

Tackling the Problem of State Dependent Execution Probability: Empirical Evidence and Order Placement

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July 12, 2023

Abstract

Order placement tactics play a crucial role in high-frequency trading algorithms and their design is based on understanding the dynamics of the order book. Using high quality high-frequency data and survival analysis, we exhibit strong state dependence properties of the fill probability function. We define a set of microstructure features and train a multi-layer perceptron to infer the fill probability function. A weighting method is applied to the loss function such that the model learns from censored data. By comparing numerical results obtained on both digital asset centralized exchanges (CEXs) and stock markets, we are able to analyze dissimilarities between the fill probability of small tick crypto pairs and large tick assets – large, relative to cryptos. The practical use of this model is illustrated with a fixed time horizon execution problem in which both the decision to post a limit order or to immediately execute and the optimal distance of placement are characterized. We discuss the importance of accurately estimating the clean-up cost that occurs in the case of a non-execution and we show it can be well approximated by a smooth function of market features. We finally assess the performance of our model with a backtesting approach that avoids the insertion of hypothetical orders and makes possible to test the order placement algorithm with orders that realistically impact the price formation process.

Keywords: Optimal Execution, Fill Probability, Survival Analysis, Limit Order Book, High Frequency

I Introduction

In continuous double-auction markets, one can choose to add liquidity by posting a limit order or to take liquidity by sending a marketable order. Depending on the agent's utility function and objective, a trading algorithm may favor one order type over the other and adjust the price of the limit order according to some predefined rules. For example, a market making strategy manages the inventory risk and avoids adverse selection by adjusting the posting distances around a fair price. An execution algorithm seeks for a discounted execution price on one side of the book while aiming at a target quantity to execute under a specific time horizon that can be arbitrarily small, *i.e.* of the order of milliseconds to seconds. Although these strategies are of different nature, they share a central source of uncertainty that is the randomness of execution.

The characterization of the execution risk of a limit order is usually made by specifying the probability of execution (or fill probability). The dynamics of

the limit order book (LOB) is well characterized by the fill probability function, provided that it depends on informative market variables such as the bid-ask spread, the volatility, the order flow regime, etc. The order flow is highly sensitive to many market variables and they may be clustered in two classes of features: *snapshot variables* which compose the Markovian part of the model (bid-ask spread for example) and *differential variables* which compose the non-Markovian part of the model (traded volume variation computed over a period or realized volatility for example). A fill probability model should find its predictive power in both classes of variables in order to capture the main explanatory features of the order flow dynamics.

When trading in a limit order book, agents face a *to limit or not* dilemma in which the decision to post a limit order or to immediately execute must be taken. When opting for the placement of a limit order in the book, the non-execution risk combined with price volatility may cause additional costs if the target quantity to trade is not reached at the end of the time horizon. Assuming that the target quantity

must be traded at all costs, a marketable order sent at the end of the period will potentially trade at a worse price than if it had been sent at the beginning due to adverse price movements. The estimation of such a price move in the case of non-execution is capital since it draws the decision boundary of the order placement tactic, and its sensitivity to market variables leads to a complex interaction with the fill probability function in the decision-making process. As an example, one easily intuits that both fill probability and clean-up cost functions increase with the realized volatility which will force the agent to make a trade-off between higher market risk and lower execution risk.

In this paper, we focus on the estimation of both the fill probability and the clean up cost functions for a fixed trading horizon and demonstrate how they can be integrated within an execution algorithm in practice. Numerical experiments are carried on level 3 high frequency data including small tick crypto pairs and large tick – large, relative to cryptos – equities, both with a price-time priority rule. After providing empirical evidence of fill probability state dependence using survival analysis, we show that its non-linear yet regular structure is well adapted to inference by neural network models. We discuss the importance of taking into account the market impact of limit orders insertion and a training procedure with an application of the inverse probability of censoring weighting (IPCW) is presented. Interestingly, the same observation holds about the clean up cost function and we provide a methodology for its estimation using level 3 data. Both models are integrated in a fixed horizon execution framework in which an agent aims at buying a small amount of an asset under a short time horizon, *i.e.* one to several seconds. The small tick order placement policy suggests that posting a limit order in the spread near the opposite best price could be optimal when taker fees are high, which is the case in CEXs for example. A feature importance analysis of the cost function demonstrates the key differences between large tick and extremely small tick assets. Finally, our findings suggest the use of a latency risk model to penalize excessive aggressiveness of the small tick order placement router.

II Literature review

The fill probability function estimation is generally carried out using two main classes of methods. The first class encompasses survival analysis tools and is applied to financial data in [1], [2], that carry out empirical analysis of the role played by multiple key market variables. The second class of estimation procedure encompasses the insertion of hypothetical limit orders and the computation of their first passage time, either using transaction data or the crossing of a reference price. First passage time distributions of non-Gaussian dynamics are widely used in the literature and their empirical scaling properties are carefully studied in [3]. The tail exponents of the first passage

time and empirical time-to-fill and time-to-cancel are analyzed in [4] and the authors suggest that the latter tails observed for the first passage times are explained by cancellations. They also provide a simple model that succeeds in capturing the above stylized facts. A state dependent model of the fill probability function was proposed in [5] for which a recurrent neural network was used to compute the fill probability of synthetic limit orders as a function of market features. In a more recent work [6], the authors develop a deep neural network structure for the estimation of the full survival function. They use the right-censored likelihood as a loss function for training and proper scoring rules for model performance assessment.

In [5], the authors pointed out the fact that using real limit orders for fill probability computation brings a selection bias to the analysis, which finds its nature in the heterogeneity of information contained in the order flow. Since limit orders are posted by both informed and uninformed traders with various strategies and time scales, the analysis of such orders cannot be used for the purpose of fill probability computation of an uninformed agent with a specific horizon. The problem is that in practice, the insertion of a limit order impacts the order book and consequently the price process itself, see [7], [8], [9], [10], [11] for both empirical studies and price impact models. Last but not least, the posted size affects market depth and order flow imbalances and thus the execution probability itself. This assertion is even more true in high frequency perspectives, and the classical first passage time method fails at capturing these stylized facts as the main hypothesis is the absence of impact of posted limit orders. Following the findings of [2] and the above reasoning, we decide to favor market impact adjusted fill probabilities, *i.e.* real order flow fill probabilities over selection bias correction.

Optimal limit order placement tactics are studied in various frameworks [12], [13], [14], [15], [16], [17], [18]. The LOB dynamics are often characterized with an execution intensity that is a decreasing function of the distance. An univariate representation of the execution probability is certainly unrealistic and restrictive, but it still provides enough information to develop optimal order placement algorithms. In [19], the authors build a stochastic control framework and study the impact of adverse selection risk and latency on optimal order placement algorithms. Work has also been done to tackle the queue position valuation problem [20], [21].

The structure of this paper is as follows: Section 3 presents the high-frequency data sets that will be used in the numerical experiments. Section 4 discusses two survival analysis methods to compute fill probability on level 3 data and provides empirical evidence of smooth non-linearity of the fill probability as a function of market features. Once these ingredients are gathered, we present our fill probability model in Section 5 and fi-

nally study an order placement algorithm that incorporates both the fill probability function and a clean up cost model. Results about the optimal placement policy and an execution-specific backtesting approach are displayed before discussing the importance of latency and how it can be integrated in the cost function.

Main Contributions

- ◊ Using tick-by-tick data, we apply survival analysis tools to compute the fill and cancellation probabilities as functions of market features. The use of such data instead of first passage time probability allows us to adjust the probability for market impact. We discuss the difference between fill probabilities used in execution algorithms and the ones used for market simulation.
- ◊ We propose a methodology for fill probability estimation using a neural network model and the inverse-probability-of-censoring weighting to handle canceled orders.
- ◊ We analyze the hierarchical importance of market features and expose the main differences between large tick and small tick assets.
- ◊ We set an optimal execution framework and display feature importance results of the cost evaluation process. We highlight the predictive power of features that are sensitive to manipulation in crypto markets.
- ◊ We demonstrate how to compute an optimal order placement policy and show in the case of cryptos that the taker-maker fee gap forces the algorithm to post limit orders in the spread. We discuss the latency risk in such algorithms.

III Data

Our work uses a high-frequency data set divided in two parts:

- ◊ A digital asset centralized exchange part with tick-by-tick data feed;
- ◊ An equity part with level tick-by-tick data feed.

The digital asset data is provided by SUN ZU Lab's proprietary feed handlers. There are few centralized exchanges who provide a tick-by-tick data API among which Coinbase, Bitstamp and Bitfinex are the most popular. We chose to use Coinbase data for two reasons. Firstly, the tick size remains extremely small during the covered period of time which is not the case for Bitstamp for example where tick sizes have been enlarged in 2022. Secondly, Coinbase provides a timestamp with microsecond precision and a sequence number with each message allowing us to be confident

about the order book reconstruction process. We have 1 month of data at our disposal, from November 5th 2022 to December 5th 2022 on 2 pairs: BTC-USD and ETH-USD.

For the equity part of our analysis, we use the BEDOIH data base (Base Européenne de Données Financières à Haute-fréquence), built by the European Financial Data Institute (EUROFIDAI). This comprehensive data base offers highly detailed order data for all stocks traded on Euronext Paris between 2013 and 2017. To conduct our analysis, we will concentrate on the most recent year available. We analyzed one year of data from January 2017 to December 2017 for two liquid French stocks BNPP and LVMH.

As these data feeds provide raw messages, it is referred to as a level 3 data feed, giving the most details possible regarding each order. We are able to trace the life of each order and identify when it was completed or canceled. Initially, message order is all that makes up the raw data. The order book reconstruction stage is the first crucial part of the data preparation process. The trading day is replayed and each message type is carefully examined to achieve this.

Table 1 summarizes several descriptive statistics of the data we used. Statistics of the spread clearly differentiate small tick assets (BTC-USD, ETH-USD) from large tick assets (BNPP, LVMH) databases. In [22], [23], the differentiation is made for an average spread limit of 1.6 ticks but we still classify BNPP as a large tick asset since the orders of magnitude are far more important for the digital assets.

We gather some descriptive statistics of the lifetime of orders in Table 2.

IV Non-parametric evidence of state dependence

The aim of this Section is to extract some stylized facts about the fill probability function empirically estimated on real order flow. A non-parametric estimation is carried out and two methodologies are provided: one that allows to analyze the cancellation tactics of participants by placing ourselves in a competing risks framework, and another that outputs an exploitable fill probability for execution purposes, considering cancellation as censoring. The former is typically well-adapted to the design of a market simulator while the latter is integrated in high-frequency trading strategies. Some key variables that characterize market regimes are used to emphasize the importance of microstructural features in computing the fill probability of a given order when inserted in the book.

		Spread	Volatility	Trade size	Daily volume	Price amplitude
BTC-USD	5%	84.31	0.11	1.01	197,146,101	0.50
	Median	163.40	7.51	127.86	470,617,696	43.50
	95%	394.25	74.47	8,619.66	1,635,066,155	420.50
	Mean	192.51	18.96	1,725.54	612,757,871	106.29
	St.d.	128.05	39.43	8,425.79	461,655,911	207.71
ETH-USD	5%	8.31	0.00	0.84	221,517,905	0.00
	Median	15.61	14.86	371.70	431,407,671	5.50
	95%	37.34	113.26	7,915.72	1,459,323,150	44.00
	Mean	18.07	31.04	1,836.98	563,289,467	11.92
	St.d.	10.40	57.16	7,633.24	399,987,452	21.28
BNPP	5%	1.00	9.78	20	496,135	0.00
	Median	2.00	19.14	140	194,8473	0.50
	95%	3.00	40.27	386	3,716,829	2.50
	Mean	1.70	21.52	180	2,092,348	0.84
	St.d.	0.66	11.87	328	1,116,719	0.92
LVMH	5%	1.00	6.72	7	99,870	0.00
	Median	1.00	12.51	49	319,678	0.00
	95%	2.00	27.69	147	573,315	1.00
	Mean	1.24	14.38	61	324,954	0.31
	St.d.	0.43	8.38	99	146,173	0.47

Table 1: The units of the descriptive statistics are the following: bid-ask spread is expressed in number of ticks, volatility in percentage of the mid-price, trade size is expressed in quote asset (USD) for both crypto pairs and in number of stocks for the equities, daily volume in quote asset for the cryptos pairs and number of stocks for the equities, and price amplitude in number of ticks. The tick size is 0.01 USD for BTC-USD and ETH-USD pairs and depends on the price value for both BNPP and LVMH with respect to MIFID rules. The trade size metrics are computed on the volume of the recorded market orders and serves as a reference for the average trade size (ATS). The volatility estimator we used is the high-frequency one described in [24], computed over windows of 1 second for cryptos and 10 seconds for equities, and then annualized. The price amplitude is defined as follows: if (p_t) is the mid price process, its amplitude over an interval $[0, h]$ is $\sup_{t \in [0, h]} |p_t - p_0|$. The horizon used for the price amplitude computation is set to 1 second for the crypto pairs and 10 seconds for the equities.

		Lifetime	Time to fill	Time to cancel
BTC-USD	5%	0.003	0.0005	0.003
	Median	0.063	0.125	0.059
	95%	8.499	5.697	7.107
	Mean	152.931	53.672	4.176
ETH-USD	5%	0.003	0.0006	0.003
	Median	0.069	0.205	0.065
	95%	9.035	11.274	7.839
	Mean	117.453	65.914	4.347
BNPP	5%	0.0004	0.0007	0.0004
	Median	1.296	4.069	1.193
	95%	100.08	106.56	96.31
	Mean	48.19	70.73	32.68
LVMH	5%	0.0002	0.0009	0.0002
	Median	4.173	8.375	3.983
	95%	292.92	231.69	281.28
	Mean	85.13	84.91	70.16

Table 2: The time statistics are displayed in seconds and were computed over 493,272,703 orders for the BTC-USD pair, 435,960,156 orders for the ETH-USD pair, 49,654,295 orders for BNPP and 25,517,476 orders for LVMH among which respectively 477,635,085, 423,765,442, 47,341,070 and 24,333,212 were cancelled, 12,185,946, 9,695,863, 2,203,945 and 1,085,395 were filled, residual orders being right-censored.

A Cancellation as a competing risk

In the rest of the paper, we place ourselves in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F} := (\mathcal{F}_t)_t)$. Let the state of a pending limit order be modeled by the \mathbb{F} -adapted process $(X_t)_{t \geq 0}$ with values in a discrete state space \mathbb{S} and such that $X_0 = 0$ a.s.. The order is said to be alive at time t if $X_t = 0$ and dead if $X_t \neq 0$. Hence, the lifetime of this order is fully characterized by the random variable $L := \inf\{t > 0, X_t \neq 0\}$. Naturally, all the states (except for 0) are absorbing since they all signify the death of the underlying order. We now discuss the specification of the state space \mathbb{S} in the continuous trading paradigm. We will also denote, for $t \geq 0$, $F(t) := \mathbb{P}(L \leq t)$ and $S(t) := 1 - F(t)$ respectively the cumulative distribution function and the survival function (or complementary cumulative distribution function) of the order's lifetime L .

This limit order is seen as a birth-death entity for which death – removal from the order book – can be triggered by either its full execution or its cancellation. It is noteworthy that the case of partial execution is not considered here and should be tackled differently since it is not an absorbing state, the order remaining alive in the book. When the death of the order is not observed, we say that the order is right-censored. Censoring is present in both financial markets and crypto CEXs data sets. Another typical example of censored orders is when they are neither cancelled nor fully executed at the end of the observation period. In the case of CEXs, censoring often happens for orders posted far from the mid price. For example, some orders in ETH-USD order books are posted at \$1 and \$10 M whereas the mid price is around \$2000. One can simply fix a threshold δ and only keep the orders whose prices are within the mid price $\pm \delta\%$. From the crypto practitioner's point of view, the main causes of censoring are feed handler disconnections that happen frequently depending on the venue (this also happens on traditional financial markets but to a lesser extent). In the case of a disconnection, data feed is stopped for a random time that ranges from milliseconds to seconds or even hours for API shutdowns. Pending orders that were close to the mid price are thereby subject to censoring since chances are high that their death happened during the disconnection. The censoring issue in the equity dataset occurs at the close of the trading day and is therefore much smaller.

As outlined in previous work [25], the presence of cancellations plays a significant role in the difference observed between empirical first passage time (FPT) and time to fill (TTF) distributions, leading to fatter tails for the FPT. When analyzing lifetimes of real posted orders, cancellation acts as a competing risk with respect to execution rather than as censoring since once the order is cancelled, its future execution cannot be observed anymore.

Based on this, we specify the state space as $\mathbb{S} :=$

$\{0, 1, 2\}$ and define $\mathbb{S}^\dagger := \{1, 2\}$ the death state space, where 1 indicates a fully executed order and 2 indicates a cancelled order. We define the cause-specific hazard rate $\forall t \geq 0, i \in \mathbb{S}^\dagger$:

$$\lambda_i(t) := \lim_{\Delta t \rightarrow 0^+} \frac{\mathbb{P}(L \in [t, t + \Delta t], X_L = i | L \geq t)}{\Delta t}. \quad (1)$$

We define the cumulative incidence function (CIF), $\forall t \geq 0, i \in \mathbb{S}^\dagger$:

$$F_i(t) := \mathbb{P}(L \leq t, X_L = i) \quad (2)$$

$$= \int_0^t \mathbb{P}(L > s) \lambda_i(s) ds, \quad (3)$$

where the last equality is established with elementary calculus using Equation (1).

B Non-parametric estimation

We suppose we either observe the lifetime or the censoring of N limit orders which are exposed to the 2 mutually exclusive causes of death we discussed previously, namely execution and cancellation.

Previous notations are naturally indexed in the following way for convenience. We denote the lifetime of the n -th order by L_n and the distribution function of L_n is denoted by F . We assume the order can be subject to independent right censoring modeled by a random variable C_n and the censoring variable is assumed independent from L_n . Thus, what we effectively observe is the realization of the random variables $T_n := \min(L_n, C_n)$ and $\mathbb{1}_{\{L_n \leq C_n\}}$ and t_n denotes our observation of the censored lifetime T_n .

Let $(t_{(k)})_{k \in \{1, \dots, K\}}$ be the ordered sequence of observed and censored lifetimes such that $\forall n \in \{1, \dots, N\}, t_n \in \{t_{(k)}, k \in \{1, \dots, K\}\}$. In the seminal paper [26], the Kaplan-Meier estimator was introduced as a non-parametric estimator of the survival function in the presence of censored data:

$$\forall t \geq 0, \hat{S}(t) = \prod_{k, t_{(k)} < t} \left(1 - \frac{d_k}{n_k}\right), \quad (4)$$

where d_k is the number of deaths at time $t_{(k)}$ and n_k is the number of pending orders at time $t_{(k)}^-$. Thus, after each $t_{(k)}$, the number of pending orders becomes $n_{k+1} = n_k - (c_k + d_k)$ where c_k is the number of right censored observations occurring at $t_{(k)}$.

A non-parametric estimator of the cumulative incidence function was proposed in [27] and [28]. The Aalen-Johansen estimator writes $\forall t > 0, i \in \mathbb{S}^\dagger$:

$$\hat{F}_i(t) = \sum_{k, t_{(k)} < t} \hat{S}(t_{(k-1)}) \frac{d_k^i}{n_k}, \quad (5)$$

using the convention $t_{(0)} := 0$ and denoting d_k^i the number of deaths from cause i observed at $t_{(k)}$. Naturally, the Kaplan-Meier curve $\widehat{S}(.)$ is computed without distinguishing between the causes of death: $\forall k \in \{1, \dots, K\}$, $d_k = d_k^1 + d_k^2$. The Aalen-Johansen estimator is built by summing products of the Kaplan-Meier survival function and increments of the Nelson-Aalen cumulative hazard rate estimator (see [29], [30] and [31]).

Counting process theory brings mathematical expressions of confidence intervals for the Aalen-Johansen estimator. The procedure used here is based on the Gray's estimator [32] for the CIF's variance, $\forall t > 0$, $i \in \mathbb{S}^\dagger$:

$$\begin{aligned} \widehat{\text{Var}}(\widehat{F}_i(t)) &= \sum_{k, t_{(k)} < t} \frac{(\widehat{F}_i(t) - \widehat{F}_i(t_{(k)}))^2 d_k}{(n_k - 1)(n_k - d_k)} \\ &\quad + \sum_{k, t_{(k)} < t} \frac{\widehat{S}(t_{(k-1)})^2 d_k^i m_k^i}{(n_k - 1)n_k} \\ &\quad - 2 \sum_{k, t_{(k)} < t} \frac{(\widehat{F}_i(t) - \widehat{F}_i(t_{(k)})) \widehat{S}(t_{(k-1)}) d_k^i m_k^i}{(n_k - d_k)(n_k - 1)}. \end{aligned} \quad (6)$$

with the notation: $m_k^i := \frac{n_k - d_k^i}{n_k}$.

This estimator is known to slightly over-estimate the true variance [33]. As suggested in [34], we use the log-log method to compute the confidence interval's boundaries. It restricts the values of the boundaries to $[0, 1]$ as it is not necessarily the case when computing linear confidence intervals. Let α be the confidence level and q_x the Gaussian x -percentile. If we define, $\forall t > 0$, $i \in \mathbb{S}^\dagger$:

$$C_\alpha^i(t) := q_{\frac{\alpha}{2}} \frac{\sqrt{\widehat{\text{Var}}(\widehat{F}_i(t))}}{\widehat{F}_i(t) \log(\widehat{F}_i(t))}, \quad (7)$$

then a α -confidence interval $\text{CI}_\alpha(\widehat{F}_i(t))$ may be approximated using the following formula:

$$\text{CI}_\alpha(\widehat{F}_i(t)) := \left[\widehat{F}_i(t)^{e^{-C_\alpha^i(t)}}, \widehat{F}_i(t)^{e^{C_\alpha^i(t)}} \right]. \quad (8)$$

C Methodology

We describe a method for the computation of state dependent non-parametric execution probabilities. The goal is to estimate a fill probability function which depends on state variables such as liquidity imbalance, bid-ask spread, etc. Let $Z \in \mathbb{R}^d$ be the state variable of interest at the evaluation time. What we call evaluation time is the moment fill probabilities have to be computed: for example, the starting time of an execution schedule.

Conditionally on Z , we estimate the Kaplan-Meier curve $(\widehat{S}^{KM}(t))_t$ using all deaths caused by both risks, i.e. cancellations and executions. Then the CIFs F_1 and F_2 are estimated by combining the Kaplan-Meier curve and the empirical increments of cause-specific cumulative hazard rates.

What we have done so far is formulating a mathematical framework for the computation of the following quantities, conditionally on an observed state $z \in \mathbb{R}^d$:

◊ Empirical fill probability under time t :

$$\widehat{F}_1(t|Z=z) := \mathbb{P}(L \leq t, X_L = 1 | Z=z), \quad (9)$$

◊ Empirical cancellation probability under time t :

$$\widehat{F}_2(t|Z=z) := \mathbb{P}(L \leq t, X_L = 2 | Z=z). \quad (10)$$

The analysis of these two quantities provides insights on agents behaviors and cancellation tactics. This framework provides a way to fully characterize states that guarantee high fill probabilities and identify orders that are likely to be cancelled.

D Fill probability for trading

We now discuss the computation methodology for optimal trading purposes. In a competing risk framework induced by cancellation risk, the fill probability function cannot be used in a trading strategy as it stands. Indeed, the cumulative incidence function is useful to help designing a market simulator for instance but it does not reflect the true probability of an agent that is willing to stay in the book for, at least, the time horizon at which the probability is computed. A data set of orders that would best describe the execution outcomes of a trader who places a Good Til' Cancelled order with horizon 1 second would be exclusively composed of orders that are either filled within 1 second or whose lifetime is greater than 1 second. This leads us to compute a second estimator of the fill probability as follows.

The post-and-wait fill probability under a time horizon t is defined as the fill probability of an order that is not cancelled before t . Therefore, considering cancellations before horizon t as censoring, the Kaplan-Meier estimator is sufficient to compute the post-and-wait fill probability. Keeping the previous notations in mind, the only changes in the computation of \widehat{S} is that d_k is the number of filled orders and c_k is the number of cancelled orders plus right-censored orders at time $t_{(k)}$.

E Numerical experiments

In the following experiments we analyze both passive limit order flow, i.e. limit orders that do not affect the bid-ask spread when posted, and aggressive limit order flow, i.e. limit orders which form a new best queue when posted. In the latter case, marketable limit

orders that are partially executed are not studied and are left for further investigation. Since these orders would cause both best prices to change, they could be treated as a special case of aggressive limit orders. We decide to discard orders that were posted too far in the book and set the threshold at 50% of the mid price. For the equities, we focus on the first 10 limits of the limit order book.

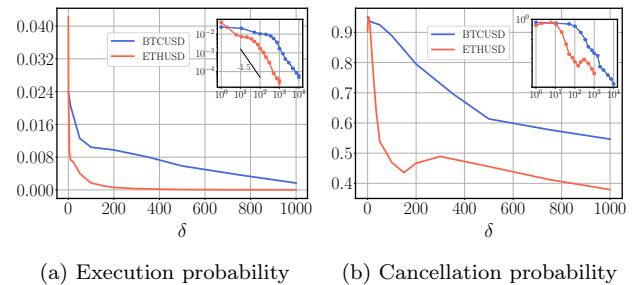
We analyze the sensitivity of the fill probability and cancellation probability functions to some of the most used microstructural variables in the literature. We used the Aalen-Johansen estimator in the competing risk framework developed in C. Other variables such as high-frequency volatility estimators, signed trade order flow, etc. will be considered when designing the fill probability model in Section V. The results are displayed in 1 dimension and 95% confidence intervals are shown. The horizon is set to 1 second for crypto pairs and 10 seconds for equities. For each variable, we either display the results for the small tick or the large tick assets depending on the relevancy of the feature for the illustration (*e.g.* the bid-ask spread and distance plots are more relevant for small tick assets). In appendix, we display additional results in two dimensions.

Our goal is to investigate the influence of microstructure features on the fill probability. Thus, it is crucial to select an appropriate time horizon that strikes a balance between considering microstructure effects and ensuring a substantial percentage of executions relative to cancellations. Indeed, choosing a too short time horizon for the fill probability would lead to extremely imbalanced classes, resulting in high (relative) estimation error. The time horizon depends on the asset considered and especially on the frequency of trades. We observed that a 1 second time horizon for the crypto pairs and a 10 seconds time horizon for the equities gave satisfactory results. Given these time horizons, the percentage of observed executions among posted limit orders is 2% for the crypto pairs and 4% for the equities (see Table 2 for additional descriptive statistics). The same methodology can be used for different time horizons – and other types of horizons such as event time, price move time or transaction time.

- ◊ **Distance from the best quoted price before insertion (δ)** Expressed in terms of ticks, it is the price difference between the price p of the posted bid (respectively ask) limit order at insertion and the best bid p^b (respectively ask p^a) price right before insertion, or in other words, just before its insertion in the LOB:

$$\delta := \begin{cases} p^b - p, & \text{on bid side} \\ p - p^a & \text{on ask side.} \end{cases} \quad (11)$$

The fill probability decreases with the distance parameter δ and asymptotically scales as a power law



(a) Execution probability (b) Cancellation probability

Figure 1: 1 second execution and cancellation probabilities as functions of distance to the best quoted price δ expressed in number of ticks (0.01 USD for both pairs). We observe a power law asymptotic behaviour for the fill probability of both assets with a similar exponent. The cancellation probability function starts to decrease from 10 ticks for ETH-USD and 100 ticks for BTC-USD, indicating the LOB’s depth within which most of high-frequency trading’s activity is concentrated.

function for both crypto pairs. The implication of this observation is two-fold. Firstly, it illustrates the sparsity of limit order books: especially in digital asset venues, a tiny tick size leads to the presence of what we call “gaps of liquidity”. In this case the tick size is so small that market makers simply cannot quote every prices – which is also one of the main sources of short-term volatility as pointed out in [35]. Moreover, crypto venues allow agents to post limit orders of very tiny sizes of 10 USD magnitude, strengthening the LOB’s sparsity. That being said, a bid-ask spread of 5 basis points does not represent the same thing in crypto venues and in traditional financial markets. The same rule applies for distance to the best price at insertion. Secondly, the low decreasing rate of the fill probability as a function of the distance indicates traders can post their order further in the book and still expect to be filled within a small horizon. This stylized fact for small tick crypto assets could lead to interesting consequences on the results of classical optimal trading frameworks such as [17] in which the specification of an exponential decreasing rate prevents the strategy to post orders far in the book.

- ◊ **Bid-ask spread before insertion (ψ)** Expressed in terms of ticks (or bps), it is the price difference between the two quoted best prices, namely the best ask price and the best bid price, just before the insertion of the limit order:

$$\psi := p^a - p^b. \quad (12)$$

It is noteworthy that we consider the bid-ask spread before the insertion of the limit order instead of the bid-ask spread after its insertion. Both quantities are equal in the case of passive order flow. But for the case of aggressive limit orders, one might decide to use the former, the latter or both since they provide different information. We will show later that the aggressiveness of each order can be well

described by a specific aggressiveness index.

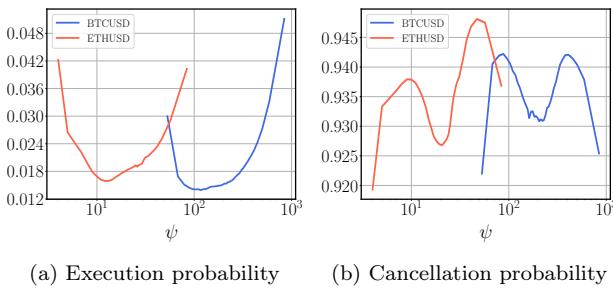


Figure 2: 1 second execution and cancellation probabilities of passive orders as functions of the bid-ask spread ψ expressed in number of ticks (0.01 USD for both pairs). Note the logarithmic scale of the x-axis. We observe a U-shape for both fill probability functions, emphasizing the important activity of liquidity takers in extreme spreads scenarios. Interestingly, the minimum is attained near the median spread. The cancellation probability function exhibits a similar shape but with a decrease for extreme spreads.

The fill probability function is decreasing at low spreads and increasing at large spreads. This U-shape contrasts with some other results in the literature such as [1]. Our interpretation is the following: a high fill probability for small and large spreads is explained by the greater trading activity that is usually observed at these market states. Typically, large spreads indicate both low liquidity and high market activity and such a market regime is of high uncertainty for traders, leading to more liquidity taking. Small spreads represent lower transaction costs for liquidity takers, thereby inciting them to grab the excess of liquidity causing the spread to revert back to its equilibrium. One may notice that the increase in fill probability does not occur only beyond extremely large spreads. For example, in the case of BTC-USD, the change of monotony happens at a bid-ask spread of around 250 ticks and spreads which exceed this value are observed for a substantial amount of event time – see Table 1.

◊ **Best limits liquidity imbalance after insertion ($\mathcal{I}_{\text{best}}$)** It quantifies the liquidity imbalance at the best limits after the insertion of the limit order and indicates if the liquidity at the best queues is concentrated on one side of the order book. Noting Q_{bid} (respectively Q_{ask}) the volume available at the best bid (respectively best ask), we write:

$$\mathcal{I}_{\text{best}} := \frac{Q_{\text{bid}} - Q_{\text{ask}}}{Q_{\text{bid}} + Q_{\text{ask}}}. \quad (13)$$

Since this variable might be sensitive to the insertion of the limit order *i.e.* in the case of a distance $\delta \leq 0$, we only consider the state variable after the insertion. Intuitively, in crypto markets, the presence of small orders and the size of the tick make this measure less informative than in traditional stock markets. However, it is still a key indicator for next

trade sign prediction and plays a non-negligible role in computing the fill probability of an order posted in the current best queue or near.

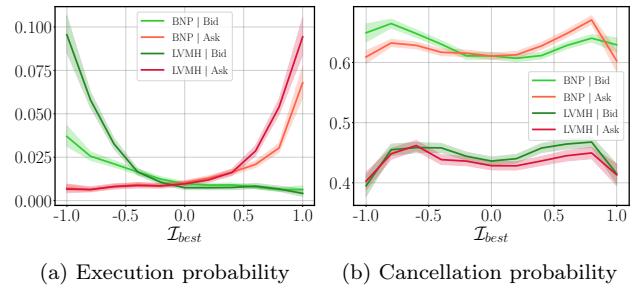


Figure 3: 10 seconds execution and cancellation probabilities as functions of the imbalance at the best limits. We compute separately the probabilities for the orders at the best bid and the best ask.

We observe a symmetric relationship between fill probability at best quotes and imbalance of best queues. For an order placed at the current best bid, the lower the imbalance, the greater the probability of being filled within the time horizon. The importance of liquidity imbalance as a feature for next trade’s side prediction is a well-known phenomenon – see [19]. This stylized fact and our observation are in fact two sides of the same coin. Interestingly, we note that cancellation probabilities is not impacted by the imbalance which can be surprising. As the imbalance is recorded at the insertion of orders, we do not observe any excess of cancellation at extreme imbalances that would be caused by agents to avoid adverse selection. The latter would certainly be obtained if we had looked at pending orders instead of new insertions.

◊ **Limit order flow imbalance after insertion ($\mathcal{I}_{\text{add}}^m$)** It quantifies the volume imbalance of limit orders that were posted in the last m events, right after the current order insertion, and indicates if the new bid/offer intentions are concentrated on one side of the LOB. At the insertion of the order, we sum the volumes inserted on the bid side $\Delta_m Q_{\text{bid}}$ and on the ask side $\Delta_m Q_{\text{ask}}$ on the last m events (including this order). Notice that $\Delta_m Q_{\text{bid}} \geq 0$, $\Delta_m Q_{\text{ask}} \geq 0$ and $\Delta_m Q_{\text{bid}} + \Delta_m Q_{\text{ask}} > 0$ since the current posted limit order is taken into account. We thus define the indicator with the following expression:

$$\mathcal{I}_{\text{add}}^m := \frac{\Delta_m Q_{\text{bid}} - \Delta_m Q_{\text{ask}}}{\Delta_m Q_{\text{bid}} + \Delta_m Q_{\text{ask}}}. \quad (14)$$

To our knowledge, our work is the first to study the influence of this variable in fill probability computation. Its tuning could lead to a relevant indicator when the traded asset is prone to price manipulation strategies involving punctual liquidity bumps such as spoofing. A proper extension of this indicator would be the use of a

weighting function that would give more weights to both relevant limits and more recent observations.

The number of events m is set to 50. The m used here was not chosen in an optimal way and several values must be tested in the context of feature selection.

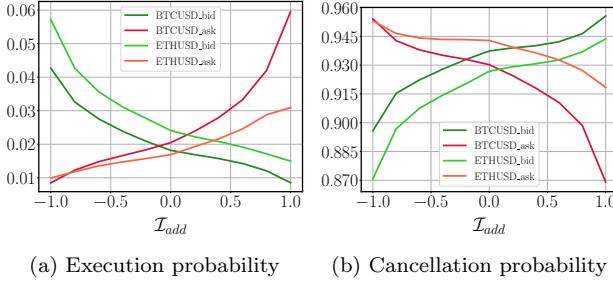


Figure 4: 1 second execution and cancellation probabilities as functions of the limit order flow imbalance I_{add} . The observed shape is of the same kind as the one obtained for the BBO imbalance as far as fill probability is concerned. For the cancellation probability, we observe an inverse relationship.

The fill probability is symmetrically monotonous as a function of the limit order flow imbalance and the smoothness of this relationship is a strong evidence of the effectiveness of price manipulation strategies that involve fake liquidity such as spoofing. By manipulating the order flow imbalance, a manipulative agent can easily expect to multiply the fill probability of a *bona fide* limit order in the opposite side by a factor 5. The inverse relationship observed for the cancellation is quite interesting. We believe this illustrates an immediate rise in aggressive insertion activity following an important market depth variation: if an agent manipulates the liquidity by artificially inflating the bid market depth, this will certainly provoke a rise in buying incentives and thus in aggressive bid limit order posting rate. In this case, agents who previously inserted a bid limit order may cancel their order and post aggressively in order to gain price priority.

◊ **Aggressiveness index (ω)** For the purpose of analyzing the fill probability of aggressive order flow, we define a metric that will quantify the degree of aggressiveness of a new posted order. We therefore place ourselves in the case of a bid-ask spread before insertion that is greater than 1 tick. We include the flow of orders that were posted at the current best price, setting $\delta \leq 0$. In that case, they are passive but can be classified as aggressive orders with a zero aggressiveness index for convenience. We thus define:

$$\omega := \frac{\delta}{1 - \psi}. \quad (15)$$

A 0 index indicates the order was posted at the current best price while a value of 1 indicates a narrowing of the bid-ask spread to its minimal value, *i.e.* 1 tick. We also write this quantity in terms of the new bid-ask spread after insertion $\psi^* \leq \psi$:

$$\omega := \frac{\psi - \psi^*}{\psi - 1}. \quad (16)$$

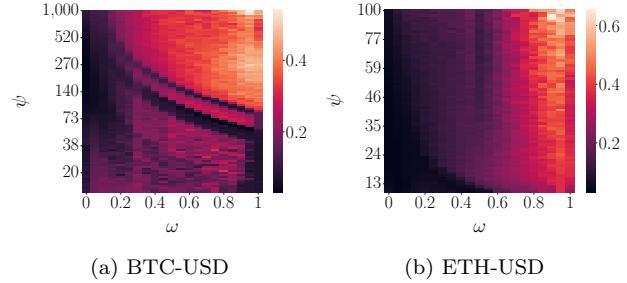


Figure 5: 1 second execution probability as a function of the aggressiveness index ω and the bid-ask spread ψ expressed in number of ticks (0.01 USD for both pairs).

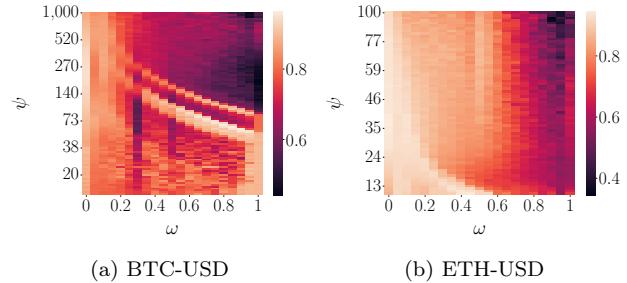


Figure 6: 1 second cancellation probability as a function of the aggressiveness index ω and the bid-ask spread ψ expressed in number of ticks (0.01 USD for both pairs).

When posting an order inside the bid-ask spread (and thus creating a new best queue), traders expect a higher execution probability and thus minimize the risk of non-execution while trading at a better price than if they had sent a market order – without mentioning the saved taker fees. It is especially the case for small tick assets such as a large majority of cryptocurrencies in CEXs. Therefore, traders who want to execute fast with minimum slippage may choose to send aggressive orders, leading to a significant tightening of the bid-ask spread as there might be many of them trading at the same time. As outlined in [25], aggressive orders will cause an instantaneous rise in the intensity of liquidity taking and will rapidly get the same state as pending orders in best queues. We observe a significantly high execution probability of aggressive orders: the greater the aggressiveness index, the higher the fill probability. Moreover, the competing risk framework allows us to extract the probability of cancellation as a function of the aggressiveness index. The more aggressive the order is, the less likely it is to be cancelled: this emphasizes at

least two phenomena. The first one is the propensity of impatient agents to optimize their price priority by placing their order at a better price than the current best limit. By doing so, a cascade effect could cause the successive insertion of multiple fleeting orders as the loss of price priority of aggressive agents forces them to cancel their order and insert it again inside the bid-ask spread and so on. The second one is the pinging activity in crypto venues, where orders are submitted inside the spread and cancelled shortly thereafter.

◊ **Priority volume at the insertion (V_{prior})** The priority volume (or volume prior) quantifies the priority of an order in the order book relative to other orders at the same price level or at “better” prices (from a liquidity taker’s point of view). Orders are generally executed on a first-come, first-served basis, meaning that the orders at the front of the queue are executed before orders behind them. Therefore, orders with a lower queue position are executed before orders with a higher queue position at the same price level. This feature plays a different role whether the underlying asset is a large tick equity or a small tick cryptocurrency.

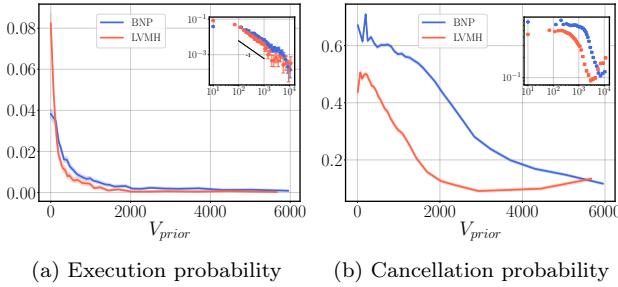


Figure 7: 10 seconds execution and cancellation probabilities as functions of the volume ahead (V_{prior}) of the order. The volume is expressed in terms of the number of stocks. We observe a similar shape for both fill probability functions.

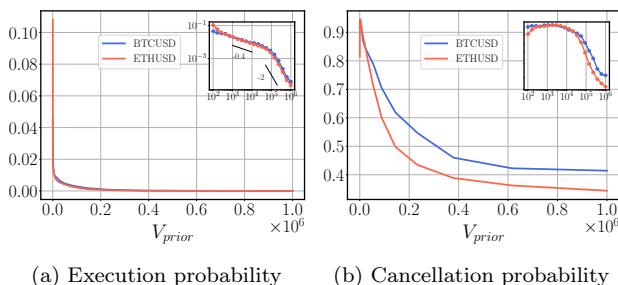


Figure 8: 1 second execution and cancellation probabilities as functions of the volume ahead (V_{prior}) of the order. The volume is expressed in USD. We observe two distinct power law regimes for the fill probability function with a breaking point at 100,000. Both pairs fill probabilities are almost indistinguishable.

A remarkable observation is that the fill probability functions shapes are identical among asset class

(whether we look at the two large tick equities or the 2 small tick crypto pairs). We believe there might be a kind of universal property characterizing the sensitivity of the fill probability function to V_{prior} with respect to the asset class, the tick size and the market activity. The shape of the crypto pairs fill probability function is particularly different than the one of equities. This shows that there is a major difference between both asset classes concerning the microstructural mechanisms that lie behind the execution of orders with respect to the prior pending liquidity.

V A fill probability model for trading

A Motivation

Our goal is to estimate the fill probability associated to an order with a feature vector Z of high dimension and with a fixed time horizon h that we set to 1 second for crypto pairs and 10 seconds for equities. The order is not cancelled within the time horizon h , therefore we discarded limit orders cancelled less than a second after their placement and kept them for the loss weighting procedure that we will describe later. Our procedure differs from [5] in the sense that we do not generate any synthetic order but rather keep the historical order flow. Despite our exposition to selection bias, the benefit of our approach is threefold.

Firstly, as pointed out in [2], fill probabilities of hypothetical limit orders do not lead to accurate estimates of the actual fill probability. As it is an evidence for market orders, posting limit orders also generates market response and price impact – see [4] for a consistent price impact framework. Using a kind of first passage time method would certainly inflate the true fill probability. Things get even worse when posting the order near the mid price since it modifies the liquidity imbalance, a key feature in next trade sign prediction. If computed on “infinitesimal” orders, using such a fill probability model on orders that may inverse the best liquidity imbalance (as it is often the case with small tick assets for example) would lead to biased results. It is thus of paramount importance to take this stylized fact into consideration when designing a fill probability model. The intuitive way of doing this is training the model on a data set of real limit orders. In practice, a trader could use her own trading history in order to take into consideration a cancellation tactics. Secondly, using real-life order flow allows one to build a model for aggressive limit orders, a task that is impossible to carry out when using a first passage time method. Last but not least, the size of the posted order becomes a key feature of the model as it obviously plays a significant role in a high-frequency setting.

B Training procedure

The fill probability estimation is formulated as a binary classification problem with a suitable loss weighting procedure that will be described later. For a data set with matrix representation $\mathbf{Z} := (z_{ij})_{(i,j) \in \{1, \dots, N\} \times \{1, \dots, d\}}$ of N observations (rows) and d features (columns), we denote by $\mathbf{y} := (y_i)_{i \in \{1, \dots, N\}}$ the vector of labels, where $y_i = 0$ indicates that the i -th order was not executed under time horizon T and $y_i = 1$ indicates this order was filled. Since the time horizon of interest is small, we discarded limit orders that were posted too far in the book in order to remove noisy observations. The market depth threshold was fixed at 20 basis points of the mid price for the digital asset data base and 5 price limits for the equity one.

It was shown in Section IV that the fill probability function presents smooth non-linear dependencies with continuous features which makes the problem well-adapted to the training of neural networks with a sigmoid activation function. We tested several architectures that all provided very similar results. For the results displayed in this paper, we used 3 layers of 32 neurons with ReLU activation functions and a sigmoid activation function for the output layer.

In the crypto data set, a large proportion of orders are cancelled under 1 second. We treat cancellations as right-censoring since we are interested in computing fill probabilities for orders that are posted for at least the horizon – see Section IV. The significant presence of right censoring leads us to consider a loss weighting methodology. Simply discarding these orders from the data set would lead to a significant overestimation of the fill probability function. We use the “inverse-probability-of-censoring weighting” (IPCW) method to take care of this issue – see [36] and [37] for the introduction of the method and a comparative analysis. It was shown in [38] that an IPC-weighted version of the estimator of the survival function without censoring is equivalent to the Kaplan-Meier estimator, hence justifying the construction of this methodology.

Denote by w_i the weight associated to observation y_i , C_i the time of censoring which can be either a cancellation or other causes of censoring and E_i the time of execution. The IPC weights are defined as follows:

$$w_i := \frac{\mathbb{1}_{\{\min(E_i, T) < C_i\}}}{\mathbb{P}(C_i > \min(E_i, T))}. \quad (17)$$

The high-frequency activity being mainly near the mid price, we introduce a dependence to the distance of placement of limit orders. Such a modification will increase the weight applied to orders that were posted near best prices and even more for orders which manage to stay in the book until the time horizon T . Concerning orders that are posted inside the bid-ask spread, we propose to condition on the aggressiveness

index as defined in Equation (16) to take into account the high cancellation rate of less aggressive orders. The censoring survival function is computed using the Kaplan-Meier estimator of Equation (4) but in this specific case, considering cancellation and right-censoring as death and execution as right-censoring.

The IPCW procedure will deform the fill probability in a similar way that the Kaplan-Meier function does by giving extra weight to orders that were not executed under the horizon or posted in highly censored configurations.

We add complementary variables to the set of features introduced in Section IV:

- ◊ the size of the order;
- ◊ signed order flow and order flow imbalance using both addition of liquidity (limit order insertion) and removal of liquidity (cancellation of a pending order or transaction), computed over the last 50 events – note that this measure provides a different information than the limit order flow imbalance indicator \mathcal{I}_{add} defined in Section IV;
- ◊ signed traded volume and imbalance of traded volume, computed over the last 50 transactions. We adopt the liquidity taker’s viewpoint: if V_{bid} and V_{ask} are the traded volumes on the bid side and on the ask side over the last 50 transactions, then the signed traded volume is defined as $V_{\text{ask}} - V_{\text{bid}}$ and the traded volume imbalance as $\frac{V_{\text{ask}} - V_{\text{bid}}}{V_{\text{ask}} + V_{\text{bid}}}$;
- ◊ time elapsed since the last trade occurrence and the median duration of the last 50 trades, which could be characterized by the intensity of some self-exciting point process;
- ◊ a high-frequency volatility estimator based on the uncertainty zone model [24] and computed on traded prices of a moving window of 50 trades.

We focus on the bid side of BTC-USD and BNPP and analyze feature importance by the use of Shapley metrics, which are computed with the SHAP library [39]. For the sake of illustration, we train the crypto model on 6 days – 2022/11/05 to 2022/11/10 – and validate it on 1 day – 2022/11/11. All the crypto pairs we tested provided similar results which emphasizes the universality of the considered variables. For the equities, we train the model on 8 months – 2017/01/01 to 2017/08/31 – and validate it on 1.5 months – 2017/09/01 to 2017/10/15. Note that the huge amount of events in the crypto data allows us to consider a smaller time frame for training. The Shapley values are computed over out-of-sample observations.

We separate the results in three parts and investigate how the predictive power of features changes from one type of order to another type.

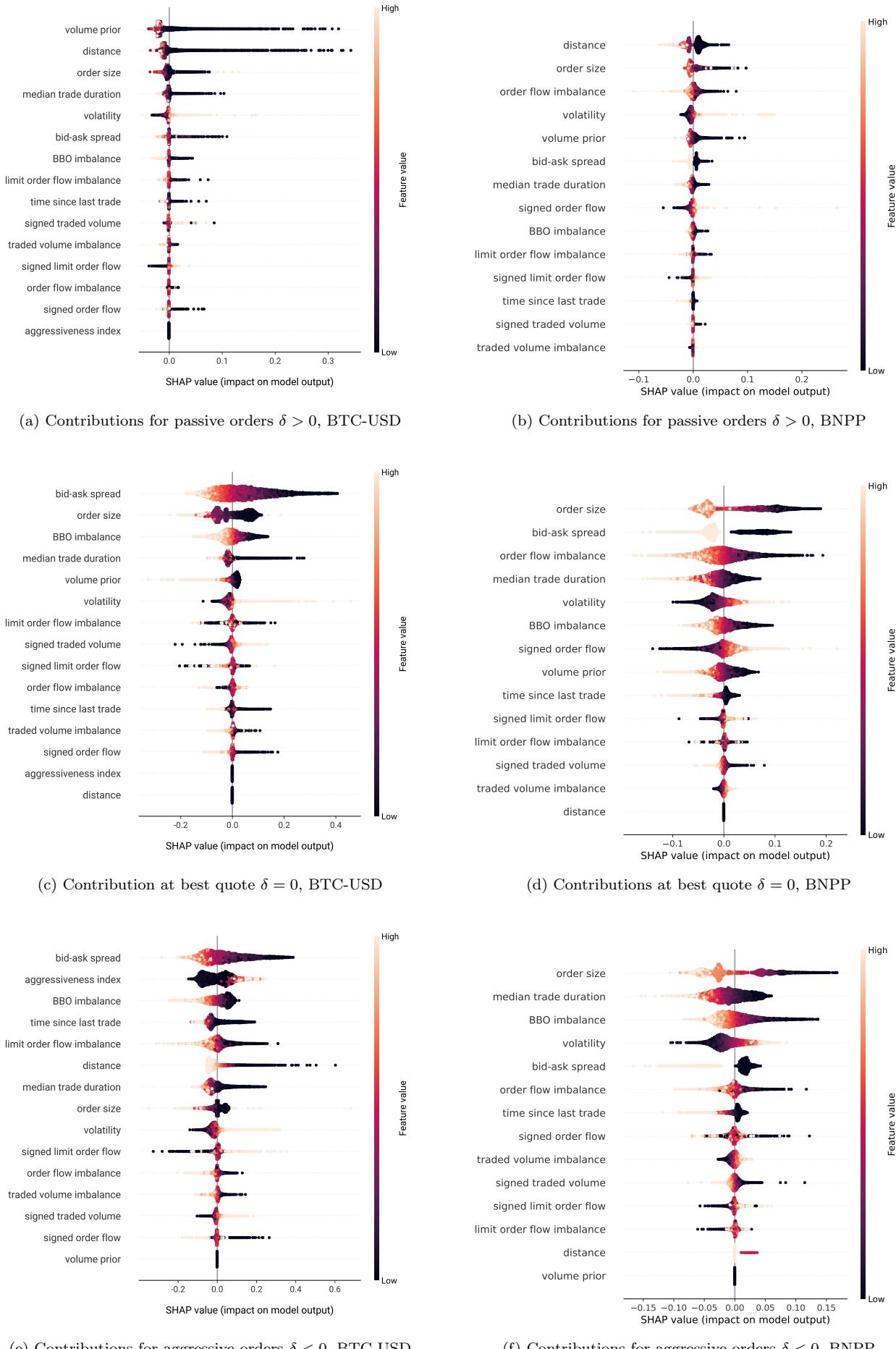


Figure 9: Market variables contributions to the fill probability magnitude using Shapley values of 10,000 predictions, BTC-USD and BNPP, bid side

- ◊ **Passively posting ($\delta > 0$):** The distance and the order size are amongst the three most important features for the BTC-USD and the BNPP. Interestingly the most important feature for the crypto pair is V_{prior} . Our understanding is that the distance in itself is not sufficient for an accurate fill probability estimation because the order book of the BTC-USD is usually sparse. For equal values of prior volume, different distances mean a different level of sparsity of the order book. If a small volume is quoted under a high distance δ , other market participants are likely to quote new prices at smaller distances than δ under the time horizon. Based on this thought, we believe both variables are inseparable when it comes to fill probability computation for small tick assets. The prior volume is also important for the equity but it comes after the order flow imbalance and the volatility.
- ◊ **Posting at the current best ($\delta = 0$):** When posting at the current best queue, the bid-ask spread and the order size seem to be the most important features for both BTC-USD and BNPP fill probabilities. Interestingly, the importance of the BBO imbalance comes after the order flow imbalance for BNPP which states the importance of the dynamic features allowing the model to capture changes in the order flow.
- ◊ **Aggressively posting ($-\psi < \delta < 0$):** When a new best queue is created, the bid-ask spread and the aggressiveness index are the most important features for the BTC-USD pair whereas for BNPP, the order size and the median trade duration are the most predictive ones. The difference between both assets is largely explained by the large spreads observed on the order books of cryptos. This offers the possibility to bump the fill probability of a newly inserted order by simply posting it at a more aggressive distance (*i.e.* a higher aggressiveness index). On the contrary, the order size and the trade intensity are the most important features for a large tick asset, together with the BBO imbalance after insertion. The limit order flow imbalance brings also an important contribution for the crypto pair, emphasizing the effectiveness of spoofing strategies.

VI Optimal order placement

A Formalisation of the problem

We consider an agent who aims at buying a small quantity q of an asset at time 0 under a fixed horizon T . Denote by $(p_t^b)_t$, $(p_t^a)_t$, $(p_t)_t$, $(\psi_t)_t$ respectively the best bid price, the best ask price, the mid price and the bid-ask spread processes of the asset. The asset tick size expressed in quote units is denoted by α and we denote the best price variation over $[0, t]$ by $(\Delta p_t^\bullet := p_t^\bullet - p_0^\bullet)_t$ for $\bullet \in \{b, a\}$. The agent observes a market state vector $z \in \mathbb{R}^d$ as previously defined in Subsection C. We place ourselves in the

Implementation Shortfall optimization framework: the reference price of the execution algorithm is the initial mid price p_0 and the execution schedule must minimize the expected difference between the execution price (including fees) and this reference price. We denote by ε^- and ε^+ the taker and maker transaction fees respectively, such that $\varepsilon^- > \varepsilon^+$. We also define the fee factors $f^- := 1 + \varepsilon^-$ and $f^+ := 1 + \varepsilon^+$. Note that in digital asset centralized exchanges, applied fees vary as a decreasing function of the traded volume which is often computed over a 30 days rolling window.

The agent has access to a limit order book and thus chooses between the two following tactics at the initial time 0:

- ◊ **Immediate execution tactic:** The agent crosses the bid-ask spread by sending a marketable order to get immediate execution at price p_0^a . The cost of this tactic is deterministic and will be denoted by \mathcal{M} . We suppose q is sufficiently small such that the corresponding execution price is the best ask price p_0^a – there is no immediate market impact.
- ◊ **Post and wait tactic (PW):** The agent posts a buy limit order at bid price $p_0^b - \alpha \delta$ and waits for its execution until the time horizon T . The parameter δ is the distance to the best bid price as a number of ticks and is negative in the case of aggressively posting. The lifetime of this order will be denoted by $L^{\delta, q}$ following the conventions introduced in Section IV. Note that it is indexed by δ and q since it is associated to an order of size q placed at a distance δ to the best bid. We will denote by $F_T^{\delta, q, z} := \mathbb{P}(L^{\delta, q} \leq T | Z = z)$ its fill probability within time horizon T conditionally on a market state z . The cost function of the post and wait tactic is random and will be denoted by $\mathcal{W}(T, \delta, q)$.

As full execution is not guaranteed in the PW case, the agent will send a marketable order for immediate execution at the end of the period if the limit order does not get filled, which will incur additional transaction costs in the case of adverse price moves. Both the fill probability and the clean-up cost increase with the volatility: the agent needs to find a trade-off between certainty of execution and management of the clean-up cost induced by market risk.

In the rest of the paper, we will remove the size factor q from the cost functions for clarity purposes and $\mathbb{E}_z[\cdot]$ will correspond to the conditional expectation $\mathbb{E}[\cdot | Z = z]$.

The expected execution costs of the two tactics are written as

$$\mathbb{E}_z[\mathcal{M}] = \mathcal{M} = f^- p_0^a - p_0, \quad (18)$$

and

$$\begin{aligned} \mathbb{E}_z [\mathcal{W}(T, \delta, q)] &= F_T^{\delta, q, z} (f^+(p_0^b - \alpha \delta) - p_0) \\ &+ (1 - F_T^{\delta, q, z}) \mathbb{E}_z [(f^- p_T^a - p_0) | L^{\delta, q} > T]. \end{aligned} \quad (19)$$

Note that the case of a partial execution is not taken into account since it is not likely to occur with a small order size. Indeed, we empirically checked that a negligible proportion of orders were partially executed with a significant fill ratio under the time horizon T . To support this hypothesis, we plot the fill ratio \mathcal{R} inverse cumulative distribution functions for reasonably “small” orders (0 to 10 times the average market order size) and “large” orders (more than 10 times the average market order size).

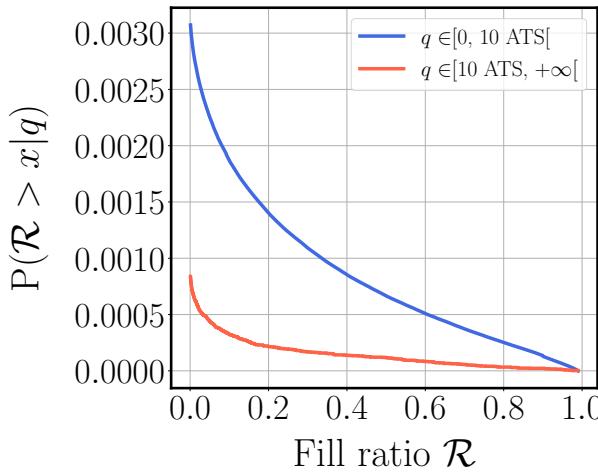


Figure 10: Inverse cumulative distribution function $x \mapsto \mathbb{P}(\mathcal{R} > x)$ of both partially executed and unfilled orders of BTC-USD inserted within a 20 basis points market depth, for a time horizon of 1 second. A very small proportion of orders - around 0.3% - was partially executed. By comparison, more than 2.5% of the orders was fully executed under the time horizon.

The expected saved cost function \mathcal{S} quantifies the expected cost reduction if the agent chooses the post and wait tactics and is defined as

$$\mathcal{S}(T, \delta, q, z) = \mathbb{E}_z [\mathcal{M} - \mathcal{W}(T, \delta, q)]. \quad (20)$$

Using elementary calculus, we write

$$\begin{aligned} \mathcal{S}(T, \delta, q, z) &= \underbrace{F_T^{\delta, q, z} (f^- p_0^a - f^+(p_0^b - \alpha \delta))}_{(a)} \\ &- \underbrace{(1 - F_T^{\delta, q, z}) f^- \mathcal{V}^{T, \delta, q, z}}_{(b)}, \end{aligned} \quad (21)$$

for which we introduce

$$\mathcal{V}^{T, \delta, q, z} := \mathbb{E}_z [\Delta p_T^a | L^{\delta, q} > T] \quad (22)$$

the expected best ask queue’s price variation at the end of the horizon T conditionally on market state z

and a non-execution of the pending order.

The right estimation of this quantity will be a crucial step of the algorithm calibration since it will drive the decision making process.

To better understand the role played by all variables, we decomposed Equation (21) into two parts:

- ◊ (a) If the limit order gets fully executed, the agent saves the spread between the initial net of fees best ask price at which an immediate liquidity taking would have occurred and the net of fees bid price of the filled order.
- ◊ (b) In the case of a non execution, the agent will incur a transaction cost that may be greater than if an immediate execution had been chosen at the beginning because of market risk. This clean-up cost is unknown at the beginning of the period and is characterized by the function \mathcal{V} . We will show that it has significant sensitivities with respect to many variables such as realized volatility and thus should not be chosen constant. To give additional intuition about the behaviour of this function, we expect it to behave as a non-increasing function of the total volume pending at better prices than the price of the posted order. Indeed, if the order is posted at the current best queue and does not get filled under the time period, it indicates that the market may have moved in the other direction thus inducing additional costs. This sensitivity is even stronger with aggressiveness. If the order is posted far from the best price, a non-execution does not necessarily indicate an adverse move of the opposite best price.

Under suitable regularity conditions, the function \mathcal{S} can then be maximized over the set of admissible distances $\mathcal{A}_{\psi_0} := \{\delta \in \mathbb{Z}, \delta > -\psi_0\}$ to find the optimal order placement strategy for fixed T and q :

$$\delta^* = \operatorname{Argmin}_{\delta \in \mathcal{A}_{\psi_0}} -\mathcal{S}(T, \delta, q, z). \quad (23)$$

The unicity of the maximum is a challenge itself since we are dealing with highly non-linear dependencies. One idea is to create a multiplicative model in which the distance component of both fill probability and clean-up cost functions would be a parametric decreasing function. The problem of such approach is that it totally discards the dependencies between the distance parameter and the others such as the liquidity imbalance. We therefore put that question aside for future investigation.

a A toy model

Before diving into the details of the estimation of \mathcal{V} , let us introduce a simplified version of the order placement model with an exponential fill probability function as specified in [17], [13]. For the sake of clarity, we set $f^- = f^+ = 1$ and we consider the modified distance

$\delta^a := \psi_0 + \delta$ that is the distance from the best ask price in ticks. We set \mathcal{V} constant and for all $\delta^a \geq 1$:

$$F^\delta = Ae^{-k\delta^a}. \quad (24)$$

The expected saved cost function writes:

$$\mathcal{S}(\delta) = Ae^{-k\delta^a}\delta^a - \left(1 - Ae^{-k\delta^a}\right)\mathcal{V}. \quad (25)$$

Setting the condition $k(1 + \mathcal{V}) \leq 1$ and differentiating with respect to δ^a , we obtain the following optimal distance of placement:

$$\delta^{a,*} := \frac{1}{k} - \mathcal{V} \quad (26)$$

and the associated maximum of the saved cost function:

$$\mathcal{S}(\delta^{a,*}) = \frac{A}{k}e^{k\mathcal{V}-1} - \mathcal{V}. \quad (27)$$

Our work will naturally compare the performance of the full model introduced in Section V to this simple strategy.

b The effect of CEXs fee policy on the strategy

When trading cryptocurrency, agents may face very different transaction fees depending on their monthly turnover. We build an example to illustrate the strong sensitivity of the strategy to the fee policy.

We consider an agent who posts a buy limit order in the book at the current best bid price $p_0^b = 19,999.50$ USD and a bid ask spread of 1.00 USD. Suppose that the model predicts a clean-up cost $\mathcal{V} = 2.00$ USD (which would correspond to an annualized volatility of approximately 56%). Following Coinbase's fee scaling presented in Table 3, we display the decision map as a function of fee level and execution probability of the posted order.

Level	Monthly turnover	ε^-	ε^+
1	[\$0, \$10K)	0.006	0.004
2	[\$10K, \$50K)	0.004	0.0025
3	[\$50K, \$100K)	0.0025	0.0015
4	[\$100K, \$1M)	0.002	0.001
5	[\$1M, \$15M)	0.0018	0.0008
6	[\$15M, \$75M)	0.0016	0.0006
7	[\$75M, \$250M)	0.0012	0.0003
8	[\$250M, \$400M)	0.0008	0
9	[\$400M, ∞)	0.0005	0

Table 3: Coinbase spot trading fee policy on April 2023

c Estimation of the expected clean-up cost function

We propose a methodology to estimate the clean-up cost function \mathcal{V} . Our approach is easy to implement and can be extended by using features representing cross-venue and cross-asset interactions.

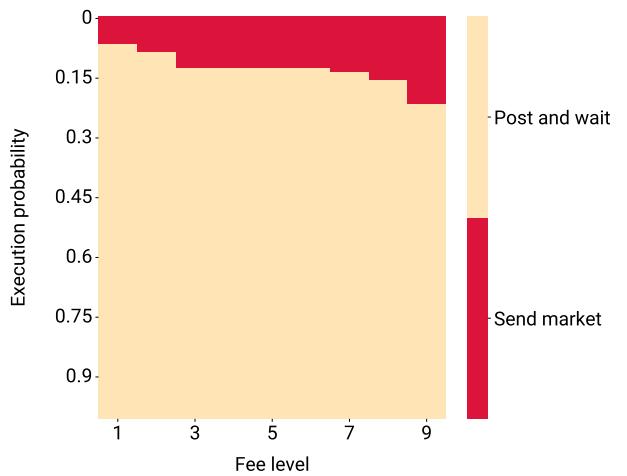


Figure 11: Decision map in the practical example as a function of fee level and execution probability - the fee scaling rule is displayed in Table 3

Using the level 3 data we track the best ask price dynamics after each order's insertion and record its variation over the time window $[0, T]$ when the order is not executed. We remove the orders that were cancelled, executed or censored before T since our only interest is in the events $\{L^{\delta,q} > T\}$. By doing so, we are able to build feature buckets and compute the average price move per bucket. An example of the shape for the non-parametric estimator of \mathcal{V} with respect to realized volatility σ is presented below. We compute this estimator by taking the empirical conditional average of observed best ask price variations following the insertion of limit orders who survived at least T .

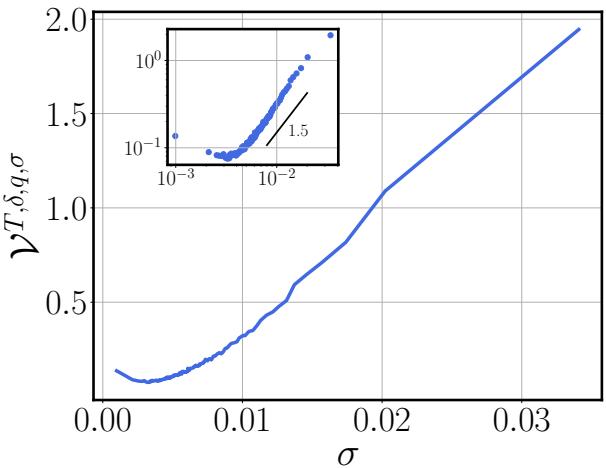


Figure 12: Empirical estimate of \mathcal{V} as a function of realized high-frequency volatility σ (expressed in %/trade), BTC-USD

The sensitivity of the function \mathcal{V} is as smooth and non-linear as the fill probability function. Thus, its inference is also well-adapted to neural network models and we use again a simple architecture to fulfill this task.

Combining this model with the fill probability one, the saved cost function \mathcal{S} can thus be estimated. We provide feature importance materials in appendix in which the sensitivity of the saved cost function to market variables as well as the implication of each feature in the model's output are displayed. In addition to the key features that we described for the fill probability function, the limit order flow imbalance turns out to play a major role for cryptos in the estimation of the saved cost function in the three considered configurations: passively posting, posting at the current best queue and aggressively posting.

B Backtest of the order placement router

We now provide material to show the relevance of our tactical execution strategy. Historical backtests are often misleading for many practical reasons such as the absence of market impact and order flow excitation that would be undoubtedly caused by the algorithm on a real market. For example, the insertion of a new limit order in the first queue would adversely modify the best queues imbalance and negatively impact the strategy execution outcome. Considering infinitesimal sizes for orders does not help that much considering it will not be the case once the algorithm is sent to production. Such approximations inflate the true out of sample performance and thus should be discarded, at least in a high frequency setting. Nevertheless, it still provides insights into the behaviour of the execution algorithm and we do not argue that the use of such backtesting approach has to be avoided at all cost.

◊ **Methodology:** We select real limit orders that were posted in the book and compute their associated saved costs \mathcal{S} . Each order should either be filled under the horizon T or left in the book at least for a time T . Moreover, the size of the order should neither be too small - it is sometimes the case in CEXs as the minimum size is much smaller than in stock markets - nor too large (we set the maximum threshold at 5 times the average trade size). Finally, the distance should not be too large since orders posted far in the book are not likely to be sent by high-frequency traders. This last point is not always true as in CEXs some manipulative large orders are sent far in the order book by high frequency traders, *e.g.* spoofers.

For each limit order in this test set, the sign of \mathcal{S} will indicate whether the algorithm would have actually inserted the limit order (positive \mathcal{S} , predicted 1) or opted for immediate execution instead (negative \mathcal{S} , predicted 0). Then the outcome at the time horizon T is observed: the limit order is either executed or not executed with a negative best ask price variation Δp_T^a (labeled 1), or the limit order is not executed under the time horizon and the best ask

price variation Δp_T^a is positive (labeled 0). This labelling procedure enables us to deduce binary classification metrics to assess the performance of the model in making the right decision. It is important to note that by proceeding so, we are able to test the “to limit or not” decision-making algorithm but not the effectiveness of the optimal distance of insertion. Nonetheless, we provide material to illustrate the sensitivity of the optimal distance with respect to market variables.

◊ **Training and testing sets:** For the CEXs data base, both fill probability and clean-up cost models are trained and validated on a full week of data and tested on the following week which represent 3 test periods over the month: 2023/11/12 to 2023/11/18, 2023/11/19 to 2023/11/25 and 2023/11/26 to 2023/12/02. The equity model is trained on 8 months - 2017/01/01 to 2017/08/31 -, validated on 1.5 months - 2017/09/01 to 2017/10/15 and tested on 2.5 months - 2017/10/16 to 2017/12/31.

We now describe the three models that will be tested and compared below. The chosen fee policy for the crypto trading algorithm is the level 9, *i.e.* $\varepsilon^- = 0.0005$, $\varepsilon^+ = 0$.

- ◊ **Model I: Exponential fill probability and constant expected clean up cost** The model introduced in [a](#) provides an expression of the saved cost for any distance of placement δ , see [Equation \(25\)](#). The fill probability parameters A and k are estimated by fitting an exponential form on the Kaplan-Meier function considering cancellation as right-censoring and \mathcal{V} is computed as the average of best ask price moves observed at horizon for every limit orders in the training set that are not executed.
- ◊ **Model II: Constant expected clean-up cost** The fill probability function used for this benchmark is the neural network model described in [V](#) and \mathcal{V} is set constant, computed as in the exponential toy model benchmark. Thus, the only difference with the full model tested in this section resides in the structure of the function \mathcal{V} .
- ◊ **Model III: Full state dependent** Combination of the two state dependent models for the fill probability and best price move at horizon.

The results from the three different models demonstrate the importance of each component in the execution algorithm. Model II enables us to evaluate the amelioration of the model with the introduction of state dependence and the model III enables us to observe the importance of using a state dependent clean-up cost instead of a constant one. The results stated in [Tables 4](#) and [5](#) clearly show the improvement of the decision making process for both assets. We note that the F-score in the BNPP backtest is slightly decreasing with the model II and model III because the model I tends to favor the immediate execution in

Action	Precision	Recall	F-score
Limit	I	0.21	0.99
	II	0.27	0.66
	III	0.37	0.81
Market	I	0.99	0.27
	II	0.91	0.65
	III	0.95	0.73

Table 4: Performance metrics of the backtest of three order placement systems, BTC-USD

Action	Precision	Recall	F-score
Limit	I	0.11	0.05
	II	0.10	0.21
	III	0.20	0.54
Market	I	0.91	0.96
	II	0.91	0.81
	III	0.94	0.79

Table 5: Performance metrics of the backtest of three order placement systems, BNPP

most of the cases. However, the model is clearly more selective in the decision to insert a limit order or not.

Concerning the optimal placement policy for cryptos, we illustrate our results with two cases. The first one is an example of the optimal distance shape as a function of the bid-ask spread in the case of fees at level 9. The second one is an example in the case of zero taker and maker fees. Our observation is the following. Firstly, the optimal distance of placement given by model III seems to be linear or at least sub-linear in the bid-ask spread. Secondly, the algorithm tends to post aggressive orders when the taker fee - maker fee gap $\varepsilon^- - \varepsilon^+$ is strictly positive and the optimal distance decreases as this gap increases. In other words, the strategy considers a distance to be optimal as long as the taker fee is almost surely saved, regardless of the price discount reduction incurred by posting far inside the bid-ask spread. This result is particularly interesting as it demonstrates that CEXs fee policies tend to push agents to be more aggressive when it comes to executing fast. We believe this could help the understanding of the impact of the fee policy on trading activity and agents behaviours in digital asset venues.

C Penalizing aggressiveness with latency risk: a practitioner viewpoint

Would we trade this strategy? We observed the resulting optimal order placement tactics often post limit orders near the best opposite price such that we are almost certain to save the high taker fees of CEXs. While posting a limit order in the spread guarantees a higher execution probability, this action brings a risk to the table: the aggressiveness of other agents or, in

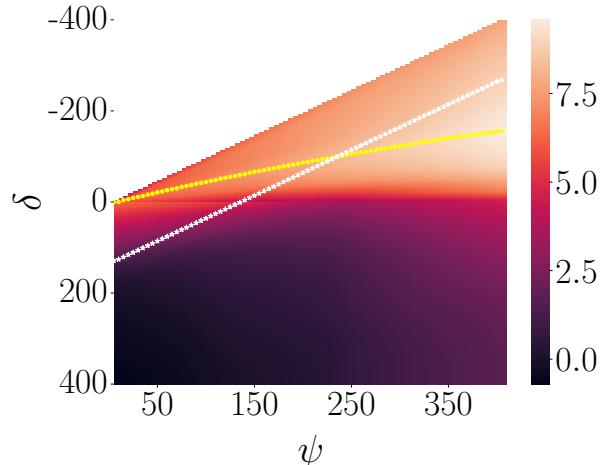


Figure 13: Example of the optimal distance policy δ^* as a function of the bid-ask spread ψ , BTC-USD setting fees to the level 9 of Table 3. The heatmap represents the values taken by the saved cost function. The optimal distance δ^* of model I given by Equation (26) is represented by a white \star and the optimal distance from model III is represented by a yellow \circ . The algorithm tends to be aggressive even if the associated price discount is low, in order to maximize the fill probability and save the taker fees.

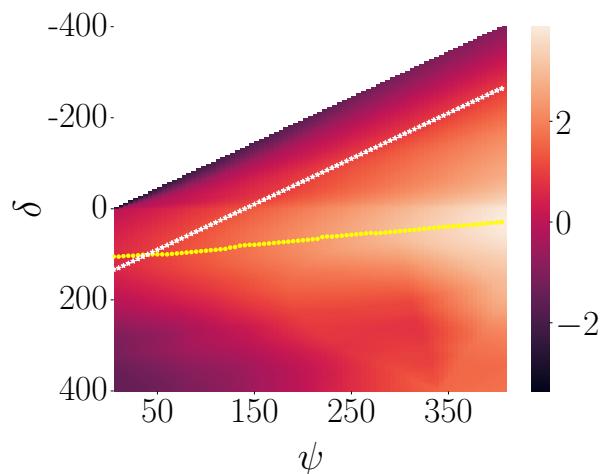


Figure 14: Example of the optimal distance policy δ^* as a function of the bid-ask spread ψ , BTC-USD, setting fees to zero. The heatmap represents the values taken by the saved cost function. The optimal distance δ^* of model I given by Equation (26) is represented by a white \star and the optimal distance from model III is represented by a yellow \circ . The algorithm generally places the order deeper in the book than in the case of non-zero taker fees.

other words, the instantaneous volatility of the best opposite price. Let us go back to the framework in which an agent is willing to buy a certain amount of a small tick asset such as BTC-USD on Coinbase. If the algorithm quotes a bid price inside the spread, there is no guarantee that between the moment the update message is sent by the exchange and the time the optimal distance is computed and the resulting order is sent, other agents have not quoted a new ask price

inside the spread too. In such cases, crossing this new best ask price would generate additional trading costs since the limit order would turn into a marketable order and incur taker fees - remind that the taker fee in Coinbase is 5 basis points whereas the maker fee is 0 basis point, provided that the monthly traded volume is sufficiently high. This latency-induced risk should be integrated in the cost function of the trading algorithm to penalize extreme aggressiveness.

To provide food for thought about modeling such a risk in our framework, let us denote by $\ell > 0$ the latency of the algorithm, *i.e.* the time in seconds that separates the moment the message is sent by the venue, processed by the matching engine and the time the order resulting from the execution tactics is posted in the LOB. We suppose in the rest of the discussion that ℓ is negligible compared to the time horizon T , for example $\frac{\ell}{T} < 10^{-2}$. This condition is not only necessary because the performance of high-frequency strategies vanishes with a high latency, but is also arranging because it allows us to assert:

$$\Delta p_\ell^a \perp\!\!\!\perp L^{\delta,q} \quad (28)$$

$$\Delta p_\ell^a \perp\!\!\!\perp \Delta p_T^a, \quad (29)$$

where $\square \perp\!\!\!\perp \triangle$ stands for “ \square and \triangle are independent random variables”.

We now denote by $\phi_\ell^z(x) := \mathbb{P}(\Delta p_\ell^a \leq x | Z = z)$ for a real number x the cumulative distribution function of the best ask price variation over $[0, \ell]$ conditionally on a market state z . The latency-sensitive saved cost function \mathcal{S}_ℓ of the post and wait tactics now writes:

$$\begin{aligned} \mathcal{S}_\ell(T, \delta, q, z) &= (1 - \phi_\ell^z(-(\psi_0 + \delta))) \mathcal{S}(T, \delta, q, z) \\ &- \phi_\ell^z(-(\psi_0 + \delta)) f^- \mathbb{E}_z [\Delta p_\ell^a | \Delta p_\ell^a \leq -(\psi_0 + \delta)], \end{aligned} \quad (30)$$

with \mathcal{S} being given in Equation (21).

The last term stands for the price discount of the marketable limit order over the immediate execution tactics due to a best ask price improvement. We clearly see that the value of this saved cost function is not necessarily smaller than the latency-free one \mathcal{S} but the maximum of the function should be attained at a different distance δ . Further research could study the impact of such an extension on the optimal placement policy δ^* which will depend on the chosen model for the probability distribution of Δp_ℓ^a .

VII Discussion and conclusion

In this work we explored the complex yet smooth dependence of the fill probability function on market variables whether it is a *Markovian* structure such as the dependence on the bid-ask spread or a *non-Markovian* one such as the dependence on volatility and past order flow. In the case of cryptos, we studied

the sensitivity of the function to limit order flow imbalance \mathcal{I}_{add} showing how adding substantial amounts of liquidity in one side of the LOB may significantly increase the fill probability of orders posted on the opposite side. This observation tends to explain the staggering spoofing activity in crypto venues and may open the door to new methods applied to market manipulation detection, involving the computation of state dependent fill probabilities. Moreover, distinguishing between small tick digital assets and large tick equities, we discussed the importance of correctly encoding the order’s position in the LOB. The priority volume and the distance from the best price are both extremely important in the case of small tick cryptocurrencies due to the sparse nature of the order book, whereas the distance seems to explain most of the variance of the fill probability function in the case of large tick equities. Using survival analysis tools, we discussed about the importance of choosing the right methodology for fill probability computation. More precisely, we argued that one must use a competing risk framework when building a market simulator since each time a new order is added to the book, its cancellation does not act as a censoring but rather as a competing risk. Thus, the exploitation of canceled orders in the fill probability computation will differ depending on whether we model the birth-death processes of order flow or we infer the execution risk for the optimization of a trading system.

We showed how, for a fixed time horizon, smooth and non-linear models such as neural networks successfully apply to fill probability inference using high-frequency level 3 data. We discussed the main advantage in exploiting such informative data for this task and strongly suggested to favor the real order flow other synthetic limit orders to eliminate the hypothesis of the absence of market impact. By designing a saved cost function for an agent who aims at buying a quantity of the asset with short time horizon, we demonstrated how to integrate such a model in a high-frequency execution algorithm and showed the empirical clean up cost exhibits a strong dependence structure that can be well inferred by standard non-linear regression models. Its interaction with the fill probability function in the cost of the agent brings an additional complexity that may have important implications for the calculation of the optimal distance. As such, it forces the agent to make a trade off between smaller execution uncertainty and higher market risk especially in high volatility regimes.

We assessed the model’s performance and compared it to benchmark models involving an exponential fill probability or a constant clean-up cost by exploiting the real past limit order flow. By gathering the decisions the models would have made at the insertion of limit orders, classification metrics can be used to study the relevancy of these execution models in the decision-making process. Examples of optimal distance of placement were provided in the case of

small tick cryptos and numerical experiments suggest that the fee policy of the trading venue plays a decisive role. Our findings suggest that in cryptocurrency venues, provided that the latency of the algorithm is zero, posting extremely aggressive limit orders is often optimal. The efficiency of such a radical tactic is explained by two main factors, the first one being the high fill probability of such an order and the second one being the fee policy that sets a significant difference between the maker fee and the taker fee (20 basis points for the lowest level, and 5 basis points for the highest level in Coinbase). Practically, inserting a bid limit order 1 tick below the best ask price when the spread is of the order of several hundreds of ticks would almost surely save 5 basis points of trading costs. We finally detailed how the latency risk is of overriding importance for such aggressive tactics and how it can be added to our framework.

In the case of a multi-horizon framework, *e.g.* execution algorithms with a trading horizon that can range from milliseconds to minutes depending on the market regime or operational constraints, the inference of the whole survival function is necessary. In this case, estimating more sophisticated models such as the ones from survival deep learning literature or the convolutional-transformer of [6] would be relevant. The performance of such models is asserted by the maximization of a censored likelihood, a quantity that cannot be estimated by fixed horizon models such as ours. This framework could also be adapted to different types of horizon such as a horizon of several ticks or several price moves.

Another extension of our approach would be to train a model for post-insertion evaluation. In a nutshell, the insertion of a limit order in the book bumps the order flow intensity and this cross-excitation vanishes over time. This causes the fill probability of an order at its insertion to differ from the fill probability of a pending order with the exact same characteristics. The insertion of liquidity reveals information about the agent’s intention and impacts – on average – the price, which is likely to move in the opposite direction. Such an extension can be carried out by simply adding pending limit orders in the training data and creating a new feature that characterizes the time elapsed since their insertion in the book.

Last but not least, the study of the mathematical properties of the saved cost function would highlight some regularity conditions that can be added as boundary constraints in the loss functions. The analysis of the optimal distance policy obtained with \mathcal{S}_ℓ compared to the one of \mathcal{S} , the integration of instantaneous market impact incurred when posting a market order and the analysis of expected fill ratios for larger traded sizes are issues that are left for future experiments.

Acknowledgments

The authors would like to express their sincere gratitude to Sophie Laruelle and Damien Challet who contributed to the refinement of this research paper through their diligent review and insightful discussions. The authors are also grateful to Maxime Garcia from the Quantitative research team at Sun Zu Lab for helpful comments.

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Appendix

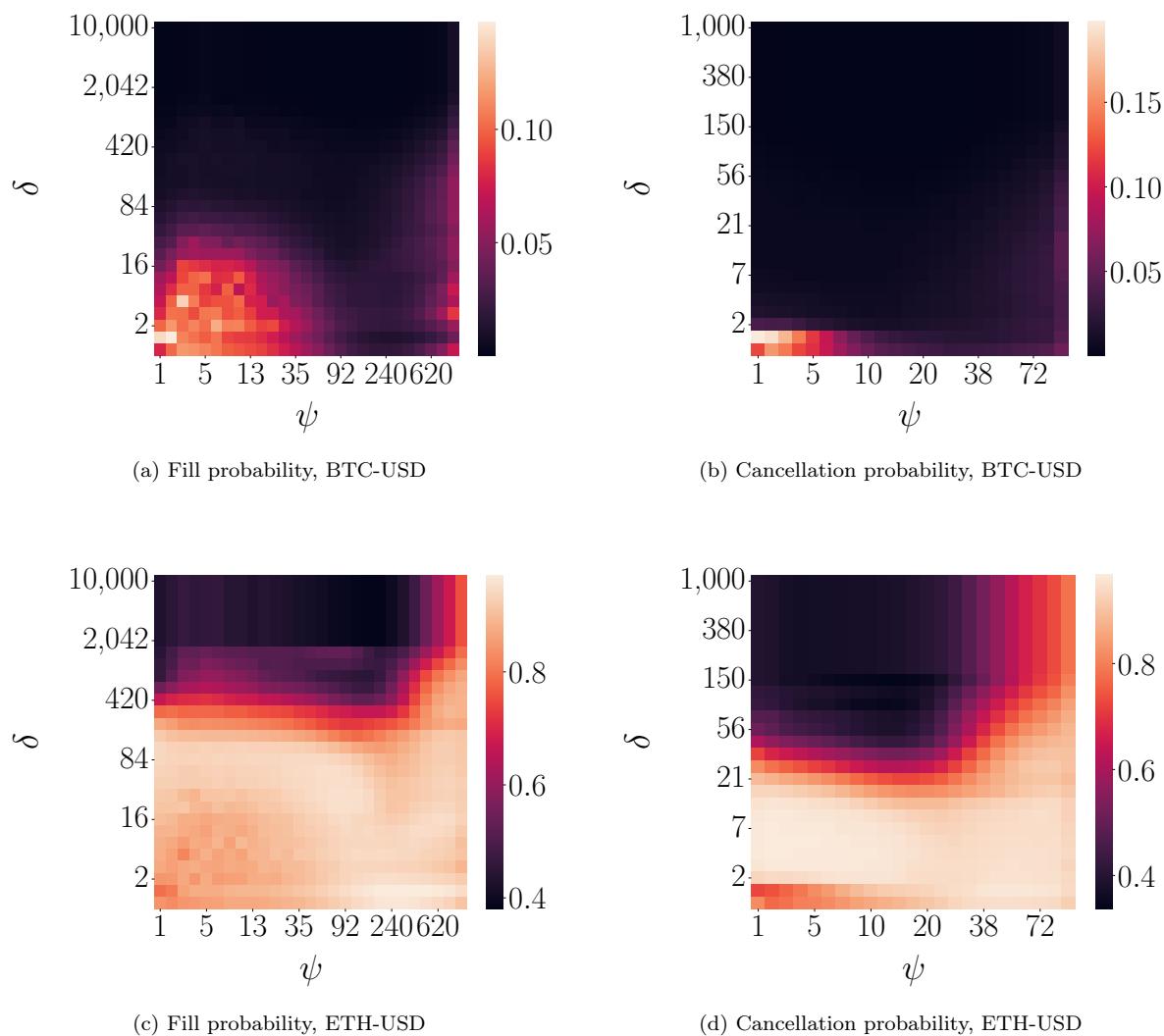


Figure 15: 1 second fill probability and cancellation probability as functions of distance δ and bid-ask spread ψ .

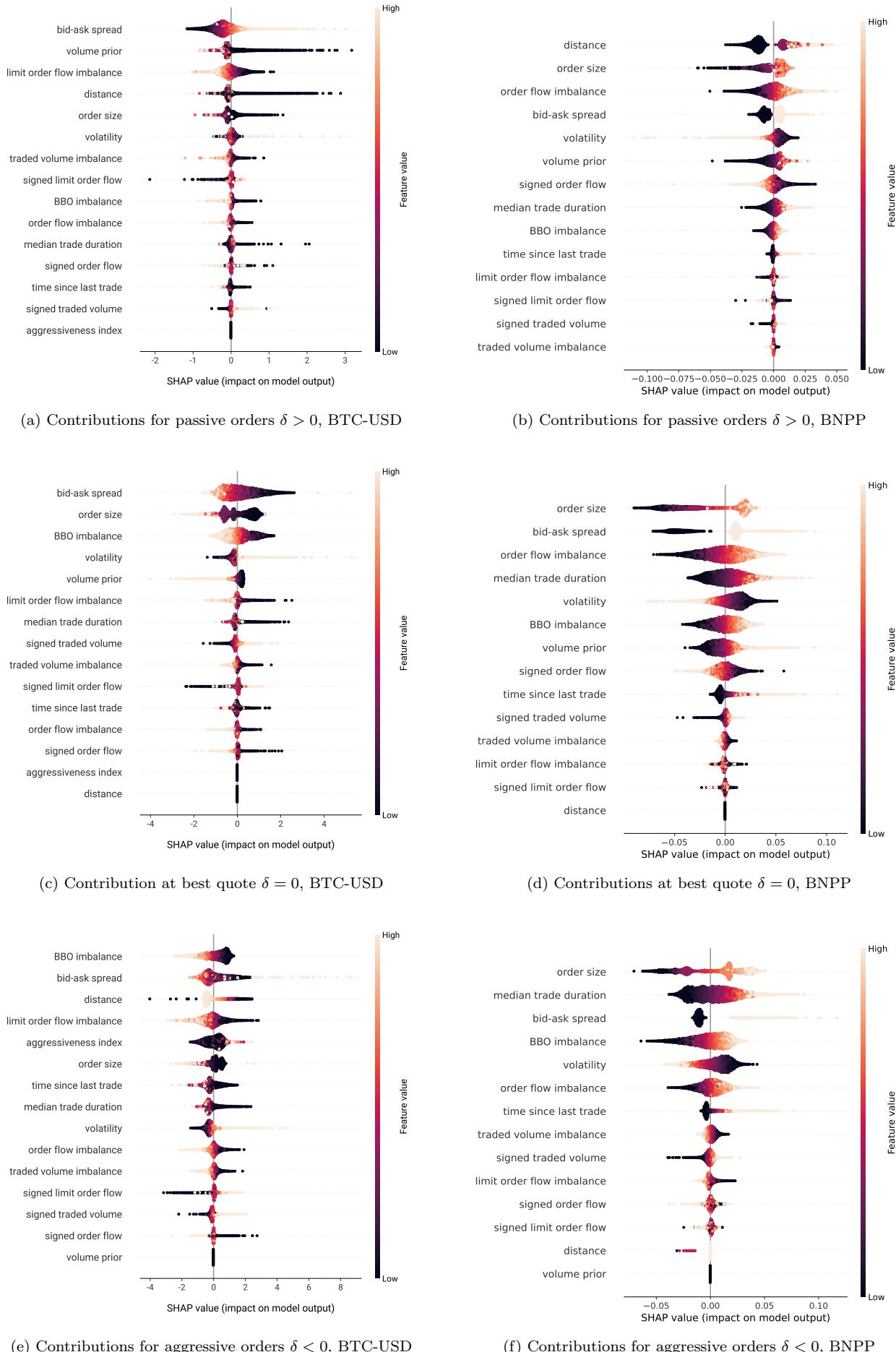


Figure 16: Market variables contributions to the saved cost function magnitude using Shapley values of 10,000 predictions, BTC-USD and BNPP, bid side