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Thank Yi Sun, Justin Drake, and Aayush Gupta for feedback and discussions.

Our full paper is here: [Octopus Contract and its Applications](#)

## TL;DR

We propose the concept of Octopus contracts, smart contracts that operate ciphertexts outside the blockchain. Octopus contracts can control the decryption of the ciphertexts based on on-chain conditions by requesting validators to sign the specified messages. Signature-based witness encryption (SWE) enables users to decrypt them with the signatures. Moreover, Octopus contracts can evaluate arbitrary functions on encrypted inputs with one-time programs built from SWE and garbled circuits. These features extend the functionality of smart contracts beyond the blockchain, providing practical solutions for unresolved problems such as a trustless bridge for a two-way peg between Bitcoin and Ethereum, private AMM, minimal anti-collusion infrastructure without a centralized operator, and achieving new applications such as private and unique human ID with proof of attribution, private computation with web data, and more.

## 1 Background and Our Contribution

Regarding the aspect of using smart contracts to operate secrets outside the blockchain, our scheme is fundamentally a generalization of the author's previous work [Trustless Bitcoin Bridge with Witness Encryption \(Leona Hioki\)](#), which also provides a practical construction of the previous work.

Compared to existing schemes to allow smart contracts to operate ciphertexts using new cryptographic schemes, e.g. [Lit protocol with threshold encryption](#), [smart contracts with secret sharing multi-party computation](#) (MPC), and [smartFHE with fully homomorphic encryption](#) (FHE), our scheme with [SWE](#) requires minimum modification to the node implementation of the validators. This is because the validators only need to sign the message specified by the Octopus contract and encrypt this signature with public-key encryption (PKE). Especially, when a circuit of the function is privately evaluated with one-time programs (OTPs), the validators' computational cost only depends on the input size of the circuit, independent of the circuit size. Besides, while both FHE-based schemes and our scheme can delegate heavy evaluation of the circuit to an untrusted third party, the latter is estimated to be faster than the former because the delegated party in the latter just decrypts the SWE encryptions of the garbled inputs and evaluates a [garbled circuit](#), which only employs a hash function and bit operations in [the optimal construction](#).

The validators' work in [vetKeys](#) is similar to ours: the validators generate BLS signatures for an ID and encrypt the signatures under the user's public key to pass the user a private key of that ID in ID-based encryption. However, it is our novel point to use the signatures for the private evaluation of functions on encrypted inputs.

Despite the above advantages of our scheme, the Octopus contract using OTPs further relies on rabble MPC, n

-of-n

MPC among randomly selected people who are different from the validators

. It is used to generate an OTP while embedding a private key unknown to humans in the evaluated circuit. The rabble MPC is more secure and feasible than existing MPC-based schemes for the following reasons:

1. If at least one participant is honest

, the MPC is secure, i.e., revealing no information other than the OTP.

1. Even if the MPC fails because some participants leave the MPC, different participants can start new MPCs any number of times

. Also, many MPCs can be performed in parallel.

1. Once the OTP is generated, there is nothing more for the MPC participant to do.

The following table summarizes the comparison between our scheme and the existing schemes.

[

Comparison\_ours\_existing

960×540 80.4 KB

](https://ethresear.ch/uploads/default/original/2X/9/90ed4de5232c71a4cb333868e5e2972471bac9e9.jpeg)

# 2 Signature-based Witness Encryption

## 2.1 Definition of SWE

We adopt a SWE scheme defined for  $t$

-out-of- $n$

BLS signatures. It provides the following algorithms. While they are based on Definition 1 of [McFly](#), some inputs are omitted or modified.

1.  $\text{ct} \leftarrow \text{SWE.Enc}(V=(vk_1, \dots, vk_n), h, m)$

: it takes as input a set  $V$

of  $n$

BLS verification keys, a hash  $h$

of a signing target  $T$

, and a message to be encrypted  $m$

. It outputs a ciphertext  $\text{ct}$

.

1.  $m \leftarrow \text{SWE.Dec}(\text{ct}, \sigma, U, V)$

: it takes as input a ciphertext  $\text{ct}$

, an aggregate signature  $\sigma$

, two sets  $U, V$

of BLS verification keys. It outputs a decrypted message  $m$

or the symbol  $\perp$

.

If more than or equal to  $t$

validators of which verification keys are in  $V$

, i.e.,  $|U| \geq t$

and  $U \subseteq V$

, generate a valid aggregated signature  $\sigma$

for the hash  $h$

, the correctness holds, i.e.,  $\text{SWE.Dec}(\text{SWE.Enc}(V=(vk_1, \dots, vk_n), h, m), \sigma, U, V) = m$

. Otherwise, the  $\text{SWE.Dec}$

algorithm returns the symbol  $\perp$

. Therefore, once the validators release the signature  $\sigma$

, anyone can decrypt the ciphertext  $\text{ct}$

.

## 2.2 Access-Control of the Signature

We use the same technique as [vetKeys](#) to control who will be able to decrypt the ciphertext by encrypting the validators' signatures. Specifically, when a legitimate decryptor provides a public key  $\text{pubKey}$

of the PKE scheme, each validator publishes an encryption  $\text{ct}_{\sigma_i}$  of the signature  $\sigma_i$  under  $\text{pubKey}$ , i.e.,  $\text{ct}_{\sigma_i} \leftarrow \text{PKE.Enc}(\text{pubKey}, \sigma_i)$ . That decryptor can recover the message by decrypting each  $\text{ct}_{\sigma_i}$  with the private key  $\text{privKey}$  corresponding to the  $\text{pubKey}$ , aggregating the recovered signatures into  $\sigma_{\Sigma}$ , and decrypting the ciphertext under SWE by  $\sigma_{\Sigma}$ . However, the other users cannot do that because they cannot obtain  $\sigma_{\Sigma}$  from  $\text{ct}_{\sigma_i}$  without  $\text{privKey}$ . Besides, even the validators cannot decrypt them as long as their honest majority does not reveal each signature  $\sigma_i$ .

As proposed in [vetKeys](#), if the PKE scheme is additive-homomorphic, e.g., EC-ElGamal encryption, the decryptor can first compute the encryption of the aggregated signature by computing the weighted sum of the encrypted signatures and then decrypt only the aggregated one. By outsourcing that computation, the decryptor can reduce the computation cost.

## 2.3 Estimated Benchmark of SWE

We estimate a benchmark of the SWE scheme based on [McFly](#) assuming a threshold  $\frac{t}{n} = \frac{2}{3}$

. For  $n=500$

and  $n=2000$

, encryption takes approximately 10 and 60 seconds, and decryption takes around 20 and 350 seconds, respectively. These results suggest the appropriate number of allocated validators for each use case. They also imply that more improvement in the SWE scheme will enhance the security of ciphertexts, i.e., increasing the number of allocated validators, without sacrificing the performance.

## 3 Octopus Contract

In our scheme, the Octopus contract helps users request the Ethereum validators to sign a specific message for the SWE decryption. Specifically, they work as follows.

1. Firstly, some validators register with and watch the Octopus contract made by an application developer.
2. An encryptor, the user willing to encrypt a message using SWE, calls the Octopus contract to register a signing target  $T$ .

1. The Octopus contract records the hash  $h := \text{Hash}(\text{PREFIX}, T)$  derived from  $T$

. Note that  $\text{PREFIX}$

is a fixed unique string, which prevents the validators from signing messages for the Ethereum consensus algorithm.

1. The encryptor generates an encryption  $\text{ct}$  of the messages  $m$  under the hash  $h$

and  $n$

validators' verification keys  $V=(vk_1, \dots, vk_n)$

allocated by the Octopus contract.

1. A decryptor, a user willing to decrypt  $ct$ ,  
, has a PKE key pair  $(privKey, pubKey)$   
and calls the Octopus contract, passing the  $h$   
and the PKE public key  $pubKey$   
to request validators' signatures.

1. The Octopus contract checks if the decryptor is legitimate based on the required on-chain conditions. If the decryptor does not pass the conditions, the contract rejects the decryptor's request.
2. More than or equal to  $t$

validators generate the encryption  $ct_{\sigma}$   
of the aggregated signature  $\sigma$   
for the hash  $h$   
in a way described in Subsection 2.2.

1. The validators provide the Octopus contract with the encryptions  $ct_{\sigma}$   
along with a proof  $\pi$   
to prove that they are valid encryptions of the aggregated signatures.

1. The decryptor first decrypts  $ct_{\sigma}$   
with  $privKey$   
to obtain the signature  $\sigma$   
, and then decrypts  $ct$   
with  $\sigma$   
to recover the messages  $m$

.

In this way, the encryptor can encrypt messages under some on-chain state conditions without knowing who satisfies the conditions in the future. As long as more than or equal to the threshold of the validators behave honestly, i.e., sign only the message confirmed by the Octopus contract, only the legitimate decryptor can decrypt the ciphertext.

When implementing our scheme, we can prepare a shared smart contract for common management of the registered validators and requests for signatures. Each application contract specifies the signing messages and checks if the decryptor is legitimate.

Its application is described in Subsection 3.1 in the full paper.

## 4 One-Time Program with Octopus Contract

### 4.1 Basic Ideas

The Octopus contract with SWE described above has the following limitations.

1. The ciphertext must be decrypted in a rather short time because the validators that can generate signatures for the decryption are fixed at the time of encryption.
2. It is impossible to apply some functions to the encrypted message  $m$   
without revealing it to the decryptor.

We solve them by introducing OTPs. The OTP is an encoded circuit that can be evaluated on at most one input. [Goyal](#) constructs a blockchain-based one-time program (BOTP) from witness encryption (WE) and garbled circuits. A generator of BOTP makes a garbled circuit of the circuit and encrypts its garbled inputs under WE. Its evaluator can decrypt each encryption of the garbled input for the bit  $b \in \{0,1\}$

of the  $i$

-th input bit by committing  $b$

as the  $i$

-th input bit on-chain. Subsequently, the decryptor evaluates the garbled circuit with the recovered garbled inputs. The decryptor can input only one bit  $b$

for each input bit to the circuit because the decryption condition of WE requires the decryptor to prove that  $b$

is committed first to the blockchain finalized by the honest majority of validators. In other words, the decryptor cannot input  $1-b$

without tampering with the finalized block containing the commitment of  $b$

While the OTP has the limitation of one-time input, it has a useful security feature that the evaluator cannot learn non-trivial information about the circuit. Therefore, the generator can embed secret data and algorithms in the circuit of the OTP. Moreover, if multiple generators use  $n$

-of- $n$

MPC, which we call rabble MPC, to generate a private key, embed it in the circuit, and output its OTP, the OTP can hold a private key that no human knows as long as at least one MPC participant and the honest majority of the validators are honest. It can be used to decrypt the encryption of the circuit input and sign the circuit output inside the circuit. For example, the OTP with the embedded private key allows us to bootstrap a SWE ciphertext, i.e., encrypting the same message under a different set of verifying keys. The OTP for the SWE bootstrap decrypts the encrypted signature with the private key, uses the signature to recover the message from the SWE ciphertext, and encrypts the same message under new verifying keys. We can generalize this approach to evaluate arbitrary functions on encrypted inputs.

## 4.2 One-time program based on SWE

Instead of existing WE constructions supporting general decryption conditions, which are [impractical or depend on heuristics cryptographic assumptions](#), we adopt SWE to build OTPs. Let  $k_{i,b}$

and  $\tilde{C}$

be a garbled input for the bit  $b$

of the  $i$

-th input bit and a garbled circuit of the input size  $|x|$

, respectively. The generator, the evaluator, and the Octopus contract managing  $\tilde{C}$

collaborate as below:

1. The generator registers  $2|x|$

signing targets  $\{(i,b) \mid i \in [|x|], b \in \{0,1\}\} = \{(1,0), (1,1), \dots, (|x|,0), (|x|,1)\}$

with the Octopus contract.

1. The Octopus contract records  $2|x|$

hashes  $\{h_{i,b} = \text{Hash}(\text{PREFIX}, (i,b)) \mid i \in [|x|], b \in \{0,1\}\}$

and allocates  $n$

validators of which the verification keys are  $V = (\text{vk}_1, \dots, \text{vk}_n)$

1. The generator generates a garbled circuit  $\tilde{C}$

and its garbled inputs  $\{k_{i,b}\}_{i \in [|x|], b \in \{0,1\}}$

.  
1. For each  $i \in [|x|], b \in \{0,1\}$   
, the generator encrypts  $k_{i,b}$   
under  $V$   
and  $h_{i,b}$   
, i.e.,  $ct_{i,b} \leftarrow \text{SWE.Enc}(V, h_{i,b}, k_{i,b})$

.  
1. The evaluator registers the input  $x$

with the Octopus contract.

1. The Octopus contract checks if the other inputs have not been registered before. If so, it requests the allocated validators to sign the  $|x|$

hashes  $\{h_{i,x_i}\}_{i \in [|x|]}$

without specifying a public key to encrypt the signatures.

1. The evaluator obtains aggregated signatures  $\{\sigma_i\}_{i \in [|x|]}$   
and uses them to decrypt  $\{ct_{i,x_i}\}_{i \in [|x|]}$   
, i.e.,  $k_{i,x_i} \leftarrow \text{SWE.Dec}(ct_{i,x_i}, \sigma_i, U, V)$

.  
1. The evaluator evaluates  $\widetilde{C}$

on  $\{k_{i,x_i}\}_{i \in [|x|]}$

.  
Notably, in formal security proof, the garbled circuit is secure only against a selective adversary that chooses the input  $x$  before seeing the garbled circuit  $\widetilde{C}$

. However, as far as our knowledge, it does not mean that there is a practical attack on the garbled circuit scheme when  $x$

is chosen adaptively. Besides, [Yao's garbled circuit](#) without modification is proven to be [adaptively secure if the circuit is an NC1 circuit](#), i.e., a low-depth circuit. To bootstrap it to a polynomial-sized circuit, we may be able to use a similar technique in [this paper](#) that bootstraps an indistinguishability obfuscation of NC1 circuits with [a randomized encoding such as Yao's garbled circuit](#).

## 4.3 Rabble MPC for Key-Embedded OTPs

OTPs of key-embedded circuits are generated through the rabble MPC,  $n$

-of- $n$

MPC among randomly selected people. The Octopus contract manages the participants of the rabble MPC and randomly assigns their subset to each generation of the OTP. These participants are different from the validators, and the Octopus contract can require a lower stake to participate in the rabble MPC than that of validators.

After registering the signing targets with the Octopus contract as described above, the selected  $n$

participants perform the  $n$

-of- $n$

MPC to privately generate a new OTP for a circuit  $C$

taking  $s$

inputs as follows:

1. Each participant provides the randomness  $r_i$  as input.

1. They derive private and public keys  $(\text{privKey}, \text{pubKey})$  from the XOR of all randomnesses  $\bigoplus_{i=1}^n r_i$ .

. These keys are assumed to be usable for both PKE and digital signature schemes.

1. They construct a key-embedded circuit  $C[\text{privKey}]$  that takes  $s$  encryptions of inputs  $(ct_{x_1}, \dots, ct_{x_s})$  under  $\text{pubKey}$ , decrypts them with  $\text{privKey}$ , provides the  $s$  inputs  $(x_1, \dots, x_s)$  for  $C$ , signs the output  $y = C(x_1, \dots, x_s)$  with  $\text{privKey}$ , and outputs  $y$  and the signature  $\sigma_{\text{otp}}$ .

. Let  $u$  be the input bits size of  $C[\text{privKey}]$ .

1. They generate a garbled circuit of  $C[\text{privKey}]$  denoted by  $\widetilde{C}[\text{privKey}]$  and its garbled inputs  $\{k_{i,b}\}_{i \in [u], b \in \{0,1\}}$ .

1. They encrypt each garbled input  $k_{i,b}$  under the allocated validators' verification keys  $V$  and the hash  $h_{i,b} = \text{Hash}(\text{PREFIX}, (i,b))$ , i.e.,  $ct_{i,b} \leftarrow \text{SWE.Enc}(V, h_{i,b}, k_{i,b})$ .

1. They outputs the OTP  $(\text{pubKey}, \widetilde{C}[\text{privKey}], \{ct_{i,b}\}_{i \in [u], b \in \{0,1\}})$ .

The way of the SWE bootstrapping and the applications with OTP are described in Subsections 4.4 and 4.5 in the full paper.

## 5 Selecting a Subset of Validators

There are several methods to select a validator set from the consensus layer of Ethereum as follows:

1. Hard fork Ethereum to force all validators to sign messages from Octopus contracts.
2. Soft fork Ethereum, allowing any validators to sign messages from Octopus contracts.
3. Use a re-staking mechanism such as [Eigen Layer](#), enabling validators to have dual roles.

Even in the first case, which imposes the greatest burden on the Ethereum network, the validators' signatures for the same messages can be aggregated, so that the additional cost of pairing is at most for each ciphertext. However, in that case, we should note that security is not completely inherited because the validators cannot be penalized in the same way as in the case of double voting when they sign messages not specified by the Octopus contracts.

In the second and third cases, we can maintain the existing protocol of the consensus layer as the modification to the node implementation for our scheme is optimal in similar to MEV-related protocols. While the restaking in the third case is easy to introduce, the soft fork supported by many validators will improve the security of our scheme more significantly.

## Applications and the other notes

The on-chain conditions for the decryption in Octopus contracts can be implemented in Solidity. By customizing the conditions for each use case, we can build various novel applications, including a trustless bitcoin bridge, private AMM, and more. The OTP extends the application of the Octopus contracts because their functionalities are almost equivalent to what TEEs can do, in particular verification of computations by private conditions, private unique human IDs with proof of attributions, and private computation with web data. They are described in our full paper.

Read our full paper here: [Octopus Contract and its Applications](#)

Sora Suegami wrote the sections about the idea of using OTPs for private function evaluation and its applications. Leona Hioki wrote the sections about a trustless bitcoin bridge and private AMM. The other sections are written together.