In this post, we discuss implications of selling multiple[TimeBoost](TimeBoost) licenses, as opposed to a unique such license. One obvious drawback of selling multiple licenses is that license holders still engage in latency war, therefore, invest in the latency improvement, which is a waste from the protocol perspective. However, we focus on a different angle of it, in particular, we analyze the simplest, public value, setting and argue that with considerable probability one of the players will buy all licenses in the equilibrium. An argument in favor of having multiple licenses is that holders of such licenses need to compete in a First-Come First-Serve (FCFS) race, and thus extract less arbitrage value. The single license holder can wait until the end of the time advantage window and typically extracts more value, see the empirical evidence. We question the implicit assumption in this argument that once multiple licenses are sold, they will be bought by independent parties. In particular, we argue that if there is additional value derived from holding all licenses and therefore, monopoly on having a time advantage, one party will buy all licenses in the equilibrium, at least with strictly positive probability.

Suppose there are k

players participating in an auction with n

TimeBoost licenses. The auction is $(n+1)$

-st price. The auction format allows us to compare it to single TimeBoost license selling via 2nd price auction. The total value derived from the FCFS competition is v

, which is a common knowledge. The total value derived from owning all TimeBoost licenses is equal to $v+\Delta$

, for some $\Delta>0$

, which is also a common knowledge. Players' strategy sets consists of choosing how many bids they make and what are these bids.

We consider two particular strategies:

1. s_1

: make one bid equal to $\frac{v}{n}$

.

1. s_2

: make n

bids equal to $\frac{v+\Delta}{n}$

.

Note that playing s_1

can not be sustained in the equilibrium, since it is profitable for any player to deviate to s_2

, pay $n\frac{v}{n}=v$

and get paid $v+\Delta$

, thus making profits equal to Δ

.

We assume that if there are $n\geq t\geq 1$

players who play strategy s_1

, they compete in the FCFS race, their latencies are equal, and they obtian the expected value $\frac{v}{t}$

.

We assume that if there are $n\geq t>1$

players who play strategy s_2

, each of them gets at least one bid chosen, which readily implies that each get paid in expectation $\frac{v}{t}$

fraction of the FCFS value v

. If, however, $t\geq n$

, then exactly n

of them are chosen uniformly at random. Then, playing s_2

can also not be sustained in the equilibrium, as the total profits to make is equal to v, while they pay $v+\Delta$

in total.

Next, we look into scenario in which players randomize between these two strategies. We check the conditions when the strategy profile in which all players try the first strategy with probability α

and the second strategy with probability $(1-\alpha)$

is an equilibrium.

Consider any player. The expected utility from the first strategy should be equal to the expected utility from the second strategy: $E[s_1] = E[s_2]$

. This indifference condition is needed to have mixed strategy profile be equilibrium.

Assume k

is a random variable that is distributed according to a Poisson distribution with parameter λ

, that is $k\sim Poisson(\lambda)$

.

By environmental equivalence

of the Poisson random variable (see [the paper by Myerson] for the properties of the games with Poisson random variable number of player), once a player learns she's a player, she has the same beliefs on the number of other players as the outsider has in the original game, namely, the number of other players is distributed as $Poisson(\lambda)$

.

Let k_1

and k_2

denote the number of players applying s_1

and s_2

strategies. Then, by decomposition property, k_1

and k_2

are independent

random variables distributed according to Poisson distributions $Poisson(\alpha\lambda)$

and $Poisson((1-\alpha)\lambda)$

, respectively.

The player's expected utility of playing strategy s_1

is equal to:

$E[s_1] = P[k_2>0]\cdot 0 + P k_2=0=e^{-(1-\alpha)\lambda}(e^{-\alpha \lambda}(v+\Delta)+\sum_{t=1}^{n-1}\frac{e^{-\alpha \lambda}(\alpha\lambda)^t}{t!}\frac{v}{t+1}).$

Let's go through all terms one by one. If there is at least one player playing strategy s_2

, then playing strategy s_1

has payoff 0

. From now assume nobody plays strategy s_2

. If there is nobody playing strategy s_1

, the payoff is $v+\Delta$

, as getting one license gives monopoly. If the number of players playing strategy s_1

, denoted by t, satisfies $0<t<n$

, then playing strategy s_1

gives payoff $\frac{v}{t+1}$

. If there are more than n players playing strategy s_1

, their expected utility is equal to 0

.

Similarly, expected utility of playing strategy s_2

is equal to:

$E[s_2] = P[k_2=0]+\ \sum_{t=1}^{\infty}P[k_2=t] = \ P[k_2=0]-\sum_{t=1}^{n}P[k_2=t]\frac{\Delta}{t+1}=e^{-(1-\alpha)\lambda}(e^{-\alpha \lambda}(v+\Delta)+(1-e^{-\alpha \lambda})\Delta)-\sum_{t=1}^{\infty}\frac{((1-\alpha)\lambda)^t e^{-(1-\alpha)\lambda}}{t!}\frac{\Delta}{t+1}=e^{-(1-\alpha)\lambda}(e^{-\alpha \lambda}v+\Delta)-\sum_{t=1}^{\infty}\frac{((1-\alpha)\lambda)^t e^{-(1-\alpha)\lambda}}{t!}\frac{\Delta}{t+1}$.

Our goal is to solve the only variable α

in the indifference condition $E[s_1]=E[s_2]$

. $\alpha\in (0,1)$

from the indifference condition ensures to have totally mixed equilibrium state. No player prefers to deviate to a strategy that tries to capture one or all licenses, but with different bids. Also, no player prefers to deviate to a strategy that tries to capture more than one license but less than all licenses.

Next, we list some analytical observations about this equilibrium state:

1. $E[s_1]$

is always positive and increasing in α

.

1. $E[s_2]$

is negative for $\alpha=0$

when $\frac{\Delta}{v}$

is large enough. It is first decreasing in α

and then increasing. For $\alpha=1$

, $E[s_2]=e^{-\lambda}v+\Delta$

is positive, and in particular it is strictly larger than the corresponding value of $E[s_1]$

. This ensures that there is a unique equilibrium of this type.

It remains an open question is there are other equilibria solutions of the game, but we conjecture there are no.

In the end, we list some observations obtained by numerical simulations:

1. If λ

is increasing, α

is increasing. This makes sense intuitively, because increasing λ

means increasing the expected number of players, and this leads to each player “shooting for exclusivity” less often.

1. Consider $\lambda =9$

, $n=4$

, $v=1$

and $\Delta\in [0.1,10]$. Then, $\alpha \approx 0.858$ is a solution of $E[s_1]=E[s_2]$. The probability everyone places one bid $P[k_2=0]$ is approximately equal to 0.278594 ("good" state), the probability someone gets all licenses $P[k_2=1]$ is approximately 0.35806 ("bad" state). With the rest probability, more than one player try to buy all licenses, but they fail.

1. Consider $\lambda=9$, $n=4$, $v=1$ and $\Delta\in [0,0.02]$, then there is no equilibrium solution of the game in totally mixed strategies.