Game Theory by Gary Bolton @ UTD: first session

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Pennstate

Decision Making
Inidividual Ex Handling uncertainty Interactive (Games)

- autonomous e.g. Trust reciprocity
- Joint negotiation

Gift game:
configuraiton:
can't influence the price and ask more by letting the other person to guess
Dominate strategy of truth telling, since only one time game
Only trade if the price on the dice is favorable

Price theory: supply curve, demand curve, and it defines price int he market
very successful in the market when:
Many "small buyers" and sellers
Firms produce homogenous products
price and demand crosses and this determines who trades or not
Traders have perfect information
no transaction costs
free entry and exist

Price theory is limited in explanation therefore we have game theory
assumptions could be broken
Monopoly is the breaking of this condition
In the competitive market is ruthless, and you don't want to play in this market
You want to be in the market that you have some control to do some strategic interaction

Game theory starts with observation that there are lot of strategic interaction
We have set of actors, and we are looking to find out what they want to achieve
We try to abstract strategic interaction of actors
There is some rule that define what is going on
This isperspective of game theory
John Nash Schizoferny

Game theory began late 40's. 1953 John Von Neuman and Oskar Morgenstern wrote the book of theory of games and economic behavior
They tried to tackle head on the lot of things that price theory does not tackle with, since it does not look at the strategic interaction
price is not enough
Game theory is very clear language that says precisely what strategic interaction is
It allows replication, and just by defining payoff, strategic intraction and actors
It may not tell you anything about fiscal cliff, and so on

Allocation games LIoyed S. Shaply, UCLA
The deferred acceptance algorithm for stabilized dating
The algorithm says:
on one hand you have boys having preference over girls
on the other hand you have girls having preference over boys
They want to find stable allocation
If there would be match between boy and girl, if you

don't like what you got, you couldn't do anything better
He found the algorithm that does this match
First we will have boys proposed to girls
The first day Mary keeps one out of 3, and kate has got no proposal, and jane only got bob
On the second round adam stays with mary, sinc ehe was chosen, then don and charlie go to kate, and charlie got rejected, and bob stays with jan since there was no other proposal
Charlie comes with jane, and on the third round it proposes to Jane
Then Bob that got rejected on the next round goes to Mary, and got rejected
On the next round it goes to the next remaining girl and got rejected

This algorithm ends in the finite amount of time for serial allocation

The Theory and Practice of Market Design
This algorithm of diferred acceptance algorithm for stabilized dating
It will say that I want to work vetenary work in masachusete but they say that sorry we got better ones
You can not reach full stability but you can get close
This algorithm was adopted by different groups

Kidney match you need to do multiple switches at the time
The algorithm trades the kidney across groups, and it is much more complex problem than gabe and shaply, but the same concept is used
Game theory used in economics, finance for IPO pricing, in marketing for sales force management, negotiating
In accounting optimal contracts, auditing behavior, and in supply chain for auction procurement
Managerial decision making, and strategic management, and decision theory also works here
Game theory works in psychology, biology, philosophy, why don't deers kill each other, or spiders web can be described as stable state of game theory
It is also used in Law

Game theory: is a set of analytical tools for studying strategic interactions between rational deicision makers

Willingness to pay defines the indifferences, but individually each person will be different. Some people see it is sovenier, but some do not care. The average should make them equal. Some have pencil, and some have dollary. The average seller willigness to pay was .56, and the buyer willigness to pay was .34.

You were asked to choose. Endowment effect is the tendency to appraise value more highly if you own the object.

Endowment effect is the vilation of rationality.

Madrian and Shea on QJE 2001 found that employee participlation in a large company's retirement plan went from 25% when employees needed to opt in to 61% when participation was default option.

We will try to start small here, and then try to build up.

We look at the rational model, and then we will find out that people deviate, and we will check the reason.

We will talk about loss aversion next time

Game theory is a set of analytical tools for studying strategic interaction between deicison makers.

Today we are going to talk about rational choice, and next session we will talk about theory that incorporates some of the things that people do. In other word how the deviaiton from the theory happens.

Rational rule 1: if yo give presented with the choice, you take the one that is best.

Usually we would like to wrap that rationality in the utility function. In order to do that person's preference should satisfy certain assumptions.

Two mains are:

1. preference should be complete: if a set of objects is given you would be able to order all of them
X P Y: Prefer
X I Y: Indifferent
X R Y: Prefer at least (is weaker statement)

2. We also assume that they have transitivity:
X P Y and Y P Z → X P Z
If there would not be a transitive by three moves from X, Y, Z the other person would be at initial state, and you would earn \$3 for the three trades that you made him move from initial state to return to it.

you can move from glass of sugers with little difference in term of suger, and you move to the point that you become different. This example shows transitivity work under discrte choice.

A set is countable if you can assign natural number to them.

Theorem: If A is finite or countable, then completeness and transitivity are sufficient as well as necessary condition for a preference to be representable by a utility function.

The form of the class would be on the first part we will talk about concept and on the second part we will talk about papers, and these set of tools, as we set game theory is set of tools, application on them.

Next session we will talk about propsect theory.

For next time we need to collect group of papers, so you can come back with the research paper that uses game theory.

After prospect theory we will talk about, dominance notion.

midterm, final, 30% each.
assignment 30% and are really critical to work out, and you will learn through them.
class participation 10%.

These utility functions are ordinal. This means when one is 3, and the other is 6, then you prefer second one twice as much. You can do whatever you want when youa ssume ordinality, and you do not need cardinality.

This theory has the problem that it does not say anything about peoples preference on risk. Yet there is inherently risk involved. Expected utility takes this basic idea, and tries to understand person's taste on risk. Risk is taste, and is subjective matter, and you can incorporate it into the subject matter. People attitude toward risk will be put in the utility theory.

Assume a gamble of \$2000 win, and \$1000 loose with %50, and sure gain of \$400.

Expected value is \$500, but it also depends on the circumstances. As a result just looking at the expected value does not suffice.

Three hundred years ago Bernoulli defined a problem that you get \$1 when it lands on heads the first time, \$2 when it lands on head the second time, \$4 when it takes three tosses, \$8 when it takes four tosses. Name the greatest certain amount that you pay to play this game once.

Bernoulli said this has infinite expected value, showing that there should be some way of fixing thing, since expected value is not everything. If you simply convert their payoff, so that their utility would be diminishing. Utility function should be increasing but has asymptote. In other word value that we get from additional money would be deminishing.

This diminishing assumption leads to convergence of expected value. People will have utility over itmes, and it would be concave. This shows why people are not willing to take chance of \$500, while there is \$400 absolute chance.

If a preference relation R is defined on M (with strict preference P and indifference I), then an expected utility function u MR is a function u such that:

(i) $u(a) \geq u(b)$ if and only if aRb; and
(ii) $u[pa; (1-p)b] = pu(a) + (1-p)u(b)$

Axioms: Let X, y and z be lotteries (to assign certain probabilities to certain payoffs). The following conditions apply to the preference for lottery:

Order
The preference are exhaustive (either xPy or yPx, or xIy) and transitive (if xPy and yPz then, xPz)

Archimedian

For all xPyPz there exists a definite probability p so that [px; (1-p)z]Iy.

Independence

If xPy, then $[px; (1-p)z]P[py; (1-p)z]$ for all values of z and all p when $0 < p < 1$.

This last part is kind of saying that if there would be the tree of conditional branching with 0.5 split of 100 definite, and a chance that splits to two branch of 0.5 and 0.5 of 500 or 0, then it could be rewritten as main root that branched to 0.25, 0.25, 0.5 with values of 500, 0, and 100.

This has problem, since behaviorally people do different thing, we will see in prospect theory. Archimedian theory also has problem.

$x =$ finding a \$ 20 bill as you leave class
$y =$ leaving class today as you expected
$z =$ getting biled in oil

Independance is operationalized, yet this third choice is not operationalized many time.

Risk averse utility function is concave. The expected value of lottery for you would be less than its real value. Concavity helps us to capture risk averse nature of people.

People who are risk neutral only take care of the expected values. Those guys have utility function that is straight line. Expected value therefore represents.

There are people who likes to take big chances. Their curve would be concave, and the utility that expected value gives would have higher value for them than real value.

Usually we assume that people have consistent expectations, and risk behavior. People are risk seeking in contronting with losses.

Game Theory by Gary Bolton @ UTD: Second session

Meisam Hejazinia

1/29/2013

This class and next class we will work on static game on complete information.

Second two weeks we will have dynamic game of complete information. Problem set would be due on feb 12.

Midterm would be on around march 12, before spring break.

First year obsessed with courses, but then socialize, and it is peer driven profession like medicals.

Phone calls, and letters, people constantly judge you.

Game theory "the princess brid" from youtube.

Game theory is a tool to build a model.

It is used in many places to build different theory.

Many ways to describe games. Games in normal form and extensive forms.

Normal form game is set of components:

1. Player $i = 1, , n$ of n- person game

2. Each player i has a utility function over outcomes of the game.

Each player feels different preferences over set of outcomes.

3. Each players selects the strategy S_i would be strategy set. $U_i(s), s = (s_1, , s_n), s_i \in S_i$

Statistical game means two players choose simultaneously.

Game would be defined in the form of $G = S; u$ where $S = (S_1, , S_n)$, and $u = (u_1, , u_n)$

Each player would have utility function.

Small s would be particular choice

Static so means, simultaneous actions. There would be no repitition.

It is complete information. Means I know your utility function, and you know my utility function. In many circumstances it is roughly true estimation.

We assume common knowledge of game structure. We know common knowledge of rationality. These are strong assumptions. So, everybody is rationa, and I know you are rationa, and I know that you know that I am rational, and I know that you know that I know that you are rationa.

Common knowledge of rationality is very strong.

Common knowledge in rationality is due to Robert Aumann who won Nobel price 10 years ago, and he was early practitionaire of game theory.

Assume 99 people sit in the room and they are in the circle, and professor would be master.

Everybody puts baseball, and there would be red and blue baseball cap. You can see everybody, but not yours. You see everybody else is wearing red cap, and you wonder whether I am blue or red. If you think you are read after ringing the bell red hats should move, and no body is going to leave, since people know others, not himself. If it would be claimed that there is at least one red ball in the room. Everybody is rational in term of I know that you know, and everybody gets up and leave. Proof would be by induction. People can see others cap, but not themselves, and they can not talk. People learned the first time. You are rational, and if I were the blue cap you woul leave, and you didn't, so second time everybody leave. It is symmetric.

Assuming that you and I are rational changes the actions. Each person can see a red cap, and at most one blue cap in the room. Another time the bell rings, and nobody leaves mean that nobody can see the blue cap in the room. For the third time everybody leaves. When there are n people in the room, then after n time ringing the bell everybody leaves. Auman point is that it is very strong assumption. In 4 people you first remove the possibility of 2 caps in the room, and then remove the possibility of one cap in the room.

Prisonar Dillema:

Represented in the form of matrix. Two player, and each has two strategies (cooperate, defect). Payoffs relative to others is important. If row guy cooperates, and the column guy defects, the row guy takes 1, and column guy will take 4, (1,4) in the matrix of row cooperate, and defect column. Diggy and Ziggy problem in museum.

Critical point of game is that the pie is greater if we cooperate, rather than free ride.

It has the dominate strategy, since no matter what the other do, it is better to defect. The money is left on the table.

People have strategy of their own interest, to free ride, and that is why frontier concept of pricing theory does not work, and people leave money on the table. This is where the notion of repeated game comes in.

Golden ball 100,000 split or steal

SPlit, split $(50,000, 50,0000)$, and $(steal, steal) = (,)$, and $(spl, stl) = (,100,000)$, and (stl, spl) strategy. Here the dominant strategy does not exist, but there would be weakly dominant strategy.

environmental goods, national defense, oligopolistic price fixing, advertisement blocking device on TV, are example of static game with complete information.

$u_i(x) = w - x_i + m\sum_{j=1}^{n} x_j$

They can pay part of their gain to public good. The additional gain would be part of amount that is payed by everybody.

endowment would be w

Strategy $x_i \in S_i = [0, w]$

If we all pay efficiency is there, and we get all back.

n players

$\sum_{j=1}^{n} X_j$ would be public good

m would be social marginal utility of private contribution

When someone thinks rationally to keep, then for all others dominant would be not to pay, and means the system will collapse.

Tragedy of commons is in this form.

Another game with no dominant strategy.

Put 1 to 100, and it will be turned in to trustworthy. The winner is the person whose number is close to two third of the average. People will think about 50, and two third of it would be 33, and then people will figure that out and it would be 22. People will not figure this out and that is why we do not iterate more. Iteratively you sould reach to 0, but behaviorally it is different. People do not get that far. If we play this game repeatedly we will reach to zero.

Repeated elimination strictly dominated strategy: iterative dominance. You begin to remove the possibilities

(l, m, r)

$o : (1, 0)(1, 2)(0, 1)$
$u : (0, 3)(0, 1)(2, 0)$

l and m domina r always by the column player

On the second round u will be eleminated by the row player

Then the column player will remove l

And therefore the iterative dominate equlibrium would be (o,m) for row and column player with (1,2)

There would be problem with this.

We have bounded rationality for the games

Keynes, 1936: intelligence what average opinion expectes the average opinion to be

You should not go too far ahead of the crowd

Problem with solution concept of 'Eliminating dominated strategies'.

Rationality must be common knowledge through elimination.

(l, m, r)
$o : (0, 0), (6, 6), (2, 2)$
$v : (6, 6), (1, 1), (0, 2)$

...

This game does not have any dominant strategy by elemination.

Zero sum Game

$(left, right)$

$up : (3, -3), (1, -1)$

$down : (-2, 2), (0, 0)$

Worse thing that would happen, would be up that maximizes the pay off.

The safety strategy for player two is also play right.

This is playing safety strategy

In zero sum game it makes sense

If I know the other would play safety strategy, in another configuration other than zero sum game, I want to leverage it here, and go for the choice that I take more.

Nash comes to the story at this point:

The idea behind equilibrium is intuitively that, things are in equlibrium is that given what you did, I am satisfied with what I did.

Given the other players plan I did the best things I could do.

It means when you see what the other person did, you do not want to deviate.

Everyone in the game has a nash equlibrium, and intuition that it is stable was main work of Nash. Under uncertainty these things always exist. For one page paper he won the Nobel price.

Dominant strategy equilibrium is nash equilibrium. In prison dillema you had it. Split-split is not equlibrium, and if you know the other is split you want to steal.

You usually guess, and you will check if somebody deviates or not.

In the game that pay off is dependent upon the other players strategy (minimum of them), and my deviation from that, the whole diagonal is Nash equlibrium. There is always more than one Nash Equlibrium. Pareto efficient Nash Equlibrium would be 7, since everyone would be better off, yet risk dominant equlibrium maximin would be 1.

The problem with this game is that it is dependent upon each single individual.

If you change the structure to compare with median, or mean rather than minimum, it will stabilize things.

This is basic coordination game, in contrast to cooperation game. This game we played was coordination game. This is why people do not invest when the economic crash, since investment, real engine of economy will stop. People invest, when there would be grwoth, and if nobody is investing, there would be no grwoth, so I will not invest. I am frighten because as I walk around, I will see bunch of frighten people.

Repeating the game will lead to convergence of equilibrium to the risk dominance part, which would be one.

Example of Cournot-Duopoly.

Everybody is small, and nobody can affect was in the form of compeitition, and when there would be one company that controls everything it would be monopoly. When there are firms that have market power we have duopoly.

Cost structure: no fixed cost. We have 2 firms and homogeneous goods. There would be constant marginal cost. Identical marginal per unit cost. Each firm simultaneously decide what amount they want to produce. Demand would be in the form of $P(Q) = a - bQ$, with $a > c > 0; b > 0, Q = q_1 + q_2$.

Positive number production, and they put in the market and they will see what price it will catch.

Nash equlibrium $max_{s_i \in S_i} u_i(s_i, s^*_{-i})$

I know firm one has put q_1, and I am going to understand what would be his amount given amount for second firm q_2^*, you treat this as fixed number, and you do this for firm two, and there you treat q_1^* as fixed number. You solve this problems simultaneously.

You calculate $q_2^*(q_1^*)$, and $q_1^*(q_2^*)$. This will be called best response functions. You solve them simultaneously. They are only two linear functions.

When both firms simultaneously generate specific quantity, we will have the equlibrium amount. You can plot the reaction functions and they cross, and that would br result. I am not suprised by what you did, therefore the equlibrium would be stable.

Profits are positive, and in the perfectly competitive market, they would become zero.

The big assumption here is that companies control quantity, and not actually the price.

Bertrand Duopoly, the firm controls the price. The marginal cost we assume it c, or zero. They will not get any economic profit, and nash equlibrium would be consistent with the perfect competition. Control over price and quantity will have huge differences. If the compeitition would be on price, it will not result in anything other than perfect competition. In cournot, control over quantity will affect the market power. Small number can exert power.

For next time read the next time read the rest of chapter one, mixed strategy.

H,T

$H : (-1, 1), (1, -1)$
$T : (1, -1), (-1, 1)$

There is no Nash Equlibrium, and you put randomness into it, and nash equlibrium is when probability would be $p = 0.5$. Slides would be there a night before the class.

Game Theory by Gary Bolton @ UTD: Third session

Meisam Hejazinia

02/05/2013

We talked about the static games, and said it should be simultaneous, yet it is not the case, in the sense that you only need to not see other persons action, until you take an action.

We talked about public good game, and Prisoner's Dilemma, on both people may free ride, so you decide not to cooperate.

One way that human solve this is to turn it into the repeated game.

Dominant strategy needs little rationality, since I know what the other player does. Defect in prisoner dillema is dominant strategy.

Iterative Elimination: you should not play the strategy that is not the best reply, so you eliminate them.

Zero was only Nash equlibrium of the game on the pick a number game based on the iterative elimination was 0. Nash equlibrium was that given what other did what would be your best response. Nash equilibrium: Each player plays best reply given the other players played their best.

There could be multipe Nash equilibrium. In the smallest value X, we saw that people played safety, and risk dominance, rather than Pareto efficient. It was behavioral matter, and people pessimistic. If you cluster people to groups you may be able to get full coordination, under some conditions.

Talked about Maximin: you play the strategy that maximizes the minimum. It is not stable in the lot of games. The problem is that it is not stable, since if I know that the other person is playing maximin, why don't I play another strategy, since it improves my payoff.

In zero sum game, maximin leads to Nash equilibrium.

The head or tail. If both head or tail column wins, and if they don't match the rwo wins. You would find out that you must be unpredictable to the certain extent. This is the mix strategy.

Battle of sexes. The couple wants to go out. She wants to go to opera, and he wants to go to boxing. If both go to oprah, (she,he) payoff would be (2,1), and if both go to boxing the payoff couple would be (1,2), and if they do not follow each others way (no match) the payoff couple would be (0,0). This is coordination game. There are two equilibrium to the point that both follow the same action. There is no dominant strategy, and this is example that requires mixed strategy.

In Rock, Paper, and Scissors that there is no pure strategy equilibrium.

There are places that people randomise because they do not want you to have certain information.

Mixed Strategy

You will decide for example 80% of time you will play rock, 10% of time scissor, and 10% of time paper. This is the decision on last second based on those probabilities. We create mixed strategy $p_i = (p_{i1}, , p_{iK})$. For example $p_c = (\frac{1}{2}, \frac{1}{3}, \frac{1}{3})$ for $(p(rock), p(paper), p(scissor))$, we are not abondoning the pure strategy, but we generalized it. When we are talking about mixed strategy, it means it is random event as opposed to deterministic strategy.

For Nash equilibrium, you will test given what other guy has done, is there any better strategy for me or not?

p_1 is the probability that player 1 plays 'Head', and p_2 would be the probability that player 2 plays 'Head', so $1 - p_1$ would be probability that player 1 plays tail, and $1 - p_2$ would be for player 2.

If you you were the column player (matcher) then $p_1(H) = \frac{2}{3}$. If column player plays head the expected pay off would be. $\pi_2(Head) = \frac{2}{3}1 - \frac{1}{3} = \frac{1}{3}$, and for $\pi_2(tails) = -\frac{1}{3}$. You are not willing to mix here, and you just want to play heads, yet if the payoffs were so that $\pi_2(head) = 0$, and $\pi_2(tails) = 0$ then you are willing to mix. You are only willing to mixe when the other player is playing 0.5, and 0.5.

What does row player have to do to have column player indifferent. It would be that $E_{col}[Tail] = E_{col}[Heads]$.

$(1 - p_1).1 + p_1(-1) = p_1(1) + (1 - p_1)(-1)$ The solution is $p_1 = \frac{1}{2}$,

For the row player you need to have the column player to mixe to make rwo player indifferent. $E[Tails] = E[Heads]$, $(1 - p_2).1 + p_2(-1) = p_2(1) + (1 - p_2)(-1)$ The solution is $p_2 = \frac{1}{2}$.

There are two things that seem troublesum. Do you really believe that people can really randomize in certain proportion.

When we set this up, I am indifferent. The reason that I am playing this is that there is equlibrium with the other side, yet there are many best responses. This proves that you are indifferent. He plays this because he is also indifferent too. It does not show strong intuition here.

Sometimes we can disguise what we do by randomization. In some games there is always at least one Nash equlibrium.

There is also a mixed strategy equilibrium in the battle of sexes, in which each person plays his favorite. There is the third mix strategy here. Is there a probability that makes the other side indifference.

In stage game, the game that repeated several time different patterns based on other persons pattern could be the dominant strategy.

O'Neil zero sum game, $1, 2, 3, J$ are strategy sets, and the winning and loosing is in the form of $+, -$. There is unique nash equilibrium that (J,1,2,3)=(.4,.2,.2,.2), and row winse .4 of the time.

The game satisfies the conditions:

1. normal matrix

2. exactly two levels of pay off for each

3. is not true that a player has two identical

4. neither player dominant

5. ...

This is minimal size game that satisfies all these conditions.

They are pretty close to the equilibrium, and row player is close to win 40% of the time.

At the aggregate level it works. There is a bias, but it is mainly strategic learning here.

Simplicity of the game, higher subject motivation, careful control of expected utility consideration, is the reason that the result was close to theory.

Individuals data is less confirming.

There is a concession that they are not, and some of them appeared to be playing one strategy too much, and there are variances, but at aggregate level they wash out. There is heterogeneity at the individual level therefore.

Rebuttal by Brown and Rosenthal, they analyzed the individual data carefully. They show that there is clear correlation in people's play. They are actually predictable. In randomization you try not to be predictable, yet at individual level it is predictable.

Soccer goal, and penalty kicks paper. When penalty kicker has to decide where the ball will go, and they need to make the judgement. The good succor has to play good mix strategy. They tested this, and the golee not only at aggregate level, but at the individual level they did pretty well. At the lab they really did not show that they understand what they are doing.

At the aggregate level it looks really close, in many games, yet at in individual level it looks really different.

Once you add the mixed strategy to the mix, you can show that there is at least one mixed strategy in the n-person normal form game G=S;u with finite n and S_i for all i.

Curve of strategies and the cross at equilibrium that no one will play better given the best response of competitor.

Fixed point

To show the eqilibrium is that basically you look for fix point. You use the theorem that shows that on the function of particular point. There is a convex set function that maps (0,1) and maps it to the convex function of (0,1). For continuity assumption any function should cross the 45 slope line. Lots of fix point exists.

Consider game with finite number of players, fore each player i, $B_i(p_{-i}) = argmax_{p_i} v_i(p_i, p_{-i})$. He said we form the bigger best response $B(p) = B(p_1, p_2, , p_n) = (B_1(p_{-1}), B_2(p_{-2}), , B_n(p_{-n}))$

A fixed point is called eqilibrium point.

Kakutani's theorem of fixed point.

Assignment is posted for the next time.

Finance game of client and auditor, stay and swerve:

Game of chicken.

Welfare Game

Probability that the auditor chooses a strategy is independent of its own payoff.

SCM

$D = Unif[1, 100]$

$12 = r =$ price/ unit sold

$3 = c =$ cost/ unit made

$max_Q \pi$

$Q^* = F_D^{-1}(\frac{r-c}{r})$

$\frac{r-c}{r} = \frac{12-3}{12} = \frac{9}{12} = \frac{3}{4}$

Next time we will look at the extensive form games. It is dynamic game. This will change the analysis. We will see that nash equilirbia has problem in this context. The solution and accepted solution is refinement of Nash equilibrium. This solution will be called subname game Nash equilibria. We will build on it in term of extensive game. Slides are up.

Game Theory by Gary Bolton @ UTD: Fourth session

Meisam Hejazinia

02/12/2013

We talked about stati game. Move before seeing other persons move, or simultaneous.

Cournot and bertrand are static games. You don't necessarily put in matrix, but it is static. Game of coordination, are also static games.

Dynamic games: somebody moves, and somebody responses, and they go forward from there.

Once, it was lot of argument whether it was necessary?

Static game was reduce normal form, static game. The question was that couldn't we use static game always?

We are not going to abondon Nash equilibrium, but we try to refine it.

Estensive game:

1. finite number of player.

2. order that each player moves (who first, who second)

3. Which action for each player

4. Which knowledge the move making player has (perfect or imperfect info).

5. Payoff of combination of the moves, when the game abondoned.

We use tree representation here.

Start with simple game. Ultimatum game.

Two player game. Proposer and responder. Bargaining with strict rule. Proposer altimatum game, and responder can accept or reject. Divide $100 with a person. During the game or after you will not know.

Stage 1: proposer offers the other guy portion of money, and takes the rest.

Responder accepts, or rejects. If he rejects, both will go with nothing. Money will transfer to smoke.

It is single shot game.

Supress social network here.

Two pieces of information. What would you do if you were proposer.

Imagine as responder, and what will you accept.

Standard way we can represent is as a game tree.

You start at the top, there would be nodes. and node at the top says that player 1 plays first. After he chooses player 2 will decide. This the game in perfect information. Mean that the player know where they are in decision tree. Player 2 knows that player 1 took action 1. He response by selecting its own action. Then there would be pay offs.

In altimatum game. There would be six possibilities. In the lab sometimes you say they are many, and you put fan. Then the responder accepts or rejects. The first payoff is player one's and second one is second ones.

Proposer here pays X, it would be in the form of tree with a sector under it. Then responder is a point below the sector with two edges to two new nodes with action a, and another with action r. Then the payoff couple would be $(100-x)$, and $(0,0)$.

In social science, and bilogy also dids many experiment on the ultimatum game. Chims and bannana also has been done. In bargaining, and political science also this has been done.

The middle nodes are Decision nodes, and the leaf of tree is end nodes, and sometimes called payoff nodes.

Strategy specify the way we talk about it.

There are versions of the game that player one can come back and do the natural counter offer.

Strategy: specifies for the player at every decision, what they can do. Player 2's strategy is condition on all strategies of player 1.

There has been long debate about why to use this form, when you can use normal form. There is one to one match here.

This will create $2*4$ since the assumption of matrix was that they are simultaneously, since assuming the other would not know he will have different payoff, so both are the same.

Sometimes players do not know other players move.

Imprefect information means that I say it is your move, but you don't know what the guy before you di. How do you represent?

You put a line between decision nodes that are the samy but in two different branches.

He does not know where in the game tree he is. He does not know what player 1 and player 2 did.

In prisoner dillema dynamic tree, but you don't know what move first player has done.

What can player 1 say, if looks at the tree and not the matrix.

Players have common knowledge means they know the decision tree of the game.

Player 2 has dominant strategy, if he things he is in each of the nodes resulted from decision of player 1, defect is better for him.

There is ambuguity about who does it first when you go to static game.

Principal agent problem: Principal has an agent and wants to monitor it. Principle is busy guy, and can not watch the agent. The agent has a moral hazard. He decides to say that he is doing something, and do something else.

Agent moves, and the principle decides how to reward that, but agent one could have deviated from his promiss, and you have not seen it.
Agent has moral hazard, and you can not monitor it, yet he goes to right, and you give the payoff of Rl, while assuming that youa re at Ll.

Buyer and seller. If don't send the money the payoff is $(1,1)$, and for the seller when he recieved the money can keep or send and respectively the payoffs would be $(keep, send) = ((-1,5),(3,3))$. In internet they introduced reputation to overcome this moral hazard. There is also problem on the buyer side as well.
Feedback system stabilized the market.

Information assymetry is game of incomplete information. These two were complete information, and for moral hazard buyer seller it was perfect information. Imperfect means that somebody in the game does not know where they are. This means principle agent is imperfect information. Incomplete information means, one player does not know the payoff the other person.

Complete vs. incomplete: uncertainty of payoff;

Perfect vs. imperfect: Does not know where they are in term of state of the game.

The question for the moral hazard is what institutions we need to introduce? For example here introducing the repeated game notion was helpful. Sending warning and so on is also another way. This is kind of signal revealing.

The principle agent problem principle did not see what the agent did, yet on the buyer seller, the seller has seen the money.

Incentive to say something and do something else is called moral hazard.

How to introduce institution that get me around of this problem. This is about contracts that get you out of problem.

Ultimatum game:

Extensive game.

Ronald Selten 1965.

Great experimentalist, and winner of noble, who never published in the journal, since he was 10 years from everbody think about the problem.

In this game does not matter, since both left and right have same payoff. If player 2 says I will go right, then player 1 will go left.

Nash equilibrium when you fold game up would be rediculus, as we convert it to extensive form.

There is way out of Dillema. There is way to eliminate the nash equilibria that has this problem by checking the tree. You can use 'Backward induction' for this purpose.

You start at the buttom of the tree, and say rationally what should he do. Then you start to go back to the tree, and say given what the guy last move was what should he do.

The idea is that it forces every node to do the rational thing. This is common knowledge.

Common knowledge here is that player 2 says he will go right, but I know that he is rational, although he said something else. As a result you say that I know that he will go left. Highlight back shows that there is nash equilibrium.

We call this subgame Nash equilibrium.

It is true that subgame perfect equlibrium is Nash equlibrium, but not all nash equlibrium is subgame perfect equilibrium.

In finite game, finite player, and finite move, there is always subgame equilibrium.

Unsettling consequences would be here.

The argument is if we get to this node, what would player one 2? $(3, 0)$ and $(0, 2)$ he will go to the first to take 3.

This is easy since you work backward in the game.

Other things in the utility function.

There are limits to the empirical validity of backward induction.

Notion of mixed strategy equilibrium in the dynamic game exists.

Most games we play in reality is a dynamic game. It is one part of larger game, and that will change the decision.

There may be no consistency in terms of offering and accepting.

People become more generous when they can be rejected than they are not. In Ultimatum game they paid more than they do when they are dictator.

Framing of dictator game could make things different.

If you reduce dynamic game to static game, there would no more be obvious that some equilibriums should not exist anymore.

Stackelberg game in supply chain

application: withought inventory (centralized, decentralized), with inventory centralized.

After observing leader move, the follower moves.

Sensitivity to quantity.

Backward induction. For given q_1 maximize q_2.

Aggregate outcome of stacklberg greater than counrot equilibrium. Leader will gain more, while follower gain less.

Feedback Nash, and time variant closed loop (TVCL).

Stochastic learning.

Some knowledge when produced in the first period will be gained from producer.

Inventory cost for buffering things produced in first period.

c_1 for first period.

$c_2 = c_1 - \Lambda q_1$ where Λ is random variable of support.

Demand is function of p_i. $D_i(p_i) = a_i - bp_i$.

Withoug inventory no carry forward.

No stackleberg, since there is only one person.

$max_{p_1,p_2}\pi_(p_1, p_2) = E[(p_1 - c_1)(a_1 - bp_1) + (p_2 - c_2)(a_2 - bp_2)]$.

$c_2 = c_1 - \Lambda q_1 = c_1 - \gamma X(a_1 - bp_1)$.

Dynamic programming principle $\pi^* = max_{p_1}(p_1 - c_1)(a_1 - bp_1) + E\{\pi_2^*(c_2)\}$

As soon as first period is over you have optimal value for c_2.

price of whole sales w_1 and $w_2(c_2)$ for period one and two respectively.

Retailer responding with $p_1(w_1)$ and $p_2(c_2|w_2(c_2))$

Solve for both retailer and manufacturer.

Use backward induction to find best strategy for retailer.

Solve for second period for retailer, then plug in to manufacturer and solve for him. (Since dynamic programming is backward). Then you use to solve for the first period.

Contracting, and hierarchy of manufacturer retailer and customers. Use stackleberge nash equilibrium and use for contract desing.

Φ_1 fraction of profit for the retailer and Φ_2 profit for the manufacturer.

There is no backlog in all scenarios for the demand that was not met previously.

Holding cost.

Does it converge for multiple periods (more than 2).

What happens in contineous? In financial market?

production pricing, contracts.

Multiple manufacturers and more retailers.

Bargaining game versus stuckleberg.

Manufacturer is leader in this game.

If manufacturer controls supply chain maximizes its profit.

Efficiency point of view.

Here it is sequentially solved in backward induction form, in contrast to simultaneous solving of cournot model.

Next time we will have guest speaker. Game theorist. Polished speaker. Part of experiment. German university.

Then we will go over incomplete information.

Game Theory by Gary Bolton @ UTD: Fourth session

Meisam Hejazinia

02/12/2013

U does not depend on the t on the repeated game.

The strategies can depend on the whole history of the game.

You can condition on what has happened on the first round.

Prison dillema, defect and cooperated, and dominant strategy.

Is there dominant strategy for two period? For second period subgame dominant is defect.

You will start cooperate and then you will get punished, and if you coopererate and the other person defects, then on the second period tit for tat player will not defect, so you will get cooperation, so defect defect is not dominant.

There are situations in which best response is not to defect.

If everybody is rational you will not expect tit for tat, but in real world people do the tit for tat.

In equlibrium you need to define whether you want to defect in each and every possible node or not; in other owrd, you need to define your strategy on each and every node.

Subgame equlibrium, and elimination strategy to get rid of all not equilibria strategies.

Game theory sometimes accepts rationality that may not be acceptable.

This happens to evolutionary case, as animals die since they do not cooperate.

People adapt heuristics, but in real world more often the cooperations fails than not.

We deviate from the assumption that rationality becomes the common knowledge.

If you ove from common knowledge you will reach the equilibrium.

Knowing that someone else is tit for tat players, results cooperation, and there would be point that everybody deviates.

You don't know when to die, and given that you will live 90 years, the probability of dying will have its own bound, and this leads backward induction.

Whether time is continuous is not clear, casting doubt on whether it is finite game or infinite.

More complex games may shift people to become nice to eachother rather than selfish, as simple game may thought them.

People believe life after death may shift the game to infinite game.

Prisoner dillema could also be deemed as oligopoly.

If the equlibrium of stage game is not definite (unique), then things would chang.e

Homework Try to solve centipede game by yourself by backward induction.

Auctions

In auction you have many buyers, people who may have value below the cost of seller, or above it.

English auction that the price will be paid equal to the the strongest bid below him.

Dutch auction: the price drops continuously until the first bidder accepts? Theoritically higher than english or not? Everybody know the value. The price would be the same, and you will still pay to the price of valuation of the strongest competitor.

In this case you have assumed that you will win, and you compute expected second highest value. You will not bid on your valuation, since there would be no profit for you.

First price auction: Every bidder submits, and the highest bid wins.

Again assuming you are the highest, the final price is the same, and all this auction leads the same.

Second price auction, that you will only pay the second price, and this leads the same price.

William Vickery shows that all these acutions yield the same revenue.

It is deep insight. If you want to become more clever than others, you will fail, since they are clever too. They always know that they do not have to pay more than the seocnd price bidder.

Auction design matters a lot.

Usually highest bidder understimate problems.

This is why some go for second bid, or bid that is close to average bid.

Changing rule completely changes incentive.

Biking contest when you peak the second one, everybody will go with eachother, and winner will be random, but who will understand the psychology of auction.

Adding 20% will not help, since people will think reversely and reduce their bid.

Institutional matter such as reputation and empiricals on the characteristic of the firm could also be answer to this question, but main point of auction is transparency.

design and behavior in spectrum auctions.

Spectrum for LTE. This is related to backward induction.

They sold 10 blocks of spectrum, and they sold them simultaneously, and each have one block.

you have to exceed prior beed by 10 percent. The auction ends if there is no bidding in any of the auctions. You stop when all the auction is end, since as long as there is bid on one, you may move to the other block.

You would need this stopping rule here for this purpose.

Typically take several weeks, and bidders could not communicate with eachother, and they will come and bid, from their different hotl.

T-mobile, Mannesmann, and vodafone.

Everybody could see the bids that are comming.

This was kind of sharing the market, and it was like negotiation.

Communication is done by their offer, and by clever bid.

By backward induction was the only subgame perfect equlibrium that could have been recommended here.

The threat was that I will distroy your prices if you distroy my prices.

What can you do in auction design to prevent this behavior like this?

The problem of simultaneous game, there would be coordination problem, and you give information for some coordination.

For gas station you need to bid on individual.

Randomizing the sequence will change the game, and your willingness to pay is depend on the price of later blogs.

There would be much more expensive on first auction than later ones.

If you slightly change the rule, mean if there is no bid, bidds will not permitted anymore, then the compeition on the first blogs would be higher, and not compeition on the first block would not be subgame equilibrium anymore.

Small changes in the rule could make differences.

Adding noise to feedback makes the acution to make actors to not be sure whether there is signal or not.

Once auction end you need to have one other round which would be sealed auction.

Then you will not be sure where each would be.

This is very good way to avoid the colusion, since on the last round everybody has to defect.

The last three digit of waterloo auction of USwest was difference in Rochester of McLeod, and this was a signal, saying that stay away or I will ruin your price on the other block. They distroyed million dollars, but it was signal.

Today you force people to choose from the menuse, so that numbers wouldn't be signal, and this makes signalling expensive, e.g. coupld of millions.

When you are at duopoly price you make no money, but if you go forward and bid one more you would be monopoly, and the other will not bid more. Then it was disaster, since the industry was monopoly. Then they changed the rules, but it was difficult. Doing backward induction also works.

Design and behavior on ebay

eBay is second price, means buyer wins but pays the price of second bidder.

In second price auction you need to bid equal to your willingness to pay.

Art auctions are different, since there is uncertainty about the value of item.

Why you should pay your highest willingness to pay, since the price is determined by highest.

If the price is below willingness to pay you will only win.

Assuming you know your willingness to pay you need to pay it.

It is dominant strategy. You don't have to think about what other are doing, and how rational or irrational others are.

Second price auction in generalized form is called vickery auction and are used in the spectrum auctions as well.

If you bid late, others will not have time to respond, mean at time 1 everybody can only respond, and thats over.

If you are late, then the bid will not come through.

eBay recommends early bidding in auction. In egnlish auction the last bidder wins, but eBay says highest bid wins always.

Also bids run the risk of coming in too late.

Esnipe: you bid very late and you hide.

Out of equilibrium play and rational play of others you should bid late.

It is puzzle that many people don't bid late on ebay. If you know your value, and when everybody knows sniping could be bayesian subgame equilibrium strategy.

If both bid the value, then payoff is zero.

If both bid at last second there would be probability that the other bid will not come through.

If I see early I will immediately bid more, and one of my best response is to pay more. This is therefore subgame perfect equlibrium.

Sniping best response versus incremental bidding.

If they are incremental naive bidders, you'd better to bid late.

Shill bidding: problem on e-bay

Chaotic bidding on the last minute of e-bay. This is why spectrums are not sold on.

Suppose they want to prevent this, what should they do?

One could be to discount the early ones, in germany do. After certain time you can bid after the horizon, but market with some probability will change. They increase cost of bidding late in this way. This is inefficient, since still some bids will not come in. This creates inefficiency.

Extnesion of bidding time when bidds come late could be solution. Amazon did this, telling that if your bid comes late, then everybody will have time to response, since the time will be extended.

Economic engineering insight: small changes can have huge impact.

On the antique auction there is much uncertainty leading to late bids.

One time bidders bid late, and they are sophisticated, in contrast to naive incremental bidders.

Revenue on Amazon is higher, while on eBay the highest does not come through.

Question is why does eBay change this rule?

eBay has much more bidders, and may be it is because eBay is Fun, while Amazon is not, and Amazon is boring.

Timing is big issue, and there are a lot to do.

Game Theory by Gary Bolton @ UTD: Sixth session

Meisam Hejazinia

02/26/2013

We will talk about games of incomplete information. State where matrix are proper to describe game. Static game not that much complicated, as we go over the dynamic game it would be more complicated.

Next week would be midterm exam.

What are they? So far we have assumed that not only you know your own payoff, but also payoff of the other guy, but in most of the cases you don't know. How do you solve the game, when one or more payoff functions are not known to other players.

The basic idea is to turn this game to the game that you know, mean the game of complete informaiton.

We will talk about the word bayesian today. Basic game plan is to turn this plan to the game that you alread know. If you have done this you will win the Nobel price.

Company 1, a monopolist decide whether to build another factory. Simultaneously company 2 is deciding whether to enter. Company 2 know their cost, but don't know cost of company 1 mean the monopolist who wants to make a factory.

Two scenarios: 1. company one's cost are high. The payoff depends on what they do. If they both enter it would be over capacity. If you are company one you may want to know whether enter or not. Company 1 will have the dominant strategy not to build, since their cost are high. If company one do not build, company 2 will enter. In the second scenario costs are low. Equilibrium depends sensitively on the dynamic of company one and two. Don't build is still in equilibrium. It is also equilibrium when don't enter, it is good for company one to build. This case would be coordination game. As a result we have two euilibria.

How we are going to deal with that? Look at each game we don't know what is the cost of company one to build.

When company one assumes entrace probability of y company one will get 1.5 and with probability of (1-y) company one will get 3.5 if it builds, so the y could be calculated .5.

We add third player to this game. We call it nature. There is some probability p that the company 1 is high cost, and (1-p) the company 1 is low cost. Strong assumption is that both parties one and two know what p is so it is common knowledge.

We transform the game so that nature moves first. Bascially nature randomizes. Then the game is played out accordingly.

This is game of imperfect information. Company 2 makes move, and don't know what company 1 and nature did.

We will create player types, and we will assume that true type is pulled from the distribution, then the game is played accordingly.

Cedibility problem is the reason we move from static game to dynamic game.

We assume fictitious player as nature that makes random move. There is common knowledge about prior (common prior).

Response functions can be calculated through the maximization of expected payoffs.

This new equilibrium is called Bayesian-Nash equilibrium, since we added common prior. (Showing we made assumption about prior).

For every P there are basically two equilibria here. One is regardless of p don't build. Regardless of cost high or low. Don't enter when p is small enough.

Action space. Strategies will be more than actions. We have type space. We have some expectation which is common prior. We have payoff function for three players now.

Rarely we assume you don't know your own payoff, since that would be schisopherny.

Nature chooses type vector, we call the type private information. Every player plays simultaneously.

You know your own payoff, so it would be private information.

Strategy specifies for every type an action.

Strategy is not just what you want to do, but what type you are when you want to do that.

1967-1968 John harsanyi published three articles entitled game with incomplete information played by Bayesian players.

Harsanyi doctorine, beliefs can be determined from a shared distribution p(t). This is strong assumption.

Bets about uncertainty does not make sense if we all have the same prior.

In the game we discribed everybody moved at the same time.

Later on when we consider dynamic game, you would not be able to observe move of the other player. On that case you will need bayesian updating. Bayesian updating is: you observe a move, and that move is more likely to happen when you be at the high type. How much higher? It is according to the mathematic formula.

Monty Hall: there are three doors, and behind two are junk prize, and behind one is car.

You take random. Then if you take 3, then he will show you that behind door 2 is goat. You should unambigiously change your mind after that. Since he has shown you information. You should switch mathematically.

Three possibility that you win one third at the time when you do not switch. But if you switch you will win two third of the time.

Intuition? There is information, since Monte has given you information, and there is two third chance it is behind the other door. Monte never shows you the door with car. You know that there is two third chance that it is behind the other door.

Cournot duopoly with incomplete information. Probability p that the cost is high, and $1-p$ that the cost is low c_L.

$maxq_i(P-c_i) = q_i(a-b(q_i+q_j)-c_i)$

We have $q_2^*(C_L)$, $q_w^*(C_L)$, and $q_1^*(c)$. Where for the company one best response you need to consider the probability two different response of the second company.

Mathematic will be different for the third one and more complex.

Reputation, Information and Matching: The Effectiveness of Electronic Reputation

Mechanisms

Repeated Prisoner dillema, that on the last round people will not cooperated, and you wind back, and you should not cooperate on the current game.

Two fixes to problem:

1. Turn it into the game of incomplete information: with certain probability payoff are such that they get utility from cooperating, just by the fact that they cooperate independent of what the other guy will do. It turns out that teverybody else that have usual payoff will cooperate for the while. This is inuition.

2. Another intuition is loosen up the issue that there is finite to the game. The game if goes to infinite future we will not have that problem. Suppose the game does not have any endtime. We have to posit that people will discount their payoff in future. We have infinite game, and you play the same prisoner dillema for inifinite time. You have to introduce discounting. You put discounting, and to the point that discount rate is lower than one, you will cooperate. If you defect, I will not cooperate again. If delta is large (close to one), that I care about future, and it would not be infinite. I would care about future. Intuitively I cooperate today, because I want you to cooperate in the future. Or instead of discount, you have some probability p that this is the last time, meand with higher probability we will continue the game.

Tit for tat is the strategy in which I cooperate today, conditional on you have cooprated last time.

As long as we do not have infinite horizon, or it goes into infinite future.

Experiments and some theory paper.

Internet market. How do you know if you buy book from amazon from non-amazon seller.

They use feedback system. Confident that this guy will not cheat you.

You can make decision based on information.

This is indifinite game played. Care about future transaction, but different kind of system, since the third person involved. You play with the same person again and again. I cooperate with you today, because you cooperated with this guy previously. This is circumstance where repuation is important.

Lot of data that suggest this system work fairly and effectively.

They work well enough to have internet market going. Used to be lot of identitiy fraud. There are still issues.

This is a simple game that the coopration is issue, but there is a twick of moral hazard. Buyer can choose to buy something or not buy, and if they don't buy there would be status que.

There are several obligation in the market. Whether they ship or not. They send you a good that is not up to the specification. All comes down to that seller not do something.

Moral hazard: one of the player in the game has an incentive to do something that they promissed not to do. The equlibrium for seller is not to ship, when get money, and the buyer should not buy. The market should shut down.

Three treatments:

1. Reputation: Random pairing. Buyer is given feedback on the seller.

2. Strangers: Random pairing, no feedback.

3. Partners: Fixed pairing. They played with one over and over again.

Then we give them feedback.

In experiment we show what they have done in the past. It was because we assumed that it would be perfect recall.

Memento movie, last part just shows the last stage, like backward induction.

It is built in z-tree.

What do you expect on strangers? No incentive for seller to ship. The market should crash.

On partners? You can play tit for tat. Be careful. If there was indifinite int he game, yet there is definite into the game.

Few honest sellers out there. Incentive to everybody else. Maybe they are a bit myopic. 30 round is long time, not infinite, but I don't do backward induction, until I am close to the end. If that is the case I would expect cooperation.

You can argue about reputation. I can play tit for tat if you have been honest or dishonest with other people.

Efficiency in the strangers it goes down quicly, and reputation worked better, but finally partner worked better than all. On the last round it goes down for all.

It always crashes at the end.

People notice that when the game is at the end personal relationship, and repuation is not important anymore. People do backward induction a the end. They also did not learn as they played again. The behavior is incomplete information. Most people are non-cooperator, but they want to mimic the cooperation. People fake it. We both fake it, and at the end it does not pay to fake it, when the continuation pay off is not big enough at the end.

Another explanation is that people of myopic, they can do the backward induction for short period of time.

Solution to problem set 2 would be available after the class.

Centipede game:

Both the same action set (take or pass).

The taker does the best always, the pie doubles if they pass.

It always has the finite game.

It can be solved in many way, and they all lead to the same conclusion. Backward induction says: every chance they get they take.

You can also solve it by iterative dominance. If pass is the dominated strategy, again take.

There are not other Nash equlibrium.

Game Theory by Gary Bolton @ UTD: Seventh session

Meisam Hejazinia

03/19/2013

Beer quiche game.

Guy sitting in bar resturant and he hast to order. Another guy comes in, and he is a bully, and want to beat somebody. He has to decide whether to order beer or quiche. He knows that the bully will come and he is weak. The bully does not know that whether the man is weak or strong. He could decide to duel or not. The beer guy does not prefer to be bit up. For ordering quiche he gets extra, and for not being beat up he gets more. Quiche makes you weak, and it is correlated. Strong people like to drink beer. In nature 10% of people are weak, and this is only thing the bully guy knows. As a result he prefers not to attach stronger, and he gets up enjoyment from beating up weak people. From food consumption he infers the person is weak or strong.

If strong guy buy beer the weak guy wants to buy beer. We are moving a little bit about subgame equilibria. The type is not observed, but what they drink is observed.

This is not simultaneous game, since the first one moves, and then second.

Eqilibrium means you can not deviate, but when he is purchasing beer, he is signaling his type.

We are not talking about simultaneous game, we are talking about sequential game.

One party knows something about himself that the other doesnt, this makes the game incomplete information.

The agent knows his own effort, or something that the principle does not know, and this raises moral hazard, and adverse selection problems raises in principle agent problem.

There is the cost that the other person does not know.

Two companies, incumband and entrant. The cost of building the factory is not clear.

You will have two equilibria, on the high cost (enter, don't build), but on low cost you will have two equilibria (enter, don't build), or (don't enter, build).

Extensive form game can help you to visualize, but you may have problem to solving them, and need to see the normal form.

Idea: transform the game with incomplete information into a game with imperfect but complete information.

Condition on observed action, and not the nature, since the nature is not observed.

You will write the permutations of all the actions, by conditioning on the state of nature, mean you will put two columns each for each type. Mean one for low type, and one for high type and you will put the action in each of the rows.

The actions will be put in the separate column.

Player 2 on columns, and row player would be player 1

This is the normal form of complete information.

$p(action|nature) = \frac{prob(nature|action)*prob(action)}{p(nature)}$

There is a dash line or dotted line. This represents the information set.

Player 1 observes the state of nature, since he knows his type, but player 2 does not know his type.

Every extensive form name has normal form presentation.

Because of the dotted lines we can not use the subgame equilibrium, since there is no subgame.

Whether sequential or simultaneous is not important. The only thing that is important is the information set of where you are.

p is the probability that nature is high or low, and it will be common prior.

you need to define q as probability of $pr(build|low)$, and you will then have $U(enter) = p(1) + (1-p)(q(-1) + (1-q)(1)$

These were sequential games, mean you were conditioning things on things that happened. Also dash line represents things that you do not know.

For bayesian you need to know the prior, means know the state of the words, and distributions before.

auction

Auction the game of assymetric information, since buyer does not know the private valuation, since if it was not, he would charge that.

second price sealed bid. First price sealed auction.

For uniform it would be $b_i = v_i \frac{N-1}{/} N$

Random valuation, and you needed to give a bid, and you would gain the difference.

You reverse the mapping between price and valuation in the auction.

Information assymetry in credit market: banks

Suppose that you are not break even and borrowers are supposed to default, so you need to increase interest rate. What will happen?

"good borrowers" and "bad borrowers" are two different types.

Adverse selection. Information assymetric exists, and the "bad" customers are more likely to be selected, and as interest goes up, and there would be more incentive to high contracts, more bad customers will be selected.

Milgram 1989 in the JPE journal is fundamental for auction.

X: your valuation
E: expected bid
P: probability of winning

$U[P,E|X] = P.(X-E)$

Denote the maximal (optimized) expected profit by $U^*(X) = U(P*, E*|X) = P^*(X).(X - E^*(X))$

Applying envelop theorym, we will have $U^{*\prime}(X) = U_X[P^*(X), E^*(X)|X] = P^*(X)$

Revenue equivalence: There would be no correlation between what you do and what othe rpeople do.

L	H	EE	ED	DE	DD
B	D	$(1.5(1-p), -1)$	$(1.5(1-p), -1)$	$2p + 3.5(1-p)$	$(2p + 3.5(1-p), 0)$
B	D	$(1.5(1-p) + 2.p, -1(1-p) + 1p)$		$2p + 3.5(1-p)$	$(3p + 3.5(1-p), 0)$
D	B	$((0, -1) * p + (2, 1)) = ((2, 1-p), (1-2p))$			$(2p + 3(1-p), 0)$
D	D	$(2, 1)$			$(3, 0)$

Game Theory by Gary Bolton @ UTD: Eighth session

Meisam Hejazinia

03/26/2013

1 Procurement Auctions

Theory and Experiments

Talk about applications.

Small detail matters. Unexpected consequences of some design decisions.

RFP, RFQ mechanism equivalent to sealed auction.

The competing forces: 1. Ensure quality and conformance 2. Minimize total cost

In procurement auction is just beggining the transaction, since after the contract awarded the right product, with right quality on the right time should be delivered.

Bidding contest rather than auction, since the bid is on price, but the winner is buyer deterimined. Supplier performance is of critical importance.

Non price attributes controlled by supplier characteristics such as: 1. reputation 2. location 3. access to expertise 3. established relationships. They can not change these attributes a lot.

Non price attributes trade off against price. Non monetary attributes of bidder i can be represented by a single parameter Q_i. The cost of bidder i is C_i, may be related to Q_i. Buyer i bids B_i if i wins. $B_i - C_i$ is i's profit. $\pi = Q_i - B_i$ buyer's surplus. $S_i = Q_i - C - i$ is bidder i's score. Make the model tractable.

Award bidder to the one who gives higher surplus.

Truthful bidding would be dominant strategy, under second price mechanisms.

Expected cost (and surplus) equivalence holds across first and second price versions.

But NOT sealed-bid and dynamic versions unless all bidders know all Q's.

Bidder more agressively in sealed bid auction, compared to english auction.

No revenue equivalence between Dutch auction and sealed auction.

Also in 2008 they found out that speed of the clock also matters even in the Dutch auction, showing that there is no equivalence.

In perishable goods, you want it to be fast.

When it is not fast people become impatient.

Perishable you want it to be fast and lot of transactions: e.g. flowers, fish.

Information rent: BD Mechanism: The buyer pays the winning bidder i an amount of Q_i minus the second highest score, and gets a unit worth Q_i in return.

PD Mechanism: The buyer pays the winner the amount of the second lowers C recieved .

$W_{(k)}$ denote the kth largest of N independent sample W.

Equilibrium bidding strategies depends on the distribution of costs.

If C and Q are perfectly negatively correlated, then BD winner captures his contribution to the surplus but under PB he does not (unrealistic condition).

If C and Q are perfectly positively correlated (more realistic), buyer surplus is the same for all bidders; BD captures all of it for the buyer, but PB does not.

In reality we will have in between.

Proposition 1: if N is sufficiently small, and there is enough negative relationship between C and Q then $\pi_{BD} < \pi_{PD}$, and if it is enough positive it would be reverse.

Number of bidders increase, the markup over cost decreases for both mechanisms.

Intuitively, BD mechanism should usually be better for large N.

Expected buyer surplus higher, under large number of bidders.

Existanc of agressive bidders, web based program with risk neutral computer bidders, before the session, to decrease them.

Regret aversion rather than risk aversion has more support in describing behavior.

A lot of noise and heterogeneity in the price.

Result: Buyer determined generates higher surpluse, as long as there is enough bidders.

Google keep people confused, to get more money.

Inexperienced bidders find it difficult to bid in buyer dtermined auctions, so suppliers should invest in training.

2 Increasing Revenue by Decreasing Information in Procurement Auctions

Not enough theory to publish in the top tier journal.

Open Bid (O), are entered dynamically and contracts goes to the bidder with the highest buyer surplus. $\pi = Q_i - B_i$

RFP, or Sealed-Bid (SB) each bidder places a single bid and mechanism awards the contract to the bidder with the highest buyer surplus. $pi = Q_i - B_i$.

RFP: No price visibility.

Open bid: Full price visibility.

Dominant strategy the dominant strategy is lower the price (since you are bidding against yourself), bidders in open-bid BD auctions do not have the dominant bidding strategy.

Buyer can adjust the Q, or tell it to all the suppliers. The question is whether he should do that?

When the buyer does not know his quality valuation could also be interesting topic to study.

Full information (F), bidders also know the Q_i's of all their competitors. Bidder i knows Q_i and also Q_j, $j \neq i$

Private information (P) bidders do not know the Q_i's of all their competitors (but know the distribution of Q's).

Equivalence of first and second price sealed bid BD auctions in terms of surplus.

$\pi_F^O = \pi_P^{SB}$ where SB: sealed bid, and O: Open bid.

In equilibrium, bids cannot fully reveal quality or score.

Open bid auction, everything is counfounded so the bid would be dependent upon the average price.

Efficiency of the auction is the percentage of the time that the highest value winner always wins.

Comparison of the coefficients between two kind of auctions.

Conceiling quality was good for the buyer in term of surplus. Sealed bid auctions are better for the buyer than open bid auctions. The interaction was not significant. However, the buyer may select to engage in different auctions, so if it would be completely blind they may not select to take part in.

Sometimes theory is enough, but experiment strengthens it.

There is a perception that rank feedback leads to less adversarial relationships.

Two kind of bidders Low and High, instead of range.

You know your own type and what are other people's type.

Treatments:
Full feedback
Rank feedback
Sealed bid

Assymetric auction you will not learn, since bidders are from different types in different spaces.

Simple type of buyer determined auction, since we only have two types.

Sealed bid effect: Sealed bid prices are lower than open bid prices.

Sealed bid has lowest quality.

Bid decrements are small.

They like rank auction, since they think rank auction are better since bid decrements are lower.

To check robustness they tried to check the overlap, means when the distributions had overlap.

Give bidders less information appears to result in lower prices.

Theorists: Good medelers.

Game Theory by Gary Bolton @ UTD: Ninth session

Meisam Hejazinia

04/02/2013

1 Dynamic Games with incomplete information

Nash equluilibrium
Sub game equlibrium
Extensive form: repeated form
Dynamic game. Time structure. One addition mean dynamic game and incomplete information (Bayesian)

Perfect Bayesian equilibrium. Next week signalling game, and refinement of bayesian equilibrium.

Type can be signalled in dynamic game. In static game it was not possible. By choosing an action, in static game you were not able to signal who you are.

By choosing optimal action I signal that I am high type or low type, or no type. IN this was pooling equilibrium.

In dynamic game information can be communication by choices.

Three important application:

1. Signalling game. Different cost. Good type has lower cost of signalling that the low type.

2. Cheap talk game. Signalling does not cost anything. Everybody wants to signal they are good or bad. Degree of info transmission is based on the degree to which players have shared interest.

3. Reputation game. By choosing an action your type is not revealed, but over time you build repuation.

Dynamic game with incomplete information. Want to eliminate some equilibrium. In subgame equilibrium you introduced that to remove some equilibriums. Adding timing structure you would be able to say this nash equilibrium is coming with uncredible threat. The threat is fighting back is not credible. The monopolist has to made the deision.

entrant and monopolist was not credible. Now we will talk about implausable threat.

Player do not know the type of opponent. Nature picks the type with common prior. Low type with probability third and high type with probability two third. Player one might be without the knowledge of type of opponent.

If the entire game does not start with singleton then we can not break down the game to subgames anymore. There is no subgame that is not whole game.

Continuation game can begin at multiple element information set. The player does not know at which sub node he is. The player has to form some belief where the game he is.

In subgame player knows where exactly he is. We llow continuation, but player has to form some belief about where in the information set the player is located. Then we can analyze sequentially rational behavior. Means there is player that is optimal in continuation and continuation that is out of equilibrium.

So you need to form some belief, with some probability the opponent is specific type. Throught the game you would be able to update the belief, posterior beliefs that is created as a result of actions.

The actions have to be optimal given beliefs. The actions have to be optimal in all the continuation game, and off the equilibrium games.

Perfect bayesian equilibrium is the one that is perfect nash equilibrium in all the continuation game.

Have to form belief in which node the player find himself, and sequentiality.

Which type of equilibria is plausible?

They are not arbitrary, and they are built on each other.

Perfect bayesian equilibrium is consistent when we are talking about incomplete information.

If we start with static game and add time structure then bayesian nash equilibrium does not rule out implausible nash equilibrium.

First apply idea of sequential rationality to the game complete but imperfect information.

Information set (dashed line) shows at which level I am, but don't know at which of them.

Sequentially rational in the form of subgame analysis.

Just like nash equilibrium describe optimal behavior in any subgame, here we have nash equilibrium describe optimal behavior in the continuation.

Requirements of the perfect bayesian equilibrium:

Condition1. The player should have belief of where he/she is at any information set.

Condition2. Given these belief the behavior of every player should be sequentially rationa. Means the strategy should be best response.

Condition3. Each players belief in every information set results from the equilibrium strategy of the player and from Bay's rule.

The problem is when bays rule could not be applied, leading us to be free to select any belief.

Perfect Bayesian equilibrium (PBE) is a strategy profile and a set of beliefs (means not only action, but also beliefs (s, μ), so that conditions 1-3 hold true.

μ is the probability that specific node is reached given the equilibrium strategy. This was neat game, but in other game you may have off-equilibrium strategies and given and calculate the probability of reaching nodes given that off-equilibrium strategies.

Belief is about player 2's belief about where he was (mean based on the action of the previous player: here player one)?

Action of player 2 depends on the belief.

Threat of playing 'right' is credible only if at that node is reached you want to play 'right'. Only on that case player 1 will not choose left, or middle, but right.

If selecting different actions could be reached with positive probability then we can always apply Bayes rule.

On the case that ranomly assigned belief then we just need to make sure that those assignments make sense, and on that case you need to refine. This happens when mixed strategies are defined.

Strategy profile as full contingent plan, by mean of anlyzing every case.

We have to also anlyze off equilibrium path, since first player may not like the result and moves in another way, so it is combination of player 1 other strategy, and players reaction based on their believes (mean first reaction).

You have candidate for equilibrium and you check what would be out of equilibrium believes, and check whether it supports equilibrium.

We have if information set of specific player is reached, how will that person react.

$\frac{p}{(p+q)}$ where p is selection of specific action given the information set is reached $(p+q)$.

We analyze with random belief say that for example $\mu < \frac{1}{3}$. ON this case the two actions that will the last player will do, what would be the best response of one before it, and then given that response of seciond player what would be the best respone of the first player. This is weak best response.

Player 2 could be indifferent, so he may do randomization.

Whether μ is plausible, the perfect bayesian equilibrium doest not talk about. Here comes refinement.

It is about player's belief about the belief of their opponents, since their opponents will take action based on their believes.

We analyze by saying that if we end up at specific node (information set is reached) then what would be the choice of the current player.

It is iterative process you limite the probability to sub spaces then select a strategy, and then we go backward to the previous player, and that player will then select action based on probability, and if it is in the subspace that we have selected before then this is part of PBE (perfect bayesian equilibrium), but if not that belief could not be hold, so we rule it out.

These believes that we discuss here is not based on nature as we had in dynamic game complete information, but it is about the actions that have been selected based on payoffs by the player.

Perfect Bayesian equilibrium introduced by Fundenberg and Tirole in 1991.

Sequential equilibrium by Kreps and Wilson 1982.

In signalling game both are the same (mean PBE and Sequential equilibrium).

Sequential equilibrium: strategy profile S, and belief system (σ, μ), so that (σ, μ) is a perfect Bayesian equilibrium, and there is sequence of complete mixed strategies. Let it run to infinity and check whether that converges to the perfect bayesian equilibrium. If not converge the perfect bayesian equilibrium will not be sequential equilibrium.

You purturb the strategy, mean you get ϵ out of some strategies and add to the others, and at the limit again the go back to the original one, then we will have sequential equilibrium.

In this case players update out of equilibrium beliefs consistently together.

Kreps and Wilson showed that sequential equilibrium exists in all finit games.

Trembling hand perfect equilibrium (want to push right, but you push left), means every action has with at least probability of ϵ.

You let ϵ converge, and see whether the quilibrium would also be equilibrium. You let the ϵ go to zero to check whether the equilibrium stay equilibrium. Means the equilibrium if survived would be called perfect equilibrium.

Perfect equilibrium is strictly defined for games with incomplete information.

2 Perfect Bayesian equilibria in "conversation with secrets"

Idea sharing decision, on idea exposure.

When no new idea is shared the game ends.

1. What are the conditions for cooperative conversational equilibrium.

2. Add a secret. I have a patent and you don't know I have the patent.

Question: How does threat of secret affect agents communication incentive.

Question: When does the agent reveal the secret.

Pooling equilibrium when two players select the same action. Separating equilibrium when one type chooses one action and the other another action.

Semi separate equilibrium means mixed strategy.

Game Theory by Gary Bolton @ UTD: Tenth session

Meisam Hejazinia

04/09/2013

1 Dynamic Games with incomplete information

social preference

Signaling games: Different types of players signal their type. Insurance market for example. Incentive to say that I am good. Broad class of applications.

Cheap talk games: incentive to take information they know and distort them.

Reputation game: build reputation. Distortion. Information signal, and how I use that signal.

They involved assymetric information.

On technical side different from previous game. You do not have formal subgames. Problems in applying subgame perfection. Reason is because of incomplete information we do not have subgame. We have credible threat.

Different machinery to talk about this game. Equilibrium concept to build on subgame perfection. Credibility threat.

No clear subgame since due to information set player 2 does not know where he is at.

Off equilibrium means the action that will not be taken in equilibriu.

Bayesian equilibrium instead of looking at nodes for subgame looks at the information sets to cut from.

Demand consistency, as a rationality condition.

Perfect Bayesian Equilibrium (PBE) demands that all strategies be sequentially rational.

Credibility problem helps to reduce the number of equilibriums. It will hep to have exact prediction.

Sequential rationality was extention of sub game equilibrium to put more constraint to have exact prediction.

Signaling game

Job market signaling. Produced in non game theoretic format. It is basis of vary famous model of workforce. Relevance to finance. Related to initial public offering (IPO), and annomalies. Established monopolist and limiting pricing are also example of this type of game.

Insurance market, and market of lemons are example of assymetric information. Potential market of assymetric information. Market spence and Stiglitz.

Show basic set up of the model. Analyze the model. Tricky part doing the analysis of the model. Basically until now we tried to guess the equilibrium. You have to guess equilibrium and then you validate it with best response. We have moving parts yet here. Education as signal to employers.

Signal on the wall showing that I have high skill, so that employer offer good wage. Why education good signal? Because if skills low, not be able to go to good school, or get lower score. One employee E (EE). Will be high skill or low skill. EE will know what his skill level is. We will have two firms.

The model starts with chance to move, started from nature. Nature decides whether the employee is high type or low with probablity of μ, and $1-\mu$ respectively.

At some point graduate and go to the market. Two firms signle E's will start, and they will offer wages. They can offer wage that is conditional on how much education the guy gets. They can not see whether he is high or low skilled guys. The employer can not see. I can not see whether you are low or high skill. You just education signal. The assumption is that the amount of education will not affect your skill level. Firm offers the wage and the employee gets to choose. The employee will choose the highest wage. The employee goes to work, and employer discovers whether they are high or low. Plaoyers where $\{E, Firm1, Firm2\}$. The employee will get $w_1 - \frac{e}{H}$, firm will get productivity H, or L is productivity, and they will give up the wage. so for example the payoff would be $H - w_l$, means productivity minus wage. Employee gets the wage and they subtract the cost of education so $w_l - \frac{e}{H}$. We want the education be the case that high productivity type find it less expensive to go to good school, than lower type. For example loan helps in this. w_1 and w_2 are also functions of e.

Identify equilibria. Guess and verify game, but complicated. Historical concept. Market for lemons paper in 1970s QJE, rejected from AER. He had toy model and reviewer said it is rediculous. It is about assymetry of information in market. Market for used cars. You go to buy a car. What you worried about? You worried about selling the lemon. You came up with production line. You just buy somebody else problem You try to distinguish from type of lemon, and the one that is not. It is important to be able to separate the types. There are two type of outcomes:

1. Separating outcome: market can see the different types at ttime of purchuse (at transaction). I have reliable information. This one is lemon, and the other is not. I will pay less for lemons.

2. Pooling outcomes: The market cannot "see" the differences. It tends to lower the price that I am willing to pay. Both are saying that you are not lemon. There is no information. I am not willing to pay one more than others, but I have to factor lemon, so the amount I am willing to pay would be lower that I pay for non lemon.

The market will close. What I am willing to pay has factors of lemon in it. Good cards will get out of market, proportion of lemons will increase, and the market will crash. We will end up with no market transaction.

Pooling equilibra are relatively easy, but separating outcome will need much more work. Is there are pooling equilibrium?

One when both side of the market do the same thing. Both time of employees do the same thing, so nothing will distringuish them. High and low type make the same choice. Is there equilibrium. Is there equilibrium when both types select the same education level, and what would be the education then?

So $w_1(e) = w_2(e) = constant$. If the education is not rewarded, so it will not factor in my decision to give you wage. I don't care whether you have education, and how much. In this case the amount of education would be zero for both types, so $e_1 = e_2 = 0$. This is best response.

Do we have full strategy profile?

EE will get no education, and E (eployer) will not have factor of education in his wage decision, and gives zero wage?

We guess and then put it inside to find whether it is a result.

Employee accepts w if payoff is greater than zero. As employee I will accept $w \geq 0$. Backward induction says this is optimal.

$w_i \geq 0$, and $w_i \geq w_j$, where $i, j \in \{1, 2\}$

Eomployee gets the offer of zero and flips the coin and selects, but the firm that gets the low quality employee has incentive to increase.

As a result the equilibrium could be $\mu.H + (1-\mu).L$. 50% chance that I could get high guy or low guy. Payoff suggests risk neutrality. Now we try again. Suppose he offered me the wage of $\mu.H + (1-\mu).L$, the guy who made the wage is happy, then why doesn't the other firm look enviously? There is only 50% chance that you are going to end up with high type. The profit to firm would be $\pi_{firm} = \mu(H - [\mu.H + (1-\mu)L]) + (1-\mu)(L - [\mu.H + (1-\mu)L]) = 0$. This is payoff based on the wage of $[\mu.H + (1-\mu)L]$, which is fair.

Doing this backward we found out that it is optimal. This is consistent with the economic that the company is paying the worker its productivity.

This is costless with no education, and there is not reason to get educated, since firm will not recognize it. GE for long time did not like MBA's. They won't pay for it, case closed. We call this perfect bayesian equilibrium. It is pooling equilibrium. There is no room for academician in it, so the sad one.

Now we want to check whether there is separating equilibrium.

Two guys will do different things. Intuitively we hope that somebody gets more education than others, we hope it would be high type that gets more education than low type.

Let e^* be the level of education high type gets.

Conditions:

1. It should be high enough that the low type doesn't want to get in. It should be such that $w(e^*) - e^*/L \leq w(0)$. Low guy won't go to school.

2. The high payoff should want to go to school. $w(e^*) - \frac{e^*}{H} \geq w(0)$.

You try to find intuitive story, and then nail down the parameters.

$w(0)$ is the wage that whould be offered if it does not go to school. In the separating equilibrium what we envision is that the firm will be presented with two kinds of group.

This is bertrand game, and they are competing on price. Consequently if you leave anything in term of profit, then the looser will increase price with ϵ more. As a result the profit should be zero.

As a result $w(0) = L$, and as you put this you will understand what e^* is. $w(e^*) = H$ in this case the only guy show up with dimploma is high guy. Now we can figure out what e^* is. $L(H-L) \leq e^* \leq H(H-L)$

We know if there is separating equilibrium the amount of education is e^* between two values, and since $H > L$ we always satisfy this.

EE. if tye H gets education $e = e^*$ where $H(H-L) \geq e^* \geq L(H-L$, and if type L, you do not get any education $e = 0$.

Employer will have wage table that says if they have diploma I offer you high, and if you do not have diploma I will offer you low.

Accept wage if it is highest of two offered, and the wage should be greater than zero, from pooling (off the equilibrium path), since the reservation price would be zero, since the cost of education is sunk.

Pooling equilibrium will work if e^* would be out of this interval.

You have to provide strategy not what if it is provided with wage of equilibrium, but on what you are going to do in response to any other action that might be taken.

We will have multiple solution for the model. We finally do the backward induction to check the solution we found.

This is general theme of all the signaling games. We will have pooling equilibrium, and we have separation. In pooling this information has no value. If there is enough information in the market I am willing to take a chance, so market may not crash.

Next time we will discuss cheap talk model. Also social preferences will be discussed. They are sort of reputation model. The paper will be posted up. They both use perfect bayesian equilibrium.

Game Theory by Gary Bolton @ UTD: Eleventh session

Meisam Hejazinia

04/16/2013

1 Estimating the Influence of Fairness on Bargaining Behavior

Out of sample estimates from a social utility model with quantal response

We will talk about social utility. We will look at dynamic game with incomplete information.

We will talk about what it is about, and how it is modeled. The paper is about bargaining. Three get more intention than others. Two of them are by Nash. Two are cooperative game theory. Most of game theory has been non-cooperative. Cooperative you assume everybody cooperates.

It is better stated a game theory that starts from different set of propositions. Actions, strategies and payoff. We have a bargaining game here. Here are some axioms about how we think cooperation should be. We will talk about some cooperative game theory next time.

The second bargaining game is also due to Nash, sometimes called Nash demand game. It is 50 years old. It is simple normal game. Both sides get demand, and if it matches both get better off, and if not we will have no deal.

It is the game that chrystalizes game in negotiation. If you agree to what I want we have a deal. The basis of everything we are going to talk about is back to Rubinstein 1982. Sequential offer bargaining is non bargaining game. Bargaining game find their ways into multiple other games.

Special game of sequential bargaining is ultimatum game. We have two bargainers α first mover, and β is second mover. α proposes portion of the game, and if β accept they go home with their portions, and else both go home with nothing.

In equilibrium α gets the entire cake up to the crum. There is two bargainer extension. Two round game that in the first round if β rejects, the pie shrinks, based on discount factor. Then β offers counter offer. Both α and β have discount factors. $_\alpha$, and δ_β. On the second round if α rejects both get nothing, and if accept the shrinked cake would be splited according to the proportion proposed. We solve this by backward induction.

The entire cake is 1, and the entire cake on second round would be $1.\delta_\beta = \delta_\beta$. In the round 2, beta offers a crum $\epsilon > 0$, and α accepts any positive offer. May be zero too. That implies that the solution to the game would be α get zero or close to the zero, and α gets δ_β.

In round 1 what should beta do? β accepts $\geq \delta_\beta$, so subgame perfect equilibrium is α gets $1 - \delta_\beta$, and β gets δ_β.

In the ultimatum game β gets no leverage at all, which is gets nothing. In the two round game, beta leverage is that now they can say no, and get counter offer, which offering α could force α to accept something. In two raound game β has subcounter leverage.

In three round game, α makes the counter offer on the end of second round after rejecting counter offer. The subgame perfect equilibrium in this case is $\delta_\beta(1-\delta_\alpha^2)$ offered to β by α, and α will keep $1-\delta_\beta(1-\delta_\alpha^2)$.

What drives this model is time, and that people care about time, and those who are more patient can leverage that, and get more.

Nash's model has different explanation which is risk. It makes the game of chicken. I dare you not to take it. Each looking for some way to break the tie. In Nash demand game everything is solution. These games try to add some increment to explain what will settle on. In Rubenstein it was about time.

Sequential bargaining game. In two round game δ_β defines what the solution is. If $\delta=0$ means nobody cares about future, and just care about today, which makes ultimatum game. $\delta=1$ in infinitely patience? If the pie does not shrink, it again becomes ultimatum game, and the only thing related is second offer, since the proposer at the last round will have leverage.

What the ultimatum game happens? They tend to reject larger than a proportion. Difference from the theory. The dictator game responder can't reject. Can only accept the offer. The proposers are less generous in dictator game. It is important clue, since despite explanation that people are irrational, people understand the power of the second size. They still give some money in the dictator game. It was a surprise too. They leave something even in dictator game.

On multiple round sequential bargaining game. They give discount factor of both sides and they see deviation away from what the equilibrium should be. Although counter offer, and discount factor exists in multiple round game.

We have list of regularities:

R1: There is a consistent first mover advantage. α bargainers recieves more than β bargainers, regardless of the value of β.

R2: Observed mean opening offers deviate from the pecuniary equilibrium in the direction of the equal money division.

R3: A substantial proportion of first-period offers are rejected.

R4: A substantial proportion of rejected first period offers are followed by disadvantageous counteroffers.

R5: The discount factor of α influence the outcome in two round games.

Social preference theory has its rule in the regularities stated above. Deviation from the prisoner dillema in many rounds except the final one was also explained by it.

They break down into two classes. Distribution models of Bolton-Fehr and Shmidt, and Moels that add intention, like Rabine, and Levine. The second type just have different machineries.

We try to fit social utility model to ultimatum game data. Social preference model can be laid down. You wonder how robust the model is. You try to forecast data from experiments.

When we have multiple round games, people play subgame perfect. This was saying what people will do, and not what they ahve done. On extreme values 'pies shrinks slowly: .1' or 'pie shrinks rapidly: .9'. The result was different. Two and three round games. They ran on the variety of discount factors on multiple round games on 2 round (.25,.25) close to subgame equilibrium. On the 5 round games, yet we have deviation. Some myopia is going on.

Maybe in longer game people have trouble doing backward induction.

Learning and that peoples rejection threshold changes.

$U_i(\sigma, c) = c(\sigma - \frac{b_i}{2}(\sigma - \frac{1}{2})^2)$ if $\sigma < \frac{1}{2}$.

Magic point is 50% of time.

$c.\sigma$ if $\sigma \geq \frac{1}{2}$.

b_i the amount of weight I attach to the loss. $\sigma - \frac{1}{2}$ would be relative payoff.

σ is absolute payoff.

$c.\sigma$ is assymetric loss function.

Assymetric loss function. I care about inequality of I am on the short side. People care about fairness when it is against them.

Take utlity function and embed into the game of ultimatum game with incomplete information. $U_i(\theta_i, 1) = 1 * (\theta_i - \frac{b_i}{2}(\theta_i - \frac{1}{2})^2) = U_i(0, 0) = 0$.

Share of the game that makes me indifferent between accepting and rejecting.

$F(\theta)$ is cumulative distribution of θ across the population. We assume $F(0) = 0, F(= .5) = 1$, F is strictly increasing.

$F(\theta) = Pr\{$ i will accept an offer of $\theta or more as \beta$ in the ultimatum game $\}$.

Nature draws α and β types from $F(\theta)$.

We will solve by backward induction. We will have perfect Nash equilibrium. β accepts x if and only if $x \geq \theta_i$.

α offers argument$_x(1 - x)F(x)$; $F(x)$ is the probability that x will accept it.

In practice x that maximizes utility tend to be .4. The offer that maximizes the profit f the first mover is 40%. There is some learning going on in reality. People experiment proposals, and those proposals converge.

We are going to go back to the data. We are going to fit model, means figure out F function using ultimatum game. Then we take the model and use it to predict what would happen in the multiple raound ultimatum game. The major complication is to deal with the fact that there is learning.

People learn about F function. We have to account for that. What data appears to show is that people going to learn F.

We embed the mover function to logistic function. This gives us the probability that any player going to make particular move. τ is going to be error term. They are called coefficient certitude.

As we go along error term will shrink. People will become more certain about what they are going to play.

The game was played repeatedly, so we had learning. It was game with two pie sizes.

The data was gathered over four countries.

Most people learn from getting rejected and they pull back.

Out of sample estiamtes: Take the model and do the simulation, and do the backward induction and see what the outcome is. Due to the error ter there would be learning.

They did it for multiple treatments. Pecuniary equilibrium: look for the money. Did not fit at all, and it fits much better here. You could not rject that slope of the line is equal to one.

Samall pie game we had rejection.

If cake is small we see more arbitrary behavior.

If you are not playing for too much, it makes more random noises.

The players will learn distribution of types, and then become optimal by backward induction.

Fehr Schmidt model has linear loss function. Standard linear regression. Easier fit. The potential downside is that it has equilibrium in boundaries.

Social preference factor adds value to the fit.

3-person ultimatum game: 1. First person proposes 2. second accepts or rejects 3. Third person recieve portion agreed by two other parties. (XYZ), where z is dummy.

z-condition is the condition that responder say's what dummy should get.

The result of this has been robust.

Social preference model explains. Point of reference in two player is 50%. This game the symmetric point is one third.

ERC model is saying that your care about fairness is celf centered.

Fehr shmidt cares about the distribution.

Game Theory by Gary Bolton @ UTD: Twelfth session

Meisam Hejazinia

04/23/2013

Next fall course of behavioral operation management. Deliverables would be proposal for behavioral study.

Problem: $0 - 100\%$ based on the discovery. 50% profit.

Game: $V_T Unif[0, 100]$

Acquirer makes a bid for T, b.

T either accepts or rejects.

Payoffs.

Target payoff T : if accept bid $b - V_T$ otherwise V_T
A : acquirer if accepts $1.5 * V_T - b$; otherwise 0

Backward induction. Another way: Guess, and check whether it works. Most common guess is 50. On average it will worth 50 to target and 1.5 to me. Some people say 60 to win. This line of reasoning does not work. Write down and check.

Propose equilibrium:

$b = 50$. Acquirer bids this. Target accepts if the value to him is $V_T \leq 50$, rejects otherwise.

Is this equilibrium? Payoff to the target is: $.5\pi_{accept} + .5 * \pi_{reject} = \frac{1}{2}.(50) + \frac{1}{2}75 = 62.5$
Payoff to acquirer? $.5*[1.5*E_{V_T}[Taccepts]-50]+.5*0$ expected value is 25. $\rightarrow .5[1.5*25-50] = -12.5/2 = -6.5$

For any number it is negative. Any number for the acquirer has positive profit.

There is information in that second party accepts. If payoff is greater than 2, every bit has positive value, so 100 is fine. It is like jar auction. What is the chance that you are the only person in the room who realizes that it has more value?

As a result accept by the second party has information in it.

1 Cheap talk game

Two players. Sender and reciever. Message sender could have three types. Receiver recieves the massage and takes an action. Types are equally likely.

$t_3 \rightarrow t3$;
$t1 + t2 \rightarrow t_2$

Recieverse beliefs: if recieves $t_3 \rightarrow t_3$ otherwise $t_1 + t_2$ each with both $\frac{1}{2}$.

If he does not revieve $t3$ he will belief it is either t_1 or t_2.

Reciever action: if recieves $t_3 \rightarrow a_3$
otherwise $\rightarrow a_2$

Difference between signaling game and cheap talk is that it is costly to signal, mean it requires commitment, in signaling game. In cheap talk,

	t1	t2	t3
a1	0,1	0,0	1,0
a2	1,0	1,2	1,0
a3	0,0	0,0	2,1

Table 1: Cheap talk game

signaling is cheap.

Is there equilibrium that have everybody revealed?

In terms of action space if you are t_1 or t_2 you both want action 2. I want what he wants, so both of us will send the same signal.

There is always pooling equilibrium in cheap talk. Reciever beliefs after recieving messat that I don't know t_1, t_2, or t_3 each with probability of $\frac{1}{3}$

Reciever best response in this case would be a_2. In pooling equilibrium sender will randomize t_1, t_2, t_3 each with probability of $\frac{1}{3}$

In the case of separating equilibrium signal makes the reciever to do something, but the payoff of result would be different to both parties, since it is defined based on true value.

Whether it is separating equilibrium or pooling equilibrium is assumption of Nash equilibrium. In labratory quite often people find their way to equilibrium. In industry we have learning. Certain payoff structure dynamic will get people to equilibrium, and in some it does not.

2 Cooperative Game Theory

Matching market as an exception that does not study non-cooperative game theory.

Game of contract of S, C, T. Cooperative game. Characteristic function.

Specification of two objects $\{N_j V\}$. N is set of players. V is characteristic function. $V(S,C,T) = 100$. V is function of how the coalition works. This is called grand coalition.

$V(S,C) = 90$

$V(S,T) = 70$

$V(C,T) = 40$

$V(S) = V(C) = V(T) = 0$

There is no specification of action space and moves.

Try to solve the game on the basis of same principles. Look at potential payoff and try to solve by appealing of principles.

The core: (the most poular solution concept). Forecast of what happens in this game is the coalition that won't be blocked. A grand coalition $\{\pi_s, \pi_c, \pi_T\} = \{33\frac{1}{3}, 33\frac{1}{3}, 33\frac{1}{3}$ is not in the core. The reason of not being in the core is that it can be blocked by a coalition of $\{\pi_s, \pi_c\} = \{45, 45\}$.

In other word it is not stable outcome since it can be blocked by another. Parties can pareto improve.

It is not in the core if there is break away coalition that can pareto improve it.

We need a Grand Coalition $\{\pi_s, \pi_c\} = \{45, 45\}$, such that $\pi_s + \pi_c + \imath_T = 100$.

$\pi_s + \pi_c \geq 90$
$\pi_s + \pi_T \geq 70$
$\pi_c + \pi_T \geq 40$

In this case you can guess, and check whether it could be blocked or not, rather than doing the math from the beggining.

$\{60, 30, 70\}$ is in the core here, since each would not be able to do better. It is the minimum that

	S	C	T
S-C-T	0	90	10
S-T-C	0	30	70
C-T-S	60	0	40
C-S-T	90	0	10
T-S-C	70	30	0
T-C-S	60	40	0
	$\frac{280}{6} = 46\frac{2}{3}$	$31\frac{2}{3}$	$21\frac{2}{3}$

Table 2: Shapley Value

anybody can get.

The basic idea is that you are looking for something that is stable, and that nobody will do something that makes pareto optimal.

Core does not have to be unique.

It does not even have to be unique.

The Shaply value
Always exists and is always unique.

Assigns average marginal contribution to a player brings to the table. Player do not come to the table at the same time.

$V(S) = V(C) = V(T) = 0$, you start and say room is empty or have someone and someone else just comes in.

People come to the room with different orders. In reality people do not come to the room in random way.

Shebley value moves in the action space, but not necessarily in the correct action space.

Axioms behand Shapley value:

(i) efficiency: grand coalition

(ii) symmetry: all at the same position same payoff

(ii) Additivity: the solution to the sum of two tu games must be the sum of what it awards to each of the two games

(iv) Dummy player: if a player contributes nothing to every coalition, the solution should pay him nothing.

If each controls we will have multiple cors that are different. Shebley value has strong assumption of what these would factor in.

Strucutre of these coalition matters. How fights are formed as a result of changing mind. Sequential bargining is another way that the outcome is analyzed.

Matching market concept is about core. Two papers about matching markets is posted.

Game Theory by Gary Bolton @ UTD: Thirteenth session

Meisam Hejazinia

04/30/2013

1 Matching Theory

Area with lot of real word applications. Assigning kids to schools, in cities when school have preferences, and children and parents have to. Recently it is used in much elaborate way in matching kidnies with patients. Swapping kidney between hospital will happen here.

We will look at some basic principles behind these algorithms. The keyword here is stability.

It is about 1984 paper.

Workers must be allocated to companies, student to universities. Organ donors to patients, authors to publishers, etc. It is shably algorithm, and all principles come from there.

These types of allocation problems are anlyzed under the concept "matching".

Matching theory belongs, along with auction theory to the most active areas of applied game theory.

It has elements of cooperative game theory in it. What is different here? We usually talked about markets, and price was clearing house, but here we talk about prices. We will talk about markets that price is secondary. For example get the job in the best institution. They do not give you more money to you. Early in your career is important to be in the good institution rather than earn couple of more dollars.

	w1	w2	w3
m1	1,3	2,2	3,1
m2	3,1	1,3	2,2
m3	2,2	3,1	1,3

Table 1: Matching problem

We don't allow kidneys, and school offers to be sold in the open market. Therefore not all algorithms are created equally.

It is still rational choice model, but the difference is that people and institutions have well defined preferences. The market clearing mechanism is just different here, although it is still rational choice.

Gale and Shably called this marriage problem. There is a group of women looking for male mates, and group of men looking for men mates. Each men and women have preference complete transitive preferences. It is strict, and there is no indifferences.

One way of portraying this is using a matrix.

You read in the form that for example man one prefers women 1 first best, and woman one prefers man 1 third best.

You have $n!$ mean here 6 possible permuations. Not all these permuations are stable.

Unstable Matching definition: A matching is considered unstable, if there are two men and two women who are paired in the form (m1,w1) and (m2,w2), although m1 would prefer w2 and w2 would prefer m1:

Formal: $w_2 \succ_{m_1} w_1$

Mean can they do better by running away from match.

If side deals can be cut, we will have stable match. If two hospital think they are matched with inferior and they do the side deal then the match will break.

It is not that the pair will break up, but it is about that one member of the pair wants to break up.

Instability comes when there is both mean and women who want to leave what they are matched to to match eachother.

You can do the matching to many other situations such as roommate problem, for sorority sisters.

For the marriage algorithm it is always one stable match.

Sometimes there are multiple stable solutions, but always there is one.

Procedure:

1. Every man proposes to his most preferred woman.

2. Every woman accepts the proposal from the man she likes best and rejects the rest.

Then you keep doing this.

In each step:

1. Every man, who was rejected in the last step, proposes to the woman whom he likes the best out of all the women he hasn't proposed to yet.

2. Every woman accepts the one whom she likes the best out of all the men who propose and rejects the others.

Stage 1			
w1	w2	w3	w4
m1			m2
Stage 2			
w1	w2	w3	w4
m1	m5	m3	m4
Stage 3			
w1	w2	w3	w4
m1	m2	m3	m4

Table 2: Matching steps

After you proposed the whole cycle the procedure is end.

$P(m1) = w1, w2, w3, 24$
$P(m2) = w4, w2, w3, w4$
$P(m3) = w4, w3, w1, w2$
$P(m4) = w1, w4, w3, w2$
$P(m5) = w1, w2, 24$
$P(w1) = m2, m3, m1, m4, m5$
$P(w2) = m3, m1, m2, m4, m5$
$P(w3) = m5, m4, m1, m2, m3$
$P(w4) = m1, m4, m5, m2, m3$

On final step man 5 will be by himself bachelor.

Man 5 tried every single one.

Man 5 was also picky, but left bachelor.

Now lets do from the other side, mean women propose to man.

We are assuming that these preferences are common knowledge.

Time and discounting could be interesting. The problem that these algorithms do not work sometimes is that timing is not included.

Three of the women get their first choice, and the other get her second choice, so women did better here.

There is proposer advantage in this algorithm.

Stage 1				
m1	m2	m3	m4	m5
w4	w1	w2		
Stage 2				
m1	m2	m3	m4	m5
w4	w1	w2	w3	[m4]

Table 3: Matching steps

These two are pareto efficient.

Strategic manipulation

You may suspect that if you are women and men is doing all the proposing, you might suspect that you are better by doing strategic manipulation of preferences.

We always talk about direct mechanism: every man and women initially take the differences.

We are considering that rather than assuming your preference is true we put the manipulated preferences.

There is always room for manipulation in this algorithm.

The problem with matching is that it assumes all the preferences is revealed and is common knowledge in strategic manipulation.

If you are men or women proposing it is dominate strategy to tell the truth, since it will give you best shot.

There are two sources for matching:

1. Al Roth's homepage.

Under rubric and market design. Market design is bigger.

2. In terms of theory of matching the go to book is two sided matching. Al Ruth and ....

Economic Design of the Entry Level Job Market

In the medical market the matching is not one to one, but is one to many.

Hospital hire multiple interns, but each intern only works in one hospital. The general result is the same, and not different from marriage problem.

Basic store is that if you are in medical school, you want to go to internship which is important. You want to intern for few years, but that increases your prospect for long term employment. Your primary concern is go to the prestigous school.

In legal market you want a clerk for supreme court justic, and there is also the same.

They start as a market free for all. There are great deal of market failiure early on.

There is strong competition among students. Student start to get offers earlier in their education. People found that if they offer earlier then they will have leverage. They give time limit. I know you think I am okay, and you may get better offer, but you may not.

Other hospitals see the same pattern and they try to reach four month before. Students get job proposal two years before, when they do not have any education.

In the legal markets for clerks. By the end of the call you need to tell me what is your decision.

These was not healthy, and the markets went to failiure. Institutional problem when exists, all the hospitals pledge to not conduct this behavior.

Nobody followed. The problem is that it is in everybody's interest to stop this behavior, but given that everybody stops I will not stop.

It is prisoner dillema kind of structuret was a failiure doing this. They try to have some algorithm that matches interns with hospitals.

Hospitals have preferences over interns.

Interns could only take one job. You have strict preferences.

Stable outcome is that you can not move anybody away. No applicant would not be able to get something better, means there would be no way for blocking.

Algorithm:

1. Every worker applies at his/her first choice firm.

2. Every firm f (with qf positions) rejects all unacceptable offers and in case it recieves more the qf acceptable offers, it keeps qf most preferred offer and rejects the rest.

...

Theorem 1: The set of stable matching is never empty.

The side that has advantage is proposing side.

In every stable matching, the same worker are distributed and the same positions filled.

Everybody left out in stable match is left out in every stable match, and everybody left in in one stable match is left in in every stable match.

When side deals happen these systems crash.