University of Liège

Study of an active mass damper

Linear control systems

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Contents

1 Control problem

1.1 Choice of the topic

The chosen topic is : **Active mass damper**.

1.2 Context

The current engineering prowesses allow us to construct buildings higher and higher. These constructions are subject to various disturbances (mainly wind, but also earthquakes) that make them oscillate. They turn into giant pendulum and swing from left to right, sometimes moving several meters at the top ! [1]

To reduce these oscillations, one uses a passive system, called *tuned mass damper*, which consists of concealing a tuned and harmonic oscillator at the top of the tower. It is coupled to its movement and oscillates in phase opposition to recover the kinetic energy of the tower and thus reduces the oscillations. [2]

An active version of this system exists : the *active mass damper*. It consists of the same principle as the tuned mass damper but it is equipped with sensors and actuators to measure the oscillations of its environment and, via an algorithm, generate a movement for the mass that reduce, or totally remove, these oscillations. [3]

The study field of this project focuses on the active mass damper systems used to reduce the oscillations caused by the **wind** on **tall** buildings.

1.3 Control problem diagram

The diagram of our control problem is shown in figure 1.

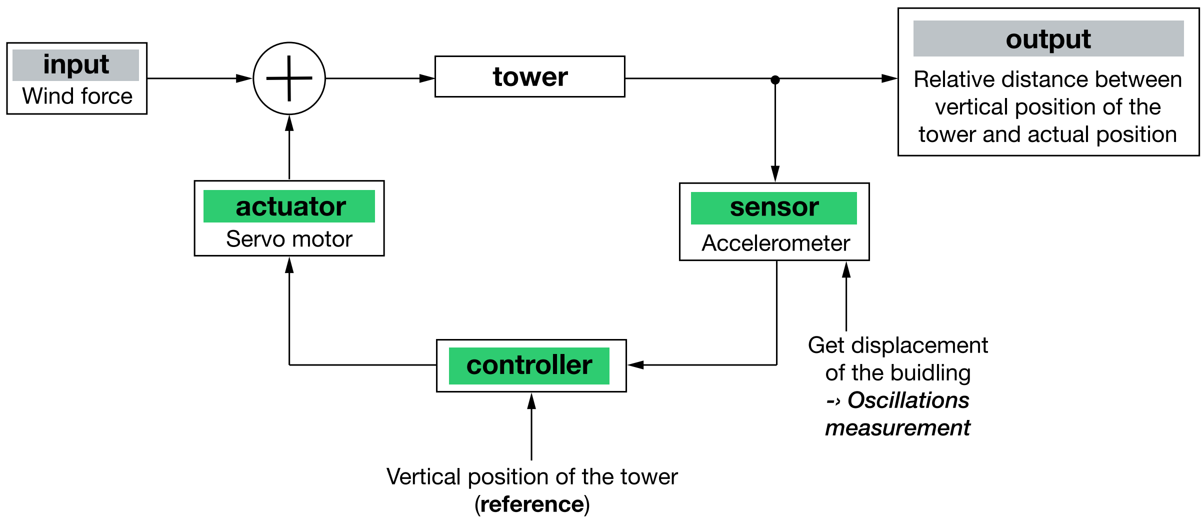

Figure 1 – Control problem diagram of the active mass damper for tall buildings

1.4 Control problem description

- **Utility of the controller** : the controller (the algorithm) allows the system (the tower) to be active, *i.e.* to measure the oscillations to which it is subjected and to cancel it. Thanks to a servo-motor connected to the controller, the mass can move and reduce, or even eliminate totally, the oscillations.
- **System to be controlled** : the tower (and the position of the tower is the signal)
- **Inputs of the system** : wind force acting on the tower (uncontrollable) and force acting on the mass damper (controllable).
- **Outputs of the system** : the relative distance between the vertical position and the displacement of the tower.
- **Reference** : the vertical position of the tower.
- **Actuators** : servo-motor to move the mass that reduces the oscillations.
- **Constraints and limitations** : to simplify our system, we consider a tower 200 m high, perfectly vertical when it undergoes no disturbance. The only disturbance on this tower is the strength of the wind. The wind, ranging from a few tens of km/h to a hundred km/h, can swing the tower from a few centimetres to several meters.

1.5 Open loop system diagram

The detailed schematic of the open loop studied system is shown in figure 2.

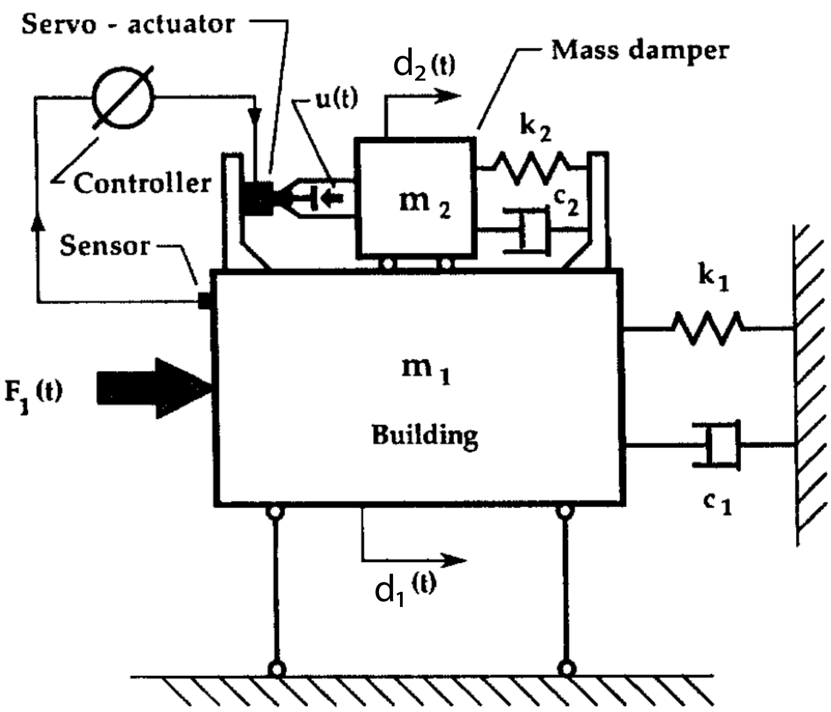

Figure 2 – Detailed schematic of the open loop studied system [3]

The building is represented by the mass m_1 and its oscillation motion is simulated by the spring k_1 and the damper c_1.

The mass damper is represented by the mass m_2 and its movement is simulated by the spring k_2 and the damper c_2.

The force $F_1(t)$ represents the wind force (uncontrollable) on the building.

The force $u(t)$ represents the force applied on the mass damper by the controller (controllable).

At first, the system is studied without a control mechanism. The controllable input $u(t)$ will therefore be 0 for all our simulations in this section.

2 State-space representation

2.1 Open loop model description

Input vector U and state vector X are given by :

$$U = \begin{pmatrix} F_1 \\ u \end{pmatrix} \qquad\qquad X = \begin{pmatrix} d_1 \\ \dot{d}_1 \\ d_2 \\ \dot{d}_2 \end{pmatrix}$$

2.1.1 Inputs

- $F_1(t)$, the force of the wind (uncontrollable), between 500 and 1500 kN (which corresponds to a wind speed of 10 to 15 $\mathrm{m\,s^{-1}}$ on our building).
- $u(t)$, the force applied on the mass damper (controllable), in the order of 1000 kN.

The sensor considered is a measurement of the horizontal position of the top of the building relatively to the vertical position $d_1 = 0$.

The actuator provides a force on the mass of the dampener ($u(t)$), sets it in motion.

2.1.2 Outputs

$y = d_1(t)$ the relative position of the building with respect to the vertical position.

2.1.3 States

- $x_1 = d_1$, as described above, ranging from a few millimeters to a few meters.
- $x_2 = \dot{d}_1$, the speed of the building, ranging from about 0.1 to 5 $\mathrm{m\,s^{-1}}$.
- $x_3 = d_2$, the relative displacement of the mass damper, ranging from a few millimeters to a few meters.
- $x_4 = \dot{d}_2$, the speed of the mass damper, ranging from about 0.1 to 10 $\mathrm{m\,s^{-1}}$.

2.1.4 Output law

The output is one of the states : $y = x_1$.

2.1.5 Update law

The update law is given by [3] :

$$\begin{cases} m_1\ddot{d}_1 + c_1\dot{d}_1 + k_1 d_1 = c_2\dot{z} + k_2 z + F_1(t) - u(t) \\ m_2\ddot{z} + c_2\dot{z} + k_2 z = -m_2\ddot{d}_1 + u(t) \end{cases}$$

with $z = d_2 - d_1$.

2.2 State-space model

The system is **linear**. The ABCD matrices can be easily derived.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & \frac{-c_2-c_1}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{-k_2}{m_2} & \frac{-c_2}{m_2} \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ \frac{1}{m_1} & -\frac{1}{m_1} \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

2.3 Constraints, limitations and numerical choice of parameter values

To model and study the system, a series of constraints have to be defined, as well as assumptions and limitations.

2.3.1 Basic system constraints

The basic modeling constraints of our system are presented in table 1.

Building	height of 200 m, width of 30 m movement along a single axis (horizontal)
Mass	no friction between m_1 and m_2

Table 1 – Basic system constraints

2.3.2 Constraints on signals

The constraints on the different signals of our system are presented in table 2.

Reference	a few millimeters or even a few centimeters at most
Controllable input	in the order of 1000 kN
Uncontrollable input	between 500 and 1500 kN
Output	from a few millimeters to a few meters

Table 2 – Constraints on signals

2.3.3 Numerical values for simulations

To simulate the system (without control mechanism), a series of numerical values, presented in table 3[1] were chosen.

Mass	$m_1 = 1 \times 10^7\,\mathrm{kg}$	$m_2 = 3 \times 10^4\,\mathrm{kg}$
Spring	$k_1 \approx 4 \times 10^8\,\mathrm{N/m}$	$k_2 = 10^5\,\mathrm{N/m}$
Damper	$c_1 \approx 1.3 \times 10^6\,\mathrm{Ns/m}$	$c_2 = 10^4\,\mathrm{Ns/m}$
Wind	$F_{max} = 810\,000\,\mathrm{N}$	

Table 3 – Numerical values of the system

The stiffness and viscosity values for the building were obtained using the formulas :

$$k_1 = (2\pi f)^2 m_1$$
$$c_1 = m_1 \pi f 0.04$$

where $f = 1\,\mathrm{Hz}$ is the natural frequency associated with the mass of the building.

The maximum wind force, on the other hand, was approximated by

$$F_{max} = \frac{1}{2}\rho v^2 A$$

with

- $\rho \approx 1.2\,\mathrm{kg/m^3}$, the air density;
- $v = 15\,\mathrm{m/s}$, the wind speed;
- $A = 200 \times 30 = 6000\,\mathrm{m^2}$, the area of one side of the building.

Scenarios considered For the uncontrollable input (wind), multiple scenarios were considered :

$$F_1 = F_{max} \quad \forall t \qquad \text{Constant wind force}$$
$$F_1(t) = F_{max}\sin(2\pi t) \qquad \text{Sinusoidal wind force}$$
$$F_1(t) = F_{max}\texttt{rand()} \qquad \text{Random wind force}$$

[1] We would like to thank Professor Denoël for discussing these values with us.

A constant force over time is not very realistic for wind but allows us to observe the behaviour of our system in the face of a very simple entry.

A sinusoidal force is also not very realistic but could correspond to multiple wind gusts on either side of the building.

A random wind is a rather realistic scenario that can model multiple wind gusts of varying intensity on either side of the building.

2.4 Stability and eigenvalues

To study the stability of the system, one computes the eigenvalues of the dynamic matrix A thanks to Matlab function (`eig`) :

$$\begin{aligned}\lambda_1 &= -0.0634 + 6.2837\mathrm{i}\\ \lambda_2 &= -0.0634 - 6.2837\mathrm{i}\\ \lambda_3 &= -0.1666 + 1.8179\mathrm{i}\\ \lambda_4 &= -0.1666 - 1.8179\mathrm{i}\end{aligned}$$

The system is stable if the real parts of the eigenvalue are all negative. In this case, the system is thus stable.

We notice that λ_1 and λ_2, being closer to the imaginary axis than λ_3 and λ_4, are two dominant eigenvalues. They therefore govern the dynamics of the system.

We also note that λ_3 and λ_4 are also very close to the imaginary axis. So our system is very responsive.

2.5 Open loop system simulations

We simulated during 30 seconds, in open loop, the different scenarios presented in section 2.3.3. We also varied the initial conditions. Since we have not yet implemented a control mechanism, the controllable input of the system is at 0 for all simulations.

2.5.1 Initial conditions

For this simulation, we changed the initial conditions of our system without applying wind force. We have defined that the initial displacement of our building (state x_1) is 0.003 m.

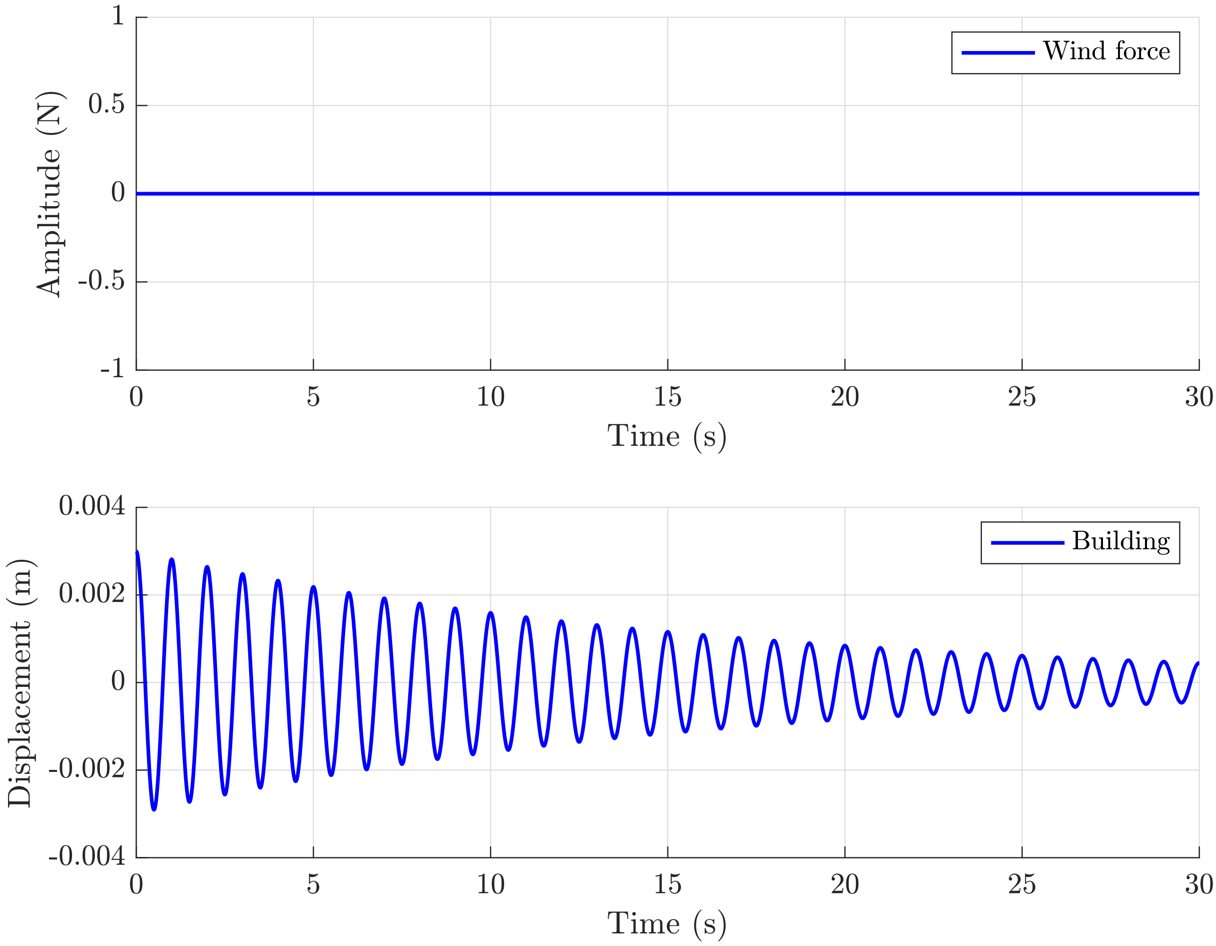

Figure 3 – Open loop simulation during 30 s with initial conditions

We can see that our system is very reactive: the building oscillates quite quickly. He very quickly regains his reference position.

2.5.2 Constant wind force

For this simulation, we applied a constant wind with zero initial conditions.

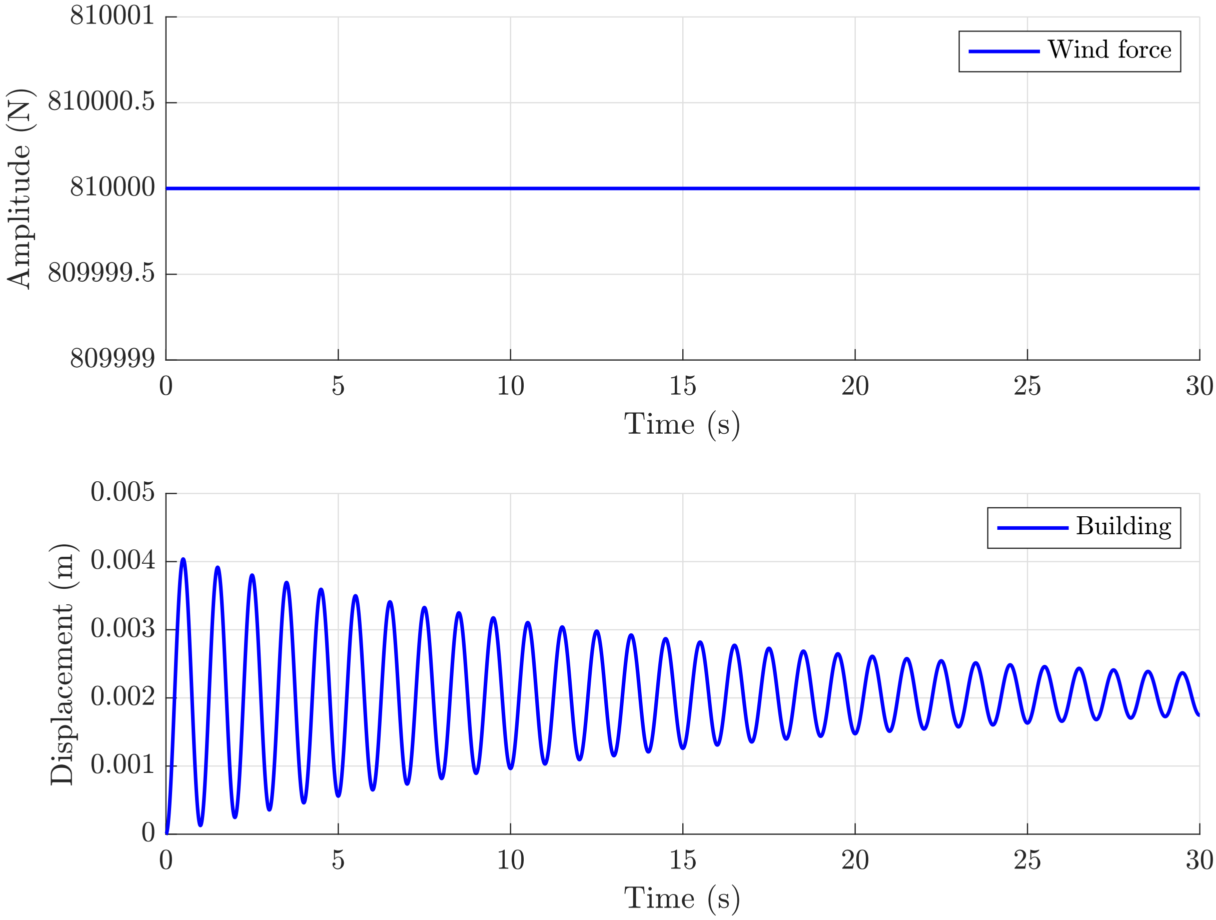

Figure 4 – Open loop simulation during 30 s for a constant wind force

In this case, we can see that the building also oscillates very quickly. These oscillations decrease quickly enough over time to finally stabilize at a position slightly different from its reference position.

2.5.3 Sinusoidal wind force

For this simulation, we applied a sinusoidal wind with zero initial conditions.

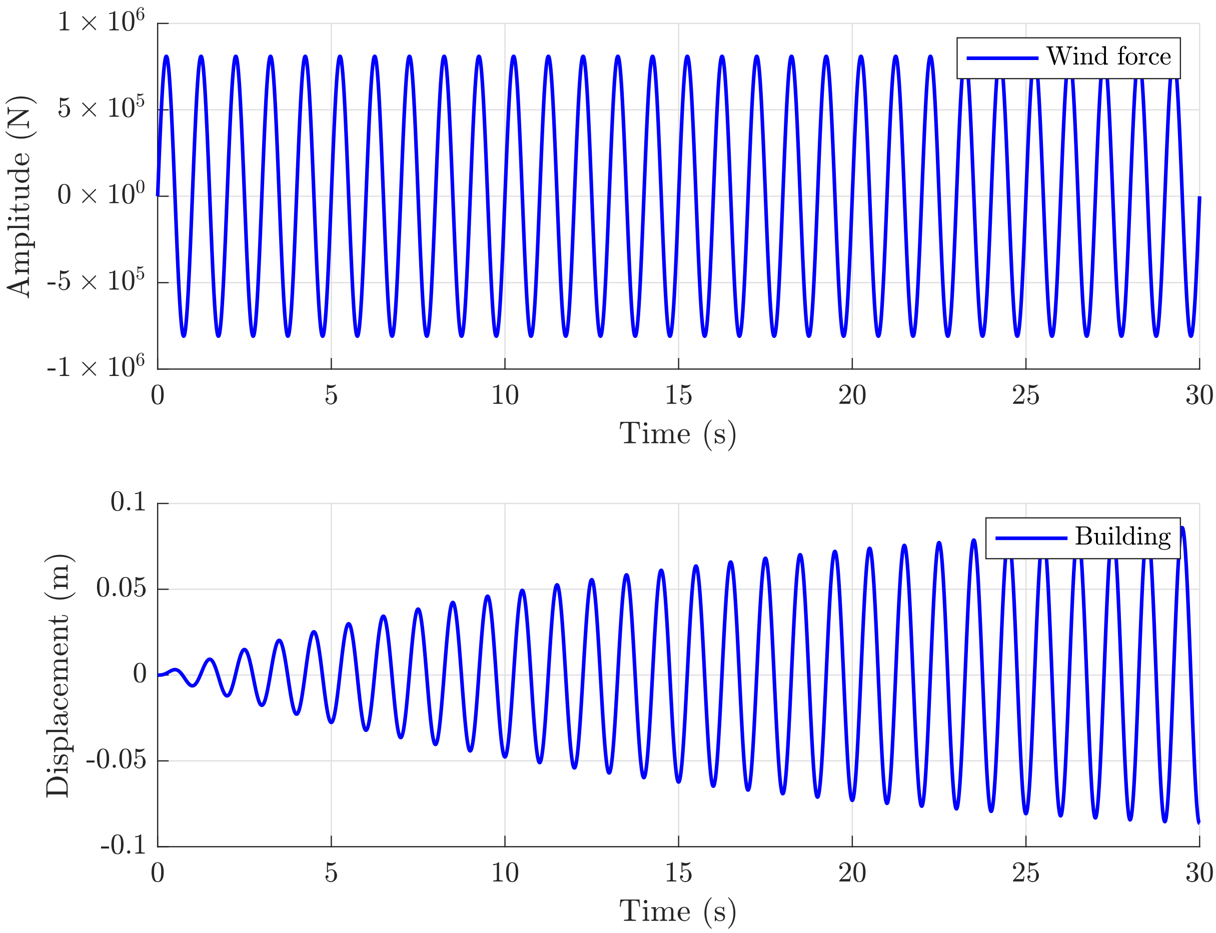

Figure 5 – Open loop simulation during 30 s for a sinusoidal wind force

We can see that the building's oscillations gradually increase, but quite rapidly, over time. A longer simulation has shown that these oscillations stabilize at a maximum value of 0.1 m after about 30 seconds.

2.5.4 Random wind force

For this simulation, we applied a random wind with zero initial conditions.

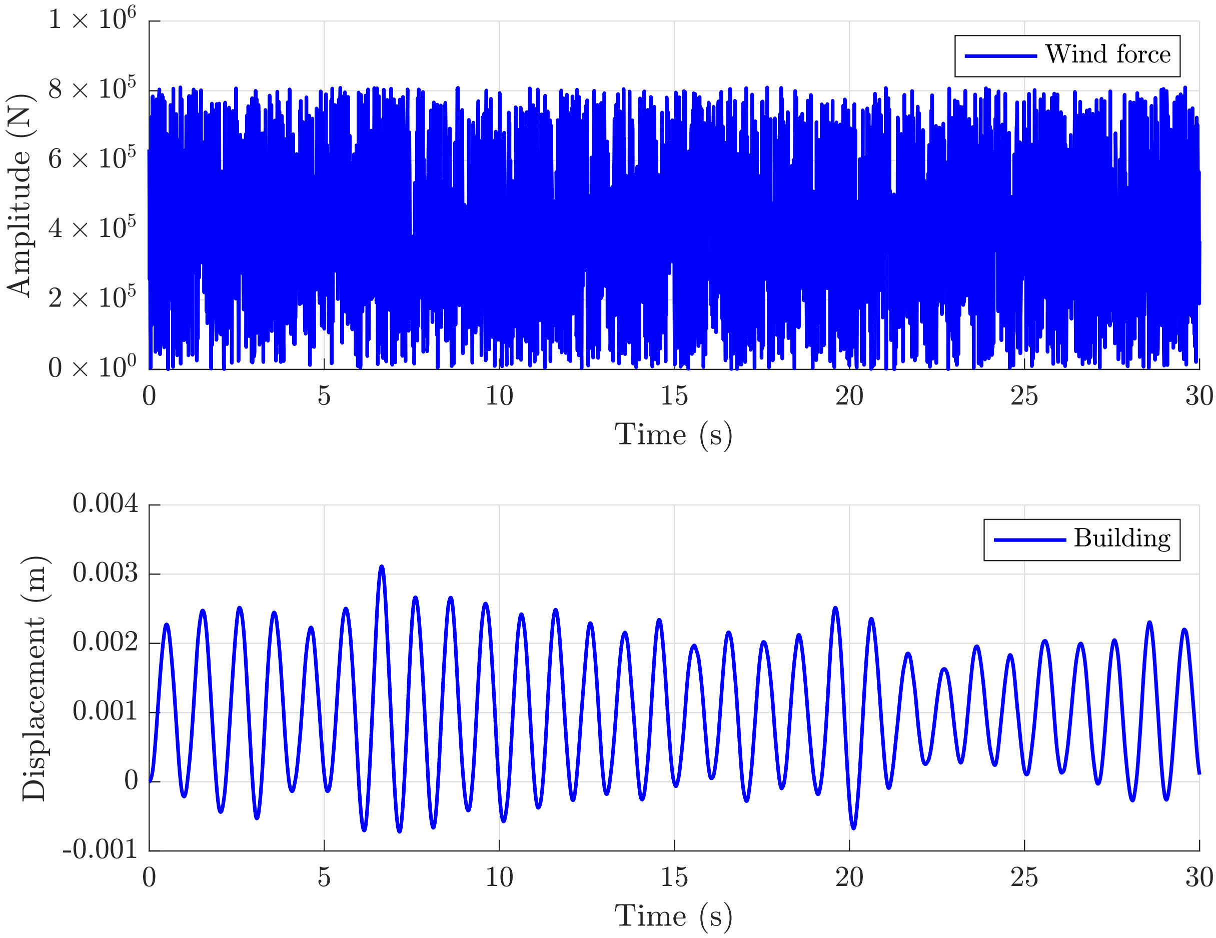

Figure 6 – Open loop simulation during 30 s for a random wind force

It can be seen that the building oscillates more or less identically and rapidly over time around a position slightly offset from its reference position.

2.5.5 Relation with the eigenvalues and utility of a controller

As mentioned in section 2.4, the eigenvalues of our system suggest that it is very reactive. Indeed, this has been observed on the different simulations : the building oscillates very quickly.

Eigenvalues also indicate that the system is stable. Again, this information could be found on the simulations : for a variation in initial conditions, or when the wind force is constant (or decreasing), the system tends to return to its reference position.

Although the movements of our building are quite small, a controller could reduce them even more, or even eliminate them completely after a while. Another role of the controller could be to slow down our system so that the building oscillates less quickly.

2.6 Observability

To determine whether or not the system is observable, one computes the observability matrix thanks to Matlab function (`obsv`).

The matrix is full rank (verified with Matlab), the system is thus fully observable.

As seen on the matrix C, one needs one sensor. According to the place of the non zero value, this sensor has to measure the x_1 state, namely the horizontal position of the top of the building d_1. This state is indeed the objective of the active mass damper and has thus to be observed. The sensor chosen is an accelerometer.

2.7 Controllability

To determine whether or not the system is controllable, one computes the controllable matrix thanks to Matlab function (`ctrb`). In order not to take into account the uncontrollable input (wind), only the second column of the B matrix was kept for the calculation.

The matrix is full rank (verified with Matlab), the system is thus fully controllable.

As seen on matrix B, one needs only one actuator. The first column of the B matrix represents the wind, while the second one concerns the damper. This latter is indeed the only controllable input and contains two non-zero elements. As a result, only one actuator is needed, and acts on two states, the speed of the building and the speed of the damper, as they take place on x_2 and x_4. The actuator chosen is a servo motor.

3 Controller in time domain

3.1 State feedback controller

In a first time, one needs to compute the gain matrix K.

In order not to apply a gain on the wind force, the matrix K is as follows :

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ g_1 & g_2 & g_3 & g_4 \end{pmatrix}$$

Indeed, the first column of matrix B concerns the uncontrollable input, so matrix K cannot affect these values.

The new dynamic matrix of the closed-loop system is $A_{CL} = A - BK$. Let's determine the eigenvalues of that matrix.

As one has a matrix of dimension 4, the approximation of the dominant poles will be done. Indeed, one has, from the previous matrix A, the eigenvalues :

$$\begin{aligned} \lambda_1 &= -0.0634 + 6.2837\text{i} \\ \lambda_2 &= -0.0634 - 6.2837\text{i} \\ \lambda_3 &= -0.1666 + 1.8179\text{i} \\ \lambda_4 &= -0.1666 - 1.8179\text{i} \end{aligned}$$

As already discussed in section 2.4, one can see that λ_3 and λ_4 are about 10 times bigger than the last two, and so they do not require any modification. Those two will therefore remain in A_{CL}.

Imposing that $(s-\lambda_3)(s-\lambda_4)$ is part of the decomposition, one gets that the determinant of A_{CL} is equal to :

$$(s-\lambda_3)(s-\lambda_4)(s^2+2\xi\omega_c s+\omega_c^2)=0$$

Since λ_3 and λ_4 are fixed, one only needs to solve the equation of the second degree in s in order to find the expressions of λ_1' and λ_2' as a function of ξ and ω_c.

The solutions of the equation are given by :

$$\begin{cases}\lambda_1' = -\xi\omega_c - \omega_c\sqrt{\xi^2-1}\\ \lambda_2' = -\xi\omega_c + \omega_c\sqrt{\xi^2-1}\end{cases}$$

The values of ξ and ω_c will be determined by simulations in the following sections. When these have been fixed, the values of the 4 poles of A_{CL} will be obtained. Then one will just have to use the `place` function of Matlab to obtain the values g_i of matrix K associated with the eigenvalues.

However, as previously noted via simulations, our system is very reactive. The two eigenvalues have x and y have also been modified to slow down the system. We finally obtain the expressions of the 4 eigenvalues of the controller :

$$\begin{cases}\lambda_1' = -\xi\omega_c - \omega_c\sqrt{\xi^2-1}\\ \lambda_2' = -\xi\omega_c + \omega_c\sqrt{\xi^2-1}\\ \lambda_3' = \mathbb{R}(\lambda_3)0.5+\mathbb{I}(\lambda_3)i\\ \lambda_4' = \mathbb{R}(\lambda_4)0.5+\mathbb{I}(\lambda_3)i\end{cases}$$

As the reference is 0, k_r has not to be considered, so it can fixed to 0.

However, if the reference was to change, one could compute k_r, it would be nice. Some tests of a change in reference will be performed in this report. We therefore calculated k_r using the following formula :

$$k_r = \frac{-1}{C(A-BK)^{-}1B}$$

where only the controllable part of matrix B (second column) was considered. Indeed, if the entire matrix were used, it would mean that we would have an action on the uncontrollable input, which is not possible.

3.2 Observer

The controllable input being a linear combination of the different states multiplied by gains, the control system requires the different states as inputs. However, the open loop system only provides one output.

Considering that the real states cannot be measured in practice, the observer is a tool that allows, based on the output of the open loop system only, to approximate the different states of the system to control it.

Its realization must be such that the convergence of the estimated states with the real states is as fast and correct as possible. One thus needs to compute the gain matrix L :

$$L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{pmatrix}$$

The new dynamic matrix is given by $A_{obs} = A - LC$.

As previously, one will keep the same two dominant eigenvalues and determine the two other via the same method that has been used for K.

Imposing that $(s-\lambda_3')(s-\lambda_4')$ is part of the decomposition, one gets that the determinant of A_{obs} is equal to :

$$(s-\lambda_3')(s-\lambda_4')(s^2+2\xi\omega_c s+\omega_c^2)=0$$

Since λ_3' and λ_4' are fixed, one only needs to solve the equation of the second degree in s in order to find the expressions of λ_1* and λ_2* as a function of ξ and ω_c.

The solutions of the equation are given by :

$$\begin{cases} \lambda_1* = -\xi\omega_c - \omega_c\sqrt{\xi^2-1} \\ \lambda_2* = -\xi\omega_c + \omega_c\sqrt{\xi^2-1} \end{cases}$$

The poles of the observer are determined by taking the poles of the controller and moving them. To do this, the real parts of each pole are multiplied by a constant α. In the case of poles λ_1' and λ_2', this amounts to multiplying ω_c by α.

One finally has :

$$\begin{cases} \lambda_1* = -\xi\omega_c\alpha - \omega_c\alpha\sqrt{\xi^2-1} \\ \lambda_2* = -\xi\omega_c\alpha + \omega_c\alpha\sqrt{\xi^2-1} \\ \lambda_3* = \mathbb{R}(\lambda_3')\alpha + \mathbb{I}(\lambda_3')i \\ \lambda_4* = \mathbb{R}(\lambda_4')\alpha + \mathbb{I}(\lambda_4')i \end{cases}$$

The values l_i of the matrix L are then obtained by using the `place` function of Matlab.

3.3 Simulations and discussion

3.3.1 Parameter determination

In order to best achieve our control system, we must determine the values of ξ, and ω_c. We know that the system control is done via the controllable input $u(t)$, and that this input will influence the variation of the output and the different states of the system.

In order to obtain a coherent system, *i.e.* physically possible state values and an attenuation of building oscillations, we must choose values of ξ and ω_c that will lead to a control input making the system coherent.

We tested several values of ξ as well as several values of ω_c with a constant wind force :

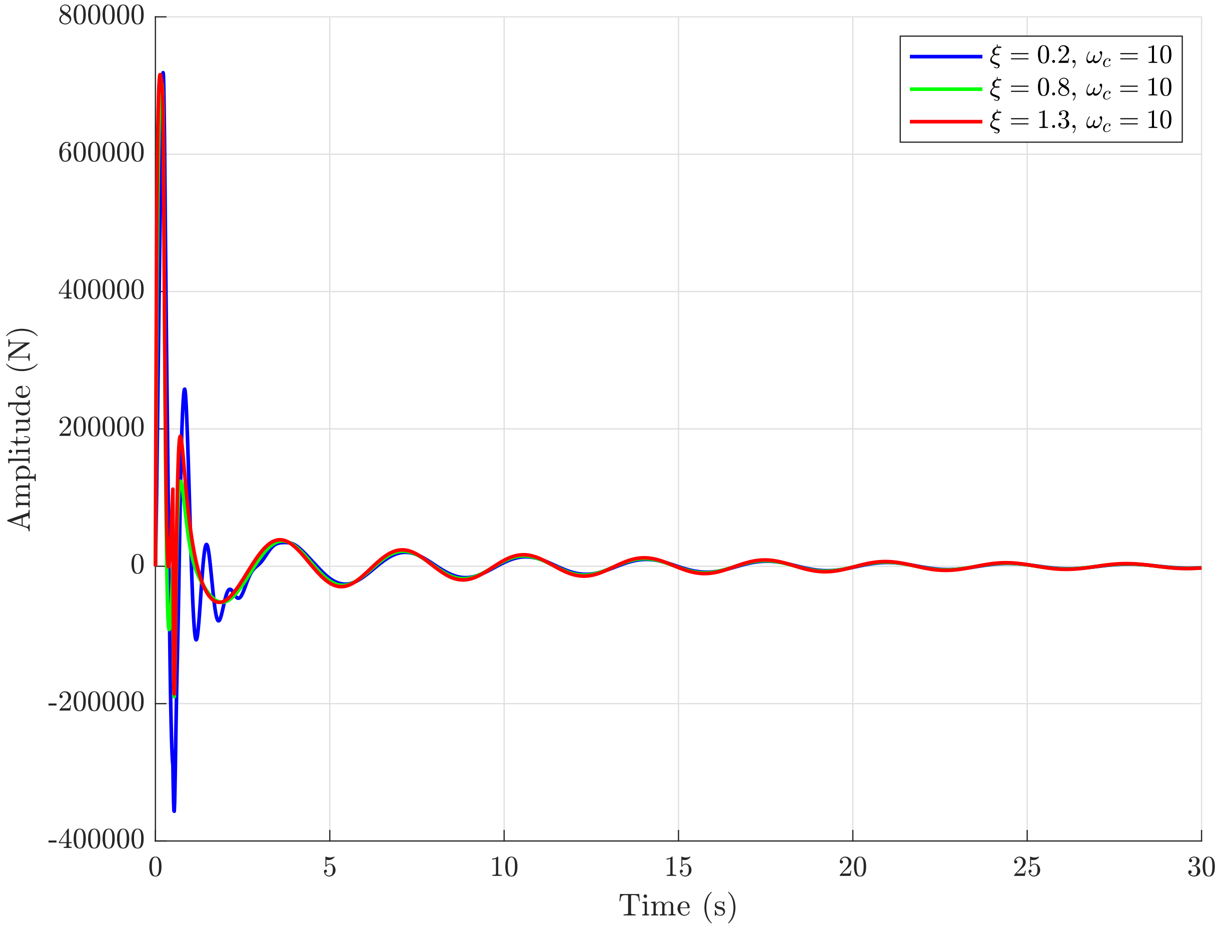

Figure 7 – Control input $u(t)$ for different variations of parameter ξ

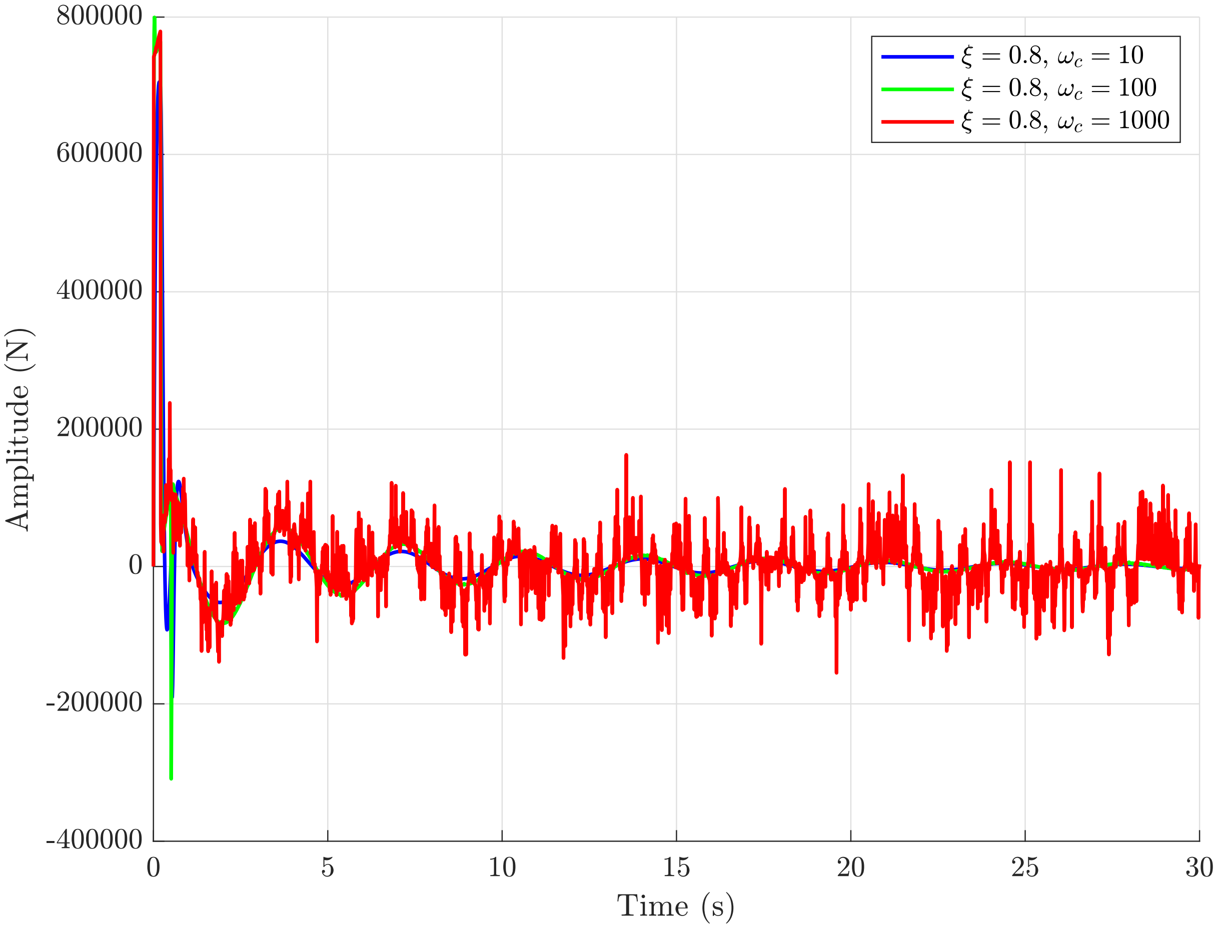

Figure 8 – Control input $u(t)$ for different variations of parameter ω_c

It can be seen that the variations in parameter ξ have little influence on the controllable force : after a few seconds, it is identical in all cases.

Concerning the parameter ω_c, we observe that the higher it is, the faster the control force oscillates (which is an undesirable behaviour).

We therefore choose to take the following values of the parameters :

$$\begin{cases} \xi = 0.8 \\ \omega_c = 10 \end{cases}$$

We also arbitrarily choose our parameter $\alpha = 5$. This choice, in view of the simulations presented later, is proving to be a good one.

With these different values, our control input is as follows :

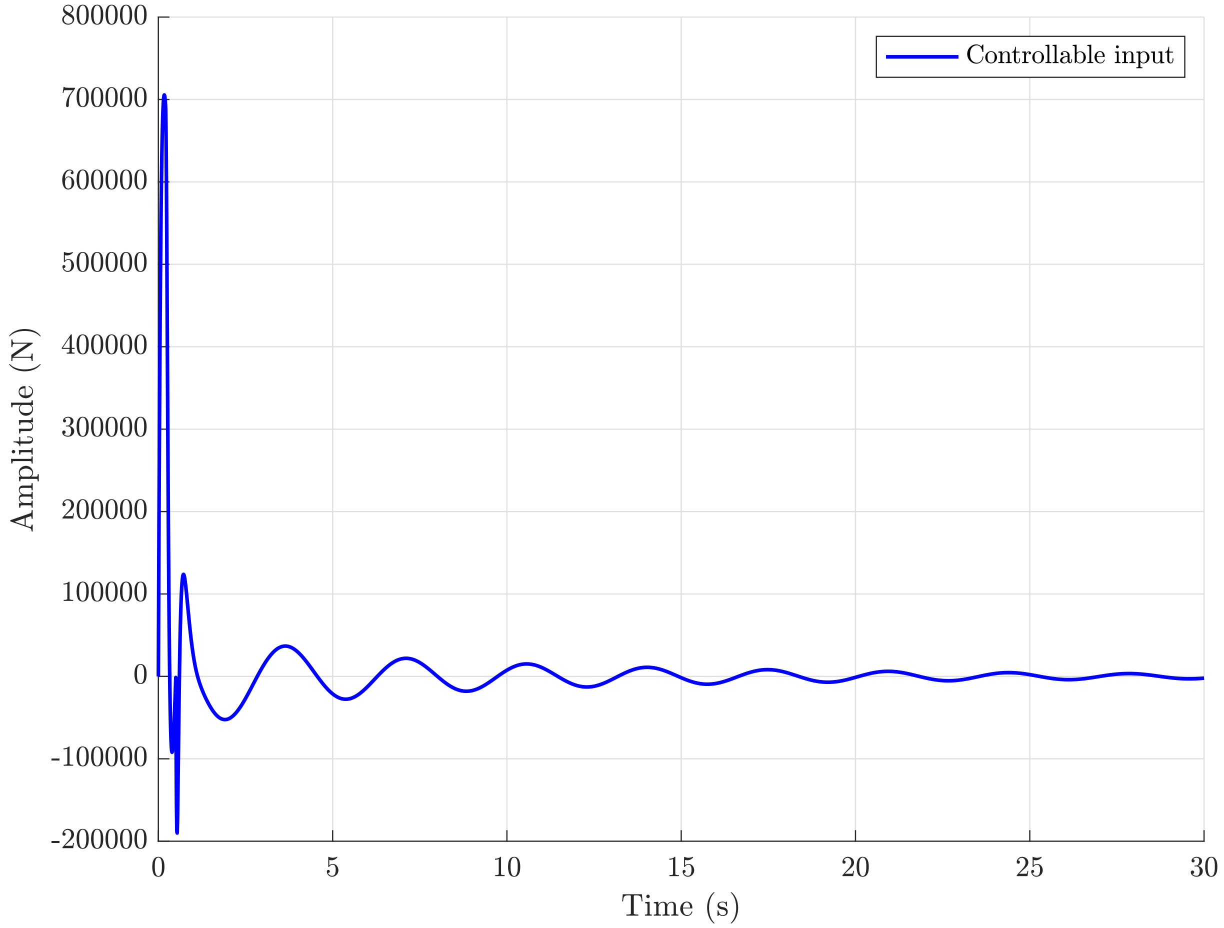

Figure 9 – Control input $u(t)$ for $\xi = 0.8$ and $\omega_c = 10$

An abnormally high peak is observed at the beginning. This peak is due to the unrealistic simulations performed : the simulation goes from a zero wind to a constant wind of several thousand newtons in an instant (similar to a step). However, it can be seen that after this peak, the control force oscillates around much more realistic values within our previously defined acceptable range of values.

This control input does allow a reduction in building oscillations, and this in a relatively slower way (the system has been slowed down).

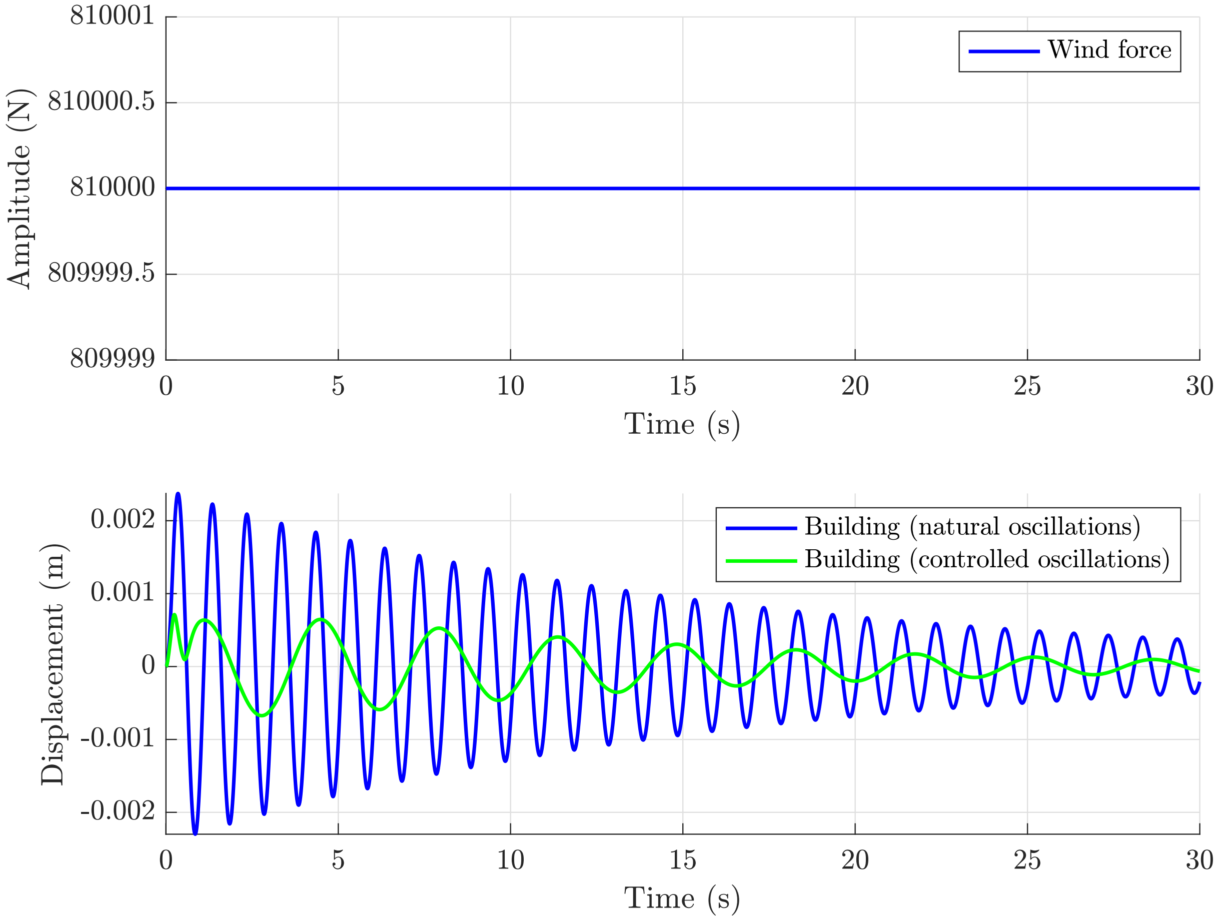

Figure 10 – System simulation with and without control input

We will now study the behaviour of the system in several situations. In order to judge its quality, the results of the observation will be displayed. The output of the system being also its first state, it can be studied by observing the first state to observe it.

3.3.2 Response to a reference variation

For this simulation, we have set the uncontrollable input to 0 and changed the reference to 0.002 m.

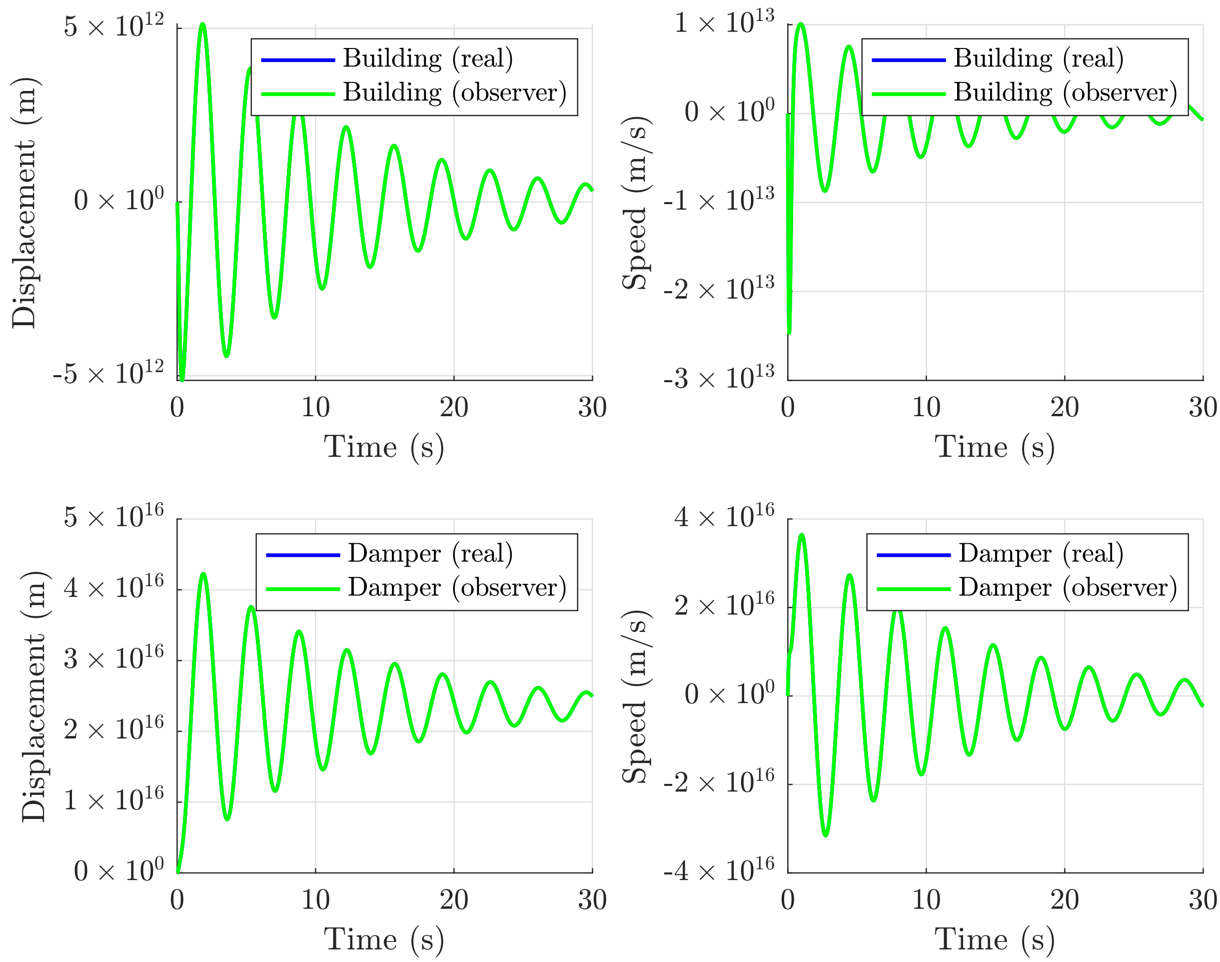

Figure 11 – System and observer simulation with a reference variation of 0.002 m

comments to do

3.3.3 Response to a perturbation (disturbance)

In order to study the convergence of the observer towards real states, we added a delay to the entry of the observer.

We simulated our different wind scenarios to see how the controlled system reacts and to check if the convergence of the observer is good.

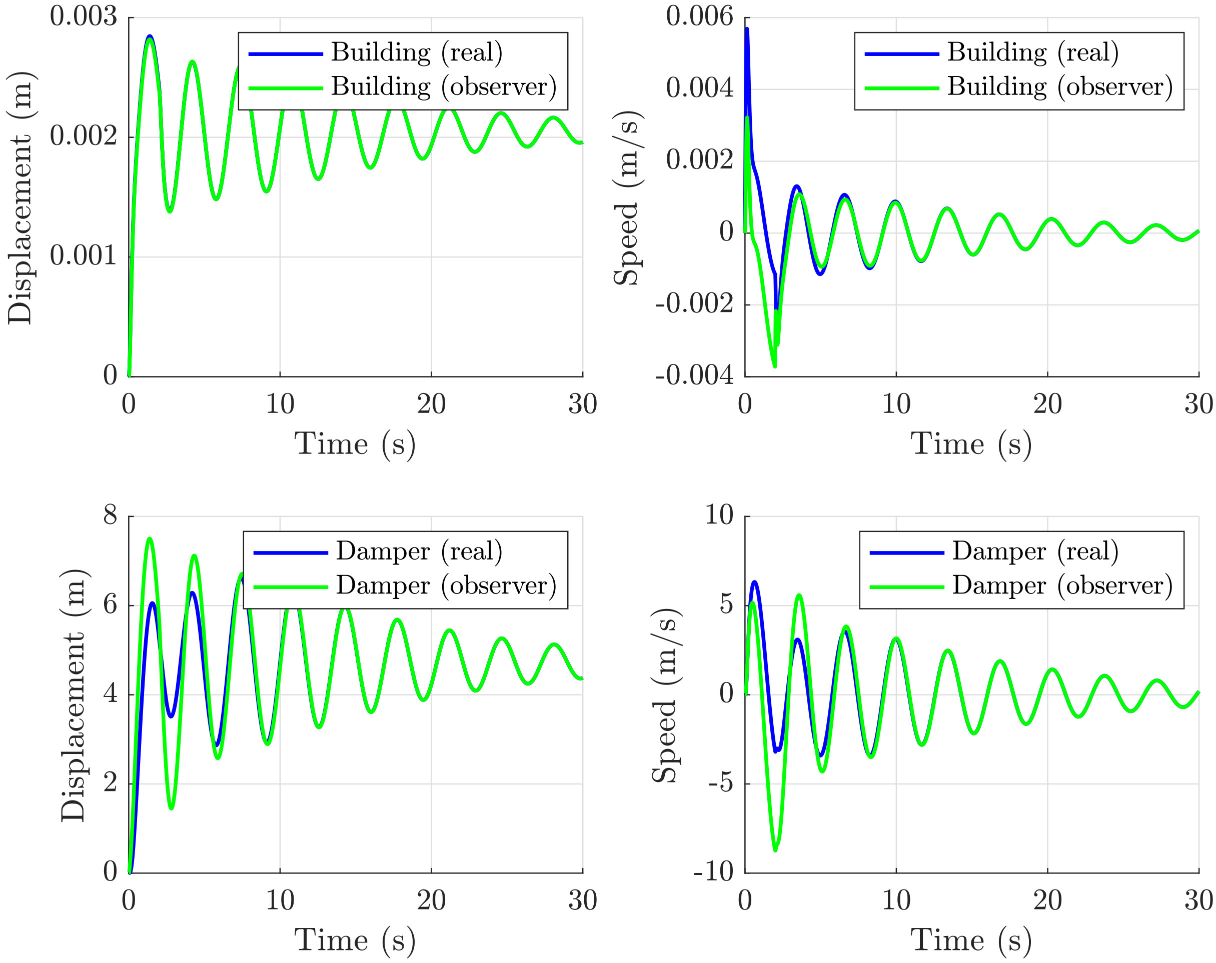

Figure 12 – System and observer simulation with a constant wind force

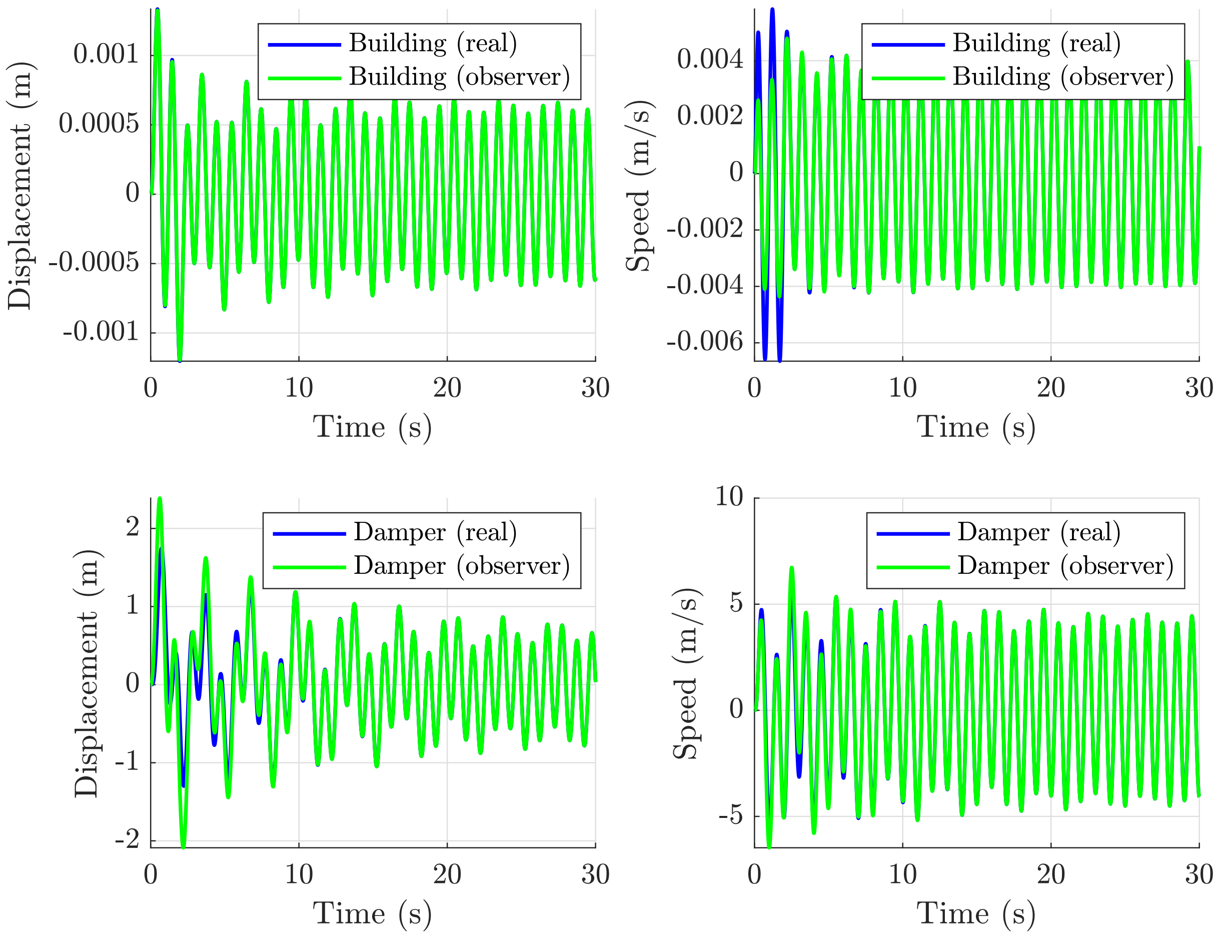

Figure 13 – System and observer simulation with a sinusoidal wind force

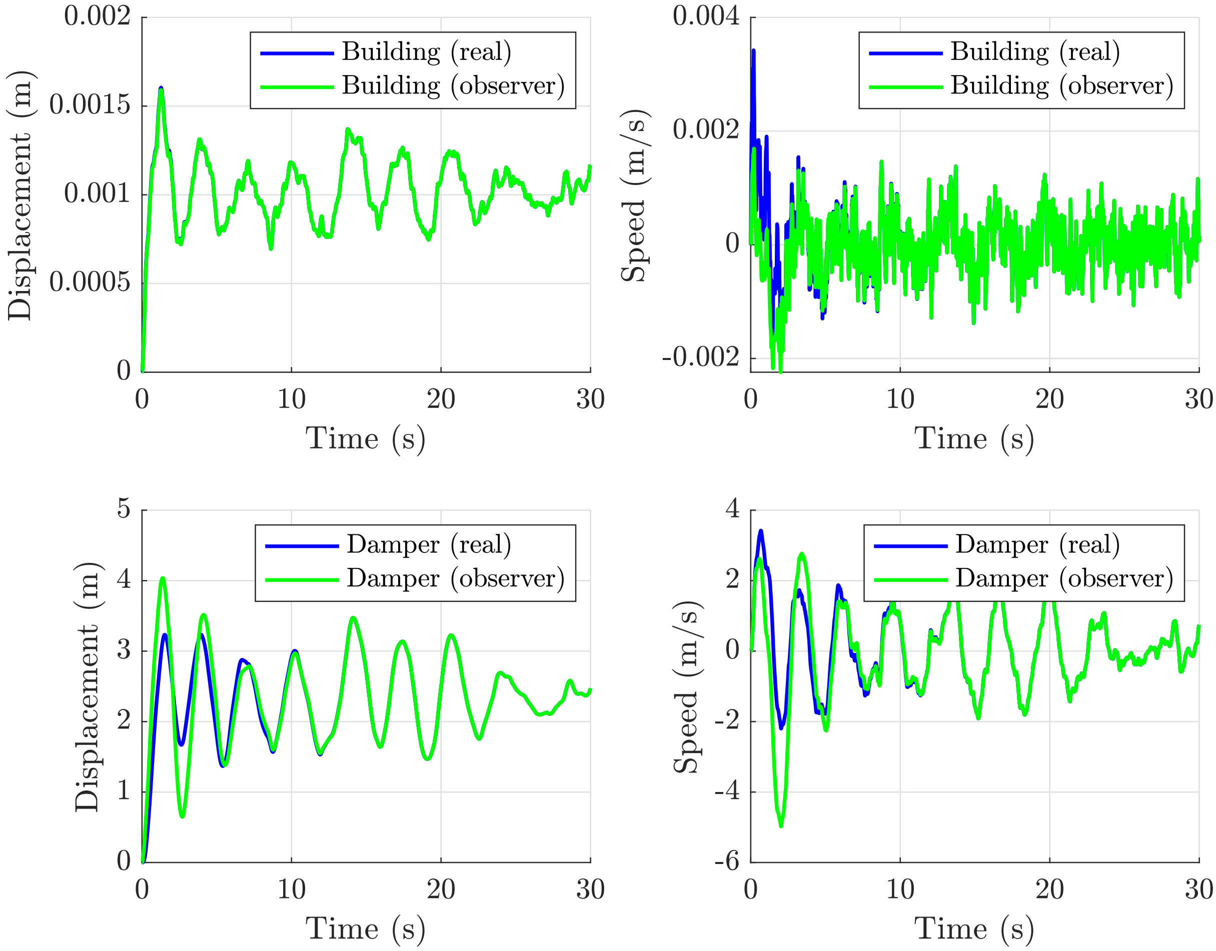

Figure 14 – System and observer simulation with a random wind force

comments to do

3.3.4 Presence of noise

For this simulation, we added a random noise to the input to the observer. This one does not receive the real output of the system, but a noisy output.

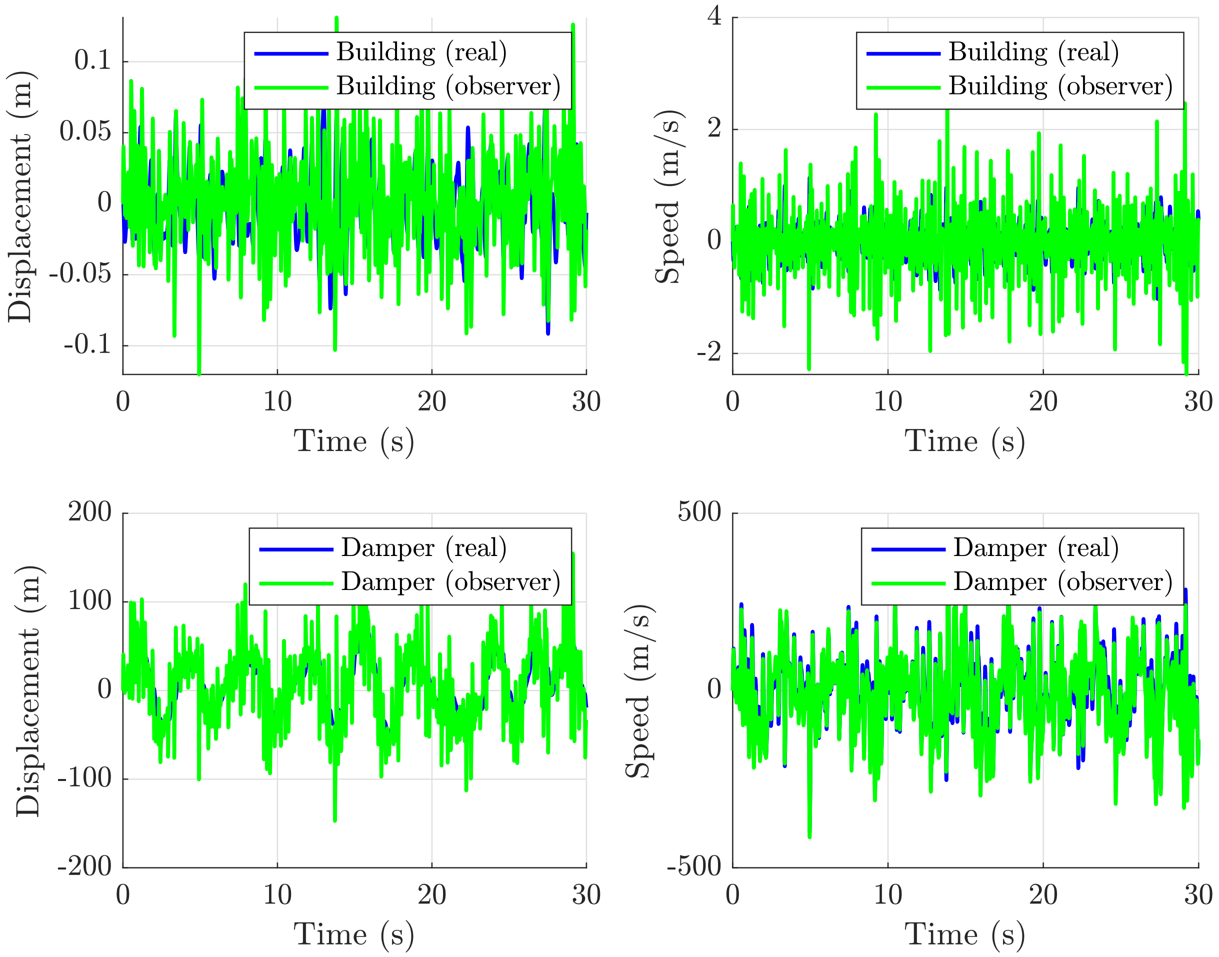

Figure 15 – System and observer simulation with noise in the observer

comments to do

4 Controller in frequency domain

4.1 Framework

For this part of the work, it has decided to simplify our system and use 2 states instead of 4. The position and speed of the damper are therefore hidden in the force of the actuator, which is still the controllable input of the system.

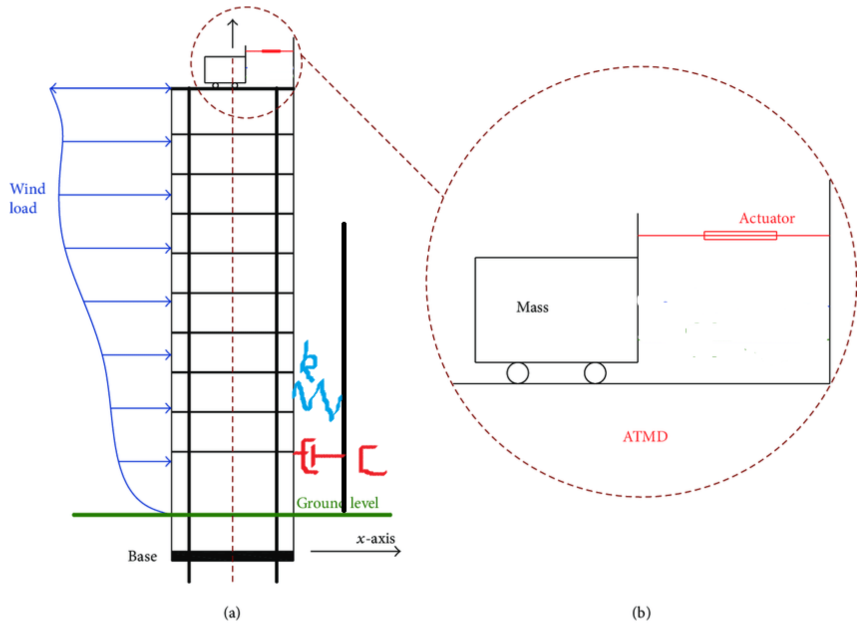

Figure 16 – Simplified system of an active mass damper.

The law that governs that system is the following :

$$m_{tot}\ddot{x} + c\dot{x} + kx = F_{wind} + F_{damper}$$

where

- $F_{damper} = m_{damper}a_{damper}$
- $m_{tot} = m_{building} + m_{damper}$
- x is the position of the building relative to its rest position ($x = 0$)

Let's now define the input, output and states :

- $u_1 = F_{wind}$
- $u_2 = F_{damper}$
- $x_1 = x$
- $x_2 = \dot{x}$
- $y = x_1$

By doing so, the ABCD matrices are the following :

$$A = \begin{pmatrix} 0 & 1 \\ \dfrac{-k}{m_{tot}} & \dfrac{-c}{m_{tot}} \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ \dfrac{1}{m_{tot}} & \dfrac{1}{m_{tot}} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

4.1.1 Constraints and simulation specifications

One has the following constraints :

- Acceleration of the mass damper between 0.3 and $0.6g$, as advised by Prof. Denoël.
- Power injected in the mass of below 10 kW so as to not have too much electrical consumption.
- Lateral movement of the top of the building not above 1 m.

The two scenario we look at are the following : a turbulent wind of maximum 810 kN, that is represented as a sine function and a constant wind of the same intensity.

Here are the values of the different parameters that have been chosen :

Mass	$m_{building} = 1 \times 10^7$ kg	$m_{damper} = 3 \times 10^3$ kg
Spring	$k \approx 4 \times 10^8\,\mathrm{N\,m^{-1}}$	
Damper	$c \approx 1.3 \times 10^6\,\mathrm{N\,s\,m^{-1}}$	
Wind	$F_{max} = 810$ kN	

Table 4 – Numerical values of the system.

4.1.2 Choice of cross-over frequency

The frequency of the building is of about 1 Hz, as advised by Pr. Denoël, and the frequency of the sinusoidal wind studied is also of 1 Hz. So the cross-over frequency was set at 5 Hz. All frequencies above that, probably coming from noise and unwanted phenomena, will be attenuated, while the amplitudes of the frequencies below that, which correspond to the internals of the system, will be amplified.

$$w_{co} = 2\pi f_{co} \approx 30\,\mathrm{rad\,s^{-1}}$$

4.2 Transfer function of the open-loop system

The Bode plots of the open-loop system are given at figure 17.

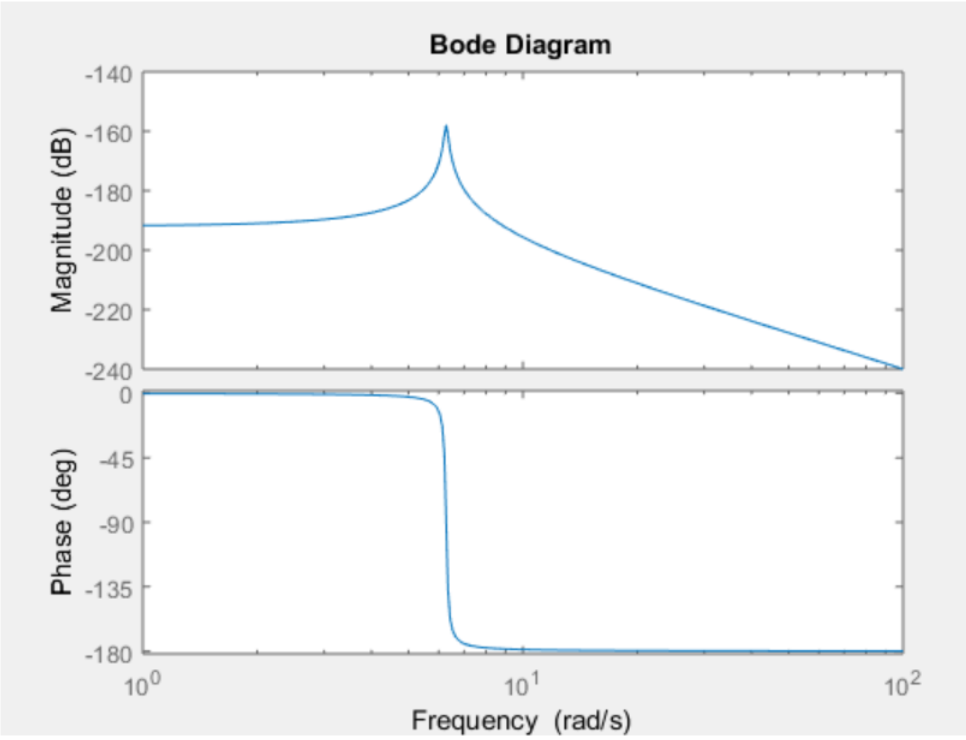

Figure 17 – Bode plots for 2D system.

As can be seen, every frequency is well attenuated, the high ones as well as the low ones.

At the cross-over frequency, the gain of the system is of about -215 dB. This is not what is wanted. One would like low frequencies to have a positive gain, high frequencies to have a negative one and the gain at the crossover frequency to be of 0 dB.

Furthermore, one needs a big enough phase margin at the crossover frequency to be resistant to the delays we will have in the system. In order to do that, one has decided to use a lead compensator as well as a gain. A low-pass filter will not be needed as high frequencies will be well attenuated without it.

4.3 Loop shaping

4.3.1 Lead compensator

Let's first start with the desired phase margin. Delays are discussed after, but one wants to be able to respond at least to 0.02 s delays, which correspond to the 50 Hz of the actuator's piston [4].

One has decided to have a phase margin of 70°. In order to increase the phase margin at the crossover frequency, one has decided to use a lead compensator.

Its transfer function is given by :

$$G(s) = \frac{\frac{s}{w_z} + 1}{\frac{s}{w_p} + 1}$$

One now has to determine the values of the parameters G_{LC}, w_z and w_p. For a given crossover frequency ω_{co} and a phase margin ϕ_m, one can determine the two w in the following way :

$$\begin{cases} w_z = \tan(\alpha) w_{\text{co}} \\ w_p = \frac{w_{\text{co}}}{\tan(\alpha)} \end{cases}$$

with $\alpha = \frac{\pi}{4} - \frac{\phi_m}{2}$.

For the crossover frequency and the desired value of ϕ_m, one gets that :

$$w_z = 5.2898 \qquad w_p = 170.1385$$

4.3.2 Gain

After that, one needs to add a gain to our system in order to increase the amplitude gains for all frequencies and make it so that the amplitude is at 0 dB at the crossover frequency. That is done by using a constant gain of 1.5178×10^9. This does not affect the phase but increases the amplitudes of about 183.6 dB, which positions our Bode plot to where one wanted it to be.

4.3.3 Trade-offs

The Bode and Nyquist plots of the controlled system are given at figures 18 and 19. As can be seen, the desired results are obtained, and one has a phase margin of 70° on the Nyquist plot.

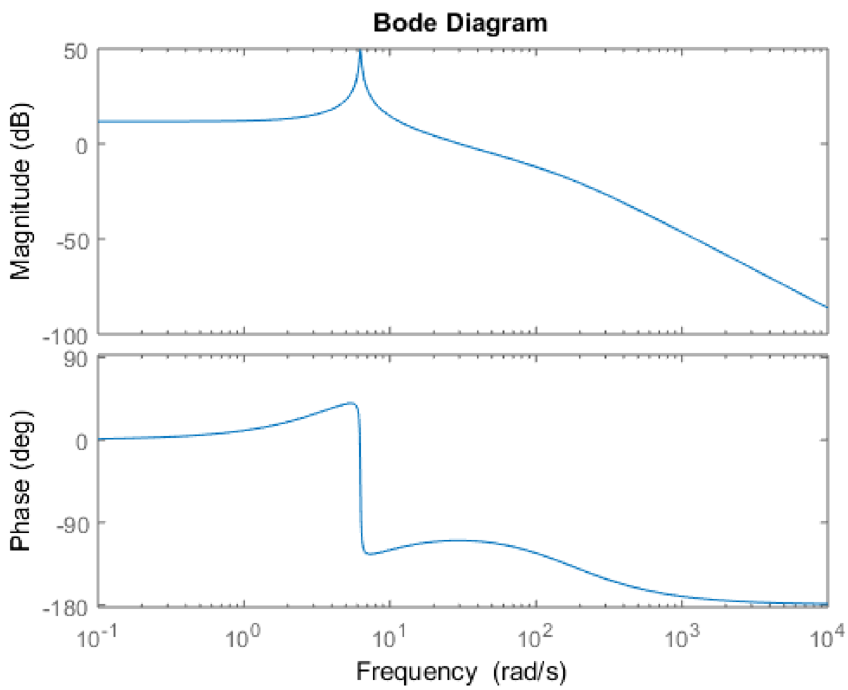

Figure 18 – Bode plots of the controlled system.

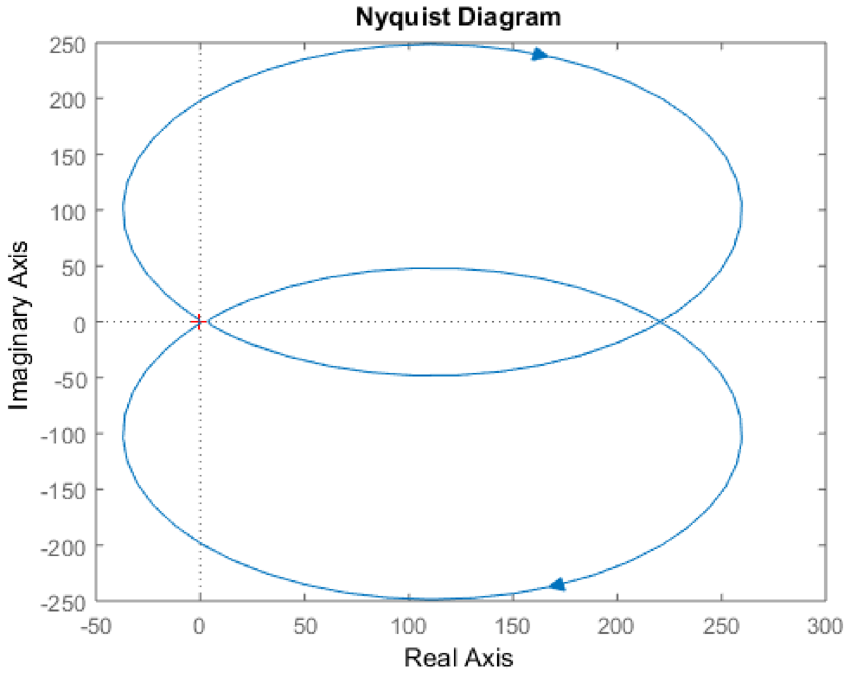

Figure 19 – Nyquist plot of the controlled system.

Concerning the impacts on the output signal and control input signal, they are in an acceptable range of values with the parameters one has chosen, as can be seen in figures 20 and 21.

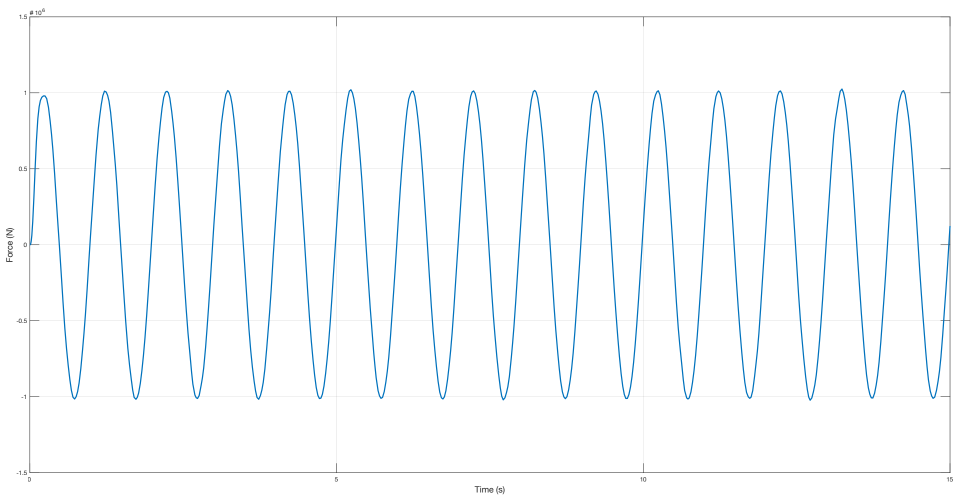

Figure 20 – Plot of the controllable input of the controlled system.

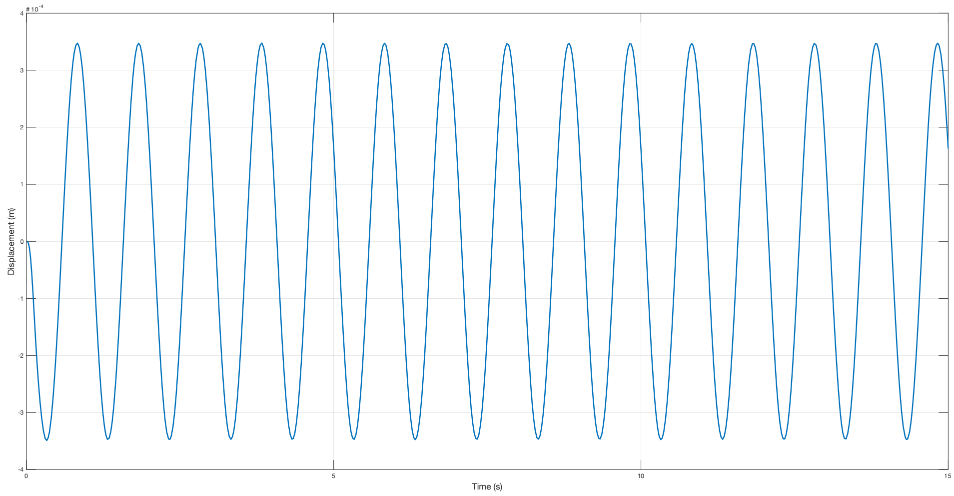

Figure 21 – Plot of the output of the controlled system.

4.4 Gang of four

4.4.1 Sensitivity function

$$S(s) = \frac{1}{1 + PC}$$

The Bode plots of the sensitivity function are given at figure 22. That function tells how the noise acts on the output. One does not want the system to react to the noise, as it is actually the measurement noise that must stay in the output.

That noise is a high-frequency phenomenon and, as can be seen on the Bode plots, there is no attenuation for high frequency, which is what one wants.

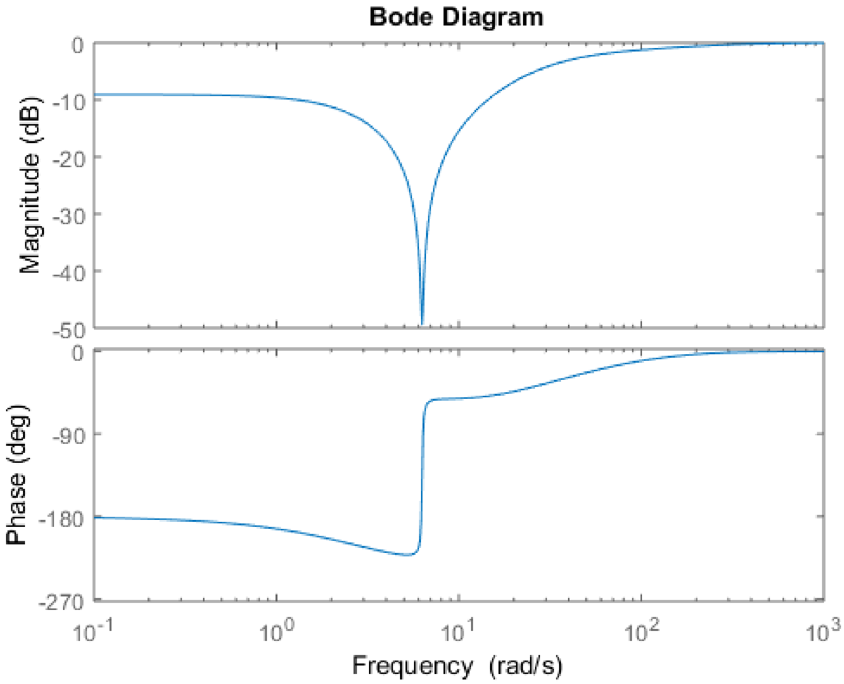

Figure 22 – Bode plots of the sensitivity function.

4.4.2 Load sensitivity function

$$PS(s) = \frac{P}{1 + PC}$$

This function tells how the disturbances act on the output and the Bode diagrams are given at figure 23. The system needs to be robust against disturbances. In the present case, these disturbances are low frequency phenomena (frequency of the wind, which we have either chosen constant or a sine function of frequency equal to 1). One can see that we have a very good reaction concerning the effect of the wind on the output of the system (attenuation of more than $-200\,\mathrm{dB}$).

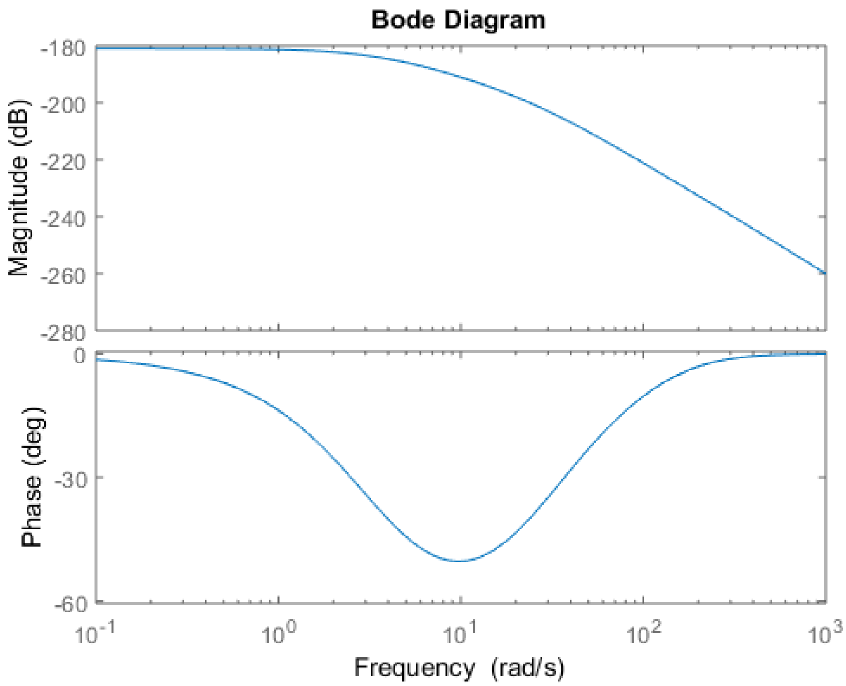

Figure 23 – Bode plots of the load sensitivity function.

4.4.3 Complementary sensitivity function

$$T(s) = \frac{PC}{1+PC}$$

This function tells how the disturbances act on the controllable input and the reference acts on the output, and the Bode diagrams are given at figure 24.

The control signal must be reactive to disturbance and the output should be able to track the reference. Amplitudes at low frequency should therefore not be dampened, and one sees that they are not attenuated on the plots.

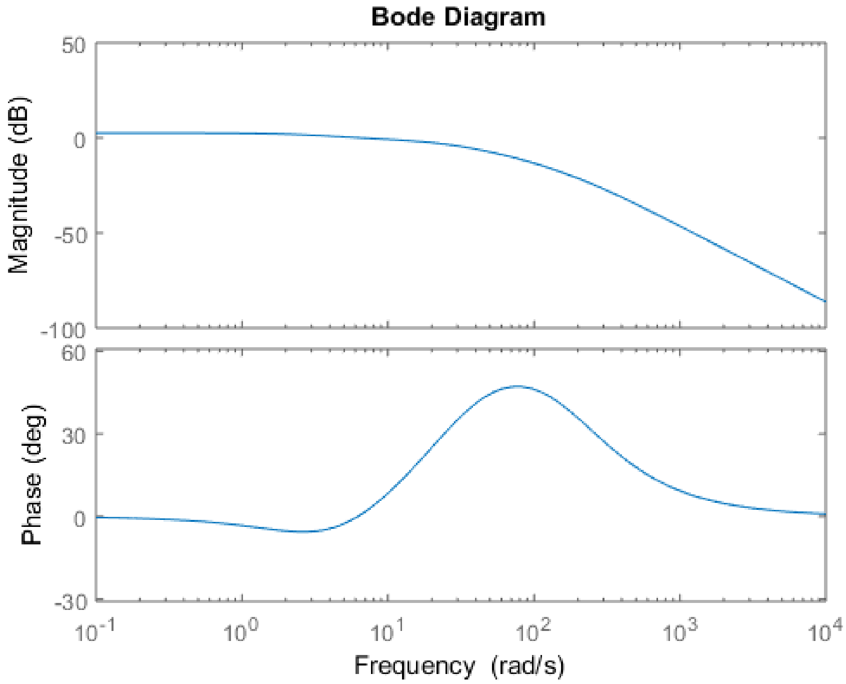

Figure 24 – Bode plots of the complementary sensitivity function.

4.4.4 Noise sensitivity function

$$CS(s) = \frac{C}{1 + PC}$$

This function tells how the noise and the reference act on the controllable input, and the Bode diagrams are given at figure 25.

That function should be reactive to reference changes, but not to noise, and so have a high magnitude at low frequency and low magnitude at high frequencies. One can see that it is not the case here. Indeed, one has high amplitudes for high frequencies. However, as our reference does not change in our system (we do not plan on dampening the oscillations in a Pisa Tower), it does not really matter.

It is also known that temporal domain controllers are better at reacting to reference changes than frequency domain ones.

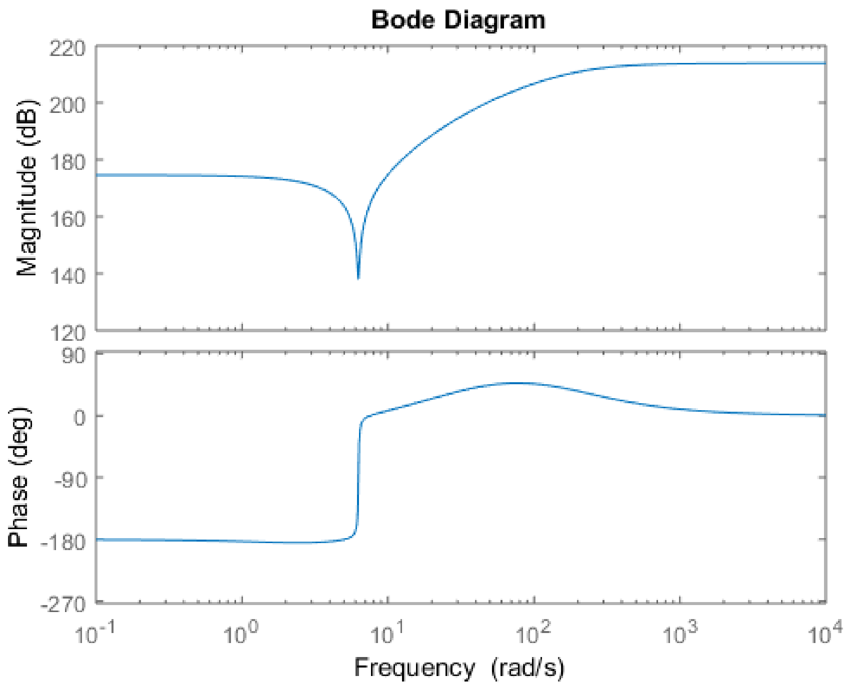

Figure 25 – Bode plots of the noise sensitivity function.

4.5 Delays

One have delays in the system that are due to the sensor transmitting the information, the microcontroller processing that information and the actuator (piston) actually moving our mass damper. One has chosen that last source of delay as being the biggest one, with a frequency of 50 Hz, which induces delays of 0.02 s.

Some Bode and Nyquist plots showing delays are given at figures 26, 27 and 28.

As can be seen, for a phase margin of 70°, the delays in our system do not bring instabilities. However, something strange is that a bigger phase margin seems to make the system less robust to delays, as can be seen on the two Nyquist plots given below. That is not how it should be, theoretically speaking. However, one has compared with the way you have designed your components, and do not see a difference.

the lead compensator and gain have the same “form” as the ones you have used. One has therefore included the Matlab script one has used to plot these figures in the mail we sent.

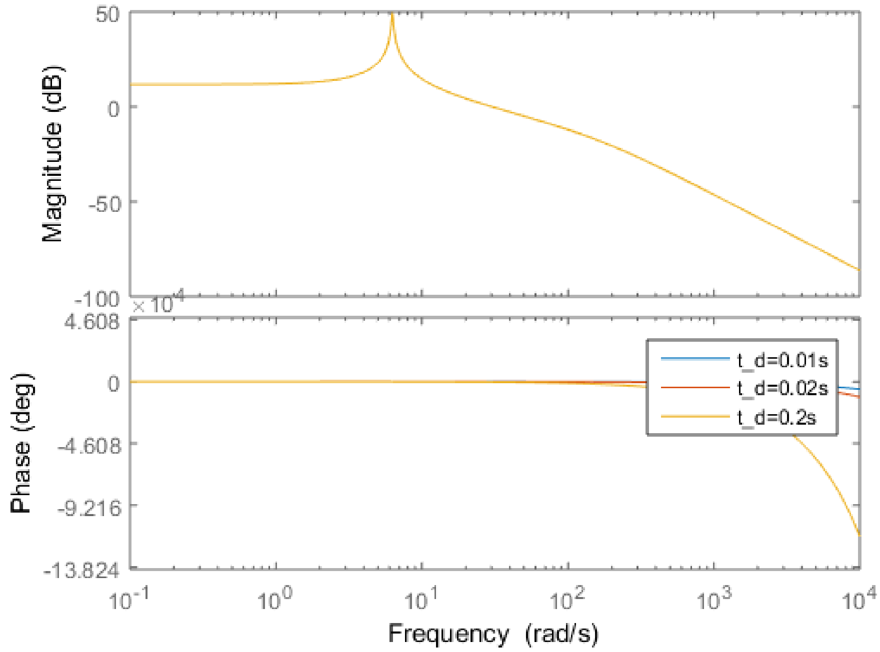

Figure 26 – Bode plots for various delays and a phase margin of 70 degrees.

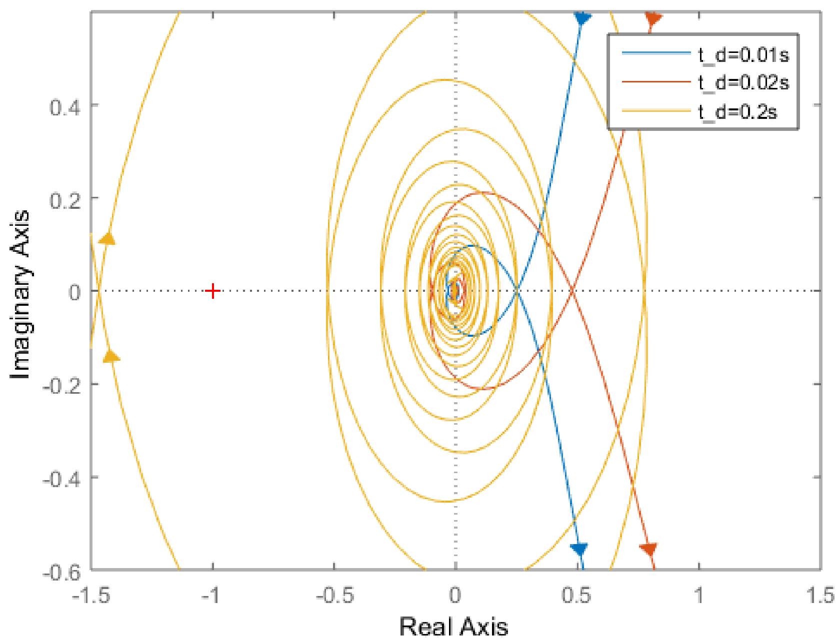

Figure 27 – Nyquist plots for various delays and a phase margin of 70 degrees.

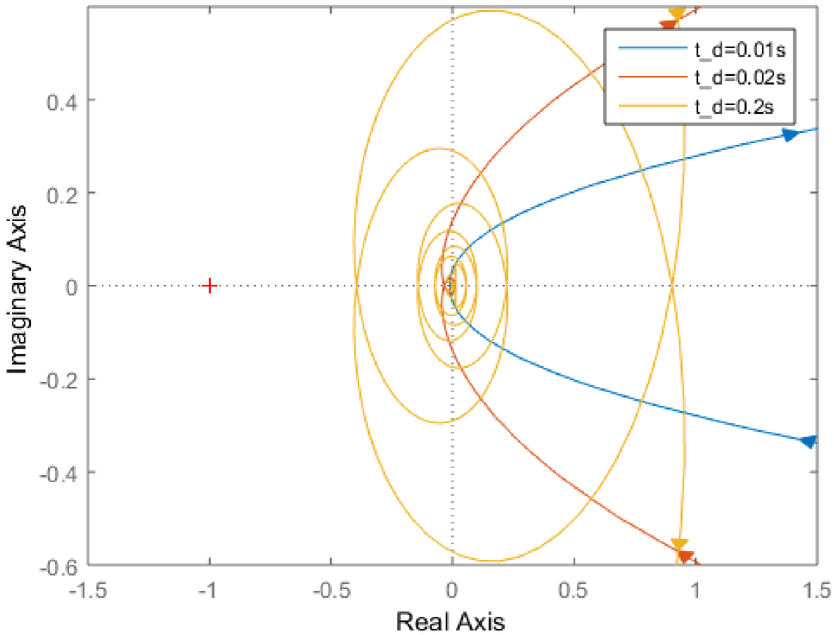

Figure 28 – Nyquist plots for various delays and a phase margin of 1 degree.

4.6 Noise

to do

4.7 Feedforward

to do

5 Conclusion

5.1 Time and frequency domains

to do

5.2 General conclusion

to do

6 References

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[4] A Hassan, A Torres-Perez, S Kaczmarczyk, and P Picton. "The effect of time delay on control stability of an electromagnetic active tuned mass damper for vibration control". In: *Journal of Physics: Conference Series* 721 (May 2016), p. 012007. DOI: 10.1088/1742-6596/721/1/012007. URL: https://doi.org/10.1088%5C%2F1742-6596%5C%2F721%5C%2F1%5C%2F012007.

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