TL;DR

: Oligopoly is better than Monopoly. Variable supply of execution rights can help achieving oligopoly in the proposer market.

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Execution tickets have been proposed as an alternative mechanism to Proof-of-Stake (PoS) to allocate proposer rights in Ethereum with the goal to separate attestation from proposing. As potential downsides of the execution tickets proposal, it has been argued that it could lead to strong centralization in the proposer market and that this effect would be exacerbated if there is a secondary market for proposal rights, which would give performant builders a benefit from acquiring a substantial share of tickets.

On the other hand, we don't observe meaningful integration of proposing and building in the current PBS market structure in PoS Ethereum, even in the presence of a secondary market (aka MEV boost), and the proposer set is decentralized in a meaningful way. There are arguably several reasons for this, the bundling of two roles (attesting and proposing) makes integration of proposers and builders less attractive. Moreover, there are path-dependent factors and reputational factors for validators in play. These factors have already been discussed extensively. Instead, I want to propose another reason for why execution tickets could lead to proposer monopoly whereas PoS in its current form merely leads to proposer oligopoly. The point is basic, and has been implicit in previous work[1]

, but I think has never been pointed out succinctly in this context:

The original execution tickets proposal does not specify how many tickets should be supplied at each point in time and suggests a parameterisable dynamic pricing mechanism. However, subsequent research has usually considered variants of the mechanism where there is a constant number of tickets, N

, in circulation (where $N=1$

in the extreme case of execution auctions). In other words, we have a perfectly inelastic supply

of execution tickets at any point in time. Contrast that with the current world of PoS, where nodes can run as many validators as they want, in other words, we have elastic supply

(of the right to run validators). This naturally raises the question: does the choice of inelastic versus elastic supply make a difference? I will argue that it potentially does for the question of market concentration.

Fixed v Variable Supply

Let's look at the case of i.i.d. rewards and risk neutral validators. Denote the reward from proposing at time t

for validator i

by $R_{i,t}\geq0$

. Denote the (time independent) expected reward for validator i

by $\bar{R}_{i}\equiv\mathbb{E}[R_{i,t}]=\mathbb{E}[R_{i,t'}]$

. Moreover, denote validator i

's capital cost by r_i

.

Common knowledge of these expected values and the capital costs is a bit of a far fetched assumption admittedly, but let's assume it for the moment. Let's compare two scenarios:

- Perfectly In-Elastic Supply

: In the execution tickets world this means a constant

supply of N

tickets and a variable

price P

, determined in equilibrium.

- Perfectly Elastic Supply:

In the execution tickets world this would mean a variable supply of N tickets, determined in equilibrium, and a constant price P. Proof of Stake is equivalent to a version of execution tickets in which there is a variable supply of “tickets” (validators), drawn “tickets” don’t become invalid for the future (proposing a block doesn’t decrease the chance of a validator to propose the next block), and we have a constant price to buy a “ticket”.[2]

We can follow the calculation of Burian et al. to find that in the execution ticket case a (risk neutral) validator chooses their execution ticket holdings k_i (by the i.i.d assumption the optimal holding won’t depend on time) so that it solves

$$\max_{0\leq k_i\leq N}I_{i,t}\left(\mathbb{E}[R_{i,t}]-P\right)-r_iPk_i,$$

where $I_{i,t}$ is the probability that a ticket of i is selected in period t.

Note that for the case of perfectly inelastic supply of execution tickets, we have:

$$I_{i,t}=\frac{k_{i}}{N}.$$

whereas for the case of perfectly elastic supply we would have:

$$I_{i,t}=\frac{k_i}{\sum_{j}k_j}$$

Let’s order indices by the expected value per ticket for the holder $\tfrac{\bar{R}_1}{1+r_1N}\geq \tfrac{\bar{R}_2}{1+r_2N}\geq \tfrac{\bar{R}_3}{1+r_3N}\geq\ldots$. It is straightforward to see that for perfectly in-elastic supply of execution tickets, in equilibrium the highest value bidder(s) will buy all the execution tickets and the equilibrium price is $\tfrac{\bar{R}_{1}}{1+r_{1}N}\geq P\geq\tfrac{\bar{R}_{j}}{1+r_{j}N}$ for the smallest index j for which the value is strictly smaller than $\tfrac{\bar{R}_1}{1+r_1N}$. Thus if all expected values per ticket are different we have one proposer in equilibrium.

Next let’s look at the case of PoS. In that case, the optimization problem of determining how much to stake k_i is simply

$$\max_{0\leq k_i}I_{i,t}\mathbb{E}[R_{i,t}]-r_ik_i.$$

Now let’s derive the equilibrium for PoS with perfectly elastic supply (if we would have execution tickets with perfectly elastic supply similar results would hold, see below).

Order indices by expected return per unit of stake for the staker, $\tfrac{\bar{R}_1}{r_1}\geq \tfrac{\bar{R}_2}{r_2}\geq \tfrac{\bar{R}_3}{r_3}\geq\ldots$. One can show that there is a unique (Nash) equilibrium[3] of the game, where the number of participants N in equilibrium is

$$N=\max\left\{n\geq 1: \frac{\bar{R}_n}{r_n}>\tfrac{n-1}{n}H\left(\frac{\bar{R}_1}{r_1},\ldots,\frac{\bar{R}_n}{r_n}\right)\right\}$$

where H

denotes the harmonic average of values among participants. Now is N

small or large? That perfectly depends on how much dispersion there is in values, but generally we have more than one proposer in equilibrium

.

The same logic applies for execution tickets with perfectly elastic supply: Order indices by discounted value of expected returns such that $\tfrac{\bar{R}_1}{1+r_1N}\geq \tfrac{\bar{R}_2}{1+r_2N}\geq \tfrac{\bar{R}_3}{1+r_3N}\geq\ldots$

. In the unique (Nash) equilibrium the number of participants N

in equilibrium is

$$N=\max\left\{n\geq 1: \frac{\bar{R}_n}{1+r_nN}>\tfrac{n-1}{n}H\left(\frac{\bar{R}_1}{1+r_1N},\ldots,\frac{\bar{R}_n}{1+r_nN}\right)\right\}.$$

Thus, a similar effect could be obtained with a version of execution tickets with elastic supply. For example, we could imagine a market design where execution tickets are sold along a convex bonding curve where parameters of that curve are adapted over time based on demand.

Private Value

Commonly known values (or costs of capital) are of course un-realistic assumptions. However, I want to argue that the previous observation is driven by the fixed supply assumption for the execution tickets and not by the complete information assumption. More precisely, one can show that even in a world where the value and/or cost of proposing is private information to the proposers and those proposers are risk-neutral, any reasonable mechanisms that would sell a fixed supply

of execution tickets would in equilibrium necessarily sell the whole supply to the proposer with the highest value for them. This follows from work in mechanism design: any incentive compatible, sybil-proof and non-wasteful mechanism necessarily does so. Non-wastefulness here means that the whole supply of execution rights is allocated by the mechanism. Note that this last requirement directly relates to the fixed supply assumption: if instead we have a possibly infinite or (large finite) supply of execution rights and don't require all of them to be allocated immediately, we might have reasonable mechanisms for allocating proposal rights that do not lead to a monopoly for the highest value proposer.

Conclusion, Further Directions and Related Work

The question that I tried to illuminate above of how to allocate proposal rights is largely independent of how the proposer role is defined exactly. Thus, the recommendations are independently relevant of whether or not we should separate attestation and proposing, or maybe even make some more radical changes to block proposing in Ethereum. For instance, we could adopt a version of APS that offers a flexible supply of execution rights and the above logic would predict that it would lead to broader participation in the proposer market similar to PoS-style leader selection.

Proposals for multiple concurrent proposers tackle the problem of proposer monopoly more fundamentally, by addressing it at the individual slot level. However, the points made above equally apply to the selection of multiple concurrent proposers. A mechanisms for selecting them shouldn't allocate all of the concurrent proposal rights to the same entity and the same recommendation - PoS-style selection mechanisms can avoid this while auction type of mechanisms could lead to proposer monopoly (the same proposer proposing all concurrent blocks) - should apply.

From a different, but related angle, instead of looking at the trade-off between efficiency and broad participation, we can also look at the trade-off between efficiency and fairness: mechanisms that use a variable supply can enhance fairness and much of what I discuss is of course very much related to that discussion, see On block-space distribution mechanisms - Block proposer - Ethereum Research

There are some obvious directions for further research: It would be interesting to extend the analysis to non i.i.d. proposal rewards. There has been work on pricing of execution tickets under mean reversion but not on equilibrium participation under mean-reverting rewards. In the PoS (or PoW) world there has been work on participation and entry dynamics if the rewards have a common value and are distributed according to a GBM, but non if there is heterogenous value. From the point of view of design, it would be interesting to explore how elastic the supply should be to achieve a sweet spot in the design space that balances efficiency and broad participation. Finally, the analysis makes the simplifying assumption of linear utility: obvious extensions would look at risk-aversion and at non-linear multi-block value, where a proposer might extract more rewards if they propose multiple blocks in a row than if they would propose the same number of non-consecutive blocks.

1. The linked articles discuss miner competition in the context of PoW which leads to similar market dynamics as PoS.↩
2. In reality, we don't exactly have perfectly elastic supply in Ethereum, as attestation rewards depend on the total

number of validators. However, we have high elasticity of supply. ↩

3. This result can be found in the literature on Tullock contest, see Mike Neuder's post for more context on the connection between Tullock contest and proposer selection. ↩