A Simple Theory of Vampire Attacks*

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Abstract

Firms find it valuable to lock in repeat consumers, and they often do so via loyalty programs such as frequent-flyer miles or discounts for returning consumers. We introduce a model of loyalty programs in the presence of competition and show that, surprisingly, such programs result in higher prices for all consumers in equilibrium.

However, competitors may also find it valuable to identify repeat consumers of other firms and convince them to switch; such efforts are called "vampire attacks"—and are particularly easy to execute in the context of Web3 platforms that record their transactions on a public blockchain ledger. We show that vampire attacks can facilitate price competition; this implies that Web3 platforms may face greater competitive pressure than more traditional firms with loyalty programs.

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1 Introduction

Blockchain-based services by nature record every consumer transaction publicly; this has enabled a new type of entry strategy—a "vampire attack," in which the entrant uses this public information to directly recruit the dominant firm's best customers, in effect "sucking the lifeblood" out of the dominant firm. For instance, in 2022, three different non-fungible token (NFT) marketplaces launched by offering various incentives to especially active users of other NFT marketplaces, particularly OpenSea, which was the dominant NFT platform as of 2021. These vampire attacks appear to have generated competition for active users: LooksRare, then X2Y2, and then Blur all entered via vampire attack, and at the start of 2023, all three firms enjoyed significant market share even though NFT trading is characterized by (positive) network externalities (Kominers, Bernstein, and Gonzalez, 2022; The Block, 2023). It is not clear whether the entry of these particular firms is sustainable—in particular, the legality and regulatory status of the incentives they provide to users is uncertain;[1] but, in any event, the latter half of 2022 and the early part of 2023 saw NFT trading platforms competing fiercely on both price and quality dimensions (Shimron, 2022; Hayward, 2023). Similar dynamics have occurred in the competition between marketplaces for trading fungible tokens (Keoun, Godbole, and Sinclair, 2020; Liu, Chen, and Zhu, 2022).

This idea of poaching a competitor's most valuable customers is not new: Airlines, for example, have long offered "status match," whereby a current frequent flyer at one airline can transfer their status to another airline; however, in that case, verification of prior status and onboarding are costly (Petersen, 2012; Rosen, 2018). The public ledger nature of a blockchain makes customer poaching far easier.

In this paper, we examine how the possibility of vampire attacks affects equilibrium

[1]The form of vampire attack employed by these platforms involved issuing users tokens of uncertain regulatory status/legality. Of course, the impact of that particular form of vampire attack may be different in scenarios where regulatory clarity and consistent enforcement exists—but, at the same time, token rewards are only one mechanism among many by which vampire attacks can occur. Our analysis here only relies on the ability of firms to provide benefits to loyal customers of competing firms if they switch, not on the particulars of how they do so.

pricing when firms have loyalty programs. To do so, we must first understand the effects of loyalty programs in oligopolistic environments. We then investigate how firms' pricing changes when (potentially costly) vampire attacks are possible.

Unsurprisingly, loyalty programs result in higher prices for consumers who do not use them. But we show that they can also result in higher prices for consumers who *do* use them, relative to the case in which loyalty programs are not present. Without loyalty programs, when raising its price, a firm faces a trade-off between more fully exploiting price-insensitive consumers and recruiting more of the price-sensitive consumers. Meanwhile, loyalty programs enable a form of price discrimination under which a firm can charge price-insensitive consumers the standard price while offering some savvier, price-sensitive consumers a lower-than-standard price. But the lower-than-standard price offered to a firm's price-sensitive consumers need only compete with other firms' standard prices—and so if all firms are offering higher standard prices, the loyalty prices can be higher as well. Thus, in equilibrium, the price paid by loyalty program consumers can be higher than the price they paid before loyalty programs were introduced.[2]

We show that vampire attacks partially undo the anti-competitive effects of loyalty programs. When vampire attacks are possible, firms can compete for each others' loyalty program members directly; this in effect converts the competition for loyalty program customers into a differentiated Bertrand competition game in which the other consumers do not play a role. As a result, in equilibrium, prices for loyalty program customers are lower than when vampire attacks are not possible; in fact, prices for such customers are lower than when loyalty programs do not exist.

In short: When loyalty status is not portable between firms, firms can use loyalty status to identify price-sensitive consumers and so raise prices on other consumers; but here, unlike in monopoly settings, all consumers can be worse off in equilibrium when firms engage in

[2]There are, of course, many other mechanisms by which firms might use loyalty-type programs to blunt the effects of price competition; and such mechanisms also likely lead to higher prices. For example, Gans and King (2006) showed that two firms in different industries can, by bundling their products, make consumers less price-sensitive and thus enhance their market power. By contrast, in our model, price sensitivity stays constant but loyalty programs nevertheless dampen price competition in equilibrium.

this type of price discrimination. But when loyalty status is fully transparent and portable, competition for price-sensitive consumers reemerges because each firm can now identify all price-sensitive consumers—and compete for them directly—instead of having to offer its competitors' loyalty program members the same price as its price-insensitive consumers.

Revenues associated with loyalty programs are at least tens of billions of dollars:[3] In addition to airlines, almost 60% of restaurants offer loyalty programs (Wolff, 2022), as does nearly every major international hotel chain (Skift, 2020). There are also subscription programs for food delivery and ride-hailing services that have a similar structure: consumers can pay a monthly or annual membership fee to have access to per-transaction discounts and the like. Similar reward structures are present on many Web3 platforms; but in that setting the public nature of blockchain transactions makes our analysis particularly important (Catalini and Kominers, 2022). Yet the effects of loyalty programs on consumer welfare and competitive dynamics are not well-understood. Our analysis suggests that, perhaps surprisingly, loyalty programs may reduce price competition in industries that feature them. However, our work also suggests that as the cost of recruiting a competitor's loyal consumers falls, this dynamic may be reversed: Loyalty programs may facilitate substantial price competition when consumers can shift across loyalty programs easily.

Related Literature

The vampire attacks we discuss here are reminiscent of a literature on customer poaching; for example, in the model of Fudenberg and Tirole (2000), firms offer all customers the same price in the first period, but in the second period each firm offers new customers a better price, as a customer who did not buy from that firm in the first period has demonstrated that he prefers another firm.[4] Thus, in these settings, conditioning price on previous purchasing behavior is a form of price discrimination. We instead focus on the direct competitive effects

[3] Indeed, airlines alone apparently take in over $10 billion through their frequent flyer programs (Statista, 2020).

[4] Farrell and Klemperer (2007) provide an overview of the literature on firms offering new and old customers different prices.

of vampire attacks, showing that they reduce prices for all consumers, even without consumers switching firms in equilibrium.

There is also an existing literature on the effects of loyalty programs on prices: In the frameworks of Caminal and Matutes (1990) and Caminal and Claici (2007), firms can pre-commit to future prices for returning consumers; moreover, consumer preferences over firms change each period. This can lead to enhanced competition between firms because consumers are more price-sensitive relative to offers from firms that fix future prices because consumers' preferences over firms may change in the future. Thus, loyalty programs make consumers better off at the cost of total surplus (since a consumer now no longer necessarily chooses their most-preferred firm each period).[5]

Our work, by contrast, does not suppose any pre-commitment on the part of firms or changing consumer preferences; instead, loyalty programs just help firms to identify price-sensitive consumers with a natural preference for that firm. Firms are able to leverage that preference information to coordinate their pricing strategies, even in the absence of repeated game dynamics: each firm chooses a price for its loyal customers that is low enough that the other firm does not wish to compete for them, but this can result in prices that are higher for both unaffiliated and affiliated customers than when loyalty programs did not exist.

To the best of our knowledge, the work most closely related to our analysis here is that of Kim, Shi, and Srinivasan (2001). In the Kim et al. (2001) framework, like in ours, there are price-sensitive users and price-insensitive users, and loyalty programs only appeal to the price-senstive users. This means that, like in our setting, loyalty programs are a method of price discriminating between consumer groups and so price-insensitive consumers pay higher prices when loyalty programs are present. Kim et al. (2001) do not provide a full characterization of the set of equilibria, and so do not come to any clear conclusions regarding the effect on price-sensitive users. By contrast, we obtain a clear comparative static that the

[5] Another strand of this literature (Banerjee and Summers, 1987; Cairns and Galbraith, 1990) focuses on coupons as a device to artificially induce differentiation in settings in which consumers are otherwise indifferent between firms.

price charged to price-sensitive consumers can be higher when loyalty programs are present.

Our paper also indirectly contributes to the literature on how the information available to firms affects pricing outcomes. The literature on information and collusive outcomes has focused on repeated-game settings—see the work of Sugaya and Wolitzky (2018) and references therein. Here, we show that the extent of firms' information about customers can have significant pricing implications: prices are higher in our model when firms gain information on which customers naturally prefer them (and so are likely to become loyal), but prices fall when firms also gain information on who their competitors' loyal customers are.[6]

Outline of the Paper

The remainder of this paper is organized as follows: Section 2 lays out our economic framework. Section 3 describes the baseline game without loyalty programs. We examine the impact of loyalty programs in Section 4, and then in Section 5, we show how vampire attacks affect pricing. Section 6 presents brief concluding remarks.

2 Economic Framework

There are two firms $\{f, g\} \equiv F$. Each firm $f \in F$ has *captive demand* $C^f(p^f) \equiv \alpha - \beta p^f$, where p^f is the *standard price* faced by *captive consumers* of f.

Additionally, there exists a continuum of *mobile consumers* distributed uniformly on $[-1, 1]$. A consumer at $z \in [-1, 1]$ obtains a utility of $-|z-1| - \sigma p^f$ if he purchases service from f and $-|z+1| - \sigma p^g$ if he purchases service from g, where σ is the *price-sensitivity* of mobile consumers.[7] Thus, *mobile demand* $M^f(p)$ for each firm f relies not only on the price that firm f chooses but the price of the other firm as well. We can then compute the

[6] Note also that the concept of a "vampire attack" considered here is not to be confused with the type involving exsanguination, which Snower (1982) has examined in the context of macroeconomic policy optimization, building on earlier work of Hartl and Mehlmann (1982) and Stoker (1897) (see also Drezner (2014)).

[7] Intuitively, customer preferences are as in a standard Hotelling model with f at 1 and g at -1.

mobile demand (on the relevant domain) as $M^f(p) \equiv \lambda(1 - \sigma(p^f - p^g))$, where λ is the *size of the mobile market* (per firm). Thus, the total size of the mobile market is 2λ regardless of prices, but the fraction of consumers who go to firm f depends on its price and the price of its competitor.[8]

The profits of firm f are thus given by

$$\Pi^f \equiv (p^f - c)(\alpha - \beta p^f) + (p^f - c)\lambda\Big\langle 1 - \sigma(p^f - p^g)\Big\rangle_{[0,2]},$$

where $c < \frac{\alpha}{\beta}$ is the *cost* of providing service and

$$\langle x \rangle_{[a,b]} \equiv \begin{cases} a & x \leq a \\ x & a \leq x \leq b \\ b & b \leq x. \end{cases}$$

We assume that mobile consumers are sufficiently price-sensitive (i.e., σ is large enough) that equilibrium prices are lower when more mobile consumers are present, i.e., that the *price responsiveness* $\rho \equiv \alpha - \beta c - 2\frac{\beta}{\sigma}$ is greater than 0.

If firm f faced only its captive demand, then its profit-maximizing price would be the *monopoly price* $p^\star \equiv \frac{\alpha + \beta c}{2\beta}$. If the two firms faced only mobile demand, the Nash equilibrium *competitive price* would be $p^\circ \equiv \frac{1}{\sigma} + c$.

3 Competition without Loyalty Programs

In this section, we analyze the *baseline game* in which both firms simultaneously announce a price that applies to both their mobile and captive consumers. In the baseline game, firm

[8]Relaxing the assumption that mobile consumers always purchase from one of the two firms would not significantly change the results; we could allow for mobile consumers who, when faced with high prices from both firms, choose not to purchase.

$f \in F$ chooses its price p^f according to

$$\max_{p^f \in [c,\infty)} \left\{ (p^f - c)(\alpha - \beta p^f) + (p^f - c)\lambda \Big\langle 1 - \sigma(p^f - p^g) \Big\rangle_{[0,2]} \right\}.$$

Thus, each firm faces a tradeoff with respect to its price: A higher price more profitably monopolizes that firm's captive consumers, but cedes a larger share of the mobile consumer market to the other firm.

Proposition 1. *Under the baseline game, in the unique pure-strategy Nash equilibrium, each firm sets a price*

$$p^\dagger \equiv \frac{\alpha + \lambda + (\beta + \lambda\sigma)c}{2\beta + \lambda\sigma}.$$

It is immediate that the price $p^\dagger$ is increasing in the size of the captive market (α) and the cost (c). Moreover, given our assumption that mobile consumers are sufficiently price-sensitive, the price is also decreasing in the size of the mobile market (λ).

Figure 1 shows how price varies with the size of the mobile market in the baseline game: When the mobile market is very small, each firm prices as if it were just a monopoly. As the mobile market becomes large, the firms price as if they had no captive consumers and were simply engaging in differentiated Bertrand competition over the mobile consumers.

4 Loyalty Programs

In this section, we analyze the game in which both firms first simultaneously announce a *preferred customer price* d^f. We let *firm* f*'s preferred customers* be those mobile consumers who prefer f to g when facing the same prices; that is, a mobile consumer $z > 0$ is considered to be a preferred customer for f and a mobile consumer $z < 0$ is considered to be a preferred customer for g.[9] The price that firm f charges its preferred customers is its preferred customer

[9] Alternatively, we could allow at the beginning of the game for each mobile consumer to choose which loyalty program, if any, to join. If we did so, our results would be unchanged because each mobile consumer

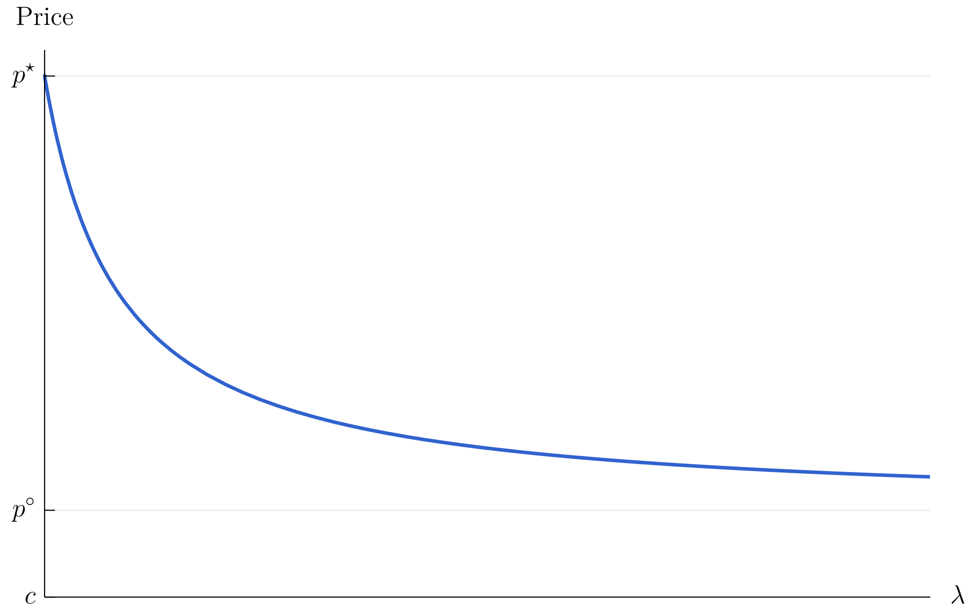

Figure 1: The Nash equilibrium price $p^\dagger$ in the baseline game as a function of the size of the mobile market in blue. Recall that $p^\star$ is the monopoly price if each firm faces only its captive consumers, and p° is the price if both firms face only the mobile consumers. Here, $\alpha = 1$, $\beta = \frac{1}{2}$, and $\sigma = 6$.

price, although those consumers could also buy at the standard price at either firm. All other consumers—i.e., f's captive consumers and the preferred customers of g—can only purchase from f at its standard price.

The game now proceeds as follows:

1. Firms f and g simultaneously announce their preferred customer prices d^f and d^g.

2. Firms f and g observe each other's preferred customer prices and then announce their own standard prices, p^f and p^g.

with $z > 0$ would choose to be a preferred customer of f and each mobile consumer with $z < 0$ would choose to be a preferred customer of g.

The profits of firm f are thus given by

$$\Pi^f \equiv \underbrace{\Big(\min\Big\{d^f, p^f\Big\} - c\Big)\lambda\Big\langle 1-\sigma\Big(\min\Big\{d^f, p^f\Big\} - p^g\Big)\Big\rangle_{[0,1]}}_{\substack{\text{profits from } f\text{'s} \\ \text{preferred customers}}} +$$

$$\underbrace{(p^f - c)(\alpha - \beta p^f)}_{\substack{\text{profits from } f\text{'s} \\ \text{captive demand}}} + \underbrace{(p^f - c)\lambda\Big\langle \sigma\Big(\min\{d^g, p^g\} - p^f\Big)\Big\rangle_{[0,1]}}_{\substack{\text{profits from } g\text{'s} \\ \text{preferred customers}}}. \quad (1)$$

We call the game just described *competition with loyalty programs*.

We now identify an equilibrium for competition with loyalty programs under which the firms exploit loyalty programs to enhance profits.

Proposition 2. *Under competition with loyalty programs, there exists a (subgame-perfect) Nash equilibrium in which:*

1. *Captive consumers pay the monopoly price* $p^\star$.
2. *Mobile consumers pay*

$$d^\star \equiv p^\circ + \frac{\Big(\sqrt{1+\frac{\lambda\sigma}{\beta}} - 1\Big)\rho}{\lambda\sigma}. \quad (2)$$

In particular, in this equilibrium both captive and mobile consumers pay higher prices than in the unique pure-strategy equilibrium of the baseline game.

Proposition 2 shows that—as expected—prices for captive consumers rise with the introduction of loyalty programs. When loyalty programs are present, competition via standard pricing is muted because each firm can "protect" its share of mobile consumers by offering them a preferred customer price low enough that the other firm will not compete for them. Thus, each firm treats its standard price as the price it is offering to its captive consumers only, and so chooses the monopoly price for those consumers.

Proposition 2 also shows that—surprisingly—prices for mobile consumers rise with the introduction of loyalty programs. After preferred customer prices have been set, each firm has to decide whether to either

- compete for the other firm's preferred customers by setting an aggressive standard price; or
- choose its standard price to monopolize its captive consumers and thus cede the other firm's preferred customers to the other firm.

In the equilibrium identified in Proposition 2, each firm chooses a preferred customer price just low enough to induce the other firm to choose the second option; $d^\star$ is the highest preferred customer price that satisfies this requirement, and so we call it the *deterring* preferred customer price.

The deterring preferred customer price $d^\star$ is in fact greater than $p^\dagger$: In the baseline case, the same price is charged to a firm's mobile consumers and its captive consumers; thus, lowering its standard price slightly to capture a larger share of the mobile consumers has only a small (negative) effect on the firm's profits from its captive consumers. By contrast, under competition with loyalty programs, each firm f charges one price—its standard price, p^f—to its captive consumers and the other firm's preferred customers, and a different price—its preferred customer price d^f—to its own preferred customers. Thus, in equilibrium, capturing a larger share of the mobile consumers requires the firm to drop its standard price substantially—from $p^\star$ to a price less than $d^\star$. Hence, choosing a standard price to compete for the other firm's mobile consumers is more costly in terms of foregone profits from captive consumers than choosing that same standard price would be when loyalty programs do not exist. Thus, a higher preferred customer price can be maintained when loyalty programs do exist because competing for mobile consumers is now more costly for the firms.

In effect, when firms can price discriminate between their preferred customers and others, they can set a very high price for other consumers; and it is in their interest to do so, since

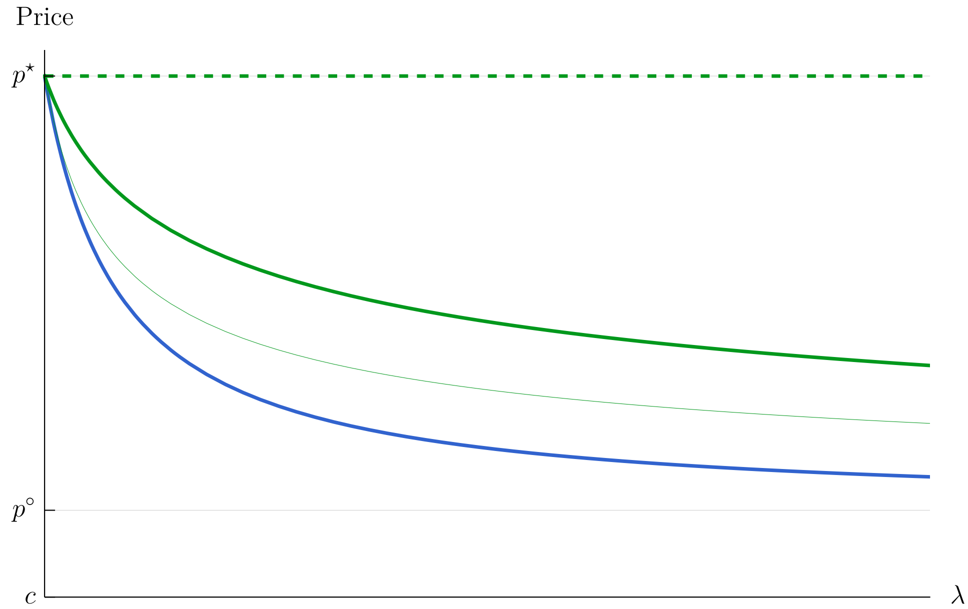

Figure 2: Equilibrium prices under competition with loyalty programs. The green line is the equilibrium price for mobile consumers; the dashed green line is the equilibrium price for captive consumers; the thin green line is the "deviation price," i.e., the best standard price if a firm chooses to compete for the other firm's preferred customers. The other lines are as in Figure 1. Here, $\alpha = 1$, $\beta = \frac{1}{2}$, and $\sigma = 6$.

many of their other customers are captive consumers. In that case, the price faced by mobile consumers considering switching to their non-preferred firm is very high; this makes it possible for each firm to raise the price faced by its own preferred customers.

Figure 2 shows how prices vary with the size of mobile market when loyalty programs are present. Regardless of the size of the mobile market, prices for captive consumers remain at their monopoly level (as shown by the green dashed line). The green line shows the price offered to mobile consumers: each firm always chooses a preferred customer price to disincentivize the other firm from cutting its standard price to take a larger share of the mobile consumer market. When the mobile market is small, each firm only has little incentive to cut its standard price to take a larger share of the mobile consumer market, and so preferred customer prices can remain high. When the mobile market is large, forestalling competition for mobile consumers requires that the firms drop their preferred customer prices. However, for any positive size of the mobile market, the equilibrium preferred customer price is higher than the standard price in the baseline game. In fact, the preferred customer price is so high that, even if a firm decides to compete for the other firm's preferred customers, the standard price that maximizes profitability under this plan is *still* higher than the standard price of the baseline game; this is shown by the thin green line in Figure 2.

Proof of Proposition 2

To prove Proposition 2, we show that there exists a subgame-perfect Nash equilibrium in which:

- Each firm announces the deterring preferred customer price $d^\star$ as its preferred customer price.
- If each firm announces $d^\star$ as its preferred customer price, then each firm announces the monopoly price $p^\star$ as its standard price.
- If either firm does not announce $d^\star$ as its preferred customer price, then each firm announces the baseline price $p^\dagger$ as their standard price.[10]

To prove that this is an equilibrium, we need to show:

1. If each firm announces $d^\star$ as its preferred customer price, then $(p^\star, p^\star)$ is a Nash equilibrium of the induced standard pricing subgame.
2. If one firm announces $d^\star$ as its preferred customer price and the other firm announces a preferred customer price of at least $p^\dagger$, then $(p^\dagger, p^\dagger)$ is a Nash equilibrium of the induced standard pricing subgame.[11]

[10]In the special case in which one firm announces a preferred customer price less than $p^\dagger$, we do not explicitly delineate the strategy profile of the induced subgame; nevertheless, we show that announcing a preferred customer price less than $p^\dagger$ can not be profitable; see Footnote 11.

[11]If a firm f announces a preferred customer price less than $p^\dagger$, then $(p^\dagger, p^\dagger)$ may not be a Nash equilibrium of the induced standard pricing subgame, but then f's profits must be less than its equilibrium profit; see Appendix A.2.

3. If firms play the Nash equilibria of the induced standard pricing games described in points 1 and 2 above, then a firm announcing $d^\star$ as its preferred customer price is a best response to the other firm announcing $d^\star$ as its preferred customer price.

Proof of the first claim: It is not optimal for firm f to announce any standard price in $[d^\star, \infty)$ other than $p^\star$; as doing so would reduce f's profits from its captive consumers, yet would not change its profits from its preferred customers and would not induce any of g's preferred customers to choose f. If firm f chooses a standard price lower than $d^\star$, then the optimal price p^f must maximize (1) given that $d^g = d^\star$ and $p^g = p^\star$; that optimal price is[12]

$$\hat{p}^f(d^\star) \equiv \frac{\alpha + (\beta + \lambda\sigma)c + \lambda(1 + \sigma d^\star)}{2(\beta + \lambda\sigma)}. \tag{3}$$

The deterring preferred customer price $d^\star$ is chosen so as to exactly deter firm f from choosing the price given in (3): Profits for f from choosing that price are the same as from choosing $p^\star$; that is

$$\underbrace{(p^\star - c)(\alpha - \beta p^\star) + (d^\star - c)\lambda}_{\text{Profits from adhering}} = \underbrace{(\hat{p}^f(d^\star) - c)\Big((\alpha - \beta \hat{p}^f(d^\star)) + \lambda(1 + \sigma(d^\star - \hat{p}^f(d^\star)))\Big)}_{\text{Profits from optimal deviation}}. \tag{4}$$

Solving (4) for $d^\star$ yields (2). Thus, each firm choosing the monopoly price $p^\star$ is a best response in the standard pricing subgame induced by each firm choosing $d^\star$ as its preferred customer price.

Proof of the second claim: If at least one firm announces $d^\star$ and the other firm announces a preferred customer price of at least $p^\dagger$, then $(p^\dagger, p^\dagger)$ is a mutual best response, just

[12] This calculation assumes that $\hat{p}^f(d^\star) \geq d^\star - \frac{1}{\sigma}$, i.e., that the optimal deviation by f is at a standard price greater than $d^\star - \frac{1}{\sigma}$, the price at which f would take all of g's preferred customers. If this is not the case, then our calculation below in (4) of the profits f would obtain deviating is too high; but then firm f would have a strict incentive not to deviate.

as in Section 3: If a firm is announcing an aggressive price to attract the other firm's preferred customers, it is a best response by the other firm to do the same.[13]

Proof of the third claim: By announcing $d^\star$, firm f obtains a profit of

$$(d^\star - c)\lambda + (p^\star - c)(\alpha - \beta p^\star).$$

Announcing any other preferred customer price results in a profit of at most

$$(p^\dagger - c)\lambda + (p^\dagger - c)(\alpha - \beta p^\dagger),$$

which is less than $(d^\star - c)\lambda + (p^\star - c)(\alpha - \beta p^\star)$ because $d^\star$ is greater than $p^\dagger$ and $p^\star$ is the price that maximizes profits from the captive consumers.

5 Vampire Attacks

We now augment our game to allow for vampire attacks. The game proceeds as under competition with loyalty programs, except that each firm also announces a *vampire price* v^f and each firm's preferred customers have access to the vampire price at the other firm. That is, the game now proceeds as follows:

1. Firms f and g simultaneously announce their preferred customer prices d^f.
2. Firms f and g observe each other's preferred customer prices and then announce their own standard prices, p^f and p^g, and their own vampire prices v^f and v^g.

[13] In fact, $(p^\dagger, p^\dagger)$ is the unique Nash equilibrium of any subgame in which f offers a preferred customer price of $d^\star$ and g offers any preferred customer price other than $d^\star$. If g were to offer a preferred customer price greater than $d^\star$, then f would find it profitable to offer $\hat{p}(d^g)$ instead of $p^\star$ (assuming g is still offering a standard price of $p^\star$), and so the only mutual best response of the subgame is for both firms to offer $p^\dagger$. Meanwhile, if g were to offer a preferred customer price less than $d^\star$, then g expects lower profits from its preferred customers even if both it and f offered a standard of $p^\star$, and so g would find it profitable to offer $\hat{p}(d^\star)$ instead of $p^\star$ (assuming f is still offering a standard price of $p^\star$), and so the only mutual best response of the subgame is for both firms to offer $p^\dagger$.

As with standard prices, we assume that each firm's preferred customers have access to that firm's vampire price when it is lower than the preferred customer price. Thus, the profits of firm f are now given by

$$\Pi^f \equiv \underbrace{\left(\min\left\{v^f, d^f, p^f\right\} - c\right)\lambda\left\langle 1 - \sigma\left(\min\left\{v^f, d^f, p^f\right\} - \min\{v^g, p^g\}\right)\right\rangle_{[0,1]}}_{\substack{\text{profits from } f\text{'s} \\ \text{preferred customers}}} +$$
$$\underbrace{(p^f - c)(\alpha - \beta p^f)}_{\substack{\text{profits from} \\ \text{captive demand}}} + \underbrace{\left(\min\left\{v^f, p^f\right\} - c\right)\lambda\left\langle \sigma\left(\min\{v^g, d^g, p^g\} - \min\left\{v^f, p^f\right\}\right)\right\rangle_{[0,1]}}_{\substack{\text{profits from } g\text{'s} \\ \text{preferred customers}}}. \quad (5)$$

Relative to (1), when vampire attacks are possible, there are three notable differences in the expression for the profits of firm f: First, f's preferred customers have access to the vampire price of firm g, and so f's demand from its preferred customers is now given by

$$\left\langle 1 - \sigma\left(\min\left\{v^f, d^f, p^f\right\} - \min\{v^g, p^g\}\right)\right\rangle_{[0,1]}$$

instead of $\left\langle 1 - \sigma\left(\min\left\{d^f, p^f\right\} - p^g\right)\right\rangle_{[0,1]}$. Second, g's preferred customers have access to firm f's vampire price, and so f's demand from g's preferred customers is given by $\left\langle \sigma\left(\min\{v^g, d^g, p^g\} - \min\left\{v^f, p^f\right\}\right)\right\rangle_{[0,1]}$ instead of $\left\langle \sigma\left(\min\{d^g, p^g\} - p^f\right)\right\rangle_{[0,1]}$. Finally, f's margin on g's preferred customers now depends on which of f's prices those consumers enjoy. We call the game just described *competition with vampire attacks.*

Vampire attacks can substantially heighten competition for mobile consumers.

Proposition 3. *Under competition with vampire attacks, in every pure-strategy subgame-perfect Nash equilibrium:*

1. *Captive consumers pay the monopoly price $p^\star$.*

2. *Mobile consumers pay the competitive price p°.*

In particular, in any pure-strategy equilibrium, captive consumers pay the same price and

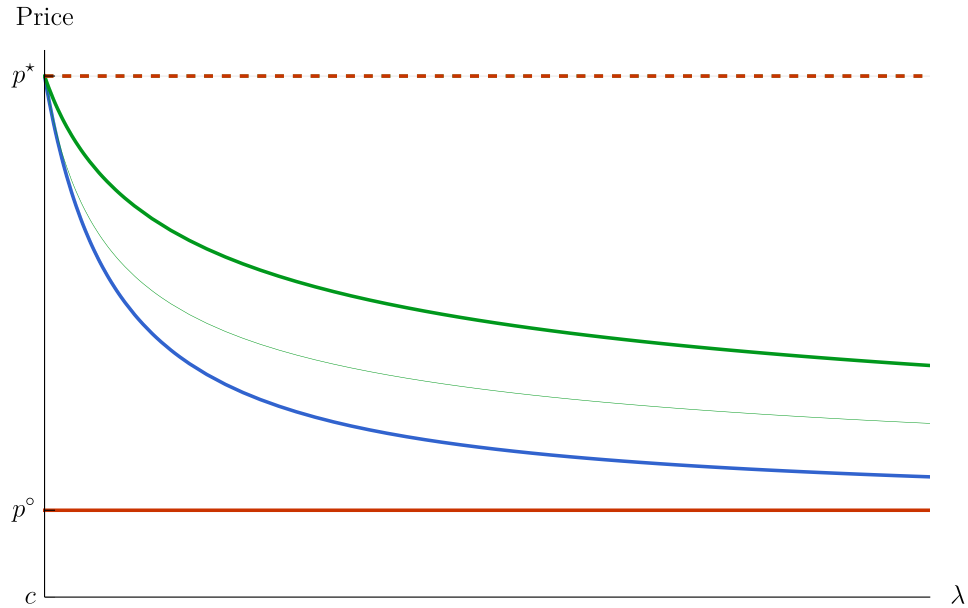

Figure 3: Equilibrium prices under competition with vampire attacks. The red line is the equilibrium price for mobile consumers; the red dashed line is the equilibrium price for captive consumers. The other lines are as in Figure 2. Here, $\alpha = 1$, $\beta = \frac{1}{2}$, and $\sigma = 6$.

mobile consumers pay strictly lower prices than in the equilibrium of the competition with loyalty programs game described in Proposition 2.

In the presence of vampire attacks, a firm f can attract the other firm's preferred customers without lowering its price for its captive consumers. Firm f does this by offering an attractive vampire price, which allows it to compete more fiercely for the other firm's preferred customers; simultaneously, firm f must choose a preferred customer price to dissuade such attacks by its rival g. This effectively transforms the game into two separate games: Each firm chooses its standard price to monopolize its captive consumers, while the result of firms offering preferred customer and vampire prices is isomorphic to the outcome of a standard differentiated Bertrand game. As a result, vampire attacks lead to dramatically different prices for the two types of consumers: captive consumers pay the monopoly price, while mobile consumers now pay a price lower than the price they paid when no loyalty programs were present. Essentially, when loyalty status is transparent, firms are naturally induced to compete fiercely for mobile consumers.

Thus, in every equilibrium under competition with vampire attacks, the price that mobile consumers pay is strictly less than prices that can be sustained in the competition with loyalty programs game.

Figure 3 shows equilibrium prices in the presence of vampire attacks. In this case, prices no longer depend on the size of the mobile market. The red dashed line shows that captive consumers are still subject to monopoly pricing. Meanwhile, the solid red line shows that mobile consumers enjoy much lower prices—prices fall to the level of classical differentiated Bertrand competition for any size of the mobile market.

Proof of Proposition 3

We show that there exists a subgame-perfect Nash equilibrium in which:[14]

- Each firm announces the competitive price p° as its preferred customer price.
- If each firm announces the competitive price p° as its preferred customer price, then each firm announces the monopoly price $p^\star$ as its standard price and the competitive price p° as its vampire price.

To prove that such an equilibrium exists, we need to show:

1. If each firm announces p° as its preferred customer price, then announcing $(p^\star, p^\star)$ as standard prices and (p°, p°) as vampire prices is a Nash equilibrium of the induced pricing subgame.
2. If any f firm announces a price other than p° as its preferred customer price, then a Nash equilibrium of the resulting subgame provides that firm with no greater profits than along the equilibrium path.

Proof of the first claim: Suppose each firm announces p° as its preferred customer price; then firm f expects firm g to announce $p^\star$ as its standard price and p° as its vampire

[14] The uniqueness argument is deferred to Appendix A.4.

price. It is then immediate that it is optimal for f to announce a standard price of $p^\star$ so as to earn monopoly profits from its captive consumers (since choosing a lower standard price to steal the other firm's preferred customers is dominated by instead choosing a vampire price to steal those customers).

Meanwhile, p° corresponds to the price that would arise when firms face only mobile demand. Thus, the profits of f are strictly increasing in its vampire price for any vampire price less than p°: firm f's profits increase with v^f, as f makes more profits from the mobile consumers since p° is a best response to p°. For vampire prices more than p°, f's profits are invariant with v^f as the other firm's preferred customers will not switch to f for any such price.

Proof of the second claim: Regardless of the preferred customer price announced by the deviating firm f, it is optimal for each firm to announce a standard price of $p^\star$ (for the same reasons as in the proof of the first claim).

If firm f announces a preferred customer price higher than p°, then in our equilibrium firm g expects firm f to announce a vampire price of p° and so the outcome is as if both firms announce p° as both their preferred customer price and their vampire price; that is, both firms choose vampire prices equal to the prices chosen when the two firms face only mobile demand. If firm f announces a preferred customer price strictly less than p°, then in the resulting subgame both firms will offer a vampire price less than p° and have lower profits than if they offered preferred customer and vampire prices of p°; see Appendix A.3 for details.

Costly Vampire Attacks

Our results for vampire attacks assume that it is costless for firms to verify whether a consumer is a preferred customer of the other firm. Realistically, such verification may be costly for the firm: For example, airlines "status match" but require documentation of the flyer's status with a competitor, and onboarding the new consumer is administratively costly. Thus, we now consider the case in which verification is costly; each firm incurs an extra cost η whenever it recruits a preferred customer of its competitor. Under costly vampire attacks, profits are given by:

$$\Pi^f \equiv \underbrace{\left(\min\left\{v^f, d^f, p^f\right\} - c\right)\lambda\left\langle 1 - \sigma\left(\min\left\{v^f, d^f, p^f\right\} - \min\{v^g, p^g\}\right)\right\rangle_{[0,1]}}_{\substack{\text{profits from } f\text{'s} \\ \text{preferred customers}}} +$$

$$\underbrace{(p^f - c)(\alpha - \beta p^f)}_{\substack{\text{profits from} \\ \text{captive demand}}} + \underbrace{\left(\min\left\{v^f, p^f\right\} - \eta \mathbb{1}_{[v^f < p^f]} - c\right)\lambda\left\langle \sigma\left(\min\{v^g, d^g, p^g\} - \min\left\{v^f, p^f\right\}\right)\right\rangle_{[0,1]}}_{\substack{\text{profits from } g\text{'s} \\ \text{preferred customers}}}.$$

Intuitively, the costly vampire attack case falls between the cases of competition with loyalty programs and competition with vampire attacks.

Proposition 4. *Under competition with costly vampire attacks, there exists a subgame-perfect Nash equilibrium in which:*

1. *Captive consumers pay the monopoly price $p^\star$.*

2. *Mobile consumers pay $p^\circ + \eta$ when $\lambda \leq \hat{\lambda}$ and $d^\star$ when $\lambda \geq \hat{\lambda}$, where $\hat{\lambda} \equiv \frac{\rho(\rho - 2\beta\eta)}{\beta\eta^2\sigma}$.*

When the size of the mobile market is small, the preferred customer price is determined by the threat of a vampire attack. Suppose firm g were to choose a preferred customer price d^g and vampire price v^g higher than $p^\circ + \eta$; then it would be optimal for firm f to respond by choosing a vampire price (strictly) between $p^\circ + \eta$ and $\min\{d^g, v^g\}$ as firm f could then take a few of firm g's consumers at a positive profit. Only when firm g lowers its preferred customer price to $p^\circ + \eta$ is it a best response for firm f to do the same. Intuitively, when firm f is considering whether to recruit firm g's preferred customers it treats its per-unit cost as $c + \eta$. Thus, the outcome is as-if the firms played a differentiated Bertrand competition game with cost $c + \eta$.

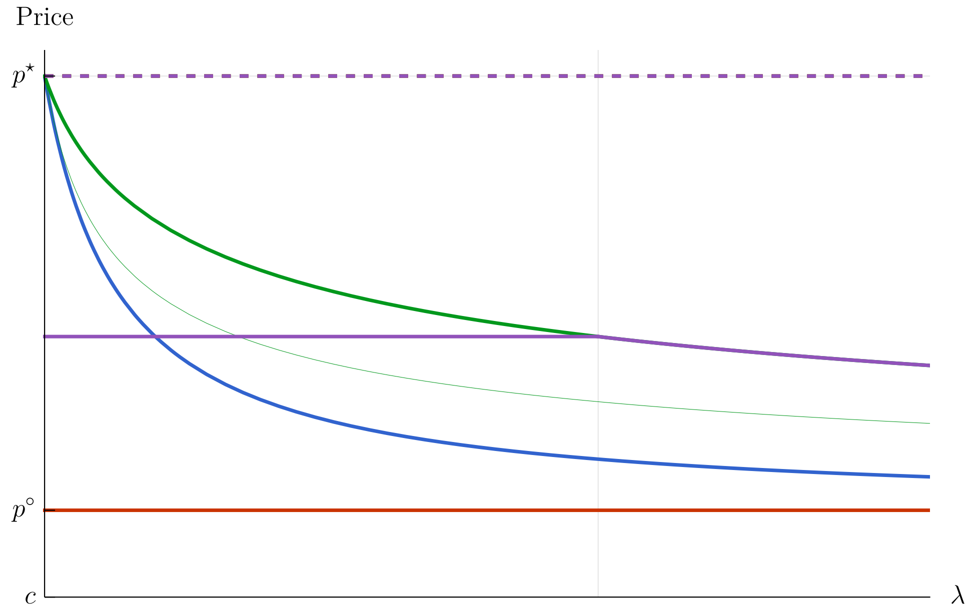

Figure 4: Equilibrium prices under competition with costly vampire attacks. The purple line is the equilibrium price for mobile consumers; the dashed purple line is the equilibrium price for captive consumers. The other lines are as in Figure 3. Here, $\alpha = 1$, $\beta = \frac{1}{2}$, and $\sigma = 6$. and $\eta = \frac{1}{3}$.

When the size of the mobile market is large, however, it is no longer cost-effective for f to take g's preferred customers via a costly vampire attack—if such an attack were profitable, it would be even more profitable for f to lower its standard price to attract g's preferred customers directly. Thus, g must set a low enough preferred customer price that f will not find it profitable to choose a low standard price to attract g's preferred customers, just as under competition with loyalty programs.

Figure 4 shows the Proposition 4 equilibrium prices in the presence of costly vampire attacks. The purple dashed line shows that captive consumers are still subject to monopoly pricing. Unsurprisingly, the prices enjoyed by mobile consumers are now higher than under costless vampire attacks, as shown by the purple line: When the mobile market is small, the price faced by mobile consumers is invariant to the size of the mobile market: Since the relevant threat is a vampire attack, and the cost of a vampire attack is per consumer, the size of the mobile market does not play a role. When the mobile market is large, the relevant constraint on preferred customer prices is the concern that the other firm will lower its standard price to increase market share; thus, prices for mobile consumers are the same as when vampire attacks are not possible.

6 Conclusion

This paper shows that loyalty programs can dampen competition not only for those consumers who are not enrolled in loyalty programs but also for the loyalty program members themselves, resulting in higher prices for all consumers. Vampire attacks change this dynamic by making the competition for loyalty program customers fierce, since vampire attacks make it easier for firms to recruit their competitors' loyal customers.

Per-user transaction data being public on the blockchain promotes competition for high-volume users between crypto platforms; by contrast, loyalty status in legacy loyalty programs such as those of airlines and hotels is highly opaque. Our analysis suggests that "blood-sucking" platform competition can enable consumers to get better prices. While our model only considers price competition, we would also expect that vampire attacks would lead firms to compete more fiercely on the quality of the services offered to preferred customers.[15] And, indeed, for crypto marketplaces for both fungible and non-fungible tokens, we have seen both incumbents and entrants reduce fees and introduce features that are particularly valuable for heavy users (Liu et al., 2022; Kominers et al., 2022).

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[15] One caveat is that for intensified competition via vampire attacks to benefit consumers it is necessary that it does not incentivize firms to engage in malbehavior; for example, Bernstein and Kominers (2022) have argued that the intense competition between centralized crypto financial service providers led them to take on excessive risk in order to offer higher returns to account holders.

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A Proofs

A.1 Proof of Proposition 1

The problem of firm f is to solve

$$\max_{p^f \in [c,\infty)} \left\{ (p^f - c)(\alpha - \beta p^f) + (p^f - c)\lambda \left\langle 1 - \sigma(p^f - p^g) \right\rangle_{[0,2]} \right\}.$$

The first-order condition for f's problem is (assuming that $1 - \sigma(p^f - p^g) \in (0, 2)$) is

$$0 = \alpha + \lambda + (\beta + \lambda\sigma)c - 2p^f(\beta + \lambda\sigma) + p^g \lambda\sigma.$$

Thus, there exists a symmetric subgame-perfect Nash equilibrium in which each firm sets a standard price of

$$p^\dagger = \frac{\alpha + \lambda + (\beta + \lambda\sigma)c}{2\beta + \lambda\sigma}.$$

(Note that $1 - \sigma(p^f - p^g) = 1 \in (0, 2)$ in this symmetric equilibrium.)

To show that this equilibrium is unique, we proceed in three steps:

No firm offers a price $p^f > p^\star$ in an equilibrium: Suppose some such equilibrium (p^f, p^g) existed with $p^f \geq p^g$. Now consider a deviation by f to the price $p^f - \epsilon$ for some small $\epsilon > 0$.

If $\sigma(p^f - p^g) > 1$, then firm f does not obtain any mobile consumer at either p^f or $p^f - \epsilon$ (for ϵ small enough); hence, f's profits at $p^f - \epsilon$ are higher as it is closer to the monopoly price (for ϵ small enough).[16]

[16] If p^f is so high that the firm does not get any captive or mobile consumers, then f could deviate to the $c + \epsilon$ and obtain positive profits.

If $\sigma(p^f - p^g) \leq 1$, then we calculate the change in f's profits as

$$\begin{aligned}
((p^f - c)\sigma - (1 - \sigma(p^f - p^g)))\lambda\epsilon + O(\epsilon^2) &\geq ((p^f - c)\sigma - 1)\lambda\epsilon + O(\epsilon^2) \\
&> \left(\frac{\alpha + \beta c}{2\beta} - c\right)\left(\frac{2\beta}{\alpha - \beta c} - 1\right)\lambda\epsilon + O(\epsilon^2) \\
&= \lambda\epsilon + O(\epsilon^2).
\end{aligned}$$

The first inequality follows from the fact that $\sigma(p^f - p^g) \geq 0$ as $p^f \geq p^g$; the second follows from the facts that $p^f > p^\star = \frac{\alpha+\beta c}{2\beta}$ and that $\sigma \geq \frac{2\beta}{\alpha-\beta c}$ as price responsiveness is positive.

In any equilibrium, both firms obtain mobile consumers: Suppose not. Then there exists an equilibrium (p^f, p^g) such that firm g obtains all the mobile consumers. Thus, $p^g = p^f - \frac{1}{\sigma}$ as at any higher price g does not obtain all of the mobile consumers and at any lower price g could improve its profits by increasing price by ϵ, as this would increase its margin on the mobile consumers without changing its level of demand from mobile consumers, and also increase its profits from captive consumers as $p^f \leq p^\star$ (from the preceding step).

Thus, if firm f does not find dropping its price by ϵ to be profitable, then

$$\frac{\partial_\downarrow \Pi^f(p^f, p^g)}{\partial p^f} = -(\alpha + \beta c - 2\beta p^f) + (p^f - c)\lambda\sigma \leq 0, \tag{A.1}$$

where $\frac{\partial_\downarrow \Pi^f(p^f,p^g)}{\partial p^f}$ is the directional change in f's profits from dropping its price. Since $p^f > p^g$, (A.1) implies that

$$-(\alpha + \beta c - 2\beta p^g) + (p^g - c)\lambda\sigma \leq 0. \tag{A.2}$$

We calculate the directional derivative of g increasing its price as

$$\frac{\partial_{\uparrow}\Pi^f(p^f,p^g)}{\partial p^g} = \alpha + \beta c - 2\beta p^g + 2\lambda - (p^g - c)\lambda\sigma$$
$$\geq 0 + 2\lambda$$
$$> 0,$$

where the first inequality follows from (A.2). Thus, firm g will find it profitable to raise its price, and so (p^f, p^g) cannot be an equilibrium—a contradiction.

There exists a unique equilibrium: From the preceding step, we know that in any equilibrium, both firms obtain some mobile consumers. Thus, we have that (p^f, p^g) must satisfy the first-order conditions for both firms, i.e.,

$$0 = \alpha + \lambda + (\beta + \lambda\sigma)c - 2p^f(\beta + \lambda\sigma) + p^g\lambda\sigma$$
$$0 = \alpha + \lambda + (\beta + \lambda\sigma)c - 2p^g(\beta + \lambda\sigma) + p^f\lambda\sigma;$$

the determinant of this linear system is non-zero, and hence we see that it has a unique solution.[17]

A.2 The Low Preferred Customer Price Case of the Proof of Proposition 2

If a firm f chooses a preferred customer price less than $p^{\dagger}$, there is not necessarily a pure-strategy equilibrium of the induced subgame. Nevertheless, it is immediate that f's profits are less than with equilibrium play: Even if g chooses to not compete for g's preferred customers

[17] The coefficient matrix for this system is $\begin{pmatrix} 2(\beta+\lambda\sigma) & -\lambda\sigma \\ -\lambda\sigma & 2(\beta+\lambda\sigma) \end{pmatrix}$ with determinant $4\beta^2+8\beta\lambda\sigma+3\lambda^2\sigma^2$.

by setting a standard price less than g's preferred customer price, f's profits are bounded by

$$\Pi^f \equiv \Big(\min\Big\{d^f, p^f\Big\} - c\Big)\lambda + (p^f - c)(\alpha - \beta p^f) + (p^f - c)\lambda\Big\langle\sigma\Big(d^\star - p^f\Big)\Big\rangle_{[0,1]};$$

note that since g is setting $p^g = p^\star > d^\star$, firm f obtains all of f's preferred customers for sure and g's preferred customers decide between f's standard price and g's preferred customer price. The maximum over p^f of the preceding expression cannot be greater than the maximum over p^f of

$$\Pi^f \equiv \Big(\min\Big\{d^\star, p^f\Big\} - c\Big)\lambda + (p^f - c)(\alpha - \beta p^f) + (p^f - c)\lambda\Big\langle\sigma\Big(d^\star - p^f\Big)\Big\rangle_{[0,1]} \tag{A.3}$$

as $d^f < d^\star$. But we know that the maximal value of (A.3) is no more than the profits gained by playing the equilibrium strategy (as maximizing (A.3) over p^f was analyzed in the first step of the proof of Proposition 2); thus f cannot be better off by choosing $d^f < p^\dagger$ than if it chooses $d^f = d^\star$.

A.3 The Low Preferred Customer Price Case of the Proof of Proposition 3

If firm f announces a preferred customer price $d^f = \bar{d}$ lower than p°, then there exists an equilibrium of the subgame in which:

- firm f announces a vampire price of $v^f = \max\Big\{\bar{d}, \frac{1}{3\sigma} + c\Big\}$; and
- firm g announces a vampire price of $v^g = \frac{p^\circ + v^f}{2}$.

That is, when firm f's preferred customer price $d^f = \bar{d}$ is only a little less than p°, firm f implements $\bar{d}$ as its vampire price as well; but when firm f's preferred customer price $\bar{d}$ is a lot less than p°, firm f chooses a vampire price that is higher to maximize its profits over firm

g's preferred customers.[18] Thus, using the prices v^f and v^g specified previously, we calculate from (5) that the profits for f from all mobile consumers are $\frac{1}{2}(\bar{p}-c)\lambda(3-(\bar{p}-c)\sigma)$ when $\frac{1}{3\sigma}+c \leq \bar{p} \leq p^\circ$ and at most $\frac{4\lambda}{9\sigma}$ when $\bar{p} \leq \frac{1}{3\sigma}+c$. Both of these are less than $\frac{\lambda}{\sigma}$, and so the profits for firm f from mobile consumers from choosing a preferred customer price other than p° are no higher than from following f's prescribed strategy.

A.4 Uniqueness of Outcomes in Proposition 3

Consider any pure-strategy equilibrium of competition with vampire attacks. In the subgame induced by a general (d^f, d^g), suppose that firm f sets a standard price $p^f \neq p^\star$. We show that there then exists a profitable deviation for f. Let $\hat{p}^f = p^\star$ and $\hat{v}^f = \min\left\{v^f, p^f\right\}$; under this new strategy in the subgame, f obtains exactly the same mobile consumers at exactly the same margin as under (p^f, v^f) and has strictly higher profits from its captive consumers.

From the analysis of Appendix A.3, we know that each firm will never offer a preferred customer price less than p° in equilibrium. Thus, in any subgame induced by a general $(d^f, d^g) \geq (p^\dagger, p^\dagger)$, each firm f will choose a standard price of $p^\star$ and some vampire price v^f. To see that there exists a unique equilibrium outcome of the subgame, we proceed in two steps:

In any equilibrium, both firms obtain mobile consumers: Suppose not. Then there exists an equilibrium (v^f, v^g) such that firm g obtains all the consumers. If $v^g \leq c$, then firm g can increase it price by ϵ to increase its profits. If $v^g > c$, then firm f can set its vampire price equal to v^g, and obtain half of the mobile consumers at a positive margin.

[18] We obtain these prices by noting that in this subgame firm f's profits are given by

$$\left(\min\left\{v^f, \bar{p}\right\} - c\right)\lambda + (p^\star - c)(\alpha - \beta p^\star) + \left(v^f - c\right)\left\langle \sigma\left(\min\{v^g, d^g\} - v^f\right)\right\rangle_{[0,1]}$$

and firm g's profits are given by

$$(p^\star - c)(\alpha - \beta p^\star) + (\min\{v^g, d^g\} - c)\left\langle 1 - \sigma\left(\min\{v^g, d^g\} - \min\left\{v^f, \bar{p}\right\}\right)\right\rangle_{[0,1]},$$

and solving for the Nash equilibrium. (Note that in the above expressions, we have already incorporated the fact that it is not optimal for g to respond by setting a vampire price lower than v^f.)

There exists a unique equilibrium: For each firm f, if $v^f > p^f$, it is weakly more profitable to set its vampire price equal to its preferred customer price, i.e., to deviate to $\hat{v}^f = p^f$. Thus, we can treat firm f's problem in the second stage as

$$\max_{v^f \in [c, p^f]} \Big\{ (v^f - c)\lambda \Big\langle 1 - \sigma(v^f - v^g) \Big\rangle_{[0,2]} \Big\}.$$

From the preceding step, we know that in any equilibrium, both firms obtain some mobile consumers; thus, in any equilibrium, v^f must satisfy

$$\max_{v^f \in [c, p^f]} \Big\{ (v^f - c)\lambda(1 - \sigma(v^f - v^g)) \Big\}.$$

Hence, we have that (v^f, v^g) must satisfy the first-order conditions for both firms, i.e.,

$$\begin{aligned} 0 &= 1 - \sigma c - 2\sigma v^f + \sigma v^g \\ 0 &= 1 - \sigma c - 2\sigma v^g + \sigma v^f. \end{aligned}$$

The determinant of this linear system is non-zero and thus the system has a unique solution.[19]

Thus, for any $(d^f, d^g) \gneqq (p^\circ, p^\circ)$ the only equilibrium of the subgame is for both firms to choose a vampire price of p°. If both firms offer a preferred customer price of p°m then there exists an equilibrium in which both firms may offer higher vampire prices, but the prices enjoyed by the mobile consumers are the same as if both firms had offered a vampire price of p°.

[19]The coefficient matrix for this system is $\begin{pmatrix} 2\sigma & -\sigma \\ -\sigma & 2\sigma \end{pmatrix}$ with determinant $3\sigma^2$.

A.5 Proof of Proposition 4

When $\lambda \geq \hat{\lambda}$, the proof of Proposition 4 is the same as the proof of Proposition 2; note that in this case the possibility of vampire attacks is irrelevant since the preferred customer price is already lower than necessary to dissuade vampire attacks.

To prove Proposition 4 when $\lambda \leq \hat{\lambda}$, we show that there exists a subgame-perfect Nash equilibrium in which:

- Each firm announces $p^\circ + \eta$ as its preferred customer price; and
- If each firm announces $p^\circ + \eta$ as its preferred customer price, then each firm announces the monopoly price $p^\star$ as its standard price and any price greater than or equal to $p^\circ + \eta$ as its vampire price.

To prove that such an equilibrium exists, we need to show:

1. If each firm announced $p^\circ + \eta$ as its preferred customer price, then announcing $(p^\star, p^\star)$ as standard prices and $(p^\circ + \eta, p^\circ + \eta)$ as vampire prices is a Nash equilibrium of the induced pricing subgame.
2. If any f firm announced a price other than $p^\circ + \eta$ as its preferred customer price, then a Nash equilibrium of the resulting subgame provides that firm with lower profits than along the equilibrium path.

Proof of the first claim: Given that each firm announces $p^\circ + \eta$ as its preferred customer price, and that g will announce $p^\star$ as its standard price and $p^\circ + \eta$ as its vampire price, it is optimal for f to announce a standard price of $p^\star$ so as to earn monopoly profits from its captive consumers: Choosing a standard price lower than $p^\circ + \eta$ in order to attract firm g's preferred customers is even less profitable than in the proof Proposition 2 since the preferred customer price of g is lower. Thus, the problem of firm f is to solve

$$\max_{v^f \leq p^\circ + \eta} \Big\{ (v^f - c)\lambda + (v^f - c - \eta)\sigma((p^\circ + \eta) - v^f) \Big\}$$

which is optimized at $v^f = p^\circ + \eta$. For vampire prices more than $p^\circ + \eta$, firm f's profits are invariant with v^f as the other firm's preferred customers will not switch to f for any such price.

Proof of the second claim: If firm f announces a preferred customer price higher than $p^\circ + \eta$, then in our equilibrium firm g expects firm f to announce a vampire price of $p^\circ + \eta$ and so the outcome is as if both firms had announced $p^\circ + \eta$ as both their preferred customer price and their vampire price. If firm f announces a preferred customer price $p^f = \bar{p}$ lower than p°, first note that it is still optimal for each firm to announce a standard price of $p^\star$ (for the same reasons as in the proof of the first claim). With respect to the choice of vampire prices, there exists an equilibrium of the subgame in which

- Firm f announces a vampire price of $v^f = \max\left\{\bar{p}, \frac{1}{3\sigma} + c + \frac{2\eta}{3}\right\}$; and
- Firm g announces a vampire price of $\frac{p^\circ + v^f}{2}$.

That is, when firm f's preferred customer price $\bar{p}$ is only a little less than p°, firm f implements $\bar{p}$ as its vampire price as well; but when firm f's preferred customer price $\bar{p}$ is a lot less than p°, firm f chooses a vampire price that is higher to maximize its profits over firm g's preferred customers.[20] Thus, we calculate that profits for f from the mobile consumers are no higher than following f's prescribed strategy.

[20]We obtain these prices by noting that in this subgame firm f's profits are given by

$$\left(\min\left\{v^f, \bar{p}\right\} - c\right)\lambda + (p^\star - c)(\alpha - \beta p^\star) + \left(v^f - c - \eta\right)\left\langle \sigma\left(\min\{v^g, d^g\} - v^f\right)\right\rangle_{[0,1]}$$

and firm g's profits are given by

$$(p^\star - c)(\alpha - \beta p^\star) + (\min\{v^g, d^g\} - c)\left\langle 1 - \sigma\left(\min\{v^g, d^g\} - \min\left\{v^f, \bar{p}\right\}\right)\right\rangle_{[0,1]}$$

and solving for the Nash equilibrium. (Note that in these expressions we have already incorporated the fact that it is not optimal for g to respond by setting a vampire price lower than $\bar{p}$.)