

Statistical Predictions of Trading Strategies in Electronic Markets

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Abstract

We build statistical models to describe how market participants choose the direction, price, and volume of orders. Our dataset, which spans sixteen weeks for four shares traded in Euronext Amsterdam, contains all messages sent to the exchange and includes algorithm identification and member identification. We obtain reliable out-of-sample predictions and report the top features that predict direction, price, and volume of orders sent to the exchange. The coefficients from the fitted models are used to cluster trading behaviour and we find that algorithms registered as Liquidity Providers exhibit the widest range of trading behaviour among dealing capacities. In particular, for the most liquid share in our study, we identify three types of behaviour that we call (i) directional trading, (ii) opportunistic trading, and (iii) market making, and we find that around one third of Liquidity Providers behave as market makers.

Keywords: agent-based models, algorithmic trading, limit order book, supervision, statistical prediction.

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1 Introduction

Modelling economic agents is a fundamental part of understanding financial markets. The increasing use of electronic trading and the vast quantities of data recorded provide opportunities for modelling agents on a fine scale. However, privacy concerns and data access restrictions have led to limited academic research using such data. In this paper we employ a unique dataset to understand trading behaviour. The dataset contains member identification and algorithm identification and comprises all activity (orders, transactions, cancellations, and amendments) in Euronext Amsterdam. We model the trading behaviour of each individual algorithm that sends limit orders (passive and aggressive) to the market. We obtain an accurate description of how algorithms choose direction, price, and volume and identify important market-observable and idiosyncratic features that predict this decision. Our work provides a detailed empirical description of how algorithms respond to market conditions, their own inventory, and their presence in the limit order book (LOB). Lastly, we find clusters of trading behaviour and compare them to their dealing capacity in

Euronext Amsterdam. A dealing capacity is a contractual arrangement between the trading venue and the trading member, specifying the trading behaviour allowed by the member when acting in that capacity. This classification is determined by the trading venue, not the regulator.

The models we build have a variety of applications, both in academic and regulatory contexts. From an academic perspective, our findings provide statistical evidence for many microstructural stylised facts discussed in the literature, and our study is the first to confirm these facts using agent-based models trained on a dataset that contains both member and algorithm identification. Furthermore, by understanding the behaviour of algorithms, our models can inform mathematical and algorithmic modelling of market participants. In an agent-based modelling context, our results provide guidance for calibration of models of the behaviour of agents at a microstructure level, rather than through their aggregate behaviour in a market. Our results assess the stability of agents through time, and report the key market and agent variables which need to be modelled.

From a regulatory perspective, our models provide guidance for surveillance practices. In particular, our models show the most important features affecting individual trading behaviour. Supervisors could use this information to improve their anomaly detection, focusing on outliers in features that are especially important predictors of individual trading behaviour. Also, our work is a starting point to build an agent-based simulator where market dynamics can be replicated at a granular level to understand how each message to the order book may affect individual behaviour and therefore affect collective dynamics. Our statistical framework is devised to capture the fine microstructural facts that drive the trading decisions of individual algorithms, which is key to understanding how trading algorithms react to new market information and to messages to the LOB. With such a market simulator, firms will be able to test their trading algorithms and regulators will be able to study the effect of new trading algorithms on the quality of the market. Similarly, financial authorities will have a tool to build counterfactual trading tapes to gain insights into market behaviour in the absence of certain trading algorithms or with potential new entrants in the market.

1.1 Insights

We build models to predict the behaviour of individual trading algorithms with a dataset of beyond-level-three LOB data that contains the activity of all market participants active in Euronext Amsterdam. For each ticker and for each algorithm in our study, we propose models to predict the three principal characteristics of an incoming order: direction (buy or sell), price, and volume. By implication, the price, the time-in-force and state of the LOB, determine whether orders are aggressive or passive. We build a comprehensive feature space comprising information available in the market and information available to the algorithm at the moment of making decisions. We do not account for the latency of market participants when constructing the features; instead, we employ information from the time “just before” the order reaches the market.¹ Specifically, we endow each algorithm with 53 features that are updated in continuous time and that describe the state of the LOB, the algorithm’s past actions, and the member’s past actions. The main contributions of this paper are:

- (i) To obtain high out-of-sample accuracies when one predicts (a) the direction (buy or sell) of limit orders, (b) the price limit of the limit order (irrespective of its time-in-force), and (c) the volume of the order.
- (ii) To provide a detailed description of the features that are important to predict the direction, price, and volume of limit orders sent by algorithms. For example: (a) the imbalance of the algorithm’s limit orders resting in the LOB, the best quotes, and the intraday accumulated inventory variables, are the most important features determining the direction of the order; (b) the volumes at the best bid and ask prices, the imbalance of the algorithm’s limit orders resting in the LOB, and the bid-ask spread in the LOB are the most important features determining the price of the order, and (c) the intraday accumulated inventory and cash variables of both algorithms and members are the most important features determining the volume of the order.
- (iii) To show three clusters of ‘stylised trading behaviour’. We use regression coefficients to build the clusters

¹We expect that accounting for latency when constructing the features would improve the out-of-sample accuracies we obtain. See [Cartea and Sánchez-Betancourt \(2021\)](#) for examples of how one can compute implied latency at a market participant level.

and to show how these clusters compare with the registered dealing capacity of market participants.² For the most liquid share considered, we find that:

- (a) Liquidity Providers have the widest range of trading behaviour in the dataset.
- (b) One third of the Liquidity Provider algorithms fall within a cluster that we label “market makers”. These algorithms tend to maintain a balanced provision of liquidity in the LOB, provide liquidity inside the spread, revert their inventories to zero, and often provide liquidity at the best quotes available in the book.³ The other two thirds of Liquidity Providers are split roughly in half according to their trading behaviour and fall in clusters that we call “opportunistic traders” and “directional traders”.
- (c) Across clusters, the wider the bid-ask spread the more likely algorithms are to send eager-to-trade orders. In this paper, an “eager-to-trade” order is either a buy limit order with limit price higher than the best bid price, or a sell limit order with limit price lower than the best ask price – essentially (as we see in Subsection 4.4.8), these are orders that either cross the spread or provide liquidity inside the spread. Similarly, an increase in the number of messages over the last 1 to 100 milliseconds reduces the probability of posting at-the-touch in either direction. In this paper, an “at-the-touch” order is either a buy limit order with a limit price equal to the best bid price, or a sell limit order with limit price equal to the best ask price. Lastly, for most horizons used to compute the quadratic variation of transaction prices, the larger the quadratic variation, the less likely algorithms will show aggressive behaviour, and the more likely it is that they will post liquidity inside the spread.
 - i. Algorithms in the first cluster (directional traders) have the lowest order to trade ratios, do not maintain a balanced provision of liquidity in the LOB, seem indifferent to their own imbalance in the book when they decide to send an eager-to-trade order, have a small baseline probability of posting at-the-touch orders or improving the bid-ask spread, and often send orders in the direction of their intraday accumulated inventories (i.e., buy orders when they have a positive intraday accumulated inventory; sell orders when they have a negative intraday accumulated inventory). More than 80% of their eager-to-trade orders trade aggressively against the other side of the LOB. This behaviour is consistent with execution or position building algorithms; the split of aggressive and passive trading shows how algorithms choose to deploy their execution schedule.⁴
 - ii. Algorithms in the second cluster (opportunistic traders) trade aggressively (73% of their transactions are liquidity taking, compared to 51% for cluster 1 and 26% for cluster 3), and similar to algorithms in cluster 1, more than 80% of their eager-to-trade orders trade aggressively against the other side of the LOB. These algorithms do not mean-revert their intraday accumulated inventory and their own imbalance in the book is an indicator of their trading direction. These algorithms have the lowest (across clusters) percentage of eager-to-trade orders that improve the spread, the lowest baseline probability of posting orders at-the-touch, and the highest baseline probability of posting limit orders deeper in the LOB. These algorithms post most of their orders away from the best quotes in an attempt for higher but less frequent profits.
 - iii. Algorithms in the third cluster (market makers) have the highest order to trade ratio across clusters, are keen to maintain a balanced provision of liquidity in the LOB, have the highest baseline probability of posting at-the-touch (when compared with the algorithms in the other two clusters), and show eagerness to revert their intraday accumulated inventory to zero. Algorithms in this cluster have the highest baseline probability of sending an order that provides liquidity inside the spread. More precisely, 81% of the eager-to-trade orders sent by algorithms in this cluster were limit orders that provided liquidity inside the spread. This

²The three dealing capacities of market participants in our study are (i) Liquidity Provider, (ii) House, and (iii) Client; we return to this in Subsection 2.2.

³An agent has a balanced provision of liquidity in the LOB if the volume that the agent has posted on the ask side of the LOB is similar to the volume that the agent has posted on the bid side of the LOB.

⁴See [Cartea and Jaimungal \(2015\)](#) for a model of how traders optimally deploy such an execution schedule using aggressive and passive orders.

behaviour is consistent with that of market makers who are averse to holding inventories (e.g., due to asymmetry of information).⁵

Using the above we discuss how supervisors can use the results of our study for the benefit of orderly trading and to have oversight on the integrity of the market.

1.2 Connection with survey data

The recent article [AFM \(2023\)](#) reports the results of a survey conducted with a sample of Dutch proprietary trading firms⁶ using algorithmic trading. The article reports that (i) more than 80% of their trading algorithms use machine learning, (ii) their trading algorithms use features such as quantities in the LOB, price trends, volatility, and volume imbalances to make short-term predictions, (iii) model parameters or model weights are updated frequently, (iv) information of the recent past has predictive power in the short future, (v) the list of possible actions that an algorithm can take after a prediction is limited and hard coded, and (vi) firms tend to use simple models, such as linear and logistic regressions, to make predictions.

In this paper we find evidence for a number of these items. For example, our analysis demonstrates that one can predict their trading actions based on features containing quantities in the LOB, price trends, volatility, and their accumulated intraday inventory. We validate that more recent information has higher predictive power than older information, and that simple models, such as the one predicting the most used price range in the past or the most used volume range in the past, have high accuracies. This lends strong support to the assertion that the actions available to algorithms are hard coded and the number of actions limited. Lastly, we show that (multinomial) logistic regression models have high out-of-sample accuracies, which lends support to algorithms using simpler, faster models.

1.3 Existing literature

The LOB plays a crucial role in the microstructure of many modern financial markets. A LOB contains information available on a specific market and reflects the past trading decisions of its participants. [Abergel et al. \(2016\)](#) discuss several models for the LOB covering important techniques such as agent-based modelling. [Bouchaud et al. \(2002\)](#) investigate empirical statistical properties of equity LOBs. [Cont et al. \(2010\)](#) propose a model for LOB dynamics that recovers aspects of the empirical properties satisfied by LOBs; further work in this direction is in [Hambly et al. \(2020\)](#) where the authors provide a probabilistic description of LOB dynamics. [Gould et al. \(2013\)](#) present a detailed account of LOBs, together with statistical analyses of historical LOB data. They discuss how LOB models provide insights into a number of economic questions (e.g., questions regarding market efficiency, price formation, or the rationality of traders), but poorly resemble real LOBs and that several empirical facts have yet to be reproduced satisfactorily.

Agent-based models comprise a number of decision-makers (agents) interacting through prescribed rules. When applied to the economy as a whole, agents can be as diverse as needed — from consumers to policy-makers to investment banks. These models do not assume that the economy will move towards a predetermined equilibrium state, as other models do; see e.g., [Farmer and Foley \(2009\)](#). Instead, at any given time, each agent acts according to its current situation, the state of the environment, and the rules governing its behaviour.

[Farmer et al. \(2005\)](#) apply agent-based modelling to financial markets. They use data from the London Stock Exchange to test a simple model in which zero intelligence agents place orders to trade at random. The model recovers simple stylised facts about the arrival rates of orders; see [Lehalle et al. \(2011\)](#) for an overview of the distinction between a zero-intelligence approach to LOB modelling and agent-based models. [Byrd et al. \(2019\)](#) introduce the design and implementation of ABIDES, a multi-agent equity market simulator. [Assefa et al. \(2020\)](#) discuss the importance of having reliable synthetic data generators for financial markets. [Cohen et al. \(2021\)](#) discuss the role of synthetic data in a modern machine-learning finance context. [Vyetrenko et al. \(2020\)](#) survey the literature and collect statistics and stylised facts seen in real markets that a market simulator should be able to recreate. They show that these models explain a large part of the variance in the bid-ask spread and price diffusion rate. In our work, the coefficients of the regression models open the

⁵See the early work [Amihud and Mendelson \(1980\)](#), [Ho and Stoll \(1981\)](#), [Kyle \(1985\)](#), [Glosten and Milgrom \(1985\)](#), and the more recent work [Avellaneda and Stoikov \(2008\)](#), [Guéant et al. \(2013\)](#), and [Cartea et al. \(2015\)](#), and [Guéant \(2016\)](#).

⁶Trading firms dealing exclusively on own account.

door to taking agent-based modelling one step further; we enable building a market model with agents whose actions (direction, price, and volume) are learned from data.

Agent-based modelling can also provide relevant insights for supervisors looking at market manipulation. Wang et al. (2021) present an agent-based model where agents spoof the LOB to manipulate prices: submit spurious orders to mislead market participants. They demonstrate that traders who use historical data to predict prices improve price discovery and social welfare, but their existence in equilibrium renders a market vulnerable to manipulation. That is, spoofing strategies can effectively mislead traders, distort prices, and reduce total surplus. Our study provides the foundations to create more realistic agent-based models, because we show that the trading decisions of agents can be modelled and predicted well. Furthermore, we show which features are most important in predicting the direction, price and volume of an order; see also Cartea et al. (2020), Tao et al. (2022), Williams and Skrzypacz (2020).

Roşu (2009) presents a model of an order-driven market in which fully strategic, symmetrically informed liquidity traders dynamically choose between limit and market orders. The model makes a number of empirical predictions, such as (i) higher trading activity and higher trading competition cause smaller spreads and lower price impact, and (ii) market orders lead to a temporary price impact larger than the permanent price impact. Sirignano and Cont (2019) find evidence of a universal relationship between order flow and the direction of price moves – they find stable out-of-sample accuracies across shares and periods. Our results also show stable out-of-sample accuracies when predicting direction, price, and volume of orders sent to the exchange by market participants. The models we build in this paper can be used to answer questions in the two aforementioned studies. In particular, we report how the probability of posting liquidity inside the spread and the probability of crossing the spread change according to key features (e.g., volume imbalances). We find that the higher the recent activity the more likely algorithms are to submit orders that are eager to trade.

Bouchaud et al. (2003) uses trades and quotes data from the Paris stock market to show that the random walk nature of traded prices results from an interplay between two opposite tendencies: long-range correlated market orders and mean reverting limit orders. They define and study a model where the price is the result of the impact of all past trades. Aït-Sahalia et al. (2022) use machine learning to study the predictability of ultra high-frequency share returns. They find that, contrary to low-frequency and long horizon returns, predictability of high-frequency returns is large, systematic, and pervasive over short horizons. Our study employs a number of the features they use, specifically by taking features over various short time horizons. We show how these features remain predictive, even when one takes into account algorithm-specific information that is not visible to the market.

Hendershott et al. (2011) focus on the impact of algorithmic trading on market quality, and say that algorithmic trading narrows spreads, reduces adverse selection, and reduces trade-related price discovery. Cartea and Penalva (2012) propose that greater speed could allow fast traders to profitably intermediate between liquidity demanders and liquidity suppliers; see also Cohen and Szpruch (2012). This additional intermediation layer would increase execution costs and microstructure volatility. Furthermore, Cartea et al. (2019) find that an increase in ultra-fast machine-driven activity is associated with lower intraday market quality (greater quoted and effective spreads and lower depth).

For a review on statistical approaches to modelling high-frequency trading data see Dutta et al. (2022); they focus on modelling the aggregated publicly available data as opposed to individual agents. There are strands of the literature that find that deeper layers of the order book contain useful information; see, e.g., Libman et al. (2021). In our models, the likelihood of algorithms sending eager-to-trade orders when they have more volume posted deeper in the LOB varies among clusters, suggesting the results of Libman et al. (2021) are applicable to particular types of market participants. We also find that around one third of Liquidity Provider algorithms behave as market makers and improve the spread around 86% of the times they send an eager-to-trade order. In comparison, directional traders and opportunistic traders improve the spread only 17% and 10% (respectively) of the times they send an eager-to-trade order.

This paper also builds upon previous research on the clustering of agents. Mankad et al. (2013) develop a dynamic machine learning method that buckets 15,686 traders in the E-mini S&P 500 futures into five persistent categories. They identify the categories of high-frequency traders (14 traders), market makers (271 traders), opportunistic traders (7,126), fundamental traders (254), and small traders (8,021). Wright et al. (2022) introduce machine learning methods that cluster active traders in the market, which can be used for intraday classification. With similar data to ours, Ruan et al. (2023) classify short-term strategies and

identify market maker and directional trading clusters. [Cont et al. \(2023\)](#) present a granular representation of the LOB that accounts for the origins of different orders. They describe client order flow from a large broker, in particular, they segment clients into different clusters, for which they identify representative prototypes. Our paper provides a complementary analysis; we cluster the trading behaviour of algorithms, and compare these clusters with the dealing capacity in Euronext Amsterdam.

[Megarbane et al. \(2017\)](#) study the behavior of high-frequency traders (HFTs) and their role in liquidity provision under market stress scenarios such as high-volatility periods surrounding news announcements. Lastly, [Hagströmer and Nordén \(2013\)](#) emphasise the importance of distinguishing different HFT strategies and their influence on market quality. They use data from Nasdaq-OMX Stockholm to distinguish between two types of HFTs: market makers and opportunistic traders. In our paper, we find evidence of Liquidity Providers having a wide range of trading behaviour. In particular, we find that one third of Liquidity Providers behave as market makers and the other two thirds are roughly split into opportunistic traders and directional traders.

2 Data description

We use data from Euronext Amsterdam spanning the period 11 October 2021 to 30 January 2022 (80 trading days in 16 weeks) in four shares with tickers ASML, INGA, AD, TOM2 – we refer to these shares as ASML, ING, AHOLD, and TOMTOM respectively. The data is labelled with algorithm identification and member identification, and contains transactions, orders, cancellations, and amendments processed by Euronext Amsterdam over the relevant period. Orders are matched via price-time-priority and are displayed in a central LOB.⁷

2.1 Assets studied

In our study, the companies represent different sectors in the capital markets (technology, finance, and consumer goods) and their market capitalisation ranges from large to small. The shares differ in the number of trading venues on which they trade, the number of derivatives written on the share, and the number of indices in which the share is a constituent. Euronext Amsterdam is the principal market in which the four shares trade. Specifically,

- (i) ASML Holding (ticker ASML) specialises in the development and manufacturing of machines that produce computer chips, its market capitalisation is approximately €200B, it is traded on various trading venues, and it is a constituent in a number of indices.⁸ We focus on ASML in the main text of the paper; results for the other shares are in the Appendix.
- (ii) ING Groep N.V. (ticker INGA) is a Dutch multinational banking and financial services corporation with headquarters in Amsterdam, its market capitalisation is approximately €50B, it is traded on various trading venues, and it is a constituent in a number of indices.⁹
- (iii) Koninklijke Ahold Delhaize N.V. (ticker AD) is a Dutch multinational retail and wholesaling company, its market capitalisation is approximately €30B, it is traded on various trading venues, and it is a constituent in a number of indices.¹⁰
- (iv) TomTom (ticker TOM2) is a Dutch multinational developer and creator of location technology and consumer electronics, its market capitalisation is approximately €1B, it is traded on fewer trading venues, and it is a constituent in a few indices.¹¹

⁷See [Euronext \(2023d\)](#), p. 43) for information about the main trading session and p. 44–46 for market mechanisms.

⁸Traded on more than 100 trading venues in 2022 according to AFM MIFID-II transaction data; see [Euronext \(2023b\)](#) for more information on ASML.

⁹Traded on more than 100 trading venues in 2022 according to AFM MIFID-II transaction data; see [Euronext \(2023c\)](#) for more information on ING.

¹⁰Traded on more than 100 trading venues in 2022 according to AFM MIFID-II transaction data; see [Euronext \(2023a\)](#) for more information on AHOLD.

¹¹Traded on more than 50 trading venues in 2022 according to AFM MIFID-II transaction data; see [Euronext \(2023e\)](#) for more information on TOMTOM.

Table 1 presents summary statistics of the four shares between 11 October 2021 and 30 January 2022.

	avg daily tradecount	avg daily traded volume €	lowest price period €	highest price period €
ASML	41,847	543,088,447	566.10	770.50
ING	21,469	186,226,078	11.58	13.62
AHOLD	10,535	78,665,021	27.54	31.35
TOMTOM	1,385	4,329,991	6.00	9.23

Table 1: Statistics for the shares in the study between 11 October 2021 and 30 January 2022.

The tick sizes of the shares do not change during the period 11 October 2021 to 30 January 2022. The tick size of ASML is €0.1, the tick size of ING is €0.002, the tick size of AHOLD is €0.005, and that of TOMTOM is €0.005. We observe that ASML, ING, and AHOLD have similar tick sizes as a percentage of their share price (between 1.3 and 1.8 basis points), while TOMTOM has a larger tick size as a percentage of its share price.¹²

2.2 Euronext: members and algorithms

In Euronext’s rule book (Euronext (2023d)), a member is any individual, corporation, partnership, association, trust, or entity who has been admitted to Euronext Securities Membership or Euronext Derivatives Membership and whose membership has not been terminated. Only members can trade on Euronext Amsterdam. A member can trade on its own account or for clients. The latter includes retail-brokers who process so-called “retail orders”. A member can trade under one or multiple dealing capacities.

One of the dealing capacities is “retail liquidity provider”. Orders of members acting as retail liquidity providers can be matched only with orders submitted by “retail” members – see Euronext (2023d, p. 14). Other dealing capacities (each with its own set of obligations) are (i) Liquidity Provider, (ii) House, and (iii) Client. This classification is determined by Euronext, not the regulator.

In this study, an “agent” is the concatenation of the membership, the dealing capacity, and the “entity” that sends the order. We provide a stylised example: suppose company ABC is a member of the trading venue and trades in two dealing capacities: Liquidity Provider and House.

When company ABC trades as a Liquidity Provider, it employs either: algorithm X, or algorithm Y. When it trades as a House, it employs algorithm Z. The agents within company ABC are (i) ABC-LP-X, which refers to algorithm X within the Liquidity Provider dealing capacity of ABC, (ii) ABC-LP-Y, which refers to algorithm Y within the Liquidity Provider dealing capacity of ABC, and (iii) ABC-H-Z, which refers to algorithm Z within the House dealing capacity of ABC.

Euronext data shows which entity within the member executes a given order, but not if this entity is (i) a human trader, or (ii) a trading algorithm. However, by cross-checking with regulatory data we know that all Liquidity Provider agents, and most of the agents within House and Client, are trading algorithms. Therefore, we use the terms “agent” and “algorithm” interchangeably.

Figure 1 shows the number of algorithms associated with each of the members trading in ASML, grouped by dealing capacity (Liquidity Provider, House, and Client). We show algorithms that were active each week throughout the sixteen weeks of data. We order members according to the number of orders they sent to Euronext; highest to lowest order count appears from top to bottom.

¹²There is an extensive literature on the effect of tick sizes on liquidity; see Verousis et al. (2018) or Penalva and Tapia (2021) for relevant references.

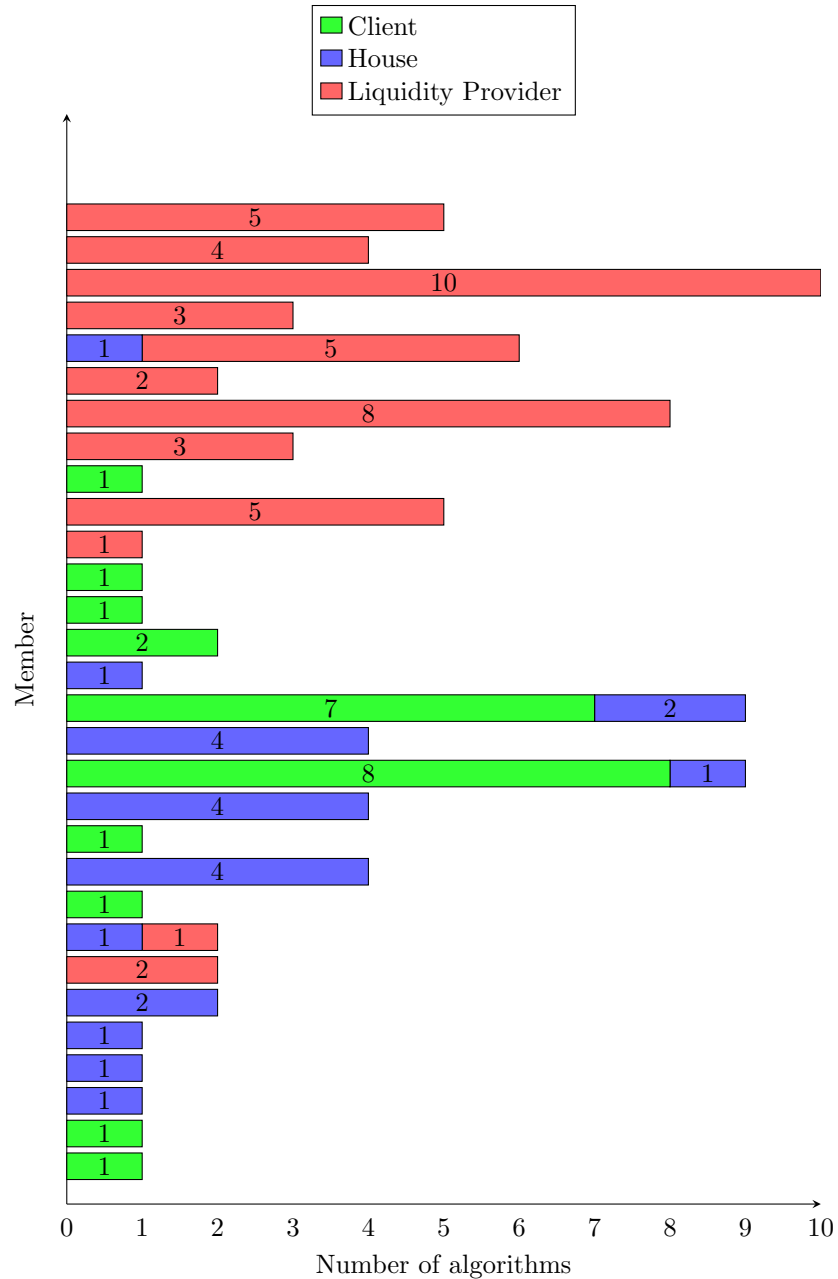


Figure 1: Number of algorithms and dealing capacities per member in ASML. The members are ordered from top to bottom using order count (high to low). We show 96 algorithms active each week for the 16 weeks of data.

Roughly, the algorithms that send the highest number of orders to Euronext have a Liquidity Provider dealing capacity. Out of the twelve members that have algorithms registered as Liquidity Providers, ten of these members trade exclusively with the Liquidity Provider dealing capacity.

2.3 Data filters and the orders we study

In this study, we apply a number of filters to the data. First, we focus on messages entered between 09:05:00 and 17:25:00. This excludes the first and last five minutes of the main trading session and also omits messages

sent during the pre-opening phase and closing phase.

Second, we ignore any undisclosed volume in the LOB when constructing the features of a given algorithm. This consists of volumes in the order book that – similar to iceberg orders – are not visible to market participants. We note, however, that under 0.1% of all orders in any of our shares include undisclosed volume.

Third, in creating our features, we ignore all orders, deletions, and amendments by agents classified as “retail liquidity providers” (retail LPs). This specific dealing capacity has the unusual behaviour that their orders, deletions, and amendments (including the resulting transactions and volumes in the order book) are visible only to “retail” members. Retail clients are able to trade with both retail LPs and the rest of the order book. In effect, this creates two LOBs, one for retail members and one for all other agents. In our dataset, we exclude retail LPs’ orders, and all retail member behaviour which matches against these orders, because we are not concerned with retail trading behaviour. The remaining orders of retail members are included in our dataset, for the purposes of determining the information available to the market, but we will not attempt to fit models for the behaviour of retail members.

2.3.1 Order types

The order types available in Euronext are (i) limit orders, (ii) market orders, (iii) stop orders, (iv) pegged orders, and (v) mid-point orders. Table 2 shows the number of trades in the first four weeks of data for ASML for the following combinations: (i) the buy order is a limit order and the sell order is a limit order, (ii) the buy order is a market order and the sell order is a limit order, (iii) the buy order is a limit order and the sell order is a market order, and (iv) all other combinations.

order type		trade count	
buy side	sell side	number	percentage (%)
limit order	limit order	752,922	97
market order	limit order	6,686	1
limit order	market order	4,974	1
all other combinations		10,446	1

Table 2: Order types of transactions in ASML during first four weeks of data.

The algorithms in our dataset mostly use limit orders, instead of market orders, to take liquidity. Approximately 97% of all transactions result from matching two limit orders. Specifically, in 97% of transactions one leg is a limit order that rests in the LOB and the other leg is an incoming limit order with a limit price generous enough to match the first leg.

2.3.2 Time-in-force

Orders submitted to Euronext have a validity parameter – also known as the time-in-force of the order. Table 3 shows the trade count grouped by the validity parameters of the buy and sell orders that were matched. More precisely, we show the number of transactions in the first four weeks of data for ASML for the following combinations: (i) the buy order has validity parameter immediate-or-cancel (“IoC”) and the sell order has validity parameter good-for-day (“DAY”), (ii) the buy order has validity parameter DAY and the sell order has validity parameter IoC, (iii) the buy order has validity parameter DAY and the sell order has validity parameter DAY, and (iv) all other combinations (for example, fill-or-kill).

validity parameter		trade count	
buy side	sell side	number	percentage (%)
IoC	DAY	283,545	37
DAY	IoC	280,615	36
DAY	DAY	195,691	25
all other combinations		15,177	2

Table 3: Validity types of transactions in ASML during first four weeks of data.

In the first two rows of Table 3, liquidity is consumed with an IoC order. In the third row, liquidity is taken with a DAY order. IoC and DAY orders account for 98% of all transactions. Naturally, in Table 3, it is always the case that one of the two legs has DAY as a time-in-force, which represents the liquidity resting in the LOB waiting to be matched; the other leg is IoC roughly two-thirds of the times and DAY roughly one-third of the times.

2.3.3 The orders that we model

In this paper we model limit orders (including all possible validity parameters). As shown in the two tables above, these orders account for almost all trades in Euronext.

Table 4 shows the order and validity types of all limit orders sent during the first four weeks of data for ASML. This constitutes a four-week snapshot of the orders that we model in this paper.

order type	validity type	order count	percentage (%)
limit order	DAY	7,350,535	97
	IoC	252,744	3
	other	10,210	0

Table 4: Order and validity types of orders in ASML during first four weeks of data.

Limit orders with validity type DAY are 97% of all limit orders sent, and orders with validity type IoC account for only 3%. We also observe that IoC orders are often filled by more than one DAY order, which explains the difference between the figures in Tables 3 and 4.

Table 5 uses the first four weeks of data for ASML and reports the (contribution to) order count, trade count, and volume traded of groups of algorithms. More precisely, the table summarises the aggregate contribution to (i) order count, (ii) trade count, and (iii) volume traded of the top five, top ten, top twenty, top fifty, and all algorithms. The ranking of algorithms is based on the order count of algorithms during the first four weeks of data. We note that each trade requires the involvement of two participants, leading to a double-counting of the number of trades in Table 5.

	order count	order count percentage of all %	trade count	trade count percentage of all %	volume €	volume percentage of all %
Top 5	2,612,307	34	382,040	25	2,332,746,616	19
Top 10	4,055,626	53	478,985	31	2,972,717,749	25
Top 20	5,652,783	74	679,310	44	4,431,976,153	37
Top 50	7,275,195	96	1,047,174	68	8,086,989,709	67
All	7,613,489	100	1,550,056	100	11,998,022,616	100

Table 5: Descriptive statistics for algorithms trading in ASML during first four weeks of data.

The top ten algorithms account for more than 50% of the orders, are involved in more than 30% of the trades, and are involved in more than 20% of the total traded volume. While algorithms below the top 50 provide only 4% of the orders, they are involved in 32% of all trades (and 33% of all traded volume by value). In the Appendix we report the above statistics for ING, AHOLD, and TOMTOM in Tables A1, A2, and A3 respectively.

3 Modelling approach

3.1 Features

In this section, we describe the variables we use in the study. Brogaard et al. (2014) find evidence that public information influences the direction of the orders submitted by HFTs, for example, news announcements, price movements, and LOB imbalances. Cont et al. (2023) suggest one should also consider the time of day and other market conditions such as momentum and volatility. Ait-Sahalia et al. (2022) use three

clocks to construct predictor variables over non-overlapping lookback windows. [AFM \(2023\)](#) indicates that proprietary trading firms report that their trading algorithms use features such as order book imbalance, volume in the order book, and price trends, all measured over various time horizons. Furthermore, firms report that the most recent data has the most predictive power – e.g., messages in the last few milliseconds have more predictive power than older messages.

In this paper, we use 53 features that reflect (i) the state of the LOB, (ii) recent activity in the LOB, (iii) the current presence of the algorithm in the LOB, (iv) the current presence of the member in the LOB, (v) the cash and inventory of the algorithm, and (vi) the cash and inventory of the member. By inventory we mean a transformation $f(x) = \text{sign}(x) \log(1 + |x|)$, where $|\cdot|$ is the absolute value, of the intraday accumulated position measured in number of shares, and the starting value of the accumulated position at the beginning of each trading day is zero by definition. Similarly, cash is the accumulated expense (in EUR) of purchases and sales of inventory; we work with expenses (so purchasing inventory increases the cash variable), to ensure that inventory and cash typically have the same sign. We also apply the above transformation f to the cash variable. Naturally, there is a strong positive relationship between these two variables. Given our data, we only consider transactions which occur on Euronext Amsterdam. All features are measured at the time just before the order enters the market.

To describe our features more precisely, let $\mathcal{A} = \{a_1, a_2, \dots, a_M\}$ be the identifiers for the algorithms on Euronext, and $\mathcal{B} = \{b_1, b_2, \dots, b_N\}$ be the identifiers for the trading members of Euronext – we have that $N \leq M$, i.e., a trading member can trade with multiple trading algorithms. The features we use for algorithm $a_i \in \mathcal{A}$ from trading member $b_i \in \mathcal{B}$ are: (i) volumes in the LOB excluding those from trading algorithm a_i , (ii) volumes in the LOB posted by trading algorithm a_i , (iii) imbalance in the LOB excluding volumes posted by algorithm a_i , (iv) imbalance in the LOB of volume posted by algorithm a_i , (v) logarithm of the volume at the best bid and the best ask, (vi) bid-ask spread, (vii) returns of transaction prices over various time windows in the past, (viii) volatility of transaction prices over various time windows in the past, (ix) number of messages over various time windows in the past, (x) number of aggressive buy orders, number of aggressive sell orders, and net aggressive order flow over various time windows in the past,¹³ (xi) logarithm of volume of last transaction, (xii) inventory of trading algorithm a_i , (xiii) inventory of trading member b_j , (xiv) cash of trading algorithm a_i , and (xv) cash of trading member b_j . In [Appendix B](#) we provide formulae for each of the above features. The categories (i) to (xv) include between one and six variables each and account for a total of 53 features.

The features we employ are measurable to regulators and trading venues. Even if variables such as inventory of trading member (i.e., intraday accumulated inventory of trading member) are not used in the decision making process of an algorithm, it is nonetheless a variable, visible to the regulator or trading venue, that can be used to predict an algorithm’s trading behaviour. Effectively, our variables can be seen as proxies for some of the private signals used by trading firms.

There is correlation between many of the above features; for example, it is well-known that there is positive correlation between the bid-ask spread and any of the measures of volatility we consider; see e.g., [Grossman and Miller \(1988\)](#), [Glosten and Milgrom \(1985\)](#), [O’Hara \(1998\)](#). For each share in our study, we use the first four weeks of data to fit a PCA transformation and project the 53 features down to 30 features, see [Figure 2](#). This captures more than 90% of the variation in the standardised features for all shares.¹⁴ For each share, we apply the transformation obtained for the first four weeks to the remaining twelve weeks. We refer to the features after the PCA transformation as the PCA-transformed features and we refer to the features before the PCA transformation as the original features. Unless stated otherwise, we work with the PCA-transformed features.

¹³We use the terms “aggressive” and “liquidity taking” interchangeably.

¹⁴The exact percentages of the variation captured with 30 features is: 93% for ASML, 92% for ING, 93% for AHOLD, and 93% for TOMTOM.

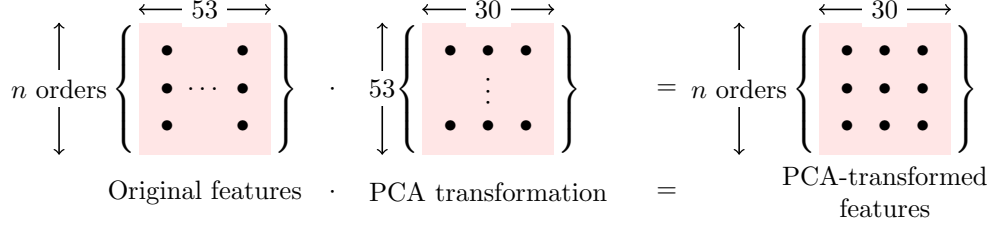


Figure 2: Transforming the original features to PCA-transformed features.

3.2 Output variable

Consider an algorithm which sends a limit order at time t . We model the choice of (a) *direction*, (b) *limit price*, and (c) *volume* of the limit order sent to the market. Recall that Table 2 shows that around 97% of all trading activity employs limit orders. Our models accommodate both provision of liquidity and taking of liquidity. We return to this point in Subsection 4.4.8 when we consider a conditional model for liquidity taking activity.

We consider all limit orders regardless of their time-in-force. To formalise the problem, let $(D_t, P_t, V_t; a_i, b_j)$ be a limit order sent by algorithm $a_i \in \mathcal{A}$ and trading member $b_j \in \mathcal{B}$ arriving at the exchange at time t , where $D_t \in \{-1, 1\}$ is the direction of the order ($D_t = 1$ if the order is to buy and $D_t = -1$ if the order is to sell), $P_t \in \mathbb{R}$ is the price limit attached to the order and $V_t \in \mathbb{R}^+$ its volume.¹⁵ The signed difference from best quote is given by

$$\mathcal{P}_t = \begin{cases} P_t - S_t^b & \text{if } D_t = 1, \\ S_t^a - P_t & \text{if } D_t = -1, \end{cases} \quad (1)$$

where S_t^a and S_t^b are the best ask price and best bid price in Euronext, respectively, and

$$\mathcal{V}_t = \log(1 + V_t), \quad (2)$$

is the log-volume of the order. For an order to buy ($D_t = 1$) we have that $\mathcal{P}_t = P_t - S_t^b$, which is: (i) greater than zero if the order has a limit price that is more generous (higher than) than the best bid price, is (ii) equal to zero if the order has a limit price that is as generous as (equal to) the best bid price, and (iii) less than zero if the order has a limit price that is less generous (lower than) than the best bid price. Similarly, for an order to sell ($D_t = -1$) we have that $\mathcal{P}_t = S_t^a - P_t$, which is: (i) greater than zero if the order has a limit price that is more generous (lower than) than the best ask price, is (ii) equal to zero if the order has a limit price that is as generous as (equal to) the best ask price, and (iii) less than zero if the order has a limit price that is less generous (higher than) than the best ask price.

We study three regimes of the variable \mathcal{P}_t and use them to define the price bucket variable \mathfrak{P}_t as follows:

$$\mathfrak{P}_t = \begin{cases} 1 & \text{if } \mathcal{P}_t \in (0, \infty) & \text{(eager-to-trade orders – more generous than current best limit price),} \\ 2 & \text{if } \mathcal{P}_t \in \{0\} & \text{(at-the-touch orders – at the current best limit price),} \\ 3 & \text{if } \mathcal{P}_t \in (-\infty, 0) & \text{(orders deeper in the LOB).} \end{cases} \quad (3)$$

For ASML, we note that $\mathfrak{P}_t = 1$ for 9.6% of orders, $\mathfrak{P}_t = 2$ for 42.8% of orders, and $\mathfrak{P}_t = 3$ for 47.6% of orders; see Table 12.

With four weeks of data from 11 October 2021 to 5 November 2021, we compute the (up to nine) unique population deciles of the variable \mathcal{V}_t , and use these population deciles to define the (up to ten) buckets associated with this variable. We denote by v^c the number of buckets for the log-volume. The bucket version of \mathcal{V}_t is denoted by $\mathfrak{V}_t \in \{1, \dots, v^c\}$. We apply this bucketing to each of the twelve subsequent weeks of data. The variables we model are

$$(D_t, \mathfrak{P}_t, \mathfrak{V}_t), \quad (4)$$

¹⁵Accounting for latency is out of the scope of this analysis and is left for future research. However, we mitigate some of the effects of latency in the features that agents observe, by measuring features over various time-intervals.

and we call them (i) direction of the order, (ii) price bucket, and (iii) volume bucket, respectively.

As discussed above, our models capture both aggressive and passive behaviour depending on the price bucket and the time-in-force of the order. For example, note that all aggressive behaviour falls within $\mathfrak{P}_t = 1$. More precisely, $\mathfrak{P}_t = 1$ includes orders that (i) cross the spread and trade against the opposite side of the LOB (aggressive orders), (ii) post liquidity inside the spread (generous liquidity provision), and (iii) are cancelled by the exchange upon entry (missed attempts) because their price limit is not generous enough to trade and their time-in-force precludes them from resting in the LOB; we return to this point below in Subsection 4.4.8 where we study a conditional model for liquidity taking activity.

3.3 Regression models

For each share c we employ $W = 16$ datasets $\{\mathcal{D}_k^c\}_{k=1}^W$, each corresponding to a given week of orders in the period 11 October 2021 to 30 January 2022. For each week k , the dataset is given by $\mathcal{D}_k^c = \{(\mathbf{x}_t^k, \mathbf{y}_t^k)\}_{t=T_1}^{T_k}$, where, $\mathbf{x}_t^k \in \mathbb{R}^{\tilde{K}}$ are the features, and $\mathbf{y}_t^k \in \{-1, 1\} \times \{1, 2, 3\} \times \{1, \dots, v^c\}$ are the order details, given by $\mathbf{y}_t^k = (D_t^k, \mathfrak{P}_t^k, \mathfrak{V}_t^k)$. The vector $\mathbf{x}_t^k \in \mathbb{R}^{\tilde{K}}$ contains the features we use for the regressions $\mathbf{x}_t^k = (x_1^{k,1}, \dots, x_1^{k,\tilde{K}})$. Here, $\tilde{K} = 30$ when we use PCA-transformed features. Note that the features \mathbf{x}_t^k implicitly depend on the algorithm placing the order. We use logistic regressions to model the decisions of agents, that is

$$(R1) \quad \mathbb{P}(D_t^k = 1) = \frac{1}{1 + \exp(\beta^{D,k} + \beta^{D,k} \cdot \mathbf{x}_t^k)}, \quad \text{on } \mathcal{D}_k^c, \quad (5)$$

$$(R\star) \quad \mathbb{P}(Y_t^k = y) = \frac{\exp(\beta_y^{Y,k} + \beta_y^{Y,k} \cdot \mathbf{x}_t^k)}{1 + \sum_{j=1}^{y^c-1} \exp(\beta_j^{Y,k} + \beta_j^{Y,k} \cdot \mathbf{x}_t^k)}, \quad \text{on } \mathcal{D}_k^c, \quad (6)$$

where $Y \in \{\mathfrak{P}, \mathfrak{V}\}$ and the operator \cdot denotes the dot product between two vectors. When Y is \mathfrak{P} then $R\star = R2$, $y \in \{1, 2, 3\}$, and $y^c = 3$. Similarly, when Y is \mathfrak{V} then $R\star = R3$, $y \in \{1, \dots, v^c\}$, and $y^c = v^c$.

We model $(D_t^k, \mathfrak{P}_t^k, \mathfrak{V}_t^k)$ with the vector of features \mathbf{x}_t^k . We also model price bucket and volume bucket conditional on the direction D_t^k ; for this, we perform logistic regressions

$$(R\star) \quad \mathbb{P}(Y_t^k = y \mid D_t^k = d) = \frac{\exp(\beta_y^{Y,k,d} + \beta_y^{Y,k,d} \cdot \mathbf{x}_t^k)}{1 + \sum_{j=1}^{y^c-1} \exp(\beta_j^{Y,k,d} + \beta_j^{Y,k,d} \cdot \mathbf{x}_t^k)}, \quad \text{on } \mathcal{D}_k^{c,d}, \quad (7)$$

where we have the following four combinations: (i) when Y is \mathfrak{P} and $d = +1$ then $R\star = R4$, $y^c = 3$, $\mathcal{D}_k^{c,d} = \mathcal{D}_k^{c,+1}$, and $y \in \{1, 2, 3\}$; (ii) when Y is \mathfrak{P} and $d = -1$ then $R\star = R5$, $y^c = 3$, $\mathcal{D}_k^{c,d} = \mathcal{D}_k^{c,-1}$, and $y \in \{1, 2, 3\}$; (iii) when Y is \mathfrak{V} and $d = +1$ then $R\star = R6$, $y^c = v^c$, $\mathcal{D}_k^{c,d} = \mathcal{D}_k^{c,+1}$, and $y \in \{1, \dots, v^c\}$; (iv) when Y is \mathfrak{V} and $d = -1$ then $R\star = R7$, $y^c = v^c$, $\mathcal{D}_k^{c,d} = \mathcal{D}_k^{c,-1}$, and $y \in \{1, \dots, v^c\}$. We also have that $\beta_3^{\mathfrak{P},k,+1}, \beta_3^{\mathfrak{P},k,-1}, \beta_3^{\mathfrak{V},k,+1}, \beta_3^{\mathfrak{V},k,-1}, \beta_{v^c}^{\mathfrak{P},k,+1}, \beta_{v^c}^{\mathfrak{P},k,-1}, \beta_{v^c}^{\mathfrak{V},k,+1}, \beta_{v^c}^{\mathfrak{V},k,-1}$ are all zero. Here we use

$$\mathcal{D}_k^{c,+1} = \{(\mathbf{x}_t^k, \mathbf{y}_t^k) \in \mathcal{D}_k^c : D_t = 1\}, \quad (8)$$

$$\mathcal{D}_k^{c,-1} = \{(\mathbf{x}_t^k, \mathbf{y}_t^k) \in \mathcal{D}_k^c : D_t = -1\}. \quad (9)$$

For each algorithm we fit the above models. Section 4.2 reports the out-of-sample accuracies and out-performances over alternative methods for ASML. The results for the other three tickers are reported in Appendix C.1.

3.4 Machine learning models

3.4.1 Random Forests

We also use random forests – introduced by Breiman (2001) – as a benchmark to assess the performance of the logistic regressions. Random forests go beyond linear models but lack explainability. We return to this point below when we discuss data privacy concerns.

We train random forests with fifty trees in the forest and a maximum depth of five for a given tree. To avoid overfitting and to obtain these parameter values, we tune hyperparameters to balance the in-sample and out-of-sample accuracies. We employ `sklearn.ensemble` in Python, and the same features as in our logistic regressions.

3.4.2 Algorithm clusters

The regression coefficients from our logistic regressions describe trading behaviour. Indeed, the value of the regression coefficients model the probability with which algorithms choose direction, price, and volume. In this paper, we use the regression coefficients of the models for direction (D_t) to cluster trading behaviour. The clustering exercise employs hierarchical agglomerative clustering; see Section 21.2 in [Murphy \(2022\)](#). In particular, we use complete linkage, cosine affinity, and a target of three clusters, to compare with the dealing capacities of algorithms – recall that we study three dealing capacities in this paper: (i) Liquidity Provider, (ii) House, and (iii) Client. We employ `AgglomerativeClustering` from `sklearn.cluster`.

4 Model fit

4.1 Train and deploy structure

As described in Section 2, we use sixteen weeks of data in the study. The models are trained over four consecutive weeks of data and applied to the week after. For example, we use weeks 1 to 4 (11 October 2021 to 5 November 2021) to train the models, then obtain out-of-sample predictions for week 5 (8 November 2021 to 12 November 2021), train models from weeks 2 to 5 (18 October 2021 to 12 November 2021), then obtain out-of-sample predictions for week 6 (15 November 2021 to 19 November 2021), and so on. We refer to each such run as a ‘train-and-deploy’ exercise – see Figure 3.

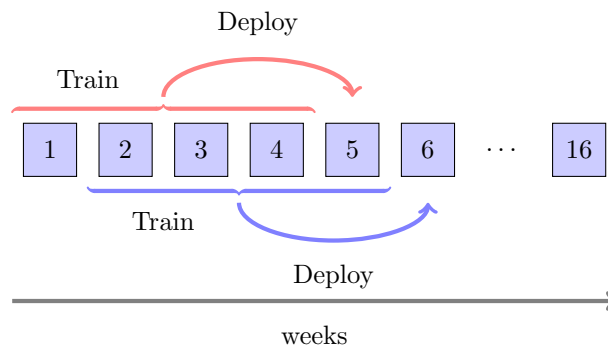


Figure 3: The models are trained on four consecutive weeks of data and applied to the week after.

With the sixteen weeks of data we perform twelve train-and-deploy exercises. Thus, there are twelve weeks of data for which we perform out-of-sample predictions. For each train-and-deploy exercise we obtain seven accuracies per algorithm (one for each model in R1 to R7; see Section 3.3). Next, we compute the order-weighted accuracy of the predictions per algorithm over the twelve weeks of data and report the results in Table 6. With these accuracies, we compute (i) the average, plus and minus the standard deviation, of the accuracies across groups of algorithms and we report it in the first five rows, and (ii) the order-weighted accuracy for all algorithms and report it in the bottom row.

4.2 Out-of-sample accuracy of predictions

Table 6 reports out-of-sample accuracies of the predictions made by the logistic regression models for twelve train-and-deploy exercises on ASML. The direction of order D_t can take two values, the price bucket \mathfrak{P}_t can take three values, and the volume bucket \mathfrak{V}_t can take nine values.

The ordering of algorithms in Table 6 is based on the order count over all deploy-weeks of the train-and-deploy exercises. For example, the ‘Top 5’ row refers to the top five algorithms that sent most orders during the twelve deploy-weeks. The same ordering method is applied to Table 7 and the tables for other tickers.

	R1 D_t	R2 \mathfrak{P}_t	R3 \mathfrak{V}_t	R4 $\mathfrak{P}_t D_t = 1$	R5 $\mathfrak{P}_t D_t = -1$	R6 $\mathfrak{V}_t D_t = 1$ (%)	R7 $\mathfrak{V}_t D_t = -1$
Top 5	65 \pm 7	80 \pm 9	60 \pm 23	85 \pm 6	84 \pm 6	61 \pm 21	60 \pm 22
Top 10	66 \pm 7	86 \pm 9	68 \pm 22	89 \pm 6	89 \pm 6	69 \pm 21	69 \pm 22
Top 20	68 \pm 9	87 \pm 12	62 \pm 23	89 \pm 10	89 \pm 10	63 \pm 22	63 \pm 22
Top 50	73 \pm 12	86 \pm 15	60 \pm 25	86 \pm 18	87 \pm 18	59 \pm 26	60 \pm 25
All	77 \pm 15	80 \pm 19	48 \pm 25	78 \pm 24	77 \pm 25	46 \pm 25	46 \pm 26
order-weighted average	70	85	62	88	88	62	62

Table 6: Accuracies of the logistic regression models for ASML. Twelve train-and-deploy exercises. Accuracies are reported in % with \pm the standard deviation.

Accuracy of predictions is high and stable over time for all algorithms. The accuracies of the logistic regression models are stable for all shares – for example, when predicting the direction of the order D_t , accuracies range between 67% and 70% according to the order-weighted accuracy for all algorithms; see Appendix C.1. The order-weighted average of the regressions for \mathfrak{P}_t that condition on direction of order perform better than regression R2. This is not the case for \mathfrak{V}_t where we have a similar performance for R2, R6, and R7.

We compare the accuracies of the logistic regression models against a benchmark where the predicted bucket is the bucket most frequently used by the algorithm in the training data. Table 7 reports the outperformance of the logistic regression models over this benchmark.

	D_t	\mathfrak{P}_t	\mathfrak{V}_t	$\mathfrak{P}_t D_t = 1$	$\mathfrak{P}_t D_t = -1$	$\mathfrak{V}_t D_t = 1$	$\mathfrak{V}_t D_t = -1$
Top 5	15 \pm 7	8 \pm 10	5 \pm 6	14 \pm 14	13 \pm 14	6 \pm 8	5 \pm 7
Top 10	15 \pm 7	4 \pm 9	2 \pm 5	7 \pm 12	7 \pm 12	3 \pm 6	2 \pm 6
Top 20	17 \pm 10	4 \pm 7	3 \pm 7	7 \pm 10	6 \pm 10	4 \pm 8	3 \pm 7
Top 50	20 \pm 13	3 \pm 6	3 \pm 6	5 \pm 8	4 \pm 8	3 \pm 7	3 \pm 7
All	21 \pm 17	3 \pm 10	0 \pm 8	3 \pm 10	4 \pm 11	1 \pm 10	0 \pm 11
order-weighted average	18	4	3	8	7	3	3

Table 7: Outperformance over benchmark for ASML. Twelve train-and-deploy exercises.

On average, the logistic regression models outperform the benchmark. The outperformance is largest for predicting the direction of an order, at 18% on average. When predicting volume, we observe the smallest outperformance over the benchmark. See Appendix C.1 for comparable results for other tickers.

Table 8 reports the outperformances of random forests when compared with the same benchmark as that used in Table 7, and compared with the logistic regressions in Table 6.

	outperformance					
	over benchmark			over logistic		
	D_t	\mathfrak{P}_t	\mathfrak{V}_t	D_t	\mathfrak{P}_t	\mathfrak{V}_t
Top 5	21 \pm 7	7 \pm 8	9 \pm 10	6 \pm 7	-1 \pm 3	4 \pm 4
Top 10	19 \pm 7	4 \pm 7	5 \pm 8	4 \pm 6	-1 \pm 2	3 \pm 3
Top 20	23 \pm 11	3 \pm 6	6 \pm 8	6 \pm 7	-1 \pm 2	2 \pm 5
Top 50	24 \pm 13	2 \pm 5	4 \pm 7	4 \pm 6	-1 \pm 2	1 \pm 4
All	22 \pm 17	3 \pm 7	2 \pm 7	1 \pm 7	0 \pm 6	3 \pm 5
order-weighted average	23	4	5	5	0	2

Table 8: Outperformance of random forests over benchmark and over logistic regressions for ASML. Twelve train-and-deploy exercises.

Overall, random forests outperform the benchmark by 23% when predicting whether algorithms send a buy or a sell order – this is 5% higher than the outperformance achieved by the logistic regression models. The outperformance over the benchmark is stable across shares; in all cases, random forests outperform logistic regression models. When predicting the price bucket, random forests outperform the benchmark by 4%, on average, which is the same outperformance attained by the multinomial logistic regression models. Lastly, when predicting the volume bucket, the outperformance of random forests over the benchmark is

5%, which is 2% higher than the outperformance of the logistic regression over the benchmark. On average, random forests tend to slightly outperform logistic regressions.

Going beyond logistic regressions comes at a cost. Machine learning models are known to have the potential to leak parts of the training data when one discloses the model (weights or coefficients obtained) as investigated in [Shokri et al. \(2017\)](#). That is, some models are vulnerable to privacy attacks – see [Rigaki and Garcia \(2020\)](#) for a comprehensive survey. For many applications of our study this suggests it is more appropriate to use logistic regressions because of their interpretability, their simplicity, and their privacy features. We therefore focus on logistic models in the remainder of this study.

4.3 Feature importance

Feature importance encompasses a set of machine learning techniques that orders features according to their contribution in explaining a given output variable. In this section, we assess the level of importance of each of the features we employ to predict D_t , \mathfrak{P}_t , and \mathfrak{V}_t – see Section 3.1 for the complete list of features used in our study.

We employ the ‘permutation feature importance’ approach in [Breiman \(2001\)](#). For a given feature, this model-agnostic technique randomly permutes the values of the feature in the training set and computes the change in model score by taking the difference between (i) the accuracy using the original data and (ii) the accuracy with the permuted values of the feature. The changes in score indicate the sensitivity of the model to the feature. The permutation of values is random, so we repeat this process ten times for each feature.

Permutation feature importance attempts to quantify the contribution of an individual feature to the predictive model. This implies that if a variable is highly correlated with others, it will typically be assigned a low importance, as the model is able to use other variables to obtain the same information.

4.3.1 Logistic regressions

We repeat the train-and-deploy exercises as described in Section 4.1 on the original features described in Section 3.1 instead of the PCA-transformed features. The use of the original features as opposed to the PCA-transformed features in the logistic regressions is restricted to the feature importance results. The results are reported below for each of the three regressions (R1, R2, R3).

First, we look at the most important feature to predict D_t , \mathfrak{P}_t , \mathfrak{V}_t for each of the top ten algorithms. The ranking of the top ten algorithms is based on the order count over all deploy-weeks of the train-and-deploy exercises. The most important feature is the feature with the highest median importance across the twelve train-and-deploy exercises, the ten algorithms, and the ten random shuffles per feature (giving us $12 \times 10 \times 10 = 1,200$ data points per feature describing their importance). See Figure 4 for the results.¹⁶

¹⁶In the figures that follow, we abbreviate the feature names for displaying purposes: “excl” is “excluding”, “chg” is “change”, “m” is “minutes”, “s” is “seconds”, “ms” is “milliseconds”, “quad var” is “quadratic variation”, “num” is “number”, “agg” is “aggressive”, and “algo” is “algorithm”.

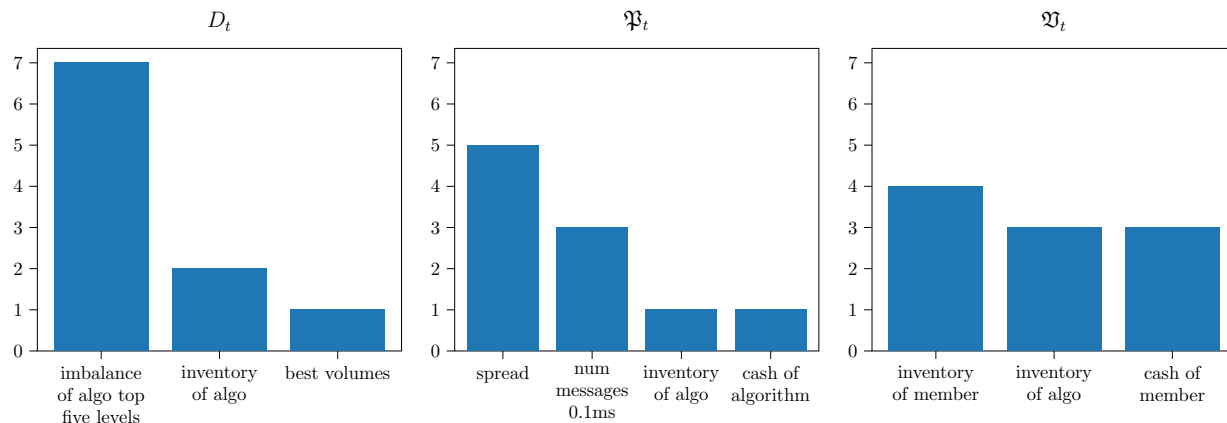


Figure 4: Most important feature for the top ten algorithms in ASML.

Although there are differences in the importance of features across predictions (D_t , P_t , and V_t), there is significant overlap on the most important feature within each of the predictions. For example, the imbalance of the volumes posted by the algorithm in the first five levels of the LOB is the most relevant feature for seven out of the top ten algorithms when predicting the direction of the order D_t . This is similar to other empirical results in the literature; see [Cartea et al. \(2018\)](#) where the authors find that the imbalance of volumes resting in the LOB is a good predictor of the sign of the next liquidity taking order sent to the exchange, that is, whether the next liquidity taking order will be to buy or to sell.¹⁷ We find that, for most algorithms, when deciding on the direction of the order D_t , the imbalance of the volumes posted by themselves is more important than the imbalance of the volumes posted by other algorithms.

To predict the price bucket and volume bucket of orders, the bid-ask spread and intraday accumulated inventory are respectively among the most important variables. Intuitively, as the spread widens (tightens), the algorithm is more likely to post orders further from (closer to) the midprice. However, as seen earlier, volumes of orders typically vary little per algorithm. We do not observe ‘directional’ variables, for example an algorithm’s current exposure in the LOB, contributing to the price model, but we see that these play a role in the price conditioned on direction model; see Subsection 4.4.7.

Next, Figure 5 shows a box-plot of the distribution of feature importances for the top ten algorithms computed over twelve train-and-deploy exercises, and ten repetitions of random permutations. We sort features according to their median importance and plot the top ten – each feature has 1,200 data points corresponding to 12 sets of four consecutive weeks, 10 algorithms, and 10 repetitions. The results in the box-plots differ from the results in Table 4 in that Figure 5 shows the distribution of feature importances between algorithms instead of showing only the most important feature for a given algorithm.

Figure 5 shows the results for ASML when predicting the direction of the order D_t .

¹⁷[Cartea et al. \(2020\)](#) show that volume imbalance helps to predict the direction of the next (non-aggressive) limit order.

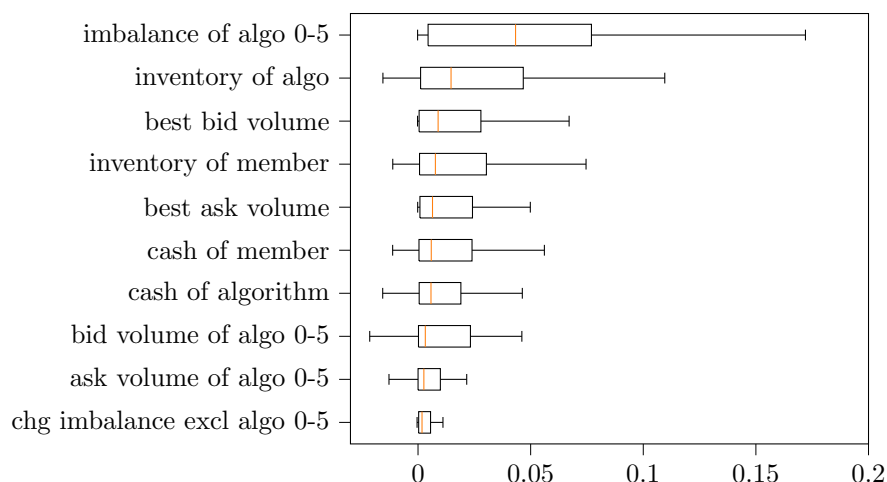


Figure 5: Importance of features to explain the direction D_t of an order for ASML (using permutation importance and logistic regressions, top features only shown).

Again, the imbalance of volumes posted by the algorithm in the first five levels of the LOB is the most important variable when predicting whether the algorithm will send a buy order or a sell order, followed by the intraday accumulated inventory of the algorithm and the intraday accumulated inventory of the trading member, and the best volumes posted in the LOB. This is similar for all shares; the same set of features appears in each of the top ten features reported. This shows that inventory related features, volumes posted on best quotes, and imbalances by the algorithm near the top of the LOB are important features to predict the direction of the order.

Figure 6 shows the most important features to predict the price bucket \mathfrak{P}_t .¹⁸

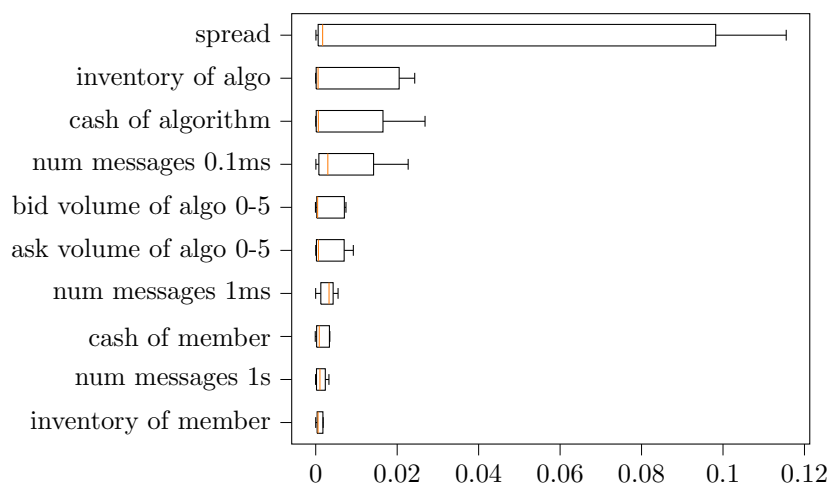


Figure 6: Importance of features to explain the price bucket \mathfrak{P}_t for ASML (using permutation importance and logistic regressions, top features only shown).

Spread is the most important feature to predict the price bucket \mathfrak{P}_t , which is in line with the results in Table 4. There is consistency across shares – five of the top ten features in ASML also feature in the top ten of the three other shares. The order of importance is consistent: spread is the most important predictor for three of our four shares, and intraday accumulated inventory and cash of algorithm appear in the top

¹⁸The figures for \mathfrak{P}_t are ordered by 75% percentile instead of median to increase interpretability.

three for ASML, ING and AHOLD. Other important features are the number of messages sent in the last 100 microseconds (i.e., 0.1 millisecond) and the volume posted by the algorithm on the first five levels of the LOB – both bid and ask side.

We draw attention to the differences and similarities between the top features that predict the direction of the order and the price bucket \mathfrak{P}_t . The main difference is the importance of spread, which is important to predict the price of an order, yet does not appear in the top ten features to predict the direction of the order.¹⁹ Another difference is that the number of messages sent by all market participants in the last 100 microseconds is not important for predicting the direction of the order but it is important to predict price bucket \mathfrak{P}_t .

Lastly, Figure 7 reports the important features to predict the volume bucket \mathfrak{V}_t .

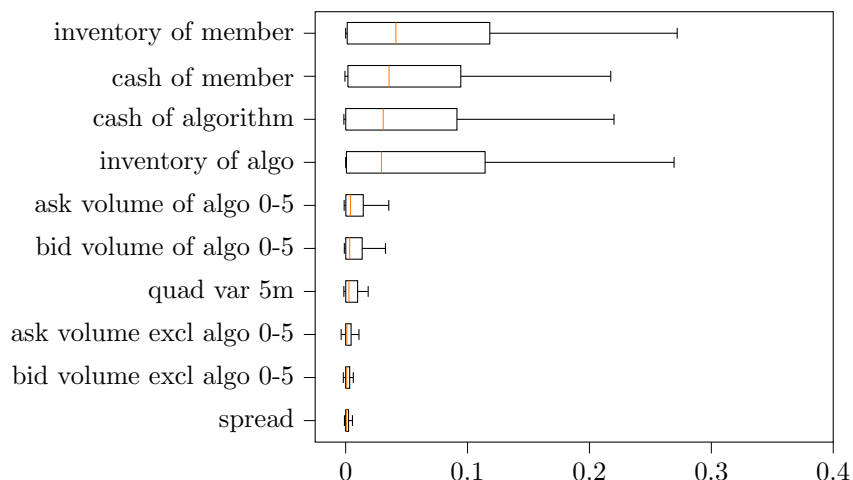


Figure 7: Importance of features to explain the volume bucket \mathfrak{V}_t for ASML (using permutation importance and logistic regressions, top features only shown).

For all shares, the four inventory variables are the most important features to predict the volume of an order. Also, the volume quoted by the algorithm on the first five levels, on both the buy side and the sell side, is an important feature for all shares. We see that the number of messages in the last second is an important feature for ING, AHOLD, and TOMTOM. Contrary to the other output variables, we observe that quadratic variation – measured over various intervals, is an important feature for ASML, ING, and AHOLD.

Inventory related features are important for predicting the three variables (D_t , \mathfrak{P}_t , \mathfrak{V}_t). Also, among order book features, those that describe behaviour closer to the top of the LOB have more predictive power than features that describe behaviour deeper in the LOB. Similarly, the number of messages sent in the recent past (e.g., in the last 100 microseconds) tends to be more important than the number of messages contained in longer horizons. This is consistent with the information that trading firms report to the AFM; see AFM (2023).

We also observe that features describing the algorithm’s presence in the LOB tend to be more predictive than features describing the presence of other participants. For example, imbalances of the algorithm and volumes of the algorithm tend to be among the most important features, yet imbalances and volumes of other agents are not – this is apart from best bid volume and best ask volume.

4.4 Clustering of algorithms

The model coefficients from predicting (i) direction of the order D_t with a logistic regression, (ii) price bucket \mathfrak{P}_t with a multinomial logistic regression, and (iii) volume bucket \mathfrak{V}_t with a multinomial logistic regression, capture information about the trading behaviour of an algorithm. More precisely, the coefficients show how

¹⁹The spread is the cost of immediacy in the LOB and it is the same cost for buy and sell aggressive orders.

the features affect the probability that an agent selects a given value for the variables $(D_t, \mathfrak{P}_t, \mathfrak{V}_t)$. Here, we use these coefficients to cluster algorithms.

We revert to using the logistic regression models on the PCA-transformed features because of their higher predicting power. In particular, we employ the model parameters from the logistic regressions to predict trade direction D_t . We focus on D_t because results are easy to interpret and because D_t is the output variable where outperformances are highest. Recall that for D_t we have only one coefficient per feature, instead of one coefficient per bucket per feature when predicting \mathfrak{P}_t and \mathfrak{V}_t .

Arguably, the clusters that arise within the regression coefficients are clusters that identify similarity in trading behaviour. In Subsection 4.4.2 we compare these clusters of trading behaviour with the dealing capacity of algorithms in Euronext Amsterdam – recall that the dealing capacities are (i) Liquidity Provider, (ii) House, and (iii) Client and are determined by the exchange, not the regulator.

We employ the clustering technique described in Section 3.4 with the coefficients of the regressions applied to the PCA-transformed features and a target of three clusters.²⁰ We do this over twelve sets of four consecutive weeks of data each and refer to this as clustering exercises. Each clustering exercise includes only the 96 algorithms present in all of the twelve clustering exercises we perform.

Figure 8 shows the size of the clusters we obtain in each clustering exercise. In this figure, the first cluster has the most algorithms and the third cluster has the least algorithms.

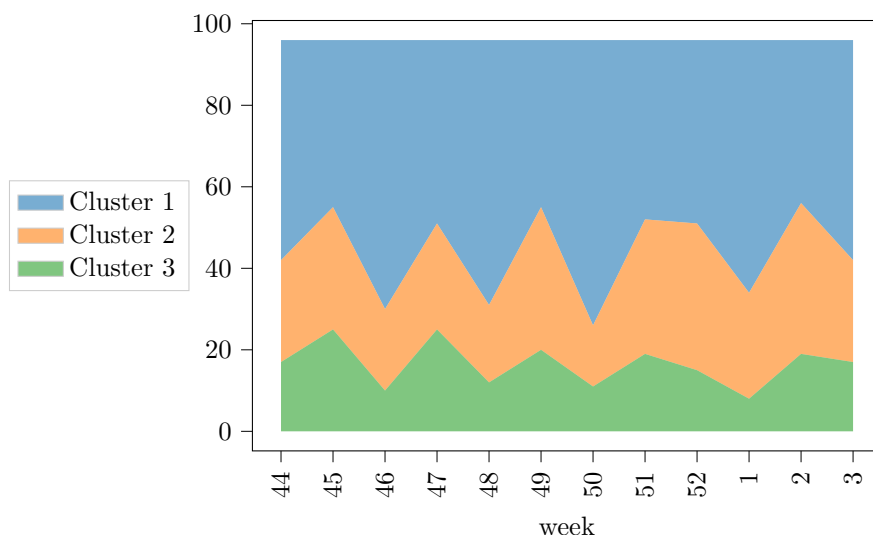


Figure 8: Size of clusters for ASML across the twelve clustering exercises. The x -axis notes the last week of the clustering exercise. For example, “44” refers to the clustering exercise on weeks 41, 42, 43, and 44 of 2021. Weeks 44 to 52 are in 2021 and weeks 1 to 3 are in 2022. The y -axis is the number of algorithms.

The number of algorithms in each of the three clusters varies for each of the twelve clustering exercises. Given that the clusters aim to capture fundamental trading behaviour, it is crucial that they exhibit a degree of consistency over time. Below, we study this point in more detail.

4.4.1 Stability through time

We explore the stability of the clusters across time. For this, given two consecutive clustering exercises A and B where the last week in B is one week after the last week in A, we compute the probability that two randomly chosen algorithms exhibit one of the following two properties: (i) they belong to the same cluster in exercise A and belong to the same cluster in exercise B, or (ii) they do not belong to the same

²⁰We choose a target of three clusters as it agrees with the number of primary dealing capacities in Euronext and it provides a trade-off between stability of clusters over time and the discretisation of trader behaviour. Using a larger number of clusters does not give significantly better separation between groups – we believe that this clustering should be better interpreted as a quantisation of the distribution of trader behaviour, rather than identifying truly discrete trader types.

cluster in exercise A and do not belong to the same cluster in exercise B. The blue line in Figure 9 shows the probability of (i) or (ii) along the twelve clustering exercises we perform across time. That is, for two consecutive clustering exercises, we count the pairs of algorithms that were clustered together and remained clustered together, and those that were clustered differently and remained in different clusters, over all pairs of algorithms. This calculation is a probability, so it lies in $[0, 1]$. The red dash-line is the theoretical value of the probability when algorithms are randomly shuffled and reassigned into the clusters while keeping the number of algorithms in each cluster fixed as those we find for the first four weeks of data.

Intuitively, if the blue line is close to 1, then the clustering does not change over time, so the clusters are stable. On the other hand, if the blue line is close to the red dash-line, then the clustering resembles that of random reshuffling of algorithms.

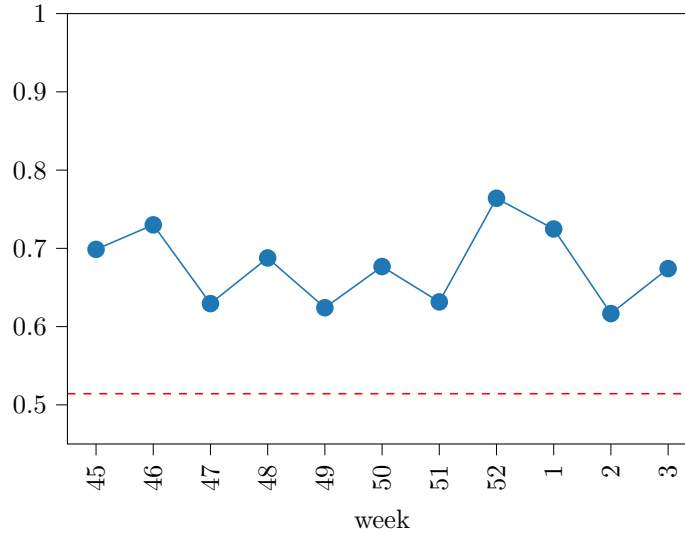


Figure 9: Stability of clusters for ASML across the twelve clustering exercises. The y -axis shows the probability of two randomly selected agents in two consecutive clustering exercises A and B (i) being members of the same cluster in A and B or (ii) different clusters in A and B. The x -axis shows the last week of the clustering exercise B.

The blue line lies above the red dotted line, which indicates stability of algorithm clusters. However, there is still a fair amount of instability in the clusters over time, as indicated by the gap between the blue line and 1. Part of the instability might be due to algorithms updating their model parameters frequently, as shown in the results of the survey (AFM (2023)). Recall that our clustering exercises include only the 96 algorithms present in all clustering exercises. Indeed, if any of these algorithms updates the model parameters they use to send orders, then we expect that the parameters we obtain will change too, resulting in unstable clusters.

We also study the link between the level of activity of the algorithms and the stability of their regression coefficients. We use the logistic regression coefficients of the 53 original features described in Section 3.1, and assign each algorithm according to the clustering obtained on the coefficients of the logistic regressions on the PCA-transformed features. The first column of Table 9 lists the most important features to predict the direction D_t of an order. The second and third columns report the temporal deviation, which for a given feature is defined as the standard deviation of the regression coefficients over time. We report the average temporal deviation for the top ten algorithms (second column) and for the bottom ten algorithms (third column) – the ranking is according to the number of orders sent during the first four weeks of data.

	Top 10 algorithms	Bottom 10 algorithms
imbalance of algo top five levels	0.27	0.33
inventory of algo	0.25	0.84
ask volume of algo top five levels	0.19	0.36
bid volume of algo top five levels	0.15	0.26
best bid volume	0.04	0.21
best ask volume	0.04	0.18
chg in imbalance excl algo 0-5	0.03	0.14
spread	0.03	0.22
net agg buy-sell last 1s	0.02	0.15
number of messages 1ms	0.02	0.21
number of messages 0.1ms	0.02	0.22

Table 9: Average temporal deviation of coefficients.

Given the large number of data points used to fit the models of the top 10 algorithms, we expect that the observed temporal deviation is principally due to changes in the algorithms themselves, for example due to re-calibration of parameters. Conversely, for the bottom 10 algorithms, the number of observed actions is relatively small, so the systematically higher temporal deviation for these algorithms may be due to statistical estimation error, rather than systematic differences between the top and bottom algorithms. Regardless of the reason for the large temporal deviation of the coefficients for the bottom algorithms, Table 9 lends support to the claim that the instability of our clusters is due primarily to variation in the estimated coefficients of the smaller algorithms.

4.4.2 Types of market participants

Here, we explore how the clusters relate to the dealing capacity of market participants. Recall that we study three dealing capacities: (i) Liquidity Providers (LP), (ii) House, and (iii) Client. Figure 10 shows the confusion matrix between dealing capacity and clusters.

Client	20	3	1
House	16	7	0
LP	18	15	16
	Cluster 1	Cluster 2	Cluster 3

Figure 10: Confusion matrix between the clusters obtained with the parameters from the first four weeks of data and the dealing capacity for ASML.

Cluster 1 consists of an almost equal number of Liquidity Providers, Houses, and Clients. Furthermore, the majority of algorithms with type House or Client are in cluster 1. Therefore, one would expect this cluster to show trading behaviour associated with directional trading firms, such as hedge funds or pension funds – see Subsection 4.4.4.

Clusters 2 and 3 consist mostly of Liquidity Providers. Hence, one would expect these clusters to exhibit behaviour most often associated with traditional market makers. Furthermore, the L-shaped confusion

matrix in Figure 10 lends support to the claim that Liquidity Providers exhibit the highest variability in trading behaviour.²¹

4.4.3 Clusters and members

Figure 11 shows the number of algorithms that each of the members have, grouped by cluster. The sorting of members is similar to that of Figure 1 (by order count); we reproduce Figure 1 on the left-hand side for convenience.

²¹Here, L-shaped refers to the higher values in Figure 10 being arranged along the left column and bottom row.

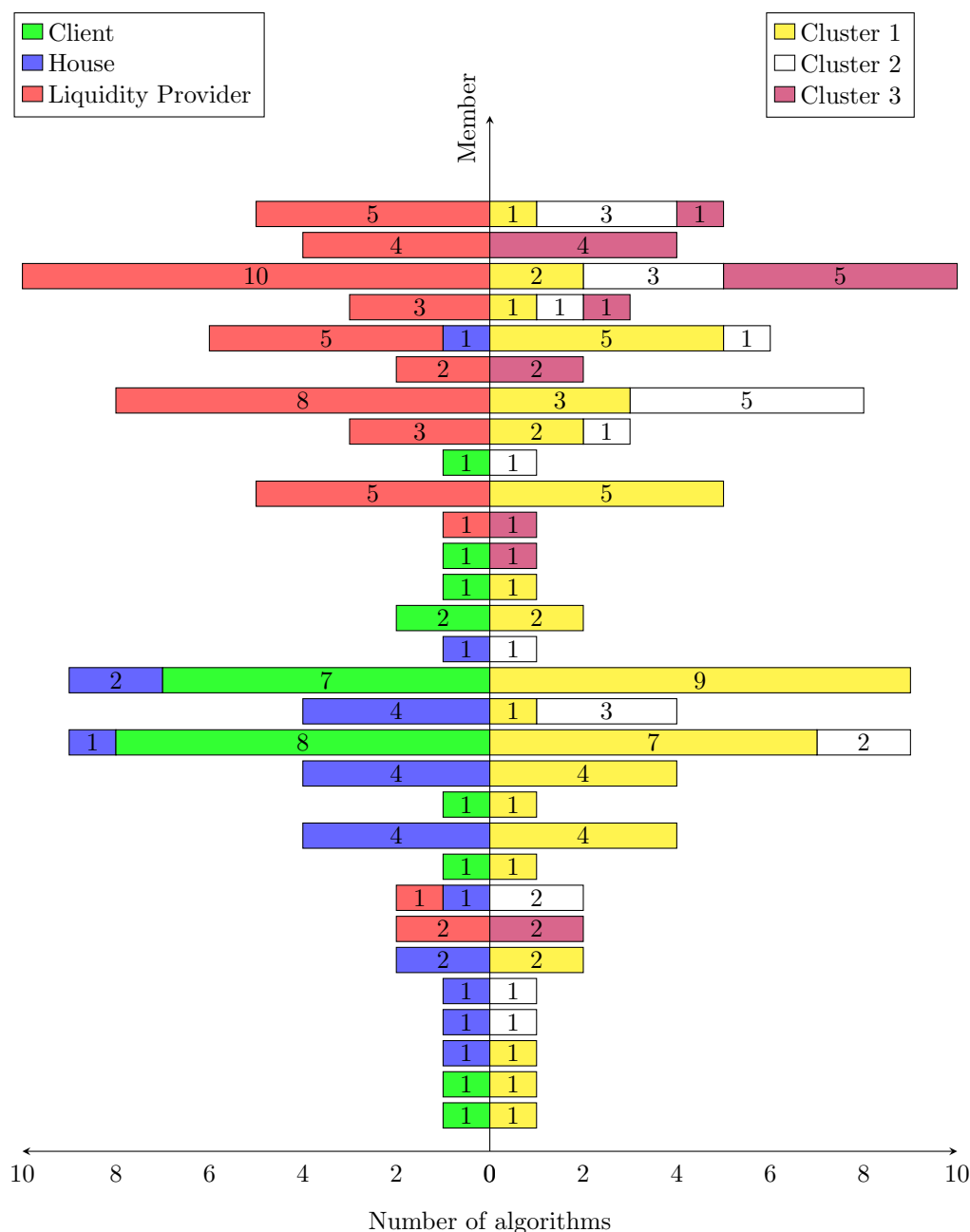


Figure 11: Number of algorithms per dealing capacity (left) and cluster (right) per member in ASML. The members are ordered from top to bottom using order count (high to low). We show 96 algorithms active each week for the 16 weeks of data.

Only three members (the first, third, and fourth from top to bottom) have algorithms in each of the three clusters, five members have algorithms in two of the three clusters, and the remaining twenty-two members have algorithms in only one cluster.²² Comparing the left-hand side to the right-hand side of Figure 11 we see that the three members with algorithms in each of the three clusters are members with algorithms that are all registered as Liquidity Providers in Euronext.

²²Out of the twenty-two members with only one cluster of trading behaviour, twelve have only one algorithm.

4.4.4 Clustered market behaviour

Next, we study each of the clusters in more detail. Table 10 provides summary statistics about the trading activity of the algorithms within the clusters.²³

	Cluster 1	Cluster 2	Cluster 3
number of algorithms	54	25	17
order count	1,639,267	1,901,239	3,712,355
trade count	573,417	343,874	461,874
order to trade ratio	2.86	5.53	8.04
order count %	23	26	51
trade count %	42	25	33
liquidity taking trade count	291,545	251,785	120,353
liquidity taking trade count %	51	73	26
average algo volume in ask side of LOB 0-5	1.43	2.07	2.47
average algo volume in bid side of LOB 0-5	1.49	2.04	2.48
average absolute imbalance of algo 0-5	1.93	1.12	0.77
average ask volume of algo 11-20	0.43	1.69	1.07
average bid volume of algo 11-20	0.44	1.68	1.08

Table 10: Statistics for algorithms in clusters 1,2, and 3 for ASML. First four weeks of data.

Although algorithms in cluster 3 send the most orders (51%), they do fewer transactions (33% compared to 42% for cluster 1 and 25% for cluster 2). This results in a higher order to trade ratio for cluster 3 (8.04) than for clusters 1 and 2 (2.86 and 5.53 respectively). Furthermore, algorithms in cluster 3 have a relatively low fraction of liquidity taking transactions; 26% of their transactions are liquidity taking, versus 51% and 73% for clusters 1 and 2. This suggests that algorithms in cluster 3 behave as traditional market makers. Algorithms in clusters 2 and 3 post more volume (on average) on the first five levels of the LOB than algorithms in cluster 1. Similarly, algorithms in clusters 2 and 3 have more balanced volumes posted in the LOB when compared to algorithms in cluster 1.²⁴

In the next sections, we investigate how the various features we consider affect the average behaviour of algorithms in each of these clusters. We focus on the impact of features on direction and price of orders, as the model for volume does not display much benefit over a naive model (as seen in Table 7). More precisely, in Subsection 4.4.6 we study the average coefficients associated with the regressions for predicting the direction D_t , then, Subsection 4.4.7 discusses the average coefficients for predicting the price bucket \mathfrak{P}_t conditional on direction, and Subsection 4.4.8 explores liquidity taking activity and liquidity provision in more detail.

4.4.5 Summary of results about trading behaviour

Consistently, across exercises in the aforementioned subsections, algorithms in cluster 3 show behaviour associated with inventory-averse market makers. We describe the algorithms in this cluster as “market makers”. These algorithms are keen to maintain a balanced presence in the LOB (e.g., they are more likely to post an order on the bid side of the LOB if their volumes on the ask side of the LOB are higher than those of the bid side of the LOB), they revert their inventories to zero, improve the spread, post liquidity at-the-touch, and do not exhibit aggressive behaviour unless their inventory is large.

In contrast, algorithms in cluster 2 do not show inventory aversion, they send orders in the direction of the imbalance of the volumes posted at the best quotes (send sell orders if the volume in the bid is much smaller than the volume in the ask), have the lowest percentage (across clusters) of eager-to-trade orders that provide liquidity inside the spread and have the highest percentage of eager-to-trade orders that traded aggressively. We describe the algorithms in cluster 2 as “opportunistic traders”.

Lastly, algorithms in cluster 1 have position-building behaviour in terms of their own imbalances and the way they choose direction as a function of its inventory and its imbalance in the LOB. These algorithms

²³Recall that a single aggressive order may result in multiple trades (depending on the number of limit orders it executes against). The number of orders in Table 4 is larger than the number of orders in Table 10 because the later only includes the algorithms present in all of the twelve clustering exercises we perform.

²⁴Recall that if the imbalance of the volumes posted by the algorithm is close to zero then the volume posted by the algorithm on the bid side is close to the volume posted by the algorithm on the ask side.

have propensity to send eager-to-trade orders and at-the-touch orders. Around 82% of their eager-to-trade orders trade aggressively. We describe the algorithms in cluster 1 as “directional traders”.

It should be recognised that this is only a description of these algorithms’ behaviour on this particular exchange – some of them may be taking an opposite directional position in a different market, for which we do not have data.

4.4.6 Direction: average behaviour

We show the average regression coefficients per cluster for the most important features to explain the direction of the order D_t . Table 11 reports the average coefficients per feature computed on the first four weeks of data. For each of the clusters, the average is taken over all algorithms in the cluster. To obtain the coefficients of the 53 features, for each of the 96 algorithms, we perform a matrix multiplication of the coefficients of the logistic regressions on the PCA-transformed features (96×30 matrix) and the principal components matrix (30×53 matrix), both computed in Section 4.1. We represent the effects of the volume in the book with the total volume and the imbalance (rather than the separate volumes on the bid and ask side).

In the tables which follow, we exclude features where the magnitude of the average coefficients is smaller than or equal to 0.1 for all clusters. We order features by their average absolute coefficient size.

	Cluster 1	Cluster 2	Cluster 3
imbalance of algo 0-5	1.48	-1.04	-0.42
imbalance of algo 11-20	0.21	-0.97	0.81
imbalance of algo 6-10	0.48	-1.00	-0.09
cash of algorithm	0.70	0.13	-0.30
inventory of algo	0.70	0.12	-0.30
best ask volume	-0.26	-0.20	0.23
best bid volume	0.24	0.19	-0.24
volume of algo 11-20	0.16	-0.05	0.40
volume of algo 0-5	-0.07	-0.04	-0.28
chg imbalance excl algo 0-5	0.04	0.16	0.05
net agg buy-sell last 1s	-0.03	0.07	0.10
return 1s	0.01	0.08	0.10
volume of algo 6-10	0.01	-0.11	0.04
chg imbalance excl algo 6-10	0.00	0.03	0.10

Table 11: Average regression coefficients per cluster on first four weeks of training data for ASML when predicting direction D_t .

We recall the formulation for the logistic regression in equation (5), where $D_t = 1$ means that the order is to buy and $D_t = -1$ means that the order is to sell. Thus, given that all features are normalised and centred around zero, all else being equal, if an average coefficient is positive (negative) it means that positive values of the relevant feature increase (decrease) the probability of a given order being a buy order; and vice-versa for negative values of the relevant feature.

Table 11 provides a number of stylised facts about the difference among clusters to understand differences in trading behaviour.

- (D-i) Cluster 1 is the only cluster with an average positive value for the coefficient for the imbalance of the algorithm in the first five levels – recall that when imbalance is positive the algorithm has more volume posted in the bid than in the ask. Thus, algorithms in cluster 1 are inclined to send a buy order if they have already posted more volume in the bid than in the ask (and similarly with the roles of bid and ask, and buy and sell reversed). Conversely, algorithms in clusters 2 and 3 are more likely to revert their imbalance back to a level where the volume provided in the ask side of the LOB is close to that provided in the bid side of the LOB. This suggests that clusters 2 and 3 behave in some ways like traditional market makers, i.e., trading algorithms that provide liquidity in both sides of the LOB in a balanced way. In contrast, algorithms in cluster 1 tend to post liquidity with a preferred direction, which is consistent with most algorithms of House and Client type being in cluster 1.
- (D-ii) Cluster 3 exhibits mean-reversion to zero in inventories at the algorithm level, that is, when the intraday accumulated inventory is positive they are more likely to send sell orders, while the inventories of

algorithms in cluster 2 are not mean-reverting to zero.²⁵ Conversely, algorithms in cluster 1 have a strong preference to send orders in the direction of their inventory (buy orders if inventory is positive and sell orders if inventory is negative).

- (D-iii) The estimation we perform through the PCA coefficients leads to an “averaging” effect over coefficients for similar features. This is seen in the coefficients associated with the inventory and cash of the algorithm which are similar.

In summary, algorithms in cluster 3 exhibit trading behaviours that the literature describes as market makers. For example, algorithms in cluster 3 seem keen to maintain a balanced provision of liquidity and take actions to revert their inventories to zero. On the other side of the spectrum we have the algorithms in cluster 1. These algorithms do not balance the liquidity posted in the LOB, and their orders are more likely to be in the same direction as their inventory (i.e., position building and directional trading). Algorithms in cluster 2 lie somewhere in the middle of the behaviour observed by algorithms in clusters 1 and 3. More precisely, out of the coefficients shown in Table 11, there are three instances in which the sign of the coefficient associated with cluster 2 is different from both of the signs of the coefficients for clusters 1 and 3. The three instances involve volumes deep in the LOB.

4.4.7 Price limit: average behaviour

To gain further insights into the trading behaviour of the algorithms in the above clusters we study how they choose their price limits as measured through the price bucket \mathfrak{P}_t .

	Cluster 1	Cluster 2	Cluster 3	Total
$\mathfrak{P}_t = 1$	178,333	128,174	391,016	697,523
$\mathfrak{P}_t = 2$	386,678	456,828	2,257,863	3,101,369
$\mathfrak{P}_t = 3$	1,074,256	1,316,237	1,063,476	3,453,969
Total	1,639,267	1,901,239	3,712,355	7,252,861

Table 12: Order count per price bucket \mathfrak{P}_t and cluster using first four weeks of training data for ASML.

Recall that by the definition of \mathfrak{P}_t in (3), the first price bucket contains orders whose price limit is more generous than the best quotes; that is, the limit price is lower than the best ask price for a sell order or the limit price is greater than the best bid price for a buy order. The second price bucket contains orders whose limit price equals that of the best quotes; that is, the limit price is the best ask price if the order is to sell or the limit price is the best bid price if the order is to buy. The third price bucket contains orders deeper in the LOB.

Thus, given that orders in the first price bucket either (i) trade with the opposite side of the LOB, (ii) improve the spread, or (iii) get cancelled upon entry because their time-in-force precludes them from resting in the LOB and their price limit is better than best quotes, we refer to them as orders that show “eagerness to trade”. Similarly, recall that we call price bucket one the “eager-to-trade” bucket. Next, we study the average regression coefficients associated with orders in the “eager-to-trade” bucket ($\mathfrak{P}_t = 1$).

²⁵See Chapter 10 in [Cartea et al. \(2015\)](#) for a mathematical model of the market making problem illustrating how risk aversion plays a role in how market makers provide liquidity as a function of their inventory.

	$\mathfrak{P}_t = 1 D_t = 1$ (Buy)			$\mathfrak{P}_t = 1 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.21	-0.46	-0.74	-0.17	-0.48	-0.77
best bid volume	0.35	0.30	1.09	-0.21	-0.17	-0.26
best ask volume	-0.20	-0.16	-0.30	0.35	0.30	1.05
imbalance of algo 0-5	-0.00	0.29	0.68	0.08	-0.32	-0.75
spread	0.11	0.07	0.56	0.10	0.07	0.54
volume of algo 0-5	-0.25	-0.12	0.40	-0.29	-0.08	0.30
volume of algo 11-20	0.04	-0.32	-0.22	0.07	-0.26	-0.21
num messages 1ms	0.17	0.20	-0.08	0.21	0.15	-0.06
volume of algo 6-10	-0.13	-0.23	0.11	-0.13	-0.18	0.04
imbalance of algo 6-10	-0.07	0.04	-0.20	0.10	-0.08	-0.28
num messages 0.1ms	0.12	0.19	-0.08	0.15	0.13	-0.06
num messages 0.1s	0.13	0.04	0.07	0.15	0.09	0.07
imbalance of algo 11-20	-0.02	0.11	-0.15	0.02	-0.09	-0.05

Table 13: Average regression coefficients for price bucket describing eager-to-trade orders, per cluster, on first four weeks of training data for ASML, conditioning on the direction of order D_t .

We observe the following stylised features of trading algorithms in each cluster:

- (P-i) According to the intercept of the regressions, algorithms in cluster 3 are noticeably less likely than those in clusters 1 and 2 to place orders that are eager to trade.
- (P-ii) For algorithms in all clusters, the higher the imbalance of the volumes (by all algorithms) posted at the best bid and ask, the more likely a buy order is eager to trade. This effect is most pronounced for algorithms in cluster 3, where the bid volume has a much greater impact on orders to buy than the ask volume; conversely for orders to sell.
- (P-iii) Algorithms in cluster 1 seem indifferent to their own imbalance in the book when determining eagerness to trade, whereas imbalance of their own volume indicates a moderate eagerness to trade for cluster 2 algorithms, and a strong eagerness for cluster 3. The direction of the order for cluster 3 is in the direction of the imbalance (so if an algorithm has a higher bid than ask volume posted, which means they have a positive imbalance, its buy orders are typically more eager to trade and its sell orders are typically less eager to trade).
- (P-iv) Regardless of the direction, the wider the bid-ask spread the more likely algorithms are to send orders that are eager to trade.²⁶ This is particularly pronounced for cluster 3 (below we see that this cluster often refills the book when the spread is above one tick).
- (P-v) For algorithms in clusters 1 and 2, the higher their own posted volumes at levels 0 to 5 on either the bid or ask side, the less likely they are to send an order (either to buy or to sell) that is eager to trade. Algorithms in cluster 3 exhibit the opposite behaviour; already having volume at the first five levels of the LOB makes it more likely for them to send eager-to-trade orders. For algorithms in clusters 2 and 3, having volume posted deeper in the book decreases the likelihood of sending eager-to-trade orders.
- (P-vi) For algorithms in clusters 1 and 2, the higher the number of recent messages, the more likely an order is eager to trade, in either direction.
- (P-vii) For all algorithms, there is a reasonably clear symmetry when the roles of bid and ask, and buying and selling, are reversed.

We also look at the second price bucket ($\mathfrak{P}_t = 2$), which describes orders adding volume to the LOB at the current best bid or best ask (also known as posting ‘at-the-touch’), and at the third ($\mathfrak{P}_t = 3$, whose coefficients are simply the negative sum of those for $\mathfrak{P}_t = 1, 2$), which describes orders adding volume deeper within the LOB.

²⁶We refine this result in Subsection 4.4.8. In particular, we find that the larger the bid-ask spread, the more likely all clusters are to provide liquidity inside the spread, and the less likely they are to cross the spread.

	$\mathfrak{P}_t = 2 D_t = 1$ (Buy)			$\mathfrak{P}_t = 2 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.15	-0.58	1.25	-0.13	-0.68	0.94
volume of algo 0-5	0.13	0.52	0.16	0.24	0.52	0.21
volume of algo 6-10	-0.03	0.38	0.19	0.00	0.33	0.17
volume of algo 11-20	-0.19	0.06	0.14	-0.27	-0.01	0.06
imbalance of algo 0-5	-0.02	0.22	0.09	0.01	-0.21	-0.15
imbalance of algo 11-20	-0.05	0.09	0.07	0.06	-0.07	0.13
best ask volume	0.08	0.03	0.04	-0.06	0.08	-0.12
imbalance of algo 6-10	-0.04	0.09	0.18	0.02	-0.05	-0.04
num messages 1ms	-0.09	-0.01	-0.04	-0.11	-0.02	-0.05
num messages 0.1s	-0.10	-0.02	-0.01	-0.12	-0.03	-0.03
spread	-0.00	0.04	-0.10	-0.01	0.04	-0.09

Table 14: Average regression coefficients for price bucket describing posting at-the-touch per cluster on first four weeks of training data for ASML conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 3 D_t = 1$ (Buy)			$\mathfrak{P}_t = 3 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	0.36	1.04	-0.51	0.30	1.16	-0.17
imbalance of algo 0-5	0.02	-0.51	-0.76	-0.09	0.52	0.90
best bid volume	-0.31	-0.36	-1.00	0.13	0.15	0.22
best ask volume	0.11	0.13	0.26	-0.29	-0.38	-0.92
volume of algo 0-5	0.11	-0.40	-0.55	0.05	-0.44	-0.51
spread	-0.11	-0.11	-0.46	-0.09	-0.11	-0.45
volume of algo 11-20	0.14	0.26	0.09	0.19	0.26	0.15
volume of algo 6-10	0.16	-0.15	-0.30	0.13	-0.16	-0.21
imbalance of algo 6-10	0.11	-0.13	0.02	-0.11	0.14	0.31
num messages 0.1ms	-0.07	-0.23	0.11	-0.09	-0.16	0.09
num messages 1ms	-0.09	-0.20	0.12	-0.10	-0.13	0.11
imbalance of algo 11-20	0.07	-0.20	0.08	-0.08	0.16	-0.08

Table 15: Average regression coefficients for price bucket describing posting deeper within the order book, per cluster on first four weeks of training data for ASML conditioning on the direction of order D_t .

By looking at Table 14 and Table 15, we observe the following stylised facts for the typical behaviour of each cluster. It should be emphasised that these observations are for a given direction of order, and so the conclusions should be read in the light of Table 11.

- (P-viii) Cluster 3 algorithms are much more likely to place their orders at-the-touch than either of the other clusters (see the intercept of Table 14). Cluster 2 is less likely to place its orders at-the-touch than cluster 1 (all other things being equal).
- (P-ix) Algorithms in cluster 2 have the highest propensity to place orders deeper in the book.
- (P-x) For all algorithms, the higher the volume at the best ask price and the lower the volume at the best bid (i.e., the lower the imbalance of the best volumes in the LOB) the more likely that a buy order will be placed deeper in the LOB; conversely for sell orders. This indicates that algorithms pre-empt that when a large volume builds at the best ask (bid) price, then prices will likely move down (up), hence why they choose to place their buy (sell) orders deeper in the LOB.
- (P-xi) The likelihood that an algorithm in cluster 2 posts at-the-touch is affected by its own imbalance near the touch. The likelihood is impacted in the direction of the imbalance (so if an algorithm has a higher bid than ask volume posted, which corresponds to a positive imbalance, it is more likely that the algorithm posts buy orders at the best bid price). A weaker effect is observed for cluster 3, but little effect for cluster 1.
- (P-xii) For all algorithms, an increase in the recent number of messages reduces the probability of posting at-the-touch in either direction.

In summary, for algorithms in the first two clusters, the higher the number of recent messages (sent to the exchange by all market participants) the more likely they are to send an eager-to-trade order (in either direction), similarly, for algorithms in all clusters, the higher the spread the less likely an algorithm is to place eager-to-trade or at-the-touch orders. Again, algorithms in cluster 1 seem indifferent to their own imbalance in the book when deciding to send an eager-to-trade order. Algorithms in cluster 2 have a moderate eagerness to trade according to their imbalance in the LOB (similar for posting at-the-touch). Algorithms in cluster 3 have a strong eagerness to trade according to their imbalance in the LOB, and they also have the highest baseline probability of posting at-the-touch. These results are consistent with our previous findings about cluster 3 exhibiting behaviour associated with market makers.

The low value of the coefficients for the inventory of the algorithm (this predictor is absent from Tables 13, 14, and 15, indicating that all clusters had an average coefficient below 0.10 in magnitude) may suggest that this has limited effect on the eagerness to trade. However, this may be due to the limitation of our statistical model to linear-logistic relationships. The plots in Figure 12 show the percentage of orders which were eager to trade, as a function of (transformed, intraday accumulated) inventory. In the rare cases where transformed inventory exceeds 6 (in absolute value), there is a noticeable increase in the eagerness to trade for all algorithms. For algorithms in cluster 3, this effect is smoother as inventory changes, and larger than in clusters 1 or 2.

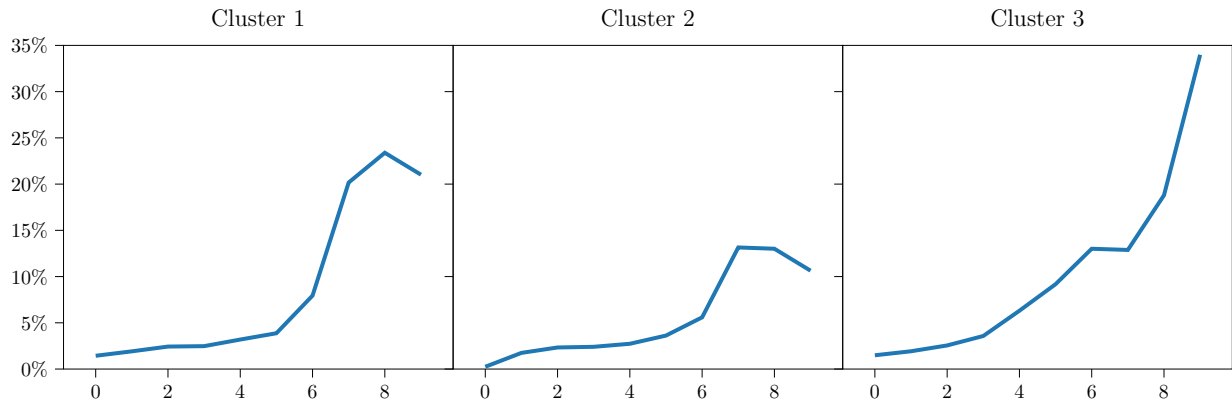


Figure 12: Percentage of eager-to-trade orders as a function of the absolute value of the transformed intraday accumulated inventory of the algorithm. See Appendix B for the formula of the log-inventory of the algorithm.

4.4.8 Liquidity taking activity

We now focus on orders in price bucket $\mathfrak{P}_t = 1$, that is, orders with prices better than the current best bid or ask price. When such an order is sent, there are three possible scenarios, which we label by a variable A_t describing whether it takes liquidity, provides liquidity, or is a missed attempt.

- (i) The case $A_t = 0$ encompasses aggressive trading, i.e., orders that cross the spread. These are buy orders with limit price greater than or equal to the best ask price, or sell orders with limit price less than or equal to the best bid price. Almost all orders within this category are executed upon entry (or partially executed) against liquidity on the other side of the LOB depending on the time-in-force, the limit price of the order, the volume of the order, and the liquidity available in the LOB.²⁷ From Table 3 we know that orders with FoK time-in-force are negligible so for all practical purposes $A_t = 1$ is aggressive behaviour.
- (ii) The case $A_t = 1$ covers generous provision of liquidity. These are orders that provide liquidity inside the spread. The time-in-force of the orders is DAY and their limit price is higher than the best bid price and lower than the best ask price if the order is to buy; conversely for sell orders.

²⁷We provide an example. Assume the best ask price in the LOB is 100 and the volume available is 50 units. An IoC order arrives willing to buy with price limit of 100 and volume of 100, then, 50 units will get executed against the available liquidity and the remaining 50 will be cancelled. Alternatively, if the order had FoK time-in-force then it is cancelled upon arrival.

- (iii) The case $A_t = 2$ describes missed attempts. These are orders that are cancelled upon entry because their price limit is not generous enough to trade with the available liquidity in the LOB and their time-in-force precludes them from resting in the LOB. This category covers (i) orders to buy with limit price less than the best ask price and IoC time-in-force; it also covers (ii) sell orders with limit price greater than the best bid price and IoC time-in-force. Missed attempts are likely aimed at (a) liquidity that disappeared over the latency period, (b) hidden liquidity, or (c) new liquidity being added over the latency period; see [Cartea and Sánchez-Betancourt \(2023\)](#).

For example, during the first four weeks of data for ASML we find that algorithms in cluster 1 (respectively cluster 2 and cluster 3) sent 178,333 orders (respectively 128,174 and 391,016 orders) to buy or to sell that were eager to trade; around 82% of these orders (respectively 89% and 18%) traded against the opposite side of the LOB, i.e., show aggressive behaviour, similarly, 17% of the orders (respectively 10% and 81%) provided liquidity inside the spread, i.e., generous liquidity provision, and 1% of the orders (respectively 1% and 1%) were cancelled by the exchange upon entry because of their lack of generosity and time-in-force, i.e., missed attempts. These results are summarised in Table 16 together with the usage of IoC associated with each of the clusters.

	Cluster 1	Cluster 2	Cluster 3	Total
number of orders $\mathfrak{P}_t = 1$	178,333	128,174	391,016	697,523
IoC orders	68%	84%	4%	245,988
$A_t = 0$ (aggressive trading)	82%	89%	18%	333,073
$A_t = 1$ (generous liquidity)	17%	10%	81%	364,650
$A_t = 2$ (missed attempt)	1%	1%	1%	2,800

Table 16: Percentage of orders sent with $A_t \in \{0, 1, 2\}$ out of all orders with $\mathfrak{P}_t = 1$.

These percentages support our findings that algorithms in cluster 3 make markets for ASML. Algorithms in cluster 3 have the lowest percentage of aggressive behaviour, and they have the highest proportion of provision inside the spread. On the other hand, algorithms in clusters 1 and 2 render similar proportions for aggressive behaviour and missed aggressive behaviour, but they differ in their proportions for providing liquidity inside the spread. Here, algorithms in cluster 1 provide liquidity inside the spread more often than algorithms in cluster 2. For both clusters 1 and 2, the vast majority of IoC orders successfully trade, and only a small fraction are cancelled.

Next, we perform a regression similar to those above to predict if the order will take liquidity, conditioned on direction and conditioned on the order being in the first price bucket. We recall that if an order sent at time t consumes liquidity we write $A_t = 0$, if the order provides liquidity inside the spread we write $A_t = 1$, and we write $A_t = 2$ otherwise. Table 17 shows the average coefficients for the regressions for $A_t = 0$, Table 18 shows the average coefficients for the regression associated with $A_t = 1$, and Table 19 shows the average coefficients for the regressions associated with $A_t = 2$.

	$A_t = 0 \mid \mathfrak{P}_t = 1, D_t = 1$ (Buy)			$A_t = 0 \mid \mathfrak{P}_t = 1, D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	0.73	0.97	-0.00	0.82	1.08	0.10
spread	-1.09	-0.95	-1.37	-1.29	-0.88	-1.35
best bid volume	-0.36	-0.35	-0.50	-0.93	-0.65	-0.78
best ask volume	-0.77	-0.65	-0.77	-0.44	-0.26	-0.47
volume of algo 0-5	-0.49	-0.84	0.08	-0.43	-0.77	-0.02
volume excl algo 0-5	0.30	0.31	0.45	0.30	0.29	0.42
volume of algo 6-10	-0.39	-0.58	0.10	-0.30	-0.58	0.03
quad var 5m	-0.20	-0.13	-0.34	-0.27	-0.16	-0.36
quad var 1m	-0.17	-0.12	-0.28	-0.21	-0.12	-0.30
quad var 60m	0.14	0.25	0.16	0.16	0.19	0.17
quad var 15m	-0.14	-0.04	-0.25	-0.19	-0.09	-0.26
last volume transacted	-0.08	-0.12	-0.15	-0.10	-0.12	-0.12
imbalance of algo 0-5	0.12	-0.02	0.21	-0.12	-0.00	-0.20
num messages 0.1ms	-0.21	-0.12	0.01	-0.16	-0.14	0.02
volume of algo 11-20	-0.18	-0.09	-0.00	-0.14	-0.19	-0.01
net agg buy-sell last 1s	-0.09	-0.12	-0.05	0.13	0.10	0.10
imbalance excl algo 0-5	-0.09	-0.11	-0.01	0.11	0.17	0.08
num messages 0.1s	0.07	0.11	0.08	0.11	0.09	0.08
agg sell last 360s	0.10	0.09	0.10	0.08	0.05	0.06
agg sell last 60s	0.09	0.06	0.13	0.07	0.05	0.09
agg sell last 1s	0.03	0.06	0.02	-0.10	-0.10	-0.09
num messages 1ms	-0.15	-0.03	0.01	-0.10	-0.05	0.03
agg buy last 60s	0.04	0.04	0.06	0.10	0.07	0.06
return 300s	0.04	0.04	0.12	0.02	0.01	0.08
volume excl algo 6-10	-0.04	-0.01	-0.04	-0.06	0.02	-0.13
cash of algorithm	0.03	0.07	-0.01	-0.07	-0.10	0.01
inventory of algo	0.02	0.06	-0.00	-0.07	-0.10	0.01
imbalance of algo 6-10	0.05	-0.00	-0.10	-0.06	-0.04	-0.01

Table 17: Average regression coefficients. Orders take liquidity upon arrival in the exchange conditioned on orders being eager to trade and conditioned on the direction D_t of orders.

	$A_t = 1 \mid \mathfrak{P}_t = 1, D_t = 1$ (Buy)			$A_t = 1 \mid \mathfrak{P}_t = 1, D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	0.05	-0.25	0.94	0.10	-0.24	0.83
spread	0.85	0.57	1.25	0.95	0.55	1.25
best bid volume	0.33	0.28	0.50	0.71	0.39	0.71
best ask volume	0.62	0.37	0.70	0.36	0.23	0.49
volume of algo 0-5	0.21	0.68	-0.17	0.17	0.55	-0.15
volume of algo 6-10	0.09	0.47	-0.16	0.02	0.40	-0.13
volume excl algo 0-5	-0.12	-0.15	-0.36	-0.13	-0.15	-0.34
quad var 5m	0.17	0.08	0.32	0.15	0.04	0.29
quad var 60m	-0.14	-0.16	-0.14	-0.20	-0.19	-0.18
quad var 1m	0.13	0.07	0.25	0.11	0.05	0.25
imbalance excl algo 0-5	-0.14	-0.11	-0.08	0.20	0.12	0.11
quad var 15m	0.11	0.02	0.23	0.07	-0.02	0.20
num messages 1ms	-0.17	-0.09	0.02	-0.22	-0.07	0.05
num messages 0.1ms	-0.17	-0.04	0.01	-0.22	-0.05	0.04
inventory of algo	0.12	0.11	-0.03	0.05	0.11	0.05
cash of algorithm	0.12	0.11	-0.03	0.05	0.11	0.05
agg sell last 60s	-0.07	-0.03	-0.08	-0.11	-0.04	-0.10
imbalance of algo 0-5	-0.06	-0.08	-0.12	0.02	0.00	0.12
net agg buy-sell last 1s	-0.10	-0.07	-0.03	0.08	0.07	-0.03
volume excl algo 6-10	0.08	0.02	0.06	0.05	-0.02	0.13
volume of algo 11-20	0.02	0.11	-0.01	-0.04	0.12	0.04
net agg buy-sell last 360s	-0.06	-0.03	0.03	0.11	0.07	0.06
return 300s	-0.03	0.03	-0.10	-0.03	-0.02	-0.12
imbalance of algo 6-10	-0.04	-0.07	0.10	0.03	0.06	-0.01
net agg buy-sell last 60s	-0.00	-0.01	0.02	0.10	0.04	0.10
volume excl algo 11-20	-0.02	-0.03	0.04	-0.02	0.01	0.10

Table 18: Average regression coefficients. Orders provide liquidity inside the spread upon arrival in the exchange conditioned on orders being eager to trade and conditioned on the direction D_t of orders.

	$A_t = 2 \mid \mathfrak{P}_t = 1, D_t = 1$ (Buy)			$A_t = 2 \mid \mathfrak{P}_t = 1, D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.78	-0.72	-0.94	-0.92	-0.83	-0.93
spread	0.24	0.39	0.12	0.35	0.33	0.11
imbalance excl algo 0-5	0.23	0.23	0.09	-0.31	-0.29	-0.19
volume of algo 0-5	0.28	0.16	0.09	0.26	0.22	0.17
num messages 0.1ms	0.38	0.17	-0.01	0.38	0.18	-0.06
volume of algo 6-10	0.30	0.11	0.06	0.28	0.17	0.10
num messages 1ms	0.32	0.12	-0.03	0.32	0.12	-0.08
net agg buy-sell last 1s	0.19	0.20	0.07	-0.20	-0.17	-0.07
volume excl algo 0-5	-0.18	-0.16	-0.10	-0.17	-0.13	-0.08
last volume transacted	0.12	0.11	0.09	0.16	0.13	0.06
best bid volume	0.03	0.07	-0.00	0.21	0.25	0.07
best ask volume	0.15	0.28	0.07	0.08	0.03	-0.02
agg sell last 1s	-0.10	-0.13	-0.07	0.15	0.12	0.03
num messages 0.1s	-0.09	-0.12	-0.07	-0.10	-0.12	-0.05
net agg buy-sell last 5s	0.12	0.07	0.05	-0.14	-0.10	-0.07
agg buy last 1s	0.14	0.11	0.01	-0.10	-0.10	-0.06
volume of algo 11-20	0.16	-0.02	0.01	0.18	0.07	-0.03
cash of algorithm	-0.15	-0.18	0.04	0.02	-0.01	-0.07
inventory of algo	-0.14	-0.17	0.03	0.02	-0.00	-0.06
imbalance of algo 0-5	-0.05	0.10	-0.09	0.10	-0.00	0.08
quad var 5m	0.03	0.05	0.03	0.12	0.11	0.07
net agg buy-sell last 60s	0.06	0.02	0.05	-0.13	-0.07	-0.07
net agg buy-sell last 360s	0.08	0.04	-0.01	-0.12	-0.09	-0.05
quad var 15m	0.03	0.02	0.02	0.12	0.11	0.06
quad var 1m	0.05	0.06	0.02	0.10	0.08	0.05
cash of member	-0.12	-0.04	-0.05	-0.06	0.03	-0.04
inventory of member	-0.12	-0.04	-0.05	-0.06	0.03	-0.04
agg buy last 360s	-0.04	-0.05	-0.04	-0.10	-0.07	-0.02
agg sell last 5s	-0.01	-0.02	-0.05	0.13	0.08	0.03
agg buy last 5s	0.13	0.06	0.01	-0.03	-0.03	-0.05
agg sell last 360s	-0.10	-0.08	-0.04	-0.02	-0.01	0.01
volume excl algo 11-20	0.06	0.04	0.01	0.02	-0.10	-0.02

Table 19: Average regression coefficients. Orders that are immediately cancelled by the exchange upon arrival conditioned on orders being eager to trade and conditioned on the direction D_t of orders.

We observe the following stylised features of trading algorithms in each cluster:

- (LT-i) According to the intercept of the above tables, algorithms in clusters 1 and 2 are more likely to send aggressive orders that consume liquidity from the opposite side of the LOB. Algorithms in cluster 3 have the highest propensity to send orders that provide liquidity inside the spread, whereas algorithms in cluster 2 are unlikely to provide liquidity inside the spread. This is consistent with the proportions reported at the beginning of this subsection, where algorithms in cluster 2 exhibit the lowest percentage of liquidity provision inside the spread (10%).
- (LT-ii) Once again, algorithms across clusters agree on their behaviour according to spread. Given they post eager-to-trade orders, algorithms are more likely to provide liquidity inside the spread when the bid-ask spread is wider. Similarly, algorithms are more likely to have missed attempts when the spread is wider. On the other hand, the wider the spread is, the less likely algorithms are to consume liquidity from the other side of the LOB. This observation is unsurprising, as for ASML the spread is usually equal to one tick, in which case it is not possible to provide liquidity inside the spread nor to miss a trade attempt, making crossing the spread relatively more likely. This should also be compared with observation (P-iv); the sums of the spread coefficients in Tables 13 and 19 show that the likelihood of an order crossing the spread is reduced when the spread is larger, unconditionally on whether the order is eager to trade or not.
- (LT-iii) The results we discuss for spread also hold for some of the variables describing recent activity (as one would expect). For example, for most horizons of the quadratic variation, the larger the quadratic variation, the less likely algorithms will show aggressive behaviour, and the more likely it is that they will post liquidity inside the spread. For both cases, the quadratic variation over the last trading hour has an inverse effect.
- (LT-iv) The best available volumes in the LOB (best bid volume and best ask volume) also have a similar consistent effect across algorithms in all three clusters. The higher the available volumes in the market,

the less likely algorithms are to consume liquidity from the opposite side of the LOB, the more likely they are to provide liquidity inside the spread, and the less likely they are to miss aggressive attempts. For liquidity taking, we observe that the magnitude of coefficients for the best ask volume is roughly twice the magnitude of coefficients for the best bid volume when describing the aggressiveness of orders to buy; conversely for orders to sell. This is consistent with the findings in (P-x); indeed, if the market builds volume in the best ask price for example, then everything else being equal, one expects prices to move down, this in turn lowers the probability of an aggressive order to buy as a consequence of the expected price move; conversely for the case of sell orders.

- (LT-v) For liquidity provision inside the spread, we observe that the coefficients for best bid volume and best ask volume swap their importance when considering buys or sells. In particular, the best buy volume is more important to determine if a buy order will be sent inside the spread; conversely for the best ask volume and sell orders. These results are consistent with what one expects from the usual models of trading activity.

Our work has focused on ASML, which is a liquid asset. Similar clustering results hold for ING, see Appendix C.3. For shares with lower volumes traded (here AHOLD and TOMTOM), the results are less clear, possibly due to lower levels of trading in these shares. See Tables A13–A21 for the clustered coefficients for other shares.

5 Implications for supervision

The results above provide a starting point to formalise and to quantify the links between a number of market features and the behaviour of trading algorithms. Our findings provide statistical evidence for many microstructural stylised facts discussed in the literature, and our study is the first to confirm these facts with a unique data set that contains both member and algorithm identification. One key contribution is to show how algorithms make trading decisions as a function of 53 market features, some of which are visible to all market participants and the others are idiosyncratic features of the algorithms and of each market participant.

We believe the findings in the above sections are of special relevance to regulators and supervisors.

5.1 Surveillance

Given the vast number of messages that are processed every day by exchanges in electronic markets, it is difficult to spot behaviour that intentionally or inadvertently may harm the integrity of markets. Either way, the regulator must develop sophisticated tools to monitor the market and understand the impact of individual algorithms on metrics of market quality.

Currently, financial regulators and supervisors use algorithms on transaction and order data to detect practices that may harm the integrity of the market. For example, in electronic trading, the Dutch Authority for the Financial Markets (AFM) relies on a combination of Suspicious Transaction and Order Reports from market participants and custom made algorithms on transaction and order data to prevent and detect market manipulation, see AFM (2021b). Similarly, the Financial Industry Regulatory Authority (FINRA) in the US uses the SONAR system to monitor suspicious trading activity such as well-timed trades occurring just before public announcements. These surveillance algorithms mine data to search for unusual patterns, some of which could be designed to manipulate markets (e.g., pump and dump), to mislead the information in the LOB (e.g., spoofing), or to trade with privileged information (e.g., front running, insider trading).

Most detection algorithms tend to look for “misleading” signals in the market.²⁸ To learn what is misleading, one should first know which features agents take into account when making a trading decision. As a substitute for misleading signals, supervisors often look for outliers or unusual trading behaviour. But this ignores the fundamental issue at stake: is the unusual trading behaviour misleading market participants?

One should not mistake features that predict algorithmic trading behaviour for features that algorithms use when making trading decisions. Anecdotally, trading firms often use a variety of non-market information sources when making decisions, while our data is restricted to quantities which are based in the market (in

²⁸See Article 12 “Market Manipulation” in EU (2014).

particular, which are observable to a regulator). This suggests that some of our conclusions will be affected by confounding factors. For example, a firm with an off-market signal which encourages them to take a directional position may often trade in a consistent direction – in this case, their intraday accumulated inventory may be a good predictor of their trading behaviour in the near future, as it acts as a proxy for their unobserved signal. This distinction is important when drawing causal conclusions from our results.

Our work has implications for both (i) data-driven detection of market manipulation, and (ii) supporting claims about misleading behaviour in cases of market manipulation. With regards to (i), our findings lend support to the claim that supervisors should look at “unusual” behaviour in the most important features as listed in Section 4.3. Regarding (ii), supervisors have difficulties backing up why some trading behaviour could be considered misleading. It is often difficult to show that some behaviour (e.g., creating an imbalance in the LOB), causes a reaction in other agents. We believe our findings show how changes in the most important features affect trading decisions, hence could be informative in cases of market manipulation. More precisely, our models can be used to simulate the sequence of events that follow a case of potential market manipulation. By comparing simulations including and excluding the potential manipulation, one could estimate if and how market participants reacted to the potential manipulation, thereby quantifying the effect of the potential manipulation.

5.2 Testing of trading algorithms

Our results provide a unique starting point to build more sophisticated trading models than those used in the extant literature. These models will be useful for market participants to build simulators to test strategies and will help financial regulators to understand market dynamics, individual behaviour, and the impact of trading algorithms and strategies on the integrity of markets.

Trading firms are required to test their trading algorithms to make sure they do not behave in an unintended manner or contribute to disorderly trading conditions, see [EU \(2016b\)](#).²⁹ To do so, firms can use their own testing environment or one provided by the trading venue. Trading venues are required to provide members with simulation facilities which reproduce as realistically as possible the production (i.e., the real) environment. The simulations should allow members to test a range of scenarios that they consider relevant to their activity, and realistically reproduce disorderly trading conditions, see [EU \(2016a\)](#).³⁰

[AFM \(2021a\)](#) notes that there is room for improvement for both trading firms and trading venues in testing trading algorithms. Current testing environments tend to range from one-off order books without new orders entering the market over time, generated by the trading venue at the request of a trading firm, to markets generating orders with a predetermined frequency and set parameters in real-time. Overall, [AFM \(2021a\)](#) notes that trading venues have difficulty creating sufficiently realistic simulation facilities. Furthermore, this report stresses that testing against disorderly trading conditions should be designed with a view to addressing the reaction of the algorithm or strategy to conditions that may create a disorderly market. Agent-based modelling is not currently being used widely in simulation facilities, nor agents trained on real data. [AFM \(2021a\)](#) encourages trading venues to explore innovative ways to make their simulation environments more realistic so that they are in line with the requirements.

²⁹Article 5(4): The methodologies referred to in paragraph 1 shall ensure that the algorithmic trading system, trading algorithm or algorithmic trading strategy:

- (a) does not behave in an unintended manner;
- (b) complies with the investment firm’s obligations under this Regulation;
- (c) complies with the rules and systems of the trading venues accessed by the investment firm;
- (d) does not contribute to disorderly trading conditions, continues to work effectively in stressed market conditions and, where necessary under those conditions, allows for the switching off of the algorithmic trading system or trading algorithm.

³⁰Article 10(2)(a) Trading venues shall provide their members with access to a testing environment which shall consist of any of the following:

- (a) simulation facilities which reproduce as realistically as possible the production environment, including disorderly trading conditions, and which provide the functionalities, protocols and structure that allow members to test a range of scenarios that they consider relevant to their activity;
- (b) testing symbols as defined and maintained by the trading venue.

This study is a starting point to build a simulator where market dynamics can be replicated at a granular level to understand how each message to the order book may affect individual and therefore affect collective dynamics. In particular, our statistical framework is devised to capture the fine microstructural facts that drive the trading decisions of individual algorithms, which is key to understanding how trading algorithms react to new market information and to messages to the LOB. With such a market simulator, firms will be able to test their trading algorithms and regulators will be able to study the effect of new trading algorithms on the quality of the market. Similarly, financial authorities will have a tool to build counterfactual trading tapes to gain insights into market behaviour in the absence of certain trading algorithms or with potential new entrants in the market.

5.3 Clustering

In terms of the clustering exercise, there are a number of relevant insights for supervisors. Irrespective of the trading venue, a supervisor could impose its own criteria on what behaviour would be expected of any Liquidity Provider, House or Client. Then, our clustering methodology can be used to highlight trading firms whose algorithms might show behaviour contrary to what one would reasonable expect based on their dealing capacity.

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Contributions

The authors contributed to the paper as follows: SC and RG initiated the collaboration; ÁC, SC, RG, and LSB developed the modelling framework, with input from SL and LV; RG and LSB implemented the primary modelling code, with input from SC, SL and LV; ÁC, SC, RG and LSB wrote the paper.

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A Description of shares

Similar to Table 5, we compute the order count, trade count, and volume traded of groups of algorithms during the first four weeks of training data for ING, AHOLD, and TOMTOM. Table A1 displays the results for ING, Table A2 for AHOLD, and Table A3 for TOMTOM.

	order count	order count percentage of all %	trade count	trade count percentage of all %	volume €	volume percentage of all %
Top 5	2,326,236	42	190,042	24	970,242,121	19
Top 10	3,542,047	64	268,601	35	1,338,267,032	27
Top 20	4,495,680	81	328,265	42	1,675,327,497	34
Top 50	5,388,559	97	600,684	77	3,631,692,876	73
All	5,555,240	100	777,720	100	4,986,039,098	100

Table A1: Descriptive statistics for algorithms trading in ING during first four weeks of training data.

	order count	order count percentage of all %	trade count	trade count percentage of all %	volume €	volume percentage of all %
Top 5	925,564	49	68,711	24	294,230,009	25
Top 10	1,250,004	66	97,494	34	378,566,543	33
Top 20	1,590,981	84	121,131	42	429,239,323	37
Top 50	1,837,989	97	228,241	80	918,281,638	79
All	1,891,709	100	285,440	100	1,163,250,600	100

Table A2: Descriptive statistics for algorithms trading in AHOLD during first four weeks of training data.

	order count	order count percentage of all %	trade count	trade count percentage of all %	volume €	volume percentage of all %
Top 5	151,036	59	6,266	17	12,802,205	14
Top 10	193,241	75	9,173	25	24,460,828	28
Top 20	230,321	89	18,410	49	41,265,509	48
Top 50	253,284	98	32,989	88	75,158,861	87
All	257,902	100	37,284	100	86,580,402	100

Table A3: Descriptive statistics for algorithms trading in TOMTOM during first four weeks of training data.

B Feature specifications

In this section, we provide a detailed description of the variables we use in the study; these were introduced in Section 3.1.

We denote time by t and let the trading horizon be $[0, T]$ on any given day. At any time $t \in [0, T]$, let \tilde{S}_t be price of the last transaction that took place before time t ; if the trade involved multiple orders, \tilde{S}_t is the volume weighted average price of the transaction. Let S_t^a be the best ask price available in the LOB at time t and V_t^a the aggregated volume at price level S_t^a . Similarly, let S_t^b be the best bid price available in the LOB at time t and V_t^b the aggregated volume at price level S_t^b . Thus, the spread at time t and the midprice at time t are given by

$$\mathfrak{S}_t = S_t^a - S_t^b, \quad S_t = \frac{S_t^a + S_t^b}{2}, \quad (10)$$

respectively. For a given number of levels $n \in \mathbb{N}$ denote by $V_t^{a,n}$ the total volume available in the first n levels of the LOB on the ask side; similarly, denote by $V_t^{b,n}$ the total volume available in the first n levels of the LOB on the bid side. Recall that $\mathcal{A} = \{a_1, a_2, \dots, a_M\}$ are the identifiers for the algorithms on Euronext, and $\mathcal{B} = \{b_1, b_2, \dots, b_N\}$ are the identifiers for the trading members of Euronext with $N \leq M$ because a trading member can have multiple trading algorithms. For $a_i \in \mathcal{A}$ (similar for $b_i \in \mathcal{B}$) let $V_t^{a,n,-a_i}$ be the

volume available in the first n levels of the LOB on the ask side when we *exclude* any volume posted by algorithm a_i (similar definition for $V_t^{b,n,-a_i}$). Likewise, we use V_t^{a,n,a_i} and V_t^{b,n,a_i} for the volume posted by algorithm $a_i \in \mathcal{A}$. We define the imbalance between positive volumes V^a and V^b by

$$\mathfrak{I}(V^b, V^a) = \log(1 + V^b) - \log(1 + V^a), \quad (11)$$

For a time window $\delta > 0$, we define the volatility of the transaction prices at time t of period δ by

$$\mathfrak{V}_t^\delta = \sqrt{\sum_{\Delta \log \tilde{S}_u \neq 0; u \in [t-\delta, t]} |\Delta \log \tilde{S}_u|^2}, \quad (12)$$

where

$$\Delta \log \tilde{S}_u = \log \tilde{S}_u - \log \tilde{S}_{u-}, \quad \text{and } \tilde{S}_{u-} = \lim_{v \nearrow u} \tilde{S}_v. \quad (13)$$

The above definition extends to other stochastic processes in the paper, that is $Y_{u-} = \lim_{v \nearrow u} Y_v$ for any càdlàg stochastic process Y . The return at time t of period δ is given by

$$\mathfrak{R}_t^\delta = \log \left(\frac{S_{t-}}{S_{t-\delta}} \right). \quad (14)$$

The number of messages at time t and period δ is denoted by \mathfrak{U}_t^δ and records the number of messages in the LOB during the window $[t-\delta, t)$. Here, a “message” is an order entry, an order deletion, or an order amendment.

Lastly, for algorithm $a_i \in \mathcal{A}$ and the associated trading member $b_j \in \mathcal{B}$, we let $\mathfrak{Q}_t^{a_i}$ and $\mathfrak{Q}_t^{b_j}$ denote the intraday accumulated inventory up to time t with the assumption that $\mathfrak{Q}_0^{a_i} = 0$ and $\mathfrak{Q}_0^{b_j} = 0$, and we let $\mathfrak{C}_t^{a_i}$ and $\mathfrak{C}_t^{b_j}$ be the accumulated expense (in EUR) of purchases of inventory with the assumption that $\mathfrak{C}_0^{a_i} = 0$ and $\mathfrak{C}_0^{b_j} = 0$. Note that for $x \in \{a_i, b_j\}$, the variables \mathfrak{C}_t^x and \mathfrak{Q}_t^x change in the same direction, that is, if inventory goes up then cash goes up and vice-versa. Cash and inventory variables are computed for trading in Euronext Amsterdam.

We are now able to describe the variables we use as features to describe the way algorithms make decisions in the market. The set of variables that an algorithm $a_i \in \mathcal{A}$ (of trading member $b_j \in \mathcal{B}$) has at its disposal at time $t \in [0, T]$ is given by:

- (i) Available volumes in the LOB excluding trading algorithm a_i (six variables).

In particular, we use $\log(1 + V)$ where V is one of the following variables: $V_{t-}^{x,5,-a_i}$, $V_{t-}^{x,10,-a_i} - V_{t-}^{x,5,-a_i}$, $V_{t-}^{x,20,-a_i} - V_{t-}^{x,10,-a_i}$, for $x \in \{a, b\}$.

- (ii) Available volumes in the LOB by trading algorithm a_i (six variables).

In particular, we use $\log(1 + V)$ where V is one of the following variables: $V_{t-}^{x,5,a_i}$, $V_{t-}^{x,10,a_i} - V_{t-}^{x,5,a_i}$, $V_{t-}^{x,20,a_i} - V_{t-}^{x,10,a_i}$, for $x \in \{a, b\}$.

- (iii) Imbalance in the LOB excluding trading algorithm a_i (three variables).

In particular, we use: $\mathfrak{I}(V_{t-}^{b,5,-a_i}, V_{t-}^{a,5,-a_i})$, $\mathfrak{I}(V_{t-}^{b,10,-a_i} - V_{t-}^{b,5,-a_i}, V_{t-}^{a,10,-a_i} - V_{t-}^{a,5,-a_i})$, and $\mathfrak{I}(V_{t-}^{b,20,-a_i} - V_{t-}^{b,10,-a_i}, V_{t-}^{a,20,-a_i} - V_{t-}^{a,10,-a_i})$.

- (iv) Imbalance in the LOB of trading algorithm a_i (three variables).

In particular, we use: $\mathfrak{I}(V_{t-}^{b,5,a_i}, V_{t-}^{a,5,a_i})$, $\mathfrak{I}(V_{t-}^{b,10,a_i} - V_{t-}^{b,5,a_i}, V_{t-}^{a,10,a_i} - V_{t-}^{a,5,a_i})$, and $\mathfrak{I}(V_{t-}^{b,20,a_i} - V_{t-}^{b,10,a_i}, V_{t-}^{a,20,a_i} - V_{t-}^{a,10,a_i})$.

- (v) Change of imbalance in the LOB excluding trading algorithm a_i (three variables).

In particular, we use: $\mathfrak{I}(V_{t-}^{b,5,-a_i}, V_{t-}^{a,5,-a_i}) - \mathfrak{I}(V_{s-}^{b,5,-a_i}, V_{s-}^{a,5,-a_i})$, $\mathfrak{I}(V_{t-}^{b,10,-a_i} - V_{t-}^{b,5,-a_i}, V_{t-}^{a,10,-a_i} - V_{t-}^{a,5,-a_i}) - \mathfrak{I}(V_{s-}^{b,10,-a_i} - V_{s-}^{b,5,-a_i}, V_{s-}^{a,10,-a_i} - V_{s-}^{a,5,-a_i})$, and $\mathfrak{I}(V_{t-}^{b,20,-a_i} - V_{t-}^{b,10,-a_i}, V_{t-}^{a,20,-a_i} - V_{t-}^{a,10,-a_i}) - \mathfrak{I}(V_{s-}^{b,20,-a_i} - V_{s-}^{b,10,-a_i}, V_{s-}^{a,20,-a_i} - V_{s-}^{a,10,-a_i})$ where $s < t$ is the time of the last message before t and as usual s^- is the time just before s .

(vi) Log volume of quantity at best bid and best offer (two variables).

In particular, we use $\log(1 + V)$ where V is one of the following variables: $V_{t-}^{x,1}$ for $x \in \{a, b\}$.

(vii) Spread in basis points (one variable).

In particular, we use $\mathfrak{S}_{t-}/S_{t-} \times 10,000$.

(viii) Returns over a number of periods (four variables).

In particular, we use: \mathfrak{R}_t^{1s} , \mathfrak{R}_t^{5s} , \mathfrak{R}_t^{60s} , and \mathfrak{R}_t^{300s} .

(ix) Volatility over a number of periods (four variables).

In particular, we use: \mathfrak{V}_t^{1m} , \mathfrak{V}_t^{5m} , \mathfrak{V}_t^{15m} , and \mathfrak{V}_t^{60m} .

(x) Number of messages over number of periods (four variables).

In particular, we count the number of messages sent to Euronext in the last: 1 second, 100 milliseconds, 1 millisecond, 100 microseconds.

(xi) Aggressive buys minus aggressive sells over previous 1, 5, 60 and 360 seconds (four variables).

(xii) Aggressive buys over previous 1, 5, 60 and 360 seconds (four variables).

(xiii) Aggressive sells over previous 1, 5, 60 and 360 seconds (four variables).

(xiv) Volume of last transaction (one variable).

In particular, if the volume of the last transaction is $V > 0$, we employ $\log(1 + V)$.

(xv) Inventory of trading algorithm $\mathfrak{Q}_{t-}^{a_i}$ (one variable).

In particular, we employ

$$\text{sign}\left(\mathfrak{Q}_{t-}^{a_i}\right) \times \log\left(1 + \left|\mathfrak{Q}_{t-}^{a_i}\right|\right). \quad (15)$$

(xvi) Inventory of trading member $\mathfrak{Q}_{t-}^{b_j}$ (one variable).

In particular, we employ

$$\text{sign}\left(\mathfrak{Q}_{t-}^{b_j}\right) \times \log\left(1 + \left|\mathfrak{Q}_{t-}^{b_j}\right|\right). \quad (16)$$

(xvii) Cash of trading algorithm $\mathfrak{C}_{t-}^{a_i}$ (one variable)

$$\mathfrak{C}_t^{a_i} = \int_0^t \tilde{S}_u d\mathfrak{Q}_u^{a_i}, \quad \mathfrak{C}_0^{a_i} = 0. \quad (17)$$

In particular, we employ

$$\text{sign}\left(\mathfrak{C}_{t-}^{a_i}\right) \times \log\left(1 + \left|\mathfrak{C}_{t-}^{a_i}\right|\right). \quad (18)$$

(xviii) Cash of trading member $\mathfrak{C}_{t-}^{b_j}$ (one variable)

$$\mathfrak{C}_t^{b_j} = \int_0^t \tilde{S}_u d\mathfrak{Q}_u^{b_j}, \quad \mathfrak{C}_0^{b_j} = 0. \quad (19)$$

In particular, we employ

$$\text{sign}\left(\mathfrak{C}_{t-}^{b_j}\right) \times \log\left(1 + \left|\mathfrak{C}_{t-}^{b_j}\right|\right). \quad (20)$$

C Model fit results for other shares

C.1 Model performance

Similar to Table 6, Table 7, and Table 8, we show (i) the accuracies of the logistic regressions, (ii) the outperformance of the logistic regressions over the most frequent bucket, and (iii) the outperformance of random forests over the most frequent bucket and over the logistic regression; we do this for the shares ING, AHOLD, and TOMTOM.

Table A4 shows the accuracies for ING, Table A5 shows the outperformance of the logistic regressions over the most frequent bucket, and Table A6 shows the outperformance of random forests over the most frequent bucket and over logistic regression. Here, D_t has two buckets, the variable \mathfrak{P}_t has three buckets and \mathfrak{V}_t has nine buckets.

	R1 D_t	R2 \mathfrak{P}_t	R3 \mathfrak{V}_t	R4 $\mathfrak{P}_t D_t = 1$	R5 $\mathfrak{P}_t D_t = -1$	R6 $\mathfrak{V}_t D_t = 1$	R7 $\mathfrak{V}_t D_t = -1$
Top 5	64 \pm 9	88 \pm 9	55 \pm 2	91 \pm 6	90 \pm 7	56 \pm 2	56 \pm 2
Top 10	65 \pm 7	87 \pm 12	64 \pm 21	90 \pm 10	90 \pm 10	64 \pm 21	64 \pm 21
Top 20	71 \pm 13	90 \pm 12	63 \pm 19	92 \pm 11	92 \pm 10	63 \pm 19	63 \pm 18
Top 50	75 \pm 14	87 \pm 15	55 \pm 21	88 \pm 15	89 \pm 14	56 \pm 21	55 \pm 21
All	78 \pm 15	78 \pm 22	46 \pm 23	75 \pm 27	76 \pm 28	43 \pm 25	42 \pm 25
order-weighted average	69	87	60	90	90	61	61

Table A4: Accuracies of the logistic regression models for ING. Twelve train-and-deploy exercises.

	D_t	\mathfrak{P}_t	\mathfrak{V}_t	$\mathfrak{P}_t D_t = 1$	$\mathfrak{P}_t D_t = -1$	$\mathfrak{V}_t D_t = 1$	$\mathfrak{V}_t D_t = -1$
Top 5	13 \pm 8	1 \pm 1	0 \pm 8	3 \pm 4	2 \pm 4	0 \pm 9	1 \pm 9
Top 10	14 \pm 7	3 \pm 7	1 \pm 6	6 \pm 9	6 \pm 9	1 \pm 7	1 \pm 7
Top 20	20 \pm 13	3 \pm 5	3 \pm 7	5 \pm 7	5 \pm 7	3 \pm 9	2 \pm 7
Top 50	23 \pm 13	4 \pm 8	1 \pm 6	5 \pm 10	5 \pm 9	2 \pm 8	2 \pm 7
All	25 \pm 17	1 \pm 12	-3 \pm 12	3 \pm 11	3 \pm 13	-2 \pm 13	-1 \pm 12
order-weighted average	18	3	0	5	5	1	1

Table A5: Outperformance over benchmark for ING. Twelve train-and-deploy exercises.

	outperformance					
	over benchmark			over logistic		
	D_t	\mathfrak{P}_t	\mathfrak{V}_t	D_t	\mathfrak{P}_t	\mathfrak{V}_t
Top 5	22 \pm 8	0 \pm 1	2 \pm 9	9 \pm 9	0 \pm 1	2 \pm 1
Top 10	20 \pm 8	2 \pm 4	2 \pm 8	6 \pm 8	-1 \pm 2	2 \pm 3
Top 20	25 \pm 12	2 \pm 3	3 \pm 9	4 \pm 6	-1 \pm 2	0 \pm 4
Top 50	26 \pm 13	2 \pm 6	2 \pm 7	3 \pm 6	-1 \pm 2	2 \pm 4
All	25 \pm 17	2 \pm 7	1 \pm 9	1 \pm 8	1 \pm 7	4 \pm 8
order-weighted average	23	2	2	5	-1	2

Table A6: Outperformance of random forests over benchmark and logistic regression for ING. Twelve train-and-deploy exercises.

Table A7 shows the accuracies for AHOLD, Table A8 shows the outperformance of the logistic regressions over the most frequent bucket, and Table A9 shows the outperformance of random forests over the most frequent bucket and over logistic regression. Here, D_t has two buckets, the variable \mathfrak{P}_t has three buckets and \mathfrak{V}_t has nine buckets.

	R1 D_t	R2 \mathfrak{P}_t	R3 \mathfrak{V}_t	R4 $\mathfrak{P}_t D_t = 1$	R5 $\mathfrak{P}_t D_t = -1$	R6 $\mathfrak{V}_t D_t = 1$	R7 $\mathfrak{V}_t D_t = -1$
Top 5	63 \pm 7	84 \pm 8	58 \pm 14	86 \pm 6	87 \pm 6	58 \pm 13	58 \pm 14
Top 10	66 \pm 9	87 \pm 10	66 \pm 21	89 \pm 9	89 \pm 9	66 \pm 21	66 \pm 21
Top 20	71 \pm 13	89 \pm 11	67 \pm 23	90 \pm 10	91 \pm 10	67 \pm 23	66 \pm 23
Top 50	74 \pm 14	84 \pm 16	53 \pm 23	85 \pm 15	84 \pm 17	55 \pm 22	53 \pm 23
All	78 \pm 16	77 \pm 21	42 \pm 24	73 \pm 28	73 \pm 28	39 \pm 27	40 \pm 25
order-weighted average	69	86	59	87	88	60	59

Table A7: Accuracies of the logistic regression models for AHOLD. Twelve train-and-deploy exercises.

	D_t	\mathfrak{P}_t	\mathfrak{V}_t	$\mathfrak{P}_t D_t = 1$	$\mathfrak{P}_t D_t = -1$	$\mathfrak{V}_t D_t = 1$	$\mathfrak{V}_t D_t = -1$
Top 5	13 \pm 7	7 \pm 9	5 \pm 6	9 \pm 10	9 \pm 10	6 \pm 7	5 \pm 7
Top 10	16 \pm 7	5 \pm 7	4 \pm 8	6 \pm 8	7 \pm 8	4 \pm 9	4 \pm 9
Top 20	20 \pm 14	3 \pm 6	4 \pm 7	5 \pm 7	5 \pm 7	5 \pm 8	3 \pm 8
Top 50	22 \pm 14	3 \pm 7	2 \pm 6	5 \pm 8	3 \pm 11	2 \pm 8	3 \pm 8
All	22 \pm 21	0 \pm 12	-2 \pm 12	3 \pm 13	4 \pm 12	-1 \pm 11	-1 \pm 11
order-weighted average	18	4	4	6	6	5	4

Table A8: Outperformance over benchmark for AHOLD. Twelve train-and-deploy exercises.

	outperformance					
	over benchmark			over logistic		
	D_t	\mathfrak{P}_t	\mathfrak{V}_t	D_t	\mathfrak{P}_t	\mathfrak{V}_t
Top 5	20 \pm 6	5 \pm 7	3 \pm 9	7 \pm 8	-2 \pm 3	-2 \pm 4
Top 10	21 \pm 7	3 \pm 5	2 \pm 9	5 \pm 6	-2 \pm 3	-2 \pm 3
Top 20	25 \pm 13	2 \pm 4	4 \pm 9	5 \pm 5	-1 \pm 2	0 \pm 5
Top 50	25 \pm 14	2 \pm 5	3 \pm 7	3 \pm 5	-1 \pm 3	1 \pm 3
All	22 \pm 19	2 \pm 7	2 \pm 10	1 \pm 11	2 \pm 8	3 \pm 7
order-weighted average	23	3	4	5	-1	0

Table A9: Outperformance of random forests over benchmark and logistic regression for AHOLD. Twelve train-and-deploy exercises.

Table A10 shows the accuracies for TOMTOM, Table A11 shows the outperformance of the logistic regressions over the most frequent bucket, and Table A12 shows the outperformance of random forests over the most frequent bucket and over logistic regression. Here, D_t has two buckets, the variable \mathfrak{P}_t has three buckets and \mathfrak{V}_t has ten buckets.

	R1 D_t	R2 \mathfrak{P}_t	R3 \mathfrak{V}_t	R4 $\mathfrak{P}_t D_t = 1$	R5 $\mathfrak{P}_t D_t = -1$	R6 $\mathfrak{V}_t D_t = 1$	R7 $\mathfrak{V}_t D_t = -1$
Top 5	68 \pm 15	88 \pm 15	57 \pm 27	89 \pm 15	89 \pm 15	58 \pm 27	58 \pm 26
Top 10	66 \pm 13	90 \pm 13	61 \pm 23	90 \pm 13	91 \pm 12	61 \pm 23	62 \pm 23
Top 20	70 \pm 15	88 \pm 13	62 \pm 26	89 \pm 12	89 \pm 12	62 \pm 25	62 \pm 26
Top 50	73 \pm 13	78 \pm 19	51 \pm 24	79 \pm 18	77 \pm 20	50 \pm 24	50 \pm 24
All	71 \pm 24	70 \pm 23	44 \pm 28	54 \pm 38	54 \pm 39	31 \pm 30	31 \pm 30
order-weighted average	67	87	52	88	88	52	52

Table A10: Accuracies of the logistic regression models for TOMTOM. Twelve train-and-deploy exercises.

	D_t	\mathfrak{P}_t	\mathfrak{V}_t	$\mathfrak{P}_t D_t = 1$	$\mathfrak{P}_t D_t = -1$	$\mathfrak{V}_t D_t = 1$	$\mathfrak{V}_t D_t = -1$
Top 5	16 ± 15	2 ± 4	4 ± 9	3 ± 5	3 ± 4	5 ± 9	5 ± 9
Top 10	14 ± 13	1 ± 3	3 ± 7	2 ± 4	2 ± 3	2 ± 8	2 ± 7
Top 20	17 ± 16	0 ± 3	1 ± 6	1 ± 4	2 ± 3	1 ± 7	1 ± 6
Top 50	19 ± 13	1 ± 5	-2 ± 7	2 ± 7	1 ± 5	-3 ± 8	-2 ± 9
All	13 ± 19	-2 ± 15	-6 ± 15	-2 ± 16	-3 ± 16	-6 ± 17	-6 ± 20
order-weighted average	15	0	3	2	2	3	3

Table A11: Outperformance over benchmark for TOMTOM. Twelve train-and-deploy exercises.

	outperformance					
	D_t	over benchmark \mathfrak{P}_t	\mathfrak{V}_t	D_t	over logistic \mathfrak{P}_t	\mathfrak{V}_t
Top 5	23 ± 16	2 ± 3	7 ± 10	8 ± 9	-1 ± 1	2 ± 2
Top 10	19 ± 15	1 ± 2	4 ± 8	5 ± 7	0 ± 1	1 ± 2
Top 20	22 ± 15	0 ± 2	2 ± 6	5 ± 9	0 ± 1	1 ± 3
Top 50	22 ± 14	1 ± 4	0 ± 6	3 ± 7	0 ± 3	3 ± 3
All	13 ± 19	0 ± 7	0 ± 8	0 ± 13	2 ± 13	6 ± 13
order-weighted average	24	0	5	9	0	2

Table A12: Outperformance of random forests over benchmark and logistic regression for TOMTOM. Twelve train-and-deploy exercises.

C.2 Feature importance

We present the permutation feature importance using the logistic regressions. Similar to Figure 5 – where we show the most important features for predicting D_t , Figure A1 reports the analogous for ING, Figure A2 reports the analogous for AHOLD, and Figure A3 reports the analogous for TOMTOM.

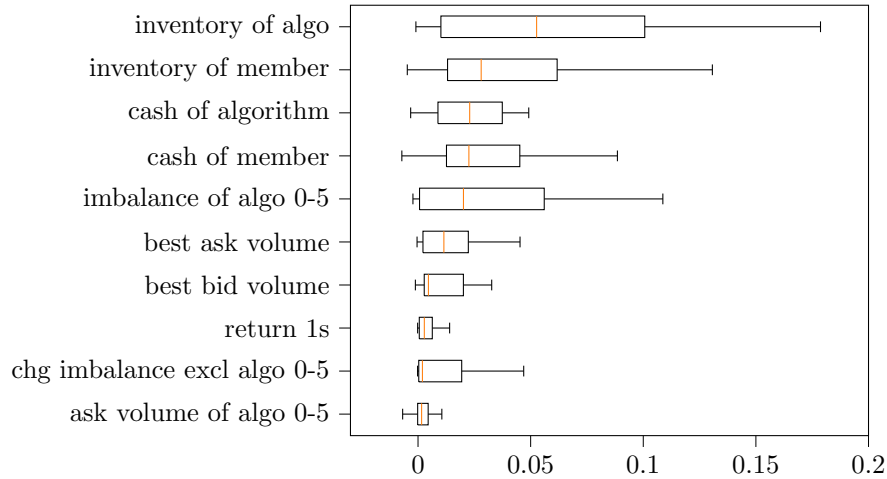


Figure A1: Most important features to explain the direction D_t of the order for ING using permutation importance and logistic regressions.

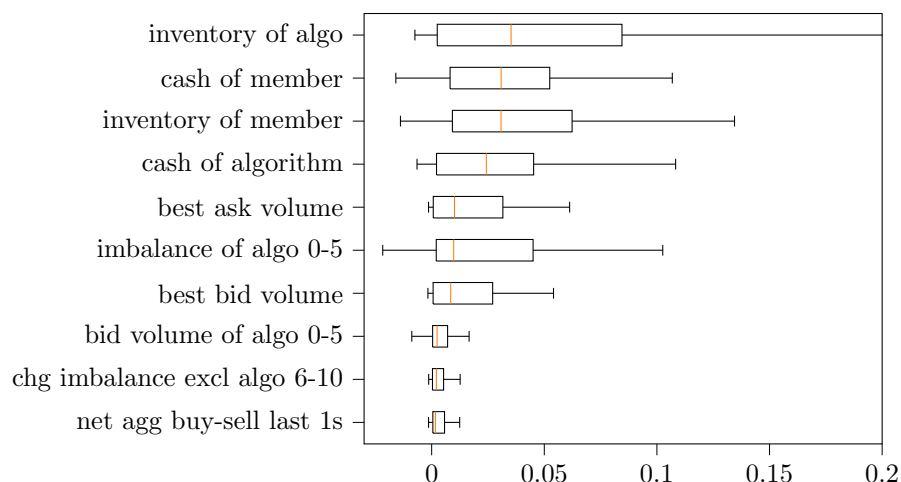


Figure A2: Most important features to explain the direction D_t of the order for AHOLD using permutation importance and logistic regressions.

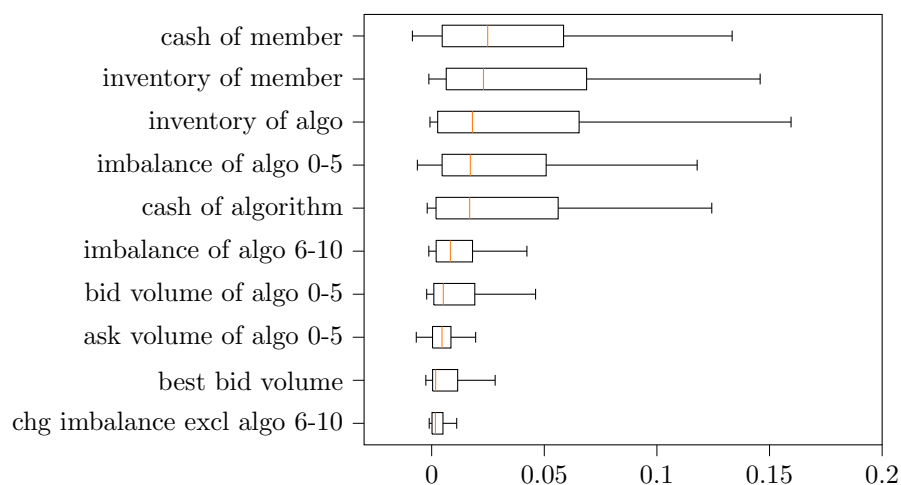


Figure A3: Most important features to explain the direction D_t of the order for TOMTOM using permutation importance and logistic regressions.

Unlike the case for ASML, inventory or cash related features are the most important features when predicting D_t for ING, AHOLD, and TOMTOM.

Similar to Figure 6 – where we show the most important features for predicting price bucket \mathfrak{P}_t , Figure A4 reports the analogous for ING, Figure A5 for AHOLD, and Figure A6 for TOMTOM.

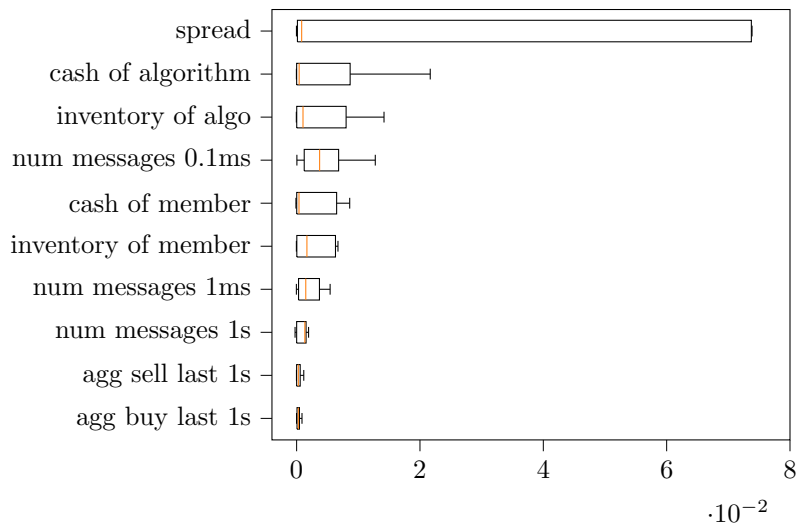


Figure A4: Most important features to explain the price bucket \mathfrak{P}_t for ING using permutation importance and logistic regressions.

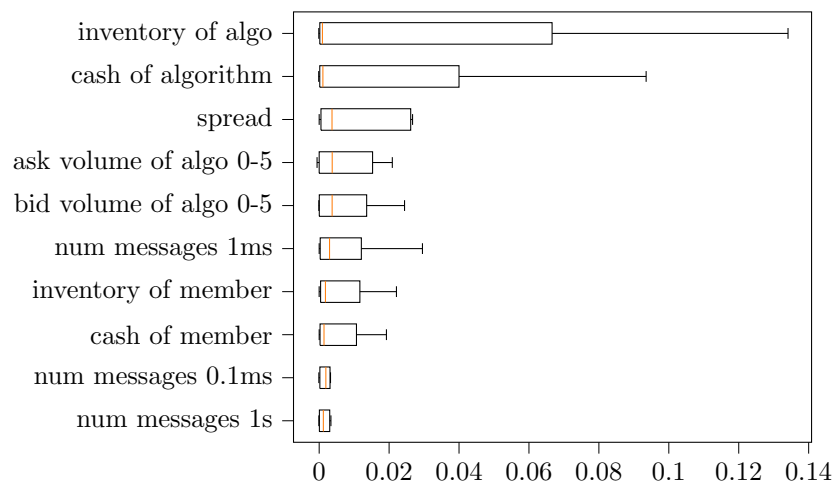


Figure A5: Most important features to explain the price bucket \mathfrak{P}_t for AHOLD using permutation importance and logistic regressions.

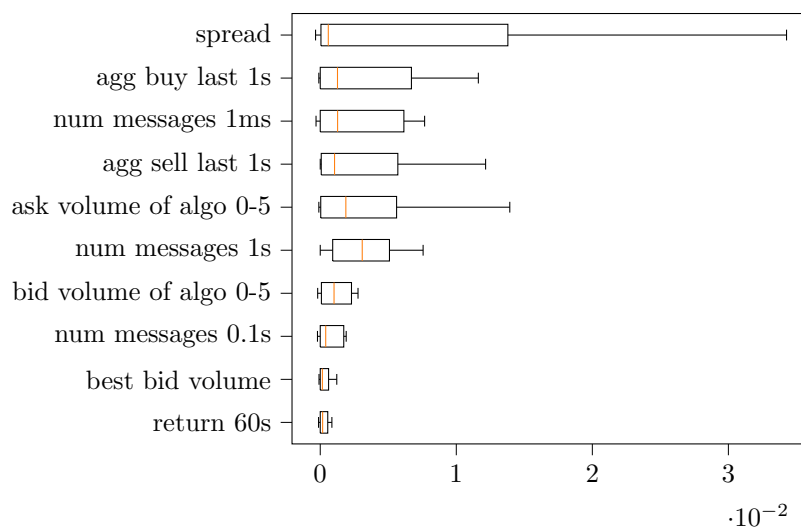


Figure A6: Most important features to explain the price bucket \mathfrak{P}_t for TOMTOM using permutation importance and logistic regressions.

Here, spread retains its place as an important feature. Inventory of algorithm and cash of algorithm are also among the most important features for ING, and AHOLD.

Similar to Figure 7 – where we show the most important features for predicting volume bucket \mathfrak{V}_t , Figure A7 reports the analogous for ING, Figure A8 for AHOLD, and Figure A9 for TOMTOM.

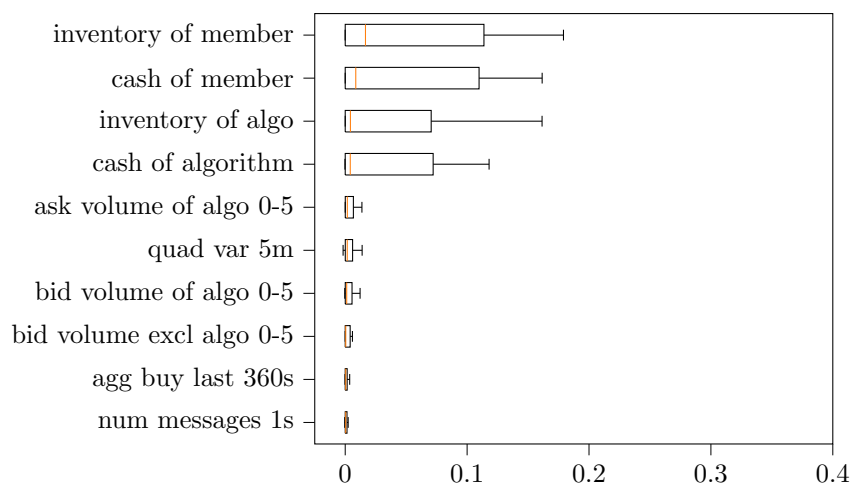


Figure A7: Most important features to explain the volume bucket \mathfrak{V}_t for ING using permutation importance and logistic regressions.

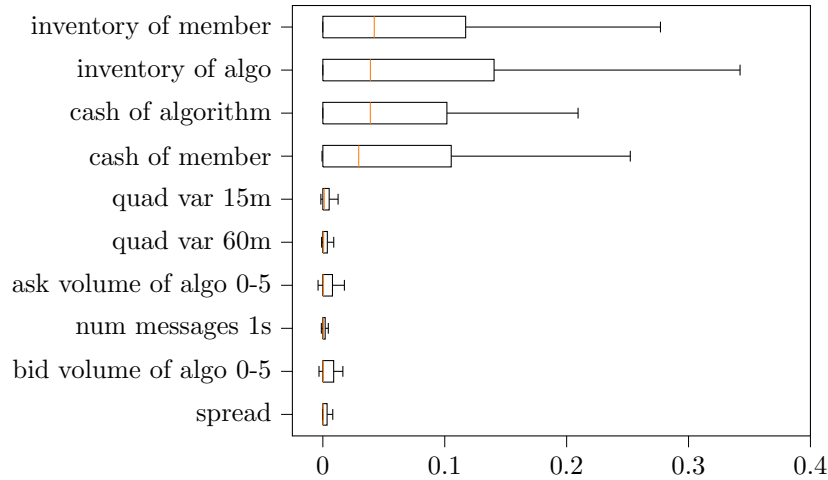


Figure A8: Most important features to explain the volume bucket \mathfrak{V}_t for AHOLD using permutation importance and logistic regressions.

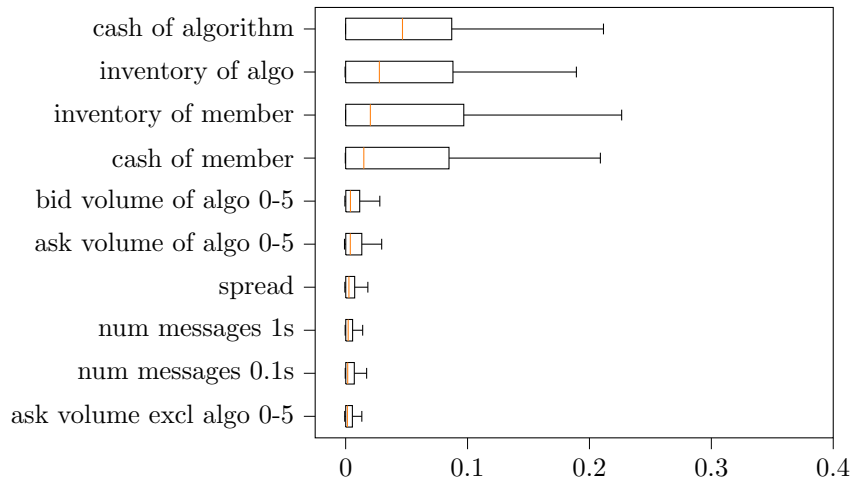


Figure A9: Most important features to explain the volume bucket \mathfrak{V}_t for TOMTOM using permutation importance and logistic regressions.

Consistently across ING, AHOLD, and TOMTOM, the four most important features are either inventory variables (inventory of algorithm and inventory of member) or cash variables (cash of algorithm and cash of member).

C.3 Clustering of agents

This section presents the clustering results for the shares ING, AHOLD and TOMTOM according to the methodology explained in Subsection 4.4. First, we show the size of the clusters we obtain in each of the twelve clustering exercises. As before, for each of the clustering exercises, the first cluster has the most algorithms, and the third cluster has the least number of algorithms.

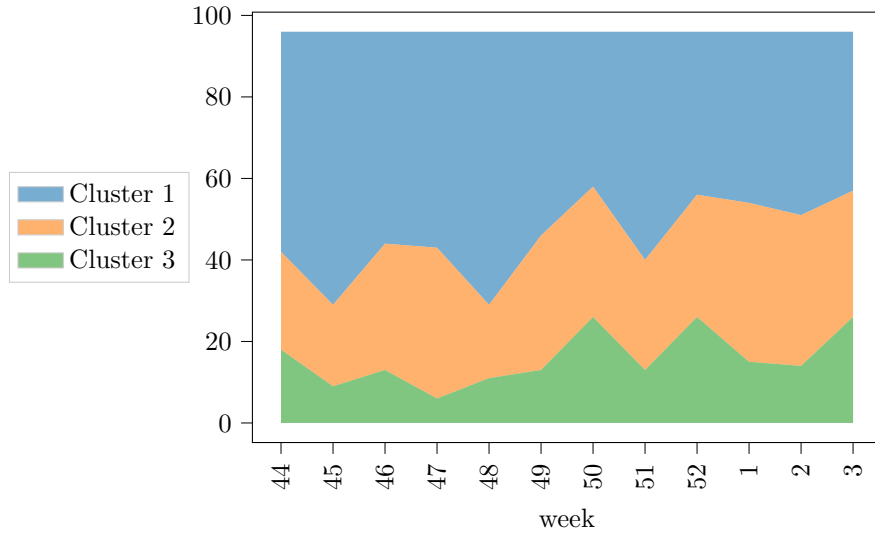


Figure A10: Size of clusters for ING across the twelve clustering exercises.

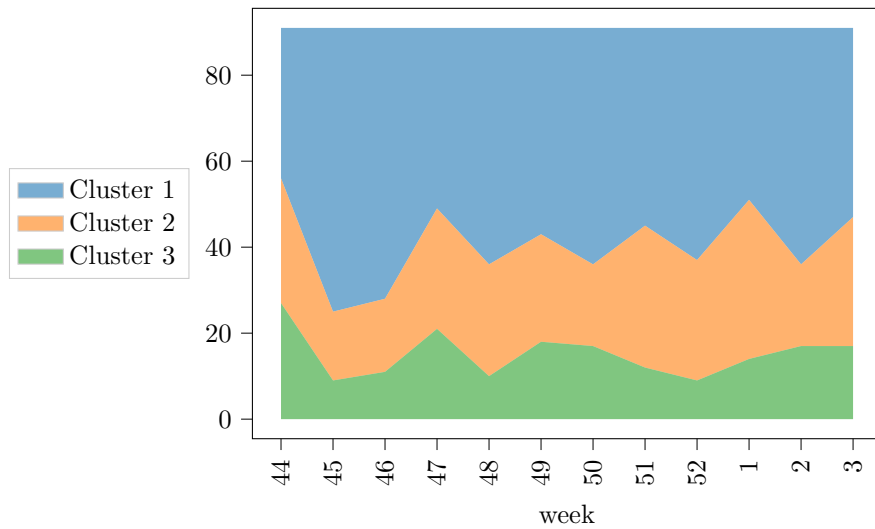


Figure A11: Size of clusters for AHOLD across the twelve clustering exercises.

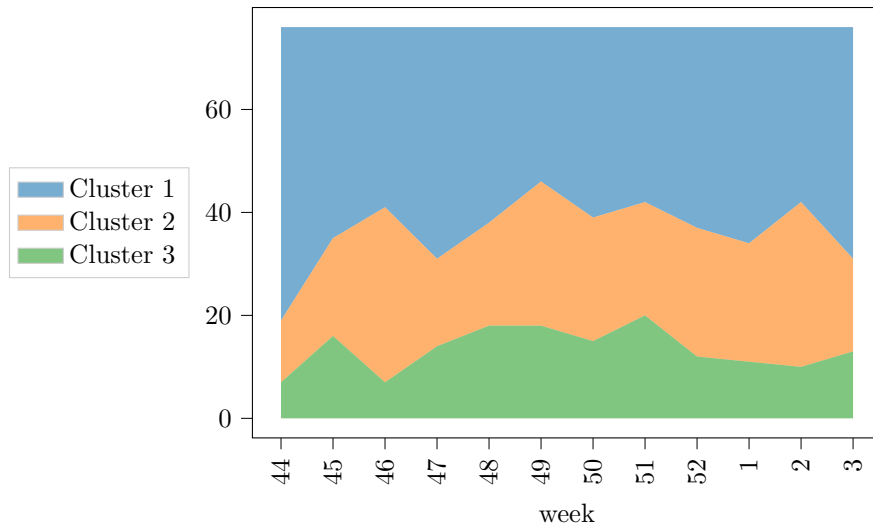


Figure A12: Size of clusters for TOMTOM across the twelve clustering exercises.

Second, we show the stability of clusters through time using the technique described in Section 4.4.1 to create Figure 9.

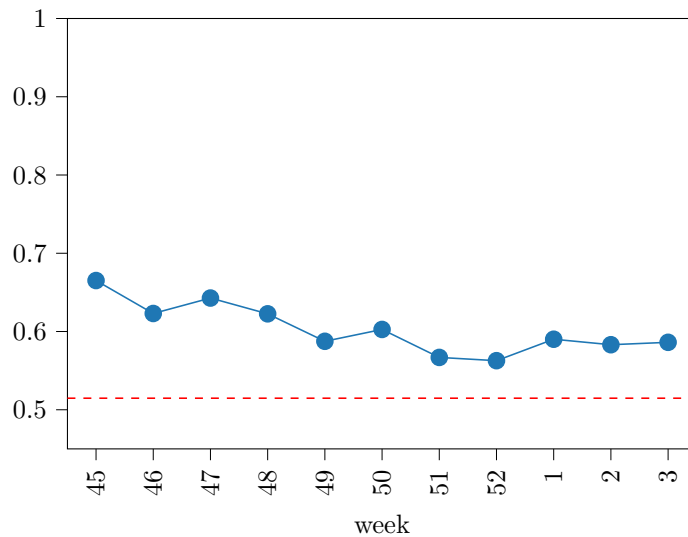


Figure A13: Stability of clusters for ING across the twelve train exercises.

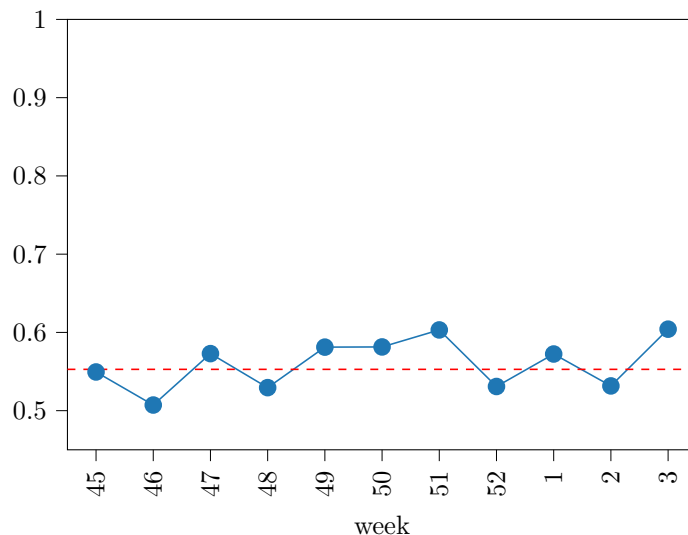


Figure A14: Stability of clusters for AHOLD across the twelve train exercises.

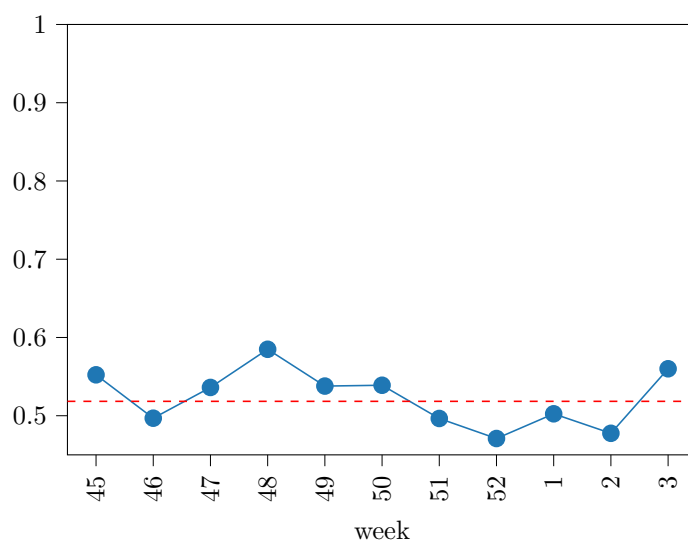


Figure A15: Stability of clusters for TOMTOM across the twelve train exercises.

Figure A14 and A15 show that the stability of the clusters we obtain for AHOLD and TOMTOM is not good. In particular, we see that random reshuffling of algorithms produces a similar stability score. Therefore we do not interpret the clustering results of these two shares. We believe the poor clustering might be due to the clusters on the first four weeks of data having substantially different sizes than the clusters obtained on other months of data of the same share (see Figures A11 and A12). If we were to take a clustering on the second month of data, for example, with clusters whose size is more representative of other months, then the stability metric would improve.

Client	14	4	5
House	13	6	2
LP	27	14	11
	Cluster 1	Cluster 2	Cluster 3

Figure A16: Confusion matrix between dealing capacity and clusters obtained on the first four weeks of data for ING.

Figure A16 shows an “L shape” similar to that in Figure 10. That is: the majority of algorithms are in cluster 1, with types Liquidity Provider, House and Client all well represented. In cluster 2 we again observe mostly algorithms with type Liquidity Provider, and the same goes for cluster 3.

Client	9	9	5
House	7	8	4
LP	19	12	18
	Cluster 1	Cluster 2	Cluster 3

Figure A17: Confusion matrix between dealing capacity and clusters obtained on the first four weeks of data for AHOLD.

Client	23	3	3
House	14	0	1
LP	20	9	3
	Cluster 1	Cluster 2	Cluster 3

Figure A18: Confusion matrix between dealing capacity and clusters obtained on the first four weeks of data for TOMTOM.

C.4 Direction: average behaviour

Here we report the average regression coefficients for ING, AHOLD, and TOMTOM using the same techniques as those described in Subsection 4.4.6.

	Cluster 1	Cluster 2	Cluster 3
imbalance of algo 0-5	1.25	-1.43	0.26
imbalance of algo 6-10	0.26	-0.55	0.98
cash of algorithm	0.58	0.25	-0.38
inventory of algo	0.58	0.25	-0.38
volume of algo 11-20	0.30	-0.11	0.44
best bid volume	0.41	-0.16	0.23
best ask volume	-0.35	0.17	-0.24
imbalance of algo 11-20	0.01	-0.47	-0.11
volume of algo 6-10	0.08	-0.14	0.24
imbalance excl algo 0-5	-0.10	-0.00	-0.22
inventory of member	0.09	-0.09	0.15
cash of member	0.09	-0.08	0.15
num messages 0.1ms	-0.02	-0.12	-0.10
num messages 1ms	-0.02	-0.10	-0.08
return 1s	0.07	0.00	0.12
volume excl algo 0-5	0.10	0.01	-0.08
net agg buy-sell last 1s	0.03	-0.04	0.11
quad var 60m	-0.11	-0.01	0.02

Table A13: Average regression coefficients per cluster on first four weeks of training data for ING when predicting direction D_t . All excluded features have average coefficients with a magnitude smaller than 0.1 for all clusters.

There are similarities between ASML and ING. For imbalance of the algorithm on the first five levels we observe a strong positive coefficient for cluster 1, a strong negative coefficient for cluster 2, and we see cluster 3 lie between the two. These results are consistent with what we observe in ASML. For inventory of algorithm we observe a strong positive coefficient for cluster 1, a strong negative coefficient for cluster 3, and cluster 2 is somewhere in between – again consistent with the clustering obtained for ASML, including the signs of the coefficients. Lastly, we see that imbalances of the algorithm near the top of the LOB have the largest positive (resp. negative) coefficients for both ASML and ING.

	Cluster 1	Cluster 2	Cluster 3
imbalance of algo 0-5	1.58	-0.24	-0.12
cash of algorithm	0.50	0.43	0.56
inventory of algo	0.50	0.43	0.55
best bid volume	0.18	-0.08	0.81
best ask volume	-0.16	0.11	-0.76
imbalance of algo 11-20	-0.04	0.02	-0.93
imbalance of algo 6-10	0.12	-0.16	-0.59
volume of algo 11-20	-0.33	0.26	-0.18
volume of algo 0-5	0.30	-0.08	0.24
cash of member	0.03	0.31	-0.02
inventory of member	0.03	0.31	-0.02
quad var 60m	-0.08	0.18	-0.01
imbalance excl algo 11-20	0.09	0.12	-0.06
chg imbalance excl algo 0-5	-0.01	-0.08	0.17
return 1s	0.10	0.01	0.15
num messages 0.1ms	0.03	-0.19	-0.03
num messages 1ms	-0.03	-0.17	-0.02
chg imbalance excl algo 11-20	0.08	0.02	0.11
imbalance excl algo 6-10	-0.00	0.06	-0.13
return 5s	0.05	0.03	0.11
volume excl algo 11-20	-0.02	0.14	-0.02
return 300s	0.02	0.03	-0.10
net agg buy-sell last 1s	0.01	0.03	0.10
imbalance excl algo 0-5	0.03	-0.00	-0.11
agg sell last 1s	-0.01	-0.02	-0.10
quad var 15m	0.01	0.10	-0.00

Table A14: Average regression coefficients per cluster on first four weeks of training data for AHOLD when predicting direction D_t . All excluded features have average coefficients with a magnitude smaller than 0.1 for all clusters.

	Cluster 1	Cluster 2	Cluster 3
imbalance of algo 0-5	0.58	-1.11	0.42
imbalance of algo 6-10	-0.05	-0.76	-0.13
cash of algorithm	0.44	0.11	-0.39
inventory of algo	0.43	0.10	-0.39
volume of algo 6-10	0.05	-0.26	-0.16
num messages 1ms	-0.02	0.04	-0.36
imbalance of algo 11-20	-0.18	-0.06	-0.18
best bid volume	0.30	-0.06	0.03
num messages 0.1ms	-0.03	0.05	-0.30
best ask volume	-0.24	0.01	-0.12
return 1s	0.15	0.08	0.14
return 5s	0.13	0.09	0.12
agg buy last 5s	0.02	0.07	0.25
return 300s	0.10	0.05	-0.16
volume of algo 0-5	0.08	-0.21	-0.02
imbalance excl algo 11-20	0.19	0.05	-0.07
num messages 0.1s	0.03	-0.02	-0.26
quad var 5m	0.05	-0.04	-0.20
chg imbalance excl algo 6-10	-0.02	0.15	-0.11
agg buy last 1s	0.04	0.06	0.16
imbalance excl algo 6-10	0.04	0.09	-0.12
agg sell last 5s	-0.06	-0.02	0.17
inventory of member	0.04	-0.03	0.18
spread	-0.09	0.03	0.12
cash of member	0.04	-0.03	0.18
volume excl algo 6-10	0.06	0.03	-0.13
volume of algo 11-20	-0.05	0.08	0.10
volume excl algo 11-20	0.15	0.00	-0.05
num messages 1s	0.01	-0.03	0.13
imbalance excl algo 0-5	-0.01	0.02	-0.12

Table A15: Average regression coefficients per cluster on first four weeks of training data for TOMTOM when predicting direction D_t . All excluded features have average coefficients with a magnitude smaller than 0.1 for all clusters.

C.5 Price limit: average behaviour

Here we report the results of Subsection 4.4.7 for ING, AHOLD, and TOMTOM. We do not comment on the coefficients of the tables below; the insights are similar to those discussed for ASML.

	$\mathfrak{P}_t = 1 D_t = 1$ (Buy)			$\mathfrak{P}_t = 1 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.09	-0.64	-0.20	-0.04	-0.64	-0.28
best bid volume	0.42	0.69	0.32	-0.22	-0.12	-0.16
best ask volume	-0.24	-0.12	-0.16	0.38	0.66	0.30
volume of algo 0-5	-0.26	0.20	-0.28	-0.26	0.28	-0.33
volume of algo 6-10	-0.20	0.19	-0.33	-0.22	0.25	-0.35
spread	0.11	0.39	0.12	0.13	0.38	0.15
imbalance of algo 0-5	0.05	0.43	0.01	-0.03	-0.58	0.09
num messages 1ms	0.19	0.12	0.09	0.16	0.18	0.12
volume of algo 11-20	-0.00	0.07	-0.28	-0.06	0.05	-0.27
num messages 0.1ms	0.16	0.10	0.06	0.13	0.17	0.09
num messages 0.1s	0.05	0.12	0.07	0.06	0.12	0.01
imbalance of algo 6-10	-0.06	0.05	-0.02	0.05	-0.17	0.04
imbalance excl algo 6-10	-0.03	0.01	0.02	0.07	0.10	0.05
inventory of member	0.03	0.01	0.10	-0.05	0.01	0.08
cash of member	0.03	0.01	0.10	-0.05	0.01	0.08

Table A16: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for ING conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 1 D_t = 1$ (Buy)			$\mathfrak{P}_t = 1 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.30	-0.59	0.11	-0.41	-0.47	0.12
best bid volume	0.45	0.76	0.40	-0.19	-0.08	-0.34
best ask volume	-0.20	-0.10	-0.30	0.42	0.62	0.46
spread	0.20	0.51	0.17	0.16	0.43	0.18
volume of algo 11-20	-0.11	-0.08	-0.38	-0.14	-0.07	-0.39
volume of algo 6-10	-0.01	-0.08	-0.50	0.00	0.01	-0.49
volume of algo 0-5	0.07	-0.02	-0.38	0.10	0.07	-0.39
imbalance of algo 0-5	-0.06	0.20	0.26	0.09	-0.11	-0.21
num messages 1ms	0.14	0.15	0.03	0.08	0.06	0.06
volume excl algo 0-5	-0.10	-0.08	-0.07	-0.02	-0.07	-0.06
num messages 0.1ms	0.12	0.12	0.01	0.08	0.06	0.04
imbalance excl algo 11-20	-0.11	-0.06	-0.08	-0.03	0.05	0.06

Table A17: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for AHOLD conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 1 D_t = 1$ (Buy)			$\mathfrak{P}_t = 1 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.02	-0.71	-0.28	-0.04	-0.76	-0.31
volume of algo 0-5	-0.40	0.01	-0.55	-0.43	-0.15	-0.44
best bid volume	0.18	0.63	0.06	-0.15	-0.19	-0.07
best ask volume	-0.16	-0.15	-0.09	0.11	0.52	0.03
num messages 1ms	0.14	0.19	0.15	0.12	0.04	0.14
imbalance of algo 0-5	-0.01	0.21	0.04	0.08	-0.30	0.01
volume excl algo 0-5	0.10	0.08	0.20	0.06	0.06	0.16
num messages 0.1ms	0.11	0.20	0.11	0.09	0.02	0.12
spread	-0.03	0.22	0.01	0.02	0.18	-0.00
num messages 0.1s	0.12	0.07	0.06	0.05	0.11	0.01
imbalance excl algo 6-10	-0.02	-0.02	0.18	0.04	0.05	0.11
volume of algo 6-10	-0.11	-0.02	-0.06	-0.07	0.09	-0.04
imbalance excl algo 11-20	-0.00	-0.04	-0.11	0.01	0.06	-0.12
imbalance of algo 11-20	0.04	0.12	-0.01	0.03	-0.10	0.03
last volume transacted	0.05	-0.01	-0.11	0.04	0.01	-0.07
volume of algo 11-20	-0.04	-0.03	0.07	0.00	0.10	0.05
imbalance of algo 6-10	-0.00	0.05	0.01	0.09	-0.10	-0.00
num messages 1s	0.02	-0.00	-0.04	-0.01	0.10	-0.07

Table A18: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for TOMTOM conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 2 D_t = 1$ (Buy)			$\mathfrak{P}_t = 2 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.07	0.08	0.09	-0.08	0.15	-0.02
volume of algo 0-5	0.17	0.26	0.27	0.25	0.27	0.34
volume of algo 11-20	-0.14	-0.23	-0.13	-0.21	-0.30	-0.10
imbalance of algo 0-5	0.01	0.53	0.02	0.02	-0.32	-0.03
volume of algo 6-10	0.05	0.10	0.12	0.09	0.06	0.19
spread	0.03	-0.16	0.08	0.05	-0.17	0.06
best ask volume	0.06	-0.02	0.11	0.04	-0.19	0.10
num messages 0.1s	-0.07	-0.08	-0.08	-0.07	-0.10	-0.08
imbalance of algo 6-10	0.00	0.24	0.00	0.02	-0.11	-0.00
best bid volume	-0.01	-0.17	0.03	0.05	0.02	0.07
num messages 1ms	-0.10	-0.04	-0.04	-0.10	-0.02	-0.03

Table A19: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for ING conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 2 D_t = 1$ (Buy)			$\mathfrak{P}_t = 2 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.22	0.10	0.11	-0.24	0.29	0.08
volume of algo 0-5	-0.03	0.18	0.33	-0.06	0.12	0.31
volume of algo 11-20	0.02	-0.09	-0.24	0.07	-0.11	-0.18
best bid volume	-0.03	-0.26	-0.02	0.08	0.03	0.11
num messages 0.1s	-0.10	-0.19	0.01	-0.07	-0.15	0.01
spread	0.00	-0.23	-0.01	0.05	-0.20	-0.03
best ask volume	0.07	0.04	0.12	0.02	-0.23	-0.04
volume excl algo 0-5	0.10	0.15	0.02	0.09	0.09	0.03
volume of algo 6-10	-0.03	0.08	0.13	-0.03	0.02	0.14
num messages 1ms	-0.13	-0.14	0.03	-0.06	-0.06	0.01
imbalance of algo 0-5	0.05	0.11	-0.07	-0.04	-0.09	0.02
num messages 0.1ms	-0.11	-0.07	0.05	-0.04	-0.00	0.03

Table A20: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for AHOLD conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 2 D_t = 1$ (Buy)			$\mathfrak{P}_t = 2 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	0.08	-0.36	-0.06	-0.02	-0.25	-0.03
volume of algo 0-5	0.13	-0.01	0.13	0.22	0.04	0.18
best bid volume	-0.06	-0.26	-0.02	0.09	0.13	0.06
volume of algo 11-20	-0.14	0.09	-0.13	-0.10	-0.01	-0.14
best ask volume	0.11	0.12	0.03	0.01	-0.24	0.04
num messages 0.1s	-0.05	-0.08	0.06	-0.06	-0.14	0.12
imbalance excl algo 0-5	0.04	0.06	-0.10	0.00	-0.08	-0.12
volume of algo 6-10	-0.04	-0.08	-0.07	-0.02	-0.13	0.06
imbalance excl algo 11-20	0.02	0.05	0.23	-0.02	-0.05	-0.01
volume excl algo 0-5	-0.00	-0.03	-0.15	0.01	0.08	-0.09
num messages 1s	-0.01	-0.06	0.04	-0.02	-0.13	0.10
volume excl algo 11-20	-0.05	0.07	0.15	-0.04	0.01	-0.04
last volume transacted	-0.05	0.08	0.11	0.01	0.07	0.02
imbalance of algo 6-10	-0.03	0.14	-0.00	0.03	-0.11	-0.01
quad var 15m	-0.06	-0.04	0.04	-0.02	-0.13	-0.02
imbalance of algo 0-5	0.01	0.09	-0.00	-0.02	-0.15	-0.04
num messages 1ms	-0.08	-0.10	-0.00	-0.05	0.01	-0.06
quad var 60m	-0.05	-0.04	0.03	-0.02	-0.14	0.00
volume excl algo 6-10	0.01	0.05	0.10	0.05	-0.00	-0.01
cash of member	-0.00	-0.00	-0.03	-0.02	0.00	0.12
inventory of member	0.00	-0.00	-0.03	-0.02	0.00	0.12

Table A21: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for TOMTOM conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 3 D_t = 1$ (Buy)			$\mathfrak{P}_t = 3 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	0.16	0.56	0.11	0.12	0.49	0.30
imbalance of algo 0-5	-0.06	-0.96	-0.03	0.01	0.90	-0.06
best ask volume	0.18	0.14	0.05	-0.41	-0.47	-0.40
best bid volume	-0.41	-0.52	-0.35	0.17	0.09	0.09
volume of algo 11-20	0.14	0.15	0.41	0.27	0.24	0.38
volume of algo 6-10	0.14	-0.29	0.21	0.13	-0.31	0.16
spread	-0.14	-0.23	-0.20	-0.17	-0.21	-0.22
volume of algo 0-5	0.09	-0.46	0.02	0.00	-0.55	-0.01
imbalance of algo 6-10	0.06	-0.28	0.02	-0.07	0.28	-0.03
num messages 0.1ms	-0.08	-0.12	-0.07	-0.08	-0.21	-0.11
num messages 1ms	-0.08	-0.08	-0.06	-0.07	-0.16	-0.08
imbalance of algo 11-20	0.06	-0.03	-0.04	-0.01	0.14	-0.04

Table A22: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for ING conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 3 D_t = 1$ (Buy)			$\mathfrak{P}_t = 3 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	0.52	0.50	-0.21	0.65	0.19	-0.20
volume of algo 11-20	0.08	0.17	0.62	0.08	0.19	0.56
best bid volume	-0.42	-0.49	-0.37	0.11	0.05	0.22
best ask volume	0.13	0.06	0.18	-0.43	-0.40	-0.42
spread	-0.21	-0.28	-0.16	-0.21	-0.23	-0.15
imbalance of algo 0-5	0.01	-0.31	-0.18	-0.06	0.20	0.19
volume of algo 6-10	0.04	-0.00	0.38	0.03	-0.04	0.35
volume of algo 0-5	-0.04	-0.16	0.05	-0.03	-0.19	0.08
imbalance of algo 6-10	0.15	-0.10	-0.12	-0.03	0.03	0.02
imbalance excl algo 11-20	0.14	0.08	0.11	0.01	-0.05	-0.06
num messages 0.1s	0.01	0.12	-0.01	0.01	0.11	-0.02

Table A23: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for AHOLD conditioning on the direction of order D_t .

	$\mathfrak{P}_t = 3 D_t = 1$ (Buy)			$\mathfrak{P}_t = 3 D_t = -1$ (Sell)		
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3
intercept	-0.06	1.07	0.34	0.06	1.02	0.33
volume of algo 0-5	0.26	0.00	0.41	0.21	0.11	0.25
imbalance of algo 0-5	0.01	-0.30	-0.04	-0.06	0.46	0.03
best bid volume	-0.11	-0.37	-0.04	0.06	0.06	0.01
best ask volume	0.05	0.03	0.05	-0.12	-0.28	-0.07
volume of algo 11-20	0.18	-0.06	0.06	0.10	-0.09	0.09
imbalance of algo 6-10	0.03	-0.19	-0.01	-0.12	0.22	0.02
volume of algo 6-10	0.15	0.10	0.13	0.09	0.04	-0.02
num messages 1ms	-0.07	-0.09	-0.15	-0.07	-0.04	-0.08
volume excl algo 0-5	-0.09	-0.05	-0.05	-0.07	-0.14	-0.07
spread	-0.04	-0.15	-0.03	-0.07	-0.13	-0.04
imbalance excl algo 6-10	0.00	0.02	-0.23	-0.01	-0.10	-0.09
num messages 0.1ms	-0.04	-0.11	-0.10	-0.07	-0.05	-0.03
imbalance of algo 11-20	-0.08	-0.13	-0.01	0.01	0.12	-0.03
num messages 0.1s	-0.07	0.01	-0.12	0.02	0.02	-0.13
cash of algorithm	0.00	0.05	-0.02	0.05	-0.07	0.14
inventory of algo	0.00	0.05	-0.02	0.05	-0.07	0.14
volume excl algo 11-20	-0.04	-0.04	-0.11	-0.04	-0.02	0.06
imbalance excl algo 0-5	-0.08	0.02	0.04	0.02	0.11	0.03
imbalance excl algo 11-20	-0.02	-0.01	-0.12	0.01	-0.01	0.13
quad var 15m	-0.01	0.12	0.03	0.00	0.10	0.03
quad var 60m	-0.01	0.12	0.04	0.00	0.08	0.01
agg sell last 360s	-0.04	0.01	-0.11	-0.04	-0.00	-0.06

Table A24: Average regression coefficient for price bucket describing eagerness to trade per cluster on first four weeks of training data for TOMTOM conditioning on the direction of order D_t .