Understanding the Value of Uniswap v3 Liquidity Positions

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3

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[part 1

](/uniswap-v3-lp-tokens-as-perpetual-put-and-call-options-5b66219db827?sk=43c071fa2796639a60fce6c9abd5aa76)and

[part 2

](/synthetic-options-and-short-calls-in-uniswap-v3-a3aea5e4e273?sk=9fa4cdb12aab88ca9ecdc4d767a4ee1e)of this series to learn about how Uniswap v3 LP tokens effectively behave like short puts and short calls.

Uniswap has revamped the way liquidity positions are created and managed in version 3 of their protocol. Compared with Uniswap v2, the process to establish a new position is fairly complex. If you're like me, you may simply click the + New Position

button and adjust the range and parameters until you get something that looks good enough. What's the best way to choose the parameters of a LP position?

In this article, we will describe what happens under the hood when a LP position is created. We will also derive a set of simple set of relationships that may help in choosing the optimal range of Uniswap v3 LP positions across many underlyings.

What is the value of a Uniswap v3 LP token?

The Uniswap v3 whitepaper describes how much of each asset has to be added when establishing a new position. The number of token0

and token1

in a new LP position will depend on the range determined by the lower tick tL

, the upper tick tH

and the price at entry P0

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I am reprinting equations (6.29) and (6.30) from the whitepaper in a slightly different notation to show how tL and tH are related to P0:

Here, the value of ΔE is determined by the initial amount token0 (denoted by x0) and token1 (denoted by y0) that is locked into the position when it is established:

Once the position is established, we can compute its Net Liquidity Value

by adding the amount of token1 to the amount of token0 times the price P:

If the price is above the upper tick tH, the Net Liq value of the LP token will converge to the geometric mean √(tL*tH). When the price is below the lower tick tL, the value of the LP token will simply be P times the size of the position.

When the value is between ticks tL and tH, the expression is a bit more complicated and depends on a function of the square root of the price P. Graphically, here's what the Net Liq value V(P) looks like:

Changing the range (tL, tH) changes the "sharpness" of the payoff curve V(P). The curve V(P) will converge to the dashed line in the figure above when (tL,tH) is a single tick wide. Again, a 1–tick wide LP position is exactly the return function of a covered call at expiration, without considering the collected fees.

Computing Delta, the rate of change in Net Liq Value

How will the value of a LP position be affected by the price of the underlyings? Specifically, we'd like to know how much would the Net Liq change if the value of token0 changes by $1. This quantity is called "delta" and represents the price sensitivity of an option.

We can obtain δ(P) by taking the partial derivative of the Net Liq value function V(P) with respect to the price P to get the following expression:

It is much easier to understand this expression if we look at it graphically and normalize by the value of the position ΔE. Since the derivative of a function is its instantaneous slope, the value of delta is simply the slope of a line that is tangential to the price curve V(P):

What this figure represents is how much the value of a LP token tracks the price of the underlying. Delta goes from 1 to 0 as the price increases, meaning that the value will match the price of the underlying with 100% correlation at low prices and 0% above the upper tick.

More concretely, let's consider a LP position deployed between (2000, 3000) that accrues 30% APR from the collected fees. You can think of delta as the slope of the blue line divided by the slope of the red line. Since the value of δ(P) is always less than or equal to 1, the return of a LP position will also be less than or equal to a holding strategy.

Notice the rather large discrepancy between the ETH price and the LP position when the price is above 3000. The area between the red and blue curve is referred to as the impermanent loss (IL). Some will see IL as "missing a great opportunity for profits" and many are extremely worried about it.

Impermanent loss doesn't worry me at all because I understand that this "missed opportunity" is a feature

of covered call positions. As I described in arecent series of tweets, while LP positions do suffer from impermanent loss, LP positions actually decreases the volatility of portfolio returns: