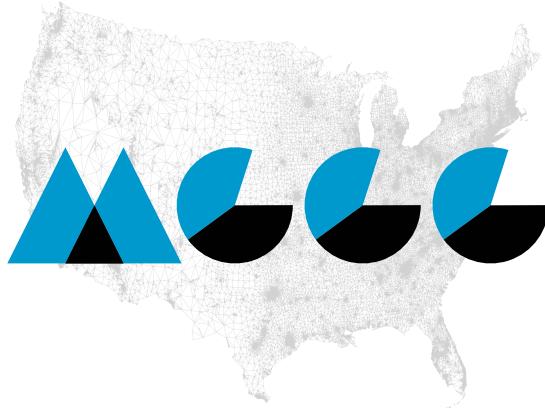


Modeling STV for the California Legislature



Data and Democracy Lab

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Contributors

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1 Understanding STV

Single transferable vote, or STV, is a ranked-choice voting system that is used to conduct multi-winner elections. STV is designed to elect a group (sometimes called a "committee") that proportionally reflects the preferences of the electorate, so it sometimes goes by the name of *proportional ranked choice voting*.

In broad strokes, STV works by beginning with a pool of candidates and using a round-by-round system to eliminate those with least support and elect those with most support, while transferring a voter's preference down their ballot as the rounds advance.¹

To be a bit more precise: there is a threshold of first-place votes (depending on the number of voters and the number of seats) that must be surpassed for a candidate to be elected. If no candidate meets the threshold initially, then the candidate with the fewest first-place votes is eliminated, and their votes are transferred to the next candidate on those ballots. This is repeated until a candidate crosses the threshold; when this occurs, that candidate is elected and any votes above the threshold are transferred to the next choice of their voters. This is repeated until all seats have been filled. More details, including a round-by-round example, can be found in Appendix A.1.

Deciding whether or not to adopt STV is highly context-dependent; the demographics of the electorate, the spatial arrangement of voters, and the overarching goals of an electoral reform must all be considered. However, some broad themes of comparison across methods can be helpful for framing the decision.

Tendency to proportionality, lower threshold for inclusion

Conventional wisdom hold that STV is "semi-proportional": while not providing guarantees, it is thought to produce more proportional outcomes than standard first-past-the-post (otherwise known as plurality) elections.² The plurality system requires that a candidate get more votes than anyone else to win—this can disenfranchise minority groups when they are evenly spread over the districts, never controlling the outcomes in any single one. Under STV, on the other hand, minority groups can achieve representation more easily, since the threshold level of support will be easier to achieve. For instance, in a district that elects four members, just 20% of the vote is needed to secure a seat.

Flexibility

STV can elect a variable number of winners by setting the *district magnitude* (or number of seats) to be any number m . Frequently, district magnitude of three to five is cited as an ideal size that balances the benefits of proportionality with the manageability of the election. So for example a nine-member city council would require nine districts using standard plurality voting, but could be configured in a 3×3 fashion (three districts electing three members each) or as a citywide at-large election in which voters choose all nine at once.

¹When electing a single candidate, this resembles a repeated runoff election where the candidate pool is reduced by one each round, so when it is used for a single winner, this mechanism usually goes by the name of instant-runoff voting (IRV).

²There is one mathematical guarantee that is well known for STV, called Proportionality for Solid Coalitions. This loosely says that if any group makes up a substantial fraction of voters and solidly ranks one group of candidates over the other, then they will get proportional representation. However, real-world voters rarely display such perfect discipline in their rankings.

In addition, STV does not require candidates to run under a party label. This can lower the barrier to entry for candidates who may not enjoy the support of a political party, and makes the system suitable for local elections that are non-partisan.

Simple or complex?

The actual mechanics of STV are relatively complex, and it can be difficult for voters to understand the algorithm that converts their rankings into an outcome. Additionally, voters are asked to rank multiple candidates, instead of simply selecting their top candidate, which may increase the research burden for voters. And the ballots themselves can be cumbersome, often asking voters to bubble in a first choice, second choice, and so on.

However, the big-picture idea of STV is easy: your preference sits with your first-place choice until that person is elected or eliminated, and then it moves down to your next favorite candidate. This allows groups to coalesce around their shared preferences, worrying less about putting long-shot candidates first. Furthermore, the idea of ranking your choices in order is extremely intuitive.³ As voting technology improves, the gap from the idea to the ballot will continue to reduce.³

Frequently, voter education efforts are geared to getting voters to cast a complete ballot, ranking as many candidates as they are allowed. This is because the transfer system is coordinated so that everyone can get the same total vote weight, with part of their support going to get a more preferred candidate to threshold and part going to their lower preferences. Short ballots can prevent that transfer process, "exhausting" the ballot before it gets its full weight.

Overall, it is important to emphasize that the specific impact of STV, particularly with respect to the promise of proportionality, depends heavily on the local political landscape and on voters' behavior. In Section 3, we explore the effects that STV could have in California under a variety of configurations and behavioral assumptions.

2 Background and current opportunities

2.1 Use in the United States and abroad

Despite sparse implementation today, STV has a long and storied history in the United States. Starting in the early 20th century, ranked choice voting in multi-member districts was adopted for use in local elections across some two dozen cities, including New York, NY; Cincinnati, OH; Cambridge, MA; and Sacramento, CA. Almost all of them eventually repealed STV; Cambridge is the notable exception, having used STV continuously since 1941 to elect its nine city councillors and six school board members.⁴

The story of STV's adoption and repeal in Cincinnati is emblematic of both the success of this reform strategy, and of the societal friction it created. Like many cities in the United States at the

³For instance, a type of voting machine called a *ballot marking device* could allow you to drag and drop the candidates into your preferred order on a screen, then print it to paper to be sure it came out as you intended before you cast the paper ballot.

⁴See cambridgema.gov.

turn of the 20th century, Cincinnati city hall was a partisan machine; here controlled by the Republicans. In the 1921 election, Republicans won 31 out of 32 seats on the city council, despite Democrats making up about 44% of the electorate. To address this inequity, reformers from the Charter Committee introduced an STV reform measure to the 1924 ballot, which passed with 69% of the vote. By most measures, the reform was a success. The city elected its first Black councilors, with 10 out of 15 councils elected under STV having at least one Black councilor. After the possibility arose that the council would elect a Black mayor, the fifth repeal effort passed in 1957, with White-majority precincts voting for repeal 2-1 and Black-majority precincts voting 4-1 to keep STV.⁵

The story in New York is largely the same, with Tammany Hall's Democratic machine controlling a supermajority of the Board of Aldermen despite only being supported by a modest majority of voters. The implementation of STV in 1937 led to a wave of women, people of color, and minority parties winning seats on the new City Council. However, STV was repealed in 1947, amidst the Red Scare, after two Communist Party candidates won seats. The two major political parties also led the charge for repeal in New York and other cities, as shifting coalitions sought to increase their influence over legislatures and voters.⁶

What is illustrated in the history of STV in the United States is that the reform worked as intended. STV, through multi-winner elections and voting thresholds, enabled candidates without majority support to be elected and gave political voice to marginalized groups, and (along with other Progressive-era government reforms) it helped break entrenched urban political machines. Ultimately, the reform was a victim of its own success.

In the 21st century, we see a renewed interest in ranked choice voting, with cities like New York, NY adopting IRV for primary elections; Portland, OR adopting STV for city council; and a very large number of municipalities moving to IRV for a wide variety of offices.⁷ Voters in two states, Maine (since 2016) and Alaska (since 2020), have adopted IRV for state elections.

STV has also been used for upwards of a century around the world (most widely in the Anglo-sphere) to elect national, regional, and local governments. Australia uses STV to elect both chambers of its Federal Parliament, one or both chambers in the parliaments of its states and internal territories, and certain local councils across the country.⁸ Ireland uses STV to elect its Dáil (lower house of parliament), delegates to the European Parliament, and every local council in the country; and single-seat STV to elect the President.⁹ New Zealand uses STV to elect 15 local councils.¹⁰ In Northern Ireland, STV is used to elect its Assembly, and both Scotland and Northern Ireland elect their local councils using STV.¹¹

⁵See [Sightline](#).

⁶For more on the New York history, see [FairVote](#).

⁷Voters in Portland, Maine approved a proportional ranked choice system for city council, but neglected to increase the district magnitude. This means that though STV is approved in principle, it functions as IRV in practice.

⁸ecanz.gov.au

⁹electoralcommission.ie

¹⁰stv.govt.nz

¹¹electoral-reform.org.uk

2.2 Opportunities in California

California, the largest and one of the most diverse states in the U.S., is an excellent site for policy experimentation because voters can amend the state constitution by simple majority through the ballot initiative process. Thus, voters could decide to reform their system of electing the 40-member Senate or the 80-member assembly, such as by shifting to a system of districts that elect five members each with STV.¹²

We will evaluate potential impacts of a system shift along racial/ethnic lines by considering the opportunity for Hispanic/Latino voters to elect candidates of choice. The Hispanic share of the population in California is 30–40%, depending on which basis for population is used. Those identifying as Hispanic/Latino made up 39.40% of total population and 35.95% of voting age population in the 2020 Census. Hispanic-identified respondents also made up 31.70% of citizen voting-age population in the 5-year American Community Survey centered on 2020. It is hard to be sure how many members of the state legislature were preferred by Hispanic voters without doing a detailed statistical analysis, but we can count members who are themselves Hispanic/Latino-identified as a rough proxy. At the moment, there seem to be 10 Latino Democrats plus 2 Latino Republicans in the Senate ($12/40 = 30\%$) and 24 Latino Democrats plus 4 Latino Republicans in the Assembly ($28/80 = 35\%$), for pretty close to proportional representation in both cases.

Next, we will consider possible partisan consequences by looking at the minority party, Republicans. As our baseline election to mark each district with its R share of votes, we will choose the Trump-Biden contest of 2020, in which Trump received 35.1% of the major-party vote. Republicans currently hold 10/40 Senate seats and 20/80 Assembly seats, or 25% in each case.

3 STV in California: Empirical results

We consider shifting California's upper legislative chamber (40 seats) and lower legislative chamber (80 seats) to an alternative system. This could be accomplished by shifting to a 10×4 or 8×5 STV system for Senate or keeping 40 districts but shifting from plurality to IRV. Likewise, the Assembly could have a 20×4 STV, 16×5 STV, or 80×1 IRV or plurality structure.

3.1 Simulating voter ranking behavior

In order to evaluate the impact of shifting to STV, we first need to generate realistic ranked-choice ballots for voters. One of the primary challenges with deciding on an electoral reform is that there is a shortage of historical ranked-choice data available to draw on in learning realistic voter ranking behavior, thus making it difficult to evaluate the possible impacts of a system shift. But there is a growing body of work in computational social choice that allows us to generate realistic preferences for voters using parameterized mathematical models.¹³

While a full discussion of the mathematical models is beyond the scope of this report, we briefly

¹²One caveat is that the state constitution contains a single-subject rule: "An initiative measure embracing more than one subject may not be submitted to the electors or have any effect." Cal. Const. Art. II, §8(d). This may raise a concern that the shift to multi-member districts is a separate subject from the shift to a ranked system with transfers. However, the STV system as such requires magnitudes of two or more, so this seems unlikely to run afoul of the single-subject rule.

¹³See for instance former MGGG case studies, compiled at mggg.org/struc_dem.

introduce them here. We use three different behavioral ranking models to generate ranked-ballots for voters: the **impulsive voter**, the **deliberative voter**, and the **Cambridge voter**. All three models start with slates; for instance, a choice featuring seven candidates from slate *A* and three from slate *B* could be written as

$$\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, B_1, B_2, B_3\}.$$

In all three models, voters first select a *ballot type* (such as *AAABABBAAA*) before filling in the choices within each slate. The impulsive voter flips a weighted coin to fill in each position, choosing *A* or *B* based on the coinflip and not looking back at the ballot overall until the candidates run out. The deliberative voter considers all the pairwise comparisons between the candidates in their ballot, effectively asking "should I swap these two?" over and over again, and using a weighted coin to decide each time. Finally, the Cambridge model uses actual historical data from Cambridge, MA to generate the ballot—so unlike the other two, it features "short ballots"—ranking just one or a few candidates instead of ranking everyone—at a realistic rate. The user can choose a cohesion parameter that controls the strength of the coinflip, and all three models are calibrated so that if group *A* has 60% cohesion, then members of that group have a 60% chance of listing an *A* candidate in first place. A more detailed description of the ballot generator models and an example can be found in Appendix B.

To study the impact of adopting voting reform, we used the mathematical ballot generators described above to conduct thousands of simulated elections. First, for each number of districts (8, 10, 16, 20, 40, or 80), we generate five districting plans with a randomized *recombination* algorithm. For each of these maps, we know the demographics and voting history of each district, using Census data for Hispanic/non-Hispanic labels and using the Trump–Biden contest for partisan labels.

3.1.1 Race/ethnicity assumptions

One of the hardest things to predict is who will run for office under a new system. As a baseline, we created a simple candidate pool

$$\{H_1, H_2, H_3, H_4, O_1, O_2, O_3, O_4, O_5, O_6\},$$

where the *H* candidates are Hispanic-preferred and the *O* candidates are preferred by Other voters (the non-Hispanic majority).

The rest of the behavioral parameters are set as follows.

- **Turnout.** Hispanic adults turn out to vote at 40% the rate of Other adults. (This takes into account lower eligibility due to citizenship, and historically lower rates of registration.)
- **Cohesion.** Hispanic voters have a 75% lean towards the slate preferred by their group overall. (So a coinflip is 75% likely to come up *H* rather than *O*.) Other voters have 80% lean toward *O* candidates.
- **Candidate strength.** We make no assumptions about whether each group views a consensus strong candidate within its own or the opposite slate; so we draw the specific strength values for candidates uniformly at random.¹⁴

¹⁴To be precise, these are set by a one-parameter symmetric Dirichlet distribution, where $\alpha \rightarrow 0$ models consensus and $\alpha \rightarrow \infty$ models indifference. The middle value $\alpha = 1$ is a neutral option. The description above sets all α values to 1.

Importantly, we do not claim that these parameter settings are historically accurate, learned from real data, or predictive. Instead, we fix these parameters as a starting point for analysis—we then vary the voting system in order to isolate system effects from behavioral effects.

In follow-up work, we will vary these behavioral choices and measure the impact on the representational projections.

3.1.2 Partisan assumptions

- **Slates.** 5 D and 5 R candidates run in each district.
- **Turnout.** Democrats and Republicans turn out at the same rate.
- **Cohesion.** D voters and R voters both have 85% lean towards their own slates.
- **Candidate strength.** As before, we make neutral assumptions on whether each group will tend to agree on strong candidates from each slate.¹⁵

Once again, we view these as baseline parameter decisions, letting us scientifically study the effects of changing the election system while preferences stay constant. Further research should certainly vary these settings.

3.2 Representational prospects for Hispanic-preferred candidates

The results of the experimental trials are shown in Figure 1. To make these, we randomly sampled five districting plans of each size needed for the study. In each plan, every district has a demographic balance of HVAP and OVAP (Hispanic and Other/non-Hispanic voting age population). By applying the turnout assumption, we infer the balance of the electorate. Then, using the settings about the candidate pool, voter lean, and candidate strength, we generate 100 preference profiles (simulated cast vote records) under each behavior model (impulsive, deliberative, or Cambridge). Finally, we feed the simulated ballots through STV, IRV, or plurality voting rules to find the winners, and tally them across the districts in the plan to record how many H-preferred and O-preferred candidates would be elected. This means that there are 1500 simulated elections in each plot in Figure 1: 5 plans \times 3 voter behavior models \times 100 preference profiles.

On the left, we see the breakdown for the number of seats projected to be filled by Hispanic-preferred candidates in the California State Senate, while the Assembly projections are shown on the right. The rows are broken out by electoral system, and the colors in the histogram highlight the differences in voter behavior (impulsive, deliberative, or drawn from Cambridge historical patterns that include short ballots).

The plots also show two lines marking the population proportion of Hispanic voting age population (VAP), and the combined support for Hispanic-preferred candidates under the assumptions about turnout and cohesion. This means that 14.18 Senate seats or 28.37 Assembly seats would be needed to match the Hispanic/Latino share of the adult population. Because of lower turnout by Hispanic voters, the electoral support received by H candidates falls several seats short of the population proportionality: the combined support level works out to 11.96 seats in the Senate or 23.93

¹⁵As before, this means Dirichlet $\alpha = 1$ across the board.

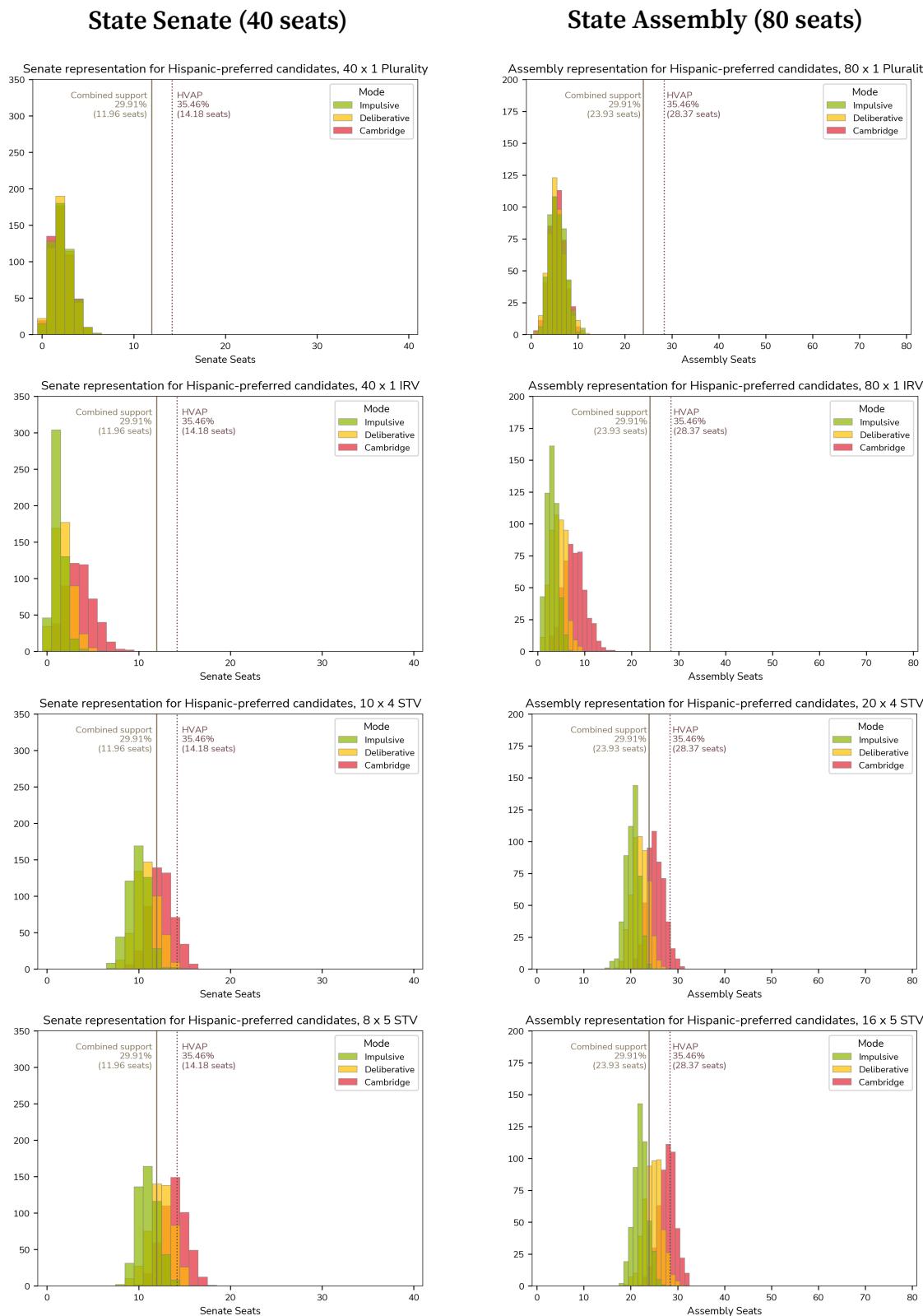


Figure 1. Comparing representational projections for Hispanic-preferred candidates across systems of election.

in the Assembly. It would be reasonable to say that a voting system secures roughly proportional representation if it meets either of these bars.

Recall from Section 2.2 that there are currently 12 Latino-identified state Senators (30%) and 28 Assembly members (35%)—already close to proportional outcomes. We first observe from the first row of Figure 1) that our plurality modeling predicts a significantly lower Hispanic-preferred seat share compared to the status quo. Reasons for the discrepancy could include the following: the design of the districts differs from our random districts in relevant ways (such as by attempting to create majority-Hispanic districts); Hispanic candidates are not the same as those preferred by Hispanic voters; we have underestimated the willingness of non-Hispanic voters to support the Hispanic-preferred slate; or we are failing to model the different kinds of support for Democratic and Republican Latinos. In any event, none of this harms our ability to learn from the shift from the first row (plurality voting) to the next rows—they hold the voter rankings constant and simply shift how winners are determined.

Some advocates argue that even while keeping the structure of 40 or 80 districts, a shift from plurality voting to single-winner ranked choice (IRV) can provide representational benefits. While still only electing a single candidate, IRV allows voters to communicate their preferences beyond the top preference. However, if the goal is proportionality, then Figure 1) shows mixed results. If California voters cast similar ballots with a similar structure to Cambridge voters, the expected share of Hispanic-preferred representation increases significantly—but if they vote complete ballots under either the impulsive or deliberative models, then IRV actually performs worse than plurality by this metric. The lack of proportionality is not particularly surprising because IRV still elects a single candidate per district, and can't award fractions of one winner.

We see a more transformational change occur when elections shift from single-winner districts to multi-winner districts. This allows for far more proportional representation for this minority voter bloc. The last two rows of Figure 1 show a drastic improvement in representation for the H slate. The proportion of seats won varies depending on the type of balloting behavior, but in all six combinations of mode and chamber, it is notable that magnitude 5 (electing five winners per district) gives a slightly higher projection than magnitude 4.¹⁶

The explanation for the proportionality performance boost from STV is fairly straightforward: to win a single-member district, you need majority support, but to win one or two seats in a multi-member district requires a far lower support threshold. Therefore we should expect STV to confer representation to a geographically dispersed minority more often. Of course, dropping the winner-takes-all property can cut both ways, but this nets out overwhelmingly in favor of STV. All told, the STV setups explored here are projected to increase the share of Hispanic-preferred representatives by 6.8 to 30.2 percentage points, depending on voter behavior.

Finally, let us briefly examine the strong performance under Cambridge-style voting. Critically, these Cambridge, MA ballots were often "truncated" (i.e., ranking fewer than ten candidates, in this case). Truncated or short ballots have sometimes unpredictable effects in STV elections, and conventional wisdom holds that the tendency to vote short ballots is bad for minority representation. Thus it is especially interesting to see that this modeling predicts that voter truncation would help Hispanic representation in California.

¹⁶This is emphatically not a general trend; previous studies have sometimes encountered scenarios where four-winner districts are better for minority representation. It depends in a detailed way on all the other facts and assumptions.

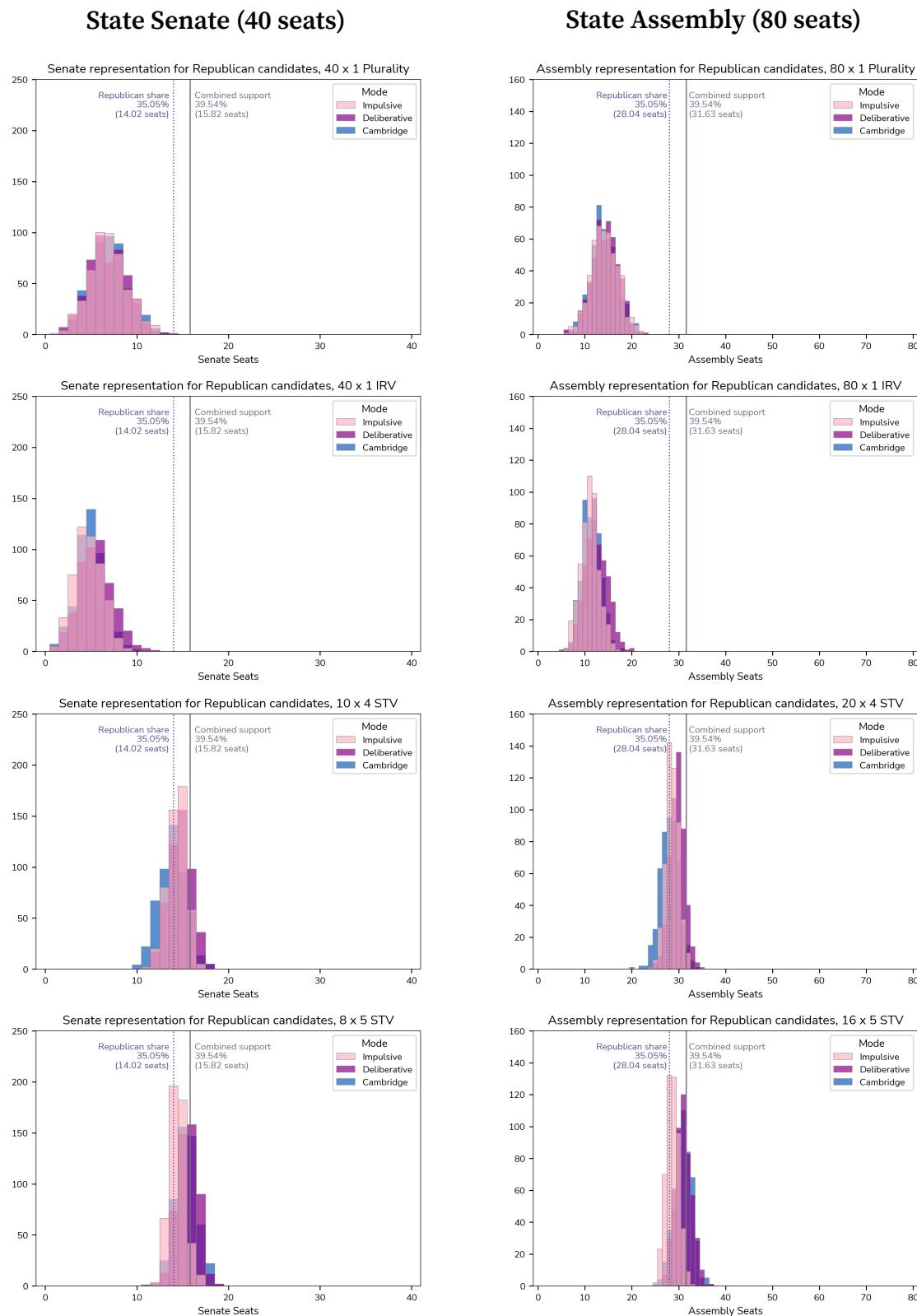


Figure 2. Comparing partisan representational projections across systems of election.

3.3 Representational prospects for Republicans

We repeat a very similar analysis to model partisan outcomes, using the Trump vs. Biden contest to tag voters with R or D labels. In the 2020 presidential election, just over 35% of major-party voters supported the Republican (Trump) and the remaining voters supported the Democrat (Biden).

We once again draw lines for two possible proportionality targets. One line marks the proportion of R votes in the state, and the other marks combined support for R candidates under the listed assumptions about turnout and cohesion. This means that 14.02 Senate seats or 31.63 Assembly seats would be needed to match the Republican share of raw votes, and slightly more would be needed to match the share of voter support. Recall that Republicans occupy 10 out of 40 Senate seats and 20 out of 80 Assembly seats (both 25%)—a significantly sub-proportional status quo.

The real-world Republican share of representation is just at the edge of what is predicted in the first row of Figure 2. The same range of explanations is possible, including the difference in actual districts from these randomized alternatives, the gap between support for Trump and for local legislative candidates, and cohesion/candidate pool/candidate strength assumptions that miss the mark. But once again this still gives us a solid set of simulated preferences from which to observe the shifts in outcome due only to a shift in voting system.

Shifting from plurality to IRV, while keeping all 40 or 80 districts, lowers the projected Republican representation this time—the reverse of what we saw for the Hispanic voter modeling. This time, substantially overlapping histograms tell us that voter ranking behavior does not significantly alter the share of Republican representatives.

Once again, the shift to multi-winner districts is much more impactful, even within the world of ranked choice mechanisms. This allows for the minority bloc—Republicans—to secure much more proportional representation. The last two rows of Figure 2, show a drastic improvement in the number of seats won by Republican candidates, for either STV magnitude we considered. Both STV setups achieve rough proportionality, but this time, there is no observable difference between $m = 4$ and $m = 5$. Finally, the ballot truncation in the Cambridge model makes a small but discernible difference in the four-winner trials, but none in the five-winner trials.

4 Conclusion

In this study, we focused on possible impacts of voting reforms that combine multi-member districts with ranked-choice voting, finding that this shift to STV would likely improve proportional representation of both of the particular racial/ethnic and partisan minorities that we considered. To evaluate potential impacts for ethnic representation of a system shift, we looked at the opportunity for Hispanic/Latino voters to elect candidates of choice. For the partisan modeling, we looked at opportunities for the Republican minority. For both scenarios, we compared four election systems, three kinds of voter ranking behavior, and a selection of different districting plans.

Across the board, while fixing structural assumptions about voter behavior and the candidate pool, STV vaults the minority group's projected representation from significant proportionality shortfalls to levels right in line with population shares and electoral support. Overall, our results robustly indicate that a shift to STV could enhance representation for several kinds of minority groups in the California legislature.

A A closer look at STV

A.1 STV election example

Example 1. Consider the following 21 cast ballots, with preferences listed from top to bottom. If $m = 2$ seats are up for election in a contest with $N = 21$ voters, the standard threshold for election is seven votes (this is called the *Droop quota*, and alternatives are defined below).

ROUND ONE

$$\begin{pmatrix} A \\ B \\ C \\ D \\ E \end{pmatrix} \times 5, \begin{pmatrix} B \\ A \\ C \\ D \\ E \end{pmatrix} \times 5, \begin{pmatrix} C \\ B \\ A \\ D \\ E \end{pmatrix} \times 6, \begin{pmatrix} D \\ C \\ B \\ A \\ E \end{pmatrix} \times 3, \begin{pmatrix} E \\ C \\ B \\ A \\ D \end{pmatrix} \times 2.$$

First-Place Totals – A:5, B:5, C:6, D:3, E:2

Since no candidate has seven votes, the candidate with the lowest total (E) is eliminated.

ROUND TWO

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \times 5, \begin{pmatrix} B \\ A \\ C \\ D \end{pmatrix} \times 5, \begin{pmatrix} C \\ B \\ A \\ D \end{pmatrix} \times 6, \begin{pmatrix} D \\ C \\ B \\ A \end{pmatrix} \times 3, \begin{pmatrix} C \\ B \\ A \\ D \end{pmatrix} \times 2.$$

First-Place Totals – A:5, B:5, C:8, D:3

Since C has eight votes, they are elected and can be removed from all ballots. One of the eight votes was a surplus over threshold, so the ballots with C in first place will transfer with a "discount" factor of $\frac{1}{8}$.

ROUND THREE

$$\begin{pmatrix} A \\ B \\ D \end{pmatrix} \times 5, \begin{pmatrix} B \\ A \\ D \end{pmatrix} \times 5, \begin{pmatrix} B \\ A \\ D \end{pmatrix} \times \frac{6}{8}, \begin{pmatrix} D \\ B \\ A \end{pmatrix} \times 3, \begin{pmatrix} B \\ A \\ D \end{pmatrix} \times \frac{2}{8}.$$

First-Place Totals – A:5, B:6, D:3

Since no candidate has seven votes, the candidate with the fewest votes (D) is eliminated.

ROUND FOUR

$$\begin{pmatrix} A \\ B \end{pmatrix} \times 5, \begin{pmatrix} B \\ A \end{pmatrix} \times 5, \begin{pmatrix} B \\ A \end{pmatrix} \times \frac{6}{8}, \begin{pmatrix} B \\ A \end{pmatrix} \times 3, \begin{pmatrix} B \\ A \end{pmatrix} \times \frac{2}{8}.$$

First-Place Totals – A:5, B:9

Since B has nine votes, they are elected. Since two candidates have reached threshold, the election is over, no further transfer is needed, and the winners are C and B.

In this example, we have hidden quite a few algorithmic decisions. Some of these implementation details are discussed in the next section.

A.2 Implementation details

The first choice to make is the threshold value. Above we used the *Droop quota*, the most common threshold used in STV elections. Let m denote the number of seats to be filled (m stands for *magnitude*) and let N be the number of voters. The Droop quota is calculated as

$$T = \left\lfloor \frac{N}{m+1} + 1 \right\rfloor,$$

where $\lfloor \cdot \rfloor$ denotes rounding down to the nearest integer. One way to interpret the Droop quota is that it is the minimum number of votes such that no more than m candidates can achieve the quota. Other thresholds can be used, including some which updated dynamically as the tabulation process progresses.

Another important decision is the method for transferring votes. The standard option is called *fractional transfer*. In fractional transfer, suppose a candidate has S votes and $S \geq T$, meaning that their support is at least the threshold. Then any ballot with that candidate in first place is discounted by a factor of $\frac{S-T}{S}$, and the vote transfers to the next candidate on the ballot. This is how we transferred votes in our example above.

Cambridge, Massachusetts currently elects its city council and school board by STV, but uses random transfer rather than fractional transfer of surplus votes. In random transfer, once a candidate is elected, they randomly select $S - T$ surplus ballots to transfer, and the votes are fully transferred to the next candidate on those ballots. (So $S - T$ ballots are transferred in full, rather than S ballots being transferred with $(S - T)/S$ weight.)

Finally, two more details must be specified. The first is the method for handling multiple candidates crossing the threshold in a single round. Either elect all of them in that round, or elect the one with the most votes, and proceed to another round of tabulation. This decision affects the order in which candidates are elected and can even alter the winner set.

The second consideration is a method for handling ties. Ties can occur whenever two or more candidates have the same number of first-place votes. See [Portland District 1](#). One method is to just randomly select one of the tied candidates; another is to use the first-place vote count from the previous round of tabulation.

These are some of the numerous variants of STV implementation. For an even more comprehensive treatment, we refer readers to [OpaVote](#) for a more comprehensive overview.

A.3 Timeline of STV transitions

Figure 3 shows a timeline of STV history in the United States.

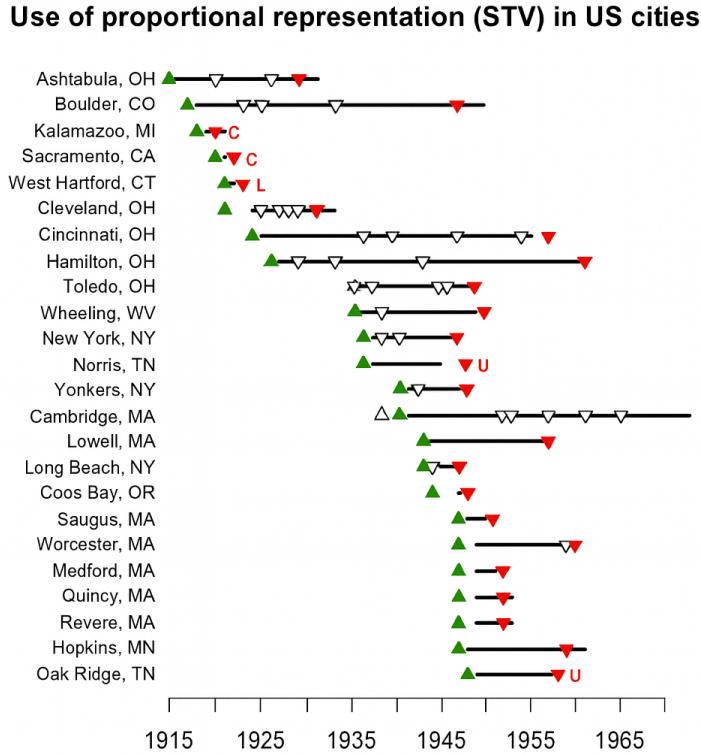


Figure 3. A timeline of STV's history in the United States. Upward green triangles denote adoption of STV, while downward red triangles denote repeal of STV. White triangles denote a failed attempt at adoption or repeal. Figure source: voteguy.com

B Generating random ballots: Models and parameters

We first assume that voters are divided into two blocs and each bloc has a corresponding slate of candidates. Each bloc of voters has an underlying preference for their slate of candidates over the other slate. Each bloc also has preferences among the candidates within their own slate. To create a ballot for a voter, a model first decides the order in which the slates appear on the ballot. Then, for each slate, the model decides the order in which the candidates appear, and fills them in on the ballot in that order. The type of model—impulsive, deliberative, or Cambridge—determines the probability of the final ballot being generated. Consider the following example.

Example 2. Suppose we have divided the electorate into two blocs of voters, Hispanic (H) and non-Hispanic, or Other (O). Each bloc has a corresponding slate of three candidates. The Hispanic slate prefers their own candidates 80% of the time, while the non-Hispanic slate prefers their own candidates 70% of the time. Within each bloc and slate, the candidates are preferred equally.

Our models all begin by sampling an order of the slates. A Hispanic voter may sample the slate order

$$\begin{pmatrix} H \\ H \\ O \\ H \\ O \\ O \end{pmatrix}.$$

Then, within the Hispanic slate, they might sample the candidate order H_1, H_3, H_2 , and within the non-Hispanic slate, they might sample the candidate order O_3, O_2, O_1 . Thus the voter's ballot is

$$B = \begin{pmatrix} H_1 \\ H_3 \\ O_3 \\ H_2 \\ O_2 \\ O_1 \end{pmatrix}.$$

This ballot has different probabilities of being generated by the different models.

These models have several parameters to control voter behavior. The first, called "cohesion," is the probability that a voter from bloc A will select for a candidate from bloc A as opposed to a bloc B candidate. When studying racial representation, we assumed Hispanic voters would select Hispanic candidates 75% of the time, while non-Hispanic voters would select non-Hispanic candidates 80% of the time.

We can also adjust the popularity of candidates inside a slate by adjusting a parameter α which controls the popularity of candidates according to a Dirichlet distribution. For the Hispanic voters, we chose α values of 1.25 and 0.75 for the Hispanic and non-Hispanic candidates, respectively, which means that Hispanic voters are relatively evenly divided among the Hispanic candidates, but they tend to have a preferred non-Hispanic candidate. For non-Hispanic voters, the α values were 0.6 and 0.5 for Hispanic and non-Hispanic candidates. In our model, non-Hispanic voters tended to have a favorite Hispanic candidate and a favorite non-Hispanic candidate.

We set similar parameter values for the partisan representation experiment. Cohesion parameters were set so that Democratic and Republican voters voted along party lines 85% of the time. α values for Democratic voters were 0.75 and 0.5 for Democratic and Republican candidates, respectively, creating voters who had a favorite Democrat and especially Republican candidate. The α values for Republican voters were 1 and 0.75 for Democratic and Republican candidates. While our Republican voters may have a preferred Republican candidate, they are evenly divided in their support for the Democratic candidates.