

# Implementing partisan symmetry: A response to a response

Daryl DeFord<sup>1</sup>, Natasha Dhamankar<sup>2</sup>, Moon Duchin<sup>2</sup>,  
Varun Gupta<sup>3</sup>, Mackenzie McPike<sup>2</sup>, Gabe Schoenbach<sup>2</sup>,  
and Ki Wan Sim<sup>2</sup>

<sup>1</sup>Washington State University

<sup>2</sup>Tufts University/Voting Rights Data Institute

<sup>3</sup>University of Pennsylvania

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## Abstract

Katz, King, and Rosenblatt recently wrote a broad survey developing and extending the theory of partisan symmetry. Our paper reviewed the implementability of the theory, focusing on simplified scores of symmetry—seemingly compatible with their formulation—that are in wide use. We analyzed these simplified scores and concluded that they are not suited for redistricting reform. By our reading of their response, Katz, King, and Rosenblatt agree.

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What should you do when your clean, abstract theory of partisan fairness needs to be translated to the real-world "political thicket" of redistricting? Should you expect that practitioners will adhere to the "statistical and social science principles of inference" in a manner that would be suitable for a graduate course or a research paper? Should you spell out a simplified protocol (or bright-line test) to create a manageable standard for courts and commissions? Or should you leave it to others to try their hand at simplification?

The partisan symmetry theory summarized and refined by Katz-King-Rosenblatt (henceforth, "KKR") has indeed been deployed as a set of simplified scores by many users in the field, across scholarly literature, expert reports, and legal text.<sup>1</sup> These scores are the subject of our analysis, and it is these scores that KKR refer to as "methodological mistakes" in our paper, though they are not of our making. Caveats and statistical machinery will be, and have been, lost in translation from academic journals to courts and state constitutions.

The main thrust of our article is not about the use of partisan symmetry with tools of statistical modeling<sup>2</sup> as a technique of social science. Instead, we believe the KKR response bolsters our argument that partisan symmetry methods belong in the research literature and will backfire if adopted as target metrics in legislation and litigation. Practitioners might have gotten a different impression from the body of amicus briefs in the last few decades, so we hope that this clarification will be helpful to policymakers and legal teams. So when KKR say that our paper "focuses solely on descriptive measures," they are correct. But they take a cut-and-dried interpretation of partisan symmetry to be our goal; instead, it is our worry.

## Reiterating premises

Readers of this exchange are by now aware of at least two competing frameworks. On one hand, one might seek a broad normative theory of electoral fairness. On the other, there is a narrower

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1. Besides the reform language discussed in our paper, examples include expert reports by leading political scientists. See for instance Supplemental Expert Affidavit of Stephen Ansolabehere in *Favors v. Cuomo*, 2012. Simplified partisan symmetry metrics are used exactly as they appear in our treatment, applying a linear shift to the 2008 presidential election in to score multiple plans; Ansolabehere calls this a "standard methodology."

2. Namely, "definitions of quantities of interest, separation of these quantities from empirical measures, uncertainty estimates, estimators, or formal statistical properties of proposed measures"

goal of identifying the redistricting maneuvers that go by the name of "partisan gerrymandering." Our framework is the latter, sticking closely to the working definition offered by KKR themselves: "Partisan gerrymanderers use their knowledge of voter preferences and their ability to draw favorable redistricting plans to maximize their party's seat share." This is why we are considering seats maximization using available data, and why we resist the slippage to larger principles of fair play.

We are treating the districting lines as movable because—well, they are. The question at hand is how to evaluate a proposed districting plan at the time of adoption. We are treating a static voting index (such as a single past election, or a pattern arrived at by combining multiple elections<sup>3</sup>) as the other input to the analysis because this, again, is what is available to line-drawers at the time of plan adoption. They do not have the ability to evaluate a plan by the lights of actual future elections, and the stakeholders are unlikely to agree on any sophisticated method of creating predicted future elections. The reform language of the last few years confirms this.

## Paradoxes

We use the word "paradox" in a sense that has a long pedigree in U.S. electoral politics. A paradox is not a logical contradiction, but just a counterintuitive consequence. Our "Utah Paradox" is a reference to the classic "Alabama Paradox" of the 1880 Census: it was observed that *adding* a seat to the U.S. House could cause the state of Alabama to *lose* a seat, under Hamilton apportionment. More seats to go around meant one fewer for Alabama. Rather than debunking any particular claim, the paradox merely highlights an undesirable property of the apportionment scheme—one that might cause the public to lose faith in the system.

In our case, since a (successful) partisan gerrymander is one in which some party maximizes its seats, it must be considered surprising if a gerrymandering metric reports the opposite advantage. There is a significant body of practitioners to whom this will be news: the scores do not mean what you think they mean.

## Characterization theorem

A few key facts are readily observed from an analysis of the partisan symmetry standard. First, no matter how you construct a seats-votes curve, a plan's deviation from symmetry can be captured in an integral. (This observation has been made in the past by Rosenblatt and others—we call this integral the *partisan Gini*, following Grofman.) Next, applying linear UPS to a vote index produces an increasing step function as the seats-votes curve, and all partisan symmetry scores can be computed knowing only the vote shares in each district of a plan.

Our partisan symmetry characterization theorem (Theorem 3) presents a different, and novel, finding that is worth repeating simply: that integral will be zero, indicating ideal symmetry, if (and only if!) the vote shares by district are symmetrically spaced on the number line. This is easy to state but tricky to prove, and it means that the blessings of the symmetry standard can be quickly eyeballed for a given plan and vote pattern.

## Targets and adversaries

The anthropologist and STS scholar Marilyn Strathern famously riffed that "*When a measure becomes a target, it ceases to be a good measure.*" To take this adage seriously, a real-world analysis must consider metrics in reductionist and adversarial conditions. Even if partisan symmetry is capturing the "quantity of interest" for a general theory of fairness, how well does that hold up in practice after the metric is announced to agenda-driven line-drawers?

Given any pre-agreed partisan symmetry setup, an adversary who is motivated by simple seats

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3. Our view is that using many elections is of course superior to using just one, but that if any combination or aggregation is attempted, the elections should also be considered individually.

maximization can easily choose a plan with an extreme seat advantage and a good symmetry score. To demonstrate this worry, we point to our Figures 4-6, which clearly illustrate the decoupling of symmetry scores from seat shares, once a particular vote index has been set.<sup>4</sup> (Replace our use of a single past election with your favorite predictive index and the point remains firmly in place.) If KKR believe that their methodological objections blunt this point, they have not told us why.

## Takeaways

Despite appearances, all ten authors are in significant agreement when it comes to our central message: there are simple scores of partisan symmetry circulating, but buyer beware!

In particular,

- When a static vote index is overlaid with a districting plan, the simple mean-median score (*the signed difference between the median and the mean vote by district*) should NOT be interpreted as quantifying the degree of advantage to a party, or even as identifying which party is advantaged.
- When a static vote index is overlaid with a districting plan, the simple partisan bias score (*the share of districts with above-average vote share, minus one-half*) should NOT be interpreted as quantifying the degree of advantage to a party, or even as identifying which party is advantaged.
- Partisan symmetry provides no reliable, self-contained, one-shot numerical test for flagging gerrymandered plans using past election results.

In addition to negative agreement, we also have positive agreement: we all endorse a careful, holistic approach to evaluating partisan advantage in a districting plan or electoral system.

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4. Whatever you think of proportionality or the efficiency gap in broad normative terms, they are quite different from symmetry in this regard. Both proportionality (based on  $S$  vs.  $V$ ) and the efficiency gap ( $S$  vs.  $2V - 1/2$ , plus turnout noise) yield scores closely coupled to the seats outcome.