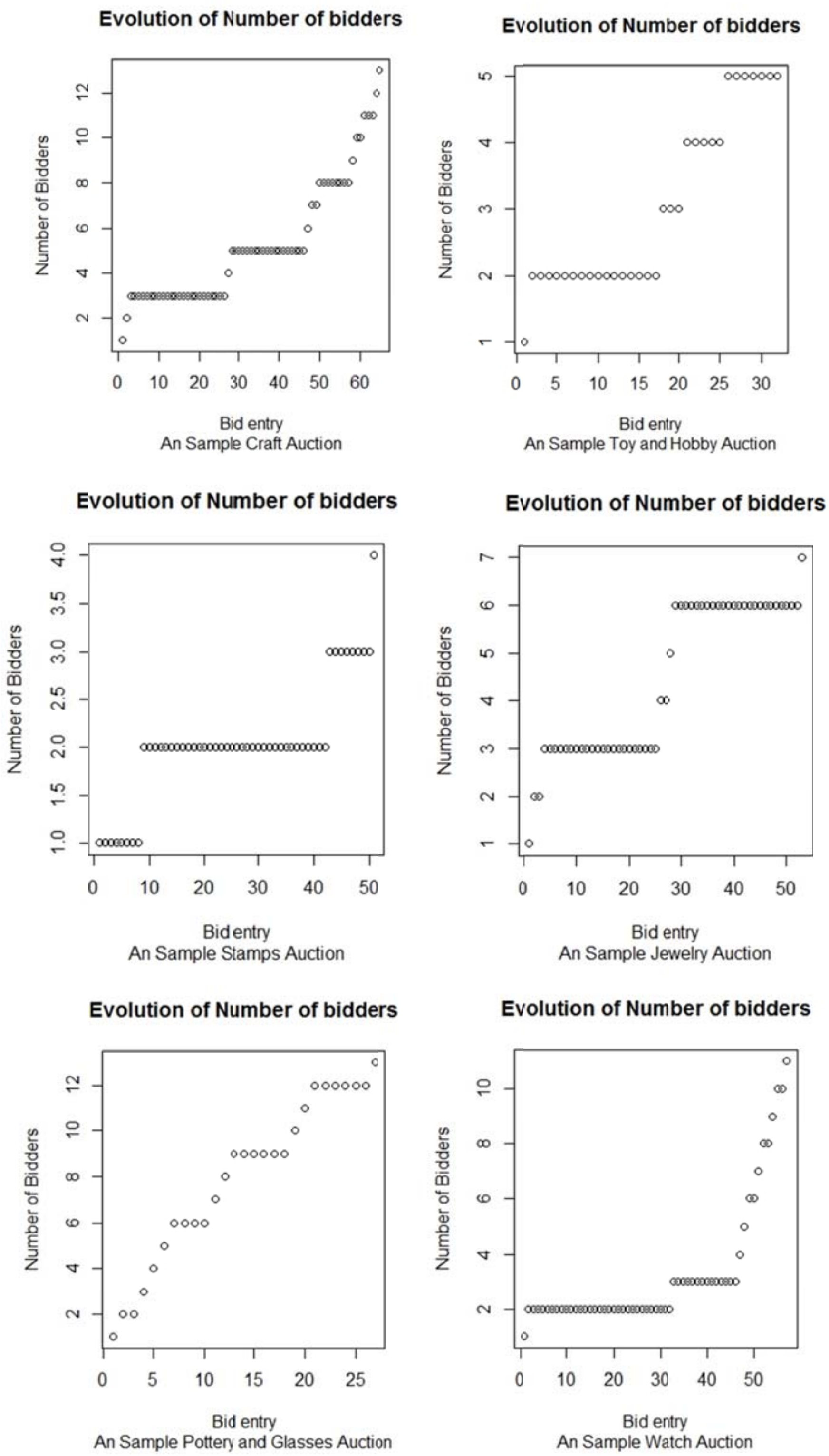


Table 1: Bidder's segment profile

Bidders 'segment Index	Segment Size	Bidders Feedback mean	STD (Bidder's feedback)	Number of Bids on This item	STD(NBTI)	total number of bids in 30 days	STD (TNB30D)	Number of items bided on in 30 days	STD (NIB30D)	Bid activity with current Seller	STD(BACS)	categories Bided on Mean	STD(NCBO)
1	488	283	292	1	1	119	92	71	64	3	4	2	1
2	120	1998	3979	20	23	1770	2143	706	942	26	34	2	1
3	463	798	1010	5	7	386	334	175	179	15	18	2	1
4	217	400	475	12	13	64	62	20	27	43	28	2	1
5	353	138	151	3	2	14	17	6	7	63	35	1	1
6	263	184	168	1	0	21	21	16	17	19	19	2	1
7	102	665	65	5	0	190	18	90	8	26	2	2	1
8	179	3661	12800	12	18	1256	1388	544	773	31	35	3	2
9	462	436	561	3	2	43	59	21	28	35	26	2	1
10	260	128	185	5	8	26	55	11	26	66	33	1	1
11	576	452	538	1	1	69	57	45	42	4	4	2	1
12	432	828	1359	9	11	317	231	151	144	10	12	2	2
13	49	658	93	5	1	189	25	89	11	25	2	2	2
14	49	658	93	5	1	189	25	89	11	26	2	2	2
15	215	900	1739	8	8	472	408	149	184	42	35	2	2
16	61	7760	24163	7	15	1188	1170	810	809	12	12	2	2
17	122	343	498	33	20	1294	1516	317	416	34	32	2	2
18	325	408	395	1	1	23	20	18	18	13	10	2	1
19	29	548	220	12	14	291	209	78	23	34	16	2	3
20	448	1102	1982	3	3	248	196	143	153	15	16	2	1
21	396	870	986	2	1	94	69	61	56	5	5	2	2
22	43	656	98	5	3	188	26	89	10	26	1	2	3
23	85	194	274	3	3	50	82	23	39	81	33	1	2
24	286	263	365	4	5	10	10	4	3	62	29	1	1
25	332	1775	2268	8	11	320	298	163	187	23	28	3	2
26	262	227	317	6	5	25	18	10	9	35	27	1	2
27	232	412	485	4	3	45	41	17	15	59	35	2	2
28	178	81	77	3	3	4	3	2	2	95	13	1	2
29	336	236	209	1	2	10	8	7	6	34	29	2	2
30	65	661	79	5	3	190	20	90	7	26	1	2	3
31	142	272	302	16	12	109	75	36	36	38	30	2	3
32	382	746	705	1	2	98	84	65	59	5	5	3	2
33	55	4168	10828	10	17	1927	2406	1004	1642	18	20	3	4
34	26	548	253	8	9	282	228	100	30	37	26	3	6
35	563	147	174	2	2	7	6	4	3	53	34	1	1
36	545	385	394	1	2	71	60	46	45	4	4	3	2
37	49	658	90	5	5	189	22	89	8	26	2	3	5
38	164	194	232	6	5	141	93	52	42	11	9	2	3
39	248	508	787	11	12	52	53	18	25	41	27	2	3
40	119	3195	11586	7	13	1580	2422	979	1747	24	26	2	4
41	319	1370	1889	7	11	444	361	194	173	5	8	3	3

42	394	442	480	3	3	48	54	23	26	34	27	2	2
43	549	120	137	4	5	7	ik3	3	2	67	32	1	2
44	263	1715	2936	15	18	554	537	205	222	28	33	3	3
45	405	681	901	6	7	224	148	101	79	7	9	2	2
46	494	219	242	2	2	54	40	29	21	5	4	3	2
47	148	3933	17383	5	4	398	802	225	463	25	11	2	4

Table 2: Auction's Cluster profile

Auction Cluster index	Cluster size	number of bidders mean	STD (number of bidders)	number of bids	STD (number of bids)	mean duration (Days)	STD (duration in Days)
1	22	9	5	38	16	5	2
2	30	9	4	41	18	5	2
3	15	9	7	39	14	5	1
4	29	8	4	44	18	4	2
5	13	9	3	37	19	5	1
6	584	11	5	58	20	5	2
7	30	9	4	35	12	5	2
8	21	8	4	35	17	5	1
9	10	6	4	32	12	6	2
10	17	11	4	50	21	4	2
11	6	3	4	3	4	3	4
12	8	7	3	32	10	6	3
13	47	8	4	43	16	5	2
14	16	8	4	42	21	5	3
15	7	9	3	34	14	7	3
16	41	10	4	49	16	5	2
17	39	10	4	49	15	5	3
18	31	10	5	48	21	5	3
19	2	15	4	14	6	11	8
20	22	9	5	47	19	6	3
21	43	10	5	52	19	5	3
22	36	10	5	50	19	5	3
23	19	9	5	39	15	6	4
24	32	10	6	49	20	5	4
25	18	13	7	45	22	5	5
26	10	9	7	37	19	7	6
27	8	8	9	34	25	7	8
28	28	8	5	39	15	6	5
29	24	10	6	41	16	6	5
30	45	9	5	44	18	5	4
31	29	9	5	44	17	5	5

32	25	10	6	49	13	6	6
33	27	9	6	41	15	6	5
34	35	11	7	45	16	5	5
35	13	10	8	38	17	7	8
36	52	10	6	45	13	5	4
37	36	10	6	43	14	6	6
38	48	10	5	46	19	5	5
39	39	11	7	49	18	5	6
40	13	12	9	40	13	7	10
41	42	10	6	45	15	5	6
42	14	11	9	40	11	8	10
43	4	14	17	22	16	15	16
44	11	12	11	36	11	8	11
45	8	12	13	35	19	10	13
46	5	15	16	31	14	14	16
47	13	12	11	47	18	8	11
48	25	11	9	48	16	7	9
49	3	23	18	38	8	20	20

Table 3: Log likelihood elements of the model

Element of the maximum a posteriori model selection criteria	Log Likelihood
Number of bidders evolution state space model	-82.2875
Bid evolution with each auction state space model	-228.623
Valuation evolution state space model	46.563

Table 4: The action regret α_i estimates across bidder's segments

Bidders Segment	Segment Size	Mean Est.	STD	5%	95%	10%	90%
Segment 1	488	-6.59788*	4.385363	-14.127	1.061332	-12.2958	-0.943
Segment 2	120	-5.74815	4.83268	-14.9388	1.976191	-11.7457	0.456597
Segment 3	463	-6.83111*	4.438043	-14.0868	0.399482	-12.6417	-1.13258
Segment 4	217	-6.39843*	4.250984	-12.8704	0.916372	-12.0758	-1.18339
Segment 5	353	-6.32749	4.564988	-13.5551	1.173994	-12.2745	0.099376
Segment 6	263	-6.93176*	4.833877	-14.6429	1.105897	-12.8324	-0.80758
Segment 7	102	-6.36438	4.879287	-14.2377	1.614317	-12.03	0.030773
Segment 8	179	-6.41941*	4.497481	-14.1729	1.132733	-12.4936	-0.64673
Segment 9	462	-6.72564**	4.263735	-14.1639	-0.0439	-12.4468	-1.33023
Segment 10	260	-6.82803**	4.332608	-14.6161	-0.2981	-12.5137	-1.72986
Segment 11	576	-6.60039*	4.473677	-13.7232	0.843274	-12.1107	-0.95723
Segment 12	432	-6.92419*	4.599975	-14.5303	1.286586	-12.8024	-1.06383
Segment 13	49	-7.06965**	4.834565	-14.1926	-0.76561	-12.922	-1.29117
Segment 14	49	-6.26347	5.711184	-14.5781	3.341684	-12.0053	0.972296
Segment 15	215	-6.74227*	5.216267	-14.9636	1.537699	-12.6673	-0.53761
Segment 16	61	-6.10349	5.552944	-14.0383	2.060269	-12.9231	0.160451
Segment 17	122	-5.92055*	4.727238	-13.9476	0.820128	-11.2676	-0.80294
Segment 18	325	-6.45741*	4.539079	-14.0632	0.162873	-12.2262	-0.97404
Segment 19	29	-5.9259	6.537305	-13.8831	12.58444	-11.3711	0.617119
Segment 20	448	-6.79864*	4.610808	-14.4232	0.652436	-12.3027	-1.27342
Segment 21	396	-6.479*	4.503804	-13.4727	0.563265	-11.975	-0.97399
Segment 22	43	-6.12586*	6.180197	-14.9384	1.326045	-13.4866	-0.07025
Segment 23	85	-7.31348*	5.36302	-14.2035	0.27507	-12.9519	-2.07948
Segment 24	286	-6.45984*	4.782971	-13.8371	1.358531	-12.3894	-0.47173
Segment 25	332	-6.96675*	4.404076	-13.4738	0.056201	-12.1412	-1.98827
Segment 26	262	-6.48957**	4.495116	-13.6588	-0.12138	-11.675	-0.92995
Segment 27	232	-6.50829*	5.168312	-14.6342	1.896561	-12.5778	-0.32229
Segment 28	178	-6.67582*	5.098924	-13.9043	1.801829	-11.9032	-0.54067
Segment 29	336	-6.60015*	4.789981	-13.8511	0.662597	-12.2426	-0.77731
Segment 30	65	-6.14262*	5.840079	-12.2442	0.441811	-11.6712	-1.08393
Segment 31	142	-6.69621*	5.679877	-14.2183	1.94436	-13.1771	-0.10525
Segment 32	382	-6.62021*	4.966391	-13.9965	0.77137	-12.5596	-1.12274
Segment 33	55	-4.39333	7.228549	-13.5963	5.484088	-12.5707	2.403765
Segment 34	26	-4.60211	8.681276	-12.6346	22.78317	-12.0711	0.871366
Segment 35	563	-6.89954*	4.696307	-13.837	0.162248	-12.5504	-1.25052
Segment 36	545	-6.84435*	4.644226	-14.1768	0.078593	-12.4737	-1.34088
Segment 37	49	-5.35242	7.59	-15.2464	1.172862	-12.2226	0.535383
Segment 38	164	-6.77371**	5.196898	-13.7503	-0.18745	-12.2146	-1.74924

Segment 39	248	-6.45754*	5.063377	-13.9684	0.044267	-12.1617	-1.11009
Segment 40	119	-6.39713*	6.021775	-14.2002	1.183805	-12.1395	-0.65236
Segment 41	319	-6.57609*	5.281095	-13.9875	0.609474	-12.8205	-0.51056
Segment 42	394	-6.37481*	5.138428	-14.088	0.603729	-12.6476	-0.72518
Segment 43	549	-6.99609**	4.782886	-14.0277	-0.40901	-12.4503	-1.71706
Segment 44	263	-6.39811*	5.179005	-13.0538	0.609576	-11.8459	-1.13029
Segment 45	405	-6.41977*	5.221899	-14.1323	1.329889	-12.2877	-0.70067
Segment 46	494	-6.73344*	5.023136	-14.1553	0.61188	-12.2872	-1.09443
Segment 47	148	-6.30799*	6.063024	-13.6135	0.474432	-12.1698	-1.66993

** Two tail 0.95% confidence interval significance

* One tail 0.95% confidence interval significance

Table 5: The inaction regret β_i estimates across bidder's segments

Bidders Segment	Segment Size	Mean Est.	STD	5%	95%	10%	90%
Segment 1	488	-3.49503	4.369664	-11.0485	3.548309	-9.14257	1.998304
Segment 2	120	-3.60917	4.477701	-11.6079	3.350969	-8.90072	2.116756
Segment 3	463	-3.49157	4.566079	-10.8883	4.155156	-9.30653	2.801798
Segment 4	217	-3.69072	4.348169	-11.5198	4.329198	-9.30478	1.943703
Segment 5	353	-3.376	4.490566	-10.6656	4.203978	-9.27767	2.67269
Segment 6	263	-3.20599	4.220277	-10.3737	3.838775	-8.5587	2.484127
Segment 7	102	-2.93112	4.57716	-10.8362	5.337365	-8.98848	3.298663
Segment 8	179	-3.57165	4.32095	-10.3003	3.57352	-9.0537	1.67986
Segment 9	462	-3.24432	4.23216	-10.1799	3.694375	-8.76424	1.982153
Segment 10	260	-3.55495	4.5838	-11.244	4.057138	-9.14529	1.995664
Segment 11	576	-3.61861	4.530564	-11.3952	3.65111	-9.34066	2.379432
Segment 12	432	-3.37635	4.391185	-10.6544	3.701603	-8.67616	2.553075
Segment 13	49	-2.94678	4.426745	-9.64687	6.182513	-7.68881	2.667957
Segment 14	49	-2.95447	4.387125	-9.96758	4.857141	-7.98116	1.738211
Segment 15	215	-3.48916	4.388858	-10.8133	3.737174	-9.18774	1.889057
Segment 16	61	-2.74469	4.888393	-10.7513	5.583138	-9.40472	2.714705
Segment 17	122	-3.20187	4.572875	-11.3042	4.137759	-8.42995	2.197123
Segment 18	325	-3.35694	4.760805	-10.9503	4.05766	-9.22607	2.229552
Segment 19	29	-4.18	5.898522	-13.2073	11.35804	-9.92599	3.124299
Segment 20	448	-3.18814	4.355796	-10.6994	3.836015	-8.70436	2.181738
Segment 21	396	-3.86751	4.546917	-11.4478	3.194066	-9.58499	1.214394
Segment 22	43	-2.68284	5.307566	-11.1252	3.699268	-8.69305	2.04559
Segment 23	85	-2.39615	4.902431	-10.0686	5.393295	-7.67153	4.163476
Segment 24	286	-3.57262	4.46745	-10.6264	3.31251	-9.00586	1.696506
Segment 25	332	-3.11764	4.665114	-10.2996	4.397015	-8.54335	2.716452
Segment 26	262	-3.61154	4.820882	-11.1293	4.018778	-9.57123	2.038403
Segment 27	232	-3.50599	4.863605	-11.7318	3.602241	-9.767	2.422991

Segment 28	178	-3.33215	4.976925	-10.7282	3.999016	-8.95647	2.049024
Segment 29	336	-3.50574	4.747387	-11.2797	3.956603	-9.29748	2.346223
Segment 30	65	-3.93065	6.308964	-10.8795	6.856421	-10.1406	2.491108
Segment 31	142	-4.07281	4.983307	-10.8663	2.635468	-9.20071	1.313656
Segment 32	382	-3.16531	4.677028	-10.0818	3.674808	-8.67447	2.280492
Segment 33	55	-1.52811	6.685849	-9.74323	8.285319	-8.57665	4.379244
Segment 34	26	-2.15118	8.356758	-10.6195	23.86687	-9.02178	3.599635
Segment 35	563	-3.38355	4.406254	-9.85132	3.280458	-8.78781	1.931637
Segment 36	545	-3.58915	4.623178	-10.8499	3.572652	-9.10851	2.224139
Segment 37	49	-2.69309	6.704936	-9.6116	3.524795	-7.84957	2.181807
Segment 38	164	-2.75089	5.299423	-10.4567	4.988713	-8.08511	2.391766
Segment 39	248	-3.1793	5.214039	-10.9252	3.39566	-9.20164	2.461528
Segment 40	119	-2.27205	5.645967	-10.5855	4.325154	-8.00871	2.95008
Segment 41	319	-3.50047	5.134404	-10.6636	4.320405	-9.28684	2.029507
Segment 42	394	-3.1766	4.911791	-10.4672	4.13051	-9.08833	2.470855
Segment 43	549	-3.28276	4.82278	-11.1672	3.853893	-9.0521	2.532665
Segment 44	263	-3.01748	5.120434	-10.862	3.361885	-9.19647	1.654234
Segment 45	405	-3.36035	4.850862	-10.5275	3.586063	-8.86302	2.186339
Segment 46	494	-3.34817	4.958881	-10.665	4.184209	-9.11308	2.196692
Segment 47	148	-3.24013	5.970723	-10.7449	4.05388	-9.99394	1.688613

** Two tail 0.95% confidence interval significance
* One tail 0.95% confidence interval significance

Table 6: The update of valuation parameters δ_i and ρ_i estimates across bidder's segments

Bidders Segment	Segment Size	Valuation growth δ_i	STD (δ_i)	Highest bid update ρ_i	STD (ρ_i)
Segment 1	488	3.69	2.74	3.65	2.74
Segment 2	120	4.03	3.08	3.19	2.42
Segment 3	463	3.38	2.65	3.62	2.79
Segment 4	217	3.60	2.79	3.40	2.51
Segment 5	353	3.53	2.65	3.49	2.59
Segment 6	263	3.55	2.70	3.40	2.64
Segment 7	102	3.30	2.68	3.56	2.43
Segment 8	179	3.53	2.65	3.24	2.41
Segment 9	462	3.56	2.67	3.63	2.73
Segment 10	260	3.83	3.14	3.57	2.81
Segment 11	576	3.63	2.75	3.56	2.66
Segment 12	432	3.65	2.89	3.43	2.80
Segment 13	49	3.57	3.13	4.42	2.97
Segment 14	49	3.82	2.97	4.07	2.80
Segment 15	215	3.90	2.97	3.80	2.80
Segment 16	61	3.56	2.80	3.28	3.00
Segment 17	122	3.67	2.89	3.72	3.10
Segment 18	325	3.93	2.91	3.78	2.76
Segment 19	29	4.14	4.04	3.94	3.61
Segment 20	448	3.66	2.84	3.49	2.81
Segment 21	396	3.60	2.89	3.41	2.82
Segment 22	43	3.66	3.92	4.20	3.68
Segment 23	85	3.94	3.51	3.68	3.37
Segment 24	286	3.66	3.04	4.14	3.04
Segment 25	332	3.67	3.07	3.73	3.03

Segment 26	262	3.37	2.82	3.68	2.90
Segment 27	232	3.76	2.98	3.41	2.94
Segment 28	178	3.23	3.07	3.82	3.44
Segment 29	336	3.70	3.07	3.61	3.00
Segment 30	65	4.36	4.30	3.40	4.22
Segment 31	142	3.75	3.51	3.86	3.50
Segment 32	382	3.76	3.21	3.87	2.97
Segment 33	55	3.89	4.96	4.03	4.72
Segment 34	26	5.35	6.58	5.59	6.52
Segment 35	563	3.55	2.99	3.68	3.06
Segment 36	545	3.60	3.00	3.51	3.02
Segment 37	49	4.37	5.79	5.31	5.51
Segment 38	164	3.59	3.86	3.46	3.57
Segment 39	248	3.74	3.74	3.66	3.47
Segment 40	119	3.86	4.30	4.43	4.30
Segment 41	319	3.80	3.27	3.56	3.46
Segment 42	394	3.88	3.32	3.37	3.12
Segment 43	549	3.81	3.20	3.59	3.14
Segment 44	263	3.67	3.56	3.68	3.57
Segment 45	405	3.66	3.56	3.18	3.25
Segment 46	494	3.63	3.44	3.57	3.20
Segment 47	148	3.91	4.75	3.92	4.56

Table 7: The growth of bids and their drift parameters, τ_i and γ_i, and the rush of bidders at the end of auction rate and average entrance rate in each period, η_j and ι_j, estimates across auction segments

Auction Cluster	Auction Cluster Size	growth of bids (τ_i)	STD (τ_i)	Drift of bids (γ_i)	STD (γ_i)	Last minute flood (η_j)	STD(η_j)	Mean entrance rate (ι_j)	STD (ι_j)
1	22	3.03	1.51	5.62	3.07	3.70	2.58	3.09	2.97
2	30	3.41	2.81	6.49	2.91	3.32	1.80	3.62	2.40
3	15	5.54	3.24	4.39	2.13	4.23	4.02	3.51	2.91
4	29	3.04	1.78	6.46	3.94	3.38	2.46	3.44	2.30
5	13	4.80	2.69	5.42	2.75	3.93	1.99	4.31	3.85
6	584	3.92	2.85	5.94	3.72	3.70	2.70	3.75	2.88
7	30	4.24	3.13	5.21	4.12	4.12	2.60	4.05	2.28
8	21	3.97	2.78	7.32	3.54	4.24	1.88	3.71	3.36
9	10	4.37	3.19	4.35	3.21	4.41	2.20	4.55	3.48
10	17	4.73	2.23	7.22	3.55	4.33	2.60	5.44	2.86
11	6	4.77	3.89	7.23	3.55	6.33	4.40	3.39	3.74
12	8	4.16	3.96	6.25	3.79	3.32	3.36	4.33	3.34
13	47	3.34	2.77	6.87	3.80	3.96	3.29	5.01	3.67
14	16	3.64	3.23	6.68	4.88	3.71	3.54	4.75	3.66
15	7	5.32	4.37	5.98	6.15	4.40	4.70	4.06	4.96
16	41	3.64	3.01	5.73	3.71	3.46	3.22	4.49	2.99
17	39	4.12	3.22	6.95	4.45	3.68	3.31	4.18	3.81
18	31	4.25	3.69	6.87	4.57	3.56	3.53	4.18	3.36
19	2	11.16	7.84	10.73	8.27	11.42	7.58	12.51	6.49
20	22	4.50	4.10	8.12	4.64	3.42	4.15	4.16	4.21
21	43	3.36	3.53	7.30	5.07	3.57	3.42	3.68	3.70

22	36	4.10	4.12	6.53	5.12	4.32	4.41	4.07	3.93
23	19	4.55	5.04	6.93	4.79	4.56	5.04	5.36	4.88
24	32	3.52	4.42	5.44	4.79	3.55	4.33	4.83	4.27
25	18	6.07	6.35	8.55	6.27	4.36	5.63	5.17	5.77
26	10	6.22	6.84	7.79	7.15	4.53	7.37	5.79	7.27
27	8	6.54	8.20	8.77	7.48	7.08	7.93	7.94	7.84
28	28	4.62	5.30	7.57	5.36	4.64	5.14	4.68	5.22
29	24	5.54	6.11	6.99	5.97	4.07	5.65	5.81	5.64
30	45	3.83	4.73	6.75	4.66	4.39	4.62	4.40	4.61
31	29	4.04	5.57	6.49	6.28	4.49	5.61	3.77	5.42
32	25	4.76	6.29	6.89	6.05	4.19	6.17	4.32	6.37
33	27	4.76	6.16	7.09	6.41	4.48	5.96	5.12	6.38
34	35	5.12	5.67	8.60	5.60	4.27	5.63	4.37	5.79
35	13	6.64	8.51	8.90	8.42	6.12	8.69	5.72	8.71
36	52	4.48	5.00	6.16	5.36	3.83	5.09	4.74	5.39
37	36	4.37	6.14	7.69	6.72	4.68	6.07	4.99	6.02
38	48	4.54	5.65	6.79	5.41	4.67	5.82	4.31	5.79
39	39	4.39	6.26	5.72	6.73	3.87	6.15	5.26	6.32
40	13	7.18	9.95	8.65	9.72	6.81	9.97	5.75	10.14
41	42	4.75	6.29	6.90	6.74	4.53	6.19	4.46	6.30
42	14	6.68	10.26	8.31	10.18	6.47	10.07	6.79	10.09
43	4	16.11	15.66	12.44	17.76	13.31	17.15	12.88	17.39
44	11	5.69	12.33	8.70	11.41	8.33	11.77	7.26	11.85
45	8	9.06	13.78	11.14	13.01	9.27	13.81	10.30	13.32
46	5	12.02	17.02	11.82	17.29	10.28	17.88	11.01	17.57
47	13	7.21	11.66	9.78	11.39	6.38	12.13	7.45	11.80
48	25	5.22	9.39	8.19	8.94	5.51	9.03	5.81	9.17
49	3	19.69	20.74	17.25	22.47	17.42	22.35	18.39	21.66

Figure 4: Histogram of valuation evolution and regret parameters of bidders across different bidder segments

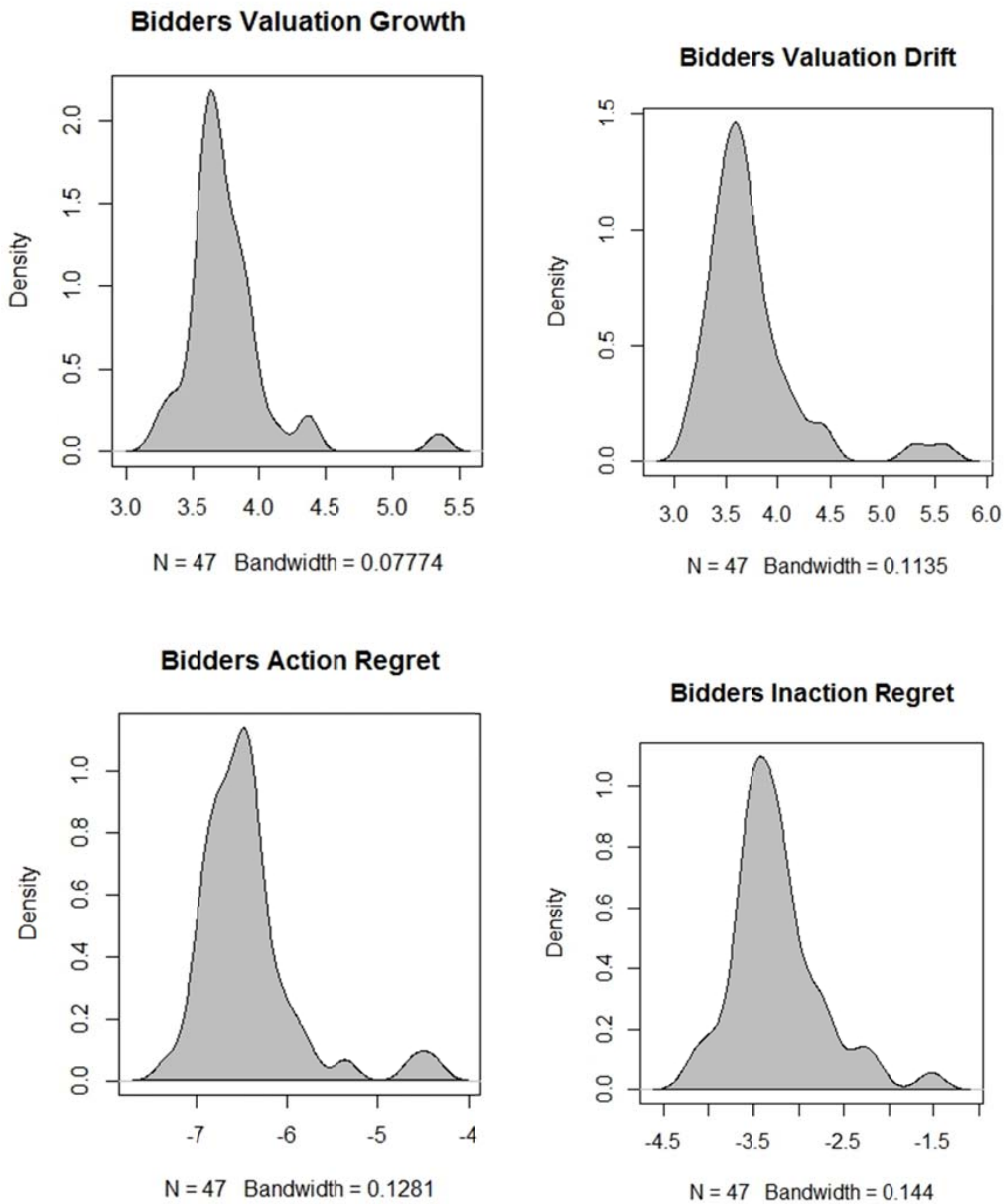


Figure 5: Histogram of bid value and the number of bidders evolution parameters across different auction clusters

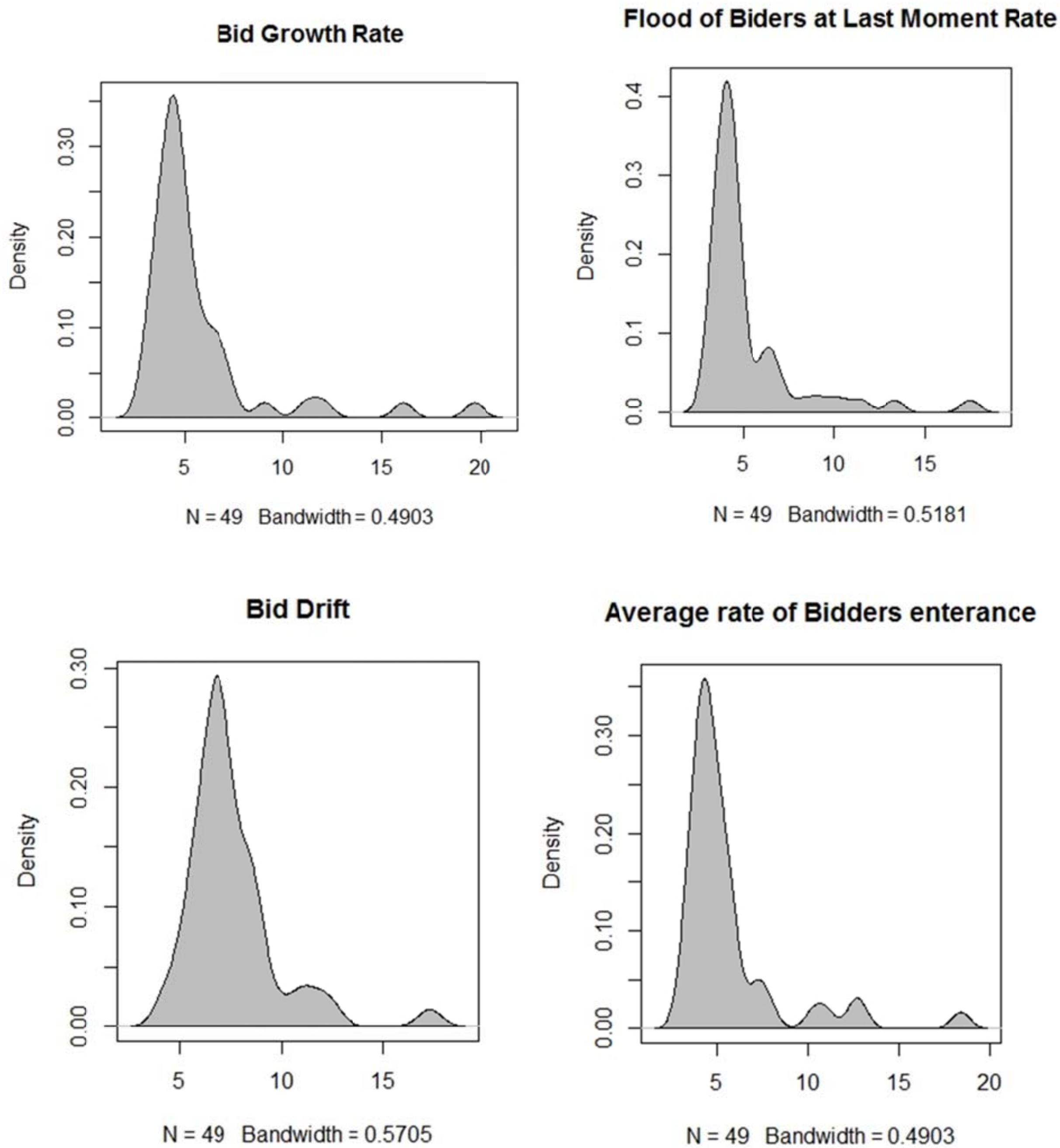
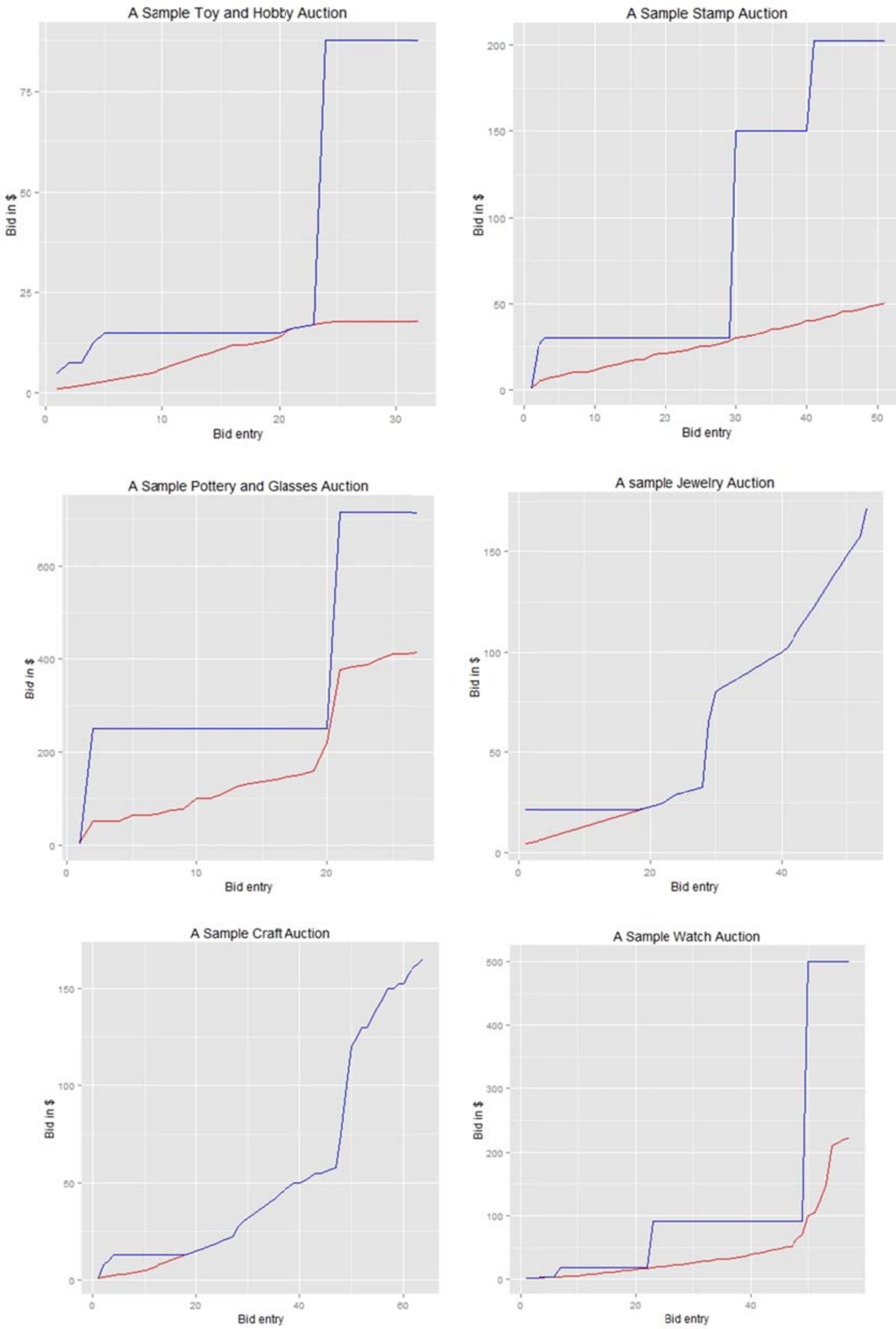


Figure 6: Counterfactual analysis of 5% less action regret impact on the bidding strategies (blue line the optimal bidding under 5% less action regret, and red line the observed bids)



Appendix A:

A.1. Latent Dirichlet Allocation

LDA is a three-level hierarchical Bayesian model, in which each item of a collection is modeled as a finite mixture over an underlying set of topics (Blei et al 2003). LDA is a generative approach; it use naïve conditional independence assumption, and it neglect the order of features by assuming exchangeability and using bag of words representation. These assumptions bring two main benefits to these approaches: simplicity, computational efficiency. Formally the LDA model assumes the following generative process for each item i in a collection C consisting of element (feature) e:

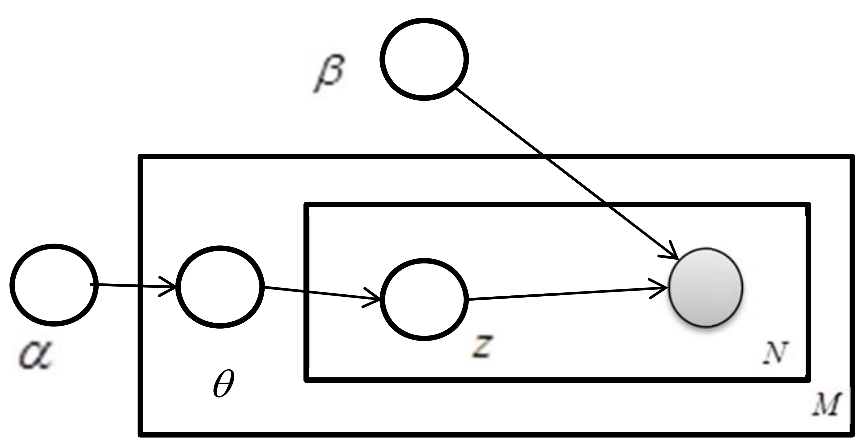
1. Choose N ~ Poisson (ξ), where N is the number of elements e
2. Choose $\theta \sim Dir(\alpha)$, where θ is the probability that a given document has primitive topic
3. For each of the N features i_n :
    a. Choose a topic $z_n \sim Multinomid(\theta)$
    b. Choose a feature i_n from $p(i_n \mid z_n, \beta)$, a multinomial probability conditioned on the topic

A k-dimensional Dirichlet random variable θ can take values in the (k-1)-simplex (a k-vector θ lies in the (k-1)-simplex if $\theta_i \geq 0, \sum_{i=1}^{k} \theta_i = 1$), and has the following probability density on this simplex:

$$p(\theta \mid \alpha) = \frac{\Gamma(\sum_{i=1}^{k} \alpha_i)}{\prod_{i=1}^{k} \Gamma(\alpha_i)} \theta_1^{\alpha_1 - 1} \backslash ... \theta_k^{\alpha_{k1} - 1}$$

We represented the Probability Graphical Model (PGM) of LDA in figure 4. As figure depicts, there are three levels to the LDA representation. The parameters α, β are collection level parameters, and they are sampled once. The variable θ_d has Dirichlet distribution, and it is document level variable, so it is sampled once per document. This variable simply defines the weight distribution of topics within the document. Finally variables z_{d_n} and w_{d_n} are feature level parameters and they are sampled once for each feature within each document. Variable z_{d_n} defines the topic of n'ths word within document d, and variable w_{d_n} defines the feature instance that appears at location n within document d. As we can see an LDA model is a type of conditionally independent hierarchical model, and it is often referred to as parametric empirical Bayes model. One of the advantages of an LDA model is that it is parsimonious, so unlike probabilistic Latent Semantic Indexing (pLSI) model, it does not suffer from over fitting.

Figure 4: Graphical model representation of LDA



To estimate LDA model we define the likelihood of model in the following:

$$p(D \mid \alpha,\beta) = \prod_{d=1}^{M} \int p(\theta_d \mid \alpha)\left(\prod_{n=1}^{N_d} \sum_{z_{d_n}} p(z_{d_n} \mid \theta_d) p(w_{d_n} \mid z_{d_n},\beta) d\theta_d\right.$$

The key inferential problem to solve for LDA is computing posterior distribution of topic hidden variables θ_d, z_d, the first one with Dirichlet distribution, and the second one with multinomial distribution. To normalize the distribution of words given α,β we marginalize over the hidden variables as following:

$$p(D \mid \alpha,\beta) = \prod_{d=1}^{M} \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \int \left(\prod_{i=1}^{k} \theta_i^{\alpha_i - 1}\right)\left(\prod_{n=1}^{N_d} \sum_{i=1}^{k} \prod_{j=1}^{V} (\theta_i \beta_{ij})^{w_n^j} d\theta\right.$$

Due to the coupling between θ and β in the summation over latent topics this likelihood function is intractable. Therefore to estimate it Blei et al. (2003) suggests using variational inference method. Variational inference or variational Bayesian refers to a family of techniques for approximating intractable integrals arising in Bayesian inference and machine learning. These family of methods are an alternative to sampling methods, and they are basically used to analytically approximate the posterior probability of the unobservable variables, in order to do statistical inference over these variables. These methods also give a lower bound to the marginal log likelihood. This family of lower bounds is indexed by a set of variational parameters. To obtain tightest lower bound we use an optimization procedure to select the variational parameters. A simple way to obtain a tractable family of lower bounds is to consider simple modifications of the original graphical model, by removing dependencies and introducing new variational parameters instead. In the LDA model we used following variational distribution to approximate posterior distribution of unobserved variables given the observed data s follows:

$$q(\theta, z \mid \gamma, \phi) = q_1(\theta \mid \gamma) \prod_{n=1}^{N} q_2(z_n \mid \phi_n)$$

Where $q_1(.)$ is a Dirichlet distribution with parameters γ and $q_2(.)$ is a multinomial distribution with parameters ϕ_n. Variational parameters are result of solving the following optimization problem:

$$(\gamma^*, \phi^*) = \arg\min_{(\gamma,\phi)} D_{KL}(q(\theta, z \mid \gamma, \phi) \,||\, p(\theta, z \mid w, \alpha, \beta))$$

where D_{KL} represents the Kullback-Leibler (KL) divergence between the variational distribution and the true joint posterior of latent parameters $p(\theta, z \mid w, \alpha, \beta)$. Formally, D_{KL} is defined as follows:

$$D_{KL}(q(\theta, z \mid \gamma, \phi) \,||\, p(\theta, z \mid w, \alpha, \beta) = \sum_{(\gamma,\phi)} q(\theta, z \mid \gamma, \phi) \log(\frac{q(\theta, z \mid \gamma, \phi)}{p(\theta, z \mid w, \alpha, \beta)})$$

As a result we can write KL-divergence in the following format:

$$Logp(w \mid \alpha, \beta) = L(\gamma, \phi; \alpha, \beta) + D_{KL}(q(\theta, z \mid \gamma, \phi) \,||\, p(\theta, z \mid w, \alpha, \beta))$$

where

$$L(\gamma, \phi; \alpha, \beta) = E_q[\log p(\theta, z, w \mid \alpha, \beta)] - E_q[\log q(\theta, z)]$$

This relation suggests that maximizing the lower bound $L(\gamma,\phi;\alpha,\beta)$ with respect to γ and ϕ is equivalent to minimizing the KL divergence between the variational posterior probability and the true posterior probability. Expanding $L(\gamma,\phi;\alpha,\beta)$ using factorization of p and q gives the following:

$$
\begin{aligned}
L(\gamma,\phi;\alpha,\beta) &= E_q[\log p(\theta \mid \alpha)] + E_q[\log p(z \mid \theta)] + E_q[\log p(w \mid z,\beta)] - E_q[\log q(\theta)] - E_q[\log q(z)] \\
&= \log\Gamma(\textstyle\sum_{j=1}^{k}\alpha_j) - \sum_{i=1}^{k}\log\Gamma(\alpha_i) + \sum_{i=1}^{k}(\alpha_i - 1)(\Psi(\gamma_i) - \Psi(\sum_{j=1}^{k}\gamma_j)) + \sum_{n=1}^{N}\sum_{i=1}^{k}\phi_{ni}(\Psi(\gamma_i) - \Psi(\sum_{j=1}^{k}\gamma_j)) \\
&\quad - \log\Gamma(\textstyle\sum_{j=1}^{k}\gamma_j) - \sum_{i=1}^{k}\log\Gamma(\gamma_i) + \sum_{i=1}^{k}(\gamma_i - 1)(\Psi(\gamma_i) - \Psi(\sum_{j=1}^{k}\gamma_j)) + \sum_{n=1}^{N}\sum_{i=1}^{k}\phi_{ni}\log\phi_{ni}
\end{aligned}
$$

Where $\Gamma(.)$ is gamma function and $\Psi(.)$ is its derivative. They key for this derivation is the following equation: $E[\log\theta_i \mid \alpha] = \Psi(\alpha_i) - \Psi(\sum_{j=1}^{k}\alpha_j)$, which is direct derivative of general fact that the derivative of log normalization factor with respect to the natural parameter of an exponential distribution is equal to the expectation of sufficient statistics. Collecting terms that are only related to each of the variational parameters γ and ϕ_{ni} from $L(\gamma,\phi;\alpha,\beta)$, and getting the derivative respectively give us an algorithm to solve the above optimization problem to find variational parameters. In particular, we can use simple iterative fixed-point method and update two variational parameters by the following equations until convergance:

$$
\begin{aligned}
&\phi_{ni} \propto \beta_{iw_n}\exp\{E_q[\log(\theta_i) \mid \gamma]\} \\
&\gamma_i = \alpha_i + \textstyle\sum_{n=1}^{n}\phi_{ni}
\end{aligned}
$$

This optimization is document specific, so we view the Dirichlet parameter $\gamma^*(w)$ as providing a representation of a document in the topic simplex. In summary we have the following variational inference algorithm for LDA (Blei et al 2003):

(1) Initialize $\phi_{ni}{}^0 := 1/k$ for all i and n

(2) Initialize $\gamma_i := \alpha_i + N/k$ for all i and n

(3) **Repeat**

a. **For** n=1 **to** N

i. **For** i = 1 **to** k

1. $\phi_{ni}^{t+1} := \beta_{iw_n} \exp(\Psi(\gamma_i'))$

ii. Normalize ϕ_{ni}^{t+1} to sum to 1

b. $\gamma^{t+1} := \alpha + \sum_{n=1}^{N} \phi_n^{t+1}$

(4) **until** convergence

This algorithm has the order of $O(N^2 k)$. Given the variational Bayesian method we have tractable lower bound on the log likelihood, a bound which we can maximize with respect to α and β. We can thus find approximate empirical Bayes estimates for the LDA model via an alternating variational EM (VEM) procedure that maximizes a lower bound with respect to variational parameters γ and ϕ, and then, for fixed values of the variational parameters, maximizes the lower bound with respect to the model parameters α and β. The VEM algorithm is defined in the following:

1. (E-step) For each document, find the optimization value of the variational parameters $\{\gamma_d^*, \phi_d^* : d \in D\}$. This is done as described in the above variational inference algorithm.
2. (M-step) Maximize the resulting lower bound on the log likelihood with respect to the model parameters α and β. This corresponds to finding the maximum likelihood estimates with expected sufficient statistics for each document under the approximate posterior which is computed in the E-step. The update for the conditional multinomial parameter β can be written out analytically as:

$$\beta_{ij} \propto \sum_{d=1}^{M} \sum_{n=1}^{N_d} \phi_{dni}^{*} w_{dn}^{j}$$

The last concern about LDA is to make sure that sparsity does not make the likelihood zero, an extended graphical model with prior on β, where β is a k*V random matrix(k number of topics and V number of features, a row for each component), with independence identically Dirichlet distributed with parameter η rows assumption. Now β_i can be treated as a random variable to be endowed to the posterior distribution of hidden variables, giving us the following variational distribution with independence assumption:

$$q(\beta_{1:M}, z_{1:M}, \theta_{1:M} \mid \lambda, \gamma, \phi) = \prod_{i=1}^{k} Dir(\beta_i \mid \lambda_i) \prod_{d=1}^{M} q_d(\theta_d, z_d \mid \gamma_d, \phi_d)$$

To account for this modification, we only need to change the variational inference algorithm by augmenting the following update of variational parameter λ as follows:

$$\lambda_{ij} = \eta + \sum_{d=1}^{M} \sum_{n=1}^{N_d} \phi_{dni}^{*} w_{dn}^{j}$$

This equation finalizes our plot of VEM algorithm to estimate an LDA model. There is an alternative approach proposed by Phan et al. (2008) that uses Gibbs sampling to estimate an LDA model. This approach draws from the posterior distribution of p(z|w) by sampling as follows:

$$p(z_i = K \mid w, z_{-i}) \propto \frac{n_{-i,K}^{(j)} + \delta}{n_{-i,K}^{(.)} + V\delta} \frac{n_{-i,K}^{(d_i)} + \alpha}{n_{-i,.}^{(d_i)} + k\alpha}$$

where z_{-i} is the vector of current topic memberships of all words without the i'th word w_i. The index j indicates that w_i is equal to the j'th term in the vocabulary. $n_{-i,K}^{(j)}$ gives how often the j'th term of the vocabulary is currently assigned to topic K without the i'th word, and the dot implies the summation over all relevant index instances. d_i indicates the document in the collection to which the word w_i belongs to. In this Bayesian formulation δ and α are the prior parameters for the term distribution of topics β and the topic distribution of documents θ, respectively. The predictive distribution of the parameter θ and β given w and z are given by:

$$\hat{\beta}_K^{(j)} = \frac{n_{-i,K}^{(j)} + \delta}{n_{-i,K}^{(.)} + V\delta} \qquad \hat{\theta}_K^{(d)} = \frac{n_{-i,K}^{(d_i)} + \alpha}{n_{-i,.}^{(d_i)} + k\alpha}$$

The likelihood for the Gibbs sampling also has the following form:

$$\log(p(w \mid z)) = k \log(\frac{\Gamma(V\delta)}{\Gamma(\delta)^V}) + \sum_{K=1}^{k} \{[\sum_{j=1}^{V} \log(\Gamma(n_K^{(j)} + \delta))] - \log(\Gamma(n_K^{(.)} + V\delta))\}$$

Dirichlet Process

As Blei and Jordan (2006) suggest, the DP can be used for nonparametric prior in a hierarchical Bayesian model. The process looks as follows:

$$\begin{aligned} &G \sim DP(\alpha, G_0) \\ &\eta_n \sim G \\ &X_n \sim p(. \mid \eta_n) \end{aligned}$$

where α is scaling parameter, and G0 is baseline Dirichlet distribution. As the parameter are drawn from G, the data themselves will partition according to the drawn values from the same parameters. It is a form of infinite mixture model, in which we draw the parameters either from one of the partitions of parameters we have seen before, or from a new partition. This process is sometimes referred to as Polya's urn or Chinese restaurant process. Another view suggests a stick breaking construction of G, by considering $V_i \sim Beta(1,\alpha)$ and $\eta_i^* \sim G$ for $i = \{1,2,...\}$. As a result formally we can define G and the proportions θ_i of each of the infinite pieces of stick relative to original unit-length stick with size proportional to number of draws from a distribution as:

$$\theta_i = V_i \prod_{j=1}^{i-1} (1 - V_i)$$
$$G(\eta) = \sum_{i=1}^{\infty} \theta_i \delta(\eta, \eta_i^*)$$

As Blei and Jordan (2006) suggest, θ comprises the infinite vector of mixing proportions and $\eta_{1:\infty}^*$ are the infinite number of mixture components. We denote Z_n as the mixture component with which X_n is associated. Therefore the data generating process for DP is as follows:

1. Draw $V_i \sim Beta(1,\alpha), i = \{1,2,...\}$
2. Draw $\eta_i \sim G_0,, i = \{1,2,...\}$
3. For each data point n:
    a. Draw $Z_n \sim Mult(\theta)$
    b. Draw $X_n \sim F(\eta_{z_n})$

To estimate this model Blei and Jordan (2006) suggest we truncate this construction at K, by setting $V_{K-1} = 1$, which translates into $\theta_k = 0, k > K$. It has shown that the truncated Dirichlet process (TDP), closely approximates a true Dirichlet process for K chosen large enough relative to the number of data. To estimates this model we use Variational Bayesian approximate of variational distribution, and its prameters. The Jensen's inequality suggests, a lower bound for log-likelihood as:

$$\log p(x) = \log \int_h p(x,h)dh = \log \int_h \frac{q(h)p(x,h)}{q(h)} dh \geq \int_h q(h)\log(p(x,h))dh - \int_h q(h)\log(q(h))dh$$
$$= E[\log p(x,H)] - E[\log q(H)]$$

The above inequality can intuitively be explained by the concavity of the log function, and it should be satisfied with an arbitrary distribution q(h). H denotes the hidden variables, and x denotes the observations. They key behind variational methods is to restrict q(h) to a parametric family such that optimizing the bound is tractable. The solution is usually straight forward by considering the natural parameter and sufficient statistic of specific family of distributions. Penny (2001) calculated Kullback-Leibler (KL) divergence or relative entropy of of Normal, Gamma, Dirichlet and Wishart densities. For DP mixture Blei and Jordan (2006) apply mean filed variational approach for the stick-breaking construction. Hidden variables of the model are V, η^* and Z, and coupling them in the likelihood makes it analytically intractable. Thus, we have to introduce a variational distribution $q(v, \eta^*, z)$, in which all the hidden variables are independent, as we factorize this variational distribution. As a result our factorized variational distribution can be written as:

$$q(v, \eta^*, z, K) = \prod_{i=1}^{K} q(v_i \mid \gamma_i) \prod_{i=1}^{K} q(\eta_i^* \mid \tau_i) \prod_{n=1}^{N} q(z_n \mid \phi_n)$$

where γ are the Beta parameters for the distributions on V_i , τ are natural parameters for the distributions on η_i^*, and ϕ are multinomial parameters for the distribution on Z_n. Therefore the lower bound on the likelihood by K-L divergence criteria can be written as:

$$\log p(x \mid \alpha, \lambda) \geq E[\log p(V \mid \alpha)] + E[\log p(\eta^* \mid \lambda)] + \sum_{n=1}^{N} E[\log p(Z_n \mid V)]$$
$$+ E[\log(p(x_n \mid Z_n)] - E[\log q(Z, V, \eta^*)]$$
$$E[\log p(Z_n \mid V)] = E[\log(\prod_{i=1}^{K} (1 - V_i)^{1[Z_n > i]} V_i^{Z_n^i}$$

As a result we will have:

$$E[\log p(Z_n \mid V)] = \sum_{i=1}^{k} q(z_n > i) E[\log(1 - V_i] + q(z_n = i) E[\log V_i]$$
$$q(z_n = i) = \phi_{n,i}$$
$$q(z_n > i) = \sum_{j=i+1}^{K} \phi_{n,i}$$
$$E[\log V_i] = \Psi(\gamma_{i,1}) - \Psi(\gamma_{i,1} + \gamma_{i,2})$$
$$E[\log(1 - V_i)] = \Psi(\gamma_{i,2}) - \Psi(\gamma_{i,1} + \gamma_{i,2})$$

Optimization of K-L divergence criteria can be done by a coordinate ascent algorithm in the variational parameters. The updates of τ_n and γ_n follow the standard recipe for variational inference with exponential family distribution in a conjugate setting (Ghahramani & Beal, 2001), so we have:

$$\gamma_{i,1} = 1 + \sum_{m=1}^{N} \phi_{n,i}$$
$$\gamma_{i,2} = \alpha + \sum_{k=i+1}^{K} \sum_{m=1}^{N} \phi_{n,i}$$

The update for the variational multinomial on Z_n, $\phi_{n,i}$ is proportional to:

$$\exp(E[\log V_i \mid \gamma_i] + E[\eta_i \mid \tau_i]^T X_n - E[a(\eta_i) \mid \tau_i] - \sum_{j=i+1}^{K} E[\log(1 - V_i) \mid \gamma_j])$$

For the Gaussian component portion, we adopted an algorithm suggested by Penny (2002). We refer interested reader to that short instruction.