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jaxparrow: a Python package solving the cyclogeostrophic balance using a variational formulation

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Summary

Sea Surface Height (SSH) variations measured by satellite instruments are widely used to estimate Sea Surface Currents (SSC) in oceanographic operational or research applications. The geostrophic balance approximation, which only consider the pressure gradient, the current velocity, and the Coriolis force, is commonly employed to link SSH and SSC. It is known that under some configurations, the centrifugial acceleration, disregarded in the geostrophic formulation, should be included to the balance (Bakun, 2006; Charney, 1955; Maximenko & Niiler, 2006), leading to the cyclogeostrophic approximation. In general, solving the cyclogeostrophic balance can not be done analytically and numerical methods are needed. However, (1) existing iterative approaches are known to diverge (Arnason et al., 1962), resulting in the use of ad-hoc iterative schemes (Ioannou et al., 2019; Penven et al., 2014); (2) publicly available implementations are missing.

To overcome these limitations, we propose the Python package `jaxparrow.jaxparrow` formulates the cyclogeostrophic balance as a variational problem and solve it using a collection of well known optimizers. Its implementation heavily relies on JAX (Bradbury et al., 2021), the Python library bringing together automatic differentation and just-in-time compilation, and the growing ecosystem around it. `jaxparrow` can be used as a package for an easy integration to existing oceanographic pipelines, or as a standalone executable working directly with NetCDF files.

Statement of need

Sea Surface Currents (SSC) can be easily approximated from satellite altimetry observations of the Sea Surface Height (SSH) using the geostrophic balance. Geostrophy describes the balance between the pressure gradient force (indirectly observed via the SSH), and the Coriolis force. Geostrophic currents satisfy this equilibrium:

$$f\left(\vec{k} \times \vec{u}_g\right) = -g\nabla\eta, \tag{1}$$

where f is the Coriolis parameter, $\vec{k}$ the vertical unit vector, $\vec{u}_g$ the geostrophic velocity, g the gravity, and η the Sea Surface Height.

However, as discussed by Charney (1955), Bakun (2006), and Maximenko & Niiler (2006), geostrophy alone is not always sufficient to accurately estimate the Sea Surface Currents, and an advective term should be considered. For example, it has been shown by Penven et al. (2014) that, in the highly energetic Mozambique Channel, the geostrophic velocity can produce errors in the order of 30%. In these conditions, the centrifugal acceleration and the inertial effects of oceanic dynamics are no longer neglectable. To account for those forces, the advective term $\vec{u} \cdot \nabla \vec{u}$ is added back to the balance. Considering a horizontal, stationary, and inviscid flow, the momentum equation linking the currents velocity $\vec{u}$ with the SSH —through the geostrophic velocity $\vec{u}_g$ from Equation 1— can be expressed as:

$$\vec{u}_c - \frac{\vec{k}}{f} \times (\vec{u}_c \cdot \nabla \vec{u}_c) = \vec{u}_g, \tag{2}$$

where $\vec{u}_c$ is the cyclogeostrophic velocity.

Because of the advective term $\vec{u}_c \cdot \nabla \vec{u}_c$, Equation 2 is nonlinear, and solving it analytically is conceivable only in idealized scenarios, making numerical approaches essential. The current state-of-the-art method to solve the cyclogeostrophic equation is the iterative formulation introduced by Arnason et al. (1962) and Endlich (1961), which consists of reaching balance using the following iterative scheme:

$$\vec{u}_c^{(n+1)} = \vec{u}_g + \frac{\vec{k}}{f} \times \left(\vec{u}_c^{(n)} \cdot \nabla \vec{u}_c^{(n)}\right), \tag{3}$$

with $\vec{u}_c^{(0)} = \vec{u}_g$. This approach is known to diverge since Arnason et al. (1962), and in practice (Ioannou et al., 2019; Penven et al., 2014) the residual $res = |\vec{u}_c^{(n+1)} - \vec{u}_c^{(n)}|$ is used to control point by point the iteration process. The iteration locally stops when the point residual is below 0.01 m/s or starts to increase.

To avoid the local divergence issue of the iterative process, and its ad-hoc control, we propose to formulate the cyclogeostrophy as the variational problem:

$$J(\vec{u}_c) = \left\| \vec{u}_c - \frac{\vec{k}}{f} \times (\vec{u}_c \cdot \nabla \vec{u}_c) - \vec{u}_g \right\|^2, \tag{4}$$

where $\|.\|$ is the discrete L^2 norm. Thanks to JAX automatic differentiation capabilities, ∇J is numerically available, and the cyclogeostrophic currents are estimated by minimizing Equation 4 using the classical gradient descent (see Equation 5), or any gradient-based optimizer.

$$\vec{u}_c^{(n+1)} = \vec{u}_c^{(n)} - \alpha \nabla J(\vec{u}_c^{(n)}), \tag{5}$$

where α is the learning rate, and $\vec{u}_c^{(0)} = \vec{u}_g$.

Application to the Alboran sea

Because the Alboran sea is an energetic area of the Mediterannean sea, it is perfectly adapted to demonstrate the need to consider cyclogeostrophy, and in particular the interest of the variational formulation allowed by `jaxparrow`. The data and results presented here can be found in the Alboran sea notebook hosted on GitHub.

We use currents from the eNATL60 configuration of the state-of-the-art NEMO ocean circulation model (Gurvan et al., 2022) as reference. NEMO takes several types of measurements as inputs —including Sea Surface Height—, to reconstruct the ocean dynamics —such as Sea Surface Currents, from which currents (normalized) vorticity can be computed (see Figure 1).

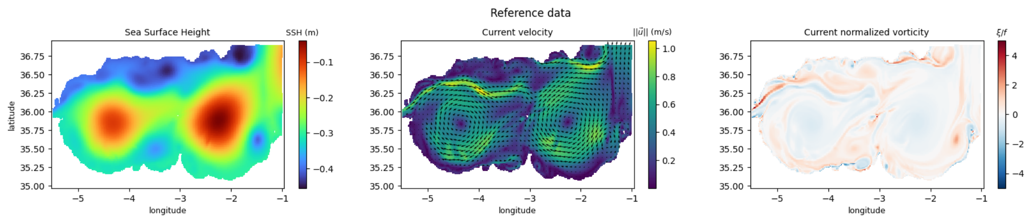

Figure 1: Reference data: on the left panel, the measured SSH used by NEMO; on the middle, NEMO reconstructed current velocity (colored by the magnitude, with arrows giving the direction); on the right, the corresponding normalized vorticity.

Using the SSH, `jaxparrow` can first estimate the geostrophic currents (Equation 1). As geostrophy is a major mechanism governing ocean dynamics, the vorticity derived from those currents present an overall similarity with the one obtain from NEMO data. However, we can clearly identify pathological areas (around (-4, 35.5), (-3, 36), and (-2.5, 35.5) in (longitude, latitude) coordinates, see Figure 2) where the geostrophic balance fails to capture the true ocean circulation.

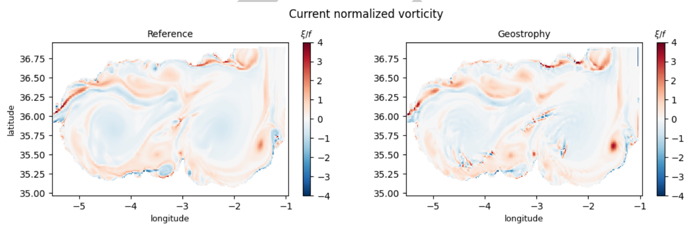

Figure 2: The qualitative comparison between reference (left panel) and geostrophic (right panel) vorticities reveals several regions with highly erroneous estimations.

From the imperfect geostrophic approximation, `jaxparrow` then solves the variational formulation of the cyclogeostrophy (Equation 4), using —in this example— gradient descent (Equation 5). As a result, almost all the problematic areas are now much more accurately reconstructed, leaving only costal or domain boundary regions with significant differences from our reference vorticity (see Figure 3, left and middle panels). By evaluating the cyclogeostrophic disequilibrium (the functional J from Equation 4) along the optimization process, we clearly see that `jaxparrow` is able to close the cyclogeostrophic balance (right panel of Figure 3).

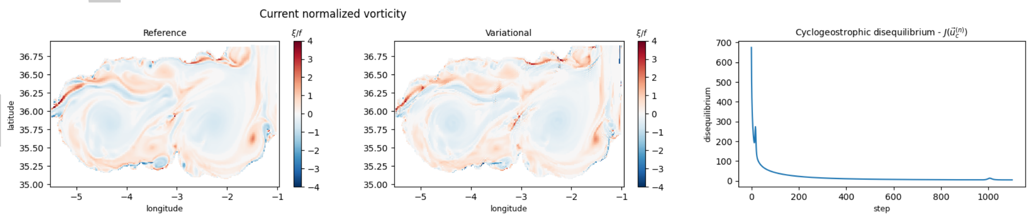

Figure 3: In contrast to the geostrophic approximation, variational cyclogeostrophy (middle panel) provide accurate reconstruction of the reference (left panel) vorticities in all the inside of the domain. The right panel demonstrates the fast convergence towards cyclogeostrophic balance.

In comparison, `jaxparrow` can also estimate the cyclogeostrophic currents using the iterative scheme (Equation 3). In this example, it is evident that the point by point update of the velocities is not able to fill the cyclogeostrophic balance, and we notice that this estimation is qualitatively worse than the geostrophy (left and middle panels of Figure 4). Even with an additional ad-hod threshold imposed on the residuals at the first iteration, divergence from cyclogeostrophic equilibrium immediately occurs (see Figure 4, right panel).

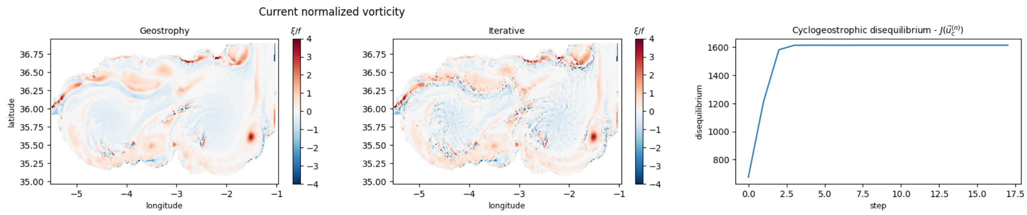

Figure 4: As exhibited in the right panel, the iterative approach diverges from the cyclogeostrophic balance; and we can notice from the two other panels that the resulting vorticity is qualitatively worse than the geostrophic one.

Those qualitative observations are inline with the quantitative evaluation we performed. We computed the 1000 first percentiles of the vorticity distributions, and we oberserve that the percentiles of the variational distribution are the closest to the ones of the reference distribution (Figure 5).

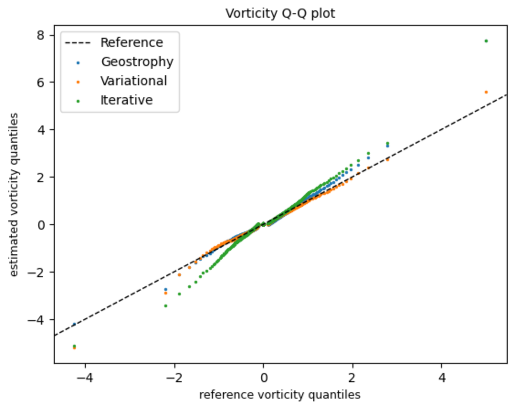

Figure 5: The percentiles of the vorticity distributions demonstrate that our variational estimation of the cyclogeostrophy (in orange) corrects the tendency of the geostrophy (in blue) to overestimate positive vorticities, while the iterative scheme (in green) diverges and systematically overestimate the absolute vorticity.

Availability

Beside the interesting paradigm shift, `jaxparrow` also offers the first to our knowledge open implementation of the cyclogeostrophy resolution. The code is available on GitHub, with the specific tag `joss` for the version matching this publication; and the documentation, with pip-installation instuctions, usage examples, and toy notebooks, is hosted on Read the Docs.

Acknowledgements

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