Looking to gain insight on the matter of the inactivity penalty quotient

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While the Serenity Design Rationale document states that

With the current parametrization, if blocks stop being finalized, validators lose 1% of their deposits after 2.6 days, 10% after 8.4 days, and 50% after 21 days. This means for example that if 50% of validators drop offline, blocks will start finalizing again after 21 days.

If we examine the current parametrization we have

penalties[index] += Gwei(effective_balance * finality_delay // INACTIVITY_PENALTY_QUOTIENT)

with INACTIVITY_PENALTY_QUOTIENT

= 2**25

(33,554,432

).

Now, if we build and run a quick python script

balance = 100.0

for i in range(4,4726): balance -= (i * balance) / 2**25 print(str(i) + "\t" + str(balance))

our results are, for an initial balance of 100

4 99.99998807907104 5 99.99997317791163 6 99.99995529652298

[snip]

4723 71.71416019772546 4724 71.70406383570766 4725 71.69396675817036

adjusting the exponent of 2

in the above script, with 2**23.94128

we get

[snip]

567 99.00533232662947 568 99.00184121683876 569 98.99834408404766

[snip]

1840 90.01886280962862 1841 90.00857450307387 1842 89.99828178458043

[snip]

4723 50.026236935763414 4724 50.01156578060706 4725 49.996895823295965

Which are closer

to the statements in the rationale, namely, losing 1% at 2.6 days (568 epochs), 10% after 8.2 days (1841 epochs), and 50% after 21 days (4724 epochs).

So the short question

is why 2**25

was chosen as the Inactivity Penalty Quotient instead of 2**24

. And the long question

is about the methodology used to compute the coefficient .

For the latter, we tried an analytical approach and end up with the following equation to compute the exponent x

:

Suppose we are looking for a value x

such that if we apply $B_i = B_{i-1}(1-\frac{i+4}{2^{x}})$

we obtain that $B_{4725} \approx 0.5B_0$

. In other words we want the balance B

to be halved after 21 days, (4,725

epochs)

$B\prod_{n=4}^{4725} (1-\frac{i}{2^{x}}) = 0.5B$

Simplifying the product,

$\prod_{n=4}^{4725} (1-\frac{i}{2^{x}}) = 1 - [\frac{4725 * 4726}{2}-6]\frac{1}{2^x}+...\approx1 - [\frac{4725 * 4726}{2}-6]\frac{1}{2^x}=1-\frac{11165169}{2^x}$

Equaling to 0.5

and solving for x

we have $x=24.412$

. We can attribute the difference between this result and the ran experiment (23.94128

) to the dismissed terms of the product.