INFLATION LINKED BONDS: THE CASE OF BTP ITALIA

Final project by

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Introduction

This thesis aims to conduct a comparative analysis between two types of Italian government bonds: the conventional BTP and the inflation-indexed BTP Italia. The study first explores the key features, risks, and valuation of bonds in general, and then provides a specific focus only on Italian bonds in Chapter 2: zero-coupon bonds (BOTs), fixed rate notes (BTPs) and inflaton linked bonds (BTPs Italia), all of which are issued by the Italian government. In particular, the functioning of BTP Italia is presented in detail. In the final chapter, pricing and scenario analysis for comparing the relative convenience of BTP and BTP Italia is presented. By examining these aspects, the thesis seeks to provide insights into the potential benefits and drawbacks of investing in inflation-indexed bonds compared to their conventional counterparts within the Italian government bond market.

Overview of Bonds Market

A bond is a security that represents a loan made by an investor to a borrower, typically a government or corporation. Bonds are a common form of investment used by individuals, institutions, and governments to raise capital and generate income.

1.1 Typologies of Bonds

This Section explores the various features and types of debt securities and their implications for investors. In particular, it will focus on zero-coupon bonds, coupon bonds, and floating-rate securities.

1.1.1 Zero-Coupon Bonds

A zero coupon bond is a type of bond that does not pay coupons to the bondholder. The investor buys the bond below its par value, consequently the return comes from the difference between the discounted purchase price and the face value received at maturity.

Period	Cash Flow
0	-95
1	0
2	0
3	0
4	0
5	100

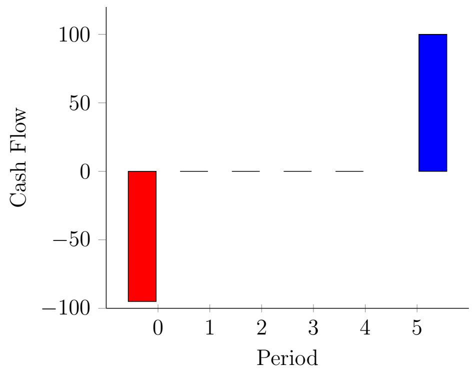

Figure 1.1: Tabular *(left)* and graphical *(right)* representation of the payment schedule of a 5 years zero-coupon bond bought at 95€

1.1.2 Coupon Bonds

A coupon bond is a type of bond that pays periodic interest payments, known as coupons, to the bondholder. The investor buys the coupon bond at the issue price or at the secondary market price, and over the bond's term, he receives the regular coupon payments. At maturity, the investor also receives the bond's par value back. The total return on a coupon bond comes from both the interest payments received and the face value received at maturity.

Period	Cash Flow
0	-97
1	5
2	5
3	5
4	5
5	105

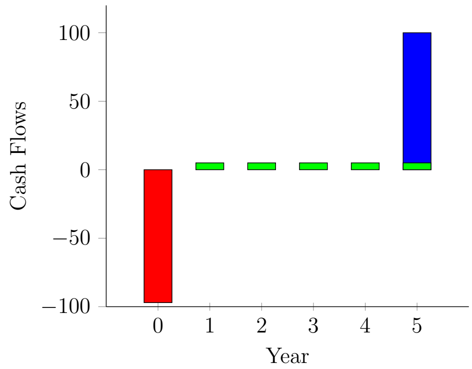

Figure 1.2: Tabular *(left)* and graphical *(right)* representation of the payment schedule of a 5 years fixed-rate coupon bond bought at 97€, annual coupon of 5€, and notional of 100€

Maturity

The term to maturity of a bond is the number of years remaining prior to final principal payment. The maturity date is the time when the issuer will redeem the bond by paying the outstanding balance. Typically, bonds with a maturity between 1 and 5 years are referred to as short-term, those with a maturity between 5 and 12 years are called intermediate-term, and long-term bonds are those with a maturity exceeding 12 years.

There are three reasons why the term to maturity of a bond is important. Firstly, it indicates the time period over which the investor expects to receive interest payments. Secondly, the yield offered on a bond depends on the term to maturity. The relationship between the yield on a bond and maturity is called yield curve, which will be analyzed later on. Finally, as the maturity of a bond increases, it becomes more susceptible to price volatility caused by changes in interest rates.

Notional Value

The notional (or par or face) value N of a bond is the amount that the issuer agrees to repay the bondholder at or by the maturity date. Bonds can have any par value, and they may trade below or above that amount.

Coupon Rate

The coupon rate c, also called notional rate, is the interest rate that the issuer agrees to pay each year. The coupon rate affects the bond's price sensitivity to changes in market interest rates. The higher the coupon rate, the less the price will change in response to a change in market interest rates.

Payment Frequency and Coupon Amount

The payment frequency m of a coupon bond refers to how often the bond issuer makes interest payments to the bondholder. It is the frequency at which the bond's coupon, representing the fixed interest payment, is paid to the investor over the bond's term. Common payment frequencies for coupon bonds include annual ($m = 1$), semi-annual (twice a year, $m = 2$), quarterly (four times a year, $m = 4$), and monthly ($m = 12$). Given the payment frequency m and the coupon dates $T_i, i = 1, \cdots, n-1$, the coupon amount is equal to

$$C(T_i) = \frac{c}{m} \times N. \tag{1.1}$$

At maturity T_n the total payment includes the coupon and repayment of the notional N

$$C(T_n) = \left(1 + \frac{c}{m}\right) \times N. \tag{1.2}$$

1.1.3 Floating-Rate Securities

The coupon rate on a bond does not need to be fixed over the bond's life. Floating-rate securities have coupon payments that reset periodically according to some reference rate. To calculate the floating-rate coupon, the reference rate is added to a specific value known as the quoted margin. The quoted margin is the additional amount that the issuer agrees to pay above the reference rate, however it doesn't necessarily need to be a positive value.

Let's consider a generic scenario where a floater pays interest semiannually, and the coupon reset date is the current day. The coupon rate is calculated using coupon formula 1.3, and it represents the agreed-upon interest rate that the issuer will pay on the next interest payment date, which is six months from now.

$$C(T_i) = (R(T_{i-1}, T_i) + \delta)\frac{1}{m} \times N, \tag{1.3}$$

where R is the reference rate observed in T_{i-1} for the payment due in T_i, δ is the quoted margin, m is the payment frequency and N is the notional.

A floating-rate coupon may have restrictions on the maximum and the minimum coupon rate paid at any reset date. The maximum coupon rate is called a cap and it is an unattractive feature for the investors, while the minimum coupon rate is called a floor and it attracts a bigger amount of investors.

Period	Cash Flow
0	-98
1	3
2	8
3	4
4	6
5	106

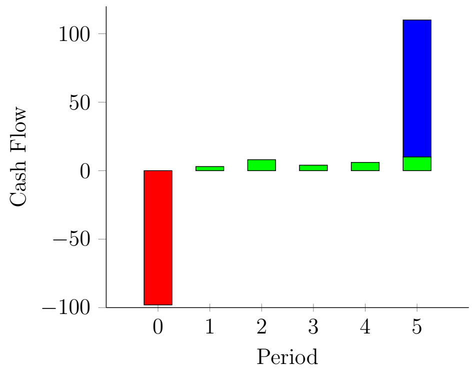

Figure 1.3: Tabular *(left)* and graphical *(right)* representation of the payment schedule of a 5 years floating-rate security bought at 98€ and notional of 100€

1.2 Bonds Instruments

In examining bond products, it is important to consider their diverse nature and characteristics.

Sovereign Bonds

Sovereign bonds represent debt obligations issued by national governments to finance their operations and activities. These bonds, including treasury bonds, government notes, and government bonds, are usually low-risk due to the state's ability to levy taxes and control monetary policy.

Semi-government/Agency Bonds

Semi-government/agency bonds, on the other hand, are issued by government-sponsored agencies or entities operating in specific sectors. While not having direct government backing, they often carry an implicit or explicit guarantee, which contributes to their relatively lower risk compared to corporate bonds.

Tax-backed Debt Securities

State and local government bonds include various types of debt instruments. Tax-backed debt bonds are secured by the taxing authority of state or local governments, ensuring that the government can raise taxes to meet its debt obligations.

Revenue bonds are, however, backed by specific revenue streams generated by infrastructure projects like toll roads, airports, or utilities.

Corporate Debt Securities

Corporate debt securities are issued by corporations to raise capital for different purposes, such as funding operations, expansions, or acquisitions. The credit risk associated with corporate debt securities is contingent upon the financial health and creditworthiness of the issuing company.

Asset-backed Securities

Lastly, asset-backed securities are backed by pools of financial assets, such as mortgages. These bonds derive their value and payment streams from the cash flows generated by the underlying assets, providing investors with exposure to specific asset classes.

1.3 Risks Affecting Bond Investors

An investor in any bond is exposed to many types of risk. We can have a first impression of the bond's quality thanks to ratings done by bond rating agencies. Securities ratings are assigned based on a variety of metrics. Bonds are assigned letters or letter and number combinations corresponding to their quality. Bonds with an AAA rating are typically considered the most reliable, while securities with a D rating are considered to have the highest likelihood of default.
The following Sections will analyze the various risks affecting bond investors.

1.3.1 Interest Rate and Yield Curve Risk

When interest rates rise, the price of bonds with fixed interest rates experiences a decrease in value. Conversely, when interest rates fall, these bonds increase their price. Consequently, since the price of a bond fluctuates with market interest rates, the risk that an investor faces is that the price of a bond held in a portfolio will decline if market interest rates rise. This is the major risk faced by investors in the bond market.

However, it is important to note that interest rate risk extends beyond individual bond prices. It also encompasses the risk associated with changes in the yield curve, that is the graphical depiction of the relation between interest rates on debt securities and their respective maturities. Yield curve risk refers to the potential for shifts in the shape and slope of the yield curve to impact the value of bonds. Changes in the yield curve can occur due to shifts in market expectations of future interest rates. A steepening yield curve indicates that longer-term interest rates are increasing at a faster pace than shorter-term rates. This affects bonds differently depending on their maturities. Bonds with longer maturities are typically more sensitive to changes in the yield curve. In contrast, a flattening yield curve occurs when the difference between longer-term and shorter-term interest rates decreases. This also impacts bond prices, particularly those with longer maturities.

Investors face yield curve risk because shifts in the yield curve can result in changes to the relative value of different bond maturities.

1.3.2 Inflation Risk

Inflation risk arises from the decline in the value of a security's cash flows due to inflation, which is measured in terms of purchasing power. For example, if an investor purchases a bond with a coupon rate of 5%, but the inflation rate is 3%, the investor's purchasing power has increased by only 2%.

Not considering inflation protection bonds, an investor is exposed to inflation risk because the interest rate the issuer promises to make is fixed for the entire life of the bond.

1.3.3 Liquidity Risk

Liquidity risk is the risk that the investor will have to sell a bond below its indicated value. The primary measure of liquidity is the size of the spread between the bid price and the ask price. The wider the bid-ask spread, the greater the liquidity risk.

1.3.4 Credit Risk

An investor who lends funds by purchasing a bond issue is exposed to credit risk. There are three types of credit risk: default, credit spread and downgrade risk.

Default risk is defined as the risk that the issuer will fail to satisfy the terms of the obligation with respect to the timely payment of interest and principal.

Credit spread risk refers to the potential for changes in the difference (spread) between the yields of riskier bonds and safer bonds. It is the risk that the creditworthiness of bond issuers may deteriorate, causing the spread to widen and potentially resulting in a decrease in the value of bonds.

Downgrade risk is the risk that a bond's credit rating may be downgraded, usually due to a decline in the issuer's creditworthiness, leading to a potential decrease in the bond's value.

1.3.5 Sovereign Risk

When an investor acquires a bond issued by a foreign entity, he faces sovereign risk. This is the risk that there may be either a default or an adverse price change.

Sovereign risk consists of two parts. First is the unwillingness of a foreign government to pay, the second is the inability to pay due to unfavorable economic conditions in the country.

1.3.6 Other Types of Risk

Volatility risk refers to the potential for significant and unpredictable fluctuations in the value or price of an asset, resulting in potential losses or uncertainty for investors.

Call risk refers to the risk that a bond issuer will redeem a bond before its maturity date, potentially leaving the investor with reinvestment risk. Similarly, repayment risk refers to the risk associated with early repayment of a loan or debt instrument, which can impact the expected cash flows and returns for investors.

Exchange rate risk refers to the potential for losses or uncertainties that arise from fluctuations in the value of one currency relative to another, impacting the value of international investments or transactions.

Event risk is the risk of unexpected and significant events, such as natural disasters or political changes, that can adversely affect the issuer's ability to fulfill its obligations to bondholders.

1.4 Primary and Secondary Market

This Section explores the primary and secondary markets for bonds, which play a crucial role in the issuance and trading of these financial instruments.

1.4.1 Primary Market

The primary market for bonds involves the distribution to investors of newly issued securities by central governments, corporations and other entities. There are many ways to allocate bonds.

A bought deal refers to a transaction in which an underwriter or investment bank purchases the entire issuance of bonds from the issuer and then resells them to investors. This allows the issuer to raise funds quickly and efficiently, transferring the risk of selling the bonds to the underwriter.

An auction process involves the sale of bonds to investors through a competitive bidding process. The issuer sets the terms and conditions of the bond offering, and potential investors submit their bids specifying the quantity and price they are willing to pay. The bonds are then allocated to the highest bidders.

A private placement is a direct sale of securities to a specific group of investors, such as institutional investors or wealthy individuals, rather than offering them to the general public through a public offering. It is a more exclusive and tailored approach to raising capital, typically involving negotiations between the issuer and the investors.

1.4.2 Secondary Market

The secondary market for bonds refers to the marketplace where already issued bonds are bought and sold among investors. Unlike the primary market, where bonds are initially issued and sold by the issuer, the secondary market enables investors to trade bonds with each other.

The secondary market provides liquidity to bond investors by offering a platform to buy or sell bonds before their maturity. This allows investors to exit or adjust their bond positions as needed, guaranteeing an efficient allocation of capital. Moreover, it facilitates price discovery, as bond prices are determined through the interactions of buyers and sellers.

Italian Government Bond Market

This Chapter introduces the three most important debt securities commonly used in Italy, and subsequently, it will give a precise focus on the BTP Italia. All of these bonds are issued and denominated in Euros, and can be subscribed for a minimum nominal value of 1,000 Euros or multiples of this amount.

Chart 2.1 illustrates the composition of Government Bonds in circulation as of June 30, 2023. BTPs account for 72.05% of the total, followed by Indexed BTPs (CCTs) at 8.10%, CCTeus at 6.23%, BOTs at 5.04%, BTP Italia at 3.52%, Foreign Currency Bonds at 2.02%, BTP Green at 1.42%, BTP Futura at 0.86%, and finally BTP Valore at 0.76%.

2.1 Zero Coupon Bonds: BOTs

BOTs[1] are short-term securities with a maturity not exceeding one year. The remuneration is entirely determined by the issuance discount, that is the difference between the face value and the price paid, so there is no coupon. BOTs offer a fixed interest rate, providing investors with predictable returns over the investment period. These particular fixed-income securities are actively traded in the secondary market, allowing investors to buy or sell them before their maturity date. BOTs are often utilized by investors for cash management purposes, as they provide a relatively low-risk option with a short-term investment horizon.

Quotations on the secondary market

Table 2.1 presents the quotations of BOTs with different maturities ranging from a few days (August 2023) to one year (July 2024) on the secondary market. Quotations are taken on Friday July 27th, 2023. For BOTs, the settlement period is usually 3 business days following the trade date. This means that the financial transaction becomes effective three business days after the date on which it was executed. This means that the settlement of the trade occurs on Tuesday, August, 1st 2023.

[1]BOTs stands for *Buoni Ordinari del Tesoro*

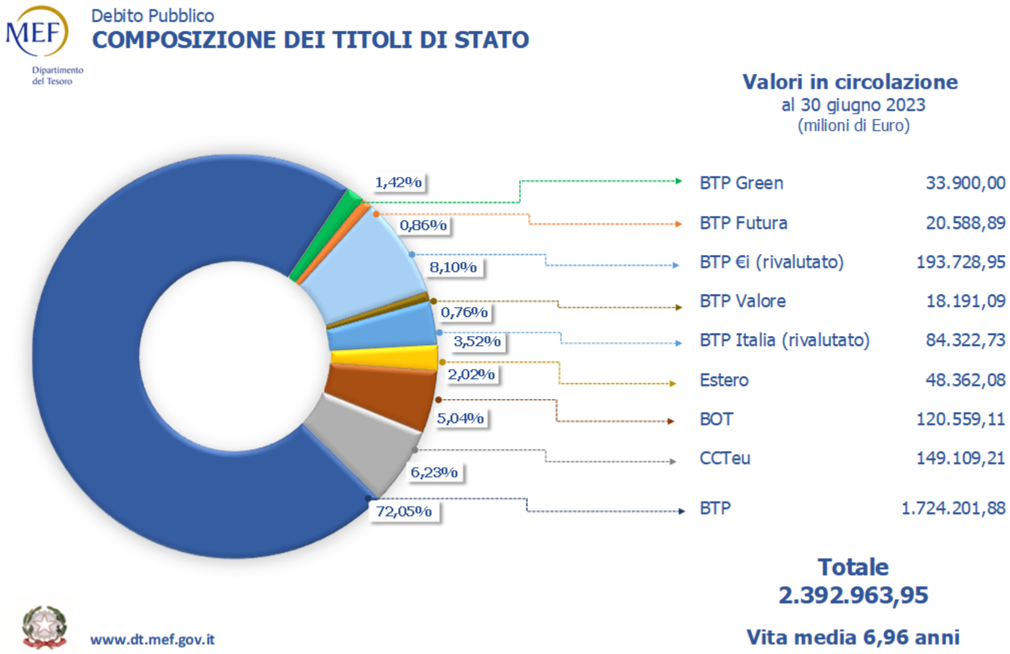

Figure 2.1: Composition of Government Bonds in circulation as of June 30, 2023. Source: MEF dati statistici

Name	Isin	Last Price	Last Contract
BOT ZC LG23 S EUR	IT5531295	99.972	26/07/23 17.06.17
BOT ZC ST23 A EUR	IT5508236	99.556	27/07/23 17.07.41
BOT ZC ST23 S EUR	IT5541278	99.411	27/07/23 17.26.14
BOT ZC OT23 A EUR	IT5512030	99.283	27/07/23 17.12.43
BOT ZC NV23 A EUR	IT5518516	98.971	27/07/23 17.21.57
BOT ZC NV23 S EUR	IT5547887	98.761	27/07/23 17.23.53
BOT ZC DC23 A EUR	IT5523854	98.68	27/07/23 17.09.07
BOT ZC GE24 A EUR	IT5529752	98.313	27/07/23 17.25.24
BOT ZC GE24 S EUR	IT5557365	98.086	27/07/23 17.29.29
BOT ZC FB24 A EUR	IT5532988	98.289	27/07/23 17.35.28
BOT ZC MZ24 A EUR	IT5537094	97.683	27/07/23 17.25.19
BOT ZC AP24 A EUR	IT5542516	97.36	27/07/23 17.28.42
BOT ZC MG24 A EUR	IT5545469	97.056	27/07/23 16.49.19
BOT ZC GN24 A EUR	IT5549388	96.784	27/07/23 17.35.10
BOT ZC LG24 A EUR	IT5555963	96.495	27/07/23 17.40.43

Table 2.1: BOTs quotations on July 27th, 2023. Source: Milano Finanza

2.2 Fixed Rate Notes: BTPs

BTPs[2] are medium to long-term securities (usually issued with a term of 3, 5, 7, 10, 15, 20, 30 and 50 years) with a fixed coupon paid semiannually. BTPs possess several key features that make them significant investment instruments. First and foremost, BTPs have medium to long-term maturities, allowing investors to make long-term commitments and potentially benefit from stable returns over an extended period. As already mentioned, these bonds pay a fixed coupon, providing investors with regular and predictable interest payments. They are also highly liquid, as they are actively traded in secondary markets, enabling investors to buy or sell them with relative ease. Furthermore, BTPs are considered relatively low-risk investments due to the creditworthiness of the Italian government. Just like BOTs, these securities can be traded on the secondary market as well.

Coupon

The coupon payment at date T_i is equal to

$$C(T_i) = c \times \alpha_{i-1,i} \times N, i = 1, \cdots, n-1, \tag{2.1}$$

and at maturity we have

$$C(T_n) = (1 + c \times \alpha_{i-1,i}) \times N, \tag{2.2}$$

where c is the annual coupon rate and N is the face value. $\{T_i\}_{i=1}^{n}$ are the bond payment dates. In general, these dates are equally spaced (aside adjustments for weekends and holidays), e.g. every quarter/semester/year. On T_n, in addition to the coupon, we receive the face value N; $\alpha_{i-1,i}$ is called coupon tenor, i.e. the year fraction (accrual factor) of time between coupon dates T_{i-1} and T_i. For BTPs the standard day count convention is

$$\alpha_{i-1,i} = \frac{1}{2}.$$

Accrued Interest

Accrued interest refers to the amount of interest that has been earned but not yet paid or received on a debt instrument. It represents the portion of the interest that has accumulated from the most recent interest payment date up to a specific point in time, typically the date of sale or settlement. It is calculated based on the stated interest rate, the principal amount, and the number of days between the last interest payment and the date of calculation.

[2]BTPs stands for *Buoni del Tesoro Poliennali*

The invoice price IP (or dirty price or full price or gross price) consists then of two components:

- Clean Price CP: quoted price of the bond without the accrued interest.
- Accrued Interest AI: this amount compensates the seller of the bond for the coupon interest earned from the time of the last coupon payment to the settlement date of the bond.

Therefore
$$IP = CP + AI.$$
When a bond is traded below its par value ($CP < N$), it is considered to be trading at a discount. Conversely, if a bond is traded above its par value ($CP > N$), it is referred to as trading at a premium. Finally, when a bond is traded at its par value, it is said to be trading at par.

In the majority of cases the Accrued Interest which has to be added to the clean price is computed using the formula

$$AI = \frac{c}{m} \times \frac{d_1}{d_2} \times N,$$

where c is the annual coupon rate, m the coupon frequency, d_1 is the number of days accrued since last coupon payment, and d_2 is the number of days in the coupon period.

Quotations on the secondary market

Table 2.2 presents the secondary market quotations (clean prices) of BTPs with different maturities. Quotes refer to Friday July 27th, 2023. Notice that in the purchase of BTPs, typically, the settlement period is 2 business days following the trade date. This means that the financial transaction becomes effective two business days after the date on which it was executed. In the present case, it would be Monday July 31st, 2023. Figure 2.3 illustrates the time variation of the yield to maturity of benchmark BTPs with different maturities (1, 10 and 50 years). The figure confirms that the yield risk can be quite significant, mainly for bonds having long maturities. For example, the yield on the 50-years bonds has registered wide oscillations over the last 3 years, moving from 4% down to 2%, and up again to 5%.

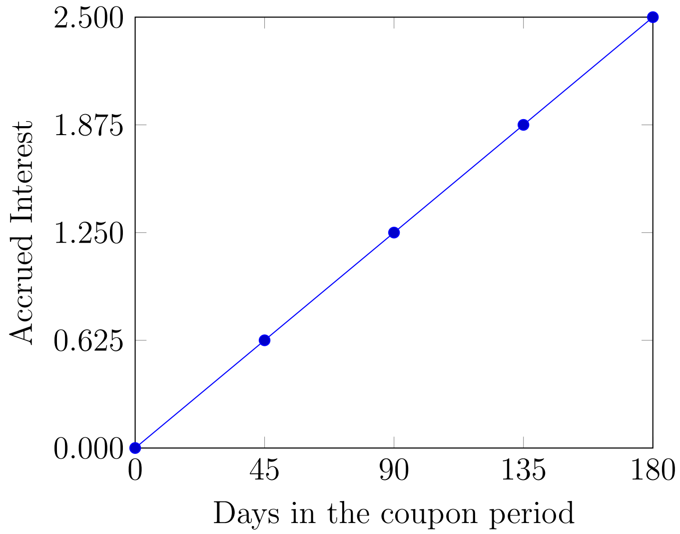

Figure 2.2: Accrued Interest for a bond having semi-annual coupons, 5% coupon rate, notional value of 100€, and 180 days in the current coupon period

Name	Isin	Last Price	Last Contract
BTP TF 0% GE24 EUR	IT5424251	98.33	28/07/23 10.34.25
BTP TF 0% AP24 EUR	IT5439275	97.419	28/07/23 10.43.09
BTP TF 0% DC24 EUR	IT5474330	95.19	28/07/23 10.43.23
BTP TF 1,2% AG25 EUR	IT5493298	95.35	28/07/23 10.41.49
BTP TF 0% AP26 EUR	IT5437147	91.04	28/07/23 10.39.02
BTP TF 0% AG26 EUR	IT5454241	89.97	28/07/23 10.40.31
BTP TF 1,1% AP27 EUR	IT5484552	91.68	28/07/23 10.35.11
BTP TF 0,95% ST27 EUR	IT5416570	90.1	28/07/23 10.36.52
BTP TF 1,35% AP30 EUR	IT5383309	85.96	28/07/23 10.32.02
BTP TF 0,95% MZ37 EUR	IT5433195	66.96	28/07/23 10.41.23

Table 2.2: BTPs quotations on July 28th, 2023. Source: Milano Finanza

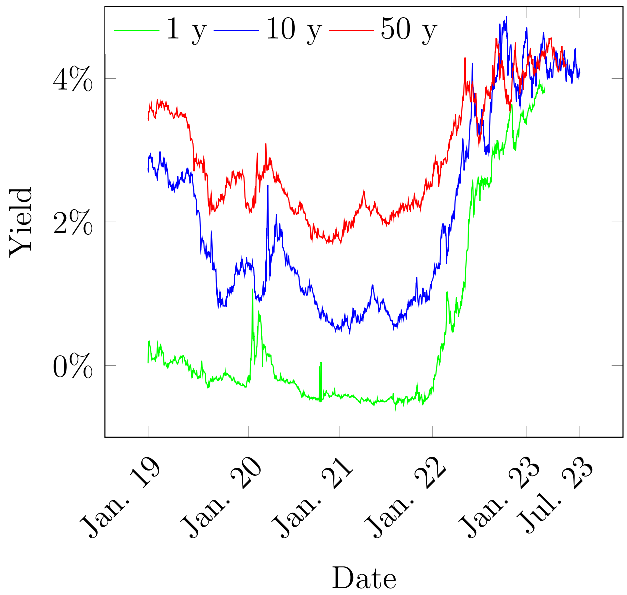

Figure 2.3: The yield on benchmark BTPs with different maturities. Source: Investing.com

2.3 Inflation Linked Notes: BTP Italia

The BTP Italia, first launched in March 2012, is a government bond issued by the Italian Ministry of Finance (MEF) with returns indexed to the national inflation rate. In principle, BTP Italia bonds are designed to provide investors with protection against inflation, so they are tougth to support private savings and safeguarding the purchasing power of citizens and families. Their principal value is adjusted for inflation using the Italian consumer price index (CPI), ensuring that the purchasing power of the investment is maintained over time. BTP Italia bonds are often marketed to retail investors, making them accessible to a wide range of individuals. BTP Italia bonds are also of interest to institutional investors, including pension funds and insurance companies, due to their inflation-linked characteristics and predictable cash flows. In addition, BTP Italia bonds can serve as a hedging tool against inflation, providing a degree of protection for investors' portfolios when inflationary pressures rise.

BTP Italia bonds are the only Italian government securities indexed to the national inflation rate.

Term

BTP Italia is designed as a short to medium-term investment, as it has a multi-year duration with maturities of 4, 5, 6, or 8 years, depending on the issuance.

Coupon

Every semester, BTP Italia bonds pay fixed-rate interest on the capital, which is adjusted to the inflation with reference to the ISTAT[3] index for the consumer price index for blue-collar and white-collar workers (FOI), excluding tobacco.

In detail, the coupon amount $C_{d,m}$ for day d in month m is calculated using the following formula:

$$C_{d,m} = \frac{r}{2} \times N \times IN_{d,m}, \tag{2.3}$$

where r is the annual real coupon rate, N is the subscribed nominal capital, and $IN_{d,m}$ is the reference Index Number (*Numero indice di riferimento*), i.e. the estimated inflation on a specific day d, and is computed using a linear interpolation of the FOI index (see figure 2.4). In particular:

$$IN_{d,m} = FOI_{m-3} + \frac{d-1}{dd} \times \left(FOI_{m-2} - FOI_{m-3}\right), \tag{2.4}$$

where dd is the actual number of days of the month m. Therefore, IN is computed using FOI index registered 3 and 2 months in advance. The capital revaluation and coupon adjustment are carried out through the Indexing Coefficient ($IC_{d,m}$), that is calculated using the following formula

$$IC_{d,m} = \frac{IN_{d,m}}{IN_{\bar{d},\bar{m}}}, \tag{2.5}$$

where $\bar{d}$ and $\bar{m}$ refer to the day and month of the previous coupon payment. In general, $\bar{d}$ and $\bar{m} = m - 6$.

On top of the coupon payment, as mentioned before, there is also an additional remuneration due to the capital revaluation. The Revalued Capital $RC_{d,m}$ is calculated as follows:

$$RC_{d,m} = N \times (IC_{d,m} - 1). \tag{2.6}$$

The total amount paid to the subscriber each semester is then:

$$TA_{d,m} = C_{d,m} + RC_{d,m}. \tag{2.7}$$

[3] ISTAT stands for *Istituto Nazionale di Statistica*

In case of deflation, i.e. $IC_{d,m} < 1$ or, equivalently, negative inflation is observed, the coupons are still calculated based on the nominal invested capital, providing a minimum amount given by the real coupon rate. This means that the equation (2.5) becomes

$$IC_{d,m} = \max\left(1, \frac{IN_{d,m}}{IN_{\bar{d},\bar{m}}}\right). \tag{2.8}$$

In the subsequent period, if the Indexing Coefficient is higher than one, the previous semester's price index will be used for the calculation, as long as it is higher than the last maximum value observed in previous semesters. However, if the price index of the previous semester is increasing but not higher than the previous maximum value, the base will continue to be constituted by the last maximum value. In practice, the formula to be used is

$$IC_{d,m} = \max\left(1, \frac{IN_{d,m}}{\max IN_{\bar{d},\bar{m}}}\right). \tag{2.9}$$

To enhance comprehension of the entire process, a comprehensive calculation will be presented in Section 2.3.2.

Accrued Interest and Accrued Capital Revaluation

The Accrued Interest as of the settlement date (day d of month m) must distinguish between the minimum coupon amount and any additional coupon payments, as well as account for Capital Revaluation resulting from inflation.

Concerning the minimum coupon the accrued interest is computed as for standard BTPs

$$AI^1_{d,m} = \frac{r}{2} \times \frac{d_1}{d_2} \times N \times IC_{d,m},$$

where d_1 is the number of days accrued between the last coupon date and the settlement date, and d_2 is the number of days in the coupon period.

The accrued interest on the additional coupon requires the calculation of the *Coefficiente di indicizzazione* relative to day d of month m, i.e. $IC_{d,m}$. Then the accrued interest is

$$AI^2_{d,m} = \frac{r}{2} \times \frac{d_1}{d_2} \times N \times (IC_{d,m} - 1),$$

The rate of capital revaluation on the settlement date of the transaction is calculated as follows

$$AI^2_{d,m} = N \times (IC_{d,m} - 1),$$

In conclusion, if PR indicates the real market quotation price at the time of the exchange, the invoice price is

$$GP = PR + AI^1_{d,m} + AI^2_{d,m} + AI^3_{d,m}.$$

Loyalty Premium

Investors who purchase the securities during the first phase of the placement period and hold them until maturity are entitled to a loyalty premium, which is a percentage of the nominal value of the investment and usually inferior to the 1%.

2.3.1 BTP Italia and Inflation Protection

To what extent do BTP Italia bonds provide inflation protection? Let's consider an investor who purchased a BTP Italia bond issued at par. Let us assume that the 12-month inflation remains constant at 6 percent in April and May 2022 (assuming a decrease from the 6.7 percent in March, partly due to government interventions on energy price hikes).

In this scenario, over the first two years (from May 2020 to May 2022), the bond would achieve an average annual real yield of 1.05 percent (Table 1). An identical non-indexed BTP over the same period would yield an average annual real return of -2.40 percent, with -0.16 percent in the first year and -4.61 percent in the second year (Table 2).[3] The advantage of BTP Italia's indexation is evident, although the effective yield is lower than the nominal yield of 1.40 percent that would be obtained in the absence of inflation. This occurs due to the lag in the indexation mechanism, as discussed in the following paragraph.

2.3.2 A Detailed Calculation

The Ministry of Economy and Finances of Italy (MEF) routinely releases the IC (Indexation Coefficient) values associated with each BTP Italia bond. An illustration of this can be observed in figure 2.4. The previously presented formulas allow for the reconstruction of IC values provided by the MEF. To illustrate, take into consideration the BTP Italia with ISIN number IT5351660, which was initially released on November 26, 2018. This bond pays semi-annual coupons on the 26th of both May and November. The minimum coupon rate for this bond is 1.45%. The procedure for calculating the coupon amount to be disbursed on November 26, 2022, is outlined as follows. At first, it is applied the formula (2.4), where $m = 11$. Then, from the ISTAT website, the relevant FOI values are:

$$FOI_8 = 113.2,\ FOI_9 = 113.5.$$

Therefore,

$$IN_{d,11} = 113.2 + \frac{d-1}{30} \times (113.5 - 113.2)\,.$$

In particular: on the first day of the month IN is equal to the FOI index recorded 3 months earlier; on the last day of the month IN is equal to the FOI index recorded 2 months earlier; on the coupon payment date, we have

$$IN_{26,11} = 113.45.$$

Moreover, $IN_{\bar{26},\bar{11}}$ was equal to 109.68673 (this value refers to the $IN_{\bar{26},\bar{5}}$ number computed on 26.05.2022). Consequently,

$$IC_{d,11} = \frac{113.2 + \frac{d-1}{30} \times (113.5 - 113.2)}{109.68673},$$

so that $IC_{26,11} = \frac{113.45}{109.68673} = 1.03431$, that corresponds to the bold value in Table 2.3. The coupon amount paid in November 2022 can be finally computed using the formula (2.3)

$$C_{26,11} = \frac{1.45\%}{2} \times 1, \times \max(1.03431, 1) = 7.498748€,$$

while the Revaluated Capital is equal to

$$RC_{26,11} = 1, \times (1.03431 - 1) = 34.31€. \tag{2.10}$$

In conclusion, on November 26, 2022, the BTP Italia IT5351660 made a total payment of

$$7.498748 + 34.31 = 41.808748€.$$

In fact, the bond has matured on this date, resulting in the repayment of the nominal value of € 1,000.

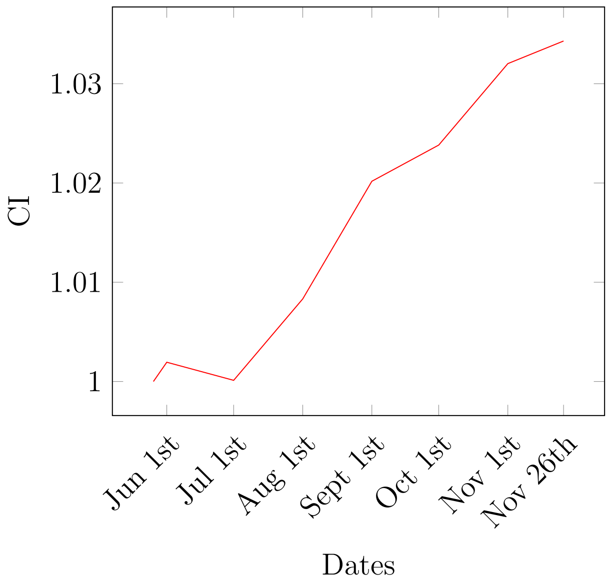

Figure 2.4: The *reference index number* during the coupon period commencing on May 26, 2022, and concluding on November 26, 2022, for the BTP Italia with identification IT5351660 issued on November 26. Source: our own calculations to be compared with BTP Italia IT5351660

Date	$FOI(m-3)$	$FOI(m-2)$	Day d	$IN_{d,m}$	$IC_{d,m}$	
01/11/22	113.2	113.5	1	30	113.20000	1.03203
02/11/22	113.2	113.5	2	30	113.21000	1.03212
03/11/22	113.2	113.5	3	30	113.22000	1.03221
04/11/22	113.2	113.5	4	30	113.23000	1.03230
05/11/22	113.2	113.5	5	30	113.24000	1.03239
06/11/22	113.2	113.5	6	30	113.25000	1.03248
07/11/22	113.2	113.5	7	30	113.26000	1.03257
08/11/22	113.2	113.5	8	30	113.27000	1.03266
09/11/22	113.2	113.5	9	30	113.28000	1.03276
10/11/22	113.2	113.5	10	30	113.29000	1.03285
11/11/22	113.2	113.5	11	30	113.30000	1.03294
12/11/22	113.2	113.5	12	30	113.31000	1.03303
13/11/22	113.2	113.5	13	30	113.32000	1.03312
14/11/22	113.2	113.5	14	30	113.33000	1.03321
15/11/22	113.2	113.5	15	30	113.34000	1.03330
16/11/22	113.2	113.5	16	30	113.35000	1.03339
17/11/22	113.2	113.5	17	30	113.36000	1.03349
18/11/22	113.2	113.5	18	30	113.37000	1.03358
19/11/22	113.2	113.5	19	30	113.38000	1.03367
20/11/22	113.2	113.5	20	30	113.39000	1.03376
21/11/22	113.2	113.5	21	30	113.40000	1.03385
22/11/22	113.2	113.5	22	30	113.41000	1.03394
23/11/22	113.2	113.5	23	30	113.42000	1.03403
24/11/22	113.2	113.5	24	30	113.43000	1.03412
25/11/22	113.2	113.5	25	30	113.44000	1.03421
26/11/22	113.2	113.5	26	30	113.45	1.03431

Table 2.3: Indexing Coefficient for BTP Italia IT5351660 for each day of November 2022. Source: Own calculation

m	y	d	dd	FOI_{m-3}	FOI_{m-2}	$NI_{d,m}$	$\hat{NI} = \max\ NI_{\bar{d},\bar{m}}$	$IC_{d,m} = \max\left(1, \frac{NI_{\bar{d},<\bar{m}}}{\hat{NI}}), \hat{NI}\right)$
11	2018	26	30	102.900	102.40	102.48333		
5	2019	26	31	102.30	102.50	102.46129	102.48333	1.
11	2019	26	30	103.20	102.50	102.61667	102.48333	1.001300
5	2020	26	31	102.50	102.60	102.58065	102.61667	1.
11	2020	26	30	102.50	101.90	102.00	102.61667	1.
5	2021	26	31	103.00	103.30	103.24194	102.61667	1.006090
11	2021	26	30	104.70	104.50	104.53333	103.24194	1.012510
5	2022	26	31	108.80	109.90	109.68710	104.53333	1.049300
11	2022	26	30	113.20	113.50	113.45	109.68710	1.034310

Table 2.4: Calculation of IC in correspondence of the coupon payment dates of BTP Italia IT5351660. Source: Own calculation

Assuming now deflation, the Indexing Coefficient used in this case would be less than one, however the so-called floor mechanism would be applied, therefore, the Indexing Coefficient used for the coupon payment would be equal to 1. The coupon rate would be equal to the real coupon rate and there would be no capital revaluation. In fact, the floor mechanism protects the invested capital from deflation, preventing any coefficient of indexing below one from causing a devaluation.

Start Date	End Date	$1/m$	$IC_{d,m}$	$c \times \frac{1}{m} \times N$	$c \times IC_{d,m}$	$N \times (IC_{d,m} - 1)$	Cash Flow
26/11/2018	26/05/2019	0.5	1.	7.25	7.250	0.0	7.250
26/05/2019	26/11/2019	0.5	1.001300	7.25	7.259425	1.3	8.559425
26/11/2019	26/05/2020	0.5	1.	7.25	7.250	0.0	7.250
26/05/2020	26/11/2020	0.5	1.	7.25	7.250	0.0	7.250
26/11/2020	26/05/2021	0.5	1.006090	7.25	7.2941525	6.0900	13.384153
26/05/2021	26/11/2021	0.5	1.012510	7.25	7.3406975	12.5100	19.850698
26/11/2021	26/05/2022	0.5	1.049300	7.25	7.607425	49.3	56.907425
26/05/2022	26/11/2022	0.5	1.034310	7.25	7.4987475	34.3100	41.808748

Table 2.5: Calculation of past cash flows of BTP Italia IT5351660. Source: own calculation

2.3.3 Other Features

The BTP Italia has other characteristics, which will be now briefly analyzed.

Issuance Process

The BTP Italia is issued through the MOT[4]. Unlike the auction procedure where the price is determined at the end of the auction itself, in this procedure, the

[4]MOT stands for *Mercato Obbligazionario Telematico*

coupon rate is determined on the last day of the placement based on market conditions, while the price is set at par. The BTP Italia is placed on the market during a placement period divided into two phases: the first phase, which takes place in the first days of the offering, is reserved for individual savers and other similar investors, while the second phase, that coincides with the last day of the placement period, is reserved for institutional investors. From the settlement date of the issuance, the bond will be tradable on the secondary market and will have a price determined by market conditions.

Purchase

BTP Italia bonds have a minimum lot size of 1, euros. During the first phase of the placement period, which is dedicated to individual savers and other similar investors, the minimum purchase amount for each order is 1, euros, and multiples of 1, euros can be purchased at the issuance. During the second phase of the placement period, the minimum purchase proposal is 100, euros with multiples of 1, euros.

Taxation

The taxation applicable to capital income for these securities is the same as for all other government bonds, and it is set at 12.5%.

Quotations on the secondary market

Table 2.6 presents the quotations of BTPs Italia with different maturities on the secondary market. Quotes refer to Friday July 27th, 2023. Notice that in the purchase of BTP Itlia, the settlement period is the same as for BTPs, i.e. 2 business days following the trade date. This means that the financial transaction becomes effective two business days after the date on which it was executed. In the present case, it would be Monday July 31st, 2023.

Name	Isin	Last Price	Last Contract
BTP ITALIA NV23 EUR	IT5312142	99.085	27/07/23 17.18.17
BTP ITALIA AP24 EUR	IT5174906	98.573	27/07/23 17.29.18
BTP ITALIA OT24 EUR	IT5217770	97.628	27/07/23 17.14.31
BTP ITALIA MG25 EUR	IT5410912	98.52	27/07/23 17.27.22
BTP ITALIA MG26 EUR	IT5332835	96.35	27/07/23 17.20.01
BTP ITALIA OT27 EUR	IT5388175	95.34	27/07/23 17.27.39
BTP ITALIA MZ28 EUR	IT5532723	100.66	27/07/23 17.07.13
BTP ITALIA NV28 EUR	IT5517195	97.93	27/07/23 17.24.17
BTP ITALIA GN30 EUR	IT5497	96.5	27/07/23 17.23.27

Table 2.6: BTPs Italia quotations on July 27th, 2023. Source: Milano Finanza

2.4 Floating Rate Notes: CCTs

CCTs[5] are variable-rate securities with a term of 7 years. Interest payments are made through semi-annual deferred coupons indexed to the yield of semi-annual BTPs. The indexing mechanism currently in use operates as follows: the annual gross yield observed on six-month BOTs during the last auction prior to the coupon's payment date is multiplied by 0.5, and the spread is added. The resulting figure is then rounded to the nearest five cents. The remuneration is also influenced by the potential issuance discount, which represents the difference between the face value and the price paid. Overall, these features make CCTs attractive to investors seeking variable-rate securities with a medium-term investment horizon and returns linked to the performance of the underlying benchmark securities.

Quotations on the secondary market

Table 2.7 presents the quotations of CCTs with different maturities on the secondary market. The reference index for the calculation of the floating coupon is the 6 months Euribor rate. Euribor[6] is the benchmark interest rate at which European banks offer to lend unsecured funds to other banks in the euro wholesale money market. It is a daily reference rate based on the interest rates at which a panel of European banks borrow funds from one another. On top of the floating component, these bonds pay a fixed amount ranging from 50 basis points (1 basis point=1/10,) to 185 basis points. For CCTs (Italian Certificates of Treasury), the settlement period is generally 3 business days following the trade date. This means that the financial transaction becomes effective three business days after the date on which it was executed.

[5] CCTs stands for *Certificati di Credito del Tesoro*

[6] Euribor stands for *Euro Interbank Offered Rate*

Name	Isin	Last Price	Last Contract
CCT-EU TV EUR6M+0.55% DC23 EUR	IT5399230	100.191	27/07/23 16.27.41
CCT-EU TV EUR6M+0.75% FB24 EUR	IT5218968	100.462	27/07/23 16.27.07
CCT-EU TV EUR6M+1.10% OT24 EUR	IT5252520	101.31	27/07/23 17.10.12
CCT-EU TV EUR6M+1.85% GE25 EUR	IT5359846	102.649	27/07/23 16.52.20
CCT-EU TV EUR6M+0.95% AP25 EUR	IT5311508	101.45	27/07/23 17.13.56
CCT-EU TV EUR6M+0.55% ST25 EUR	IT5331878	100.84	27/07/23 17.25.05
CCT-EU TV EUR6M+0.5% AP26 EUR	IT5428617	100.67	27/07/23 17.07.29
CCT-EU TV EUR6M+0.8% OT28 EUR	IT5534984	100.39	27/07/23 16.42.35
CCT-EU TV EUR6M+0.65% AP29 EUR	IT5451361	99.66	27/07/23 17.22.41
CCT-EU TV EUR6M+0.75% OT30 EUR	IT5491250	98.83	27/07/23 17.25.46
CCT-EU TV EUR6M+1.15 OT31 EUR	IT5554982	100.7	27/07/23 16.43.57

Table 2.7: CCTs quotations on July 27th, 2023. Source: Milano Finanza

Valuation of Debt Securities

Valuation, also known as pricing, is the process of determining the fair value of a financial asset. The fundamental principle of financial asset valuation is that its value is equal to the present value of its expected cash flows.
The pricing process consists of three steps that are described in the next Sections.

3.1 Estimating Cash Flows

Cash flow represents the anticipated future cash receipts from an investment. In the context of fixed-income securities, cash flows encompass the aggregation of cash received at each period, comprising both interest income and principal repayment. While the estimation of cash flows for a conventional BTP (assuming no default risk) is relatively straightforward, involving periodic coupon payments and the nominal amount at maturity, the assessment of cash flows for BTP Italia is more complex. This complexity arises due to the necessity of factoring in how fluctuations in the inflation rate could impact future cash flows.

3.2 Discounting the Expected Cash Flows

Once the cash flows for a fixed income security are estimated, the next step is how to discount the cash flows to the present date. A discount factor is a financial term used to calculate the present value of future cash flows. It is a mathematical factor that represents the reduction in value of money over time, reflecting the opportunity cost of waiting to receive a payment in the future instead of receiving it immediately. By applying the discount factor to each future cash flow, it is possible to determine the present value of those cash flows, which allows for a meaningful comparison of cash flows occurring at different points in time. The discount factor is typically based on the prevailing interest rate or discount rate and the time period until the cash flow is received.

The most appropriate way to value the cash flows of a bond is to use a different discount factor specific to each time period when a cash flow will be received. For example, consider a bond that matures in five years, has a coupon rate of 6%, semi-annual payments, and has a maturity value of €100. Let's assume a discount factor of

$$DF(t,T) = e^{-i(t,T)\times(T-t)},$$

where $i(t, T)$ is called the continuously compounded spot rate relative to the time to maturity $T - t$. The discount factors are applied to calculate the present value of each cash flow. The bond price is obtained via the following formula

$$BP(t) = \sum_{i=1}^{n} DF(t, T_i) \times CF(T_i),$$

where n is the number of due cash flows, i.e., periodic coupons and notional to maturity. The above procedure is implented in table 3.1, where it is assumed a flat term structure of spot rates at 4%, and the bond is paying a semi-annual coupons of 3€ and a notional of 100€ in 5 years. The resulting bond price is 108.7925€.

Time T_i	Semester i	Cash Flow $CF(T_i)$	Discount Factor $DF(t, T_i)$	Discounted Cash Flow $DF(t, T_i) \times CF(T_i)$
0.5	1	3	0.9802	2.9406
1	2	3	0.9608	2.8824
1.5	3	3	0.9418	2.8253
2	4	3	0.9231	2.7693
2.5	5	3	0.9048	2.7145
3	6	3	0.8869	2.6608
3.5	7	3	0.8694	2.6081
4	8	3	0.8521	2.5564
4.5	9	3	0.8353	2.5058
5	10	103	0.8187	84.3293

Table 3.1: Computing the present value of a hypothetical fixed-rate bond, assuming a flat term structure $i(t, T_i) = 4\%$

This example confirms the importance of knowing the cash flows and the discount factors. The crucial point is how to obtain the discount factors that should be used to value the default-free cash flows. In practice, the discount factors are not directly observable in the market. However, the prices of bonds with different maturities are quoted. Therefore, the idea is not to use the discount factors to compute the bond prices, but to do the reverse, i.e., to get the discount factors implied in the bond quotations. This procedure is called bootstrapping. To perform the bootstrapping process, the prices of bonds with different maturities and known coupon rates are used as inputs. By solving a series of equations, the discount factors for various maturities can be derived. These discount factors are then used to construct the term structure of interest rates and serves as a valuable tool for pricing and valuing other fixed income securities.

Clearly, everyday a new term structure can be observed, because the bond prices move depending on the changes of market conditions. Changes in the term structure will affect the prices of interest rate products.

3.3 Pricing of Inflation Linked Bonds

Pricing an inflation-linked bond involves considering the impact of inflation on the bond's cash flows. As illustrated in the previous Chapter, unlike conventional fixed-rate bonds, inflation-linked bonds have their principal and coupon payments adjusted based on changes in an inflation index, such as the Consumer Price Index (CPI), or, in the Italian case, the FOI index.

To price an inflation-linked bond, it is necessary to implement the following steps:

- Firstly, it is necessary to collect data on the bond, including its coupon rate, maturity date, the frequency at which inflation adjustments are made (e.g., semi-annually), and the formula to compute the actual coupon payment and the capital revaluation; this has been presented in formulas (2.3) and (2.6), with reference to BTP Italia.

- Secondly, scenarios concerning the future value of the price index are generated, which will be used to compute the bond's cash flows. Price index scenarios can be derived from historical data or by using some stochastic model.

- Then, the principal and coupon payments are adjusted according to the Indexing Coefficient for each relevant period.

- Use the appropriate discount factors to calculate the present value of each cash flow and then add up all the present values to obtain the bond's overall price.

It is important to verify if the calculated price is consistent with prevailing market prices and conditions. If discrepancies exist, it could present an arbitrage opportunity or the price scenarios forecast needs to be revised.

It is clear that the pricing of inflation-linked bonds is more complex than conventional bonds due to the inflation-adjusted nature of the cash flows. As a result, specialized financial models and tools may be used in the pricing process.

In the next Chapter, a simple procedures will be adopted in order to simulate inflation scenarios.

Scenario Analysis: BTP vs BTP Italia

In this Chapter, the application of simulation to assess the probability distribution of BTP Italia is conducted. Historical simulation, presented in 4.1, is a simulation that provides valuable insights into the potential risks faced by investments, considering real-world market conditions. However, it is essential to note that historical simulation assumes future behavior will resemble the past, which may not always hold true, particularly during unprecedented events or market regime changes. Therefore, historical simulation should be used in conjunction with other risk assessment methods, such as parametric Monte Carlo simulation or others. Regardless of the chosen simulation method, the focus is on generating plausible paths for the future values of the FOI index and obtaining simulated payments for the desired financial product, BTP Italia in the present case.

4.1 Historical Simulation

The historical simulation approach is described in Chapter 22 in Hull (2017) and it consists of the following steps:

- Data collection involves gathering historical data, specifically the FOI index time series, which can be obtained from the ISTAT website. Notably, in January 2011 and January 2016, ISTAT conducted a rebasement, resetting the FOI series to a value of 100 on these dates. To ensure comparability, linking coefficients are calculated and published by ISTAT[1]. Considering the recent surge in inflation, the analysis focuses only on data starting from January 2016. Figure 4.1 displays the series starting from this date: the significant increase in the price index is noteworthy.

- Given the FOI series is not stationary, a preliminary transformation is performed by calculating the inflation on a monthly basis, by considering the log changes in the price index. Descriptive statistics are presented in the table 4.1. It can be observed the significant increase in 2021 and 2022 in comparison with the average negative values registered in 2020[2]. The largest

[1]Refer to the document Coefficients

[2]To estimate the annual average, the monthly average has to be multiplied by 12

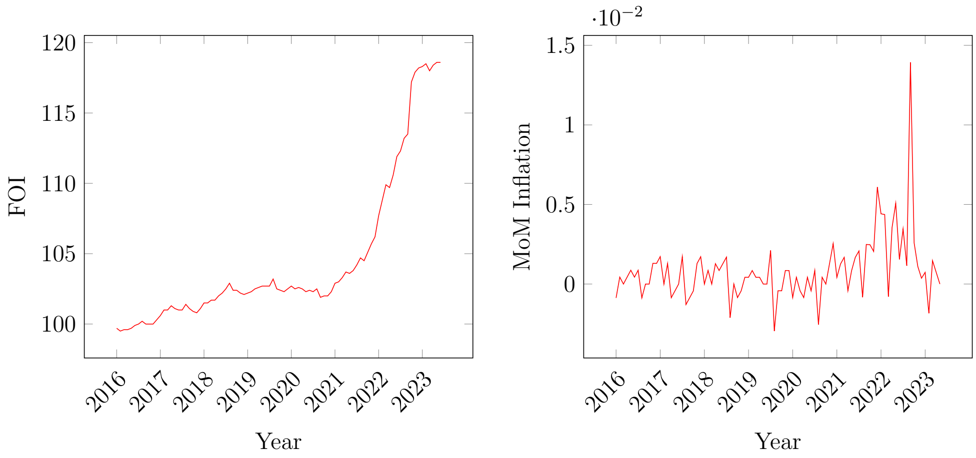

Figure 4.1: FOI Index *(left)* and Monthly Inflation (MoM) *(right)*. Source: Inflation data table January 2000 - June 2023

inflation rate MoM was observed in October 2022 (+1.39%), and YoY was at +11.8% on annual basis[3].

- Then, the monthly differences in inflation are calculated, and the bootstrap simulation is employed on these first differences. The bootstrap simulation involves repeatedly resampling the observed data with replacement to generate multiple simulated series. From the simulated first differences series, the simulated inflation and FOI index are reconstructed. Subsequently, using the simulated FOI index, the cash flows of the BTP Italia at the payment dates can be obtained.

- The simulated cash flows are utilized to calculate the present value of the expected payments as well as the potential range of values that could occur. We can subtract to this present value the bond gross price to obtain an estimate of the probability distribution of the bond net present value. This approach allows for the estimation of a plausible confidence interval for the value of BTP Italia, indicating the maximum, minimum and expected net return that the investor may realize.

4.2 An illustrative example

An illustrative example of the above procedure is presented in table 4.2 where a simulated trajectory of the FOI index is presented.

[3]MoM, i.e., between Sept. and Oct. 2022; YoY, i.e., between Oct. 2021 and Oct. 2022

Year	2016	2017	2018	2019	2020	2021	2022	2023	2016-23
Mean	0.02%	0.03%	0.04%	0.01%	-0.01%	0.14%	0.39%	0.02%	0.08%
StD	0.07%	0.11%	0.11%	0.12%	0.10%	0.11%	0.37%	0.11%	0.21%
Min	-0.09%	-0.13%	-0.21%	-0.30%	-0.25%	-0.08%	-0.08%	-0.18%	-0.30%
Max	0.13%	0.17%	0.17%	0.21%	0.13%	0.25%	1.39%	0.15%	1.39%

Table 4.1: Descriptive statistics of the monthly inflation rates. Data for 2023 are incomplete

M-Y	MoM Inflation	First Differences	M-Y	Resampling Random Integer	Resampling First Differences	Resampling Inflation	Resampling FOI
Dec-22	-0.09%		Jun-23			0	118.6
Jan-23	0.04%	0.13%	Jul-23	5	-0.04%	-0.04%	118.5
Feb-23	0.00%	-0.04%	Aug-23	3	0.04%	0.00%	118.5
Mar-23	0.04%	0.04%	Sep-23	5	-0.04%	-0.04%	118.5
Apr-23	0.09%	0.04%	Oct-23	1	0.13%	0.09%	118.6
May-23	0.04%	-0.04%	Nov-23	5	-0.04%	0.04%	118.7

Table 4.2: Example of bootstrapping historical series to simulate the FOI index starting from June 2023 using a hypothetical sample of size 5

Figure 4.2 illustrates five simulated paths of the inflation rate over the next 84 forthcoming months. Correspondingly, the remaining figures are presenting simulated values of the FOI index, of the Index Number and a possibile realized scenario for the coupon amount at different coupon dates (see Figure 4.5). Figure 4.6 illustrates, across 10,000 simulations, the maximum, average and minimum (dotted blue lines) simulated coupons and 10 additional possible paths (in green). Finally, in Figure 4.7 given the BTP Italia under examination, we consider a standard fixed rate BTP and we estimate the probability that the Net Present Value of the BTP Italia is larger with respect to the NPV of the BTP varying the fixed rate. Clearly, lower the fixed coupon rate, more convenient is to invest in the BTP Italia. Larger the coupon, smaller the probability of obtaining a larger NPV. For example, the probability is smaller than 50% if the fixed rate coupon is approximately larger than 5.

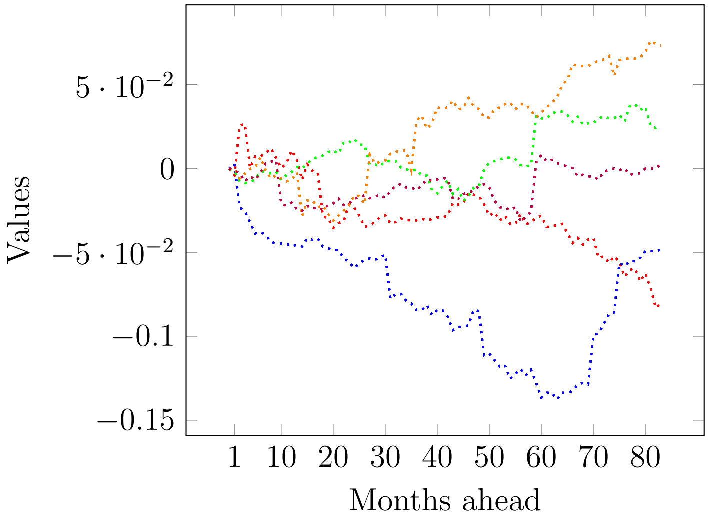

Figure 4.2: Five simulated paths of the MoM inflation

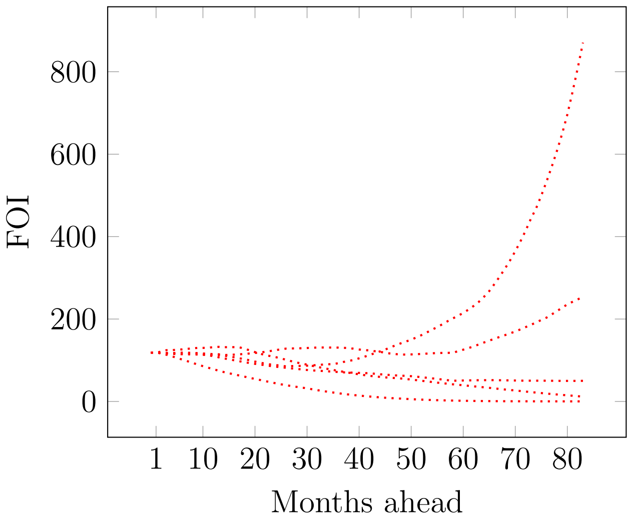

Figure 4.3: Five simulated paths of the FOI Index

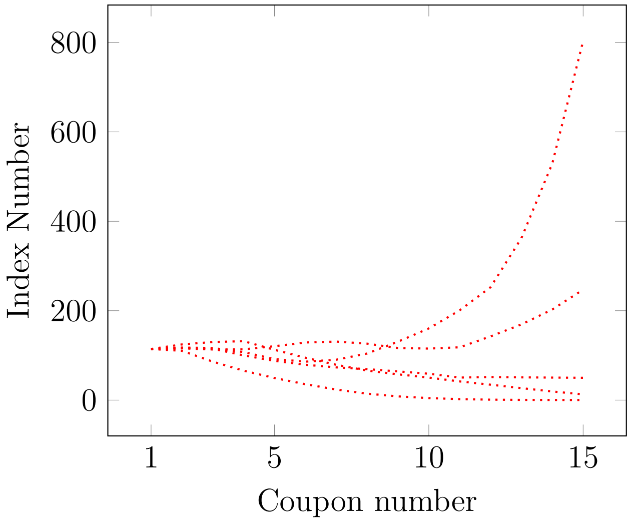

Figure 4.4: Five simulated paths of the IN number at the coupon payment dates

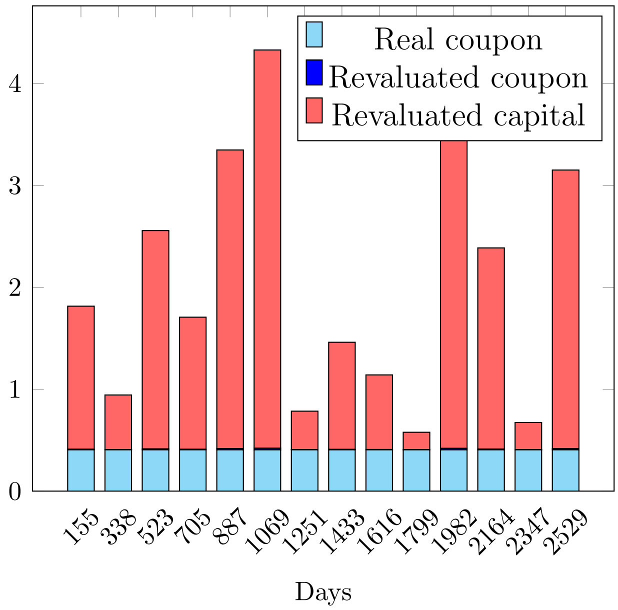

Figure 4.5: Stacked values of simulated fixed and real coupons

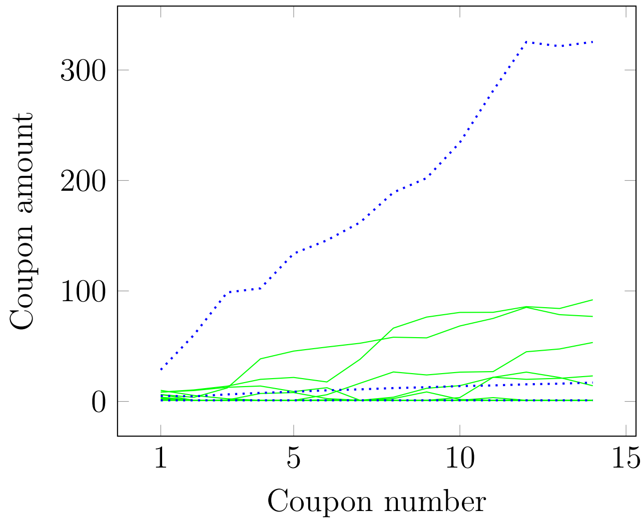

Figure 4.6: Simulated paths of the revaluated coupon and capital amount, including maximum, minimum, average values (in blue)

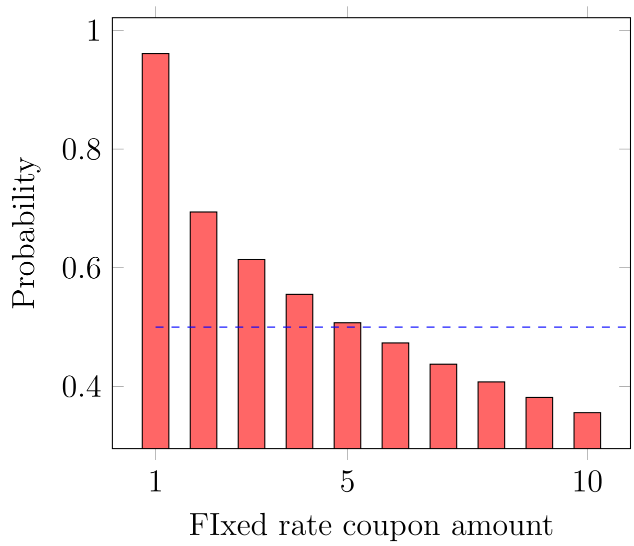

Figure 4.7: Probability that the NPV of the BTP Italia is larger than the NPV of the fixed rate BTP

Conclusions

In this thesis, we have presented the main fixed-rate securities, with a particular focus on BTPs (Italian government bonds) and BTP Italia bonds. In reference to these securities, we have provided a detailed description of how coupon payments are calculated. Subsequently, a scenario analysis has been conducted to propose a possible criterion for choosing between a traditional fixed-rate BTP and a BTP Italia bond.

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