

# Quantification

Quantification allows us to talk about all values in a set

## Universal Quantification

Syntax:

```
forall ( in ...)
```

```
forall ( in )
```

Universal quantifiers are powerful expressions that allow users to talk about all values that fit a criteria. Scribble supports 2 types of universal quantifiers - quantifiers over numeric ranges and quantifiers over arrays and maps.

## Numeric ranges

Quantifiers can talk about properties that hold over all numbers in a given range. The syntax for such quantifiers is `forall(n ...)`.

The start expression is inclusive, while the end range expression is exclusive. The most common use-case of quantification over numeric ranges is describing all indices of an array (see below example).

Example 1: Forall over arrays In the below example we check that the array `arr` is sorted.

```
...  
  
Copy /// #invariant forall(uint i in 0...arr.length-1) arr[i] < arr[i+1]; contract Token { uint[] arr; ... }
```

''' Note that the type of the quantifier variable (uint i in the above example) must always be a numeric type, and it must be able to fit both the start and end expressions. So for example you can't write: `forall (int8 i in 0...arr.length-1) ...` in the above example instead, since `int8` is too small of a type to store the array length (which is `uint256`).

## Arrays/maps

The second flavor of universal quantifiers talks about all indices in an array or keys in a map. The syntax is `forall (n )`.

Example 1: Forall over arrays Example 2: Forall over maps In the below example, we check that each stakeholder has a positive stake

```
...  
  
Copy /// #invariant /// forall(uint i in stakeHolders) stakes[stakeHolders[i]] > 0; contract Token { address[] stakeHolders;  
mapping(uint=>uint) stakes; }
```

''' In the below example, the `mapping` `authorized` is a mapping from account addresses to operators addresses, `authorized` to act on behalf of the given account. The property below states that for no account is the `0x0` address authorized to act on their behalf.

```
...  
  
Copy /// #invariant /// forall(address acct in authorized) authorized[acct] != address(0); contract Token {  
mapping(address=>address) authorized; }
```

''' Any `forall` over arrays can be equivalently expressed as a `forall` over numeric ranges. I.e. `forall(i in arr)` is equivalent to `forall(uint i in 0...arr.length)` Note that in Solidity maps are technically defined over their complete input range (i.e. a `mapping(uint=>uint)` is defined for all `uint256`s, just its 0 for most of them). Due to this, when we say that we "quantify over all keys in a map", we mean all keys that have:

1. Been set explicitly at least once
2. Have not been deleted with the `delete` keyword.

To support this, under the hood we re-write all annotated maps with our custom data type that track key insertion and deletions.

The `delete` operation is equivalent to zeroing out a value. However, we do not treat them equivalently. Given a map `m`, when you do `delete m[x]`, we will remove `x` from the keys that we store in `m`. However, if you do `m[x] = 0` we will not remove `x` from the keys we store in `m`. You can nest `forall` statements arbitrarily. For example, we can extend the example above to store a list of authorized users for a given address and require that none of the authorized users is `0x0`:

Authorized Users 2 In the below example, the `mapping` `authorizedUsers` stores a list of addresses of operators authorized to

act on behalf of a given address. The property below states that the 0x0 address is not authorized to act on anyone's behalf.

...

```
Copy /// #invariant /// forall(address acct in authorized) /// forall(uint i in authorized[acct]) /// authorized[acct][i] != address(0);
contract Token { mapping(address=>address[]) authorized; }
```

...

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