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theory Problem-3
  imports Complex-Main
begin
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Positive integers a, b, z satisfy $ab = z^2 + 1$. Prove that there exist positive integers x and y such that (* TAGS: number-theory *)

$$\frac{a}{b} = \frac{x^2+1}{y^2+1}.$$

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theorem problem3:
  fixes a b z :: int
  assumes a > 0  b > 0  z > 0
  assumes a∗b = z^2 + 1
  shows ∃ x y. rat-of-int a / of-int b = of-int (x^2 + 1)/of-int (y^2 + 1) (is ∃ x y. ?P x y)
proof (rule, rule)
  let ?x = z + a and ?y = z + b
  have a ∗ (?y^2 + 1) = b ∗ (?x^2 + 1)
    using ‹a∗b = z^2 + 1›
    unfolding power2-eq-square
    by algebra
  moreover have rat-of-int b ≠ 0
    using ‹b > 0› by simp
  moreover have rat-of-int (?y^2 + 1) ≠ 0
  proof −
    have ?y^2 + 1 > 0 by simp
    thus ?thesis by linarith
  qed
  ultimately show ?P ?x ?y
    by (simp add: frac-eq-eq flip: of-int-mult of-int-add)
qed

end
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