Liquidity Provider Strategies for Uniswap v3: Power Perpetuals

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This article in the

[series

](/liquidity-provider-strategies-for-uniswap-v3-table-of-contents-64725c6c0b10)introduces the concept of power perpetuals, and shows how they can help to hedge your liquidity pool positions.

Motivation

The <u>dynamic hedging</u> article showed how to continuously rebalance the hedge to match the asset composition in the pool. By matching the hedge with the pool's current assets, the market risk is reduced. Depending on the hedging costs, <u>the results can be nicely profitable LPing</u> even in volatile markets.

The trick is that the value and market exposure of such a hedge position must be a nonlinear function of the underlying asset, as the LP position's value is <u>itself non-linear</u>. Dynamic rebalancing is a way to solve it, but the challenge is that someone needs to take care of the rebalancing operations. They can be automated — but do you trust liquidity management automation platforms? A better alternative would be a protocol that takes care of the rebalancing, and just presents you with a dynamic hedge position. Power perpetual protocols is a step in that direction.

Power perpetuals

TradFi has futures

- , which allow to speculate on asset prices without holding the asset itself. (Of course, it's just one way out of many another is options
- , which will be covered in the next article.)

TradFi futures have fixed expiry dates, but crypto markets have an innovation of their own: [perpetual futures

](https://en.wikipedia.org/wiki/Perpetual_futures), usually called simply perps. They never settle; instead, funding cost is paid, on regular basis, to incentivize the price of the futures contract to move closer to the asset price.

Power-perpetuals is a recent idea that moves the idea of perpetual futures one step further. To summarize:

· A regular perpetual tracks the price

of some asset. Leverage allows to set a multiplier to the asset exposure, but does not change the shape of the value exposure function itself: it remains a line.

· A power perpetual

instead tracks the square, cube, or a higher-order

function of some asset's price.

See this article from Paradigm for a more technical discussion.

The connection with hedging LP positions

"But wait a second," you say. "The shape of the square or cube functions does not look like liquidity position's value function at all — it's the <u>square root function</u> for full-range positions, and inverted hockey stick graph for concentrated liquidity positions. How can I get the inverse-to-the-LP payoff from the power perps?" The answer is that hedging LP positions is

similar to cooking. Linear payoffs are like the base layer of a meal — plain potatoes or pasta: necessary but bland. Power perps are like sauce and spices — they turn the whole into a tasty dish, but you would not eat them alone. So you'd start with a relatively large short of the volatile asset. It forms the base of the hedge. Then add a smaller amount of power perps on top of that. The combined payoff function can be made as close to the inverse-to-the-LP payoff as desired by using higher-order perpetuals.

For the meaty math consult the excellent short paper 'Spanning with Power Perpuals' by Joseph Clark. The general idea is to apply Taylor series expansion (a way how to approximate any continuous function with a polynomial) to the LP position's value function, around the pointr = 0,

where r

is the price return.

As a note, if the power perp funding rate is "fair" (the market is efficient), then the funding costs of hedging an LP position should be the same as the LVR of that position.

Example

Alice has a full-range USDC/WETH liquidity position P she want to hedge. She checks the math and finds out she needs to get 50% short exposure to the ETH and 12.5% long exposure to squared ETH (the percentage is computed from V(LP)

, the total value of the capital locked in the LP position). She shorts ETH as usual (either using simple perps, or by using borrowed ETH to LP), then heads to Opyn or another power perp protocol, and buys the required amount of Squeeth. Squeeth ("squared ETH") is a derivative asset to ETH that changes it's price with the square of ETH's price.

If higher-order power perps are available, she might also get 3/48 · V(LP)

long of ETH3 and 15/384 · V(LP)

short of ETH4, and so on.

If Alice wants to match 50:50 HODL portfolio instead, she doesn't short ETH, but directly proceeds to buy the 2nd and higher order power perps (see the figure below).

The figure above shows the how the LP position's payoff function is successively more closer approximated by a series of power perpetuals. The perps still need to be rebalanced occasionally, but unlike the simple linear hedges, the LP does need to do that on every 1% or 5% price move. Even doing it just on 50% or 100% moves may give sufficient hedging accuracy.

Uniswap v3 CL example

Dr. Clark's paper works out the coefficients required to hedge a full range Uniswap position. Here I show to obtain the coefficients for CL positions.

For CL positions the value function should be separated in three parts: in-range, below range and above range. The value functions of out-of-range parts are simple lines. For the in-range part Taylor series expansion is calculated and used. Whenever the position goes out of range, the higher-order power perps should be sold in full, as simple shorting/longing is sufficient to perfectly hedge out-of-range positions.

The value V(P)

of an in-range Concentrated Liquidity position at price P

is given by:

where:

• K

is the strike price (average execution price when crossing the position),

r

is the range width,

Vmax

is the maximum value of the liquidity position (without counting fees), equal to the value of the position at upper price Pb and above.

The LP position's returns when price moves fromP0

to P1

are defined as r^LP =V(P1)/V(P0) — 1

. Via algebraic simplification:

As you can see, the coefficients are going to depend on the price range boundaries. It's too difficult to express this equation as a function of price return and find the Taylor series expansion of that function.

Instead, I solved it numerically, via NumPy's polyfit

function, using a price range endpoints p a

= \$66.7 and p_b

\$200. The resulting coefficients are: -0.615 first order (short to hedge), 0.525 second order (buy to hedge). By comparing these with the full-range results from the table (-0.5 first order, 0.125 second order), we see a much higher amount of the second order perp is required hedge the CL position than the full-range position. For more concentrated liquidity positions even higher amount is required. Narrow range positions might require to spend more in buying the power perp than the value of the position.

The source code of the algorithm is available at: https://github.com/atiselsts/uniswap-lp-articles-code/blob/main/fit power perpetual coefficients.py

Details on hedging with power-perpetuals

Practicalities

There are very few protocols that offer power perpetuals at the moment and from those only Opyn's <u>Squeeth</u> has established itself, and even Squeeth does not have large amounts of liquidity in the market.

The funding rate of power perpetual is typically higher than that of regular perpetuals. The "fair" funding rate from LPs perspective would equal to the LVR rate of the LP's position.

Results

The main benefit of PP, in the context of this article, lies in their ability to replicate any other portfolio. The higher order perpetuals are used, the more accurately they can replicate a portfolio.

For CL position I used the assumption that the power perp is sold as soon as the position goes out of range (which is from \$66 to \$200):

In order to hedge a LP position, the LP should buy power perpetuals that approximate the negative payoff

of the LP's value function. In particular, to deal with the negative gamma of LP positions, LPs should buy power perpetuals with positive gamma.

For instance, to match the value of the 50:50 HODL benchmark as in the figure above, the LP should simply buy Squeeth worth 12.5% of the initial position's value. The square-like payoff of Squeeth cancels out the square root-like payoff of the liquidity position itself. The total payoff approximates a line function again, with close-to-zero gamma.

The following figure shows the idealized scenario with 5% fee income and no hedging costs. The LP is green almost everywhere; only very large price changes (-75%) make the cumulative payoff negative. In the real world, assuming an efficient market, the funding costs will be significant. Before entering any hedged positions, the LP should compare their expected fee income with the expected costs.

Dr. Clark provides a more concrete example, backtested on the real data and using "fair" (mathematically computed / simulated) funding rates. In these conditions, the Uniswap fee yield in the WETH/USDC 0.05% pool is similar to the hedging costs most of the time, but much lower during high-volatility periods

, when most of the fee income is made — see the figure below:

In terms of the "real" funding rate, it has been 0.07% per day for Opyn's Squeeth (see below). This corresponds to APR of 25–26%; however, do note that Squeeth worth just 12.5% of the position's value must be bought, there for the real second-order funding range is just 3–3.5% APR of the position's value.

Caveats

- 1. Power perpetuals require some kind of oracle that tells the protocol the "true" price. Oracle risks are one of main sources of DeFi hacks, and the risks are amplified with the higher order perpetuals. It's also a potential centralization factor, and doesn't vibe well with the current narrative of oracle-less protocols.
- 2. Power perpetuals are at an extremely early tech stage. Their markets are very small and unfortunately even Squeeth seems to be losing popularity:

New protocols entering the power perp space are making a risky bet, but the opportunity is large, too.

References

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Summary

- 2nd order or higher order power-perpetuals allow to hedge for an arbitrary payoff, including Uniswap v2 and v3 position payoffs.
- The hedging is profitable if the fees collected by the LP position are higher than the perp's funding costs.
- The market offering of power perps is very limited, and it's not clear how the situation is going to evolve in the nearterm future.

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