University of Liège

Controller in time domain

Linear control systems

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Remark. This report is in the state in which we delivered it at the homework deadline. Changes and corrections were made in the final report.

1 Summary of project

Our project is to design an active mass damper in order to stabilize high buildings. The state representation of our system, previously determined, is as follows.

1.1 Inputs

- $F_1(t)$, the force of the wind (uncontrollable).
- $u(t)$, the force applied on the mass damper (controllable).

Our sensor is a measurement of the horizontal position of the top of the building relatively to the vertical position $d_1 = 0$.

Our actuator provides a force on the mass of the dampener, sets it in motion.

1.2 Outputs

$y = x_1 = d_1(t)$ the relative position of the building with respect to the vertical position.

1.3 States variables

- $x_1 = d_1$, as described above.
- $x_2 = \dot{d}_1$, the speed of the building.
- $x_3 = d_2$, the relative displacement of the mass damper.
- $x_4 = \dot{d}_2$, the speed of the mass damper.

1.4 ABCD matrices

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & \frac{-c_2-c_1}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{-k_2}{m_2} & \frac{-c_2}{m_2} \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ \frac{1}{m_1} & -\frac{1}{m_1} \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

2 State feedback controller

As the reference is 0, we need not care about k_r, so we can fix it to 0.

However, if the reference was to change, we could compute k_r, it would be nice. Some tests of a change in reference will be performed in this report.

In a first time, we only need to compute the gain matrix K.

In order not to apply a gain on the wind force, our matrix K is as follows :

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ g_1 & g_2 & g_3 & g_4 \end{pmatrix}$$

The new dynamic matrix of the closed-loop system is $A_{CL} = A - BK$. Let's determine the eigenvalues of that matrix.

As we have a matrix of dimension 4, we will make the approximation of the dominant poles. Indeed, we have, from the previous matrix A, the eigenvalues :

$$\begin{aligned} \lambda_1 &= -0.0645 + 6.2824\mathrm{i} \\ \lambda_2 &= -0.0645 - 6.2824\mathrm{i} \\ \lambda_3 &= -1.6655 + 5.5285\mathrm{i} \\ \lambda_4 &= -1.6655 - 5.5285\mathrm{i} \end{aligned}$$

We can see that λ_3 and λ_4 are about 100 times bigger than the last two, and so we do not need to work on them. Those two will therefore remain in A_{CL}.

Imposing that $(s - \lambda_3)(s - \lambda_4)$ is part of the decomposition, we get that the determinant of A_{CL} is equal to :

$$(s - \lambda_3)(s - \lambda_4)(s^2 + 2\xi\omega_c s + \omega_c^2) = 0$$

Since λ_3 and λ_4 are fixed, we only need to solve the equation of the second degree in s in order to find the expressions of λ_1 and λ_2 as a function of ξ and ω_c.

The solutions of the equation are given by :

$$\begin{cases} \lambda_1 = -\xi\omega_c - \omega_c\sqrt{\xi^2 - 1} \\ \lambda_2 = -\xi\omega_c + \omega_c\sqrt{\xi^2 - 1} \end{cases}$$

The values of ξ and ω_c will be determined by simulations in the following sections. When these have been fixed, we will obtain the values of the 4 poles of A_{CL}. Then we will just have to use the `place` function of Matlab to obtain the values g_i of matrix K associated with the eigenvalues.

3 Observer

We need to compute the gain matrix L :

$$L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{pmatrix}$$

The new dynamic matrix is given by $A_{obs} = A - LC$.

As previously, we will keep the same two dominant eigenvalues and determine the two other via the same method we have used for K.

Imposing that $(s-\lambda_3)(s-\lambda_4)$ is part of the decomposition, we get that the determinant of A_{obs} is equal to :

$$(s-\lambda_3)(s-\lambda_4)(s^2+2\xi\omega_c s+\omega_c^2)=0$$

Since λ_3 and λ_4 are fixed, we only need to solve the equation of the second degree in s in order to find the expressions of λ_1 and λ_2 as a function of ξ and ω_c.

The solutions of the equation are given by :

$$\begin{cases} \lambda_1 = -\xi\omega_c - \omega_c\sqrt{\xi^2-1} \\ \lambda_2 = -\xi\omega_c + \omega_c\sqrt{\xi^2-1} \end{cases}$$

The poles of the observer are determined by taking the poles of the controller and moving them. To do this, the real parts of each pole are multiplied by a constant α. In the case of poles λ_1 and λ_2, this amounts to multiplying w_c by α.

We finally have :

$$\begin{cases} \lambda_1 = -\xi\omega_c\alpha - \omega_c\alpha\sqrt{\xi^2-1} \\ \lambda_2 = -\xi\omega_c\alpha + \omega_c\alpha\sqrt{\xi^2-1} \\ \lambda_3 = \mathbb{R}(\lambda_3)\alpha + \mathbb{I}(\lambda_3)i \\ \lambda_4 = \mathbb{R}(\lambda_4)\alpha + \mathbb{I}(\lambda_4)i \end{cases}$$

We then obtain the values l_i of the matrix L by using the `place` function of Matlab.

4 Constraints and simulations specifications

The numerical values used for the simulations are identical to those used previously (homework 2).

5 Simulations and discussion

Through several tests, we have determined the following values to obtain acceptable results :

$$\begin{cases} \xi = 0.8 \\ \omega_c = 5 \\ \alpha = 5 \end{cases}$$

5.1 Response to a reference variation

The new reference has been set at $0.002\,\mathrm{m}$ for simulations.

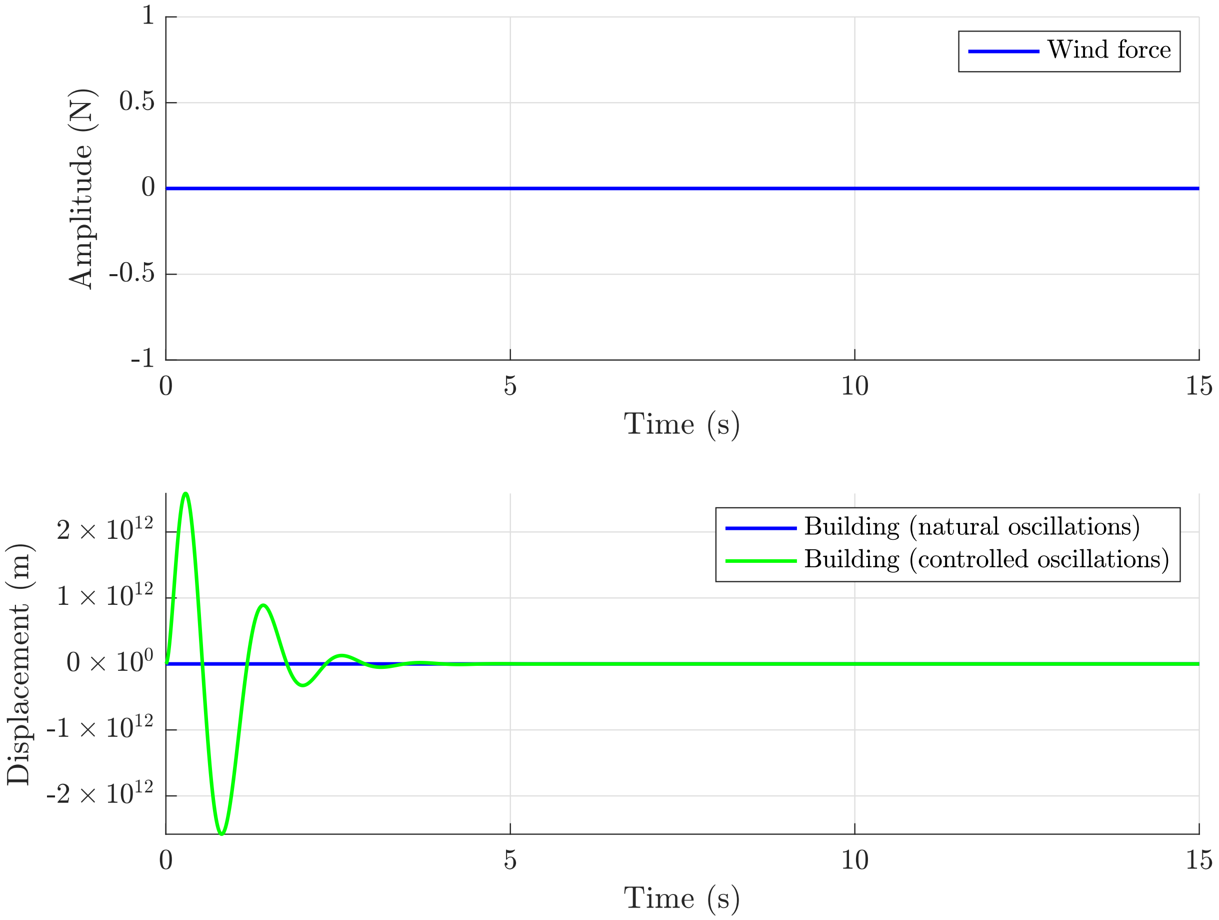

Figure 1 – Response to a reference variation - controlled output

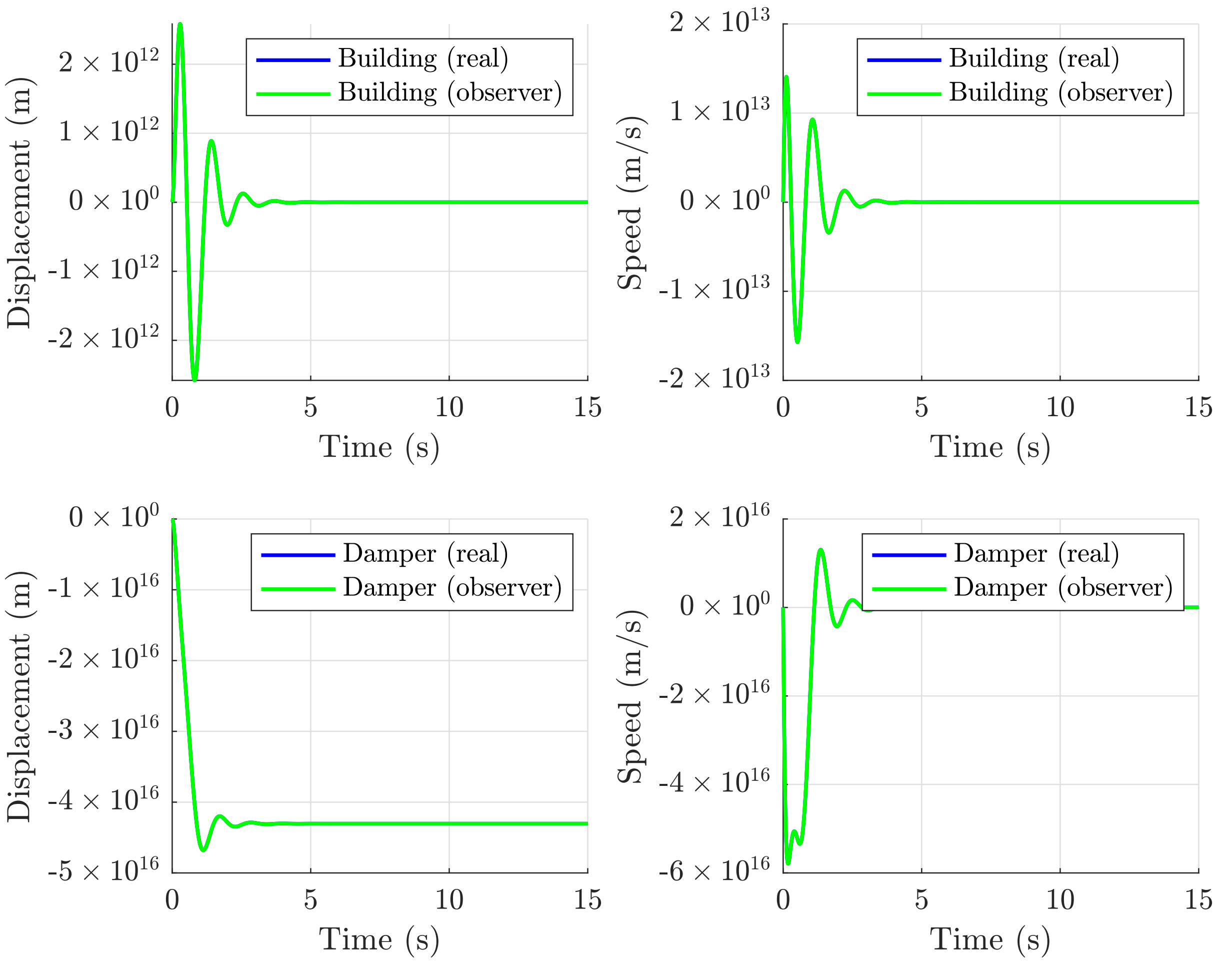

Figure 2 – Response to a reference variation - states

5.2 Response to a perturbation (disturbance)

For a constant wind force, we get :

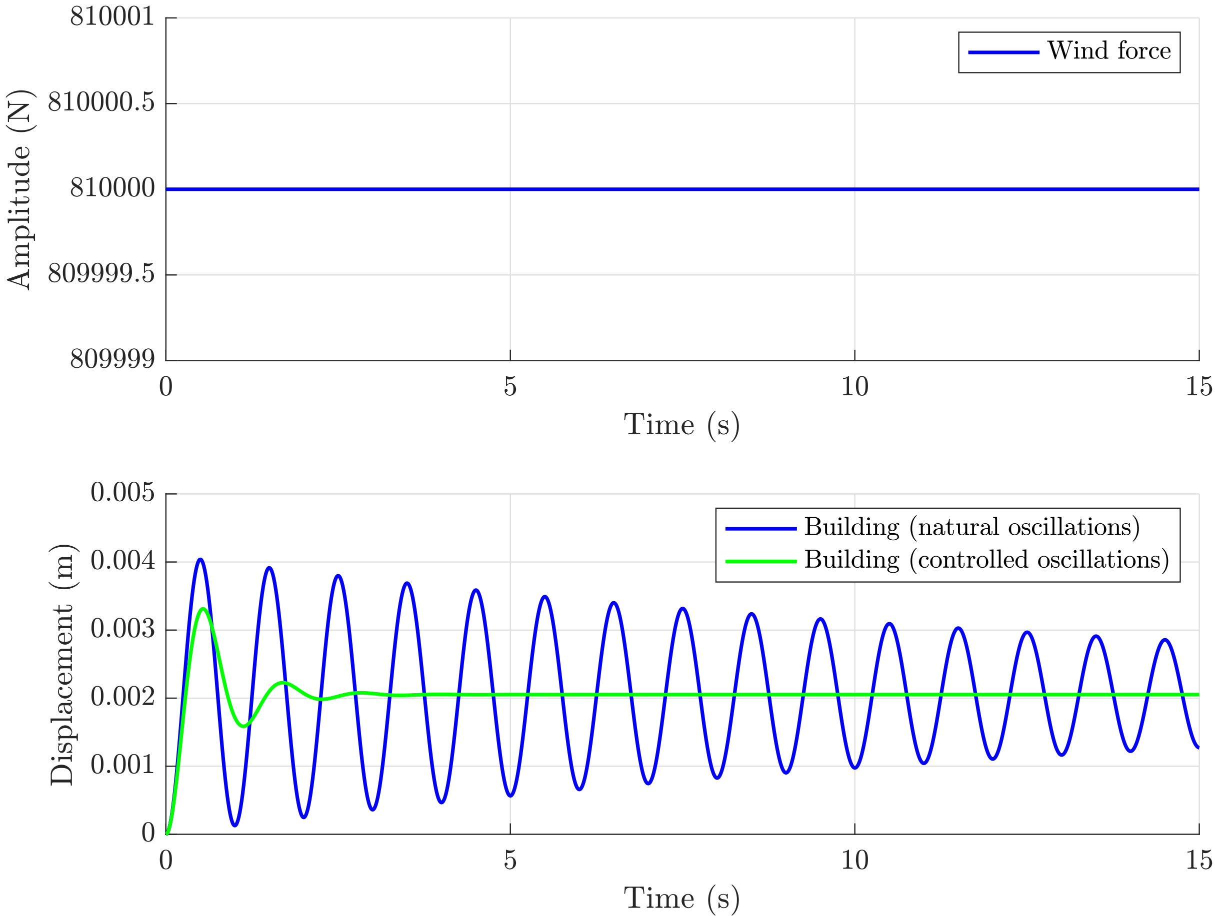

Figure 3 – Response to a constant wind force - controlled output

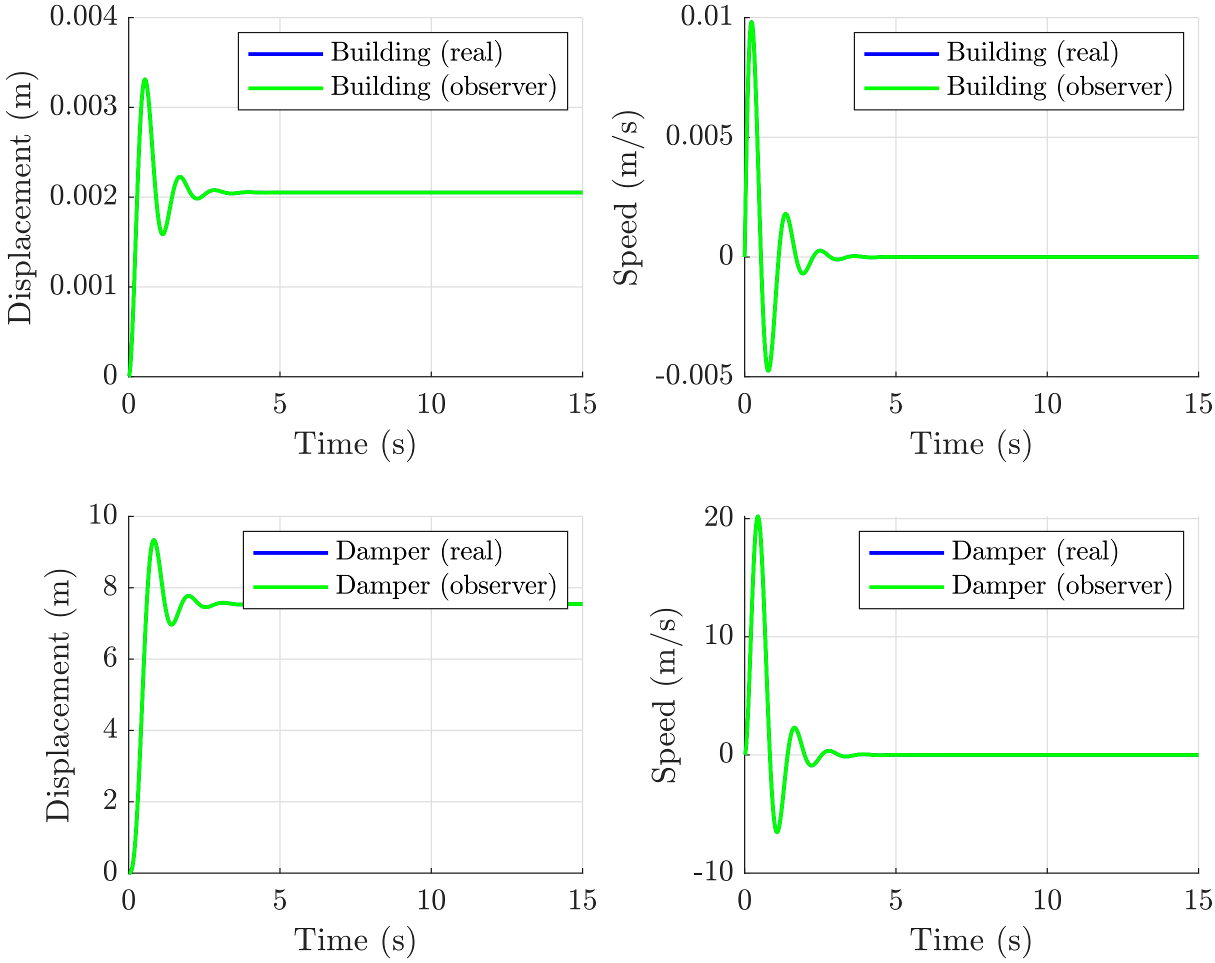

Figure 4 – Response to a constant wind force - states

For a sinusoidal wind force, we get :

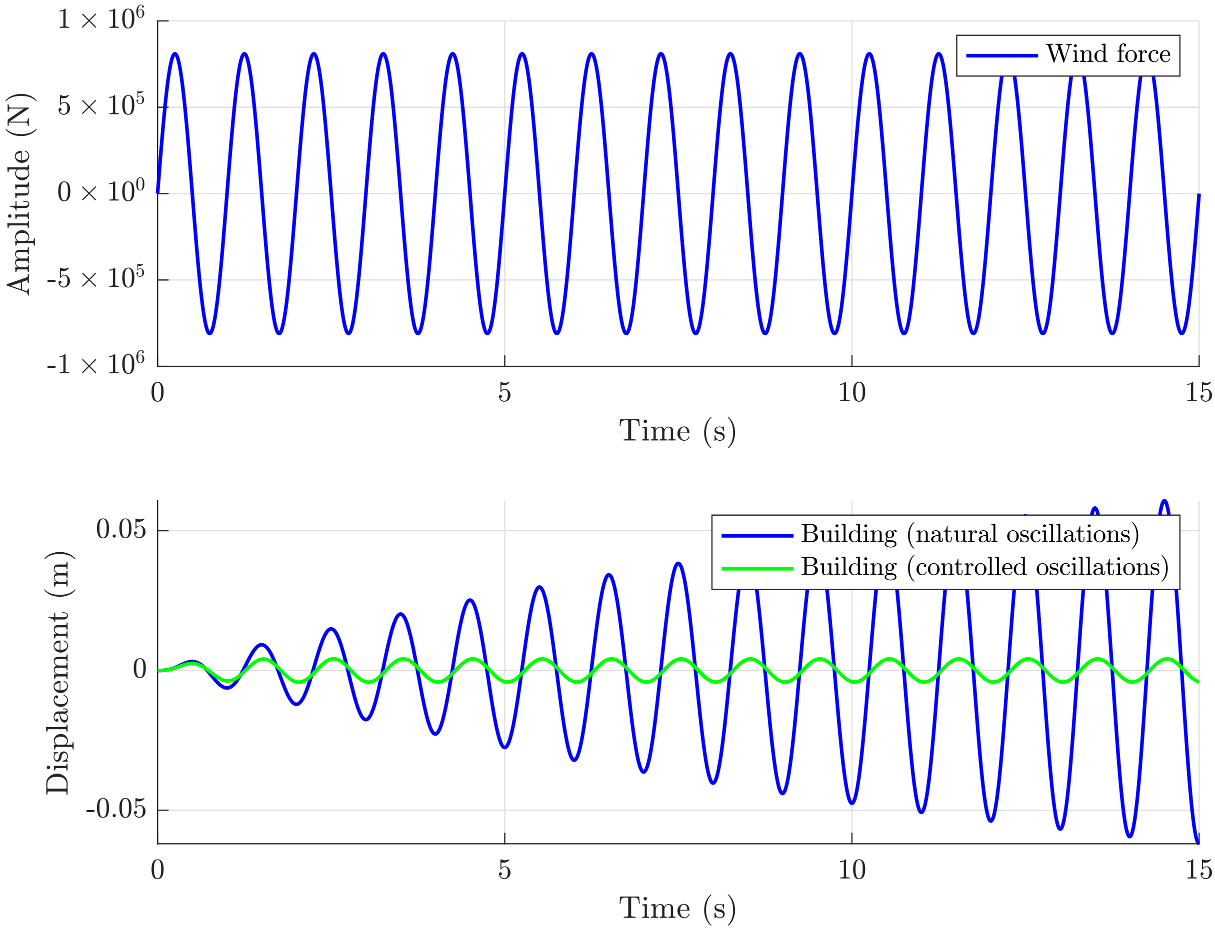

Figure 5 – Response to a sinusoidal wind force - controlled output

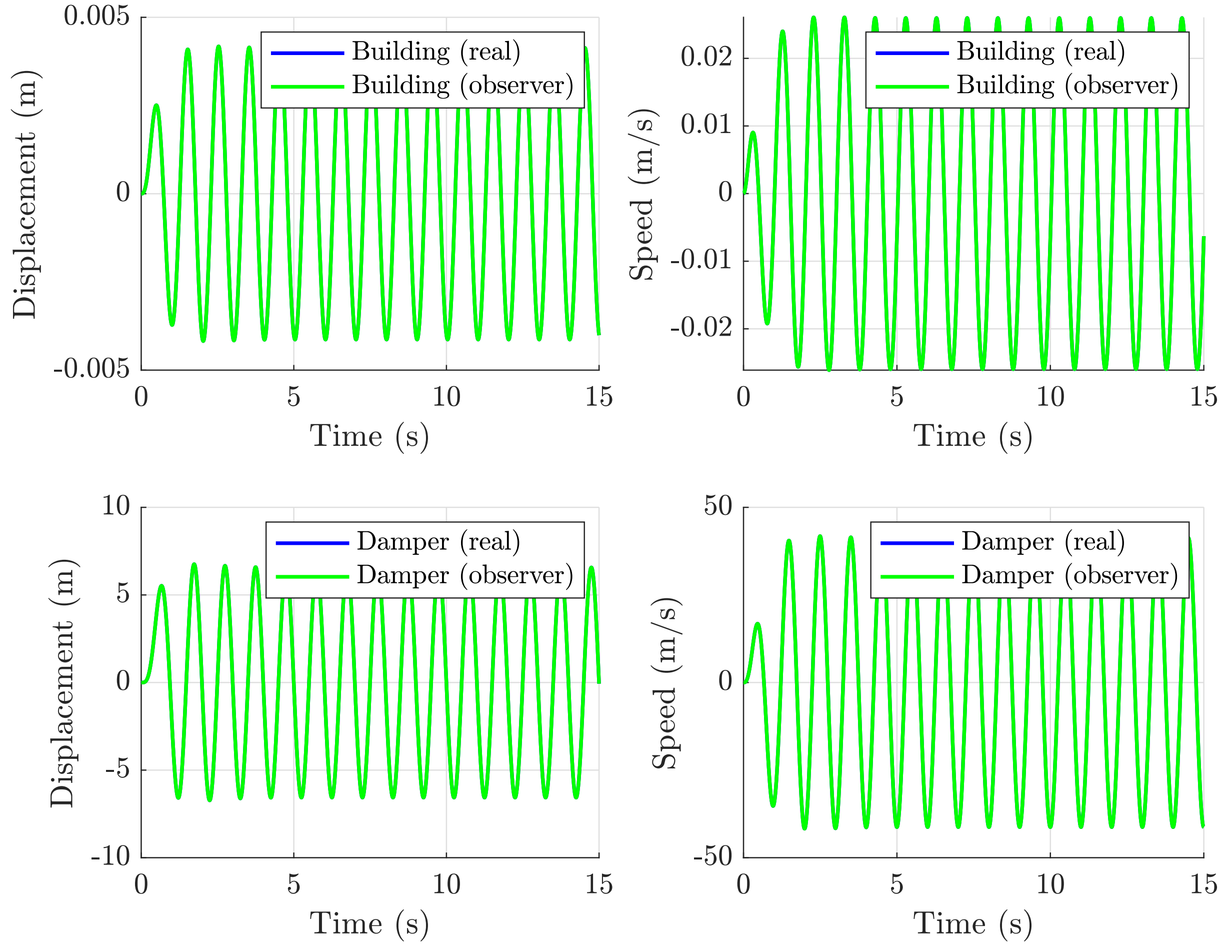

Figure 6 – Response to a sinusoidal wind force - states

5.3 Presence of noise

to do

5.4 Impact of delays

to do