

PBFT-based consensus protocols provide some good properties such as fast finality over chain-based protocols (e.g. Bitcoin, Cardano Ouroboros), but they typically don't scale well with the number of nodes participating in the consensus. This is because PBFT-based protocols require all-to-all votings to reach consensus (unless probabilistic tricks like Algorand-style committee sampling is in use). Even with latest proposals like HotStuff which avoids the complexity in view changes, each consensus decision still requires  $O(N^2)$

voting messages flowing through the network, where  $N$

is the number of nodes. Imagine a system with 100k nodes, in each of the consensus voting phases, every node has to receive and process 100k voting messages from other nodes. This could overwhelm the network, and thus limit the number of participants in PBFT-based protocols. This makes PBFT-based protocols less decentralized compared to chain-based protocols.

To address this bottleneck, we propose to leverage SNARK proof aggregation to reduce the amount of voting messages. Recently, [@vbuterin](#) wrote a [nice introduction](#) to SNARK proof aggregation/merging.

The basic idea is to use SNARK aggregation in a way similar to signature aggregation, so that an aggregated SNARK can compactly encode which nodes have voted for a block. For simplicity, let us assume the network has a fixed number of  $N$

nodes. Assume a new block has just been proposed, and the  $i^{\text{th}}$

node  $n_i$

wants to vote for the block. It does the following:

- First generate a SNARK proof  $\pi_i$

representing his yes vote (basically it just needs to create a SNARK proof  $\pi_i$

for his signature  $\sigma = \text{sign}_{\text{sk}}(h)$

, where  $h$

is the hash of the new block).

- Broadcast the SNARK proof  $\pi_i$

, and an index vector

$v_i$

to all its neighbors. The index vector is an  $N$

dimensional boolean

vector that encodes which nodes have provided SNARK proofs for their yes votes. Initially,  $v_i = (0, 0, \dots, 1, \dots, 0)$

, i.e., all but the  $i^{\text{th}}$

element are zeros, since only node  $n_i$

has provided the proof for its yes vote.

Meanwhile, Node  $n_i$

keeps aggregating the proof and index vector and then broadcast them out to all neighbors:

- Upon receiving  $(\pi_j, v_j)$

from its neighbor  $n_j$

, it first verifies the validity of the proof  $\text{verify}(\pi_j, v_j) == \text{true}$

. If so, it updates its local proof and index vector by  $\pi_i = \text{Aggregate}(\pi_i, v_i, \pi_j, v_j)$

,  $v_i = v_i$

OR  $v_j$

. Here OR is the element-wise boolean "or" operator. The node stops doing aggregation when the number of "1"s in  $v_i$

is over a certain threshold, e.g.  $\frac{2}{3}N$

- Node  $n_i$

maintains a local timer. Whenever the timer is triggered (e.g. once every 100ms), node  $n_i$  broadcasts out the latest  $\pi_i$  and  $v_i$

. Note that in the first few rounds, most of the elements of  $v_i$  are zero, so data compress can be applied to effectively reduce the message size.

The readers might wonder why we don't just use signature aggregation. The issue there is that signature aggregation typically requires  $v_i$

and  $v_j$

to be disjoint, i.e., the same element (e.g. the  $k^{\text{th}}$

element) cannot be 1 in both vectors. SNARK aggregation overcomes this restriction.

It can be shown that in a gossip network with relatively good connectivity, only  $O(\log N)$

broadcasting rounds are required for the above voting process to converge (similar to how a gossiped message can propagated through the network in  $O(\log N)$

time). Moreover, since a gossip network has  $O(N)$

number of edges, the total number of voting messages is  $O(N \log N)$

, which scales much better as  $N$

grows. The above process also has good tolerance to Byzantine nodes, since Byzantine nodes cannot forge fake proofs.

Is this a viable approach? Feedback and comments are appreciated!