University of Liège

Open Loop System

Linear control systems

Bastien Hoffmann (20161283)
Maxime Meurisse (20161278)
Valentin Vermeylen (20162864)

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1 Detailed schematic of the open loop system

The detailed schematic of the open loop studied system is shown in figure 1.

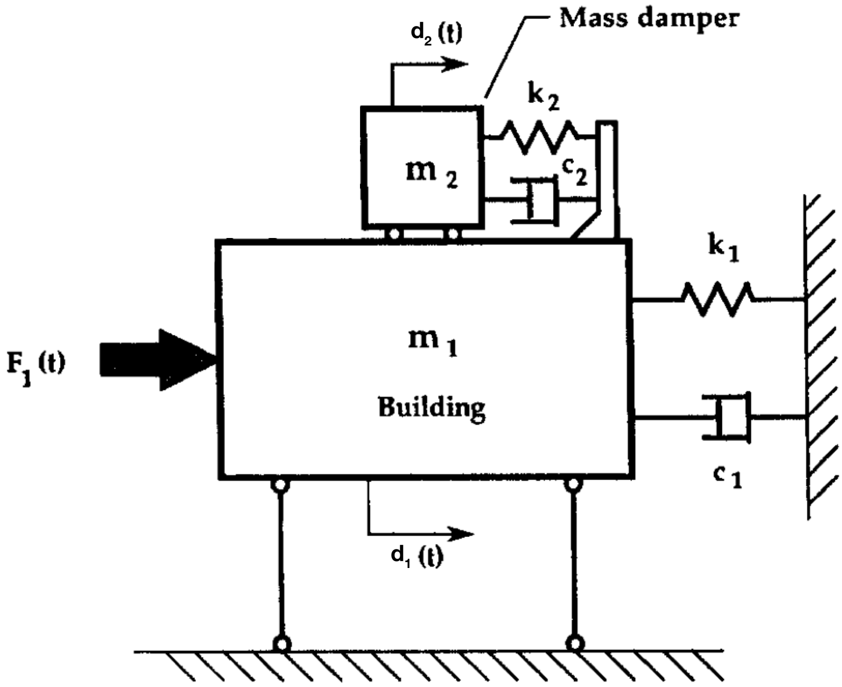


Figure 1 – Detailed schematic of the open loop studied system [1]

The building is represented by the mass m_1 and its oscillation motion is simulated by the spring k_1 and the damper c_1.

The mass damper is represented by the mass m_2 and its movement is simulated by the spring k_2 and the damper c_2.

The force $F_1(t)$ represents the wind force (uncontrollable) on the building.

2 Constraints, assumptions, limitations

To model and study the system, we defined a series of constraints, assumptions and limitations, presented in table 1.

Building	height of 250 m, width of 40 m movement along a single axis (horizontal)
Wind force	max intensity of 7.35 MN
Mass	no friction between m_1 and m_2

Table 1 – Constraints, assumptions and limitations of the system.

3 State-space representation

Input vector U and state vector X are given by :

$$U = \begin{pmatrix} F_1 \end{pmatrix} \qquad X = \begin{pmatrix} d_1 \\ \dot{d}_1 \\ d_2 \\ \dot{d}_2 \end{pmatrix}$$

3.1 Inputs

$F_1(t)$, the force of the wind (uncontrollable).

3.2 Outputs

$y = d_1(t)$ the relative position of the building with respect to the vertical position.

3.3 States

- $x_1 = d_1$, as described above.
- $x_2 = \dot{d}_1$, the speed of the building.
- $x_3 = d_2$, the absolute displacement of the mass damper.
- $x_4 = \dot{d}_2$, the speed of the mass damper.

3.4 Output law

The output is one of the states : $y = x_1$.

3.5 Input law

The input law is given by [1] :

$$\begin{aligned} m_1\ddot{d}_1 + c_1\dot{d}_1 + k_1 d_1 &= c_2\dot{z} + k_2 z + F_1(t) \\ m_2\ddot{z} + c_2\dot{z} + k_2 z &= -m_2\ddot{d}_1 \end{aligned}$$

with $z = d_2 - d_1$.

The system is **linear**. We can easily derive the ABCD matrices.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & \frac{-c_2-c_1}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{-k_2}{m_2} & \frac{-c_2}{m_2} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 \end{pmatrix}$$

4 System simulations without controller

To simulate the system, we choose a series of numerical values, presented in table 2 [2].

Mass	$m_1 = 20\,000\,\mathrm{t}$	$m_2 = 10\,\mathrm{t}$
Spring	$k_1 = 1500\,\mathrm{MN/m}$	$k_2 = 3.5\,\mathrm{MN/m}$
Damper	$c_1 = 30\,\mathrm{MNs/m}$	$c_2 = 1\,\mathrm{MNs/m}$

Table 2 – Numerical values of the system

For the strength of the wind, we considered 2 cases (in newton) :

$$F_1 = 7\,350\,000 \qquad \text{Constant wind force}$$
$$F_1(t) = 3\,675\,000\sin(2\pi t) + 3\,675\,000 \qquad \text{Sinusoidal wind force}$$

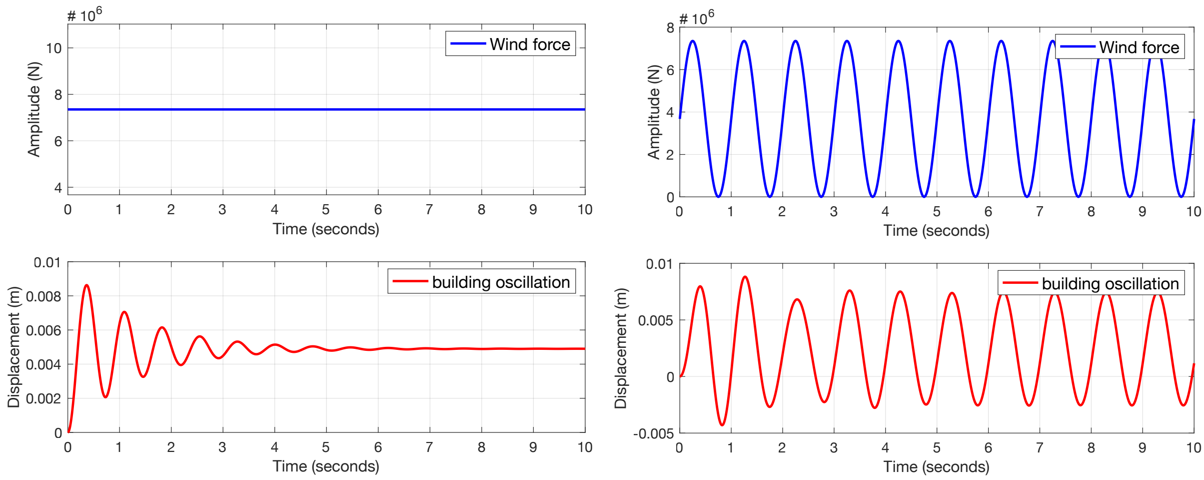
These values have been approximated via

$$F = \frac{1}{2}\rho v^2 A$$

with

- ρ, the air density;
- v, the wind speed;
- A, the area of one side of the building.

Remark We used SI units except for the newton we left as such.



(a) Constant wind force (b) Sinusoidal wind force

Figure 2 – Linear simulations results

In both cases, we observe that the building oscillates slightly at first and then moves according to the wind (in the first case, the building stabilises in an unbalanced position, and in the second case, the building oscillates steadily).

5 State-space representation analysis

5.1 Stability

To study the stability of the system, we compute the eigenvalues of the dynamic matrix A thanks to Matlab function (`eig`).

The system is stable if the real parts of the eigenvalue are all negative. In the case of the studied system, we obtain the following eigenvalues :

$$\lambda_i = \begin{bmatrix} -96.4185 \\ -0.7498 + 8.6255\mathrm{i} \\ -0.7498 - 8.6255\mathrm{i} \\ -3.6319 \end{bmatrix}$$

Given these values, the system is stable.

5.2 Observability

To determine whether or not the system is observable, we compute the observability matrix thanks to Matlab function (`obsv`) :

$$Ob = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -75.1750 & -1.5500 & 0.1750 & 0.05 \\ 134.0213 & -67.7725 & -17.7712 & -4.9025 \end{pmatrix}$$

This matrix is full rank (verified with Matlab), the system is thus fully observable.

5.3 Controllability

To determine whether or not the system is controllable, we compute the controllable matrix thanks to Matlab function (`ctrb`) :

$$Co = 1 \times 10^{-8} \begin{pmatrix} 0 & 5 & -7.75 & -338.862 \\ 5 & -7.75 & -338.862 & -1255.906 \\ 0 & 0 & 500 & -49\,024.99 \\ 0 & 500 & -49\,024.99 & 4\,690\,900 \end{pmatrix}$$

This matrix is full rank (verified with Matlab), the system is thus fully controllable.

6 Reference

[1] *Parametric study of active mass dampers for wind-excited tall buildings*. https://www.sciencedirect.com/science/article/pii/0141029695001088. Accessed : 2019-10-10.

[2] *Estimation of optimum tuned mass damper parameters via machine learning*. https://www.sciencedirect.com/science/article/pii/S235271021930364X. Accessed : 2019-10-10.