

$$\begin{array}{c}
\mathbf{A - Prop} \frac{S \models \text{len } x \approx \text{len } y}{A := A, \text{len } x \approx \text{len } y} \\
\mathbf{S - Prop} \frac{A \models_{LIA} \text{len } x \approx \text{len } y}{S := S, \text{len } x \approx \text{len } y} \\
\mathbf{Len} \frac{x \approx t \in \mathcal{C}(S) \quad x \in \mathcal{V}(S)}{A := A, \text{len } x \approx (\text{len } t) \downarrow} \\
\mathbf{Len - Split} \frac{x \in \mathcal{V}(S \cup A) \quad x : \text{Str}}{S := S, x \approx \epsilon \parallel A := A, \text{len } x > 0} \\
\mathbf{A - Conflict} \frac{A \models_{LIA} \perp}{\text{unsat}} \\
\mathbf{R - Star} \frac{s \text{ in star}(\text{set } t) \in R \quad s \not\approx \epsilon \in \mathcal{C}(S)}{S := S, s \approx \text{con}(t, z) \quad R := R, z \text{ in star}(\text{set } t)} \\
\\
\mathbf{S - Cycle} \frac{t = \text{con}(t_1, \dots, t_i, \dots, t_n) \quad t \in \mathcal{T}(S) \setminus C \quad t_k \approx \epsilon \in \mathcal{C}(S) \text{ for all } k \in \{1, \dots, n\} \setminus \{i\}}{S := S, t \approx t_i \quad C := C(C, t) \setminus \{t_i\}} \\
\mathbf{Reset} \frac{}{F := \phi, N := \phi, B := \phi} \\
\mathbf{S - Split} \frac{x, y \in \mathcal{V}(S) \quad x \approx y, x \not\approx y \in \mathcal{C}(S)}{S := S, x \approx y \parallel S := S, x \not\approx y} \\
\mathbf{S - Conflict} \frac{x \approx t \in \mathcal{C}(S) \quad s \not\approx t \in \mathcal{C}(S)}{\text{unsat}} \\
\mathbf{L - Split} \frac{x, y \in \mathcal{V}(S) \quad x, y : \text{Str} \quad S \not\models \text{len } x \not\approx \text{len } y}{S := S, \text{len } x \approx \text{len } y \parallel S := S, \text{len } x \not\approx \text{len } y} \\
\\
\mathbf{F - Form1} \frac{t = \text{con}(t_1, \dots, t_n) \quad t \in \mathcal{T}(S) \setminus (\mathcal{D}(F) \cup C) \quad N[t_1] = s_1 \dots N[t_n] = s_n}{F := F, t \mapsto (s_1, \dots, s_n) \downarrow} \\
\mathbf{F - Form2} \frac{l \in \mathcal{T}(S) \setminus \mathcal{D}(F)}{F := F, t \mapsto (l)} \\
\mathbf{N - Form1} \frac{[x] \notin \mathcal{D}(N) \quad s \in [x] \setminus (C \cup \mathcal{V}(S)) \quad Ft = Fs \text{ for all } t \in [x] \setminus (C \cup \mathcal{V}(S))}{N := N, [x] \mapsto Fs} \\
\mathbf{N - Form2} \frac{[x] \notin \mathcal{D}(N) \quad [x] \subseteq C \cup \mathcal{V}(S)}{N := N, [x] \mapsto (x)} \\
\\
\mathbf{F - Unify} \frac{F s = (w, u, u_1) \quad F t = (w, u, v_1) \quad s \approx t \in \mathcal{C}(S) \quad S \models \text{len } u \approx \text{len } v}{S := S, u \approx v} \\
\mathbf{F - Split} \frac{F s = (w, u, u_1) \quad F t = (w, u, v_1) \quad s \approx t \in \mathcal{C}(S) \quad S \models \text{len } u \approx \text{len } v \quad u \notin \mathcal{V}(v_1) \quad v \notin \mathcal{V}(u_1)}{S := S, u \approx \text{con}(v, z) \parallel S := S, v \approx \text{con}(u, z)} \\
\mathbf{F - Loop} \frac{F s = (w, x, u_1) \quad F t = (w, u, v_1, x, v_2) \quad s \approx t \in \mathcal{C}(S) \quad x \notin \mathcal{V}((v, v_1))}{S := S, x \approx \text{con}(z_2, z), \text{con}(v, v_1) \approx \text{con}(z_2, z_1), \text{con}(u_1) \approx \text{con}(z_1, z_2, v_2) \quad R := R, z \text{ in star}(\text{set } \text{con}(z_1, z_2)) \quad C := C, t} \\
\\
\mathbf{D - Base} \frac{s \in \mathcal{T}(S) \quad s : \text{Str} \quad S \models \text{len } s \approx \text{len}_B \text{ for no } B \in B}{B := B, \{[s]\}} \\
\mathbf{Card} \frac{B \in B \quad |B| > 1}{A := A, \text{len}_B > \lfloor \log_{|A|} (|B| - 1) \rfloor} \\
\mathbf{D - Add} \frac{s \in \mathcal{T}(S) \quad s : \text{Str} \quad B = B', B S \models \text{len } s \approx \text{len}_B[s] \notin B \quad \text{for all } e \in B \text{ there are } w, u, u_1, v, v_1 \text{ such that} \quad (N[s] = (w, u, u_1), Ne = (w, v, v_1), S \models \text{len } u \approx \text{len } v, u \not\approx v \in \mathcal{C}(S))}{B := B', (B \cup \{[s]\})} \\
\mathbf{D - Split} \frac{s \in \mathcal{T}(S) \quad s : \text{Str} \quad B = B', B S \models \text{len } s \approx \text{len}_B[s] \notin B \quad e \in B \quad (N[s] = (w, u, u_1), Ne = (w, v, v_1), S \models \text{len } u \not\approx \text{len } v)}{S := S, u \approx \text{con}(z_1, z_2), \text{len } z_1 \approx \text{len } v \parallel S := S, v \approx \text{con}(z_1, z_2), \text{len } z_1 \approx \text{len } u}
\end{array}$$