Signals & Systems

EEE-223

Lab # 08

.

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LAB # 08

Properties of Convolution

Lab 08- Properties of Convolution

Pre-Lab Tasks

8.1 Properties of Convolution:

In this section, we introduce the main properties of convolution through illustrative examples.

- Commutative Property

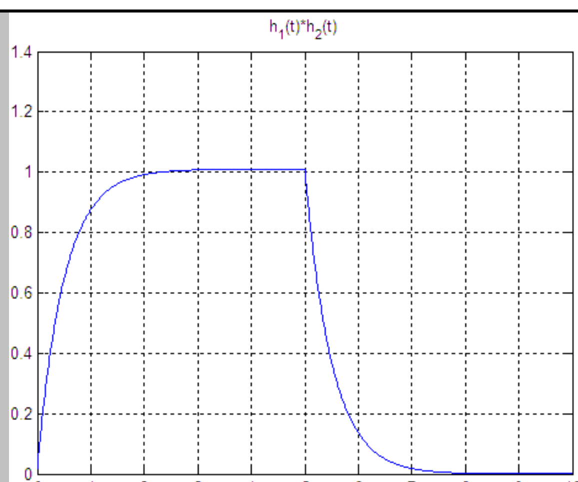
For two signals $h_1(t)$ and $h_2(t)$ the commutative property stands; that is equation 8.1, given as

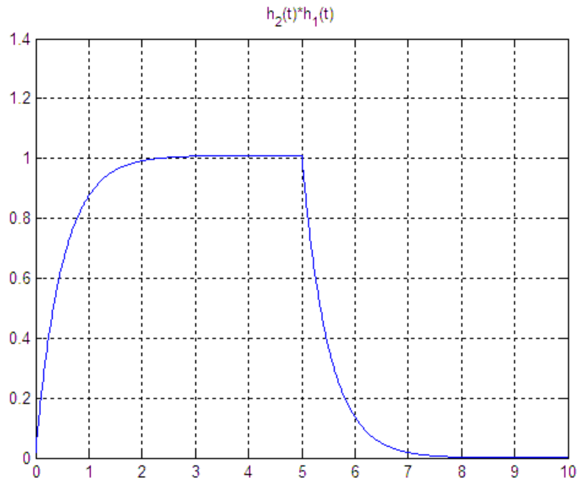
$$h_1(t)*h_2(t)=h_2(t)*h_1(t)$$

Example:

Verify the commutative property of the convolution supposing that $h_1(t)=1, 0\le t\le 5$ and $h_2(t)=2e^{-2t}, 0\le t\le 5$.

The left side of 8.1, i.e., the signal $y(t)=h_1(t)*h_2(t)$ is computed and plotted first while the right side of 8.1, namely, $z(t)=h_2(t)*h_1(t)$ is computed and plotted.

Commands	Results	Comments
t=0:0.01:5; **h1=ones(size(t));** **h2=2*exp(-2*t);** **y=conv(h1,h2)*0.01;** **plot(0:0.01:10,y),grid on** **title('h_1(t)*h_2(t)')**		The left side of 8.1.

z=conv(h2,h1)*0.01; **plot(0:0.01:10,y),grid on** **title('h_2(t)*h_1(t)')**		The right side of 8.1.

- Associative Property

For three signals $h_1(t)$, $h_2(t)$ and $x(t)$ the associative property stands; that is 8.2, given as,

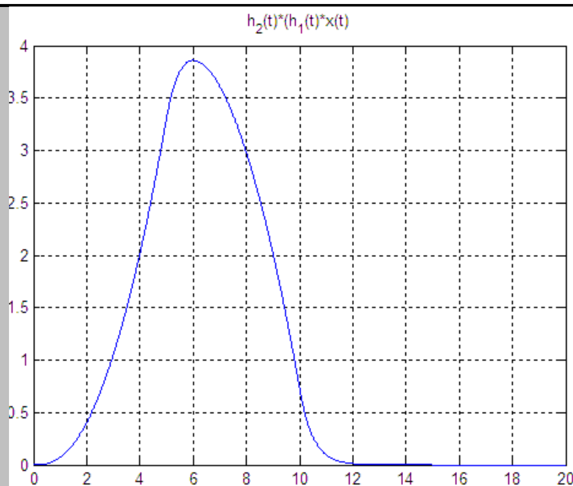
$$h_2(t)*\left[h_1(t)*x(t)\right]=\left[h_2(t)*h_1(t)\right]*x(t)$$

Example:

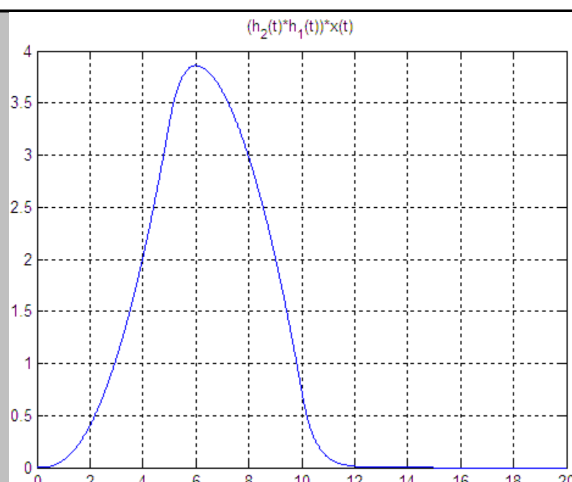
Verify the associative property of the convolution supposing that $h_1(t)=(1/\pi)t, 0\le t\le 5$ and $h_2(t)=2e^{-2t}, 0\le t\le 5$; and $x(t)=u(t)-u(t-5)$.

For the left side of 8.2, which is $y(t)=h_2(t)*\left[h_1(t)*x(t)\right]$, first the convolution $y_1(t)=h_1(t)*x(t)$ is computed. Next, the signal $h_2(t)$ is defined in the same time interval with $y_1(t)$ (at $0\le t\le 10$) and the result of their convolution is plotted.

Commands	Results	Comments

t=0:0.01:5; **x=ones(size(t));** **h1=1/pi*t;** **y1=conv(h1,x)*0.01;** **h2=2*exp(-2*t);** **th2=5.01:0.01:10;** **hh2=zeros(size(th2));** **h2=[h2 hh2];** **y=conv(h2,y1)*0.01;** **plot(0:0.01:20,y), grid on** **title('h_2(t)*(h_1(t)*x(t)')**		The left side of 8.2.

The right side of 8.2, which is $z(t)=\left[h_2(t)*h_1(t)\right]*x(t)=\left[h_1(t)*h_2(t)\right]*x(t)$, first the convolution $z_1(t)=h_1(t)*h_2(t)$ is computed. Next the signal $x(t)$ is defined in the same time interval with $z_1(t)$ (at $0\le t\le 10$) and the result of their convolution $z(t)$ is plotted.

Commands	**Results**	**Comments**
t=0:0.01:5; **h1=1/pi*t;** **h2=2*exp(-2*t);** **z1=conv(h1,h2)*0.01;** **x=ones(size(t));** **tx=5.01:0.01:10;** **xx=zeros(size(tx));** **x=[x xx];** **z=conv(z1,x)*0.01;**		The right side of 8.2.

```
plot(0:0.01:20,z), grid on
title('(h_2(t)*h_1(t))*x(t)')
```

- Distributive Property

For three signals $h_1(t)$, $h_2(t)$ and $x(t)$ the distributive property stands; that is 8.3, given as

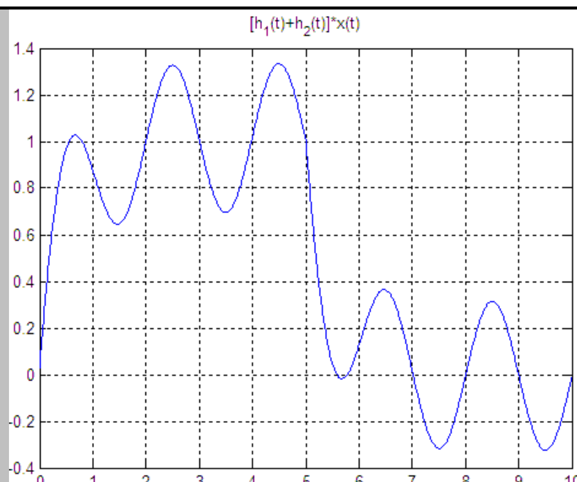
$$\left[h_1(t)+h_2(t)\right]*x(t)=h_1(t)*x(t)+h_2(t)*x(t)$$

Example:

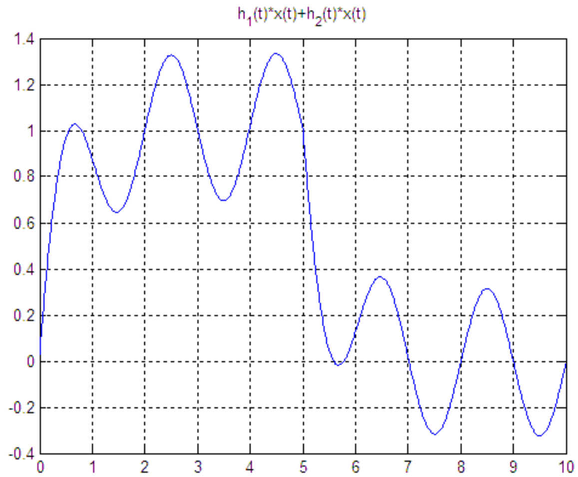
Illustrate the distributive property of convolution by using the signals; $h_2(t)=2e^{-2t}, 0\le t\le 5$; and $x(t)=u(t)-u(t-5)$.

In the similar vein to two previous examples, the left side of 8.3, that is, $y(t)=\left[h_1(t)+h_2(t)\right]*x(t)$ is compared to the right side of 7.3, that is, $z(t)=h_1(t)*x(t)+h_2(t)*x(t)$.

Commands	Results	Comments

t=0:0.01:5;
x=ones(size(t));
h1=cos(pi*t);
h2=2*exp(-2*t);
h=h1+h2;
y=conv(h,x)*0.01;
plot(0:0.01:10,y),grid on
title('[h_1(t)+h_2(t)]*x(t)')
		The left side of 8.3.

```
t=0:0.01:5;
x=ones(size(t));
h1=cos(pi*t);
h2=2*exp(-2*t);
z1=conv(h1,x)*0.01;
z2=conv(h2,x)*0.01;
z=z1+z2;
plot(0:0.01:10,z),grid on
title('h_1(t)*x(t)+h_2(t)*x(t)')
```



The right side of 8.3.

- Identity Property

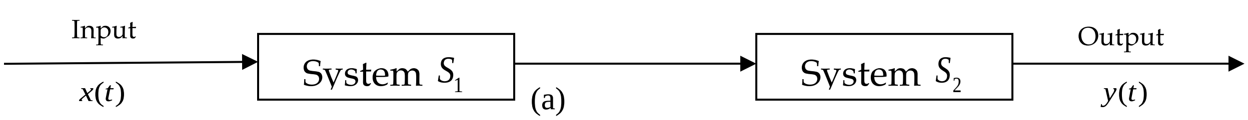
  If $\delta(t)$ is the Dirac delta function, then for any signal $h(t)$ the following expression is true:

  $$h(t)*\delta(t)=h(t)$$

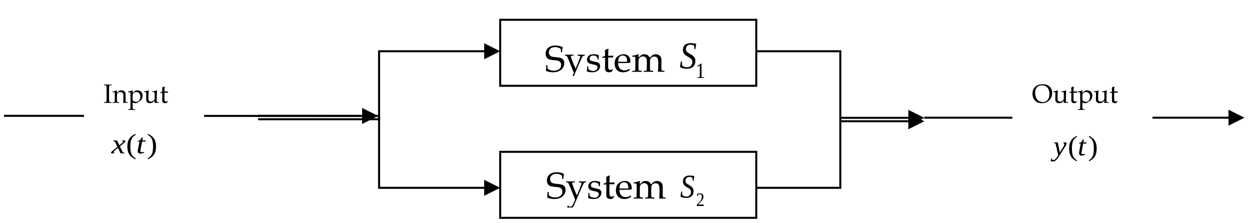
This property is straight forwardly proven from the definition of Dirac function. Nevertheless, an example will be provided in lab of discrete time convolution.

8.2 Interconnections of Systems:

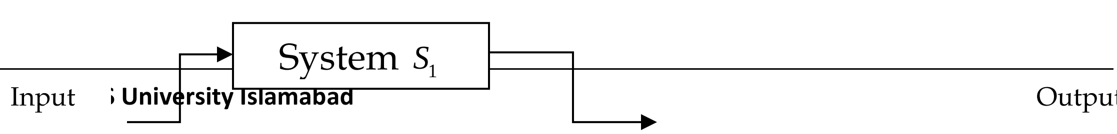
Systems may be interconnections of other sub-systems. The basic interconnections are the cascade, the parallel, the mixed and the feedback. The block diagrams are illustrated in Figure 8.1.

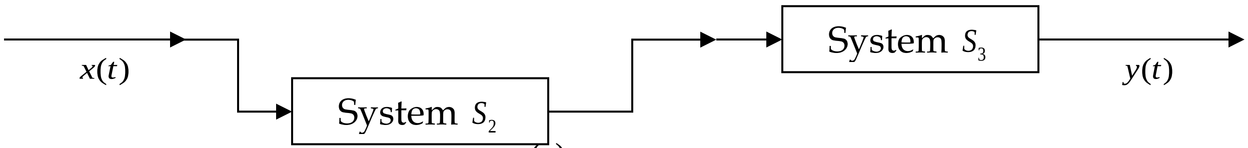


(a)

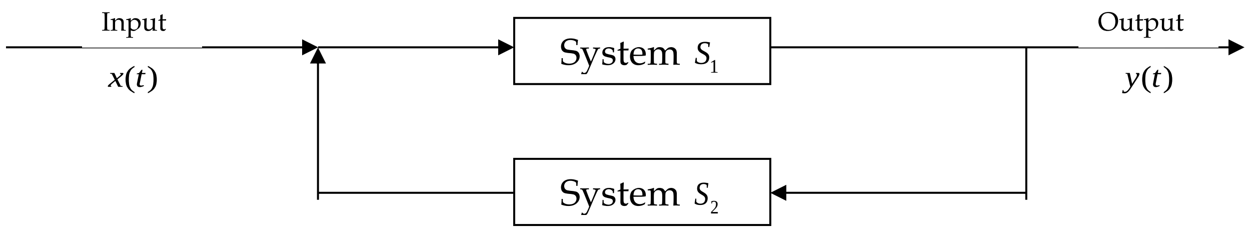


(b)





(c)



(d)

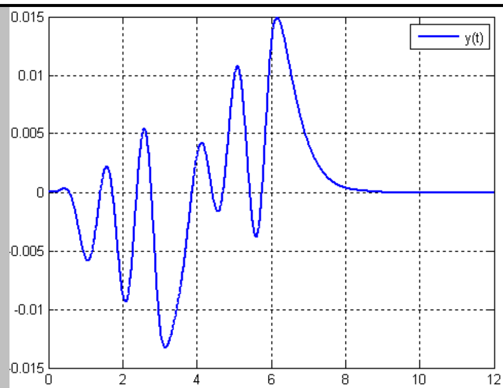
Figure 8.1: Interconnections of (sub) systems: (a) Cascade, (b) Parallel, (c) Mixed, and (d) Feedback

When two systems S_1 and S_2 are cascade (or serially) connected (Figure 8.1a), the output of the first system is the input of the second system. The block diagram off two parallel interconnected systems is presented in Figure 8.1b. The same input signals are applied to the two parallel-connected systems and the output of S_1 and S_2 are combined to generate the overall output. The mixed interconnection is a combination of cascade and parallel interconnections. In the block diagram of Figure 8.1c, systems S_1 and S_2 are parallel connected, and their output is input to the cascade-connected system S_3. Finally in Figure 8.1d the feedback interconnection block diagram is depicted. The output of S_1 is input to S_2, while the output of S_2 is fed back to S_1 and combined with the input signal produce the overall output of the system.

The interconnected subsystems can be considered as one system, i.e., an equivalent system described by one overall impulse response. In order to compute the output and the impulse response of the equivalent overall system for the various types of interconnections, suppose that the subsystems S_1 is described by the impulse response $h_1(t) = te^{-3t}\left[u(t) - u(t-3)\right]$ and the subsystem S_2 by the impulse response $h_2(t) = t\cos(2\pi t)\left[u(t) - u(t-3)\right]$. Finally, let $x(t) = u(t) - u(t-3)$ be the input signal.

- Cascade interconnection

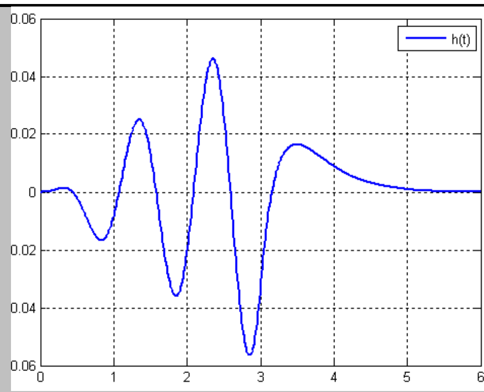
The output of S_1 is input to S_2. Thus, the output of the equivalent systems is computed as $y(t)=\left[x(t)*h_1(t)\right]*h_2(t)$; that is, the input signal is first convoluted with the impulse response of S_1 and the computed output is convoluted with the impulse response of S_2.

Commands	**Results**	**Comments**
t=0:0.01:3; **x=ones(size(t));** **h1=t.*exp(-3*t);** **y1=conv(x,h1)*0.01;** **t1=0:0.01:3;** **h2a=t1.*cos(2*pi*t1);** **t2=3.01:0.01:6;** **h2b=zeros(size(t2));** **h2=[h2a h2b];** **y=conv(y1,h2)*0.01;** **plot(0:0.01:12,y,'linewidth',2),grid on** **legend('y(t)')**		Graph of the output $y(t)=\left[x(t)*h_1(t)\right]*h_2(t)$

To compute the impulse response of the overall system, the associative property of the convolution is applied. More specifically, applying the associative property to the output relationship $y(t)=\left[x(t)*h_1(t)\right]*h_2(t)$ yields $y(t)=x(t)*\left[h_2(t)*h_1(t)\right]$. Consequently, the impulse response of the overall equivalent system, if the subsystem, are cascade connected, is given by $h(t)=h_1(t)*h_2(t)$. Straightforwardly, the systems response of $x(t)$ is given by $y(t)=x(t)*h(t)$.

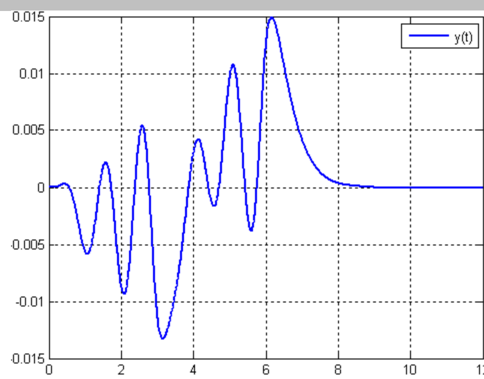
Commands	**Results**	**Comments**

```
t=0:0.01:3;

h1=t.*exp(-3*t);

h2=t.*cos(2*pi*t);

h=conv(h1,h2)*0.01;

plot(0:0.01:6,h,'linewidth',2), grid on;

legend('h(t)')
```



The impulse response $h(t)$ is obtained by $h(t)=h_1(t)*h_2(t)$

```
t1=0:0.01:3;

t2=3.01:0.01:6;

x1=ones(size(t1));

x2=zeros(size(t2));

x=[x1 x2];

y=conv(x,h)*0.01;

plot(0:0.01:12,y,'linewidth',2),grid on;

legend('y(t)')
```



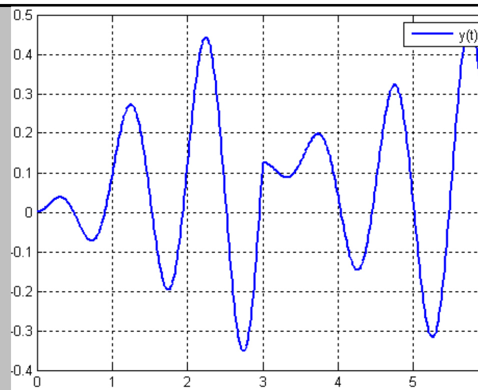
The output of the system $y(t)$ is computed from the convolution between the input signal $x(t)$ and the overall impulse response $h(t)$, which was derived using the associative property.

The two graphs are identical; hence, the computation of the impulse response and the output signal of the equivalent system are correct.

- Parallel Interconnection

In this type of interconnection, the same input is applied to both subsystems. The two outputs of subsystems are added to obtain the final output. The mathematical expression is $y(t)=h_1(t)*x(t)+h_2(t)*x(t)$.

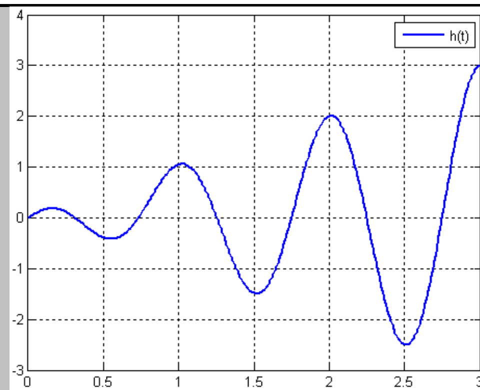
Commands	Results	Comments

t=0:0.01:3; **h1=t.*exp(-3*t);** **h2=t.*cos(2*pi*t);** **x=ones(size(t));** **y1=conv(h1,x)*0.01;** **y2=conv(h2,x)*0.01;** **y=y1+y2;** **plot(0:0.01:6,y,'linewidth',2), grid on;** **legend('y(t)')**		The response of the systems to the input signal $x(t)$ is computed by adding the outcome of the convolutions between the input signal and the impulse response of the subsystems; that is, it is computed as $y(t)=h_1(t)*x(t)+h_2(t)*x(t)$

To compute the impulse response of the overall system the distributive property of the convolution is applied. More specifically, applying the distributive property to the output relationship $y(t)=h_1(t)*x(t)+h_2(t)*x(t)$ yields $y(t)=\left[h_1(t)+h_2(t)\right]*x(t)$. Consequently, the impulse response of the equivalent system when the subsystems are parallel connected is given by $h(t)=h_1(t)+h_2(t)$. Straightforwardly, the output of the system is given by $y(t)=x(t)*h(t)$. To verify the conclusion, we consider the same signals used in the cascade interconnection case, namely, $h_1(t)=te^{-3t}\left[u(t)-u(t-3)\right]$, $h_2(t)=t\cos(2\pi t)\left[u(t)-u(t-3)\right]$ and $x(t)=u(t)-u(t-3)$.

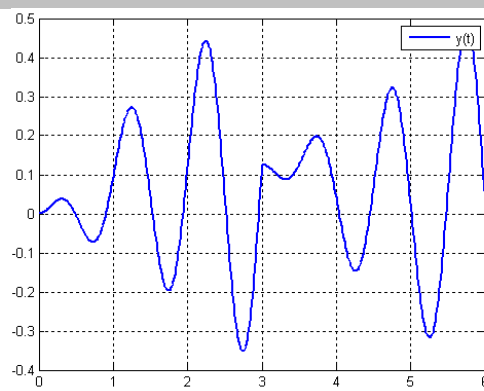
Commands	**Results**	**Comments**

```
t=0:0.01:3;

h1=t.*exp(-3*t);

h2=t.*cos(2*pi*t);

h=h1+h2;

plot(0:0.01:3,h,'linewidth',2), grid on;

legend('h(t)')
```



The overall impulse response of the system is the sum of the impulse responses of the subsystems; that it is computed by $h(t)=h_1(t)+h_2(t)$

```
x=ones(size(t));

y=conv(x,h)*0.01;

plot(0:0.01:6,y,'linewidth',2),grid on;

legend('y(t)')
```



The output of the system $y(t)$ is computed from the convolution between the input signal $x(t)$ and the impulse response of $h(t)$, which was derived using the distributive property.

The graphs of the system response are identical; hence, our computation of the impulse response and the output of the equivalent system is accurate.

Note: The implementation of the mixed interconnection is left as an exercise to the students.

8.3 Stability Criterion for Continuous Time Systems:

The concept of stability was introduced in lab session 5. A system is bounded-input bounded-output (BIBO) stable if for any bounded applied input; the response of the system is also bounded. The knowledge of the impulse response of the systems allows us to specify a new criterion about the stability of a system.

An LTI system is BIBO stable if its impulse response is absolutely integrable on $(-\infty,+\infty)$

The mathematical expression (equation 8.4) is

$$\int_{-\infty}^{+\infty}|h(t)|\,dt<\infty$$

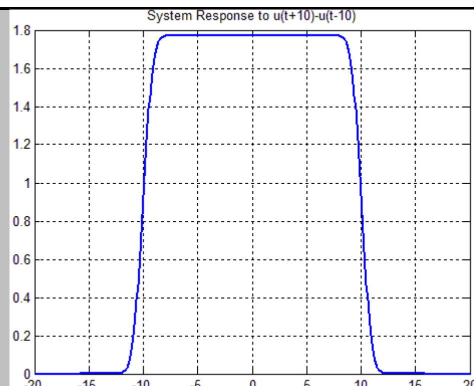
Example:

A system is described by the impulse response $h(t) = e^{-t^2}$. Tell is this system is BIBO stable and verify your conclusion.

Commands	Results	Comments
syms t **h=exp(-t.^2);** **int(abs(h),t,-inf,inf)**	ans=pi^(1/2)	Condition 8.4 is fulfilled; hence, the system under consideration is BIBO stable.

A system is BIBO stable if the condition given in 8..1 is satisfied; that is, we have to examine if its impulse response is absolutely integrable.

In order to verify that this is a BIBO stable system, the bounded input signal $x(t) = u(t+10) - u(t-10)$ is applied to the system. The response of the system is expected to be also bounded.

Commands	Results	Comments
t=-10:0.1:10; **x1=ones(size(t));** **h=exp(-t.^2);** **y1=conv(x1,h)*0.1;** **plot(-20:0.1:20,y1,'linewidth',2), grid on;** **title('System Response to u(t+10)-u(t-10)'**		The system response $y(t)$ is computed from the convolution between $x(t)$ and $h(t)$.

Indeed, the response of the systems is bounded $\left(|y(t)| < M = 2\right)$; thus the BIBO stability of the system is verified.

8.4 Stability Criterion for Discrete Time Systems:

In the previous section, we have established a criterion about the stability of continuous time LTI systems. More specifically, it was stated that a system is stable if the impulse response of the system is absolutely integrable. For discrete time systems a similar criterion can be established.

More specifically, a discrete time linear shift invariant system is stable if and only if its impulse response $h[n]$ is absolutely summable. The mathematical expression (equation 8.2) is

$$\sum_{-\infty}^{+\infty} |h[n]| < \infty$$

Example:

A system is described by impulse response $h[n] = \left(1/2^n\right)u[n]$. Tell if this is a BIBO stable system.

A discrete time system is BIBO stable if the condition given in equation 8.2 is satisfied; that is; we examine if the impulse response of the system is absolutely summable.

Commands	Results	Comments
syms n **h=1/(2^n)** **symsum(abs(h),n,0,inf)**	**ans=2**	Condition 8.4 is not fulfilled; hence, the system is not BIBO stable.

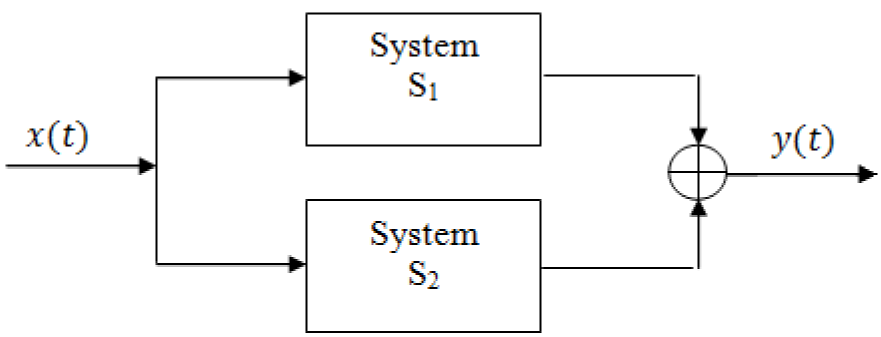
In-Lab Tasks

Task 01: A system is described by the impulse response $h(t) = t^2$. Tell if this is a BIBO stable system.

```
syms t
h=t^2;
int(abs(h),t,0,inf)

ans= inf
```

As the answer in infinite, hence the system h(t) is not BIBO stable

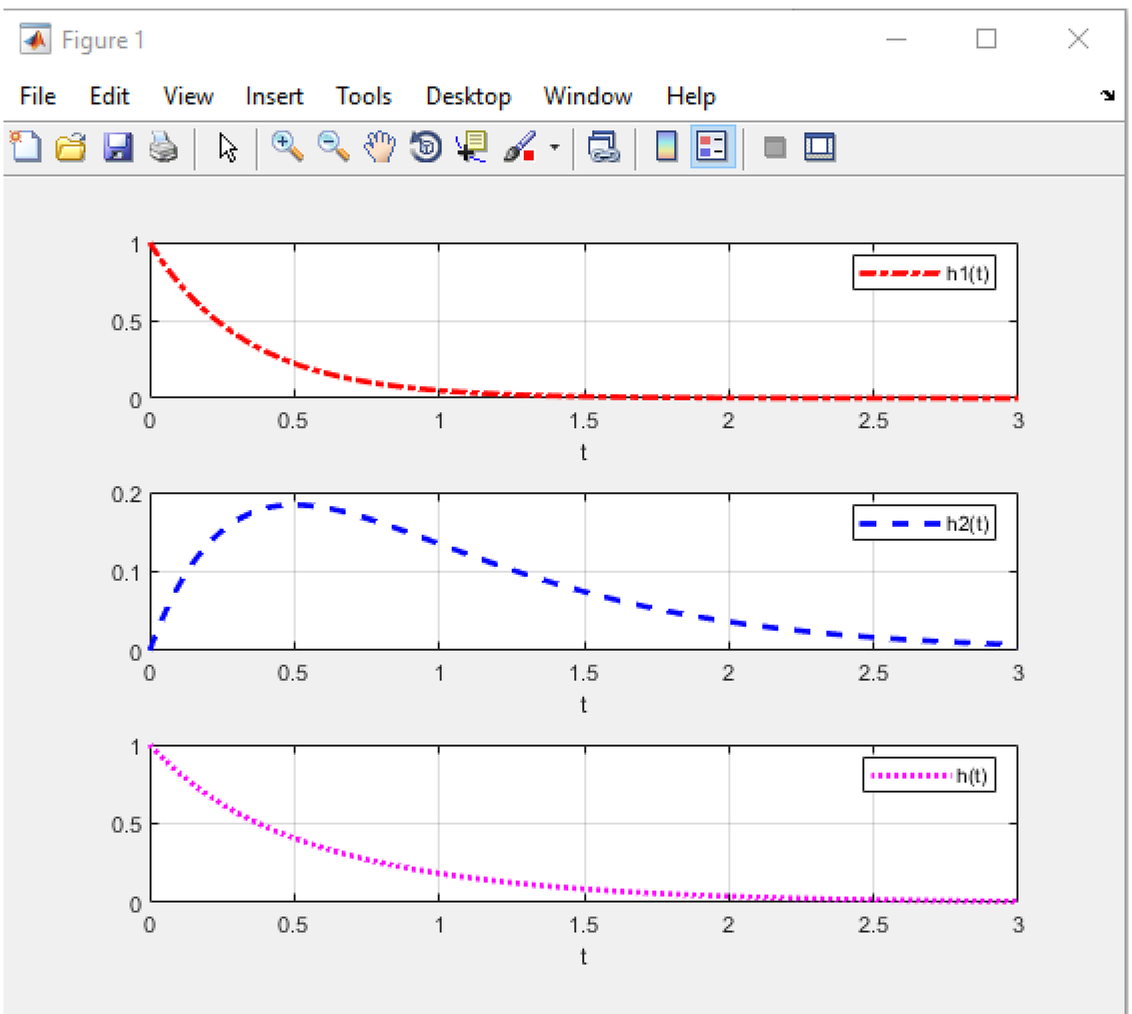
Task 02: Suppose that the impulse response of the subsystems S_1 and S_2 that are connected as shown in figure below are $h_1(t) = e^{-3t}u(t)$ and $h_2(t) = te^{-2t}u(t)$. Determine if the overall system is BIBO stable.



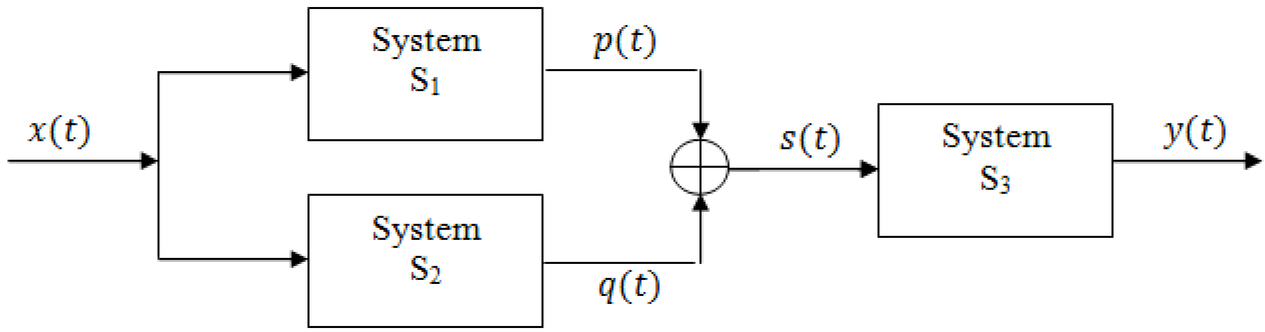
```
step=0.01;

t=0:step:3;


u=ones(size(t));

subplot(3,1,1);

h1=u.* exp(-3.*t);

plot(t,h1,'r-.','linewidth',2),grid on;

legend('h1(t)');

xlabel('t')


subplot(3,1,2)

h2=u.*(t.*exp(-2.*t));

plot(t,h2,'b--','linewidth',2),grid on;

legend('h2(t)');

xlabel('t')


h=h1+h2;

subplot(3,1,3)

plot(t,h,'m:','linewidth',2),grid on

legend('h(t)');

xlabel('t')


syms t
```

```
f=exp(-3.*t)+(t.*exp(-2.*t));

int(abs(f),t,0,inf)


ans =


7/12
```

System is BIBO Stable.



Task 03: Suppose that the impulse responses of the sub-systems S_1, S_2 and S_3 that are connected as shown in the figure below are $h_1(t) = t\cos(2\pi t)\,, 0 \le t \le 4$; $h_2(t) = t\,e^{-2t}, 0 \le t \le 4$; and $h_3(t) = u(t) - u(t-5)$. Compute and plot in the appropriate time interval the impulse response of the overall system and the response of the overall system to the input signal $x(t) = te^{-2t}\,[u(t) - u(t-2)]$.



i. Make only one file for this task.
ii. Call functions within this m-file which is needed.
iii. Compute impulse response of the overall system.
iv. Compute the response of the overall system to the given input signal $x(t)$.
v. Plot all graphs $p(t), q(t), s(t)$ and $y(t)$.
vi. Determine if the overall system is BIBO stable or not.

```
close all
clc
clear all
step=0.01;

t1=0:0.01:4;
t2=t1;
t3=t2;
 figure();
h1= t1.*cos(2.*pi.*t1);
subplot(3,1,1);
plot(t1,h1,'r-.','linewidth',2),grid on
legend('p(t)');

h2= t2.*exp(-2.*t2);
subplot(3,1,2);
plot(t2,h2,'b--','linewidth',2),grid on
legend('q(t)');
title('+','fontsize',16)
```

```
h3=ones(size(t3));

h12=h1+h2;
subplot(3,1,3);
plot(t2,h12,'m:','linewidth',2),grid on
legend('s(t)');
title('=','fontsize',16)
%overall impulse response
figure();


th=0:step:8;
h= conv(h12,h3)*step;
subplot(3,1,1)
plot(th,h,'r-.','linewidth',2),grid on
legend('h(t)')
title('overall impulse response','fontsize',14)
 %input signal
tx=0:step:2;
x=tx.*exp(-2.*tx);
subplot(3,1,2);
plot(tx,x,'b--','linewidth',2),grid on
legend('x(t)')
title('Input signal','fontsize',14)
 %the response of the overall system to the given input signal ____
ty=0:0.01:10;
y=conv(x,h)*step;
subplot(3,1,3);
plot(ty,y,'m:','linewidth',2),grid on
legend('y(t)')
title('The response of the overall system to the given input
signal','fontsize',14)
```

BIBO STABLE

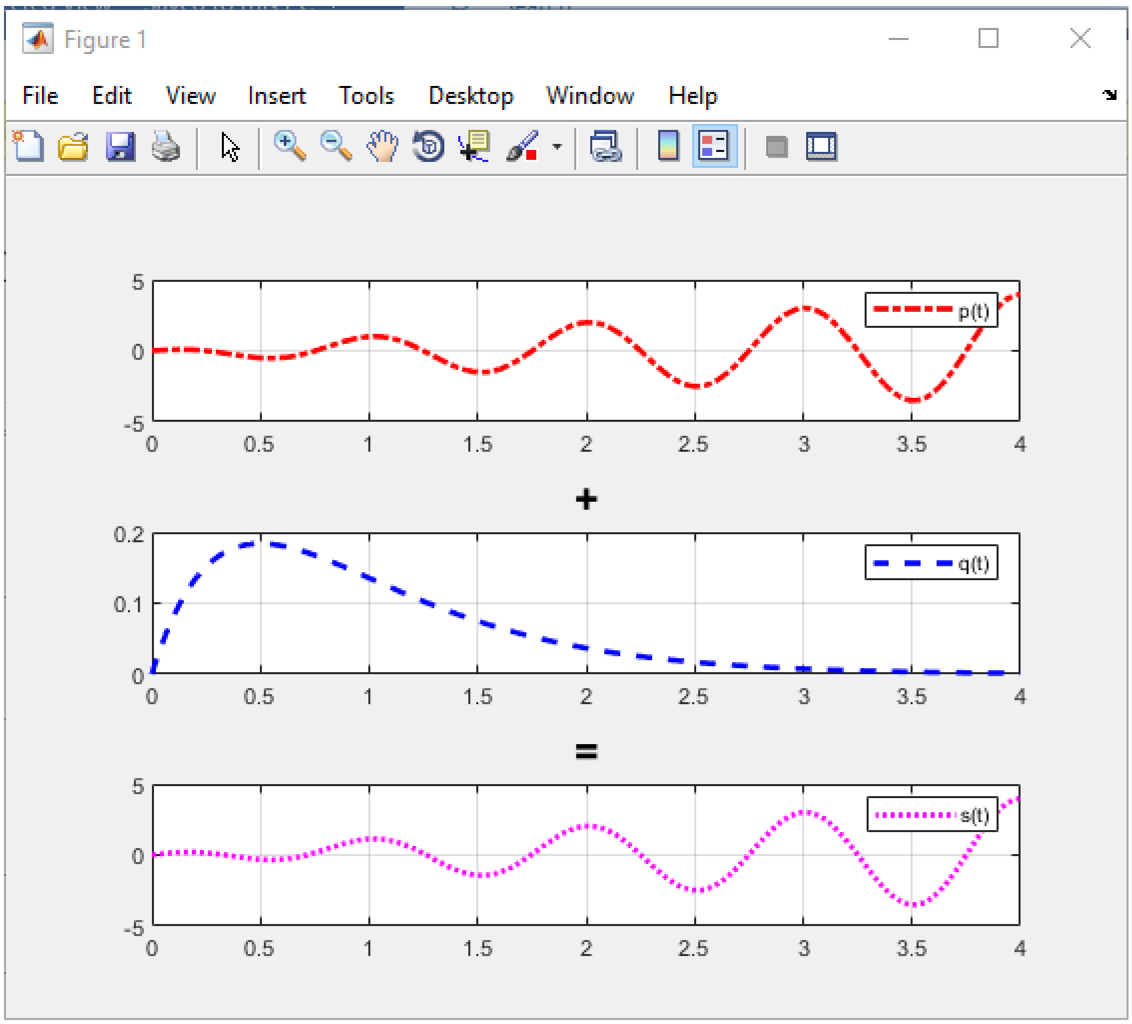


Figure 1
File Edit View Insert Tools Desktop Window Help
p(t)
+
q(t)
=
s(t)

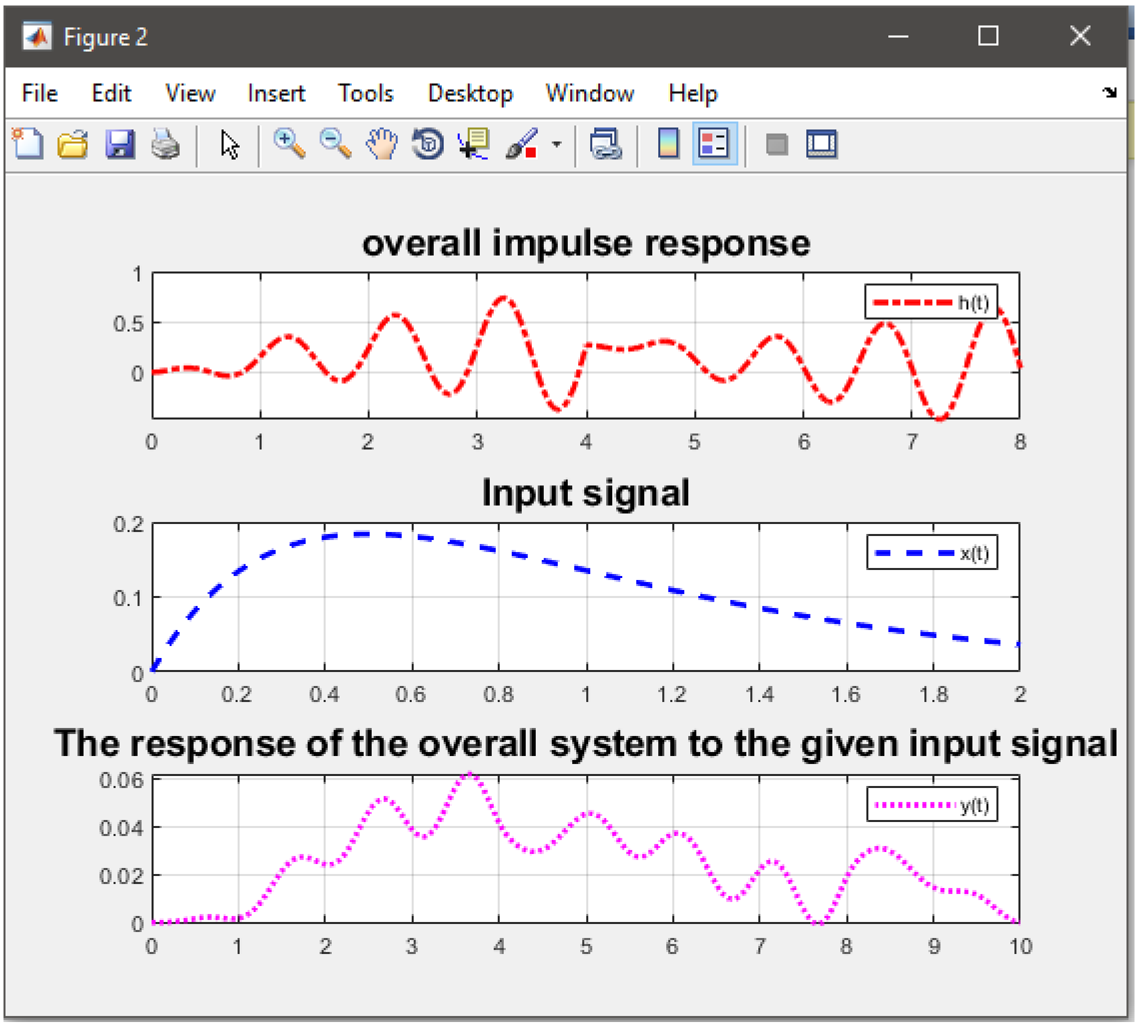
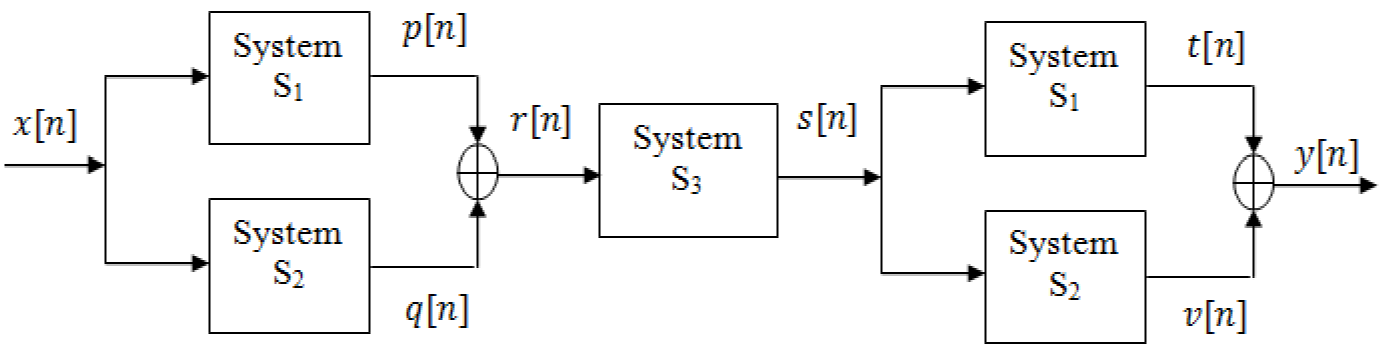


Figure 2
File Edit View Insert Tools Desktop Window Help
overall impulse response
h(t)
Input signal
x(t)
The response of the overall system to the given input signal
y(t)

Task 04: Suppose that the impulse responses of the sub-systems S_1, S_2 and S_3 that are connected as shown in the figure below are $h_1[n] = [2, 3, 4], 0 \leq n \leq 2$; $h_2[n] = [-1,3,1], 0 \leq n \leq 2$; and $h_3[n] = [1,1,-1], 0 \leq n \leq 2$, respectively. Compute



i. Make only one file for this task.
ii. Call functions within this m-file which is needed.
iii. Compute impulse response of the overall system.
iv. Compute the response of the overall system to the given input signal $x[n] = u[n] - u[n-2]$.
v. Plot all graphs $p[n], q[n], r[n], s[n], t[n], v[n]$ and $y[n]$.
vi. Determine if the overall system is BIBO stable or not.

```
n = 0:2;

h1 = [2,3,4];

h2 = [-1,3,1];

h3 = [1,1,-1];


subplot(5,1,1)

stem(n,h1,'fill','linewidth',2),grid on

legend('p[n]')


subplot(5,1,2)

stem(n,h2,'fill','linewidth',2),grid on

legend('q[n]')


h12=h1+h2;

subplot(5,1,3)

stem(n,h12,'fill','linewidth',2),grid on

legend('r[n]')


subplot(5,1,4)

stem(n,h3,'fill','linewidth',2),grid on

legend('h3[n]')


n123 = 0:4;
```

```
h123 = conv (h12,h3);

subplot(5,1,5)

stem(n123,h123,'fill','linewidth',2),grid on

legend('s[n]')


figure();

nt=0:6;

h1231=conv(h123,h1);

subplot(5,1,1)

stem(nt,h1231,'fill','linewidth',2),grid on

legend('t[n]')


nv=0:6;

h1232=conv(h123,h2);

subplot(5,1,2)

stem(nv,h1232,'fill','linewidth',2),grid on

legend('v[n]')



nh=0:6;

h=h1231+h1232;

subplot(5,1,3)

stem(nh,h,'g--','fill','linewidth',2),grid on
```

```
legend('h[n]')


xn=0:1;

x=ones(size(xn));

subplot(5,1,4)

stem(xn,x,'r:','fill','linewidth',2),grid on

legend('x[t]');


ty= 0:7;

y= conv(x,h);

subplot(5,1,5)

stem(ty,y,'r:','fill','linewidth',2),grid on

legend('y[n]')


syms n

ans=symsum (abs(h),n,0,inf)


ans =


[ Inf, Inf, Inf, Inf, Inf, Inf, Inf]
```

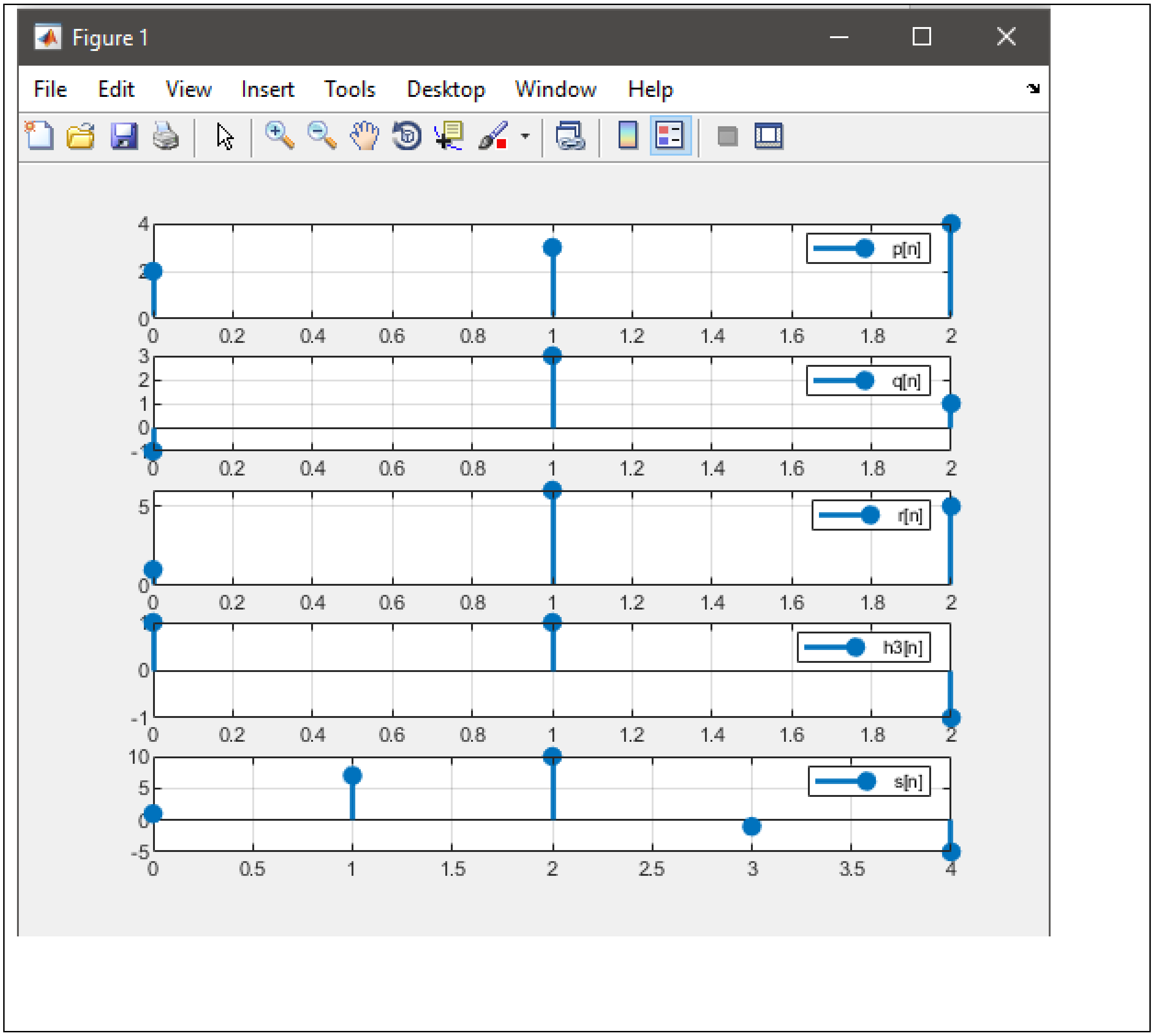


Figure 1
File Edit View Insert Tools Desktop Window Help
p[n]
q[n]
r[n]
h3[n]
s[n]

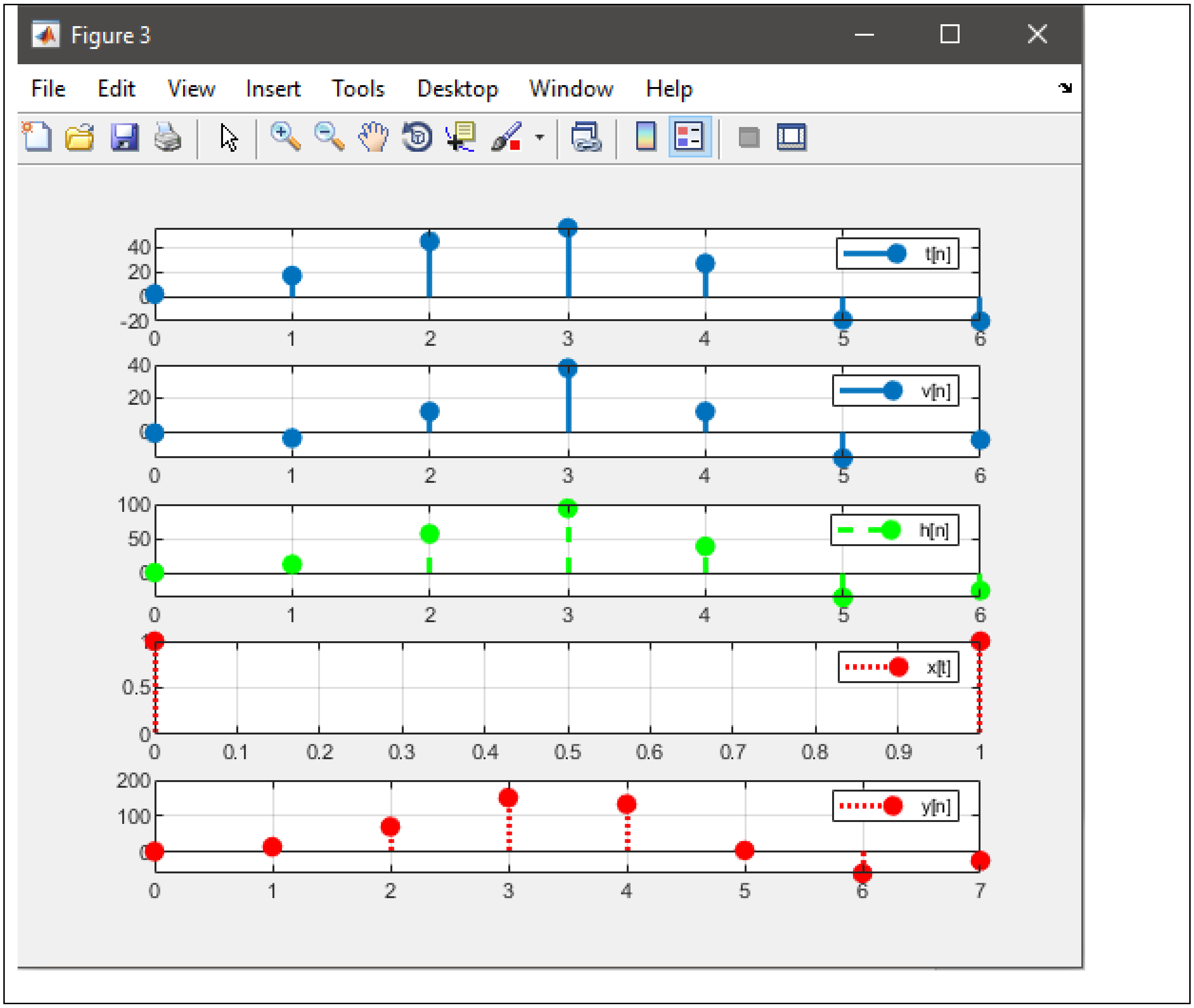


Figure 3
File Edit View Insert Tools Desktop Window Help
t[n]
v[n]
h[n]
x[t]
y[n]

Post-Lab Tasks

Critical Analysis / Conclusion

In this lab, we implemented various properties of convolution which includes commutative, associative, distributive and identity property. We also implemented them on different types of system: cascaded, parallel, mixed and feedback system and obtained the output using those properties in these systems.

Lab Assessment		
Pre-Lab	**/1**	**/10**
In-Lab	**/5**	
Critical Analysis	**/4**	
Instructor Signature and Comments		