University of Liège

Open Loop System

Linear control systems

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Remark. This report is in the state in which we delivered it at the homework deadline. Changes and corrections were made in the final report.

1 Detailed schematic of the open loop system

The detailed schematic of the open loop studied system is shown in figure 1.

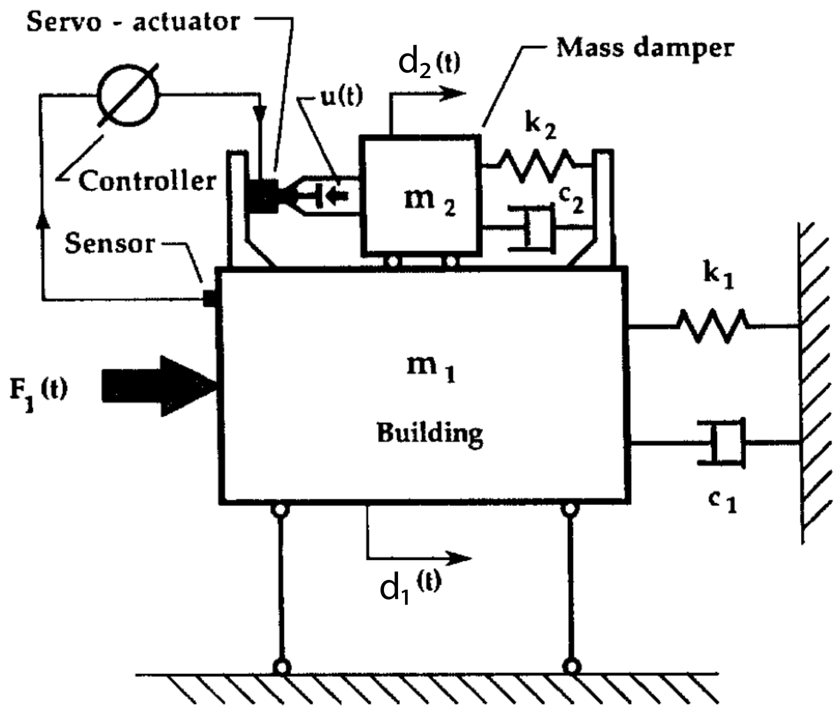


Figure 1 – Detailed schematic of the open loop studied system [1]

The building is represented by the mass m_1 and its oscillation motion is simulated by the spring k_1 and the damper c_1.

The mass damper is represented by the mass m_2 and its movement is simulated by the spring k_2 and the damper c_2.

The force $F_1(t)$ represents the wind force (uncontrollable) on the building.

The force $u(t)$ represents the force applied on the mass damper by the controller (controllable).

We are studying, at first, our system without a control mechanism. Our controllable input $u(t)$ will therefore be 0 for all our simulations in this section.

2 Constraints, assumptions, limitations

To model and study the system, we defined a series of constraints, assumptions and limitations, presented in table 1.

Building	height of 200 m, width of 30 m movement along a single axis (horizontal)
Mass	no friction between m_1 and m_2

Table 1 – Constraints, assumptions and limitations of the system.

3 State-space representation

Input vector U and state vector X are given by :

$$U = \begin{pmatrix} F_1 \\ u \end{pmatrix} \qquad X = \begin{pmatrix} d_1 \\ \dot{d}_1 \\ d_2 \\ \dot{d}_2 \end{pmatrix}$$

3.1 Inputs

- $F_1(t)$, the force of the wind (uncontrollable), approximately between 1000 and 2000 kN.
- $u(t)$, the force applied on the mass damper (controllable), approximately between 1000 and 2000 kN.

Our sensor is a measurement of the horizontal position of the top of the building relatively to the vertical position $d_1 = 0$.

Our actuator provides a force on the mass of the dampener, sets it in motion.

3.2 Outputs

$y = d_1(t)$ the relative position of the building with respect to the vertical position.

3.3 States

- $x_1 = d_1$, as described above, ranging from a few millimeters to a few meters.
- $x_2 = \dot{d}_1$, the speed of the building, ranging from about 0.1 to $5\,\mathrm{m\,s^{-1}}$.
- $x_3 = d_2$, the relative displacement of the mass damper, ranging from a few millimeters to a few meters.
- $x_4 = \dot{d}_2$, the speed of the mass damper, ranging from about 0.1 to $5\,\mathrm{m\,s^{-1}}$.

3.4 Output law

The output is one of the states : $y = x_1$.

3.5 Input law

The input law is given by [1] :

$$\begin{cases} m_1\ddot{d}_1 + c_1\dot{d}_1 + k_1 d_1 = c_2\dot{z} + k_2 z + F_1(t) - u(t) \\ m_2\ddot{z} + c_2\dot{z} + k_2 z = -m_2\ddot{d}_1 + u(t) \end{cases}$$

with $z = d_2 - d_1$.

The system is **linear**. We can easily derive the ABCD matrices.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & \frac{-c_2-c_1}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{-k_2}{m_2} & \frac{-c_2}{m_2} \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ \frac{1}{m_1} & -\frac{1}{m_1} \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

4 System simulations without controller

To simulate the system (without control mechanism), we choose a series of numerical values, presented in table 2[1].

Mass	$m_1 = 1 \times 10^7\,\mathrm{kg}$	$m_2 = 3 \times 10^3\,\mathrm{kg}$
Spring	$k_1 \approx 4 \times 10^8\,\mathrm{N/m}$	$k_2 = 10^5\,\mathrm{N/m}$
Damper	$c_1 \approx 1.3 \times 10^6\,\mathrm{Ns/m}$	$c_2 = 10^4\,\mathrm{Ns/m}$
Wind	$F_{max} = 810\,000\,\mathrm{N}$	

Table 2 – Numerical values of the system

For the strength of the wind, we considered 2 cases (in newton) :

$$F_1 = F_{max} \quad \forall t \qquad \text{Constant wind force}$$
$$F_1(t) = F_{max}\sin(2\pi t) \qquad \text{Sinusoidal wind force}$$

The stiffness and viscosity values for the building were obtained using the formulas :

$$k_1 = (2\pi f)^2 m_1$$
$$c_1 = 2m_1(2\pi f)0.01$$

where $f = 1\,\mathrm{Hz}$ is the natural frequency associated with the mass of the building.

The maximum wind force, on the other hand, was approximated by

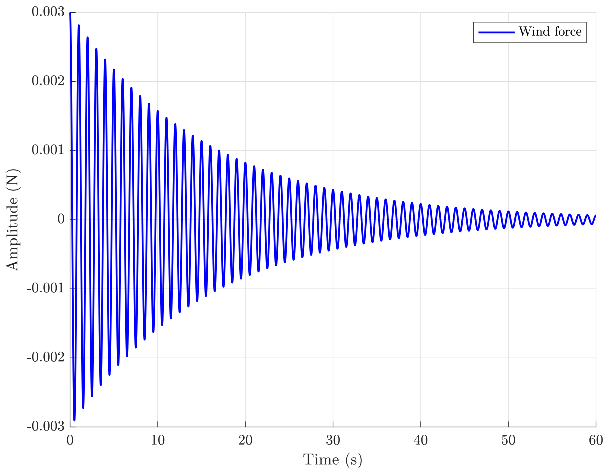
$$F_{max} = \frac{1}{2}\rho v^2 A$$

with

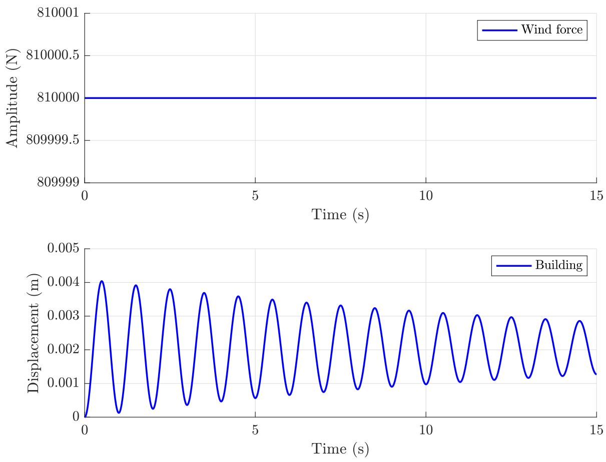
[1]We would like to thank Professor Denoël for discussing these values with us.

- $\rho \approx 1.2\,\mathrm{kg/m^3}$, the air density;
- $v = 15\,\mathrm{m/s}$, the wind speed;
- $A = 200 \times 30 = 6000\,\mathrm{m}^2$, the area of one side of the building.

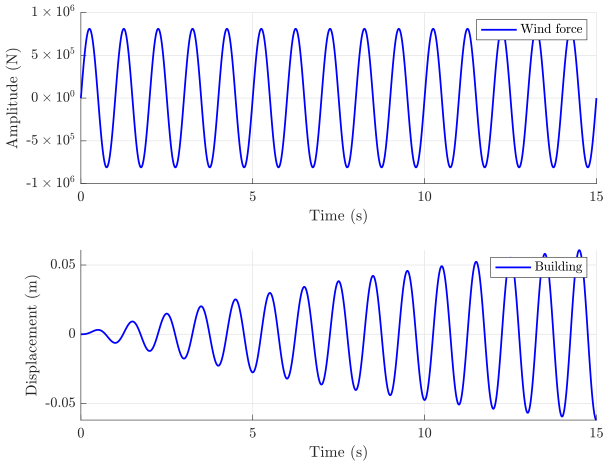
4.1 Simulation results



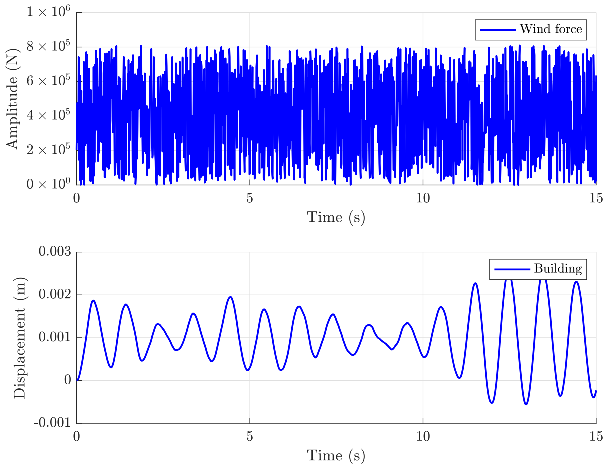
(a) Initial conditions



(b) Constant wind force



(c) Sinusoidal wind force



(d) Random wind force

Figure 2 – Simulation results

The first simulation (figure 2a) is a response of our system to initial conditions : the initial displacement of the building is defined at $0.5\,\mathrm{m}$. We observe that the building oscillates and tends to regain its reference position.

The other simulations are responses of our system to an input (the wind).

In the case of a constant force (figure 2b), the building oscillates at the beginning and then tends to stabilize (at a position different from its reference).

In cases of sinusoidal and random forces, the building oscillates and follows approximately the wind movement.

5 State-space representation analysis

5.1 Stability

To study the stability of the system, we compute the eigenvalues of the dynamic matrix A thanks to Matlab function (`eig`) :

$$\lambda_1 = -0.0645 + 6.2824\mathrm{i}$$
$$\lambda_2 = -0.0645 - 6.2824\mathrm{i}$$
$$\lambda_3 = -1.6655 + 5.5285\mathrm{i}$$
$$\lambda_4 = -1.6655 - 5.5285\mathrm{i}$$

The system is stable if the real parts of the eigenvalue are all negative. In our case, the system is stable.

5.2 Observability

To determine whether or not the system is observable, we compute the observability matrix thanks to Matlab function (`obsv`).

The matrix is full rank (verified with Matlab), the system is thus fully observable.

As seen on the matrix C, we need one sensor. According to the place of the non zero value, this sensor has to measure the x_1 state, namely the horizontal position of the top of the building d_1. This state is indeed the objective of the active mass damper and has thus to be observed.

5.3 Controllability

To determine whether or not the system is controllable, we compute the controllable matrix thanks to Matlab function (`ctrb`). In order not to take into account the uncontrollable input (wind), only the second column of the B matrix was kept for the calculation.

The matrix is full rank (verified with Matlab), the system is thus fully controllable.

As seen on matrix B, we need only one actuator. The first column of the B matrix represents the wind, while the second one concerns the damper. This latter is indeed the only controllable input and contains two non-zero elements. As a result, only one actuator is needed, and acts on two states, the speed of the building and the speed of the damper, as they take place on x_2 and x_4.

6 References

[1] *Parametric study of active mass dampers for wind-excited tall buildings*. https://www.sciencedirect.com/science/article/pii/0141029695001088. Accessed : 2019-10-10.