Provable MLR: Forecasting AAVE's Lifetime Repayments

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For this particular tutorial, we will build a Closed-Form Multiple Linear Regression algorithm and use it to forecast AAVE's (WETH Pool) future projected Lifetime Repayments as a practical example. Towards the second half-end of the tutorial, we will convert the model to Cairo enabling us to make the entire MLR system as well as the forecasts fully provable & verifiable.

Provability & Verifiability

The key benefit of this Lightweight Multiple Linear Regression Solver lies in its commitment to Provability and Verifiability. By utilizing Cairo & Orion, the entire MLR system becomes inherently provable through STARKs, offering unparalleled transparency. This enables every inference of the model construction, execution, and prediction phase to be transparently proved using e.g. LambdaClass STARK Prover. In essence, the Provability and Verifiability aspect ensures that the tool is not only for prediction but also a framework to build accountability and trust in on-chain business environments.

Brief intro to MLR

To give a brief overview of MLR, it is used to model the relationship between a single dependent variable denoted asy , and multiple independent variables, such asx1 ,x2 , etc. This method extends the principles of simple linear regression by allowing us to incorporate multiple explanatory factors to predict y. The significant advantage lies in its capability to evaluate both individual and joint linear relationships between each feature and the target variable, providing a comprehensive understanding of how changes in predictors correspond to changes in the outcome.

! $y = \beta 0 + (\beta 1 * x 1) + (\beta 2 * x 2) + ... + (\beta n * x n) + e$

y = dependent variable x 1 , x 2.. x n = independent variables β 0 = intercept β 1 , β 2... β n = regression coefficients ε = error term y = β0 + (β1 * x1) + (β2 * x2) + ... + (βn * xn) + e \quad \begin{align*} y & \text{= dependent variable} \ x1,x2..xn & \text{= independent variables} \ β0 & \text{= intercept } \ β1,β2...βn & \text{= regression coefficients} \ ε & \text{= error term} \ \end{align*}* The regression coefficients, illustrated byβ0 ,β1 ...βn play a pivotal role in quantifying the impact of each feature variable on the dependent variable. It not only enables us to discern the individual impact (magnitude and direction) but also unveils how they collaboratively combine to shape outcomes.

In summary, when incorporating multiple factors into our model, we can improve the prediction & forecasting accuracy when compared to relying solely on a single predictor, as seen with simple regression. This enhancement can be mainly attributed to the fact that real-world outcomes being typically influenced by a myriad of factors. Therefore, leveraging multiple linear regression (MLR) serves as a foundational stepping stone to adeptly capture the intricate relationships between features and labels, ultimately guiding us in building accurate and highly interpretable models.

Closed-form approach for computing MLR gradients

As outlined above, MLR still remains a powerful tool for problem-solving in many data-oriented business applications. As we step into the ProvableML domain to enhance model transparency, these algorithms still prove to be highly advantageous in on-chain environments due to their lightweight, interpretable, and cost-efficient attributes.

Traditionally, the common approach to MLR involves computing pseudo-inverses and Singular Value Decomposition (SVD). While robust, their implementation complexity can often overshadow the regression problem at hand. Consequently, gradient-based methods are often preferred in data science projects, but this also can be deemed excessive due to the resource-intensive iterative approach taken to approximate gradients which can be very costly. In addition to this, the manual hyperparameter tuning required can be a significant hindrance, especially in automated on-chain environments.

In light of these considerations, this tutorial introduces an intuitive closed-form approach to calculating MLR gradients without any hyperparameter tuning, making it easy to implement and run MLR algorithms effectively on Starknet. This approach also makes it easy to estimate computational steps/costs required to run MLR given a dataset.

The closed-form MLR comprises of three integral components:

1. Orthogonalization of Input Features: Ensures independence among the X features.
2. Gradient Calculation: Computes the exact gradient between each decorrelated X feature and y variable.
3. Forecasting & Predictions: Utilizes the computed coefficients to make new predictions.
4.

Python implementation

To demonstrate a realistic end-to-end implementation, we'll first work with the AAVE dataset before delving into the implementation of the MLR Solver. Step by step, we'll implement the full process in Python first, which should lay the groundwork to allow us to make a seamless transition to Cairo in the subsequent stages of this tutorial.

Preparing the AAVE dataset

To begin with, we will use the Aave dataset which can be accessed from this link . We will work with our cleaned-up version of the dataset which includes various business metrics such as liquidity incentives and borrowing rates, providing valuable insights for forecasting future lifetime repayments.

```

Copy importpandasaspd importnumpyasnp importmatplotlib.pyplotasplt importos fromsklearn.metricsimportr2_score

# dataset pulled from https://app.aavescan.com/

df_main=pd.read_csv('AAVE-V3-weth.csv') df_main.drop('Unnamed: 0', axis=1, inplace=True)

# Order the DataFrame from the oldest to the most recent datapoint based on the date

df_main=df_main.iloc[::-1]

# Since Most of the df values are in wei we divide all values by a fixed factor to make the data easy to work with.

# Dividing by 1e+22 converts values to thousands of ETH and prevents overflow as we transition to Cairo later.

factor=1e+22 df_main=df_main/factor

days_to_forecast=-7

# Our y variable to train on

df['lifetimeRepayments_7day_forecast']=df[['lifetimeRepayments']].shift(days_to_forecast) df=df[0:days_to_forecast]

```

accruedToTreasury availableLiquidity lifetimeFlashLoans lifetimeLiquidity lifetimeReserveFactorAccrued totalLiquidityAsCollateral totalScaledVariableDebt lifetimeRepayments variableBorrowRate lifetimeRepayments_7day_forecast 30 0.000933 7.40 12.4 204.0 0.0550 119.0 28.7 76.4 3370.0 77.4 29 0.001340 7.45 12.4 205.0 0.0550 119.0 28.7 76.4 3370.0 77.4 28 0.001740 8.02 12.4 207.0 0.0550 119.0 28.8 76.5 3320.0 77.5 27 0.002120 9.59 12.8 212.0 0.0550 124.0 28.6 77.0 3180.0 77.6 26 0.000017 10.00 14.1 215.0 0.0575 128.0 28.7 77.3 3150.0 78.0 In order to separate the feature and label of our dataset, we have replicated the lifetime repayments column into a new target variable column whilst shifting its values up by 7 rows. This aligns each repayment value with the appropriate features from 7 days prior. Consequently, thelifetimeRepayments_7day_forecast column will serve as our predictive label (y), while the other metrics across the same rows become our explanatory variables (X) for predicting future repayments.

By framing our features and labels in this format, we will be able to train the MLR model to be able to estimate the daily lifetime repayments based on current lending pool metrics.

Data normalization

We will now normalize the data using min-max scaling to transform all features and labels into a common 0-1 range.

```

Copy defnormalize_data(original_data): data_min=np.min(original_data, axis=0) data_max=np.max(original_data, axis=0) data_range=data_max-data_min data_normalized=(original_data-data_min)/data_range returndata_normalized

# Drop the y label from dataframe

features=df.drop(['lifetimeRepayments_7day_forecast'], axis=1)
```

setting our y label

target=df['lifetimeRepayments_7day_forecast']

convert data to numpy format

X_original=features.to_numpy() Y_original=target.to_numpy()

normalize the data

X_normalized=normalize_data(X_original) y_normalized=normalize_data(Y_original)

```

Computing MLR gradients

As outlined in the prior section, this closed-form approach to computing the regression coefficients does not rely on gradient descent. Instead, it orthogonalizes the x feature variables, ensuring independence across predictors. It then calculates the gradient between the orthogonalized x features and the y variable. This approach allows us to compute the exact coefficients in a single step, eliminating the need for iterative approximations.

It's very important to notice that in thedecorrelate_features function, only the last feature row is fully orthogonalized. The rest of the features are decorelated from one another but are not fully orthogonal to each other. This is done to save on computational costs and make the algorithm more efficient since we can still compute the coefficients without necessarily needing to fully orthogonalize them.

This is better illustrated in thecalculate_gradients function, as the process starts from the last fully orthogonalized X feature. Subsequently, it then computes the corresponding gradient and removes this feature's influence from the y label. By iteratively repeating this process across all features we can compute the gradient without the need to have all features fully orthogonalized since we are also removing their influences from the y label iteratively. This streamlined approach reduces computational steps and memory requirements, enhancing the algorithm's efficiency and performance.

```

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We will first transpose the X features and add a bias term.

deftranspose_and_add_bias(feature_data): transposed_data=feature_data.T transposed_data_with_bias=np.vstack((transposed_data, np.ones(transposed_data.shape[1]))) returntransposed_data_with_bias

decorrelate the features (only the last feature row will be fully orthogonal)

defdecorrelate_features(feature_data): x_temp=feature_data.copy() feature_rows=feature_data.shape[0]

Decorrelate features

foriinrange(feature_rows): feature_squared=np.sum(x_temp[i]*2) forjinrange(i+1, feature_rows): feature_cross_prod=np.sum(x_temp[i]x_temp[j]) iffeature_squared==0: print('Warning, division by zero encountered and handled') feature_squared=1e-8 feature_grad=feature_cross_prod/feature_squared x_temp[j]-=feature_grad*x_temp[i] decorelated_x_vals=x_temp returndecorelated_x_vals

compute the exact gradients for each feature variable, including the bias term

defcalculate_gradients(decorelated_x_vals,y_values,original_x_features): y_temp=y_values.copy() feature_rows=decorelated_x_vals.shape[0] gradients=np.zeros(feature_rows)

Calculate gradients

foriinrange(feature_rows-1,-1,-1): prod=np.sum(y_temp*decorelated_x_vals[i])
squared=np.sum(decorelated_x_vals[i]*decorelated_x_vals[i]) ifsquared==0: print('Warning, division by zero encountered and handled') squared=1e-8 gradients[i]=prod/squared y_temp-=gradients[i]*original_x_features[i] returngradients

X_normalized_transposed_with_bias=transpose_and_add_bias(X_normalized)
decorrelated_X_features=decorrelate_features(X_normalized_transposed_with_bias)
gradient_values=calculate_gradients(decorrelated_X_features, y_normalized, X_normalized_transposed_with_bias)

real_gradient_values_reversed=np.flip(gradient_values) print('All regression coefficient values, including the bias term: ', real_gradient_values_reversed)

    All regression coefficient values,including the bias term:[-1.270622431.159312710.173401-
    0.31112069,1.093384390.93959362-1.12956438-0.083711131.187340430.3425375]

```

Reconstructing the y labels using the calculated gradients and X feature data

Using the computed regression coefficients we can now rebuild the y labels to see how well they fit to the dataset. In order to achieve this we simply compute the dot product between the calculated coefficient values and original X feature values.

```

Copy defdenormalize_data(original_data,normalized_data): data_min=np.min(original_data)
data_max=np.max(original_data) data_range=data_max-data_min

denormalize_data=(normalized_data*data_range)+data_min returndenormalize_data

y_pred_norm=gradient_values@X_normalized_transposed_with_bias#prediction#
reconstructed_y=denormalize_data(Y_original,y_pred_norm)

Plot the denormalized y values

plt.figure(2) plt.title(" LifetimeRepayment Predictions") plt.plot(reconstructed_y) plt.plot(Y_original) plt.legend([" Actual y values","reconstructed ys (predictions)"]) plt.xlabel('Days') plt.ylabel('Lifetime Repayment (thousands ETH)')

Calculate R^2 score for denormalized prediction

accuracy_denormalized=r2_score(Y_original, reconstructed_y) print("R^2 score (denormalized):", accuracy_denormalized)

    R^2score(denormalized):0.9968099033369738

```

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Forecasting the upcoming 7-Day Lifetime Repayments Projections for AAVE's WETH Pool

With the model now fitted, we can use the most recent data points to forecast future repayments projections. Additionally, we will calculate the uncertainty bounds of a 95% confidence interval for these predictions to quantify the reliability of our repayment projections based on the model's historical accuracy across the training data. By using both estimates of prediction and confidence intervals, we provide both repayment expectations and precision guidance that can help in business planning.

```

Copy df_forecast=df_main[-7:] df_forecast_data=df_forecast.to_numpy()

normalize data

X_min=np.min(X_original, axis=0) X_max=np.max(X_original, axis=0) X_range=X_max-X_min
df_forecast_data_normalized=(df_forecast_data-X_min)/X_range

transpose the matrix and add bias

df_forecast_data_normalized_transposed=df_forecast_data_normalized.T df_forecast_data_normalized_transposed_with_bias=np.vstack((df_forecast_data_normalized_transposed, np.ones(df_forecast_data_normalized_transposed.shape[1])))

normalized forecasts

forecast_normalized=gradient_values@df_forecast_data_normalized_transposed_with_bias

denormalize forecast

Y_min=np.min(Y_original, axis=0) Y_max=np.max(Y_original, axis=0) Y_range=Y_max-Y_min

denormalize forecast

Y_min=np.min(Y_original, axis=0) Y_max=np.max(Y_original, axis=0) Y_range=Y_max-Y_min forecast_pred=(forecast_normalized*Y_range)+Y_min forecast_plot_data=np.insert(forecast_pred,0, Y_original[-1])

Calculate expanding confidence intervals

residual=Y_original-reconstructed_y stderr=np.std(residual) z_score=1.96# z-score for 95% CI intervals=z_score*stderr*np.sqrt(np.arange(len(forecast_plot_data)))

Creating the plot

plt.figure(figsize=(10,5)) plt.plot(Y_original , label='Historical Lifetime Repayments') plt.plot(len(Y_original)-1+np.arange(len(forecast_plot_data)), forecast_plot_data , color='orange', label='Upcoming 7 day forecast') plt.fill_between(len(Y_original)-1+np.arange(len(forecast_plot_data)), (forecast_plot_data-intervals), (forecast_plot_data+intervals), alpha=0.12, color='green', label='95% confidence interval')

plt.plot(reconstructed_y , label="Model's Predictions", color='lightblue')

Adding labels and title

plt.xlabel('Days') plt.ylabel('Lifetime Repayment (thousands ETH)') plt.title(" 7 Day Forecast (AAVE's total WETH lifetimeRepayment)") plt.legend()

Display the plot

plt.show()

forecast_pred values

Day 1 Forecast: 95.62317677745183

Day 2 Forecast 96.5934311440076

Day 3 Forecast 97.113932324072

Day 4 Forecast 97.5688580115012

Day 5 Forecast 98.45026776663158

Day 6 Forecast 99.42560920294711

Day 7 Forecast 100.49892105541984

```

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Transition to Cairo

Now that we have covered all the steps for constructing and fitting the MLR model using the AAVE dataset in Python, our subsequent step will be to implement it in Cairo. This transition will provide end-to-end provability across all aspects of the multiple linear regression system.

In order to catalyze our development we will leverage Orion's built-in functions and operators to construct the MLR Solver and use it to forecast AAVE's Lifetime Repayments.

Code Structure

The outlined code structure below should serve as a guide to help with our implementation as we will be working within multiple folders.

```

Copy . | ├── datasets | ├── aave_data | | ├── aave_x_features.cairo | | └── aave_y_labels.cairo | ├── user_inputs_data | | └── aave_weth_revenue_data_input.cairo | ├── aave_data.cairo | └── user_inputs_data.cairo ├── src | ├── data_preprocessing.cairo | ├── datasets.cairo | ├── helper_functions.cairo | ├── lib.cairo | ├── model.cairo.cairo | └── test.cairo ├── Scarb.toml ├── Scarb.lock | └── target

```

Setting up the Scarb project

Scarb is the Cairo package manager specifically created to streamline our Cairo development process. Scarb will typically manage project dependencies, the compilation process (both pure Cairo and Starknet contracts), and downloading and building external libraries such as Orion.You can find all the information about Scarb and Cairo installationhere .

To create a new Scarb project, open your terminal and run:

```

Copy scarbnewmultiple_linear_regresion

```

A new project folder should be created for you and make sure to replace the content in Scarb.toml file with the following code:

```

Copy [package] name="multiple_linear_regresion" version="0.1.0" [dependencies] orion={ git="https://github.com/gizatechxyz/onnx-cairo"} [scripts] test="scarb cairo-test -f multiple_linear_regression_test"

```

Now let's replace the contents ofsrc/lib.cairo with the following code. This will let our compiler know which files to include during the compilation of our code.

```

Copy modtest; moddata_preprocessing; modhelper_functions; moddatasets; modmodel;

```

Converting the dataset to Cairo

To convert the AAVE dataset to Cairo let's execute the following Python code. This simply creates a newdatasets folder for us and converts the x and y variables into Orion's 16x16 tensor format.

Orion's 16x16 tensor format was chosen for this particular tutorial, due to having a relatively good degree of accuracy for both the integer part and decimal part relative to our AAVE dataset.

```

Copy

Convert the original data to Cairo

defgenerate_cairo_files(data,name,folder_name): os.makedirs(f'multiple_linear_regression/src/datasets/{folder_name}', exist_ok=True) withopen(os.path.join('multiple_linear_regression/src/datasets',f'{folder_name}',f"{name}.cairo"),"w")asf: f.write("use array::ArrayTrait;\n"+ "use orion::numbers::fixed_point::implementations::fp16x16::core::{FP16x16Impl, FP16x16PartialEq };\n"+ "use orion::operators::tensor::{Tensor, TensorTrait, FP16x16Tensor};\n"+ "use orion::numbers::{FP16x16, FixedTrait};\n\n"+ "fn{0}() -> Tensor ".format(name)+"{\n"+ " let tensor = TensorTrait::::new(\n") iflen(data.shape)>1: f.write(" shape: array![{0},".format(data.shape[0])) f.write("{0}].span(),\n".format(data.shape[1])) f.write(" data: array![\n") iflen(data.shape)==1: f.write(" shape: array![{0}].span(),\n".format(data.shape[0])) f.write(" data: array![\n") forvalinnp.nditer(data.flatten()): f.write(" FixedTrait::new({0},{1}),\n".format(abs(int(val2*16)),str(val<0).lower())) f.write("].span() \n \n"+ ");\n\n"+ "return tensor; \n"+ "}") withopen(os.path.join('multiple_linear_regression/src/datasets',f'{folder_name}.cairo'),'a')asf: f.write(f"mod{name};\n")

generate_cairo_files(X_original,'aave_x_features','aave_data') generate_cairo_files(Y_original,'aave_y_labels','aave_data') generate_cairo_files(df_forecast_data,'aave_weth_revenue_data_input','user_inputs_data')

` ` `

The converted x and y values will now be populated intoaave_x_features.cairo andaave_y_labels.cairo , which should be found under thesrc/dataset/aave_data folder.

On the other hand, theaave_weth_revenue_data_input will populated intosrc/dataset/user_inputs_data which is a separate folder. Theaave_weth_revenue_data_input represents the latest AAVE's WETH lending pool metrics, which will be later used for performing the 7-day lifetime repayments forecasts.

Now that we have placed the files into this new folder structure, we need to make sure that the files are still accessible to the compiler. Hence, let's create the filesaave_data.cairo anduser_inputs_data.cairo and add the following module references accordingly.

` ` `

Copy // in aave_data.cairo modaave_x_features; modaave_y_labels;

` ` `

` ` `

Copy // in user_inputs_data.cairo modaave_weth_revenue_data_input;

` ` `

Data Preprocessing

Now that our dataset has been generated, it is crucial to implement data normalization before feeding it into the MLR Solver. This is highly recommended for any future MLR implementation in Cairo to mitigate potential overflow issues during subsequent stages. This is due to the MLR closed-form approach involving squaring x values, which can get very large if left unnormalized.

To facilitate this process, we will establish a dedicated Cairo file nameddata_preprocessing.cairo which should be located under the mainsrc folder. This file will store all our data preprocessing functions, including the min-max normalization function.

` ` `

Copy // importing libs useorion::operators::tensor::{ Tensor,TensorTrait,FP16x16Tensor,U32Tensor,U32TensorAdd,FP16x16TensorSub,FP16x16TensorAdd, FP16x16TensorDiv,FP16x16TensorMul }; useorion::numbers::{FP16x16,FixedTrait}; usemultiple_linear_regresion::helper_functions::{ get_tensor_data_by_row, transpose_tensor, calculate_mean, calculate_r_score, normalize_user_x_inputs, rescale_predictions };

[derive(Copy,Drop)]

structDataset{ x_values:Tensor, y_values:Tensor, }

[generate_trait]

implDataPreprocessingofDatasetTrait{ fnnormalize_dataset(refself:Dataset)->Dataset{ letmutx_values=TensorTrait::::new(array![1].span(),array![FixedTrait::new(0,false)].span()); letmuty_values=TensorTrait::::new(array![1].span(),array![FixedTrait::new(0,false)].span()); // used for multiple_linear_regression_models ifself.x_values.shape.len() >1{ x_values=normalize_feature_data(self.x_values); y_values=normalize_label_data(self.y_values); } // used for linear_regression_models ifself.x_values.shape.len()==1{ x_values=normalize_label_data(self.x_values); y_values=normalize_label_data(self.y_values); }

returnDataset{ x_values, y_values }; } }

// normalizes 2D Tensor fnnormalize_feature_data(tensor_data:Tensor)->Tensor { letmutx_min_array=ArrayTrait::::new(); letmutx_max_array=ArrayTrait::::new(); letmutx_range_array=ArrayTrait::::new(); letmutnormalized_array=ArrayTrait::::new(); // transpose to change rows to be columns lettransposed_tensor=tensor_data.transpose(axes:array![1,0].span()); lettensor_shape=transposed_tensor.shape; lettensor_row_len=*tensor_shape.at(0);// 13 lettensor_column_len=*tensor_shape.at(1);//50 // loop and append max and min row values to the corresponding array letmuti:u32=0; loop{ ifi>=tensor_row_len { break(); } letmuttransposed_tensor_row=get_tensor_data_by_row(transposed_tensor, i); x_max_array.append(transposed_tensor_row.max_in_tensor()); x_min_array.append(transposed_tensor_row.min_in_tensor()); x_range_array .append(transposed_tensor_row.max_in_tensor()-transposed_tensor_row.min_in_tensor()); i+=1; }; // convert array to tensor format for ease of math operation letmutx_min=TensorTrait::< FP16x16

::new(shape:array![1, tensor_row_len].span(), data:x_min_array.span()); letmutx_range=TensorTrait::< FP16x16 ::new(shape:array![1, tensor_row_len].span(), data:x_range_array.span()); letnormalized_tensor=(tensor_data-x_min)/x_range; returnnormalized_tensor; }

// normalizes 1D tensor fnnormalize_label_data(tensor_data:Tensor)->Tensor { letmuttensor_data_=tensor_data; letmutnormalized_array=ArrayTrait::::new(); letmutrange=tensor_data.max_in_tensor()-tensor_data.min_in_tensor(); // loop through tensor values normalizing and appending to a new array letmuti:u32=0;

loop{ matchtensor_data_.data.pop_front() { Option::Some(tensor_val)=>{ letmutdiff=*tensor_val-tensor_data.min_in_tensor(); normalized_array.append(diff/range); i+=1; }, Option::None(_)=>{break; } }; }; // convert normalized array values to tensor format letmutnormalized_tensor=TensorTrait::< FP16x16

::new(shape:array![tensor_data.data.len()].span(), data:normalized_array.span()); returnnormalized_tensor; }

```

Looking at the code above, we also have implemented a newDataset struct to encapsulate the predictor features (x_values) and target variable (y_values) into a single reusable data object. By bundling x and y into Dataset, we can easily implement new methods into it such as thenormalize_dataset() , allowing for a seamless normalization of both components simultaneously. This approach not only streamlines normalization operations in a single step but also eliminates redundant logic.

The MLR Solver in Cairo

To keep everything organized let's now make a new folder namedmodel under the mainsrc folder. Within it, we will create a dedicated Cairo file namedmultiple_linear_regression_model.cairo to host all our MLR functions in Cairo.

All of the function MLR functions implemented can be seen below:

```

Copy useorion::operators::tensor::{ Tensor,TensorTrait,FP16x16Tensor,U32Tensor,U32TensorAdd,FP16x16TensorSub,FP16x16TensorAdd, FP16x16TensorDiv,FP16x16TensorMul }; useorion::numbers::{FP16x16,FixedTrait}; usemultiple_linear_regresion::data_preprocessing::{Dataset,DatasetTrait}; usemultiple_linear_regresion::helper_functions::{ get_tensor_data_by_row, transpose_tensor, calculate_mean, calculate_r_score, normalize_user_x_inputs, rescale_predictions };

[derive(Copy,Drop)]

structMultipleLinearRegressionModel{ coefficients:Tensor }

[generate_trait]

implRegressionOperationofMultipleLinearRegressionModelTrait{ // reconstruct the y values using the computed gradients and x values fnpredict(refself:MultipleLinearRegressionModel, feature_inputs:Tensor)->Tensor { // random tensor value that we will replace letmutprediction_result=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span());

letmutresult=ArrayTrait::::new(); // for multiple predictions iffeature_inputs.shape.len() >1{ letfeature_values=add_bias_term(feature_inputs,1); letmutdata_len:u32=feature_values.shape.at(0); letmuti:u32=0; loop{ ifi>=data_len { break(); } letfeature_row_values=get_tensor_data_by_row(feature_values, i); letpredicted_values=feature_row_values.matmul(@self.coefficients); result.append(predicted_values.data.at(0)); i+=1; }; prediction_result= TensorTrait::< FP16x16

::new(shape:array![result.len()].span(), data:result.span()); }

// for single predictions iffeature_inputs.shape.len()==1&&self.coefficients.data.len() >1{ letfeature_values=add_bias_term(feature_inputs,1); prediction_result=feature_values.matmul(@self.coefficients); }

returnprediction_result; } }

fnMultipleLinearRegression(dataset:Dataset)->MultipleLinearRegressionModel{ letx_values_tranposed=transpose_tensor(dataset.x_values); letx_values_tranposed_with_bias=add_bias_term(x_values_tranposed,0); letdecorrelated_x_features=decorrelate_x_features(x_values_tranposed_with_bias); letcoefficients=compute_gradients(decorrelated_x_features, dataset.y_values, x_values_tranposed_with_bias); returnMultipleLinearRegressionModel{ coefficients }; }

//Adds bias term to features based on axis fnadd_bias_term(x_feature:Tensor, axis:u32)->Tensor { letmutx_feature_=x_feature; letmuttensor_with_bias=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span()); letmutresult=ArrayTrait::::new(); // check if feature data has multiple rows and columns ifx_feature.shape.len() >1{ letmutindex:u32=0; ifaxis==1{ index=0; }else{ index=1; } letdata_len=x_feature.shape.at(index);// 50 letmuti:u32=0; loop{ ifi>=data_len { break(); } result .append(FixedTrait::new(65536,false));//65536=ONE in FP16x16, change accordingly i+=1; }; ifaxis==0{ letres_tensor=TensorTrait::new(shape:array![1, data_len].span(), data:result.span()); tensor_with_bias= TensorTrait::concat(tensors:array![x_feature, res_tensor].span(), axis:axis); }else{ letres_tensor=TensorTrait::new(shape:array![data_len,1].span(), data:result.span()); tensor_with_bias= TensorTrait::concat(tensors:array![x_feature, res_tensor].span(), axis:axis); } } // check if feature data is 1D ifx_feature.shape.len()==1{ letmutj:u32=0; loop{ matchx_feature_.data.pop_front() { Option::Some(x_val)=>{ result.append(x_val); j+=1; }, Option::None(_)=>{break; } }; }; result.append(FixedTrait::new(65536,false));//65536=ONE in FP16x16, change accordingly tensor_with_bias= TensorTrait::::new(shape:array![result.len()].span(), data:result.span()); } returntensor_with_bias; }

// decorrelates the feature data (*only the last tensor row of the decorrelated feature data will be fully orthogonal) fndecorrelate_x_features(x_feature_data:Tensor)->Tensor { letmutinput_tensor=x_feature_data;

letmuti:u32=0; loop{ ifi>=x_feature_data.shape.at(0) { break(); } letmutplaceholder=ArrayTrait::::new(); letmutfeature_row_values=get_tensor_data_by_row(input_tensor, i); letmutfeature_squared=feature_row_values.matmul(@feature_row_values); // avoiding division by zero errors iffeature_squared.data.at(0)==FixedTrait::new(0,false) { feature_squared= TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span()); } // loop through remaining tensor data and remove the individual tensor factors from one another letmutj:u32=i+1; loop{ ifj>=x_feature_data.shape.at(0) { break(); } letmutremaining_tensor_values=get_tensor_data_by_row(input_tensor, j); letfeature_cross_product=feature_row_values.matmul(@remaining_tensor_values); letfeature_gradients=feature_cross_product/feature_squared; remaining_tensor_values=remaining_tensor_values -(feature_row_values feature_gradients);//remove the feature factors from one another // loop and append the modified remaining_tensor_values (after the correlated factor has been removed) to the placeholder array letmutk:u32=0; loop{ ifk>=remaining_tensor_values.data.len() { break(); } placeholder.append(*remaining_tensor_values.data.at(k)); k+=1; };

j+=1; }; // convert placeholder array to tensor format and update the original tensor with the new modified decorrelated tensor row values letmutdecorrelated_tensor=TensorTrait::new(shape:array![x_feature_data.shape.at(0)-1-i,x_feature_data.shape.at(1)].span(), data:placeholder.span()); letmutoriginal_tensor=input_tensor .slice(starts:array![0,0].span(), ends:array![i+1,*x_feature_data.shape.at(1)].span(), axes:Option::None(()), steps:Option::Some(array![1,1].span())); input_tensor= TensorTrait::concat(tensors:array![original_tensor, decorrelated_tensor].span(), axis:0); i+=1; }; returninput_tensor; }

// computes the corresponding MLR gradient using decorrelated feature fncompute_gradients(decorrelated_x_features:Tensor, y_values:Tensor, original_x_tensor_values:Tensor)->Tensor { letmutgradient_values_flipped=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span());

letmutresult=ArrayTrait::::new(); letmuttensor_y_vals=y_values; letmuti:u32=decorrelated_x_features.shape.at(0); // loop through Decorrelated_x_features starting from the fully orthogonalized last tensor row values loop{ ifi<=0{ break(); } letindex_val=i-1; letmutdecorelated_feature_row_values=get_tensor_data_by_row(decorrelated_x_features, index_val); letmutdecorelated_features_squared=decorelated_feature_row_values .matmul(@decorelated_feature_row_values); letmutfeature_label_cross_product=tensor_y_vals .matmul(@decorelated_feature_row_values);// multiply the tensors // avoiding division by zero errors ifdecorelated_features_squared.data.at(0)==FixedTrait::new(0,false) { decorelated_features_squared= TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span()); } // computing the feature gradient values using the y values and decorrelated x features and appending them to array letmutsingle_gradient_value=feature_label_cross_product /decorelated_features_squared;// divide the summed value by each other result.append(single_gradient_value.data.at(0)); // remove the associated feature gradient value away from y values letmutoriginal_x_tensor_row_values=get_tensor_data_by_row(original_x_tensor_values, index_val); tensor_y_vals=tensor_y_vals -(original_x_tensor_row_values single_gradient_value);//remove the first feature from the second feature values i-=1; }; // convert the gradient array to tensor format letfinal_gradients=TensorTrait::new(shape:array! [*decorrelated_x_features.shape.at(0)].span(), data:result.span());

letmutreverse_grad_array=ArrayTrait::::new(); letmutdata_len:u32=final_gradients.data.len(); loop{ ifdata_len<=0{ break(); } lettemp_val=data_len-1; reverse_grad_array.append(*final_gradients.data.at(temp_val)); data_len-=1; }; // convert gradient values to tensor format letgradient_values_flipped=TensorTrait::< FP16x16

::new(shape:array![reverse_grad_array.len()].span(), data:reverse_grad_array.span());

returngradient_values_flipped; }

```

At the core of this file lies the pivotalMultipleLinearRegression() function, which orchestrates the entire model fitting process. This function plays a central role by invoking critical functions such asDecorrelate_x_features() ,add_bias_term() , andcompute_gradients() , to calculate the regression coefficients. It is important to notice that the output of theMultipleLinearRegression function returns the newly createdMultipleLinearRegressionModel object type. This is done to encapsulate the trained model parameters into a reusable bundle that contains the fitted coefficients.

We have also implemented apredict() method into the newMultipleLinearRegressionModel struct which should enable us to generate new predictions and forecasts by simply passing the new feature X inputs to the function. This modular approach avoids the need to re-fit the model each time when making new predictions allowing us to store, access, and conveniently manipulate model coefficients.

Once again to ensure that the file is accessible to the compiler we need to also add a reference module. For this, let's create the file namedmodel.cairo under the mainsrc folder and add the following:

```

Copy // in model.cairo modmultiple_linear_regression_model;

```

Helper functions

Now let's create an additional file namedhelper_functions.cairo under the mainsrc folder which will host all our helper functions required to construct the MLR Solver. Some of these functions stored here will also be used later during the testing phase to assess the model's performance once fitted. This file consists of multiple functions some of which include:

- Function to help with retrieving tensor data by row and column index, which are essential for MLR construction
- Function to compute the accuracy of our model using the R-squared method
- A function for computing tensor means used in testing.
- Functions dedicated to normalizing feature inputs, enabling accurate predictions and forecasts.
- A rescaling function tailored to adjust prediction results to appropriate sizes.
-

```

Copy usedebug::PrintTrait; usearray::{ArrayTrait,SpanTrait}; useorion::operators::tensor::{ Tensor,TensorTrait,FP16x16Tensor,U32Tensor,U32TensorAdd,FP16x16TensorSub,FP16x16TensorAdd, FP16x16TensorDiv,FP16x16TensorMul };

useorion::numbers::{FP16x16,FixedTrait};

// retrieves row data by index in a 2D tensor fnget_tensor_data_by_row(tensor_data:Tensor, row_index:u32,)->Tensor { letcolumn_len=*tensor_data.shape.at(1);//13 // create new array letmutresult=ArrayTrait::::new(); // loop through the x values and append values letmuti:u32=0; loop{ ifi>=column_len { break(); } result.append(tensor_data.at(indices:array![row_index, i].span())); i+=1; }; letresultant_tensor=TensorTrait::< FP16x16

::new(array![column_len].span(), data:result.span()); returnresultant_tensor; }

// transposes tensor fntranspose_tensor(tensor_data:Tensor)->Tensor { lettensor_transposed=tensor_data.transpose(axes:array![1,0].span()); returntensor_transposed; }

fncalculate_mean(tensor_data:Tensor)->FP16x16{ lettensor_size=FixedTrait::::new_unscaled(tensor_data.data.len(),false); letcumulated_sum=tensor_data.cumsum(0, Option::None(()),Option::None(())); letsum_result=cumulated_sum.data[tensor_data.data.len()-1]; letmean=*sum_result/tensor_size; returnmean; }

// Calculates the R-squared score between two tensors. fncalculate_r_score(Y_values:Tensor,Y_pred_values:Tensor)->FP16x16{ letmutY_values_=Y_values; letmean_y_value=calculate_mean(Y_values); // creating the appropriate tensor shapes and empty arrays to populate values into letmutsquared_diff_shape=array::ArrayTrait::new(); squared_diff_shape.append(Y_values.data.len()); letmutsquared_diff_vals=array::ArrayTrait::new(); letmutsquared_mean_diff_shape=array::ArrayTrait::new(); squared_mean_diff_shape.append(Y_values.data.len()); letmutsquared_mean_diff_vals=array::ArrayTrait::new();

letmuti:u32=0;

loop{ matchY_values_.data.pop_front() { Option::Some(y_value)=>{ letdiff_pred=*y_value*-Y_pred_values.data.at(i); letsquared_diff=diff_pred*diff_pred; squared_diff_vals.append(squared_diff);

letdiff_mean=*y_value-mean_y_value; letsquared_mean_diff=diff_mean*diff_mean; squared_mean_diff_vals.append(squared_mean_diff); i+=1; }, Option::None(_)=>{break; } } };

letsquared_diff_tensor=TensorTrait::< FP16x16

::new(squared_diff_shape.span(), squared_diff_vals.span()); letsquared_mean_diff_tensor=TensorTrait::< FP16x16 ::new(squared_mean_diff_shape.span(), squared_mean_diff_vals.span()); letsum_squared_diff=squared_diff_tensor.cumsum(0, Option::None(()),Option::None(())); letsum_squared_mean_diff=squared_mean_diff_tensor .cumsum(0, Option::None(()),Option::None(())); letr_score=FixedTrait::new_unscaled(1,false) -*sum_squared_diff.data.at(Y_values.data.len()-1)* /sum_squared_mean_diff.data.at(Y_values.data.len()-1);

returnr_score; }

// computes the x_min, x_max and x_range. Used for helping in normalizing and denormalizing user input values operations fnnormalize_user_x_inputs(x_inputs:Tensor, original_x_values:Tensor)->Tensor { letmutx_inputs_normalized=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span());

letmutx_min=ArrayTrait::::new(); letmutx_max=ArrayTrait::::new(); letmutx_range=ArrayTrait::::new(); letmutresult=ArrayTrait::::new();

iforiginal_x_values.shape.len() >1{ lettransposed_tensor=original_x_values.transpose(axes:array![1,0].span()); letdata_len=*transposed_tensor.shape.at(0);//13 // loop through each row calculating the min, max, and range row values for each feature column letmuti:u32=0; loop{ ifi>=data_len { break(); } letmuttransposed_tensor_row=get_tensor_data_by_row(transposed_tensor, i); x_min.append(transposed_tensor_row.min_in_tensor()); x_max.append(transposed_tensor_row.max_in_tensor()); x_range .append(transposed_tensor_row.max_in_tensor()-transposed_tensor_row.min_in_tensor()); i+=1; }; letmutx_min_tensor=TensorTrait::new(shape:array![data_len].span(), data:x_min.span()); letmutx_max_tensor=TensorTrait::new(shape:array![data_len].span(), data:x_max.span()); letmutx_range_tensor=TensorTrait::new(shape:array![data_len].span(), data:x_range.span());

// for normalizing 2D user inputted feature vals ifx_inputs.shape.len() >1{ letmutj:u32=0; loop{ ifj>=*x_inputs.shape.at(0) { break(); }; letmutrow_data=get_tensor_data_by_row(x_inputs, j); letmutnorm_row_data=(row_data-x_min_tensor)/x_range_tensor; letmutk:u32=0;

loop{ ifk>=norm_row_data.data.len() { break(); }; result.append(*norm_row_data.data.at(k)); k+=1; }; j+=1; }; x_inputs_normalized= TensorTrait::< FP16x16

::new(array![*x_inputs.shape.at(0),*x_inputs.shape.at(1)].span(), data:result.span()); };

// for normalizing 1D feature input ifx_inputs.shape.len()==1{ x_inputs_normalized=(x_inputs-x_min_tensor)/x_range_tensor; }; }

iforiginal_x_values.shape.len()==1{ letmutx_min_tensor=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![original_x_values.min_in_tensor()].span()); letmutx_max_tensor=TensorTrait::< FP16x16 ::new(shape:array![1].span(), data:array![original_x_values.max_in_tensor()].span()); letmutx_range_tensor=TensorTrait::< FP16x16 ::new(shape:array![1].span(), data:array![original_x_values.max_in_tensor()-original_x_values.min_in_tensor()] .span()); letmutdiff= ((x_inputs-x_min_tensor)); x_inputs_normalized=((x_inputs-x_min_tensor))/x_range_tensor; }; returnx_inputs_normalized; }

// rescales model predictions to standard format fnrescale_predictions(prediction_result:Tensor, y_values:Tensor)->Tensor { letmutrescale_predictions=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![FixedTrait::new(10,false)].span());

letmuty_min_array=ArrayTrait::::new(); letmuty_max_array=ArrayTrait::::new(); letmuty_range_array=ArrayTrait::::new();

letmuty_max=y_values.max_in_tensor(); letmuty_min=y_values.min_in_tensor(); letmuty_range=y_values.max_in_tensor()-y_values.min_in_tensor(); // convert to tensor format for ease of math operations lety_min_tensor=TensorTrait::< FP16x16

::new(shape:array![1].span(), data:array![y_min].span()); lety_max_tensor=TensorTrait::< FP16x16 ::new(shape:array![1].span(), data:array![y_max].span()); lety_range_tensor=TensorTrait::< FP16x16 ::new(shape:array![1].span(), data:array![y_range].span());

rescale_predictions=(prediction_result*y_range_tensor)+y_min_tensor;

returnrescale_predictions; }

```

Running tests on the model

At this stage, we have already implemented all the important sections of this tutorial in Cairo. What's left is doing some testing to ensure our model is behaving as expected. To perform our test we will create a new test file calledtest.cairo under the mainsrc folder and import all the necessary libraries including our x and y values and the MLR solver traits and functions as seen below.

```

Copy // use traits::Into; usedebug::PrintTrait; usearray::{ArrayTrait,SpanTrait};

usemultiple_linear_regresion::datasets::aave_data::aave_x_features::aave_x_features; usemultiple_linear_regresion::datasets::aave_data::aave_y_labels::aave_y_labels; usemultiple_linear_regresion::datasets::user_inputs_data::aave_weth_revenue_data_input:: {aave_weth_revenue_data_input };

usemultiple_linear_regresion::model::multiple_linear_regression_model::{ MultipleLinearRegressionModel,MultipleLinearRegression,MultipleLinearRegressionModelTrait }; usemultiple_linear_regresion::data_preprocessing::{Dataset,DatasetTrait}; usemultiple_linear_regresion::helper_functions:: {get_tensor_data_by_row, transpose_tensor, calculate_mean , calculate_r_score, normalize_user_x_inputs, rescale_predictions};

useorion::numbers::{FP16x16,FixedTrait};

useorion::operators::tensor::{ Tensor,TensorTrait,FP16x16Tensor,U32Tensor,U32TensorAdd, FP16x16TensorSub,FP16x16TensorAdd,FP16x16TensorDiv,FP16x16TensorMul};

[test]

[available_gas(999999999999999)]

fnmultiple_linear_regression_test() {

//Constructing our model letmutmain_x_vals=aave_x_features(); letmutmain_y_vals=aave_y_labels(); letmutdataset=Dataset{x_values:main_x_vals,y_values:main_y_vals}; letmutnormalized_dataset=dataset.normalize_dataset(); letmutmodel=MultipleLinearRegression(normalized_dataset); letmutmodel_coefficients=model.coefficients; letmutreconstructed_ys=model.predict (normalized_dataset.x_values); letmutr_squared_score=calculate_r_score(normalized_dataset.y_values,reconstructed_ys); // r_squared_score.print(); // model accuracy around 0.9969482421875

// checking if data has been normalized correctly assert(normalized_dataset.x_values.max_in_tensor() <=FixedTrait::new(65536,false), 'normalizedx not between0-1'); assert(normalized_dataset.x_values.min_in_tensor() >= FixedTrait::new(0, false), 'normalized x not between0-1'); assert(normalized_dataset.y_values.max_in_tensor() <= FixedTrait::new(65536, false), 'normalized y not between0-1'); assert(normalized_dataset.x_values.min_in_tensor() >= FixedTrait::new(0, false), 'normalized y not between0-1'); // performing checks on the shape of normalized data assert(normalized_dataset.x_values.data.len()== main_x_vals.data.len() && normalized_dataset.y_values.data.len()== main_y_vals.data.len() , 'normalized data shape mismatch'); // performing checks on the shape of coefficient values (gradient vals + bias) assert(model.coefficients.data.len() == *main_x_vals.shape.at(1)+1, 'coefficient data shape mismatch'); // model accuracy deviance checks assert(r_squared_score >= FixedTrait::new(62259, false), 'AAVE model acc.less than95%');

// using the model to forecast aave's7-day WETH net lifetime repayments forecast letlast_7_days_aave_data=aave_weth_revenue_data_input(); letlast_7_days_aave_data_normalized=normalize_user_x_inputs(last_7_days_aave_data, main_x_vals); letmutforecast_results=model.predict (last_7_days_aave_data_normalized); letmutrescale_forecasts=rescale_predictions(forecast_results, main_y_vals);// PS. ** the rescaled forecasted outputs are denominated in thousands of ETH // (*rescale_forecasts.data.at(0)).print(); // day1 forecast: 95.66773986816406 // (*rescale_forecasts.data.at(1)).print(); // day2: 96.64869689941406 // (*rescale_forecasts.data.at(5)).print(); // day6: 99.44300842285156 // (*rescale_forecasts.data.at(6)).print(); // day7: 100.57145690917969 }

```

Our model will get tested under themultiple_linear_regression_test() function which will follow these steps:

1. Data retrieval: The function initiates by fetching the AAVE dataset's x and y values.
2. Dataset construction and normalization: A new Dataset object gets initialized by passing the x and y variables. It is then normalized using the built-innormalize_dataset()
3. method.
4. Model fitting: Using theMultipleLinearRegression
5. function we fit the normalized dataset and compute the regression coefficients.
6. Computing accuracy of the model: To calculate the accuracy we utilize thepredict
7. method to compute the dot product between the model's regression coefficients and the x values. We then compute the R-squared score to measure the accuracy of our model.
8. Perform some testing: In the subsequent step we perform some checks to ensure that the tensor shape/dimension is correct. We also check the model's accuracy deviance to see if it's still within an acceptable range.
9. Making forecasts: If our checks have passed then our model should be clear to enable us to make new predictions. For this, we will use theaave_weth_revenue_data_input()
10. values which represent the most recent AAVE datapoints which should enable us to make forecasts for the upcoming 7 days of AAVE's WETH Pool Lifetime Repayments.
11. 

Finally, we can execute the test file by running:

```

Copy

scarb cairo-test -f multiple_linear_regression_test

testingmultiple_linear_regresion... running1tests

testmultiple_linear_regresion::test::multiple_linear_regression_test... testresult:ok.1passed;0failed;0ignored;0filteredout;

```

.... And as we can our test cases have passed! Hooray!!

Congratulations on reaching this point! You are now ready to implement fully transparent and verifiable forecasting solutions using this MLR framework.

If you're looking for more examples of using the MLR Solver, look into here as it covers more easy-to-follow jupyter notebook tutorials (e.g. Boston dataset).😁

We invite you to join us in forging a future by making AI a transparent and reliable resource for all!
```