(This proposal benefits greatly from discussion with Vitalik)

Background Introduction

Since 2018, Vitalik has advocated for quadratic financing (QF) as a method to generate optimal provision of public goods in a decentralized, self-organizing ecosystem (Buterin, Hitzig, and Weyl, 2018). One of the most challenging problems faced by QF is collusion. For example, one actor or a group of coordinated actors may control multiple addresses or collude to "game the system" and extract unjustifiable subsidies in QF.

Since 2019, Gitcoin Grants has run 4 rounds of QF. Gitcoin Grants uses an innovative method: pairwise-bounded coordination subsidies (Buterin, 2019). However, I find the specification of the discoordination coefficient in Buterin (2019) not very intuitive and convincing. Besides, the tweakable parameter in Buterin (2019) mainly serves to bound extractable value for any pair of investors, but not doing enough to adjust for the collusion between them. In this post, I propose a new method to improve pairwise coordination subsidies with rich economic meaning.

A New Specification of the Discoordination Coefficient

Adopting the notation of Buterin (2019), I propose a new specification of the discoordination coefficient:

$$k_{i,j} = 1 - \frac{\sum_p \sqrt{c_{i \rightarrow p} } \sqrt{c_{j \rightarrow p} }}{(\sum_p c_{i\rightarrow p})^{1/2} (\sum_p{c_{j\rightarrow p}})^{1/2}} \tag{1}$$

Where $k_{i,j}$ is the discoordination coefficient between investor i and j, $c_{i\rightarrow p}$ and $c_{j\rightarrow p}$ stand for investor i and j's investment to project p, and $\sum_p$ means the sum over all projects. The discoordination coefficient measures how independent a pair of investors are in making investment decisions. There are two ways to understand the economic meaning of (1).

First, consider two vectors $I = [\sqrt{c_{i\rightarrow 1}}, \sqrt{c_{i\rightarrow 2}}, ... , \sqrt{c_{i\rightarrow P}}]$ and $J = [\sqrt{c_{j\rightarrow 1}}, \sqrt{c_{j\rightarrow 2}}, ... , \sqrt{c_{j\rightarrow P}}]$, where P stands for the total number of projects.

Let $||\quad ||$ be the magnitude of a vector:

$$||I|| = (\sum\nolimits_{p} c_{i\rightarrow p})^{1/2}, ||J||= (\sum\nolimits_p{c_{j\rightarrow p}})^{1/2} \tag{2}$$

Let $I\cdot J$ be the inner product of the two vectors:

$$I\cdot J = \sum\nolimits_p \sqrt{c_{i\rightarrow p}} \sqrt{c_{j\rightarrow p}} \tag{3}$$

Let θ be the angle between two vectors. Therefore,

$$I\cdot J = ||I|| \cdot ||J|| \cdot cos(\theta) \tag{4}$$

If the two vectors are orthogonal (i.e. $\theta=\pi/2$

), then $\cos(\theta)=0$

and $I\cdot J=0$

. If they are codirectional (i.e. $\theta=0$

), then $\cos(\theta)=1$

and $I \cdot j = ||I||\cdot||J||$

.

From (1)-(4), it is easy to see that

$$k_{i,j} = 1-\cos(\theta) \tag{5}$$

For (5), Let's first consider two extreme cases.

Case #1

: If for every p

, $\sqrt{c_{i\rightarrow p}}\sqrt{c_{j\rightarrow p}}=0$

, which means i

and j

have no investment in common, this corresponds to the case of $\theta = \pi/2$

and $\cos(\theta)=0$

(i.e. orthogonal vectors). In this case, $k_{i,j}=1$

, means i

and j

have maximum discoordination.

Case #2

: If there exists a number λ

such that for every p

, $\sqrt{c_{i\rightarrow p}}=\lambda\sqrt{c_{j\rightarrow p}}$

, which means i

and j

make the same investment decision. Although they may have different amounts of money to invest, they share the same asset allocation decision in term of percentage. This corresponds to the case of $\theta = 0$

and $\cos(\theta)=1$

. (i.e. codirectional vectors). In this case, $k_{i,j}=0$

, which means i

and j

have maximum coordination.

The other cases lie between Case #1

and #2:

θ

is between 0

and $\pi/2$, and $\cos(\theta)$ is between 0 and 1. Therefore, $k_{i,j}$ is between 0 and 1. The larger $k_{i,j}$ is, the higher discoordination is.

Second, we can also understand the meaning of $k_{i,j}$ from the theory of probability. Ex ante, every investor's investment decision is a random variable. Therefore, $[\sqrt{c_{i\rightarrow 1}}, \sqrt{c_{i\rightarrow 2}}, ... , \sqrt{c_{i\rightarrow P}}]$ and $[\sqrt{c_{j\rightarrow 1}}, \sqrt{c_{j\rightarrow 2}}, ... , \sqrt{c_{j\rightarrow P}}]$ are ex post realization of inverstment decisions of i and j, respectively. In (1), $\frac{\sum_p \sqrt{c_{i \rightarrow p} } \sqrt{c_{j \rightarrow p} }}{(\sum_p c_{i\rightarrow p})^{1/2}(\sum_p{c_{j\rightarrow p}})^{1/2}}$ is simply a sample estimation of the correlation coefficient between their investment decisions. The higher the correlation coefficient is, the smaller the discoordination coefficient is.

Discoordination Adjusted Subsidies

For project p, i and j's subsidy after adjusting for coordination is

$$2k_{i,j}\sqrt{c_{i\rightarrow p}}\sqrt{c_{j\rightarrow p}} \tag{6}$$

The subsidy is 0 whenever $\sqrt{c_{i\rightarrow p}}\sqrt{c_{j\rightarrow p}}=0$ or i and j are perfect correlated.

From (6), I define the discoordination adjusted subsidy (DAS) extracted by i and j from all projects as

$$DAS_{i,j} = \sum\nolimits_p{2k_{i,j}\sqrt{c_{i\rightarrow p}}\sqrt{c_{j\rightarrow p}}}\tag{7}$$

It is worthy pointed out that to estimate $k_{i,j}$ by (1), we can use sample data from past rounds of quadratic financing, not just the current round. In this way, we can make the estimation of the discoordination coefficient more robust and less constrained by current sample size.

Adjustment for Pairwise Bound

Suppose for any pair of investors, we introduce an upper bound on the total subsidies they extract from all projects. Let B be the universally applied upper bound. B is similar to the tweakable parameter in Buterin (2019) .

We need to accommodate $DAS_{i,j}$ with the upper bound B. Similar to Buterin (2019b) , I use the following formula :

$$\frac{B\cdot DAS_{i,j}}{B+ DAS_{i,j}}\tag{8}$$

Obviously, $\frac{B\cdot DAS_{i,j}}{B+ DAS_{i,j}}\leq B$ and it is an increasing function of $DAS_{i,j}$.

Also similar to Buterin (2019), if there exists a particular level of total subsidies T, we need to solve for B that satisfies the following constrains (N is the total number of investors):

$$\sum\nolimits_{1\leq i \ne j \leq N}{\frac{B\cdot DAS_{i,j}}{B+ DAS_{i,j}}}=T \tag{9}$$

It is not difficult to solve (9) with numeric methods.

Comparison with Current Method

Consider the case that i and j make the same investment decision (i.e. there exists a number λ such that for every for p, $\sqrt{c_{i\rightarrow p}}=\lambda\sqrt{c_{j\rightarrow p}}$. With my method, they will receive no subsidy. But with current method, they could extract a large amount of subsidy, being only scaled downed by the tweakable parameter.

For example, the scenario in Buterin (2019). Suppose k coordinated agents all contribute a very large amount of money W toward a project. Since they are perfect correlated, the subsidies that they can extract are 0 with my method. However, with current method, the subsidies they can extract are $\frac{2MW}{M+W}< 2M$, where M is the tweakable parameter.

Vitalik suggests the following case. Suppose we have two donors A and B and three projects A, B and C

, and the pairs of donor and project that have the same letter are colluding with each other. Now suppose A

donates X 0 X, and B donates 0 X X.

In this example, $k_{A,B}=0.5$

. If we use data from past rounds of quadratic financing, $k_{A,B}$

may have a different estimation and could be more accurate.

The discoordination adjusted subsidy is $DAS_{A,B} = 2\cdot 0.5 \cdot (\sqrt{X}\cdot 0 + 0 \cdot \sqrt{X} + \sqrt{X} \cdot \sqrt{X}) = X$

, which exactly half the level with current method. After taking the upper bound into consideration, the total subsidy is $\frac{B\cdot X}{B+X}$

. With current method, the total subsidy is $\frac{2MW}{M+W}$

.

From the above examples, it is clear to see with current method, there is no adjustment for discoordination, only adjustment for pairwise bound.

Reference

[1] Buterin, Vitalik, Zoë Hitzig, and E. Glen Weyl, 2018, “Liberal Radicalism: A Flexible Design for Philanthropic Matching Funds”. URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3243656

[2] Buterin, Vitalik, 2019b, “Pairwise Coordination Subsidies: A New Quadratic Funding Design”. URL:Pairwise coordination subsidies: a new quadratic funding design