theory *Problem-1*
  imports *HOL−Number-Theory.Number-Theory*
begin

Find all positive integers n such that $4^n + 2007$ is a perfect square.

theorem *problem1*:
  assumes $n > 0$
  shows $\nexists k{::}nat.\ k^2 = 4\hat{\ }n + 2007$
proof
  assume $\exists k{::}nat.\ k^2 = 4\hat{\ }n + 2007$
  then obtain $k{::}nat$ **where** kk: $k^2 = 4\hat{\ }n + 2007$ **by** *blast*
  with ⟨$n > 0$⟩ **have** $[k^2 = 3]\ (mod\ 4)$
    by (*simp add*: *cong-def flip*: *mod-add-eq*)
  hence $(k\ mod\ 4) * (k\ mod\ 4)\ mod\ 4 = 3$
    by (*simp add*: *cong-def mod-mult-eq power2-eq-square*)
  thus *False*
    using *mod-exhaust-less-4* [*of k*] **by** *auto*
qed

end