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theory Problem-1B
  imports HOL-Algebra.Algebra
begin

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Problem 1B asks for a special case of Lagrange's theorem, thus we avoid using the general variant.

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theorem (in comm-group) problem1B:
  assumes finite: finite (carrier G)
  assumes closed:  $g \in \text{carrier } G$ 
  shows  $g \mid \text{order } G = 1$ 
proof -
  let ?f =  $\lambda x. g \otimes x$ 
  have [simp]: ?f ' carrier G = carrier G
    by (simp add: closed group.surj-const-mult)
  have inj-on ?f (carrier G)
    by (simp add: closed group.inj-on-cmult)
  hence  $(\bigotimes x \in \text{carrier } G. x) = (\bigotimes x \in \text{carrier } G. g \otimes x)$ 
    using finprod-reindex[where  $h=?f$  and  $A=\text{carrier } G$  and  $f=\lambda x. x$ , symmetric]
    by simp
  also have ... =  $(\bigotimes x \in \text{carrier } G. g) \otimes (\bigotimes x \in \text{carrier } G. x)$ 
    using closed by (intro finprod-multf) auto
  finally have  $(\bigotimes x \in \text{carrier } G. g) = 1$ 
    using closed by (intro r-cancel-one'[THEN iffD1]) auto
  thus ?thesis
    using closed unfolding order-def by (simp add: finprod-const)
qed

end

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