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Assignment #9 MECO6315

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(a) $f(x_i|\theta)=\frac{1}{\theta}$ $0\le x_i\le\theta$

$L(x_1,x_2,\dots,x_n|\theta)=\prod_{i=1}^{n} f(x_i)=\frac{1}{\theta^n}$

to show $\hat\theta=\max(x_1,x_2,\dots,x_n)$

$-L(x_1,x_2,\dots,x_n|\theta)$ (I) monotonically decreasing since $L'=\frac{-n}{\theta^{n-1}}<0$ & $\theta>0$

if $\theta<x_i$ $1<i<n$ the $f(x_i)=0 \Rightarrow L(x_1,\dots,x_n|\theta)=0$

$\Rightarrow \max(x_1,\dots,x_n)<\theta$ due to (I)

$\Rightarrow \hat\theta=\max(x_1,x_2,\dots,x_n)$ (*)

H_0: spaces ω; $\theta=\theta_0$ $LL=-n\theta$

H_A: Ω : $\theta\neq\theta_0$ $LL_\omega=-n\ln\theta_0$

$LL_\Omega=-n\ln\theta$

$\cancel{-2(LL_\omega-LL_\Omega)=-2(-n\ln\theta_0+n\ln\theta)=2n\ln\theta_0-2n\ln\theta}$

$\cancel{=2n\ln\left(\frac{\theta_0}{\theta}\right)\sim\chi^2}$ asymptote

Not Regular

$\ln(1+z)=z-\frac{z^2}{2}+\frac{z^3}{3}+\dots$

$2n\ln\left(1+\frac{\theta_0-\theta}{\theta}\right)=2n\left(\frac{\theta_0-\theta}{\theta}\right)-\frac{(\theta_0-\theta)^2}{2\theta^2}+\frac{(\theta_0-\theta)^3}{3\theta^3}+\dots$

$=\frac{2n(\theta_0-\theta)}{\theta}-\frac{n(\theta_0-\theta)^2}{\theta^2}+\frac{2n(\theta_0-\theta)^3}{3\theta^3}+\dots$

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in class you wanted us to find direct interval & not asymptote:

(1) we got sufficient statistics $t(x)=\max(x_i)=x_m$

we found out in (*) that $\hat\theta=x_m$

to simplify we should find b where

$pr(x_m<\theta<bx_m)=1-\alpha$ since $x_m<\theta$

and likelihood is decreasing according to (I)

as a result $pr(\theta<x_m)=0$ (**)

$\Rightarrow Pr(x_m<\theta<bx_m)=pr(\theta<bx_m)-\underbrace{pr(\theta<x_m)}_{\to 0 \text{ due to } (**)}$

$=pr(\theta<bx_m)=1-\alpha$ $\Rightarrow pr(bx_m<\theta)=\alpha$

$\Rightarrow pr(\theta<bx_m)=1-pr(bx_m<\theta)$

$pr(x_m<\frac{\theta}{b})=\alpha$ (*)

$pr(x_m<z)=pr(x_1,x_2,\dots,x_n<z)=\left(\frac{z}{\theta}\right)^n$ (II)

(*) Cont. $\Rightarrow$ from (II) : $\alpha=\left(\frac{1}{b}\right)^n\Rightarrow\frac{1}{b}=\alpha^{\frac{1}{n}}\Rightarrow\boxed{b=\alpha^{-\frac{1}{n}}}$

$\Rightarrow$ $\boxed{\text{interval}: pr\left(x_m<\theta<\frac{x_m}{\alpha^{\frac{1}{n}}}\right)=1-\alpha}$ (6)

$n=50$

$x_m=35$

$\alpha=1-0{,}95=0{,}05$

$\Rightarrow \alpha^{\frac{1}{n}}=0{,}94$

$\frac{x_m}{\alpha^{1/n}}=37{,}16$

$\Rightarrow \boxed{pr(35<\theta<37{,}16)=0{,}95}$

(*) Confidence interval in direct way

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a) MLE

$= P(x_1, x_2, \dots, x_n) = \prod^{i} \binom{x_i - 1}{r-1} (1-\pi)^{x_i - r} \pi^r \Rightarrow LL = \sum^{i} [(x_i - r)\ln(1-\pi) + r\ln(\pi) + \ln(x_i - 1)!$

$- \ln(r-1)! - \ln(x_i - r)!] \Rightarrow \frac{\partial LL}{\partial \pi} = \frac{-(\sum x_i - nr)}{1-\pi} + \frac{nr}{\pi} \Rightarrow \frac{\partial LL}{\partial \pi} = 0 \Rightarrow \frac{n\bar{x} - nr}{1-\pi} = \frac{nr}{\pi}$

$\Rightarrow n\pi\bar{x} - \cancel{n\pi r} = nr - \cancel{n\pi r} \Rightarrow \boxed{\pi_{MLE} = \frac{r}{\bar{x}}}$ (*)

$\Rightarrow Var(\hat{\pi}_{MLE}) = +\frac{1}{n} I(\theta)^{-1}$ where $I(\theta) = -E \frac{\partial^2}{\partial \pi} LL(\pi)) \Rightarrow I(\theta) = E\left(+\frac{\sum x_i - nr}{(1-\pi)^2} + \frac{nr}{\pi^2}\right)$

$\Rightarrow V(\hat{\pi}_{MLE}) = \left[E\left(\frac{\bar{x} - r}{(1-\pi)^2} + \frac{r}{\pi^2}\right)\right]^{-1} = \left(\frac{\frac{r}{\pi} - r}{(1-\pi)^2} + \frac{r}{\pi^2}\right)^{-1} = \frac{1}{r}\left(\frac{\pi - \pi^2 + (1-\pi)^2}{\pi^2(1-\pi)^2}\right)^{-1} = \frac{1}{r}\left(\frac{\cancel{1-\pi}}{\pi^2 \cancel{(1-\pi)}^2}\right)^{-1}$

from (*)

we know $E(\bar{x}) = E(x)$

$E(\sigma^2_{\bar{x}}) = \frac{\sigma^2}{n}$

$= \frac{1}{r}(\pi^2(1-\pi)) \Rightarrow \boxed{V(\hat{\pi}_{MLE}) = \frac{\pi^2(1-\pi)}{r}}$

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b) Bayes estimator

$P(\underline{\theta} | \underline{x}) = \frac{P(\underline{x} | \underline{\theta}) \cdot g(\theta)}{h(x)} \Rightarrow P(x|\pi) \cdot g(\pi) = \binom{x-1}{r-1}(1-\pi)^{x-r}\pi^r$

$h(x) = \int_0^1 \binom{x-1}{r-1}(1-\pi)^{x-r}\pi^r d\pi = \frac{\Gamma(x)}{\Gamma(r)\Gamma(x-r+1)} \int_0^1 (1-\pi)^{x-r}\pi^r d\pi = \frac{\Gamma(x)\Gamma(x-r+1)\Gamma(r+1)}{\Gamma(r)\Gamma(x-r+1)\Gamma(x+2)} \underbrace{\int_0^1 Beta(\pi) d\pi}_{1}$

Beta dist

$\boxed{h(x) = \frac{r}{x(x+1)}}$

$\Rightarrow p(\pi|x) = \frac{P(x|\pi) \cdot g(\pi)}{h(x)} = \frac{(x-1)! \times x(x+1)}{(r-1)!(x-r)! \, r}(1-\pi)^{x-r}\pi^r = \frac{\Gamma(x+2)}{\Gamma(r+1)\Gamma(x-r+1)}(1-\pi)^{x-r}\pi^r$

$\Rightarrow \boxed{p(\pi|x) = Beta(r+1, x-r+1)}$

now that we have dist. of $p(\hat{\pi})$ we can use it for any attrib of π

$$\boxed{\begin{aligned} E(\hat{\pi}_b) &= \frac{\alpha}{\alpha+\beta} = \frac{r+1}{x+2} \\ V(\hat{\pi}_b) &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{(r+1)(x-r+1)}{(x+2)^2(x+3)} \end{aligned}}$$

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part 2

$E(\hat{\pi}_{MLE}) = E\left(\frac{r}{\bar{x}}\right) = r\,E\left(\frac{1}{\bar{x}}\right) \Rightarrow Bias = |E(\pi_{MLE}) - \pi| = |r\,E(\frac{1}{\bar{x}}) - \pi|$

$\boxed{Bias_{MLE} = |r E(\frac{1}{\bar{x}}) - \pi|}$

$MSE_{MLE} = \sigma^2_{MLE} + (Bias)^2 = \frac{\hat{\pi}^2(1-\hat{\pi})}{r} + r^2(E(\frac{1}{\bar{x}}) - \pi)^2 = (\frac{r}{\bar{x}^2})(1-\frac{r}{\bar{x}}) + r^2(E(\frac{1}{\bar{x}}) - \pi)^2$

$\Downarrow$

$\boxed{\hat{MSE}_{MLE} = (\frac{r}{\bar{x}^2})(1-\frac{r}{\bar{x}}) + r^2(E(\frac{1}{\bar{x}}) - \pi)^2}$

$\boxed{Bias_B = |E(\pi_b) - \pi| = |E\frac{(r+1)}{x+2} - \pi| = |(r+1)E(\frac{1}{x+2}) - \pi|}$

$\boxed{MSE_B = \frac{(r+1)(x-r+1)}{(x+2)^2(x+3)} + ((r+1)E(\frac{1}{x+2}) - \pi)^2}$

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$\boxed{\text{assignment 6}}$ → second part missed on previous

MatCad output

$$\begin{pmatrix} x \\ s2 \end{pmatrix} = \begin{bmatrix} \dfrac{\alpha}{\alpha+\beta} \\ \dfrac{\alpha\cdot\beta}{(\alpha+\beta)^2\cdot(\alpha+\beta+1)} \end{bmatrix} \text{solve}, \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \left(-\frac{x^3 - x^2 + s2\cdot x}{s2} \quad \frac{x - s2 - 2\cdot x^2 + x^3 + s2\cdot x}{s2} \right)$$

$$\begin{pmatrix} 0.442 \\ 0.046 \end{pmatrix} = \begin{bmatrix} \dfrac{\alpha}{\alpha+\beta} \\ \dfrac{\alpha\cdot\beta}{(\alpha+\beta)^2\cdot(\alpha+\beta+1)} \end{bmatrix} \text{solve}, \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow (1.9278502608695652174 \quad 2.4338019130434782609$$

$\alpha := 1.928$ $\beta := 2.433$ $x := 0.442$ $s2 := 0.046$

$$\frac{2\cdot\alpha\cdot\beta\cdot(\beta-\alpha)}{(\alpha+\beta)^3(\alpha+\beta+1)\cdot(\alpha+\beta+2)} \rightarrow 0.0016751011988922546357 \qquad Ex3 := 0.0017$$

$$\frac{2\cdot\alpha\cdot\beta\cdot(\beta-\alpha)}{(\alpha+\beta)^3(\alpha+\beta+1)\cdot(\alpha+\beta+2)} - 3\left(s2 + x^2\right)\cdot x + 3x^3 - x^3 \rightarrow -0.14567178680110774536$$

$\boxed{cov := -.0146}$

$$\frac{3\cdot\alpha\cdot\beta\cdot\left[2\cdot(\alpha+\beta)^2 + \alpha\cdot\beta\cdot(\alpha+\beta-6)\right]}{(\alpha+\beta)^4\cdot(\alpha+\beta+1)(\alpha+\beta+2)(\alpha+\beta+3)} \rightarrow 0.0047038510409691330412 \qquad Ex4 := 0.0047$$

$$Ex4 - 4Ex3\cdot x + 6\cdot\left(s2 + x^2\right) - 4\cdot x^4 + x^4 \rightarrow 1.335377122512 \qquad Ex\mu4 := 1.3354 \qquad n := 100$$

$$\frac{2\cdot s2^2}{n-1} + \frac{Ex\mu4 - 3\cdot s2^2}{n} \rightarrow 0.013333267474747474747 \qquad \frac{s2}{n} \rightarrow 0.00046$$

$Vs2 := 0.0133$

$Vx := 0.0005$

$$\begin{pmatrix} \dfrac{d}{dx}\left(-\dfrac{x^3 - x^2 + s2\cdot x}{s2}\right) & \dfrac{d}{dx}\dfrac{x - s2 - 2\cdot x^2 + x^3 + s2\cdot x}{s2} \\ \dfrac{d}{ds2}\left(-\dfrac{x^3 - x^2 + s2\cdot x}{s2}\right) & \dfrac{d}{ds2}\dfrac{x - s2 - 2\cdot x^2 + x^3 + s2\cdot x}{s2} \end{pmatrix} \rightarrow \begin{pmatrix} 5.4762608695652173914 & -2.9545217391304 \\ -51.518483931947069943 & -65.039172022684 \end{pmatrix}$$

$d\alpha x := 5.4762$ $d\alpha s2 := -51.5185$ $d\beta x := -2.9545$ $d\beta s2 := -65.0391$

$$(d\alpha x \quad d\alpha s2)\cdot\begin{pmatrix} Vx & cov \\ cov & Vs2 \end{pmatrix}\cdot\begin{pmatrix} d\alpha x \\ d\alpha s2 \end{pmatrix} \rightarrow 43.553334888385$$

$var(\alpha) := \cancel{43.55}$ .07484

$var(\beta) := \cancel{50.653}$ .12374

$$(d\beta x \quad d\beta s2)\cdot\begin{pmatrix} Vx & cov \\ cov & Vs2 \end{pmatrix}\cdot\begin{pmatrix} d\beta x \\ d\beta s2 \end{pmatrix} \rightarrow 50.653474556558$$

$$(d\beta x \quad d\beta s2)\cdot\begin{pmatrix} Vx & cov \\ cov & Vs2 \end{pmatrix}\cdot\begin{pmatrix} d\alpha x \\ d\alpha s2 \end{pmatrix} \rightarrow 47.534198082187$$

$cov(\alpha, \beta) := \cancel{47.53}$

.078195

MECO 6315
Assignment 7

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(1)

$$f(x|\mu,\lambda)=\left[\frac{\lambda}{2\pi x^3}\right]^{1/2} e^{-\frac{\lambda}{2\mu}\left(\frac{x}{\mu}-2+\mu/x\right)}$$

$$\Rightarrow \ln f(x|\mu,\lambda)=\tfrac{1}{2}\ln(\lambda)-\tfrac{1}{2}\ln(2\pi x^3)-\frac{\lambda}{2\mu}\left(\frac{x}{\mu}-2+\mu/x\right)$$

$x_1, \dots, x_n$

$$\Rightarrow L(\mu,\lambda)=\frac{n}{2}\ln(\lambda)-\frac{n}{2}(\ln 2\pi)-\frac{3}{2}\sum\ln x_i-\frac{\lambda}{2\mu^2}\overset{n\bar{x}}{\cancel{\sum x_i}}+\frac{\lambda\cdot n}{\mu}-\frac{\lambda}{2}\sum\frac{1}{x_i}$$

$$\Rightarrow (1)\ \frac{\partial L(\mu,\lambda)}{\partial\mu}=\frac{\lambda n\bar{x}}{\mu^3}-\frac{n\lambda}{\mu^2}\Rightarrow\frac{\partial L(\mu,\lambda)}{\partial\mu}=0\Rightarrow\frac{\lambda n\bar{x}}{\mu}=n\lambda\Rightarrow\boxed{\mu=\bar{x}}$$

$$(2)\ \frac{\partial L(\mu,\lambda)}{\partial\lambda}=\frac{n}{2\lambda}-\frac{n\bar{x}}{2\mu^2}+\frac{n}{\mu}-\frac{1}{2}\sum\frac{1}{x_i}\Rightarrow\frac{\partial L(\mu,\lambda)}{\partial L}=0\Rightarrow\frac{n}{2\lambda}-\frac{n}{2\bar{x}}+\frac{n}{\bar{x}}-\frac{1}{2}\sum\frac{1}{x_i}=0$$

$$\Rightarrow\frac{n}{2\lambda}=\frac{1}{2}\sum\frac{1}{x_i}-\frac{n}{2\bar{x}}\Rightarrow\lambda=\frac{n}{\sum\frac{1}{x_i}-\frac{n}{\bar{x}}}\Rightarrow\boxed{\lambda=\frac{1}{\overline{(1/x)}-1/\bar{x}}}$$

$$(3)\ \frac{\partial L(\mu,\lambda)}{\partial\mu\,\partial\lambda}=\frac{n\bar{x}}{\mu^3}-\frac{n}{\mu^2}\Rightarrow=\frac{n}{\bar{x}^2}-\frac{n}{\bar{x}^2}=0\Rightarrow\boxed{\frac{\partial L(\mu,\lambda)}{\partial\mu\,\partial\lambda}=0}$$

$$(4)\ \frac{\partial^2 L(\mu,\lambda)}{\partial\mu^2}=\frac{-3\lambda n\bar{x}}{\mu^4}+\frac{2n\lambda}{\mu^3}=\frac{-3\lambda n}{\bar{x}^3}+\frac{2n\lambda}{\bar{x}^3}=\frac{-n\lambda}{\bar{x}^3}\Rightarrow\boxed{\frac{\partial^2 L(\mu,\lambda)}{\partial\mu^2}=\frac{-n\lambda}{\bar{x}^3}}$$

$$(5)\ \frac{\partial^2 L(\mu,\lambda)}{\partial\lambda^2}=-\frac{n}{2\lambda^2}\Rightarrow\quad \text{Asympt. Cov matrix: } B^{-1}(\mu,\lambda)=\begin{bmatrix}-\frac{\partial L(\mu,\lambda)}{\partial\mu^2} & -\frac{\partial^2 L(\mu,\lambda)}{\partial\lambda\partial\mu}\\ -\frac{\partial L(\mu,\lambda)}{\partial\lambda\,\partial\mu} & -\frac{\partial L(\mu,\lambda)}{\partial\lambda^2}\end{bmatrix}$$

$$\Rightarrow B^{-1}(\mu,\lambda)=\begin{bmatrix}\frac{n\lambda}{\bar{x}^3} & 0\\ 0 & \frac{n}{2\lambda^2}\end{bmatrix}^{-1}=\begin{bmatrix}\frac{\bar{x}^3}{n\lambda} & 0\\ 0 & \frac{2\lambda^2}{n}\end{bmatrix}\Rightarrow\boxed{Var(\hat{\mu})=\mu^3/n\lambda}$$

$$\boxed{Var(\hat{\lambda})=\frac{2\lambda^2}{n}}$$

1

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Q1

$$\bar{x} = \frac{\alpha}{\alpha+\beta}$$

$$s^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

point estimate: $s^2 = \frac{\hat\alpha\hat\beta}{(\hat\alpha+\hat\beta)^2(\hat\alpha+\hat\beta+1)}$

$\hat\alpha = \bar{x}\hat\alpha + \beta\bar{x} \Rightarrow (1-\bar{x})\alpha = \beta\bar{x}$

$\hat\alpha = \left(\frac{\bar{x}}{1-\bar{x}}\right)\beta \qquad \hat\beta = \frac{(1-\bar{x})\hat\alpha}{\bar{x}}$ ①

$\Rightarrow s^2 = \frac{\hat\alpha^2(1-\bar{x})}{\bar{x}\,\frac{\hat\alpha^2}{\bar{x}^2}\left(\frac{\hat\alpha+\bar{x}}{\bar{x}}\right)} \Rightarrow s^2(\hat\alpha+\bar{x}) = (1-\bar{x})\bar{x}^2$

$\Rightarrow \bar{x}s^2 + s^2\hat\alpha = \bar{x}^2 - \bar{x}^3$

$\Rightarrow \hat\alpha = \frac{\bar{x}^2 - \bar{x}^3 - s^2\bar{x}}{s^2}$ ②

1 $\cancel{10} = \frac{\hat\alpha}{\hat\alpha+\hat\beta}$

2 6

$\cancel{12}$ ← need numerical

$\hat\alpha + \hat\beta = \left(\frac{1-\bar{x}}{\bar{x}} + 1\right)\hat\alpha = \frac{\hat\alpha}{\bar{x}}$

16 extract

Point estimates

$$\boxed{\hat\alpha = \bar{x}\left(\frac{\bar{x}(1-\bar{x})}{s^2} - 1\right)} \Rightarrow \boxed{\hat\beta = (1-\bar{x})\left(\frac{\bar{x}(1-\bar{x})}{s^2} - 1\right)}$$

to calculate variation and covariance we need to calculate following table first

① $\frac{\partial\hat\beta}{\partial\bar{x}}$	② $\frac{\partial\hat\beta}{\partial s^2}$	⑤ $var(\bar{x})$	⑦ $Cov(\bar{x}, s^2)$
③ $\frac{\partial\hat\alpha}{\partial\bar{x}}$	④ $\frac{\partial\hat\alpha}{\partial s^2}$	⑥ $Cov(\bar{x}, s^2)$	⑧ $V(s^2)$

(*) I provided detail of calc in appendix - here are just results:

① $$\frac{\partial\hat\beta}{\partial\bar{x}} = -\frac{(\alpha^2 + 5\alpha\beta + 4\beta^2 + \alpha + 3\beta)}{\beta}$$

② $$\frac{\partial\hat\beta}{\partial s^2} = \frac{-(\alpha+\beta+1)^2(\alpha+\beta)}{\alpha}$$

③ $$\frac{\partial\hat\alpha}{\partial\bar{x}} = 1 + \frac{(\beta-\alpha)(\alpha+\beta+1)}{\beta}$$

④ $$\frac{\partial\hat\alpha}{\partial s^2} = \frac{-(\alpha+\beta)(\alpha+\beta+1)^2}{\beta}$$

⑤ $$Var(\bar{x}) = \frac{\alpha\beta}{n(\alpha+\beta)^2(\alpha+\beta+1)}$$

used mathcad for this one

⑧
$$Var(s^2) = \frac{\beta^2\alpha^2}{(n-1)(\alpha+\beta)^4(\alpha+\beta+1)^2} - \frac{3\beta^2\alpha^2}{n(\beta+\alpha)^4(\beta+\alpha+1)^2} + \frac{3\beta\alpha(2\beta^2+2\alpha^2+4\beta\alpha+\beta\alpha(\beta+\alpha-2))}{n(\beta+\alpha)^4(\alpha+\beta+1)(\alpha+\beta+2)(\alpha+\beta+3)}$$

+ 2

⑥ ⑦
$$Cov(\bar{x}, s^2) = \frac{3\beta\alpha(2(\beta+\alpha)^2 + \beta\alpha(\beta\alpha-2))}{n(\alpha+\beta)^4(\alpha+\beta+1)(\alpha+\beta+2)(\alpha+\beta+3)}$$

we will use the following structure:

$$var(f(x,y)) = \begin{pmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{pmatrix}\begin{pmatrix} V(x) & Cov(x,y) \\ Cov(x,y) & V(y)\end{pmatrix}\begin{pmatrix}\frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y}\end{pmatrix} \quad (*)$$

we substitute $\hat\alpha$ and $\hat\beta$ with $f(x,y)$ and $\bar{x}, s^2$ with x, y as we calculated in ① to ⑧

$\boxed{2}$ I plugged in ① – ⑧ into ✱ and used math Cad. The result was:

$$\frac{\left[\frac{\alpha\cdot(4\beta^2+5\beta\alpha+3\beta+\alpha^2+\alpha)}{n(\beta+\alpha)^2(\beta+\alpha+1)}+\frac{3\beta(\beta+\alpha+1)(\beta^2\alpha^2+2\beta^2+2\beta\alpha+2\alpha^2)}{n\cdot(\beta+\alpha)^3(\beta+\alpha+2)\cdot(\beta+\alpha+\beta)}\right]\cdot(4\beta^2+5\cdot\beta\cdot\alpha+\ldots}{}$$

$$+\frac{\left[\frac{(\beta+\alpha)\cdot(\beta+\alpha+1)^2\cdot\left[\frac{\beta^2\alpha^2}{(n-1)\cdot(\beta+\alpha)^4(\beta+\alpha+1)^2}-\frac{3\cdot\beta^2\cdot\alpha^2}{n\cdot(\beta+\alpha)^4(\beta+\alpha-1)^2}+\frac{3\cdot\beta\cdot\alpha(2\beta^2+2\alpha^2+4\beta\cdot\alpha+\beta\alpha)}{n(\beta+\alpha)^4\cdot(\beta+\alpha+1)(\beta+\alpha+2)}\cdot\frac{(\beta+\alpha-2)}{(\beta+\alpha+3)}+2\right]}{\beta}+\frac{3\cdot\alpha(\beta\cdot\alpha^2+2\beta^2+2\beta\alpha+2\cdot\alpha^2)\cdot(4\cdot\beta^2+5\beta\alpha+3\beta+\alpha^2+\alpha)}{n\cdot(\beta+\alpha)^4\cdot(\beta+\alpha+1)\cdot(\beta+\alpha+2)\cdot(\beta+\alpha+3)}\right]\cdot(\beta+\alpha)\cdot(\beta+\alpha+1)^2}{\alpha}$$

now we again use ✱ to calculate $\underline{var(\hat{\alpha})}$ using mat cad

$$var(\hat{\alpha})=\frac{\left[\frac{\alpha\cdot(\alpha^2-2\beta-\beta^2+\alpha)}{n\cdot(\beta+\alpha)^2(\beta+\alpha+1)}+\frac{3\cdot\alpha(\beta+\alpha+1)\cdot(\beta^2\alpha^2+2\beta^2+2\beta\alpha+2\alpha^2)}{n\cdot(\beta+\alpha)^3\cdot(\beta+\alpha+2)\cdot(\beta+\alpha+3)}\right]\cdot(\alpha^2-2\beta-\beta^2+\alpha)}{\beta}+$$

$$\frac{(\beta+1)\cdot\left[\frac{(\beta+\alpha)(\beta+\alpha+1)^2\left[\frac{\beta^2\alpha^2}{(n-1)(\alpha+\beta)^4(\alpha+\beta+1)^2}-\frac{3\cdot\beta^2\alpha^2}{n(\beta+\alpha)^4(\beta+\alpha+1)^2}+\frac{3\cdot\beta\alpha(2\beta^2+2\alpha^2+4\beta\alpha+\beta\alpha)\cdot(\beta+\alpha-2)}{n\cdot(\beta+\alpha)^4\cdot(\beta+\alpha+1)\cdot(\beta+\alpha+2)\cdot(\beta+\alpha+3)}+2\right]}{\beta}+\frac{3\alpha\cdot(\alpha^2-2\beta-\beta^2+\alpha)\cdot(\beta^2\alpha^2+2\beta^2+2\cdot\beta\alpha+2\alpha^2)}{n\cdot(\beta+\alpha)^4\cdot(\beta+\alpha+1)\cdot(\beta+\alpha+2)(\beta+\alpha+3)}\right]\cdot(\beta+\alpha+1)^2}{\beta}$$

for the Covariance we have: $Cov(\hat{\beta}_1,\hat{\beta}_2)=\left(\frac{\partial f_1}{\partial x_1}\quad\frac{\partial f_2}{\partial x_2}\right)\begin{pmatrix} var\ x & Cov(x,y)\\ Cov(x,y) & var\ (y)\end{pmatrix}\begin{pmatrix}\frac{\partial f_2}{\partial x_1}\\ \frac{\partial f_2}{\partial x_2}\end{pmatrix}$

I used Mat Cad and result from step ①...⑧ to calculate Covariance and result is given in the next page

$$Cov(\hat{\alpha}, \hat{\beta}) = \frac{\left[\frac{\alpha\cdot(4\cdot\beta^2+5\beta\cdot\alpha+3\beta+\alpha^2+\alpha)}{n\cdot(\beta+\alpha)^2(\beta+\alpha+1)} + \frac{3\cdot\alpha(\beta+\alpha+1)\cdot(\beta^2\alpha^2+2\beta^2+2\beta\alpha+2\cdot\alpha^2)}{n\cdot(\beta+\alpha)^3\cdot(\beta+\alpha+2)(\alpha+\beta+3)}\right]\cdot(\alpha^2-2\beta+\beta^2+\alpha)}{\beta}$$

$$+ \left[\frac{(\beta+\alpha)\cdot(\beta+\alpha+1)^2\cdot\left[\frac{\beta^2\alpha^2}{(n-1)(\beta+\alpha)^4\cdot(\beta+\alpha+1)^2} - \frac{3\beta^2\alpha^2}{n\cdot(\beta+\alpha)^4(\beta+\alpha+1)^2} + \frac{3\cdot\beta\cdot\alpha(2\beta^2+2\alpha^2+4\beta\alpha+4\beta\alpha(\beta+\alpha-2))}{n\cdot(\beta+\alpha)^4\cdot(\beta+\alpha+1)\cdot(\beta+\alpha+2)\cdot(\beta+\alpha+3)}+2\right]}{O}\right.$$

$$\frac{\left. + \frac{3\cdot\alpha(\beta^2\alpha^2+2\beta^2+2\beta\alpha+2\alpha^2)\cdot(4\beta^2+5\beta\alpha+3\beta+\alpha^2+\alpha)}{n(\beta+\alpha)^4(\beta+\alpha+1)(\beta+\alpha+2)\cdot(\beta+\alpha+3)}\right]\cdot(\beta+\alpha)(\beta+\alpha+1)^2}{\alpha}$$

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$(X,Y)' \sim MVN$

$MVN: f(x,y) = \frac{1}{\sqrt{2\pi}\,|\Sigma|^{1/2}} e^{\frac{-(x-\mu)'\Sigma^{-1}(x-\mu)}{2}}$

$\mu = \begin{pmatrix}\mu_x\\ \mu_y\end{pmatrix}, \Sigma = \begin{pmatrix}\sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2\end{pmatrix}$

$\Sigma\Sigma' = I \Rightarrow \begin{pmatrix}B & A\\ A & C\end{pmatrix}\begin{pmatrix}\sigma_x^2 & \rho\sigma_x\sigma_y\\ \rho\sigma_x\sigma_y & \sigma_y^2\end{pmatrix} = \begin{pmatrix}1+\rho A\sigma_x\sigma_y & \frac{\rho\sigma_x}{\sigma_x}+A\sigma_y^2 \\ A\sigma_x^2+\frac{\rho\sigma_y}{\sigma_x} & A\rho\sigma_x\sigma_y+1\end{pmatrix}$

$\Rightarrow \begin{cases} B\sigma_x^2+\rho A\sigma_x\sigma_y = 1 \\ A\rho\sigma_x\sigma_y + C\sigma_y^2 = 1 \\ \rho B\sigma_x\sigma_y + A\sigma_y^2 = 0\end{cases} \Rightarrow B = \frac{-A\sigma_y^2}{\rho\sigma_x\sigma_y} = -\frac{A\sigma_y}{\rho\sigma_x} \Rightarrow B = \frac{1}{(1-\rho^2)\sigma_x^2}$

Symmetr $\Rightarrow C = \frac{1}{(1-\rho^2)\sigma_y^2}$

$\Rightarrow -\frac{A\sigma_y\sigma_x}{\rho} + \rho A\sigma_x\sigma_y = 1 \Rightarrow A = -\frac{\rho}{(1-\rho^2)\sigma_x\sigma_y}$

$\Rightarrow f(x,y) = \boxed{\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2}+\frac{(y-\mu_y)^2}{\sigma_y^2}-2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\right]}$

$P(Y|X) = \frac{f(x,y)}{f(x)} = \frac{f(x,y)}{\int f(x,y)dy}$

$f(x) = \int f(x,y)dy = A \times B \times \left(\int e^{\frac{-(y-\mu_y)^2}{2(1-\rho^2)\sigma_y^2}} \cancel{dy} \times e^{\frac{\rho(x-\mu_x)(y-\mu_y)}{(1-\rho^2)\sigma_x\sigma_y}} dy\right) =$

$B = e^{-\frac{(x-\mu_x)^2}{2(1-\rho^2)\sigma_x^2}}$ should be Normal $\Rightarrow f(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} e^{\frac{-(x-\mu_x)^2}{\sigma_x^2}}$

$\Rightarrow p(Y|X) = \dfrac{\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2}+\frac{(y-\mu_y)^2}{\sigma_y^2}-2\frac{\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\right]} \simeq e^P}{\frac{1}{\sqrt{2\pi}\,\sigma_x} e^{\frac{-(x-\mu_x)^2}{\sigma_x^2}} \simeq e^S}$

Bad notation

I think you mean $\rho^2\frac{(x-\mu_x)^2}{\sigma_x^2}$ still

$\Rightarrow \frac{e^P}{e^S} \Rightarrow P-S = \frac{-1}{2(1-\rho^2)}\left[\cancel{\frac{(x-\mu_x)^2}{\sigma_x^2}} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(x-\mu_x)^2}{\sigma_x^2}\cancel{(1-\rho^2)}\right]$

$= -\frac{\left(y-\left(\frac{\sigma_y}{\sigma_x}\rho(x-\mu_x)+\mu_y\right)\right)^2}{2\sigma_y^2(1-\rho^2)}$

$\Rightarrow p(Y|x) = \frac{1}{\sqrt{2\pi}\,\sigma_y\sqrt{1-\rho^2}} e^{\frac{-\left(y-\left(\cancel{\frac{\sigma_y}{\sigma_x}}\times\frac{\sigma_{xy}}{\sigma_x\sigma_y}(x-\mu_x)+\mu_y\right)\right)^2}{2\sigma_y^2(1-\rho^2)}} \equiv \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(y-\mu)^2}{2\sigma^2}}$

means Normal where $\sigma = \sigma_y\sqrt{1-\rho^2}$, $\mu = \frac{\sigma_{xy}}{\sigma_x^2}(x-\mu_x)+\mu_y$

$\Rightarrow \boxed{E(Y|x) = \mu_y + \frac{\sigma_{xy}}{\sigma_x^2}(x-\mu_x) \Rightarrow \beta_0 = \mu_y - \frac{\sigma_{xy}\mu_x}{\sigma_x^2},\ \beta_1 = \frac{\sigma_{xy}}{\sigma_x^2}}$

$\boxed{Var(Y|x) = \sigma_y^2(1-\rho^2) = \sigma_e^2}$

$Z_1, Z_2 \sim N(0,1)$, $Z_1 \perp Z_2$

$\Rightarrow P(\sqrt{x_1^2+x_2^2} < a) = P(x_1^2+x_2^2 < a^2) = P(x_1^2 < a^2 - x_2^2) =$

$\frac{1}{2\pi}\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-y^2/2} e^{-x^2/2}\, dy\, dx = \frac{1}{2\pi}\int_0^a e^{-r^2/2} r\, dr\, d\theta = \frac{4\pi}{2\pi} e^{-a^2/2} = 2 e^{-a^2/2}$

(a)

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ dy\,dx = r\,dr\,d\theta \end{cases}$ Polar Coordinate System

$\boxed{f(y) = y e^{-y^2/2}}$

Not clear

(b) $\mu_1 = \int_0^\infty x \times x e^{-x^2/2} dx = -x e^{-x^2/2}\Big|_0^\infty + \int_0^\infty e^{-x^2/2} dx = \frac{\sqrt{2\pi}}{2} = \sqrt{\pi/2} \Rightarrow \boxed{\mu_1 = \sqrt{\pi/2}}$

$u' = 1 \quad v = e^{-x^2/2}$

$\alpha_2 = E(x^2) = \int_0^\infty x^2 \times x e^{-x^2/2} dx = \int_0^\infty 2x e^{-x^2/2} = -2 e^{-x^2/2}\Big|_0^\infty = 2 \Rightarrow \boxed{\alpha_2 = 2}$

$u' = 2x \quad v = -e^{-x^2/2}$

$\mu_2 = \alpha_2 - \mu^2 = 2 - \pi/2 \Rightarrow \boxed{\mu_2 = \frac{4-\pi}{2}}$

$\mu_3 = E((x-\mu)^3) = E[x^3 - 3x^2\mu + 3x\mu^2 - \mu^3] = E(x^3) - 3E(x^2)\mu + 2\mu^3 = \alpha_3 - 3\alpha_2\mu + 2\mu^3$

$\alpha_3 = \int_0^\infty x^3\, x e^{-x^2/2} dx = 3\int_0^\infty x \cdot x e^{-x^2/2} = 3\int_0^\infty e^{-x^2/2} dx = 3/2\sqrt{2\pi} = 3\sqrt{\pi/2}$

$u' = 3x^2 \quad v' = e^{-x^2/2} \qquad u' = 1 \quad v = -e^{-x^2/2}$

$\Rightarrow \boxed{\alpha_3 = 3\sqrt{\pi/2}}$

$\Rightarrow \mu_3 = 3\sqrt{\pi/2} - 3\times 2\sqrt{\pi/2} + 2\,\pi/2\sqrt{\pi/2} = -3\sqrt{\pi/2} + \pi\sqrt{\pi/2} = (\pi - 3)\sqrt{\pi/2} \quad \boxed{\mu_3 = (\pi-3)\sqrt{\pi/2}}$

$\mu_4 = E((x-\mu)^4) = E(x^4 - 4x^3\mu + 6\mu^2x^2 - 4x\mu^3 + \mu^4) = \alpha_4 - 4\alpha_3\mu + 6\alpha_2\mu^2 - 4\mu^4 + \mu^4$
$= \alpha_4 - 4\alpha_3\mu + 6\alpha_2 - 3\mu^4$

$\alpha_4 = E(x^4) = \int_0^\infty x^5 e^{-x^2/2} dx = 4\int_0^\infty x^3 e^{-x^2/2} dx = 8 \Rightarrow \boxed{\alpha_4 = 8}$

$u = -x^4 \quad v' = -x e^{-x^2/2}$
$u' = -4x^3 \quad v = e^{-x^2/2}$

$\mu_4 = 8 + 12\sqrt{\pi/2}\times\sqrt{\pi/2} - \frac{6\times 2}{12} - 6(\pi/2)^2 = -4 + 6\pi - \frac{6\pi^2}{4} \Rightarrow \boxed{\mu_4 = -3/4\,\pi^2 + 8}$

[...]sam Hejazinia mxh109420 MECO 6315

$$\alpha_1 = \int_0^\lambda \frac{2x^2}{\lambda}dx + \int_\lambda^1 \frac{2x-2x^2}{1-\lambda} = \frac{2}{3\lambda}x^3\Big|_0^\lambda + \frac{(x^2-2/3x^3)}{1-\lambda}\Big|_\lambda^1 = 2/3\lambda^2 + \frac{(1-2/3-\lambda^2+2/3\lambda^3)}{1-\lambda} =$$

$$\frac{1/3-1/3\lambda^2}{1-\lambda} = \boxed{1/3(\lambda+1)}$$

$$\alpha_1^2 = \mu_1^2 = 1/9(\lambda^2+2\lambda+1) \qquad \alpha_1^3 = 1/27(\lambda^3+3\lambda^2+3\lambda+1) \qquad \alpha_1^4 = \frac{1}{81}(\lambda^4+4\lambda^3+6\lambda^2+4\lambda+1)$$

$$\alpha_2 = \int_0^\lambda \frac{2x^3}{\lambda}dx + \int_\lambda^1 \frac{(2x^2-2x^3)}{1-\lambda}dx = \frac{2x^4}{4\lambda}\Big|_0^\lambda + \frac{2/3x^3-2/4x^4\Big|_\lambda^1}{} = \frac{-1/6\lambda^3+\frac{1}{6}}{1-\lambda} = \boxed{\frac{1}{6}(1+\lambda+\lambda^2)}$$

$$\mu_2 = \alpha_2-\alpha_1^2 = \frac{1}{6}(1+\lambda+\lambda^2) - \frac{1}{9}(\lambda^2+2\lambda+1) = \frac{1}{18}(3+3\lambda+3\lambda^2-2\lambda^2-4\lambda-2) =$$

$$\boxed{\frac{1}{18}(\lambda^2-\lambda+1)} \qquad \mu_2^{3/2} = \left(\frac{1}{18}(\lambda^2-\lambda+1)\right)^{3/2}$$

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} \qquad \mu_3 = \sum_{i=0}^{3}(-1)^i\binom{3}{i}\alpha_1^i\alpha_{3-i} = \boxed{\alpha_3 - 3\alpha_1\alpha_2 + 3\alpha_1^3 - \alpha_1^3}$$

$$\alpha_3 = \underbrace{\int_0^\lambda \frac{2x^4}{\lambda}dx}_{A} + \underbrace{\int_\lambda^1 \frac{2x^3-2x^4}{(1-\lambda)}dx}_{B} =$$

$$A = \frac{2x^5}{5\lambda}\Big|_0^\lambda = \frac{2}{5}\lambda^4$$

$$B = \frac{1}{1-\lambda}\left(\frac{2x^4}{4}-\frac{2x^5}{5}\right)\Big|_\lambda^1 = \frac{1}{1-\lambda}\left(\underbrace{1/2-2/5}_{1/10}-\frac{\lambda^4}{2}+\frac{2\lambda^5}{5}\right)$$

$$= 1/10(1-\lambda) + \left(\frac{2\lambda^5}{5}-\frac{\lambda^4}{2}\right)\frac{1}{1-\lambda}$$

$$\left(\frac{2}{5}\lambda^4 - \frac{2}{5}\lambda^5 + \frac{1}{10} + \frac{2\lambda^5}{5} - \lambda^4/2\right)$$

$$= \frac{\frac{-\lambda^4}{10}+\frac{1}{10}}{1-\lambda} = \boxed{1/10(\lambda^3+\lambda^2+\lambda+1)}$$

$$\Rightarrow \mu_3 = 1/10(\lambda^3+\lambda^2+\lambda+1) - 3\times 1/3(\lambda+1)\frac{1}{6}(\lambda^2+\lambda+1)$$

$$+ 2/27(\lambda^3+3\lambda^2+3\lambda+1) = \frac{1}{270}(27\lambda^3+27\lambda^2+27\lambda$$

$$+27 - 45(\lambda^3+2\lambda^2+2\lambda+1) + 20\lambda^3+60\lambda^2+60\lambda+20)$$

$$= \frac{1}{270}(2\lambda^3 - 3\lambda^2 - 3\lambda + 2) =$$

$$\frac{1}{270}(2\lambda^3-3\lambda^2-3\lambda+2)$$

$$\gamma_1 = \frac{\frac{1}{270}\times 18\times\sqrt{18}(2\lambda^3-3\lambda^2-3\lambda+2)}{(\lambda^2-\lambda+1)\sqrt{\lambda^2-\lambda+1}}$$

$$= \frac{\sqrt{2}(2\lambda^3-3\lambda^2-3\lambda+2)}{5(\lambda^2-\lambda+1)^{3/2}} \rightarrow \frac{\sqrt{2}}{5}\frac{(1-2\lambda)(1+\lambda)(2-\lambda)}{(1-\lambda+\lambda^2)^{3/2}}$$

$$\gamma_2 = \frac{\mu_4}{\mu_2^2} \qquad \mu_4 = \sum_{k=0}^{4}(-1)^k\binom{4}{k}\alpha_1^k\alpha_{4-k} = \alpha_4 - 4\alpha_1\alpha_3 + 6\alpha_1^2\alpha_2 - 3\alpha_1^4$$

$$\alpha_4 = \int_0^\lambda \frac{2x^5}{\lambda}dx + \int_\lambda^1 \frac{2x^4-2x^5}{1-\lambda}dx = \frac{2x^6}{6\lambda}\Big|_0^\lambda + \frac{1}{1-\lambda}\left(\frac{2x^5}{5}-\frac{2x^6}{6}\right)\Big|_\lambda^1$$

$$= \frac{\lambda^5 - \lambda^6 + \frac{6}{5}x^5 - x^6}{} = \frac{\lambda^5 - \lambda^6 + 6/5 - 1 - 6/5\lambda^5 + \lambda^6}{3-3\lambda}$$

$$= \frac{-1/5\lambda^5 + 1/5}{3(1-\lambda)} = \boxed{\frac{1}{15}(\lambda^4+\lambda^3+\lambda^2+\lambda+1)}$$

$$\Rightarrow \mu_4 = 1/15(\lambda^4+\lambda^3+\lambda^2+\lambda+1) - \frac{1}{30}(\lambda^3+\lambda^2+\lambda+1)(\lambda+1)$$

$$+ \frac{10}{180}(\lambda^2+2\lambda+1)(1+\lambda+\lambda^2) - \frac{5}{45}\ \frac{3}{81}\ (\lambda^4+4\lambda^3+6\lambda^2$$

$$+4\lambda+1) = 1/90(4\lambda^4+4\lambda^3+4\lambda^2+4\lambda+4 - 90\lambda^3$$

$$-90\lambda^2 - 90\lambda - 45 + 45\lambda^4 + 300(\lambda^4+\lambda^3+\lambda^2+2\lambda^3+2\lambda^2+2\lambda+\lambda^2+\lambda+1) - 50\lambda^4 - 200\lambda^3$$

$$\boxed{-300\lambda^2 - 200\lambda - 50) = 1/90(209\lambda^4 + 614\lambda^3 + 814\lambda^2 + 614\lambda + 209)}$$

$$\mu_4 = \frac{1}{90}\left(209\lambda^4 + 614\lambda^3 + 814\lambda^2 + 614\lambda + 209\right)$$

$$\mu_2 = \frac{1}{18}\left(\lambda^2 - \lambda + 1\right)$$

$$\Rightarrow \boxed{\gamma_2 = \frac{\mu_4}{\mu_2^2} = \frac{(18)^2}{90}\,\frac{\left(209\lambda^4 + 614\lambda^3 + 814\lambda^2 + 614\lambda + 209\right)}{\lambda^2 - \lambda + 1}} \Rightarrow \boxed{\gamma_2 = 3 - 3/5 = \frac{12}{5}}$$

normal

Assignment 1, MECO 6315

Meisam Hejazinia

1/19/2013

Four plots are shown in the picture.

From the top first six plots are for log normal distribution, and the bottom six plots are for gamma distribution.

First measure to select from these charts to fit the data is the standard deviation. In case data are not too much deviated from the mean, we could select the distribution with lower standard deviation. Second, since all these charts are not symmetric, and are right skewed, mean is not good measure for selection, rather we need to use median and mode, for central tendency for selection. This skewness shows interdependance of phenomenon's under study. Third, different distributions have different kurtosis, it seems gamma distribution has higher deviation from kurtosis of the normal distribution which is 3. They are normally steeper, while the log normal distributions are more sholderless. Regarding the skewness, it seems gamma distribution has higher skewness to right, and this one could be used as a measure. All these distribution are for positive variables, therefore there is no differentiator between them. Regarding degree of freedom, both have two degree of freedom; therefore, there is no differentiator there. Generally to select these distributions based on problem we can used goodness of fit tests, such as Chi-Square, Kolmogorov-Smirnoff, or Anderson-Darling, over the data. Finally, as the standard deviation increases, the tail became longer.

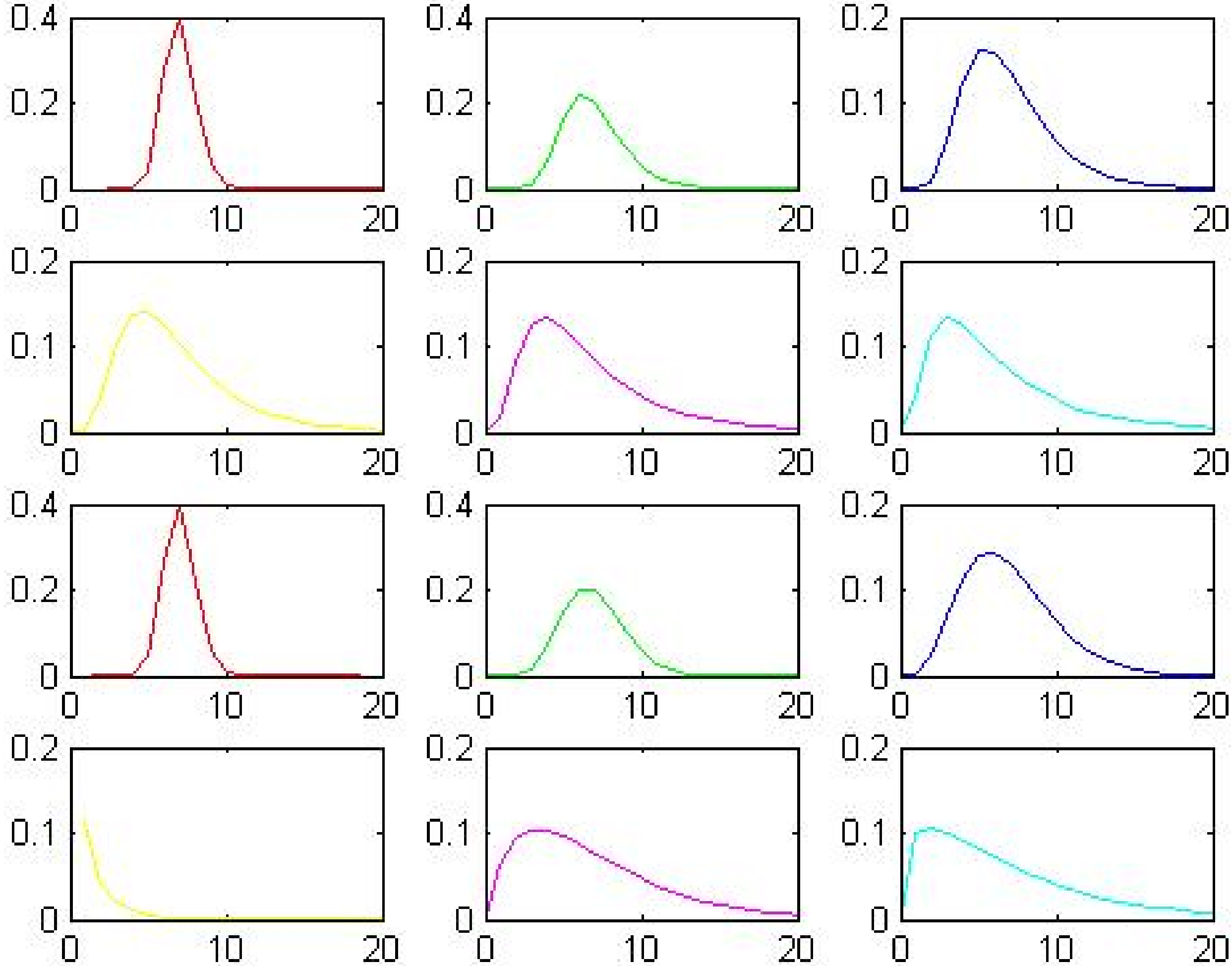

Figure 1: Log normal and gamma distribution, from top to buttom, left to write, we first have log normal distributions with standard devation varying from 1 to 6 for log normal distribution, and then the same variation for gamma distribution

Assignment 2, MECO 6315

Meisam Hejazinia

1/26/2013

a) Beta Distribution comparison with normal distribution:

As could be seen in the figure 1, beta distribution is used for data that is varied in the limited range. Stats are shown in the following:

p mean variance
1 0.500000 0.083333
2 0.500000 0.050000
3 0.500000 0.035714
4 0.500000 0.027778
5 0.500000 0.022727
6 0.500000 0.019231
7 0.500000 0.016667
8 0.500000 0.014706
9 0.500000 0.013158
10 0.500000 0.011905

Normal distribution is in red, and beta distribution is in blue. Beta distribution's kurtosis is lower than 3, which is normal distributions, therefore it has heavier tail. It is symmetric per parameters that we have selected, therefore both skewness of normal and beta distribution are the same here, and they are both symmetric, yet beta distribution's parameters (higher degree of freedom) lets us to have different shapes from U shape, to convex and concave curves. We selected both standard deviations and means to be the same. Increasing both parameters together would not touch the mean since it is $\frac{\alpha}{\alpha+\beta}$, yet will affect the variance, which has the form of $\alpha\alpha\beta(\alpha+\beta)^2(\alpha+\beta+1)$.

2. b)

Figure 2 shows gamma distribution comparison with normal distribution. The first parameter λ is assumed to be one, and there were variation in terms of $k = 7, 10, 20, 30, 40, 50, 75, 100$ from top left to buttom left. The red line is the normal distribution, and blue line is gamma distribution. The mean and variances set the same. Following table shows them:

p mean variance
7 7.000000 7.000000
10 10.000000 10.000000
20 20.000000 20.000000
30 30.000000 30.000000
40 40.000000 40.000000
50 50.000000 50.000000
75 75.000000 75.000000
100 100.000000 100.000000

Gamma distribution's kurtosis is higher than 3, which is the kurtosis of the normal distribution, and this means it has lighter tail, in comparison with normal distribution. Gamma distribution has skewness more than zero, and this means it is right skewed, in contrast to normal distribution that is symmetric. Gamma distribution has higher degree of freedom, and shape could be different, normally it is sum of exponential time of processes. So it would be combination of memoryless stages and higher level memory posesed processes. Since at high level it has memory, so it is skewed.

Source code in MATLAB is attached.
%appendix: MATLAB code
% comparison of normal vs. beta
$x = -25 : 25;$

$disp('pmeanvariance');$
$forp = 1 : 10,$
$subplot(5, 2, p)$
$holdon;$
$x = linspace(0, 1, 2 * p * 100);$
$beta = betapdf(x, p, p);$
bm,bv
$= betastat(p, p);$
$dsp = sprintf('\%d\%f\%f', p, bm, bv);$
$disp(dsp);$
$norm = normpdf(x, bm, bv);$
$plot(x, norm,' r');$
$plot(x, beta,' b');$
$holdoff;$
end
%comparison of gamma vs. normal
$disp('pmeanvariance');$
$subplot(5, 2, 1)$
$holdon;$
$x = linspace(-7, 21, 2 * 7 * 100);$
$gamma = gampdf(x, 7, 1);$
gm,gv
$= gamstat(7, 1);$
$dsp = sprintf('\%d\%f\%f', 7, gm, gv);$
$disp(dsp);$
$norm = normpdf(x, gm, gv);$
$plot(x, norm,' r');$
$plot(x, gamma,' b');$
$holdoff;$
$forp = 1 : 5,$
$subplot(5, 2, p + 1)$
$holdon;$
$x = linspace(-10 * p, 30 * p, 20 * p * 100);$
$gamma = gampdf(x, 10 * p, 1);$
gm,gv
$= gamstat(10 * p, 1);$
$dsp = sprintf('\%d\%f\%f', 10 * p, gm, gv);$
$disp(dsp);$
$norm = normpdf(x, gm, gv);$
$plot(x, norm,' r');$
$plot(x, gamma,' b');$
$holdoff;$
end
$forp = 3 : 4,$
$subplot(5, 2, p + 4)$
$holdon;$
$x = linspace(-25 * p, 75 * p, 50 * p * 100);$
$gamma = gampdf(x, 25 * p, 1);$
gm,gv
$= gamstat(25 * p, 1);$
$dsp = sprintf('\%d\%f\%f', 25 * p, gm, gv);$
$disp(dsp);$
$norm = normpdf(x, gm, gv);$
$plot(x, norm,' r');$
$plot(x, gamma,' b');$
$holdoff;$
end

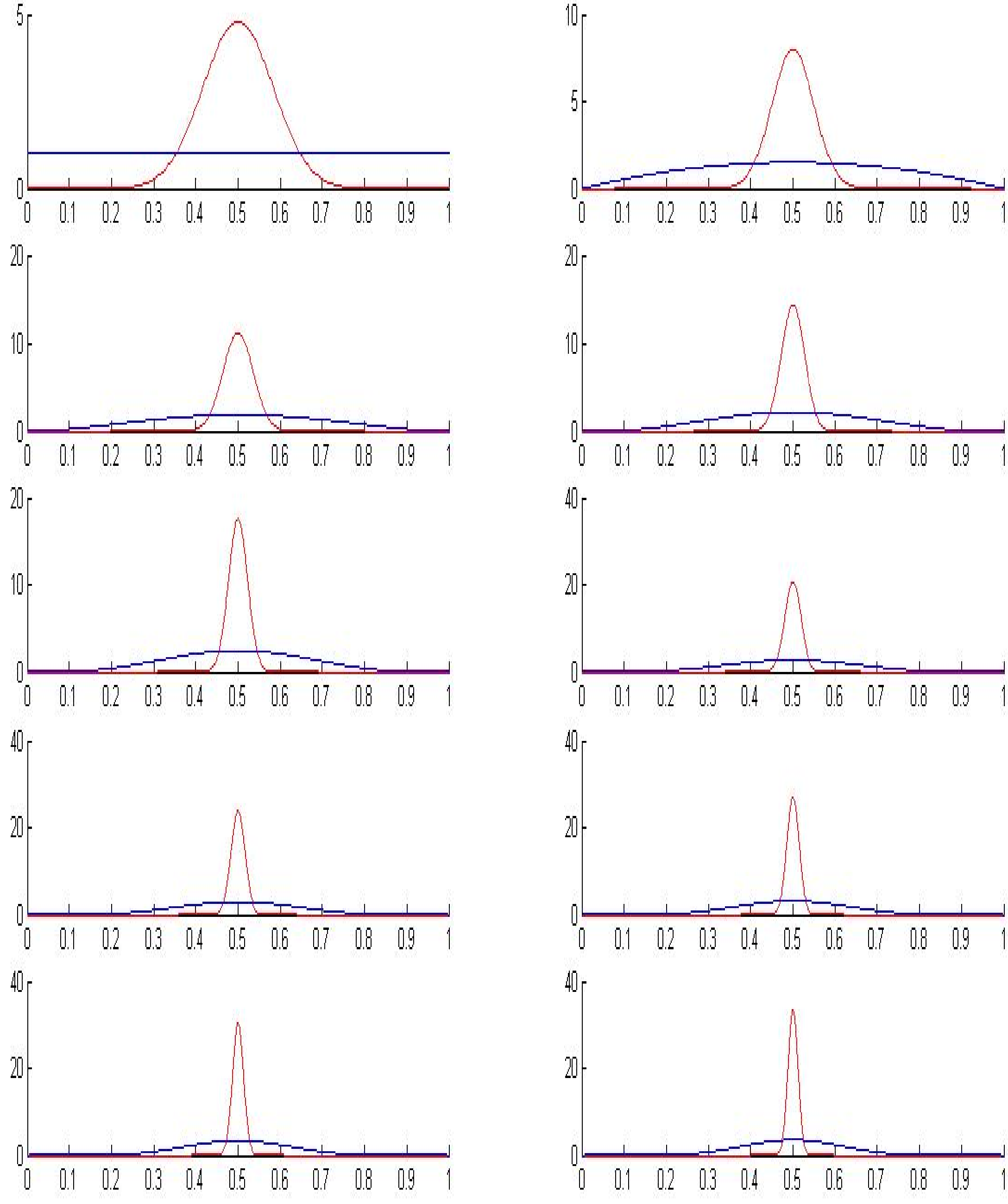

Figure 1: Beta Distribution comparison with Normal Distribution. From the top left to down right the parameters are changed from 1 to 10. The red line is normal distribution, and blude line is beta distribution. The mean and variance of both distributions are the same.

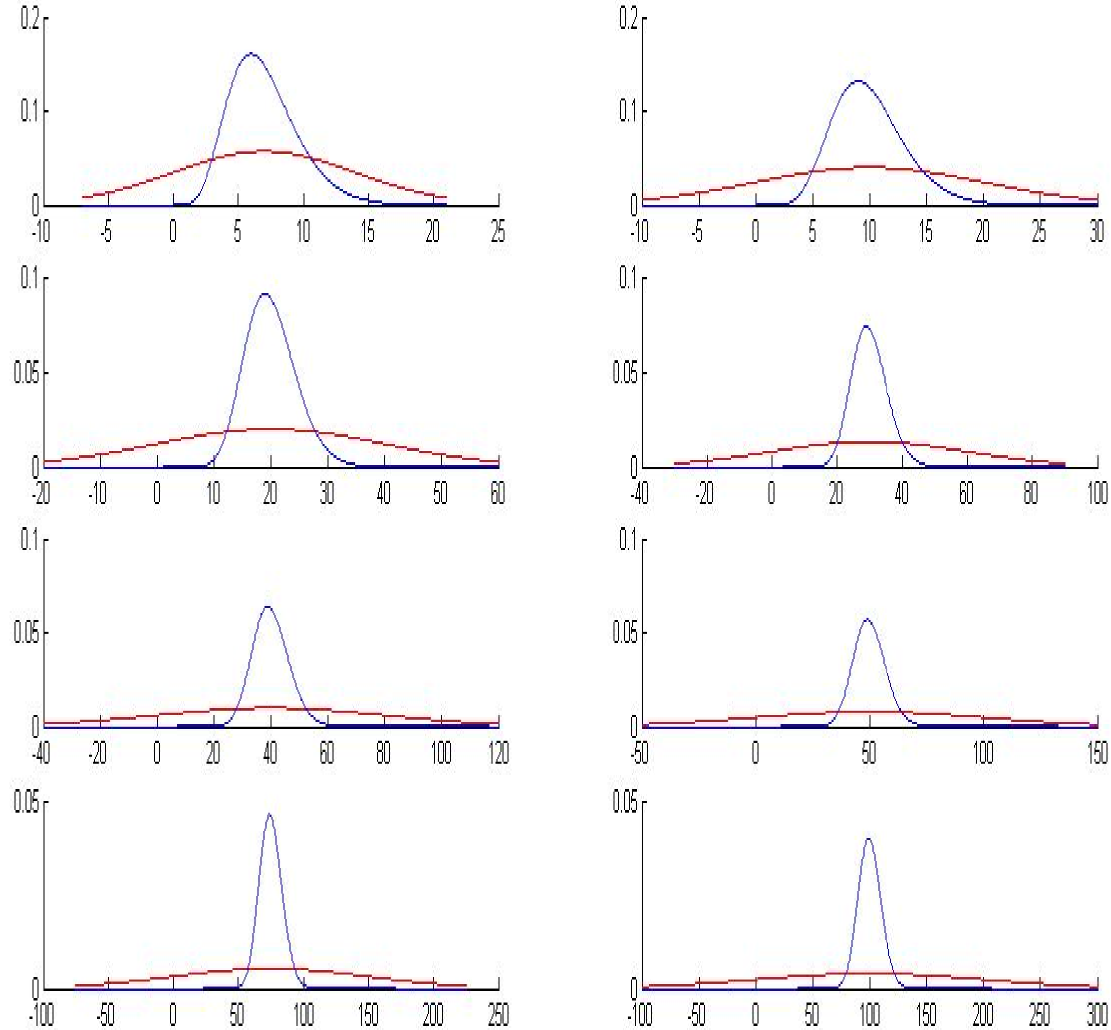

Figure 2: Gamma Distribution comparison with normal distribution. From top left to bottom right the parameter k of gamma distribution varies in the range of 7,10,20,30,40,50,75,100. Red line is the normal distribution, and blue line gamma distribution. Mean and variances are set the same.

Assignment 3, MECO 6315

Meisam Hejazinia

2/03/2013

Simulation is done both 100 times and 1000 times, for four sample sizes $n = 12, 24, 36, 50$, and the result is checked. Per central limit theorem, Lindeberg Levy, the result when sample size increases should be close to the normal distribution in the bell shape form, when the samples are i.i.d. as we had. The condition of finite mean and variance is also satisfied for our experiment. Uniform distribution has the variance of $\frac{1}{12}(b-a)^2$, and thefore the sample means variance would be $\sqrt{\frac{n}{12}}$, so z_k that is defined in the problem, considering mean of $\frac{1}{2}(a+b)$ for the uniform distribution would be normalized sample mean, and should comply central limit theorem, guiding us to normal distribution. Following is the MATLAB code used:

%appendix: MATLAB code HW#3 Meisam Hejazinia MECO 6315
% number of simulation
$ns = 1000;$
$fork = 1 : ns,$
$s = 0;$
$n = 12;$
$fori = 1 : n,$
$u(i) = unifrnd(0, 1);$
$s = s + u(i);$
end
$su(k) = s;$
$z(k) = (s - (n/2))/(sqrt(n/12));$
end
$x = -3 : 0.5 : 3;$
$subplot(2, 2, 1)$
$hist(z, x);$
$\%forsampleofsizen = 24$
$fork = 1 : ns,$
$s = 0;$
$n = 24;$
$fori = 1 : n,$
$u(i) = unifrnd(0, 1);$
$s = s + u(i);$
end
$su(k) = s;$
$z(k) = (s - (n/2))/(sqrt(n/12));$
end
$x = -3 : 0.5 : 3;$
$subplot(2, 2, 2)$
$hist(z, x);$
$\%forsampleofsizen = 36$
$fork = 1 : ns,$
$s = 0;$
$n = 36;$
$fori = 1 : n,$
$u(i) = unifrnd(0, 1); s = s + u(i);$
end
$su(k) = s; z(k) = (s - (n/2))/(sqrt(n/12));$
end
$x = -3 : 0.5 : 3;$
$subplot(2, 2, 3)$
$hist(z, x);$
$\%forsampleofsizen = 50$
$fork = 1 : ns,$
$s = 0;$
$n = 50;$
$fori = 1 : n,$
$u(i) = unifrnd(0, 1);$
$s = s + u(i);$
end
$su(k) = s;$
$z(k) = (s - (n/2))/(sqrt(n/12));$
end
$x = -3 : 0.5 : 3; subplot(2, 2, 4); hist(z, x);$

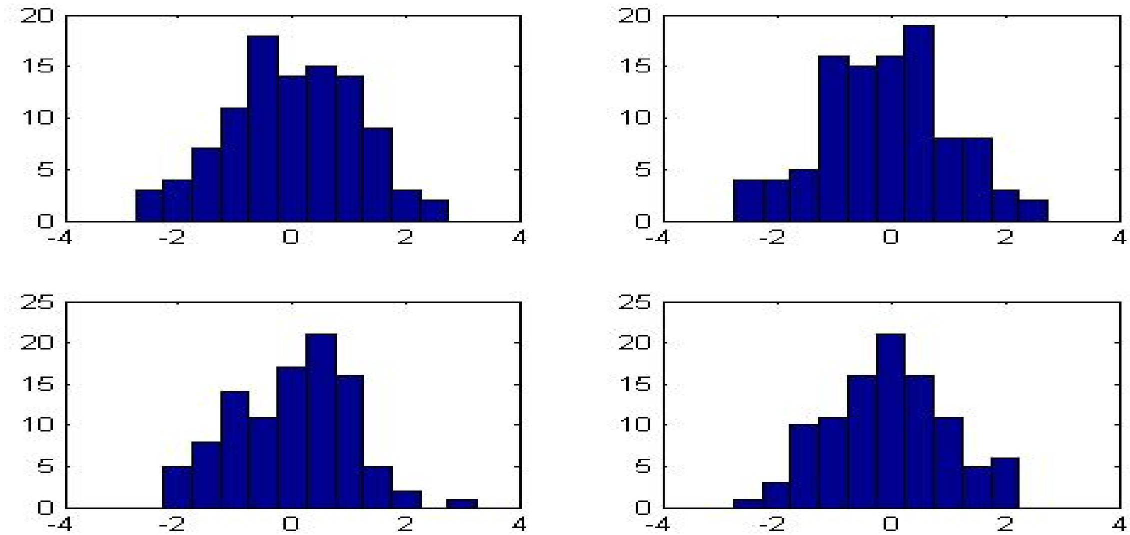

Figure 1: Simulation of 100 sample means of size 12,24, 36, and 50 respectively from top left to bottom right, showing better approximation of normal distribution histogram, as sample size increases.

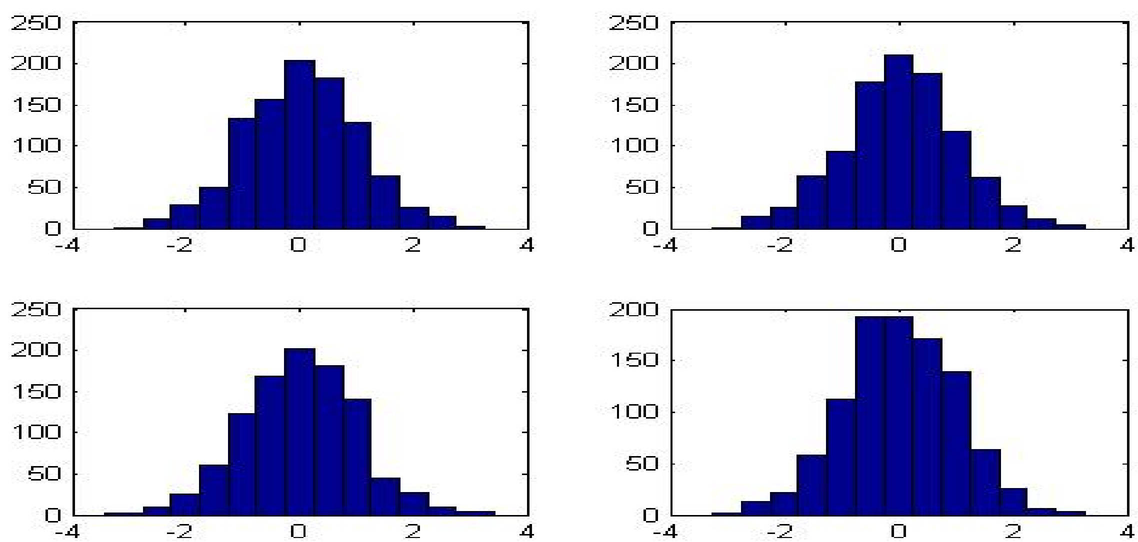

Figure 2: Simulation of 1000 sample means of size 12,24, 36, and 50 respectively from top left to bottom right, showing better approximation of normal distribution histogram, as sample size increases.

Assignment 4, MECO 6315

Meisam Hejazinia

2/09/2013

Answer to Question 1

In the sequence of random variables from sample size of 12, 24, and 36 was selected, and 1000 times simulation is run to calculate the mean distribution; then the normal plot was drawn using 'normplot(z)' function in matlab, and the result is plotted in figure 1. As could be seen the result lay on the line, showing that the distribution is normal. The result shows that although we have selected the sample from uniform distribution the mean converges to normal distribution as sample size increases. This is right the same result as central limit theorem. In other word assuming $s_n = \frac{1}{n}\sum x_i$ when x_i was selected from uniform variable, based on the figure will go to normal and in other word $p(s_n < t) \to N(\mu, \sigma^2/n)$, and we can say that $\sqrt{n}(p(s_n < t) - \mu) \to N(0, \sigma^2)$

Answer to Question 2

For $\alpha = 1.5$ and $\alpha = 2.0$ each the stable distribution is created, using two random variables of uniform and exponential distributions each independant of the other. For $\alpha = 1.5$ there is deviation from the normal line, yet for $\alpha = 2.0$ the stable distribution fits the normal probability plot line. MATLAB code is attached.

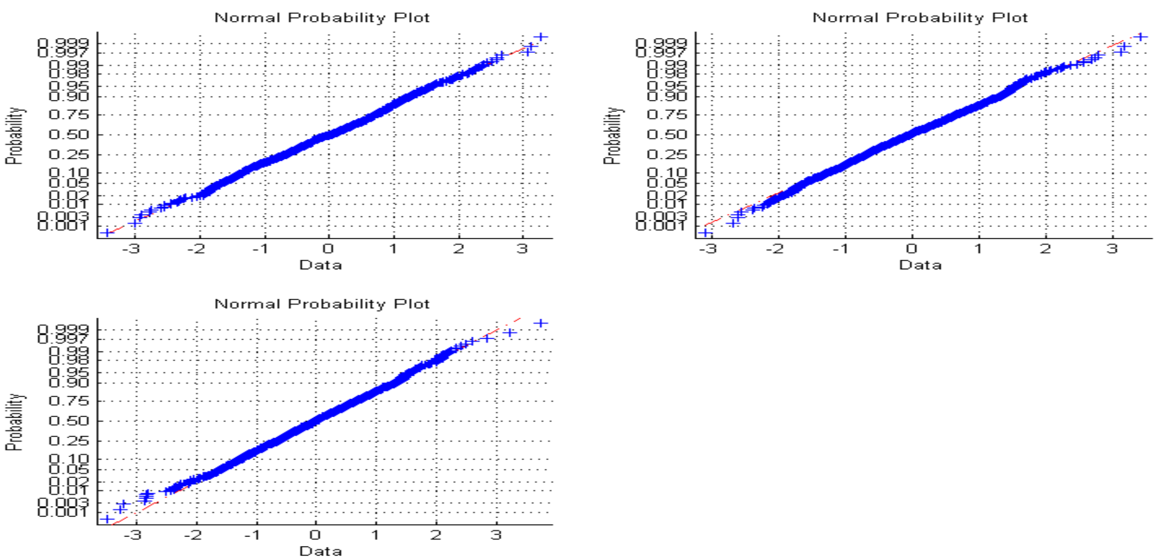

Figure 1: From top left to bottom right the sample size are varied from 12, 24 to 36. Samples were taken from the uniformly distributed random numbers. Result shows that although the distribution we collect sample from is unform the mean of samples as sample size increases will be closer to normal distribution based on central limit theorem.

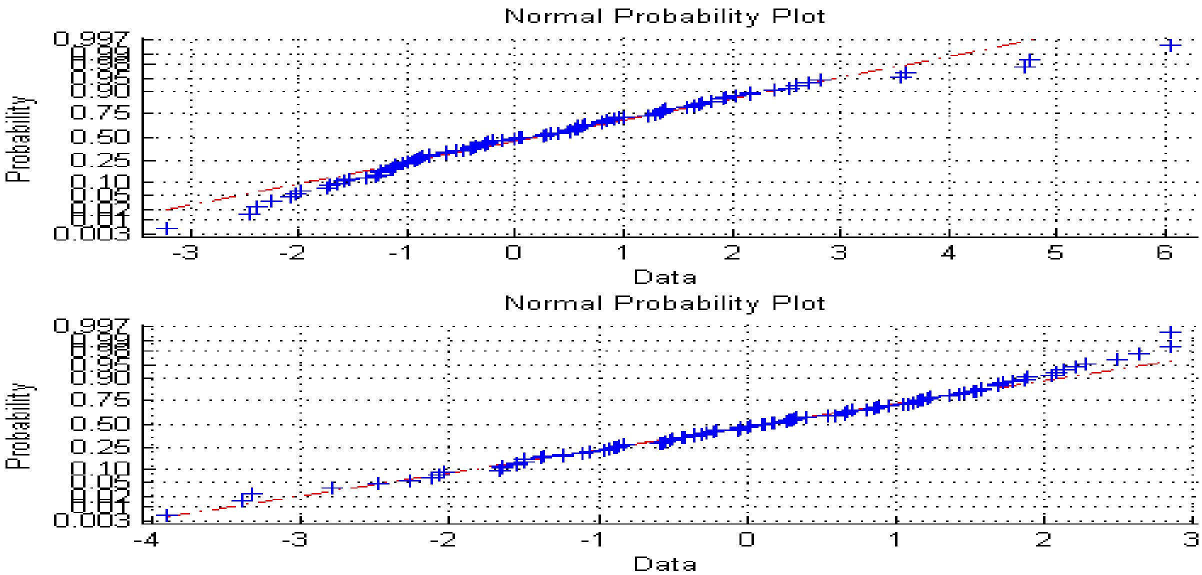

Figure 2: Stable distribution normal probability plot. Top one has $\alpha = 1.5$ while the one in the bottom has $\alpha = 2.0$. Both are result of 100 times simulation.

Assignment 5, MECO 6315

Meisam Hejazinia

2/20/2013

Answer to Question 1

I generated x from uniform distribution between zero and one, since the domain of the Burr distribution was also approximately between zero and one. I generated 10,000 value for the $xf(x)$, and $x^2.f(x)$ and calculated the mean and variance. The result of the simulated mean, and variance, and alpha and beta that calculated from them using $\alpha = (\frac{1-\mu}{\sigma^2} - \frac{1}{\mu})\mu^2$, and $\beta = \alpha(\frac{1}{\mu} - 1)$, is shown in the following table:

In the following figure I plotted beta distribution in red, and Burr distribution in blue. As λ increases beta would become better estimation for Burr distribution. Both distributions are right skewed, mean have skewness greater than zero. As mode of Burr distribution increases parameter of beta distribution mean α and β would become equal.

Answer to Question 2

Cauchy distribution is simulated five times with size 1000, and then it is plotted in the normal probability plot. As tails show there is no fit between two distributions. The figure shows the plot.

MATLAB code is attached.

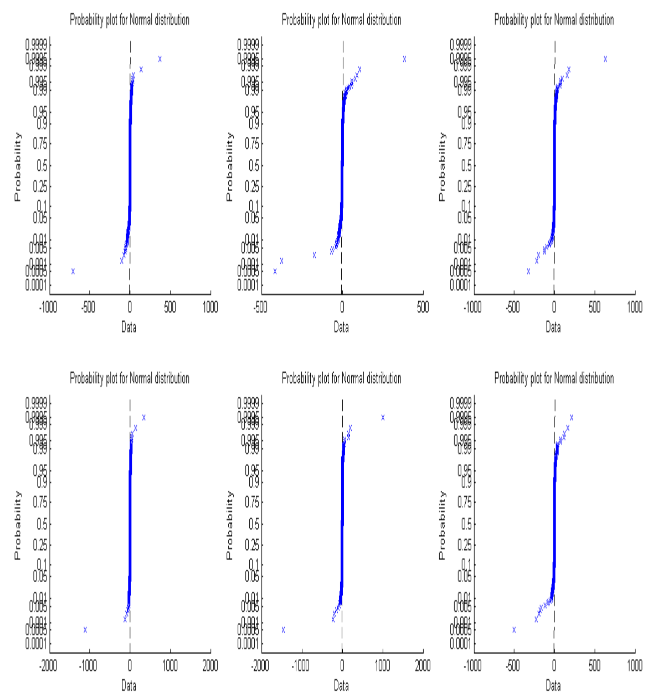

Figure 2: Cauchy distribution in probability plot; five times simulation with size 1000.

λ	μ	σ^2	α	β
0.1000	0.1051	0.0011	8.9335	76.0446
0.2000	0.2147	0.0049	7.1159	26.0298
0.3000	0.3076	0.0146	4.1704	9.3855
0.4000	0.4098	0.0250	3.5497	5.1126
0.5000	0.5019	0.0324	3.3655	3.3398

Table 1: Estimated Mean, Variance of Burr XI dist., and corresponding beta parameters

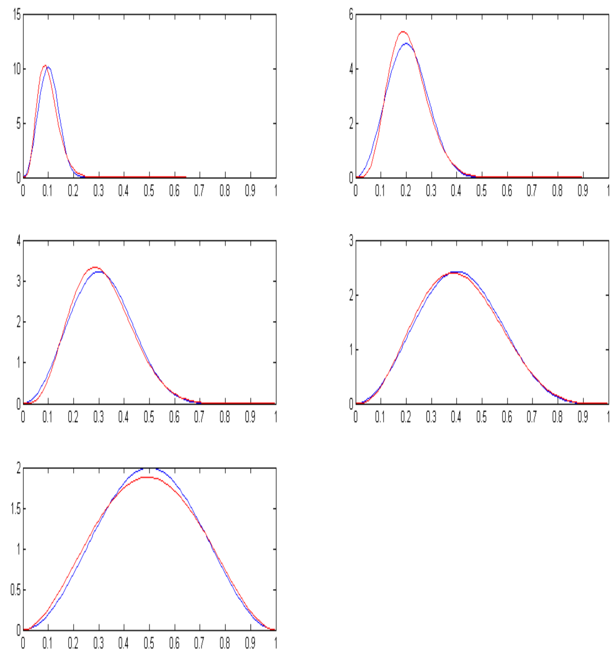

Figure 1: Burr distribution against Beta distribution. From top left to bottom right mode of burr distribution increases from .1 to .5. The mean variance and parameter corresponding to this is showen in the table.

Assignment 6, MECO 6315

Meisam Hejazinia

2/27/2013

Answer to Question 2

Based on the data following parameters were calculated:

I used the SAS for q-q plot with the following code:

data sheets;
input betas @;
datalines;
0.565
0.595

0.537
;
run;
proc capability data=sheets noprint;
qqplot betas beta($alpha$ = 1.937935 $beta$ = 2.450107);
run;

$\bar{x}$	0.44164
s^2	0.04576692
α	1.937935114
β	2.450107441

Table 1: Parameters of the data estimated for beta distribution

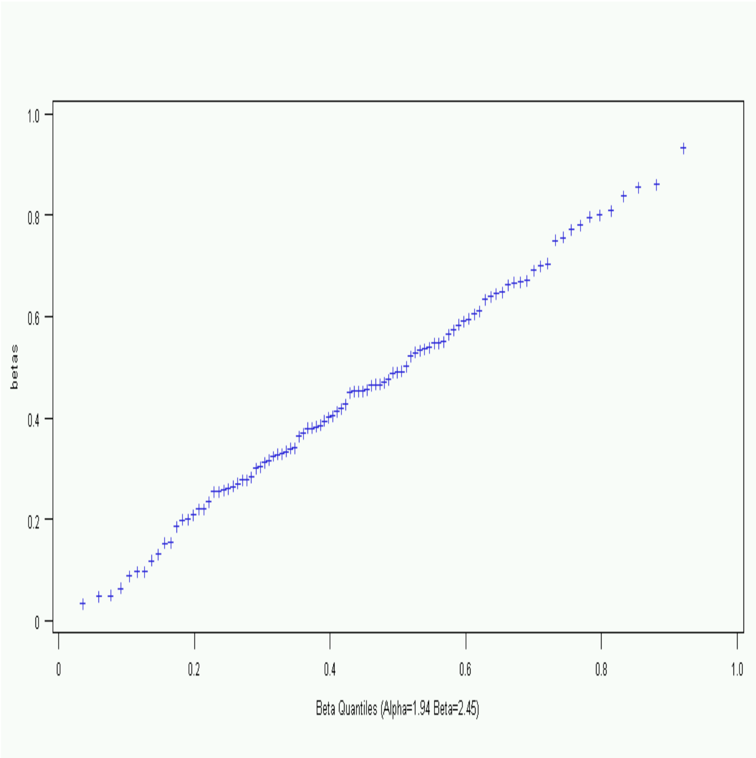

Figure 1: QQ-plot of data with beta distribution with estimated parameters of table 1

Assignment 7, MECO 6315

Meisam Hejazinia

03/06/2013

Answer to Question 2

Based on the data, and likelihood calculationfollowing parameters were calculated:

I used MATLAB for qq-plot. To do this I sorted x, and created $F^{-1}(i/n)$ and through the following code I got the qq-plot:

$Y = icdf('inversegaussian', 0.005 : 0.001 : 1.00, 34.62, 436.6686751);$
$xx = sort(x)$
$qqplot(xx, Y)$

As could be seen the data fits the distribution with the parameters.

$\bar{x}$	34.62
s^2	95.934
$\frac{1}{x}$	0.0912
$\mu = \bar{x}$	34.62
$\lambda = \frac{1}{\frac{1}{x} - \frac{1}{\bar{x}}}$	436.67
$\sigma_\mu^2 = \frac{\mu^3}{n\lambda}$	.95
$\sigma_\lambda^2 = \frac{2\lambda^2}{n}$	3813.6

Table 1: Parameters of the data estimated for wald distribution

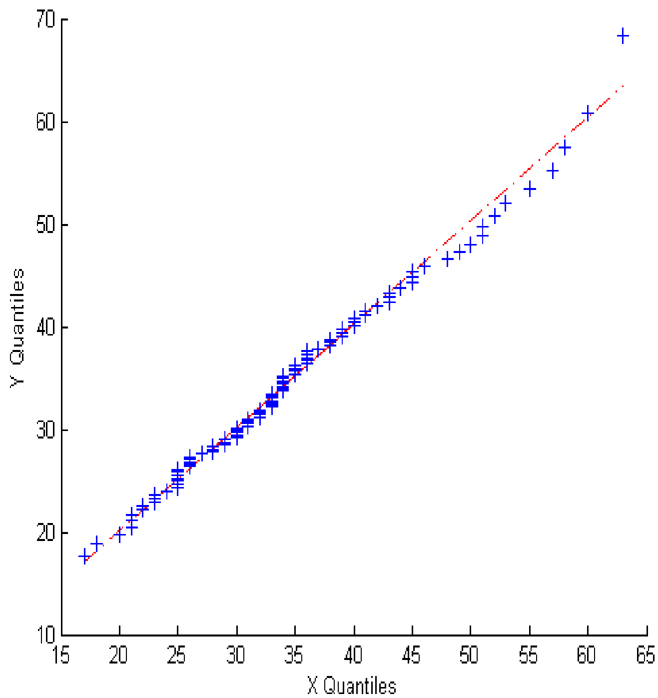

Figure 1: QQ Plot of data versus wald distribution

Assignment 8, MECO 6315

Meisam Hejazinia

03/20/2013

Part 2

Using numerical integration I calculated $E(\frac{1}{x})$, $E(\frac{1}{2+x})$, and $\bar{x}$. Then I used the the calculated formula to compute the expectation, variance, bias, and MSE for both estimators. Figure 1 shows the bias of estimators. As could be seen for $p < .3$ MLE estimator has less bias than Bays estimator. When $.3 < p < .75$ Bays estimator has less bias, and for $.75 <$ Again MLE estimators bias is lower than Bays estimator. Figure 2 shows MSE of MLE vs. Bays estimator. As could be seen if $p < .9$ bays estimator has lower MSE, while for $p > .9$ MSE of MLE would be lower. Comparison of variance of estimators is shown in third figure. As could be seen for $p < .9$ Bays estimator would have lower variance, but as $p > .9$ MLE estimators' variance would be lower. The code of MATLAB is attached.

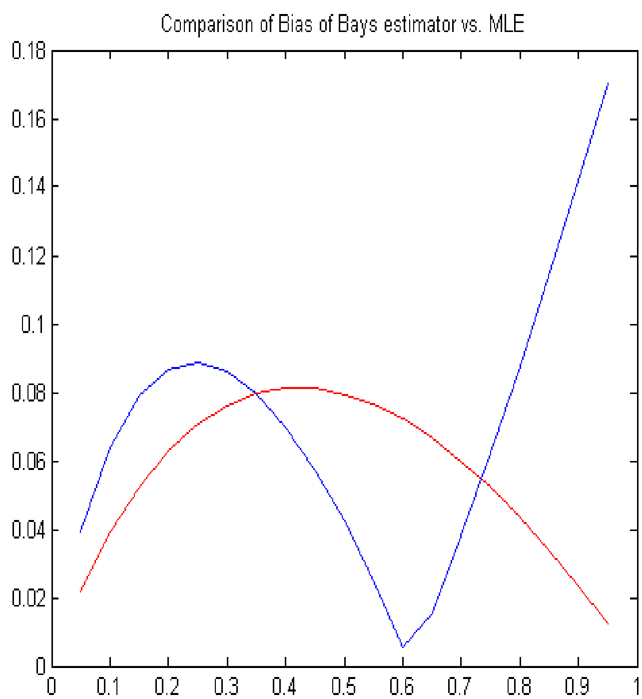

Figure 1: Bias of MLE vs. Bays estimator. Red one is MLE, and blue one is Bays estimator.

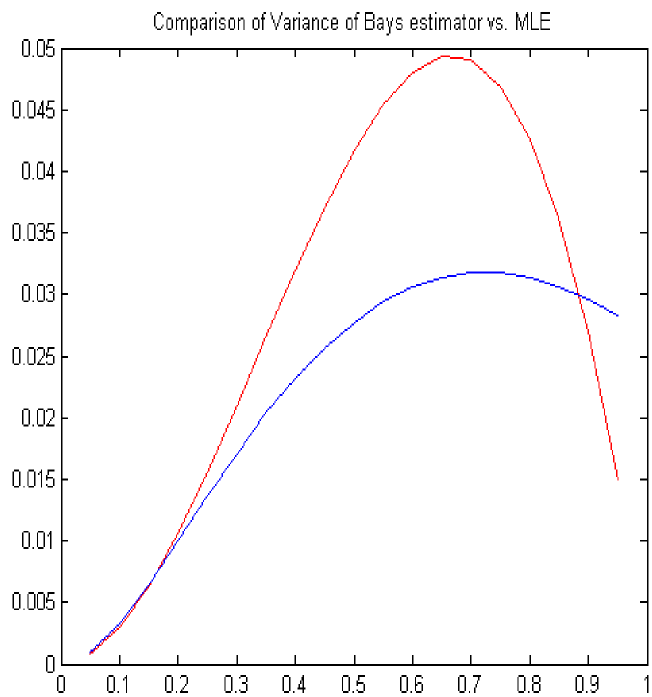

Figure 2: Variance of MLE vs. Bays estimator. Red one is MLE, and blue one is Bays estimator.

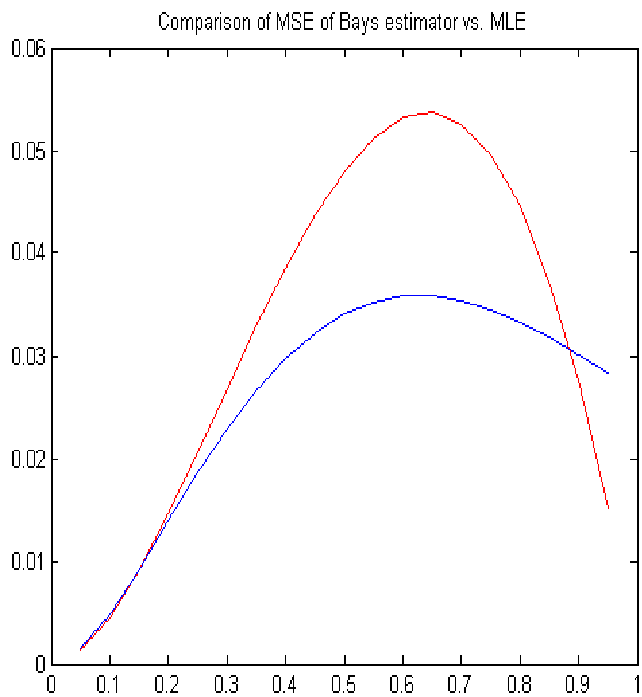

Figure 3: MSE of MLE vs. Bays estimator. Red one is MLE, and blue one is Bays estimator.

P	π_{MLE}	V(π_{MLE})	Bias$_{MLE}$	MSE$_{MLE}$	π_{Bays}	V(π_{Bays})	Bias$_{Bays}$	MSE$_{Bays}$
0.05	0.05	0.00	0.02	0.00	0.06	0.00	0.04	0.00
0.10	0.10	0.00	0.04	0.00	0.13	0.00	0.06	0.00
0.15	0.15	0.01	0.05	0.01	0.18	0.01	0.08	0.01
0.20	0.20	0.01	0.06	0.01	0.24	0.01	0.09	0.01
0.25	0.25	0.02	0.07	0.02	0.29	0.01	0.09	0.02
0.30	0.30	0.02	0.08	0.03	0.33	0.02	0.09	0.02
0.35	0.35	0.03	0.08	0.03	0.38	0.02	0.08	0.03
0.40	0.40	0.03	0.08	0.04	0.42	0.02	0.07	0.03
0.45	0.45	0.04	0.08	0.04	0.46	0.03	0.06	0.03
0.50	0.50	0.04	0.08	0.05	0.50	0.03	0.04	0.03
0.55	0.55	0.05	0.08	0.05	0.54	0.03	0.02	0.04
0.60	0.60	0.05	0.07	0.05	0.57	0.03	0.01	0.04
0.65	0.65	0.05	0.07	0.05	0.60	0.03	0.02	0.04
0.70	0.70	0.05	0.06	0.05	0.64	0.03	0.04	0.04
0.75	0.75	0.05	0.05	0.05	0.67	0.03	0.06	0.03
0.80	0.80	0.04	0.04	0.04	0.70	0.03	0.09	0.03
0.85	0.85	0.04	0.03	0.04	0.72	0.03	0.11	0.03
0.90	0.90	0.03	0.02	0.03	0.75	0.03	0.14	0.03
0.95	0.95	0.02	0.01	0.02	0.78	0.03	0.17	0.03

Table 1: Estimators expectation, variance, bias, and MSE for given p

Assignment 9, MECO 6315

Meisam Hejazinia

04/03/2013

Part b

Equation you have provided is cumulative distribution of normal variable with mean zero and variance one until $z_{\frac{\alpha}{2m}}$. So if we get the inverse normal distribution of $1 - \frac{\alpha}{2m}$ we will get $z_{\frac{\alpha}{2m}}$. For different values of m I derieved that, and it is ploted against $5 < m < 800$.

The curve shows diminishing return, or elbow on around 50. This shows after this number confidence interval is not sensitive to the value of m. It indicates that probably confidence interval of of .001 is probably close to full confidence.

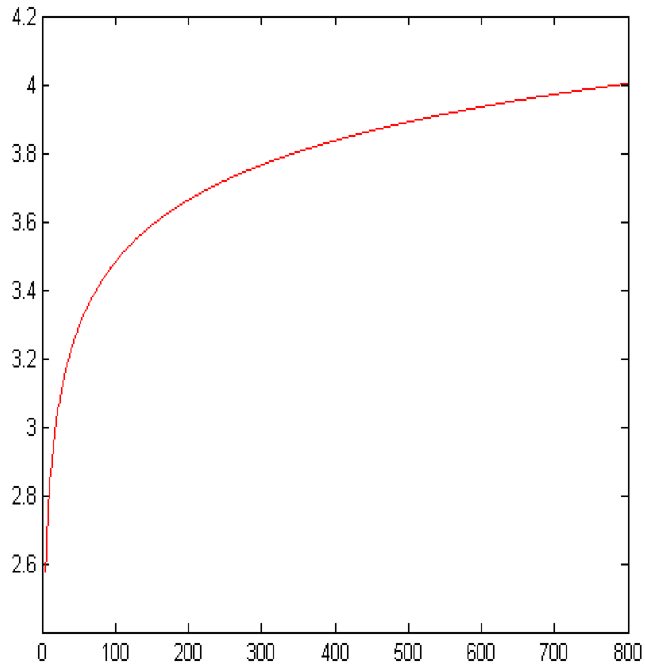

Figure 1: Confidence interval versus different z-values

Assignment 10, MECO 6315

Meisam Hejazinia

04/07/2013

Source	SS	df	MS	F	$Prob > F$
Columns(b)	867.53	4	216.88	3.70	0.01
Error(w)	2928.91	50	58.58		
Total	3796.44	54			

Table 1: One way ANOVA, independence assumption

1 Question 1

I used one way Anova for the first part, which made assumption of independance. As the p-value is significant w.r.t. 95% confidence interval, we can reject null hypothesis that nothing is going on. The code of MATLAB is attached.

2 Question 2

Repeated measure ANOVA is shown in the following table. Two way ANOVA gave me $p - val(b) < .05$ indicating that null hypothesis of no difference among word positions could be rejected.

3 Question 3

Result Bonferroni paired ttest is shown in the following table. As could be seen in table 3, null hypothesis of same mean for first and second, first and fourth, second and third, and third and fourth could be rejected.

Positions	$p - value$
x_1, x_2	0
x_1, x_3	0.3386
x_1, x_4	0.0001
x_1, x_5	0.0131
x_2, x_3	0.0002
x_2, x_4	0.4079
x_2, x_5	0.0162
x_3, x_4	0.002
x_3, x_5	0.1112
x_4, x_5	0.1021

Table 3: Bonferroni method ten paired t-test

4 Question 4

Result of multivariate analysis based on given formula and contrast vector is shown in table 4. Lower bound and higher bound is calculated, only between first and second position we have significant difference.

To select the method out of four we have the following:

1. First method is not reliable, since we know we have repeated measurement, mean with common factor.
2. Second method assumes no covariance, which we do not know anything about.
3. Fourth method assumes common covariance matrix, yet that may not also be credible.

Probably now that we do not have any information it is better to stick to the conservative approach of using Bonferroni method. The middle approach

Source	SS	df	MS	F	$Prob > F$
Columns (b)	867.53	4	216.882	9.25	$2.18E-05$
Rows	1990.84	10	199.084	8.49	$3.21E-07$
Error(w)	938.07	40	23.452		
Total	3796.44	54			

Table 2: Repeated Measure ANOVA

Positions	Low B.	High B.
x_1, x_2	0.855	20.2360
x_1, x_3	-6.093	10.0930
x_1, x_4	-1.969	19.6053
x_1, x_5	-7.090	17.8169
x_2, x_3	-18.773	1.6824
x_2, x_4	-9.499	6.0444
x_2, x_5	-15.272	4.9081
x_3, x_4	-3.398	17.0346
x_3, x_5	-5.494	12.2212
x_4, x_5	-14.565	7.6560

Table 4: Multivariate analysis

could be using Bonferroni Holm method.

MECO 6315
Assignment 11

Var1: crime rate

For the data on the other side of this page, perform a multiple regression to determine what variables are might be useful in predicting the crime rate. Make

1. sure you do pre-plots to check for curvature and potential outliers. Also make sure
2. you do residual plots for the predictor variables selected and a normal probability
3. plot to check the normality assumption.

4. In determining which variables to use as predictors do at least the forward selection method, the backward elimination method and true step-wise regression. If you did not wind up with the same models from the three approaches explain which model
5. you would use and why.

which

6. Comment as to whether the variables chosen are "sensible" and whether the sign of their coefficients seems reasonable.

7. In your report include enough computer output so that all steps in the stepwise procedure are shown. Also include all plots.

I can send you the data as an EXCEL file if you will e-mail me at wiorkow@utdallas.edu.

small figure 2

do 3 reg

- Fw sel
- Backward elimin
- stepwise

see whether get same model?

wordy Explanation

1. which model you believe
2. variables make sense of it
   what it means? → use more words

⚠ ① Plots important (residual)

② steps as computer works
history of what doing

SAS → .5 drop → reset it, since keeps everything

Datafile Name: US Crime
Number of cases: 47
Variable Names:

1. R: Crime rate: # of offenses reported to police per million population
2. Age: The number of males of age 14-24 per 1000 population
3. S: Indicator variable for Southern states (0 = No, 1 = Yes)
4. Ed: Mean # of years of schooling x 10 for persons of age 25 or older
5. Ex0: 1960 per capita expenditure on police by state and local government
6. Ex1: 1959 per capita expenditure on police by state and local government
7. LF: Labor force participation rate per 1000 civilian urban males age 14-24
8. M: The number of males per 1000 females
9. N: State population size in hundred thousands
10. NW: The number of non-whites per 1000 population
11. U1: Unemployment rate of urban males per 1000 of age 14-24
12. U2: Unemployment rate of urban males per 1000 of age 35-39
13. W: Median value of transferable goods and assets or family income in tens of $
14. X: The number of families per 1000 earning below 1/2 the median income

The Data:

R	Age	S	Ed	Ex0	Ex1	LF	M	N	NW	U1	U2	W	X
79.1	151	1	91	58	56	510	950	33	301	108	41	394	261
163.5	143	0	113	103	95	583	1012	13	102	96	36	557	194
57.8	142	1	89	45	44	533	969	18	219	94	33	318	250
196.9	136	0	121	149	141	577	994	157	80	102	39	673	167
123.4	141	0	121	109	101	591	985	18	30	91	20	578	174
68.2	121	0	110	118	115	547	964	25	44	84	29	689	126
96.3	127	1	111	82	79	519	982	4	139	97	38	620	168
155.5	131	1	109	115	109	542	969	50	179	79	35	472	206
85.6	157	1	90	65	62	553	955	39	286	81	28	421	239
70.5	140	0	118	71	68	632	1029	7	15	100	24	526	174
167.4	124	0	105	121	116	580	966	101	106	77	35	657	170
84.9	134	0	108	75	71	595	972	47	59	83	31	580	172
51.1	128	0	113	67	60	624	972	28	10	77	25	507	206
66.4	135	0	117	62	61	595	986	22	46	77	27	529	190
79.8	152	1	87	57	53	530	986	30	72	92	43	405	264
94.6	142	1	88	81	77	497	956	33	321	116	47	427	247
53.9	143	0	110	66	63	537	977	10	6	114	35	487	166
92.9	135	1	104	123	115	537	978	31	170	89	34	631	165
75	130	0	116	128	128	536	934	51	24	78	34	627	135
122.5	125	0	108	113	105	567	985	78	94	130	58	626	166
74.2	126	0	108	74	67	602	984	34	12	102	33	557	195
43.9	157	1	89	47	44	512	962	22	423	97	34	288	276
121.6	132	0	96	87	83	564	953	43	92	83	32	513	227
96.8	131	0	116	78	73	574	1038	7	36	142	42	540	176
52.3	130	0	116	63	57	641	984	14	26	70	21	486	196
199.3	131	0	121	160	143	631	1071	3	77	102	41	674	152
34.2	135	0	109	69	71	540	965	6	4	80	22	564	139
121.6	152	0	112	82	76	571	1018	10	79	103	28	537	215
104.3	119	0	107	166	157	521	938	168	89	92	36	637	154
69.6	166	1	89	58	54	521	973	46	254	72	26	396	237
37.3	140	0	93	55	54	535	1045	6	20	135	40	453	200
75.4	125	0	109	90	81	586	964	97	82	105	43	617	163
107.2	147	1	104	63	64	560	972	23	95	76	24	462	233
92.3	126	0	118	97	97	542	990	18	21	102	35	589	166
65.3	123	0	102	97	87	526	948	113	76	124	50	572	158
127.2	150	0	100	109	98	531	964	9	24	87	38	559	153
83.1	177	1	87	58	56	638	974	24	349	76	28	382	254
56.6	133	0	104	51	47	599	1024	7	40	99	27	425	225
82.6	149	1	88	61	54	515	953	36	165	86	35	395	251
115.1	145	1	104	82	74	560	981	96	126	88	31	488	228
88	148	0	122	72	66	601	998	9	19	84	20	590	144
54.2	141	0	109	56	54	523	968	4	2	107	37	489	170
82.3	162	1	99	75	70	522	996	40	208	73	27	496	224
103	136	0	121	95	96	574	1012	29	36	111	37	622	162
45.5	139	1	88	46	41	480	968	19	49	135	53	457	249
50.8	126	0	104	106	97	599	989	40	24	78	25	593	171
84.9	130	0	121	90	91	623	1049	3	22	113	40	588	160

Assignment 12, MECO 6315

Meisam Hejazinia

04/20/2013

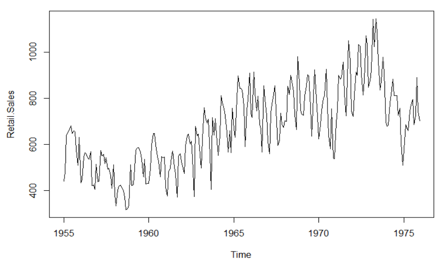

Figure 1: Monthly Retail Sales

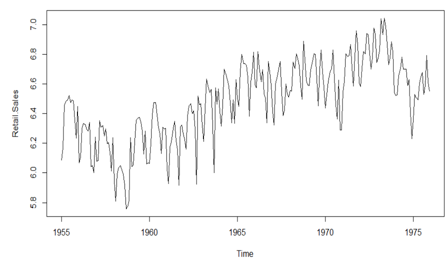

Figure 2: Log Transformed Monthly Retail Sales

1 General Analysis

From figure 1 we can infer that the model should be multiplicative, since the fluctuation in data is not constant over time. As a result, we used log transformation to transform the time series data. As figure 2 shows the transformed curve, the size of the seasonal fluctuation and the random fluctuation do not depend on the level of time series anymore. The transfomred data could be used to fit additive model. Next we tried to decompose the time series to trend and seasonal component. Figure 3 shows the result of smoothing procedure for order of 6. Still there are many random fluctuation in the data, so we smoothed by order 12, which is illustrated in figure 4. Next we used 'R' to decompose the trend and seasonal component, shown in figure 5.

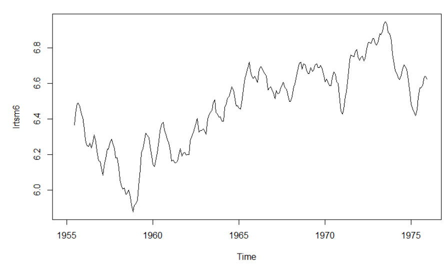

Figure 3: Result of Smoothing with Order 6

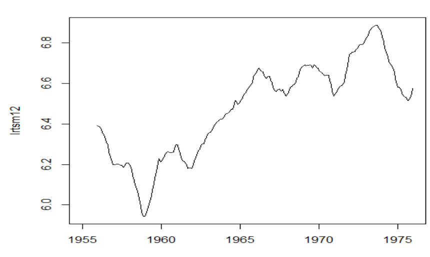

Figure 4: Result of Smoothing with Order 12

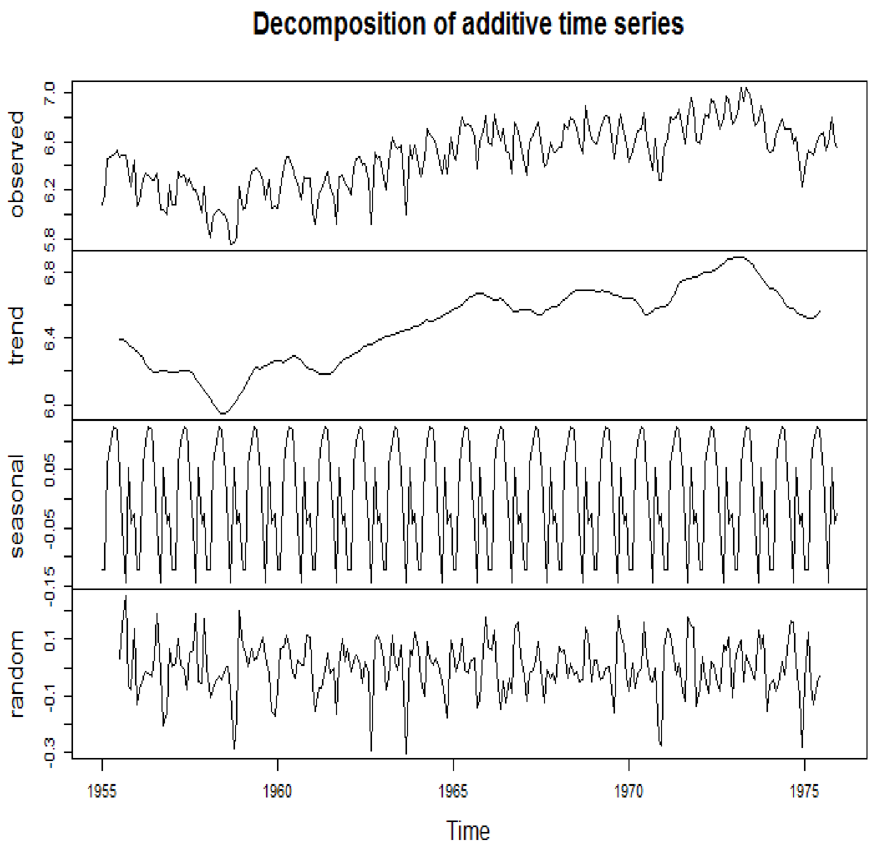

Figure 5: Result of Decomposition into Seasonal and Trend Components

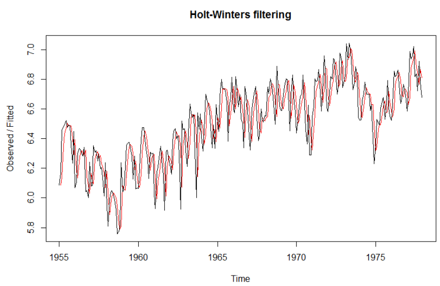

Figure 6: Holts Winter Prediction Versus Actual. Red curve illustrates predicted time series.

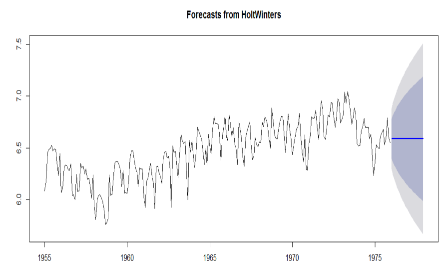

Figure 7: Holts Winter Forecast

2 Holt Winter

Holt Winter is used to forcast using exponential smoothing. Result of running Holt Winter method tells us that $\alpha = 0.6259951$. This is not close to zero, so recent observations have more effect on determining the current value. Coefficient is calculated 6.588841. Sum of square is alculate $SSE = 5.522659$ for Holt Winter multiplicative procedure.

By default, HoltWinters() just makes forecasts for the same time iginalperiod covered by our or time series. As a result we had no choice other than run the model on all years, rather than holding two last years to compare. Figure 6 shows how forecast, in red, is compared with time series. To be able to forecast another package called forecast is used. We excluded last two years, and created the model, and forecast procedure created forecast values that are shown in table 1. Figure 7 shows these forecasts. As I evaluated whether the real value is inside the confidence interval, I could not reject that they are inside of it.

Ljung-Box test is used for autocorrelation between the residuals. The result was showing that we have autocorrelation, since we had $p-value < 2.2e-16$. This tells us that we need to use another method to improve the forecast. Also I plotted corologram, on lag 2 and 3, and 7 and 8 the confidence level is touched. Holt winter residuals is illustrated in figure 8. In sample forecast error seems to have constant variance over time. Figure 9 shows normal probability plot of residuals of the model, illustrating quite fit.

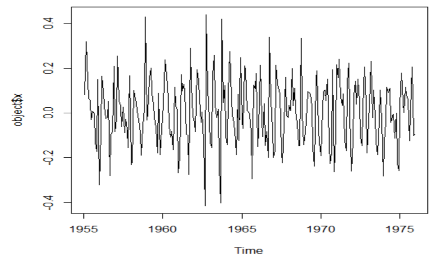

Figure 8: Holts Winter Residual Plot

Although exponential smoothing methods such as Holt Winter are useful for making forecast, they make no assumption about the correlation between time series elements. To have prediction using exponential smoothing method, it is required that forecast error be uncorrelated and normally distributed with mean zero, and constant variance. Autoregressive Integrated Moving Average (ARIMA) take into concern non-zero autocorrelation in irregular component of time series. In the next section we will run ARIMA procedure on the data.

Finally result of accuracy analysis of Holt winter method is shown in the following:

3 ARIMA

In order to run ARIMA we need to have stationary time series. To achieve stationary time series, it was

	Point	Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jan	1976	6.588841	6.398409	6.779273	6.297601	6.880081
Feb	1976	6.588841	6.364174	6.813508	6.245243	6.932439
Mar	1976	6.588841	6.334507	6.843175	6.19987	6.977812
Apr	1976	6.588841	6.307955	6.869727	6.159263	7.018419
May	1976	6.588841	6.283705	6.893977	6.122176	7.055506
Jun	1976	6.588841	6.261246	6.916436	6.087827	7.089855
Jul	1976	6.588841	6.24023	6.937452	6.055687	7.121995
Aug	1976	6.588841	6.220411	6.957271	6.025377	7.152305
Sep	1976	6.588841	6.201606	6.976076	5.996616	7.181066
Oct	1976	6.588841	6.183672	6.99401	5.969188	7.208494
Nov	1976	6.588841	6.166499	7.011183	5.942924	7.234758
Dec	1976	6.588841	6.149997	7.027685	5.917687	7.259995
Jan	1977	6.588841	6.134094	7.043588	5.893366	7.284316
Feb	1977	6.588841	6.118729	7.058953	5.869866	7.307816
Mar	1977	6.588841	6.10385	7.073832	5.847111	7.330571
Apr	1977	6.588841	6.089414	7.088268	5.825033	7.352649
May	1977	6.588841	6.075384	7.102298	5.803576	7.374106
Jun	1977	6.588841	6.061727	7.115955	5.78269	7.394992
Jul	1977	6.588841	6.048415	7.129267	5.762331	7.415351
Aug	1977	6.588841	6.035424	7.142258	5.742462	7.43522
Sep	1977	6.588841	6.02273	7.154952	5.723049	7.454633
Oct	1977	6.588841	6.010315	7.167367	5.704061	7.47362
Nov	1977	6.588841	5.99816	7.179521	5.685473	7.492209
Dec	1977	6.588841	5.986251	7.191431	5.66726	7.510422

Table 1: Holt winter forecasts

ME	RMSE	MAE	MPE	MAPE	MASE
0.003195335	0.148332824	0.118331919	0.013616809	1.836795686	1.027507875

Table 2: Holt winter model percision

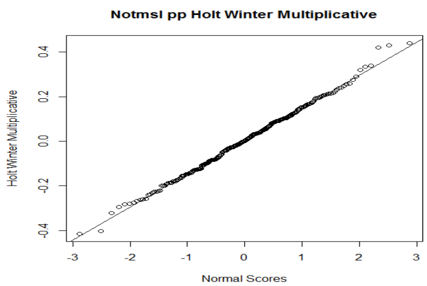

Figure 9: Holts Winter Residual Normal Probability Plot

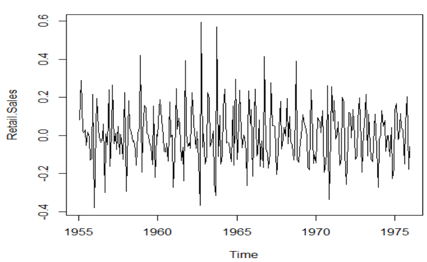

Figure 10: ARIMA First Difference Curve

need to take differences. As first difference was taken the variance becomes stationary, since according to figure 10, mean and variance become constant. Next it is needed to determine the degree of model (p,q). To determine the degree, auto correlogram is ploted in figure 11. Moreover, partial autocorrelogram is illustrated in figure 12. The two figures are indicative of either $(p, q) = (19, 0)$ or $(p, q) = (1, 11)$. We have also option with greater 'p' and 'q'. Model with less parameter is prefered to more parameter. As a result $(p, q) = (1, 11)$ is selected. The result of running the model is shown in the following.

Series: lrtsm
ARIMA(0,1,11)

Coefficients:
ma1 ma2 ma3 ma4 ma5 ma6 ma7 ma8
-0.3977 -0.2035 0.1365 -0.3439 0.1385 -0.0073 -0.1453 0.3126

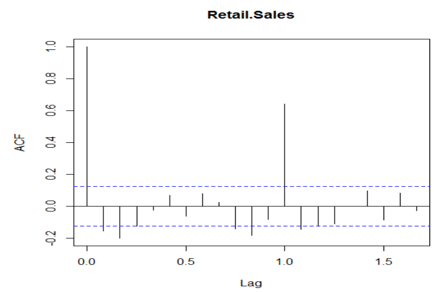

Figure 11: ARIMA Correlogram of Auto Correlation

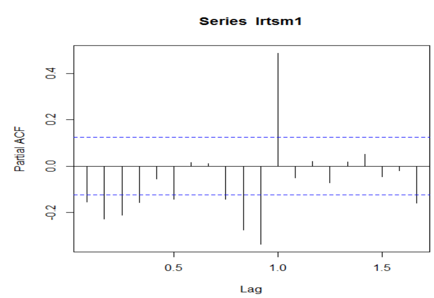

Figure 12: ARIMA Correlogram of Partial Auto Correlation

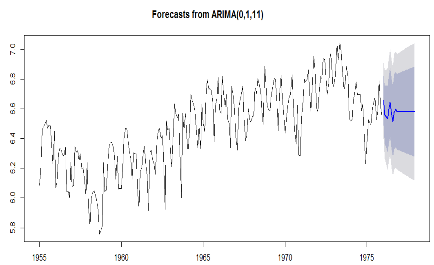

Figure 13: Manual ARIMA Model Prediction

s.e. 0.0608 0.0633 0.0624 0.0612 0.0635 0.0599 0.0705 0.0642
ma9 ma10 ma11
-0.4709 0.0076 0.2473
s.e. 0.0613 0.0593 0.0551

$sigma^2$ estimated as 0.01685: log likelihood=154.58
$AIC = -285.15$ $AICc = -283.84$ $BIC = -242.85$

Predictive accuracy of the model is given in the following:

Table 2 shows confidence interval for my prediction based on ARIMA. Due to unknow reason the model had very poor predictability. Only 7 out of 24 real values were correctly inside the intervals. This made me to test automatic ARIMA, which finds the fit automatically, rather than using my procedure. Figure 13 shows prediction of ARIMA model ran manually.

The result of automatic ARIMA is shown in the following. Compared with my model its RMSE (Root Eean Square Error)and MASE (Mean Absolute Percentage Error/Deviation) are lower, indicating lower residuals. Figure 14 shows automatic ARIMA prediction.

$alpha = 0.6145$
$gamma = 0.2317$

Initial states:
$l = 6.276$
$s = 0.0944$ -0.103 -0.0056 -0.0464 0.008 0.0073 0.0643 0.1101 0.1177 0.0868 -0.1754 -0.1582

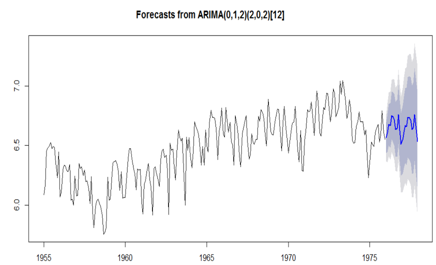

Figure 14: Automatic ARIMA Model Prediction

AIC	AICc	BIC
275.1742	276.9463	324.5862

Table 5: Automatic Exponential Fit Result

sigma: 0.1029

Table 3 shows the predicted confidence interval based on automatic ARIMA model. Interestingly, all the real parameters values are inside confidence interval of 95%.

After these I used automatic exponential model of 'R' to be able to compare result based on same measure of accuracy with ARIMA. The result is shown in the following:

ETS(A,N,A)

Call:
$ets(y = lrtsm)$

Smoothing parameters:
$alpha = 0.6145$
$gamma = 0.2317$

Initial states:
$l = 6.276$
$s = 0.0944$ -0.103 -0.0056 -0.0464 0.008 0.0073 0.0643 0.1101 0.1177 0.0868 -0.1754 -0.1582

sigma: 0.1029

Result of ETS (automatic exponential fit) of 'R' is

ME	RMSE	MAE	MPE	MAPE	MASE
0.005551083	0.129558696	0.106177122	0.053450937	1.648646594	0.921964505

Table 3: ARIMA model of $(p, q) = (1, 11)$

	Point Forecast	Low 80	Low 95	High 80	High 95
Jan-76	6.489844	6.401775	6.656209	6.822575	6.910644
Feb-76	6.366749	6.26394	6.560961	6.755172	6.857982
Mar-76	6.343575	6.234931	6.548808	6.75404	6.862684
Apr-76	6.309818	6.191387	6.533541	6.757263	6.875694
May-76	6.368988	6.249364	6.594965	6.820941	6.940566
Jun-76	6.413343	6.290241	6.645889	6.878435	7.001537
Jul-76	6.322524	6.196186	6.561182	6.79984	6.926178
Aug-76	6.274571	6.147272	6.515044	6.755517	6.882816
Sep-76	6.326108	6.191697	6.580017	6.833926	6.968337
Oct-76	6.340911	6.206489	6.594839	6.848767	6.983189
Nov-76	6.326868	6.192426	6.580835	6.834802	6.969244
Dec-76	6.322813	6.186225	6.580835	6.838857	6.975446
Jan-77	6.318821	6.180119	6.580835	6.842849	6.981551
Feb-77	6.314889	6.174106	6.580835	6.846782	6.987565
Mar-77	6.311014	6.168179	6.580835	6.850656	6.993491
Apr-77	6.307194	6.162337	6.580835	6.854477	6.999333
May-77	6.303427	6.156575	6.580835	6.858244	7.005095
Jun-77	6.29971	6.150891	6.580835	6.861961	7.01078
Jul-77	6.296041	6.14528	6.580835	6.865629	7.01639
Aug-77	6.292419	6.139741	6.580835	6.869251	7.021929
Sep-77	6.288843	6.134271	6.580835	6.872828	7.0274
Oct-77	6.285309	6.128867	6.580835	6.876362	7.032804
Nov-77	6.281817	6.123526	6.580835	6.879853	7.038144
Dec-77	6.278366	6.118248	6.580835	6.883305	7.043423

Table 4: ARIMA confidence interval for my Model

ME	RMSE	MAE	MPE	MAPE	MASE
0.001867907	0.100425412	0.077591936	0.013782450	1.206095820	0.673751645

Table 6: Automatic ARIMA Fit Result

	Point Forecast	Low 80	Low 95	High 80	High 95
Jan-76	6.565937	6.436982	6.694893	6.368717	6.763158
Feb-76	6.607679	6.446344	6.769014	6.360938	6.85442
Mar-76	6.674616	6.500495	6.848738	6.40832	6.940913
Apr-76	6.66898	6.482948	6.855012	6.384469	6.953491
May-76	6.749278	6.552054	6.946502	6.44765	7.050906
Jun-76	6.739237	6.531423	6.947051	6.421413	7.057061
Jul-76	6.717767	6.499877	6.935657	6.384533	7.051001
Aug-76	6.636195	6.408675	6.863716	6.288233	6.984158
Sep-76	6.644269	6.40751	6.881028	6.282177	7.006361
Oct-76	6.761017	6.515367	7.006668	6.385327	7.136708
Nov-76	6.624975	6.370743	6.879206	6.236161	7.013788
Dec-76	6.509787	6.247255	6.772319	6.108279	6.911296
Jan-77	6.547228	6.263095	6.831361	6.112684	6.981771
Feb-77	6.61214	6.311597	6.912683	6.152499	7.07178
Mar-77	6.666267	6.353268	6.979265	6.187576	7.144957
Apr-77	6.659518	6.33454	6.984496	6.162507	7.156528
May-77	6.7356	6.39907	7.07213	6.220921	7.250279
Jun-77	6.730885	6.383185	7.078585	6.199124	7.262646
Jul-77	6.714704	6.356183	7.073225	6.166394	7.263015
Aug-77	6.633988	6.264963	7.003013	6.069613	7.198363
Sep-77	6.64678	6.267542	7.026018	6.066785	7.226775
Oct-77	6.760761	6.371577	7.149944	6.165555	7.355966
Nov-77	6.634583	6.235701	7.033464	6.024546	7.244619
Dec-77	6.533912	6.125563	6.942261	5.909397	7.158427

Table 7: Automatic ARIMA Confidence Interval

ME	RMSE	MAE	MPE	MAPE	MASE
0.00278239	0.10286996	0.07862786	0.02780022	1.22239676	0.68274683

Table 9: Automatic Exponential Fit Result

AIC	AICc	BIC
275.1742	276.9463	324.5862

Table 8: Automatic Exponential Fit Result

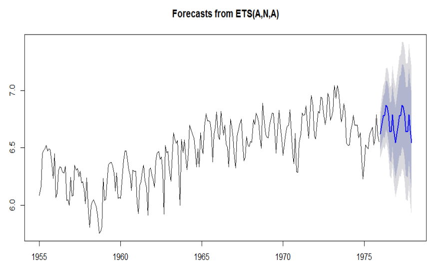

Figure 15: Automatic Exponential Model (ETS) Prediction

shown in figure 15.

The result shows that ARIMA fits the data much better than exponential model, according to its lower MAPE, and MASE. After all the models I fit the data worked worse than the models that 'R' automatically fited the time series data.

Assignment 13, MECO 6315

Meisam Hejazinia

04/30/2013

Param	estimate	STD ERR	t-value	p-value
alpha	-2.6639	0.827717092	-3.218342553	0.0013
beta	0.1094	0.030226715	3.617700091	0.0003
df	48			

Table 1: The Binary Logit Discrimination Result

	Pred. 0	Pred. 1
Actual 0	17	7
Actual 1	7	19

Table 2: Confusion Matrix for Binary Logit

1 Part 2

The result of running the binary logit model for discrimination is presented in table 1. Table 2 presents the confustion matrix for the binary logit model. Table 3 presents evaluation criteria to evaluate discrimination power of method based on Kappa and Goodman Kruskal tests. The results of both tests indicate significance.

κ	0.4391
var(κ)	0.02
zscore	3.1049
λ	0.4167
var(λ)	0.0243
zscore	2.6726

Table 3: Evaluation Criteria For Binary Logit Discr.

	0	1	Total
0	18	6	24
	75	25	100
1	7	19	26
	26.92	73.08	100
Total	25	25	50
	50	50	100
Priors	0.5	0.5	

Table 5: Conventional Discriminant Analysis Result (SAS)

2 Part 3

Table 4 presents result of discrimination based on both logit and traditional model. MATLAB is used for logit model. 'R' is used for traditional discrimination procedure, as well as SAS. Interestingly R's confusition matrix on the traditional logit was the same as logit's confustion matrix of MATLAB, but SAS's confusion matrix had minor difference. All three codes are attached. Therefore, here I will only present SAS's traditional discrimination results. Table 5 presents both Kappa and Goodman Kruskal tests' results. Based on the result for .95% confidence interval test results are significant. The only comment that could be relevant here is that z-score of the conventional discriminant analysis is greater than z-score of binary logit. This means for a greater confidence interval this method is more relevant. In other word conventional discriminant analysis provides better prediction.

xi	delta	logit	discrim	xi	delta	logit	discrim
1	0	0	0	26	0	1	1
2	0	0	0	27	0	1	1
3	0	0	0	28	1	1	1
4	0	0	0	29	1	1	1
5	0	0	0	30	1	1	1
6	1	0	0	31	1	1	1
7	0	0	0	32	0	1	1
8	0	0	0	33	0	1	1
9	0	0	0	34	1	1	1
10	0	0	0	35	1	1	1
11	0	0	0	36	0	1	1
12	0	0	0	37	0	1	1
13	0	0	0	38	1	1	1
14	1	0	0	39	1	1	1
15	0	0	0	40	1	1	1
16	0	0	0	41	1	1	1
17	1	0	0	42	1	1	1
18	1	0	0	43	1	1	1
19	1	0	0	44	1	1	1
20	0	0	0	45	1	1	1
21	1	0	0	46	1	1	1
22	0	0	0	47	1	1	1
23	1	0	0	48	1	1	1
24	0	0	0	49	1	1	1
25	0	1	1	50	1	1	1

Table 4: Discrimination of Data Based on Two Models

κ	0.48
var(κ)	0.02
zscore	3.3968
λ	0.4167
var(λ)	0.0235
zscore	2.9892

Table 6: Evaluation Criteria For Conventional Disc. Analysis