Documentation of the repository of the paper "Reduction from sparse LPN to LPN, Dual Attack 3.0"

June 6, 2024

Contents

1 Overview of the repository

References to Proposition, Figure or Model point to the eprint version of the article uploaded on December 4th:

`https://eprint.iacr.org/archive/2023/1852/1701452846.pdf`

Summary of each folder

- "Verify_Poisson_Model": A program to show that the poisson Model 1 is valid, it reproduces a figure close to Figure 2. It contains in particular parts of doubleRLPN implemented in C++. Documented in Section 2.
- "Lattice_Prediction": A program to show that we can predict the distribution of the score function of dual attacks in lattices. It essentially reproduces Figure 3 and Figure 4. Documented in Section 3.
- "Complexity_Claim": A program to verify the complexity claims relative to doubleRLPN. It contains in particular a dataset with the optimized asymptotic parameters of doubleRLPN to decode at the relative Gilbert-Varshamov distance. Documented in Section 4.

Dependencies

For "Verify_Poisson_Model"

- gcc/g++, available at `https://gcc.gnu.org`. Tested with version 13.1 but an older version with support for C++20 should suffice.

For "Verify_Poisson_Model" and "Complexity_Claim"

- python3, available at `https://www.python.org/downloads/`. Tested with version 3.11.3. Modules needed:
  - Python 3 standard Library
  - NumPy (Tested with version 1.24.3)
  - Scipy (Tested with version 1.10.1)
  - Matplotlib (Tested with version 3.7.1)

For "Lattice_Prediction"

- Jupyter notebook, available at `https://jupyter.org/`. Tested with version 6.5.4.
- SageMath, available at `https://www.sagemath.org/`. Tested with version 10.0.
- unzip.

Everything was tested on a 64 bit Arch-Linux distribution.

Acknowledgement

We would like to warmly thank the anonymous reviewers of Eurocrypt 2024's artifacts whose comments allowed to greatly improve the quality of this artifact.

2 Verification of the Poisson Model

In folder

Verify_Poisson_Model/

The goal here is to verify the Poisson Model which is used to bound the expected number of false candidates in Proposition 5, namely the quantity

$$\mathbb{E}\left(\left|\{\mathbf{x}\in\mathbb{F}_2^{k_{\text{aux}}}\setminus\{\mathbf{e}_{\mathscr{P}}\mathbf{G}_{\text{aux}}\}\ :\ \widehat{f_{\mathbf{y},\mathscr{H},\mathbf{G}_{\text{aux}}}}(\mathbf{x})\geq T\}\right|\right).$$

The goal is to show that the expected number of false candidates is the same experimentally and by supposing that the Poisson Model is true.

Remark: This section does not exactly reproduce Figure 2 of the article. The latter was generated in the case where the set $\mathscr{H}$ of LPN samples is a random subset of $\widetilde{\mathscr{H}}$ of size N. While here we focus on the framework of Proposition 5, that is when $\mathscr{H} = \widetilde{\mathscr{H}}$, which is much simpler and shows in the same manner that the Poisson Model is valid.

Overview of the folder

- "plot.py": The main script. Plot the number of false candidates given by doubleRLPN against the number of false candidates given by the Poisson Model. This script uses scripts (that can be run independently) contained in the following two folders.
- "doubleRLPN": contains parts of doubleRLPN implemented in C++. Allow to compute the expected number of false candidates in doubleRLPN. Documented in Section 2.1.
- "Poisson_Model": computes the expected number of false candidates under the Poisson Model. Documented in Section 2.2.

How to run and what is does

-python3 plot.py [--options] w t_{aux} k_{aux} s k n t N_{iter}

Options:

- --d1. Create a dataset containing the expected number of false candidates given experimentally by doubleRLPN. More specifically, it runs the script documented in Section 2.1 with the same parameters.
- --d2. Create a dataset containing the expected number of false candidates given by the Poisson Model. More specifically, it runs the script documented in Section 2.2 with the same parameters.
- --plot. Combine the two previous datasets into a plot. This option must be either combined with option d1 and d2 if the corresponding datasets do not already exist, or can be used alone if the datasets already exist.

Alternatively, the script can be run without options which is equivalent to run it with --d1, --d2 and --plot all together.
Example:

-python3 plot.py 5 2 20 28 30 60 8 100

which is equivalent to running

-python3 plot.py --d1 --d2 --plot 5 2 20 28 30 60 8 100

Executing this command can take a few hours.

Typical output

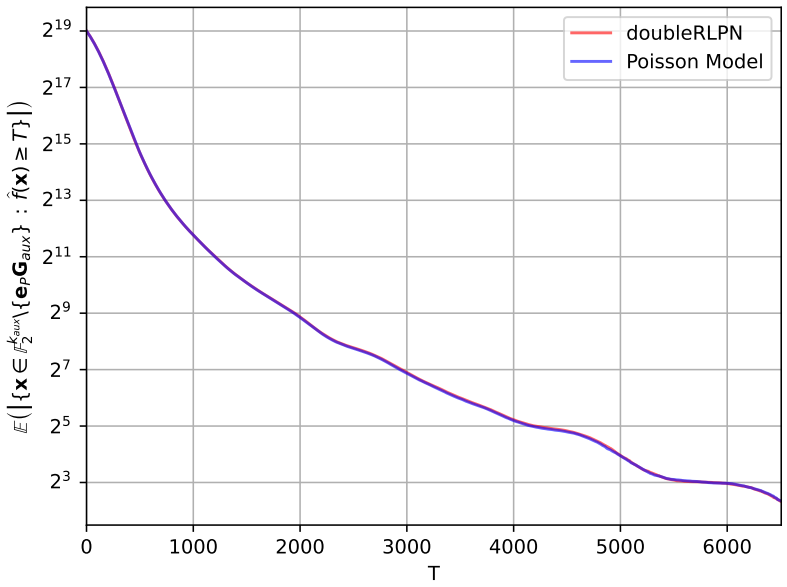
An image in

plot/plot_w_t_{aux}_k_{aux}_s_k_n_N_{iter}.pdf

Example with

plot/plot_5_2_20_28_30_60_8_100.pdf

The limit on the T axis of the plot is set to T such that the number of false candidates is equal to $\frac{500}{N_{\text{iter}}}$, this prevents the two curve from diverging from each other due to lack of data. Consider increasing N_{iter} to get information for larger T's. N_{iter} is advised to be more than 1000.



2.1 Number of false candidates in doubleRLPN

In folder

Verify_Poisson_Model/doubleRLPN/

What it does

Gives an empirical value for the expected number of false candidates in each iteration of doubleRLPN for different values of threshold T. More precisely: given the parameters of the algorithm $w, t_{\text{aux}}, k_{\text{aux}}, s, k, n, t$ and N_{iter} it runs a number N_{iter} of times the following procedure:

- **Do:**
  - Take $\mathcal{C}$ and $\mathcal{C}_{\text{aux}}$ uniformly at random in $[n,k]$ and $[s,k_{\text{aux}}]$ respectively by choosing two generator matrices $\mathbf{G}$ and $\mathbf{G}_{\text{aux}}$ uniformly at random among matrices of $\mathbb{F}_2^{k\times n}$ of rank k and matrices of $\mathbb{F}_2^{k_{\text{aux}}\times s}$ of rank k_{aux}. Compute $\mathbf{y}=\mathbf{c}+\mathbf{e}$ where $\mathbf{c}$ and $\mathbf{e}$ are taken uniformly at random in $\mathcal{C}$ and $\{\mathbf{x}\in\mathbb{F}_2^n \ : |\mathbf{x}|=t\}$ respectively. Choose uniformly at random two complementary subsets of $[\![1,n]\!]$, $\mathscr{P}$ and $\mathscr{N}$ of size s and $n-s$ respectively.

  While $\mathcal{C}_{\mathscr{P}}$ is not of dimension s.

- Compute the set of false candidates

$$\{\mathbf{x}\in\mathbb{F}_2^{k_{\text{aux}}}\setminus\{\mathbf{e}_{\mathscr{P}}\mathbf{G}_{\text{aux}}\} \ : \widehat{f_{\mathbf{y},\widetilde{\mathscr{H}},\mathbf{G}_{\text{aux}}}}(\mathbf{x})\geq T\}$$

  where for $\mathbf{x}\in\mathbb{F}_2^{k_{\text{aux}}}$,

$$\widehat{f_{\mathbf{y},\widetilde{\mathscr{H}},\mathbf{G}_{\text{aux}}}}(\mathbf{x})=\sum_{(\mathbf{h},\mathbf{m}_{\text{aux}})\in\widetilde{\mathscr{H}}}(-1)^{\langle\mathbf{y},\mathbf{h}\rangle-\langle\mathbf{x},\mathbf{m}_{\text{aux}}\rangle}$$

  and

$$\widetilde{\mathscr{H}}=\{(\mathbf{h},\mathbf{m}_{\text{aux}})\in\mathcal{C}^{\perp}\times\mathcal{C}_{\text{aux}} \ : |\mathbf{h}_{\mathscr{N}}|=w \text{ and } |\mathbf{h}_{\mathscr{P}}+\mathbf{m}_{\text{aux}}\mathbf{G}_{\text{aux}}|=t_{\text{aux}}\}.$$

It outputs a file containing, for different values of T, the experimental average (computed over the N_{iter} iterations) number of false candidates.

How to run

-python3 doubleRLPN.py w t_{aux} k_{aux} s k n t N_{iter}

Example :

-python3 doubleRLPN.py 5 2 20 28 30 60 8 100

N_{iter} is advised to be more than 1000 if possible to get the most accurate estimation as possible.

Typical output

An output file in

data/doubleRLPN_w_t_{aux}_k_{aux}_s_k_n_N_{iter}.csv

of the format

T_1,y_{T_1}
T_2,y_{T_2}
...

where y_{T_i} is the average number of false candidates for the threshold T_i.

2.2 Number of false candidates under the Poisson Model

In folder

Verify_Poisson_Model/Poisson_Model

What it does

Gives an estimate of the expected number of false candidates under the Poisson Model. More precisely, similarly to Lemma 5 we can show that the expected number of false candidates can be rewritten as

$$\mathbb{E}_{\mathcal{C},\mathcal{C}_{\text{aux}}}\left(\left|\left\{\mathbf{x}\in\mathbb{F}_2^{k_{\text{aux}}}\setminus\{\mathbf{e}_{\mathscr{P}}\mathbf{G}_{\text{aux}}\}\ :\widehat{f_{\mathbf{y},\widetilde{\mathscr{H}},\mathbf{G}_{\text{aux}}}}(\mathbf{x})\geq T\right\}\right|\right)= \left(2^{k_{\text{aux}}}-1\right)\mathbb{P}_{\mathcal{C},\mathcal{C}_{\text{aux}},\mathbf{x}}\left(\widehat{f_{\mathbf{y},\widetilde{\mathscr{H}},\mathbf{G}_{\text{aux}}}}(\mathbf{x})\geq T\right)$$

where $\mathcal{C}$ and $\mathcal{C}_{\text{aux}}$ uniformly at random in $[n,k]$ and $[s,k_{\text{aux}}]$ respectively and $\mathbf{x}$ is taken uniformly at random in $\mathbb{F}_2^{k_{\text{aux}}}\setminus\{\mathbf{e}_{\mathscr{P}}\mathbf{G}_{\text{aux}}\}$. Using Lemma 1 and Proposition 4 we have that

$$\widehat{f_{\mathbf{y},\widetilde{\mathscr{H}},\mathbf{G}_{\text{aux}}}}=\frac{1}{2^{k-k_{\text{aux}}}}\sum_{i=0}^{n-s}\sum_{j=0}^{s}N_{i,j}K_w^{(n-s)}(i)\,K_{t_{\text{aux}}}^{(s)}(j)\,.$$

Then, under the Poisson model (replacing $N_{i,j}$ by a compound Poisson variable) we have that

$$\mathbb{E}\left(\left|\left\{\mathbf{x}\in\mathbb{F}_2^{k_{\text{aux}}}\setminus\{\mathbf{e}_{\mathscr{P}}\mathbf{G}_{\text{aux}}\}\ :\widehat{f_{\mathbf{y},\widetilde{\mathscr{H}},\mathbf{G}_{\text{aux}}}}(\mathbf{x})\geq T\right\}\right|\right)=\left(2^{k_{\text{aux}}}-1\right)\mathbb{P}\left(Z\geq T\right)\tag{1}$$

where

$$Z=\frac{1}{2^{k-k_{\text{aux}}}}\sum_{i=0}^{n-s}\sum_{j=0}^{s}\widetilde{N_{i,j}}K_w^{(n-s)}(i)\,K_{t_{\text{aux}}}^{(s)}(j)$$

and

$$\widetilde{N_{i,j}}\sim\text{Poisson}\left(\widetilde{N_j}\frac{\binom{n-s}{i}}{2^{n-k}}\right)\ \text{and}\ \widetilde{N_j}\sim\text{Poisson}\left(\frac{\binom{s}{j}}{2^{k_{\text{aux}}}}\right)$$

and where the variables are independent.

Given the parameters of the algorithm $w, t_{\text{aux}}, k_{\text{aux}}, s, k, n, t$ and N_{iter}, this script estimates Equation (1) by a monte-carlo method: it draws $N_{\text{iter}}\ 2^{k_{\text{aux}}}$ variables Z to heuristically estimate $\mathbb{P}\left(Z\geq T\right)$.

How to run

-python3 PoissonModel.py w t_{aux} k_{aux} s k n t N_{iter}

Example :

-python3 PoissonModel.py 5 2 20 28 30 60 8 100

N_{iter} is advised to be more than 1000 if possible to get the most accurate estimation as possible. This part is usually the longest and can take several hours with the parameters given as example. Consider parallelizing the code.

Typical output

An output file in

data/PoissonModel_w_t_{aux}_k_{aux}_s_k_n_N_{iter}.csv

of the format

T_1,y_{T_1}
T_2,y_{T_2}
...

where y_{T_i} is the average number of false candidates for the threshold T_i under the Poisson Model.

3 Prediction of lattice score function

In folder

Lattice_Prediction/

Overview of the folder

- prediction_lattices.ipynb
  - Reproduces Figure 3 and 4 for different parameters as described in Section 8 of the article. w appearing in Equation (19) is taken here as the average length of the short dual vectors returned by the sieve. They are stored in the following file.
- out_n$\mathbf{X}$_fft$\mathbf{Y}$_enum$\mathbf{Z}$.txt
  - File containing information about the lattice and short dual vectors returned by the sieve. This file was created by showing the variables "Bprime" (before the call to the "reduce_and_sieve" function) and "dual_db" of `https://github.com/ludopulles/DoesDualSieveWork/tree/main/code/unif_score.py` with input $n = \mathbf{X}$, fft= $\mathbf{Y}$, enum= $\mathbf{Z}$ (q is set to default to 3329).
- Data_DP23/
  - is taken from `https://github.com/ludopulles/DoesDualSieveWork/tree/main/data`

How to run

First, unzip the following compressed dataset:

-unzip out_n90_fft22_enum26.zip

then, run the notebook:

-jupyter notebook prediction_lattices.ipynb

4 Verification of complexity claims

In folder

Complexity_Claim/

The files are meant to verify the complexity claims relative to doubleRLPN.

Overview of the folder

- doubleRLPN_BJMM12.csv
  - Contains, for different code rates R, the optimized relative parameters and the associated complexity of the doubleRLPN decoder to decode at the relative Gilbert-Varshamov distance when using BJMM12 technique to compute low-weight parity-checks. These parameters are used in Proposition 9 to compute the asymptotic complexity of the algorithm. The file contains, for different rates R the values of $\sigma, R_{\text{aux}}, \nu, \omega, \tau$ along with $\lambda_1, \lambda_2, \pi_1, \pi_2$, the later 4 parameters are used in Proposition 11 to compute the complexity of computing the parity-checks using BJMM12 technique. All the parameters (even $\lambda_1, ...$) are written relatively to n. τ_{aux} is implicitly set to be equal to $\sigma h_2^{-1}\left(1 - \frac{R_{\text{aux}}}{\sigma}\right)$ and N_{aux} is implicitly set to be equal to 1. The parameters relative to the two subroutines Dumer-Decoder and Solve-SubProblem will be computed on the fly in the following file.

- complexity_doubleRLPN_BJMM12.py
  - Using the relative parameters contained in the parameter file, this script re-computes, using the formula in Proposition 9, the time complexity exponent ($\alpha_{\text{doubleRLPN}}$) of the doubleRLPN decoder. This script also assert that the parameters meet the constraints of Proposition 9 and Proposition 11 (executions fails if one constraint is not verified).

How to run

-python3 complexity_doubleRLPN_BJMM12.py

Typical output

A list of complexity exponent

Rate: 0.01000; Complexity: 0.00539
Rate: 0.02000; Complexity: 0.01009
...