Order but Not Execute in Order

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Abstract. This work aims to address the general order manipulation issue in blockchain-based decentralized exchanges (DEX) by exploring the benefits of employing differentially order-fair atomic broadcast (of-ABC) mechanisms for transaction ordering and frequent batch auction (FBA) for execution. In the suggested of-ABC approach, transactions submitted to a sufficient number of blockchain validators are ordered before or along with later transactions. FBA then executes transactions with a uniform price double auction that prioritizes price instead of transaction order within the same committed batch.

To demonstrate the effectiveness of our order-but-not-execute-in-order design, we compare the welfare loss and liquidity provision in DEX under FBA and its continuous counterpart, Central Limit Order Book (CLOB). We consider three types of players for a DEX: common investors, privately informed traders, and arbitrageurs who can provide liquidity and front-run. Assuming that the exchange is realized over an of-ABC protocol, we find that FBA achieves better social welfare compared to CLOB when (1) public information affecting the fundamental value of an asset is revealed more frequently than private information, or (2) the block generation interval is sufficiently large, or (3) the priority fees are small compared to the asset price changes. The performance improvement of FBA can be attributed to several intrinsic factors. First, blockchains inherently treat time as discrete, and ensuring order fairness there is non-trivial, allowing even more room for latency arbitrage rents under CLOB. Second, a sufficiently large block creation interval allows for information dispersion under FBA. Third, higher priority fees discourage front-running under CLOB. Moreover, FBA prioritizes price in deciding execution order and fewer informed traders mean less adverse price impact, i.e., loss induced by informed traders. Further, our findings also indicate that liquidity provision is better under FBA (4) when the market is not thin, meaning that a higher number of transactions are submitted by investors and traders in a block, or (5) when fewer privately informed traders are present. Overall, in the settings mentioned above, the adoption of FBA and of-ABC mechanisms in DEX demonstrates improved performance in terms of social welfare and liquidity provision compared to the continuous CLOB model.

1 Introduction

Centralized exchanges order transactions in their receiving order and execute with two major processing methods, the predominant central limit order book (CLOB) model [15] (i.e., continuous double auction) featuring serial processing and discrete batch processing where transactions received during a pre-specified time interval are handled altogether. Here, a limit order specifies the direction, price, and quantity of trade, and a set of outstanding limit orders is called a limit order book (LOB). When the exchanges are trusted parties, they achieve receiver order fairness, where the transactions received earlier are ordered before later transactions. This inevitably and reasonably disregards the sending time of transactions. As a result, under CLOB, naïve high-frequency trading (HFT) [36] or speed technologies can then be adopted to manipulate transaction order (with respect to transactions' sending time), i.e., delivering a transaction tx2 that is sent after tx₁ to the exchange before tx₁ arrives. Such manipulations can benefit liquidity provision and price discovery [12] when HFTs implement benign trading strategies like market making (i.e., continuously placing standing orders on both sides of the market price), but can increase transaction costs for common investors [36] when they apply hostile strategies including front-running, i.e., acting on anticipation of large orders. Budish, Cramton, and Shim [7] in addition note that HFTs promote the arms race for speed, including establishing faster transmission links and co-locating servers with centralized exchanges. The race is considered as wasteful because it does not empirically improve the arbitrage size but increasingly raises the cost of obtaining the profits. They view the arms race problem as inherent to serial processing where "even symmetrically observed public information creates arbitrage opportunities". This implies that the arms race problem carries to decentralized exchanges (our focus) when sequential processing is adopted.

Decentralized exchanges (DEXes) [30,24] run by a group of parties (or validators, proposers) order transactions according to their underlying consensus or atomic broadcast (ABC) protocols. Ordered transactions can still be executed sequentially or discretely in batches when they are expressed and organized in proper data structures. As mentioned above, traditional manipulations via latency races still apply in DEXes under serial processing. The necessary speed technology now takes the form of better connections to validators, higher priority fees, etc. An extra challenge is that ensuring receiver order fairness now becomes non-trivial even if all parties running a DEX protocol are correct. This is first because the security of traditional consensus protocols only concerns safety and liveness but not the explicit order of transactions in a proposed block or whether a proposer has inserted or disregarded certain transaction(s). This means that inserting, censoring, and reordering transactions are allowed by the underlying consensus protocol. Second, Kelkar et al. [29] observe that achieving general receiver order fairness is not always possible due to the Condorcet paradox: validators may collectively have cyclic views on the receiving order of (some) transactions. General order manipulations can then be implemented even without speed technologies.

Existing defenses against order manipulation in DEXes. We summarize four main existing styles of defenses and give more detailed descriptions in §2. We address general order manipulation in DEXes as front-running for simplicity. (1) The first approach is to extend the security definitions of traditional consensus or atomic broadcast protocols to include relaxed receiver order fairness. One such proposal is batch order fairness [29,27,28] or equivalently, differential order fairness [9]: a transaction tx_1 that is delivered to a specific number of validators before another transaction tx_2 are ordered no later than tx_2 . With batch order-fair ABC, one can only successfully front-run with a certain probability that depends on the protocol specifications, transactions' sending time, and their senders' network connections to validators.

(2) Second, users can blind transactions through encryption [5,48,33,38] or commitments [35]. This eliminates manipulations dependent on transaction contents. But masking transactions does not hide network-level meta-data, does not stop validators from oblivious manipulations independent of transaction contents, e.g., ordering a transaction before others (similar to HFT) or removing transactions from certain IP addresses, and does not eliminate profitable order manipulations based on inference of transaction contents. Besides, hostile arbitrage strategies based on anticipations of future orders like traditional front-running can still be performed. (3) Tax the front-runners by charging priority fees [13]. This does not remove front-running but leaves the front-running market to reach equilibrium through actions carried out by interdependent players in an ever-changing environment. (4) Another group of approaches is from a market design perspective. Fair-TraDEX [35] has users commit to transactions and later execute revealed transactions with batch processing. As noted under approach (2), masking transactions do not eliminate profitable manipulation opportunities. P2DEX [4] utilizes a secure multi-party computation (MPC) protocol to match orders which introduces computation overhead and latencies. SPEEDEX [41] approximates the clearing price in a block in general equilibrium where demand meets supply, given a set of static orders. The equilibrium is from an economics perspective and does not aim to capture participants' strategies, acquired information, or dynamic sequential moves.

Another market design response to order manipulations. While the current predominant financial market design adopts the CLOB execution model, Farmer and Skouras [17] and Budish et al. [7] propose the alternative batch processing model, frequent batch auction (FBA) or uniform price double auction (UPDA) as a defense against HFTs in centralized exchanges. In FBA, orders are processed in batches using uniform-price auctions: trades are matched according to price, and all matched counterparties settle trades at the same price.

FBA counters HFT manipulations in centralized exchanges by rendering the exact order in a batch superficial, as long as transaction senders' latencies are compatible with the frequency of auctions. Naturally, this effect also applies to front-running in DEXes when combined with of-ABC: Recall that of-ABC limits front-runners' success probability by ordering transactions submitted to a specific number of validators first

at least in the same batch as a later transaction; FBA then ensures that trades in the same batch are executed at the same price.

Intuition. We consider the alternative processing model FBA for another reason aside from the inspiration of discussions in the literature. Blockchains inherently already treat time as discrete when ordering and batching transactions into blocks as in FBA. When transactions are executed sequentially as in CLOB, one can observe and act on the "future" if the pending transaction pool is not fully hidden. Latencies in block generation and message transmission along with priority fees allow even more space for latency arbitrage rents in DEXes.

1.1 Contribution

In this work, we explore the defense against front-running with of-ABC for ordering and FBA for execution, and compare the welfare loss and bid-ask spread under CLOB and FBA in DEXes. DEXes are often implemented as layer one blockchain protocols [30] or smart contracts [24] on general-purpose blockchains. In the latter case, we abstract away other functionalities and only focus on the exchanges. The exchange can then be modeled as ordering transactions with any of-ABC protocol, e.g., the atomic broadcast protocol achieving batch order fairness [28] or equivalently, differential order fairness [9]. Note that without fair batch ordering or other tools, FBA alone as an execution model is not an effective countermeasure. Because a victim transaction may never be committed even if FBA is the processing model. Also, note that we do not discuss how to express limit order books practically in blockchains. One can instead refer to decentralized order book exchanges built on general blockchains like Ethereum [16] and Solana [46] and decentralized CLOB designs in layer 1 blockchains like DeepBook in Sui protocol [30].

We model a single DEX, many arbitrageurs who can apply market-making and front-running strategies, common investors who submit inelastic trades, and informed traders who submit trades after observing publicly or privately available information about asset valuation jumps. Unlike common investors who are the major sources of profits for market-makers, informed traders and front-running arbitrageurs can cause loss to a market-making arbitrageur, which is called the adverse selection risk. Nature continuously decides the arrival of common investors, private information, and public information. The three types of actors then make moves in a dynamic trading game.

Overall, we find that in equilibrium, FBA imposes less welfare loss (1) if public information concerning an asset's fundamental value changes is released more often compared with private information. Because under FBA, market-makers have time to respond to public information and withdraw stale quotes. Under CLOB, an arbitrageur can front-run the liquidity providers in case of both public and private information releases. The market-maker then sets higher markups in equilibrium to counter the risk. Note that Budish et al. [7] show that for centralized exchanges, FBA is welfare-optimal when there is no private information. (2) Sufficient length for the batch auction frequency compatible with the arrival rates of different types of trading parties also decreases the welfare loss for FBA. Besides, (3) smaller priority fees for submitting transactions into the blockchain system also increase the markups under CLOB. Because the profits from front-running market-makers increase as costs decline, resulting in the liquidity providers charging higher markups. We also note that FBA has smaller bid-ask spreads in liquid markets with many transactions or when fewer privately informed traders are present.

Organization of contents. We summarize related work in §2 before concluding the discussion in §5. In §3.1, we summarize the model details of the trading system, including the DEX, three types of trading players, and the trading asset. We present the security definitions of the of-ABC protocol underlying the DEX and solution concepts adopted for solving the trading game equilibria under both market designs in §3.2. In §4, we describe the order of events, solve for the equilibria under CLOB and FBA, and compute and compare their welfare loss along with bid-ask spreads.

2 Related work

Mitigate transaction order manipulation problem. Except for taxing front-running, there are mainly three other defenses against order manipulations. The first mitigation approach is extending the security definition of consensus or ABC problem to include order fairness [29,9]. Kelkar et al. [29] define batch order fairness and present an asynchronous batch order-fair ABC protocol tolerating up to one-fourth Byzantine faults with $O(n^4)$ communication complexity, which is later improved in Themis [28]. Here n is the total number of parties. Cachin et al. [9] introduce an equivalent receiver order fairness concept, differential order fairness (Definition 1), and present an of-ABC protocol in an asynchronous network. The protocol induces quadratic communication complexity and has optimal fault tolerance of up to one-third of faults.

The second approach is blinding the transaction contents until after they have been committed [33,38]. Ferveo [5] constructs a publicly verifiable distributed key generation (DKG) scheme and combines it with a threshold encryption scheme [43] for Tendermint-based [6] Proof-of-Stake protocols to achieve mempool privacy, where transactions are encrypted until finalization. In a threshold encryption scheme, there is a single public key and each party holds a secret key share. Users can threshold-encrypt transaction encryption keys using the public key. The blockchain validators can obtain decryption shares and reconstruct the decryption key. Ferveo has high bandwidth overhead from threshold encryption and the constructed DKG protocol. F3B [48] also lets clients threshold-encrypt symmetric transaction encryption keys which are revealed by a committee after transactions are committed. To improve efficiency, Fino [33] adds to a DAG-based state machine replication protocol a threshold secret sharing [47] and threshold encryption [43] layer for users to share their transaction encryption keys. Similar to Ferveo, initially, the user encrypts her transactions using a symmetric key and commits to this key. She then secret-shares the symmetric key among the parties running the protocol (for efficient decryption on the happy path) and also threshold-encrypts it (as a fallback). They can later reconstruct the key and decrypt each transaction individually. FairBlock [38] employs an Identitybased Encryption (IBE) scheme for blinding transactions: a committee of trusted keepers runs a DKG protocol [40,18,26] to generate master secret key shares for each keeper, which are tied to a master public key. For each height, an ephemeral encryption key is generated by treating the height number as identity in the IBE scheme, and the decryption procedure involves each keeper providing a share of the ephemeral decryption key. As noted in §1, blinding does not eliminate manipulation opportunities because the networklevel identities are not masked, and the contents of transactions may be inferred, e.g., when there is positive or negative news about a cryptocurrency, the nature of transaction influx to an exchange is correctly deducible with high probability. Additionally, front-running can still be implemented in its traditional form, based on predictions instead of explicit observations of transaction arrival.

Manipulation-aware DEX design. The third approach is from a market design perspective. Fair-TraDEX [35] combines FBA with commit-then-reveal style transaction masking. Each user needs to make deposits in an escrow service so that they are incentivized to post correct commitments. In P2DEX [4] servers run a secure multi-party computation (MPC) protocol to match orders. This introduces computation overhead and latencies and adds constraints inherent to the adopted MPC machinery. SPEEDEX [41] settles trades in the same block with the same clearing price that is approximated through Tâtonnement process, which increases price when demand exceeds supply and vice versa. The solution is based on general equilibrium and has assumptions on the market including perfect competition and demand independence. The process converges to an approximated equilibrium in polynomial time. The convergence-induced latencies depend on the size of outstanding orders and elasticity of demands, i.e., how demand changes with price jumps. Overall, the general equilibrium analysis utilized is from an economics perspective and aims to locate clearing prices given orders accumulated at a static time point. We adopt solution concepts from game theory that analyze the strategies of participants in dynamic sequential games. Participants may bear private information and need to take others' (and their own) future actions into consideration.

Xavier et al. [45] alternatively consider a greedy sequencing rule in two-token constant product automatic market maker (AMM). The AMM initially has reserves for both tokens and validators need to execute a buy (or sell) order if buy (or sell) orders receive a better execution in the previous execution compared with if executed at the beginning of the block. Users can potentially strategize in submitting orders when

the reserves are known. Since this sequencing rule ensures good properties inside a block, sequencers can still push submitted transactions to future blocks. Our focus is on systems where different types of actors interact including front-runners (not limited to validators) and market-makers, so we refer the readers to more comprehensive surveys for more AMM designs like FairMM [11].

The good and bad of HFT. The effects of the classical order manipulation method, HFT, are two-fold and sometimes mixed. Cvitanic and Kirilenko [12] find that the existence of HFT actors without information advantage increases liquidity. Jovanovic and Menkveld [23] find that the effects become mixed when the HFT players are better informed than normal traders. Gomber et al. [19] consider HFT to be a natural evolution of the market and the market failures like the 2010 Flash Crash to be rooted in the U.S. market structure. On the other side, Johnson et al. [22] show that the all-machine market undergoes frequent black swan events with ultrafast durations. ¹ Patterson [39] considers the phenomenon to promote self-destructive algorithm wars. Lewis [31] thinks it leads to market injustice. Menkveld [36] through empirical evidence finds that HFTs in limit-order markets hurt the market if they only have a speed advantage but increase liquidity if they also have an information advantage. They also note that HFTs preying on large orders increase transaction costs for common investors. We focus on mitigating the downsides of harmful HFT strategies.

Comparison of CLOB and FBA in centralized exchanges. It has been shown that compared with CLOB, FBA can lead to lower transaction costs [1], decrease adverse selection and spreads [37,42], achieve an optimal trade-off between liquidity and price discovery [3], and provide a "safe haven" for slower traders [49], which promotes market justice. Existing work [32,14,25] also indicates that FBA can increase market quality. The severity of the inefficiency of liquidity provision under FBA can exceed the inefficiency from latency arbitrage under CLOB [15]. This liquidity provision inefficiency of FBA originates from bid shading ² in UPDA: every equilibrium in multi-unit uniform price auctions is inefficient due to bid shading [2] (except for bidders with unit demand).

3 Model and preliminaries

In this section, we specify the DEX along with its underlying ABC protocol, the players in the trading system, and the trading target asset, especially how its price and fundamental values evolve. We also introduce game theory concepts relevant to the equilibria in the trading game.

3.1 System and adversary model

DEX. For the blockchain system implementing a distributed exchange, we assume a set of n processes called validators $N = \{P_1, \dots, P_n\}$ instantiate a secure κ -differentially order-fair atomic broadcast protocol ($\kappa \geq 0$, Definition 1). Up to $f < \lceil \frac{n}{3} \rceil$ of the validators can be Byzantine. They may behave arbitrarily but are computationally bounded, and thus cannot break standard cryptographic primitives including digital signature schemes. Validators communicate via reliable authenticated point-to-point channels. More specifically, they are connected with Byzantine first-in-first-out (FIFO) consistent broadcast links [9] (§3.2) that securely deliver messages.

The network is partially synchronous where the network imposes some known bounded message delay only after a global stabilization time (GST). In the system, the smallest block generation interval is set to be a known constant I > 0 measured in time units consistent with the trading system. This means that a block assembles user-submitted transactions accumulated in at least I time units. We can let I be the time units that the protocol takes to output after GST.

Trading system. For the trading system, we blend the dynamic models in Eibelshäuser and Smetak [15] and Budish et al. [7] with the blockchain-based decentralized exchanges. Users submit transaction orders to

¹ "Highly consequential but unlikely events that are easily explainable - but only in retrospect" [44].

² Bidders' tendency to bid less than true valuations for later units.

the DEX by sending messages to one or more validators. Batch length in FBA equals the smallest block generation interval I.

We consider an asset X with changing fundamental value V_t at time t ($t \in [0, T], T > 0$). Note that the analysis also applies to multiple assets. We assume an observable signal equals to the asset's fundamental value and evolves according to a compound Poisson jump process, drawn from a symmetric distribution F_{jp} with arrival rate λ_{jp} , bounded support, and zero mean. The fundamental value jump can be observed in the form of public or private information. We capture the absolute value of the jump with a random variable J. There are three types of risk-neutral 3 trading players:

- 1. investors who arrive stochastically with probability λ_i and have inelastic demand, with buying and selling being equally likely.
- 2. informed traders who can observe both public information that arrives stochastically with probability λ_{pb} and private information arriving stochastically with probability λ_{pr} (both affect V_t positively or negatively with equal probability).
- 3. r arbitrageurs who can apply market-making strategy or front-running strategy and intend to sell at a price higher than V_t or buy at a price lower than V_t .

The arbitrageurs here can be trading firms, arbitrage bots, etc. All three types of players are rational and only arbitrageurs actively strategize. They are addressed as market makers (or liquidity providers) when applying market-making strategy, and as front-runners when applying front-running strategy. Specifically, a front-running arbitrageur can front-run market makers, investors, or traders. Here, we consider both arbitrageurs and privately informed traders to more comprehensively capture adverse selection risks, i.e., market makers' quotes being front-run due to a lack of information or latency in observing information concerning asset value changes.

Communication links. We make the following assumptions on latencies of different communication links. The communication channels between validators experience latencies that follow distribution F_v . The communication links between each pair of common user, i.e., investors and traders, and validator have the same latency distribution F_{it} . And the communication links between each pair of arbitrageur and validator have the same latency distribution F_a . If F_a is the same as F_{it} , then arbitrageurs have the same network connections as common users. We assume the arbitrageurs invest some constant $c \ge 0$ in establishing the communication links and other setups.

Fees. The exchange collects fees from both sides of each settled trade. There is a base fee and priority fee F which can promote the order of a transaction during tie-breaking. In the analysis, we disregard the base fee as it is the same for all transactions. We let an arbitrageur successfully front-run with probability p^* under CLOB, and describe how to calculate it after introducing of-ABC in Definition 1.

Trading system states. We capture the history information at time t with \mathcal{H}_t , and it is observable by all. The state of the trading system at time t is $S_t = (P_t, J_t, (\vec{b_t}, \vec{a_t}), \mathbf{g})$ where $P_t = \mathbb{E}[V_t | \mathcal{H}_t], J_t \sim J$, $(\vec{b_t}, \vec{a_t})$ are the bid and ask prices, and vector \mathbf{g} records the probabilities that arbitrageurs respond to investors' and traders' orders with front-running at each level of the order book.

3.2 Definitions

As mentioned in §1, FBA alone is not effective in countering order manipulations. Because validators can push victim transactions to future blocks. We adopt the following of-ABC security definition in [9] since κ -differential order fairness is shown to be equivalent to batch order fairness [28], and it simplifies the expression of front-running probability p^* .

First, a broadcast protocol allows a sender to send its value to a set of parties. Its security requires agreement where every correct party delivers the same value, validity where every correct party delivers the sender's value when the sender is correct, and termination, where correct parties eventually deliver. A

³ Obtaining return amount x for sure and in expectation generate the same utility.

secure consensus protocol where each party has an initial value also requires agreement, validity where if all correct players have the same value then they deliver this value, and termination. Atomic broadcast or ABC is closely related to consensus, and it ensures that the correct parties deliver the same messages in the same order. We next define of ABC.

Definition 1 (Differentially Order-Fair ABC [9]). A secure κ -differentially order-fair atomic broadcast protocol satisfies the following properties:

- 1. **Agreement**: If a message m is delivered by some correct process, then m is eventually delivered by every correct process.
- 2. Integrity: No message is delivered more than once.
- 3. Weak validity: If all processes are correct and broadcast a finite number of messages, every correct process eventually delivers these broadcast messages.
- 4. **Total order**: Let m and m_0 be two messages, P_i and P_j be correct processes that deliver m, m_0 . If P_i delivers m before m_0 , P_j also delivers m before m_0 .
- 5. κ -differential order fairness: If $b(m; m_0) > b(m_0; m) + 2f + \kappa$, then no correct process delivers m_0 before m.

Here $b(m_1; m_2)$ captures the number of processes broadcasting m_1 before m_2 .

In the of-ABC protocol in [9], each validator can broadcast messages over Byzantine FIFO consistent broadcast (BCCH) links. The BCCH links between validators ensure agreement, validity, and FIFO delivery, where if a correct party broadcasts a message m before m_0 , then no correct party delivers m_0 before m. Note that FIFO delivery only concerns the delivery order of messages sent by the same party. Validators collect messages for a cutoff time interval and then order the received messages.

Front-running success probability. Suppose arbitrageur j decides to front-run user i with transaction m_j after seeing i's transaction m_i . Let D be the difference of two independent random variables capturing transaction delivering latencies of i and j on each communication link between validators. Let D follow distribution \tilde{F}_{lt} . For example, if F_v is instantiated with normal distribution N(1,1), then \tilde{F}_{lt} is normal distribution N(0,2). Let \tilde{F}_{lt}^C be the CDF of \tilde{F}_{lt} . Assuming the worst case that j sees m_i immediately after it arrives at some validator, immediately delivers m_j at a validator, and always wins when neither transaction is broadcast first by more than $2f + \kappa$ validators, we can compute p^* as follows:

$$p^* = 1 - \mathbb{P}[b(m_i; m_j) > b(m_j; m_i) + 2f + \kappa] = 1 - \sum_{s=2f+\kappa-1}^{n-1} \binom{n-1}{s} [\tilde{F}_{lt}^C(0)]^s [1 - \tilde{F}_{lt}^C(0)]^{n-1-s}$$

We have n-1 in the summation instead of n because m_i first arrives at a validator and is then observed by arbitrageur i. If Byzantine validators do not relay messages, we replace n-1 in the equation with n-f-1. Note that if each validator's communication links impose latencies that follow different distributions, we can still compute p^* [10].

Solution concepts. In the trading game, r arbitrageurs are the strategic players and aim to maximize their returns. A strategy is a probabilistic distribution over possible actions, with all mass condensed at one action for a pure strategy. A solution concept captures a profile or snapshot of all players' strategies in an equilibrium. In the trading game, the solution concept essentially gives the equilibrium trading prices of the asset X. We are interested in the markups from the fundamental value of X in equilibrium when arbitrageurs play market-making strategies. Because the markups imply the welfare loss for common investors and traders.

We naturally adopt stationary Markov Perfect Equilibria (MPE) [34] for CLOB since the parameters of player and information arrival rates in the stochastic game are time-independent.

Definition 2 (Stationary MPE [34]). A Nash equilibrium (NE) is a strategy profile \vec{s} where no player increases utility by unilaterally deviating from \vec{s} . A subgame perfect equilibrium (SPE) is a strategy profile \vec{s} that forms a NE for any subgame of the original game. An MPE is an SPE in which all players play Markov strategies, i.e., strategies that depend only on the current state of the game. A stationary MPE is an MPE where strategies are time-independent.

We utilize a weaker notion, Order Book Equilibrium (OBE) for FBA since stationary MPE may not exist for FBA [8]. Intuitively, under FBA, other arbitrageurs do not have the incentive to undercut when an arbitrageur provides liquidity at the bid-ask spread that equals costs from market-making. Since others do not undercut, the arbitrageur has the incentive to widen the spread to increase profits. But this would result in others undercutting the widened quotes.

Definition 3 (OBE [8]). Given state S_t , an OBE at time t is a set of orders submitted by all arbitrageurs such that the following conditions hold: there exist (1) no safe profitable price improvements and (2) no other robust profitable deviations.

Here, a profitable price improvement is safe if it remains strictly profitable after other arbitrageurs take profitable responses, e.g., liquidity withdrawals, after the improvement. A profitable deviation is robust if it remains strictly profitable after other arbitrageurs react with profitable price improvements or liquidity withdrawals. Price improvements are liquidity provisions improving current quotes and liquidity withdrawals are cancellations of limit orders. See [8] for further formal justifications for OBE. OBE is weaker than MPE in the sense that it allows the existence of unilateral deviations that increase utility as long as they can be made unprofitable by others' reactions.

Exogenous and endogenous entry. We focus on the exogenous entry of arbitrageurs to the trading game, which means that all arbitrageurs automatically join. When we consider endogenous entry, arbitrageurs first decide whether to enter into the trading game by balancing costs and profits. Those who decide entry participate in the game. In this case, we need to first solve for the number of arbitrageurs who enter.

4 DEX under CLOB and FBA

We now model the DEXes under the CLOB and FBA market designs. We first detail the events and then provide equilibrium analysis with a focus on welfare loss in both transaction execution models.

4.1 Order of events

We abstract the DEX as run by validators connected with BCCH links. Arbitrageurs, investors, and traders submit transactions by sending messages to one or more validators. Validators then broadcast received messages to each other. In the following stochastic trading game T, we capture the arrival of the three types of players and the validators' ordering and executing transactions. Step \odot ensures κ -differential order fairness.

- (1) Repeat the following trading game for asset X with initial fundamental value V_0 .
 - (a) Arbitrageurs submit orders to the exchange. They can submit standard limit orders, withdrawals, and immediate-or-cancel (IOC) orders which automatically withdraw the portion not executed immediately.
 - (b) Nature then moves to trigger one of the following events:
 - (1) An investor arrives with probability λ_i and submits an IOC order to the exchange. This may change order book quotes $(\vec{b_t}, \vec{a_t})$ at time t according to Bayesian updating but not the fundamental value.
 - (2) A private information event occurs with probability λ_{pr} , resulting in a fundamental value jump, and an informed trader submits an IOC order to the exchange. This may change the order book quotes.
 - (3) A public information event occurs with probability λ_{pb} , resulting in a fundamental value jump, and all arbitrageurs submit IOCs to provide liquidity or front-run stale quotes. They potentially submit necessary withdrawals to the exchange. This may change the order book quotes.
 - (4) Null event. The fundamental value and order books do not change.

- © Each validator receiving a transaction message broadcasts it using BCCH links. Go to step ⓐ until the validators can order the transactions according to the of-ABC protocol. Attached priority fees help break ties, e.g., messages in the same strongly connected component in of-ABC algorithms [9,28].
- (d) CLOB. After outputting a block, the validators running the exchange execute the transactions sequentially in the proposed order.

FBA. After outputting a block, the validators in the exchange execute the transactions by clearing the market with a UPDA: (1) the validators first batch the orders in the current block and all previous outstanding orders; (2) they compute the aggregate demand and supply; (3) if there are intersections, the market clears where supply meets demand, at a uniform price. When there are conflicting ties, priority fees can lift the orders of transaction executions and further ties can be broken uniformly at random. Unmatched orders enter into the next trading game as outstanding orders.

4.2 MPE under CLOB

We denote the expected fundamental value conditioned on history and a new sell or buy order with $\mathbb{E}[V_t|\mathcal{H}_t, buy]$ and $\mathbb{E}[V_t|\mathcal{H}_t, sell]$. Arbitrageurs can provide liquidity, front-run other liquidity providers by sniping their stale quotes, or front-run common investors and traders. Let Q be the maximum number of units needed by investors and p_j be the probability of an investor transacting j units (j = 1, ..., Q).

Theorem 1. There exists a stationary MPE in trading game T under the CLOB processing model. Bid and ask prices at the k-th level of the LOB in equilibrium satisfy

$$b_t^k = \mathbb{E}[V_t | \mathcal{H}_t, sell] = P_t - \frac{s_k}{2}, a_t^k = \mathbb{E}[V_t | \mathcal{H}_t, buy] = P_t + \frac{s_k}{2}$$

where s_k satisfies

$$\lambda_i \sum_{j=k}^Q p_j \frac{s_k}{2} - \lambda_{pr} (1 + p^* \frac{\mathsf{g}_k}{\mathsf{g}_k(r-2) + 1}) \bar{J}_k - \lambda_{pb} \bar{J}_k + (\lambda_i + \lambda_{pr}) \mathsf{g}_k \mathsf{F} = 0$$

Here, $\bar{J}_k = \mathbb{P}[J > \frac{s_k}{2}]\mathbb{E}[J - \frac{s_k}{2}|J > \frac{s_k}{2}]$, and g_k is the probability that an arbitrageur front-runs traders and investors at the k-th level. In equilibrium, g_k is then updated to take the value that maximizes front-running profits, i.e., $\lambda_{pb}\frac{1}{r}\bar{J}_k - \lambda_{pb}\mathsf{F} + \lambda_{pr}p^*\frac{\mathsf{g}_k}{\mathsf{g}_k(r-2)+1}\bar{J}_k - (\lambda_i + \lambda_{pr})\mathsf{g}_k\mathsf{F}$. The expected price impact of the first unit of an incoming order is $\Delta = \tilde{J}_1\frac{\lambda_{pr}}{\lambda_{pr}+\lambda_i}$ where $\tilde{J}_k = \mathbb{P}[J > \frac{s_k}{2}]\mathbb{E}[J|J > \frac{s_k}{2}]$; its expected markup from liquidity providers is $(\lambda_{pr} + \lambda_i)(\frac{s_1}{2} - \Delta)$.

Interpretations. The markups are the extra transaction costs. They are affected by the arrival rates of investors (λ_i) , traders (λ_{pr}) , and public information (λ_{pb}) , the front-running success probability (p^*) , the priority fee F, the jump size, the number of arbitrageurs (r), and the transaction size distributions. The effects that they have on the markups are correlated and not necessarily linear. For example, more arrivals of investors compared to informed traders decrease g_k , which decreases the markup, but also decreases price impact, which increases the markup. We give more comparisons after discussing the markups under FBA.

Proof. Let s_k denote the spread at the k-th level. In equilibrium, liquidity provision and front-running have the same profits because arbitrageurs can choose their roles freely. In a DEX, unlike in centralized exchanges, an arbitrageur can also front-run privately informed traders. Since a transaction may be submitted by a common investor or privately informed trader, the front-runner can expect premiums when privately informed traders arrive. This happens with probability $\frac{\lambda_{pr}}{\lambda_{pr}+\lambda_i}$ if the arbitrageur always front-runs ($\mathbf{g}_k=\vec{1}$ for $k=1,\ldots,Q$). More meaningfully, the arbitrageur front-runs at the k-th level with probability $\mathbf{g}_k \in [0,1]$. As described, the front-running transaction beats a target transaction with probability p^* .

Suppose there are $q \leq Q$ ($Q \in N$) quantities needed, we need for each $k = 1, \ldots, Q$, s_k satisfies

Profits from investor
$$\lambda_{i} \sum_{j=k}^{Q} p_{j} \frac{s_{k}}{2} - \lambda_{pr} \overline{J_{k}} - \lambda_{pr} \overline{J_{k}} - \lambda_{pr} \overline{J_{k}} + \lambda_{pb} \overline{F}$$

$$= \lambda_{pb} \frac{1}{r} \overline{J_{k}} - \lambda_{pb} \overline{F} + \lambda_{pr} p^{*} \frac{g_{k}}{g_{k}(r-2)+1} \overline{J_{k}} - \lambda_{pr} \overline{J_{k}} - \lambda_{pr}$$

where $\bar{J}_k = \mathbb{P}[J > \frac{s_k}{2}]\mathbb{E}[J - \frac{s_k}{2}|J > \frac{s_k}{2}]$ is the expected extra value change uncovered by the spread (induced by the asset fundamental value jump), and p_j is the probability of an investor transacting j units.

In Equation (1), the left-hand side is the profits from market-making and the right-hand side is the returns from sniping stale quotes of market-makers and front-running private traders, which are the extra rents present in DEXes. The bid and ask prices at k-th level of LOB in equilibrium then satisfy this spread s_k . To update g_k stored in the current state, we can lay out the right-hand side as a function of g_k and solve for g_k that maximizes the function value in range [0, 1] for a front-running arbitrageur, by taking the first and second derivatives of the function. More specifically, after solving Equation (1), we can express each $\frac{s_k}{2}$ and \bar{J}_k . Maximizing the right-hand side of Equation (1) gives the value for g_k . One can solve it analytically or numerically if closed-form formulas do not exist for specific distributions of the jump J.

We next discuss the additional transaction costs for common investors. When one front-runs common investors, the fundamental value does not change, so the front-runner only earns premiums when private traders arrive, which happens with probability $\frac{\lambda_{pr}}{\lambda_{pr}+\lambda_i}$. To compute the welfare loss for investors, we can then treat front-running orders as part of private traders' orders. Depending on the size of the jump, they can place multi-unit orders profitably, and we first consider one unit. Let Δ be the expected price impact of an incoming unit order. Since one cannot distinguish between informed traders from investors, we have

$$\Delta = \pm \mathbb{E}[V_t | \mathcal{H}_t, \lambda_i \vee \lambda_{pr}] \mp P_t = \pm \mathbb{E}[V_t | \mathcal{H}_t, \lambda_i \vee \lambda_{pr}] \mp \mathbb{E}[V_t | \mathcal{H}_t] = \tilde{J}_1 \frac{\lambda_{pr}}{\lambda_{pr} + \lambda_i}$$

where +,- indicate buy, sell order and $\tilde{J}_k = \mathbb{P}[J > \frac{s_k}{2}]\mathbb{E}[J|J > \frac{s_k}{2}]$ (for $1 \leq k \leq Q$). After absorbing the price impact, the additional markup from the market maker for the first unit is then $M_{CLOB} = \frac{s_1}{2} - \Delta$. The expected markup is then $(\lambda_{pr} + \lambda_i)M_{CLOB}$. We can calculate for other units in the same way.

Finally, the welfare loss for arbitrageurs is the costs for implementing front-running, $r \cdot \mathbf{c} + \sum_{k=1}^{Q} \mathbf{g}_k(r-1) \cdot p^* \mathbf{F}$. The first term is the costs of establishing faster communication links with validators. The second term is the unnecessary priority fees attached in scenarios where one can originally successfully front-run due to faster transaction dissemination.

4.3 OBE under FBA

In the following, we focus on excessive demand and analyze the ask quotes for simplicity. We only need to adapt signs to analyze bid quotes. Let Z_I denote the excess demand (demand minus supply) during a batch interval and $Z_I \leq Q+1$ ($Q \in N$) is bounded. We borrow \bar{J}_k and \tilde{J}_k from Theorem 1.

Theorem 2. There exists an OBE in pure strategies in trading game T under the FBA processing model. If $|Z_I| \leq Q+1$, bid and ask quotes in equilibrium satisfy

$$b_{(I)}^k = P_I - \Delta_k - M_k, a_{(I)}^k = P_I + \Delta_k + M_k, k = 1, \dots, Q + 1$$

where $\Delta_k = \tilde{J}_k \frac{\lambda_{pr}}{\lambda_{pr} + \lambda_i}$ is the expected price impact from informed traders,

$$M_k = \sum_{u=k}^{Q} \Delta_u \prod_{v=k}^{u} \alpha_v, k = 1, \dots, Q$$

and $M_{Q+1}=0$ are the markups from liquidity providers, $\alpha_k=\frac{q_{k+1}}{q_k+q_{k+1}}$ and $q_k=\mathbb{P}[Z_I=k|\ |Z_I|\leq Q+1]$. The expected markup per unit time is $\frac{2}{I}\sum_{k=1}^Q kq_kM_k$.

Interpretations. If we fix the probability distribution of the excessive demand and the jump distribution, the markups then increase with the arrival of privately informed traders, λ_{pr} , and decrease with the arrival of investors, λ_i .

Proof. We show by induction that the markups $\{M_k\}_{k=1,\dots,Q+1}$ are the largest such that there exist (i) neither strictly profitable safe price improvements (ii) nor strictly profitable robust deviations. More specifically, (i) means that for a targeted quote at the k-th level, an arbitrageur cannot safely insert a new profitable quote at this level, i.e., pushing the original k-th quote to be the new (k+1)-th quote and so forth, and the current k-th quote issuer cannot improve this quote (after being pushed) to safely increase profits. (ii) means that the k-th quote issuer cannot robustly increase profits by issuing or updating quotes deviating from the stated markups, considering that others can react with quote withdrawals and updates.

(Q+1)-th level. We first show that $M_{Q+1}=0$ is the OBE, i.e., $a_{(I)}^{Q+1}=P_I+\Delta_{Q+1}$. We write $a_{(I)}^{Q+1}$ as a^{Q+1} in the following analysis for visual simplicity. (i) We show that safe profitable price improvements on the last quote with zero markups do not exist. First, an arbitrageur inserting a new (Q+1)-th quote with a price lower than a^{Q+1} experiences a loss in expectation. Because the spread does not cover the potential jump in asset value. Similarly, the current (Q+1)-th quote issuer does not have the incentive to lower the quote. Second, the issuer is not incentivized to increase the quote either, i.e., to $\bar{a}^{Q+1}=P_I+\Delta_{Q+1}+\epsilon$ for even a small $\epsilon>0$. Because this is not a safe price improvement: another arbitrageur can safely insert a quote at price a^{Q+1} and push this issuer's new quote to be the (Q+2)-th quote but $Z_I \leq Q+1$. The expected profit after others' reaction is zero, which is less than the original profit.

(ii) We next show that robust profitable deviations do not exist. Similar to the above reasoning, the owner of the (Q+1)-th quote is not incentivized to insert a new quote or update another quote to be the (Q+1)-th that deviates from zero markups.

(k+1)-th level. Suppose there do not exist safe profitable price improvements or robust profitable deviations on the (k+1)-th quote. The markups on the (k+1)-th level and onwards to the (Q+1)-th level are as stated.

k-th level. We show that the stated M_k is the OBE. (i) Suppose there exists a safe profitable price improvement on the k-th quote by ϵ , i.e., $\bar{a}^k = a^k - \epsilon$. First, consider an arbitrageur who does not own quotes and inserts a new k-th quote. If $Z_I = k$, then the arbitrageur expects profits, and if $Z_I > k$, then a loss is expected because the settlement price would be below \bar{a}^k . Let $\Delta_k = \tilde{J}_k \frac{\lambda_{pr}}{\lambda_{pr} + \lambda_i}$, and $q_k = \mathbb{P}[Z_I = k \mid |Z_I| \leq Q + 1]$, and we can express the profits as follows:

$$A_k = q_k(M_k - \epsilon) + \sum_{i=k+1}^{Q+1} q_i(M_{i-1} - \Delta_{i-1})$$

For this improvement to be safe, the largest markup needs to be achieved, i.e., $\epsilon \to 0$, so that no other arbitrageurs have the incentive to undercut again. If this undercutting is profitable, then $A_k \ge 0$. We know that the analog inequality for the (k+1)-th level ask quote is

$$A_{k+1} = q_{k+1}(M_{k+1} - \epsilon) + \sum_{i=k+2}^{Q+1} q_i(M_{i-1} - \Delta_{i-1}) < 0$$

For contradiction, we let the left-hand side equal to A_k for $\epsilon \to 0$. This gives us $M_k = \alpha_k \Delta_k + \alpha_k M_{k+1}$ where $\alpha_k = \frac{q_{k+1}}{q_k + q_{k+1}}$. Applying this recursively, we obtain $M_k = \sum_{u=k}^Q \Delta_u \prod_{v=k}^u \alpha_v$.

Next, consider an arbitrageur who owns some existing quotes. To implement price improvements, she can insert a new k-th quote, which results in even more loss when $Z_I > k$, compared with a loss in A_k . If she moves up a lower quote at the k^* -th $(k^* < k)$ level to the k-th (if she owns one), the expected profit is lower because some profits are lost for $k^* \le Z_I < k$. If she moves down one of the quotes to be the new k-th quote

(if she owns one), then the extra profit generated is bounded by A_k . Because A_k is the extra profits of an arbitrageur who originally expects 0 returns.

(ii) Suppose there exists a robust profitable deviation for the issuer of the k-th quote. To allow the issuer to deviate other than improve quotes (which we have just discussed above), we let the issuer own some quotes under the k-th level. The issuer can update the k-th quote to be the k'-th quote ($k' \geq k$) with a markup higher than the original $M_{k'}$ but still lower than $\Delta_{k'+1} + M_{k'+1}$. This is profitable but not robust: another arbitrageur can re-issue the original k-th level quote and push the updated k'-th quote to be the (k'+1)-th. This arbitrageur's price improvement is safe because it restores the original k-th quote.

The expected markup of the liquidity providers per unit of time can then be calculated as $\frac{2}{I} \sum_{k=1}^{Q} kq_k M_k$. The arbitrageurs do not experience welfare loss.

4.4 Welfare loss comparison

General comparison. For common investors and informed traders, when there are no privately informed traders ($\lambda_{pr}=0$) but exists public information, FBA is strictly better than CLOB because the markup in FBA becomes zero while the markups for CLOB are still positive. When there is no public information ($\lambda_{pb}=0$) but exists privately informed traders, the markup for CLOB decreases with λ_{pr} while FBA has positive markups that increase with λ_{pr} .

Compare in examples. For a more straightforward comparison of transaction costs, we can let the jump J be a constant instead of a random variable and consider unit orders. Under CLOB, let $a=1+p^*\frac{\mathsf{g}_1}{\mathsf{g}_1(r-2)+1}$ and the spread in Equation (1) satisfies $\frac{s}{2}=\frac{(\lambda_{pb}+a\lambda_{pr})J-(\lambda_i+\lambda_{pr})\mathsf{g}_1\mathsf{F}}{\lambda_{pb}+a\lambda_{pr}+\lambda_i}$. And the price impact from informed trader is $\Delta=J\frac{\lambda_{pr}}{\lambda_{pr}+\lambda_i}$. Then the markup can be computed as

$$M_{CLOB} = J \frac{\lambda_i \lambda_{pb} + (a-1)\lambda_i \lambda_{pr}}{(\lambda_i + a\lambda_{pr} + \lambda_{pb})(\lambda_{pr} + \lambda_i)} - \frac{(\lambda_i + \lambda_{pr})g_1 F}{\lambda_{pb} + a\lambda_{pr} + \lambda_i}$$

and the expected markup is $J\frac{\lambda_i\lambda_{pb}+(a-1)\lambda_i\lambda_{pr}}{\lambda_i+a\lambda_{pr}+\lambda_{pb}}-\frac{(\lambda_i+\lambda_{pr})^2\mathsf{g}_1\mathsf{F}}{\lambda_{pb}+a\lambda_{pr}+\lambda_i}$. This increases with the jump size J and decreases with priority fee F . For simplicity, we can measure F relative to the jump, i.e., let $\mathsf{F}=xJ$ for some x<1.

Under FBA, $\Delta_k = J \frac{\lambda_{pr}}{\lambda_{pr} + \lambda_i}$ for each level k, and $\frac{2}{I} \sum_{k=1}^Q k q_k M_k$ is the expected markups per unit time. It is straightforward to see that if block generation interval I is sufficiently large, FBA can always have smaller markups. But this may not be the most efficient in terms of liquidity provision, which we discuss in the next section. Suppose we consider Q=2 for simplicity, then the expected markups become $\frac{2}{I}J \frac{\lambda_{pr}}{\lambda_{pr} + \lambda_i} (q_1(\alpha_1 + \alpha_1 \alpha_2) + 2q_2\alpha_2)$. Z_I captures the excess demand and follows a truncated Skellam distribution. For simplicity and without loss of generality, we let $q_3=q_2=1/16, q_1=1/8$, FBA is better if $\frac{\lambda_i \lambda_{pb} + (a-1)\lambda_i \lambda_{pr} - (\lambda_i + \lambda_{pr})^2 \mathbf{g}_1 x}{\lambda_i + a \lambda_{pr} + \lambda_{pb}} > \frac{\lambda_{pr}}{I(\lambda_{pr} + \lambda_i)}$, i.e.,

$$\lambda_{pb} > \frac{\lambda_{pr}(\lambda_i + a\lambda_{pr}) - I(\lambda_i + \lambda_{pr})[(a-1)\lambda_i\lambda_{pr} - (\lambda_i + \lambda_{pr})^2 \mathsf{g}_1 x]}{I\lambda_i(\lambda_{pr} + \lambda_i) - \lambda_{pr}}$$

Note that $q_1 + q_2 + q_3$ is smaller than 1 because the excess demand follows the Skellam distribution. Intuitively, users' transactions can be executed against each other and the market makers only need to satisfy excessive demands. We depict the regions where FBA induces less welfare loss for common investors and traders in Figure 1. We intentionally let x or F be not too small, i.e., x = 0.15, in the figure to make the regions where CLOB imposes fewer markups more visible in this example. This means that if the price jump J = \$1, the priority fee is \$0.15. When the priority fee F decreases, the region where FBA has fewer markups expands.

Endogenous entry for arbitrageurs. When all arbitrageurs automatically enter the trading game, they experience welfare loss $(r \cdot \mathsf{c} + \sum_{k=1}^Q \mathsf{g}_k (r-1) \cdot p^*\mathsf{F})$ under CLOB and zero welfare loss under FBA. We can adapt the analysis to model endogenous entry. This involves first solving for the number of arbitrageurs that

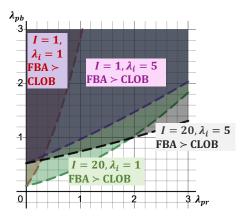


Fig. 1: Example regions where FBA has less welfare loss when F = 0.15J, $p^* = 0.8$, r = 35, truncated at $\lambda_{pr} = 3$, $\lambda_{pb} = 3$. An interactive graph for tuning parameters can be found on this page [21].

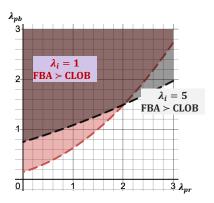


Fig. 2: Example regions where FBA has smaller bidask spreads under the same parameters as Figure 1 and additionally, $q_0 = 0.5, Q = 100$. An interactive graph for tuning parameters can be found here [20].

decide to enter the trading game in equilibrium, r^* , via the zero entry condition. For a unit order, one solves r^* that satisfies

$$\lambda_i \sum_{j=1}^Q p_j rac{s_1}{2} - \lambda_{pr} ar{J}_1 - (\lambda_{pb} rac{r^*-1}{r^*} ar{J}_1 + \lambda_{pb} \mathsf{F}) = \mathsf{c}$$

Here, the left-hand side is the profit from liquidity provision (or front-running) and the right-hand side is the cost of setup, e.g., establishing communication links with validators. Arbitrageurs' welfare loss under CLOB is then $(r^* \cdot \mathsf{c} + \sum_{k=1}^Q \mathsf{g}_k(r^* - 1) \cdot p^*\mathsf{F})$, which is still > 0.

4.5 Bid-ask spread comparison

In FBA, trades submitted by common investors and informed traders can be executed among themselves without hitting quotes from market makers, especially in liquid markets, which can result in less welfare loss. In this section, we turn to compare the two execution models from a liquidity perspective, with the bid-ask spread being the proxy quantity that we examine. The spread consists of the price impact from orders driven by information about assets' valuation changes, and markups from market makers. We have compared the expected markups upon a transaction arrival under CLOB with the expected markups per time unit under FBA in §4.4. We focus on the explicit bid-ask spreads in this section.

Compare in examples. In the same setting with constant jump size, under CLOB, the bid-ask spread is $s = \frac{(\lambda_{pb} + a\lambda_{pr})J - (\lambda_i + \lambda_{pr})g_1\mathsf{F}}{\lambda_{pb} + a\lambda_{pr} + \lambda_i}$. Under FBA, we denote the bid-ask spread as s_F and $s_F = 2(\Delta + M_1)$. Consider $Q \ge 1$. Same as before, although Z_I follows a Skellam distribution, for simplicity and without loss of generality, we consider $q_1 = \frac{1}{8}$ and $q_2 = \cdots = q_{Q+1} = \frac{1}{8Q}$. Then

$$M_1 = \sum_{v=1}^{Q} \Delta_u \prod_{v=1}^{u} \alpha_v = \Delta(\alpha_1 + \alpha_1 \alpha_2 + \dots + \alpha_1 \cdots \alpha_Q) = \Delta \frac{\frac{1}{8Q}}{\frac{1}{8} + \frac{1}{8Q}} (1 + \frac{1}{2} + \dots + \frac{1}{2^{Q-1}})$$

 $=\Delta \frac{2}{Q+1}(1-\frac{1}{2^Q})$. Let a transaction be fulfilled with transactions submitted by other common investors or traders with probability $q_0 = 1 - 2\sum_{j=1}^{Q+1} q_j$. FBA provides a smaller bid-ask spread in expectation when $(1-q_0)s > s_F$. This gives

$$2(1-q_0)\frac{\lambda_{pr}}{\lambda_{pr}+\lambda_i}\left(1+\frac{2}{Q+1}\left(1-\frac{1}{2^Q}\right)\right)<\frac{\lambda_{pb}+a\lambda_{pr}-(\lambda_i+\lambda_{pr})\mathsf{g}_1x}{\lambda_{pb}+a\lambda_{pr}+\lambda_i}$$

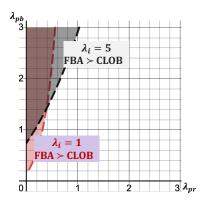


Fig. 3: Example regions where FBA has smaller bid-ask spreads in an extremely thin market under the same parameters as Figure 1 with Q = 100.

We depict the regions where FBA has a smaller expected bid-ask spread in Figure 2. When there is no private information, the spread under FBA is constant while the spread under CLOB increases with λ_{pb} . We can also consider general parameterization for Z_I 's distribution. When $\sum_{u=1}^{Q} \prod_{v=1}^{u} \alpha_v$ decreases, the region where FBA has smaller spreads expands. In Figure 3, we demonstrate the regions where FBA has smaller spreads when the market is extremely thin, i.e., a single transaction appears in the batch and potentially trades multiple units. Then each investor or trader directly trades against market makers' orders. This implies that liquidity provision under FBA worsens as the market becomes thinner.

5 Concluding remarks and future directions

In this paper, we have explored the adoption of frequent batch auction (FBA) as a market design response to mitigate general order manipulations in decentralized exchanges (DEXes) constructed using the of-ABC mechanism. The core idea behind FBA is settling trades in the same batch at the same price, while of-ABC ensures the provision of time-based batch order fairness or its equivalent. To demonstrate another benefit of adopting FBA, we have computed and compared the welfare loss under FBA and the current mainstream processing model in financial markets, CLOB, at their respective market equilibria. Our analysis reveals that FBA leads to a lower welfare loss under specific conditions. This includes scenarios where public information regarding the valuation of an asset is revealed more frequently compared to private information, when the batch auction occurs at an appropriate frequency, or when the priority fees for validators are small relative to the asset's valuation jump size.

In future research, several avenues of investigation could be pursued as extensions of this paper. It can be interesting to (1) empirically and theoretically analyze the benefits and downsides of front-running in DEXes in terms of market efficiency and quality, (2) compare other aspects of the FBA and CLOB processing models in the DEX context, e.g., price discovery, (3) introduce competition among DEXes instead of modeling a single exchange, and (4) devise and solve for stronger solution concepts for the stochastic trading game.

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