Signals & Systems

EEE-223

Lab # 11

Name	Muhammad Haris Irfan
Registration Number	FA18-BCE-090
Class	BCE-4A
Instructor's Name	Muhammad Bilal Qasim

LAB # 11

Continuous Time Fourier Transform (CTFT)

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Pre Lab

Fourier transform is used to transform a time domain signal into frequency domain. As some times frequency domain reveals more information as compared to time domain. In this lab, the Fourier transform for continuous-time signals will be discussed which is known as continuous-time Fourier transform (CTFT). By applying, Fourier transform to a continuous time signal $x(t)$, we obtain a representation of the signal at the cyclic frequency domain Ω or equivalently at the frequency domain f.
The Fourier transform is denoted by the symbol $F\{.\}$; that is, one can write (11.1) as

$$X(\Omega) = F\{x(t)\} \tag{11.1}$$

In other words, the Fourier transform of a signal $x(t)$ is a signal $X(\Omega)$. An alternative way of writing eq. (11.1) is given in eq. (11.2) and eq. (11.3) shows mathematical from of Fourier transform.

$$x(t) \xrightarrow{F} X(\Omega) \tag{11.2}$$

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt, \tag{11.3}$$

From eq. (11.3) it is clear that $X(\Omega)$ is complex function of Ω. Where Ω =2πf substituting in eq. (11.3) we get

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt, \tag{11.4}$$

In order to return from the frequency domain back to the time domain the *inverse* Fourier transform is implemented. The inverse Fourier transform is denoted by the symbol; i.e. $F^{-1}\{.\}$

$$x(t) = F^{-1}\{X(\Omega)\} \tag{11.5}$$

or alternatively,

$$X(\Omega) \xrightarrow{F^{-1}} x(t) \tag{11.6}$$

Mathematically, inverse Fourier transform is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, dt, \tag{11.7}$$

or

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j2\pi f t}\, dt, \tag{11.8}$$

The cyclic frequency Ω is measured in rad/s, while the frequency f is measured in Hertz. The Fourier transform of a signal is called (frequency) *spectrum.* MATALB command for Fourier transform is “fourier” and for inverse Fourier transform it is “ifourier”.

In-Lab Tasks

Task-1

Compute the Fourier transform of $x(t) = \mathrm{e}^{-\mathrm{t}^2}$. MATLAB code is given in following, run this code and compare your output using eq. (11.3). Write your code and results in following.

```
syms w t
x = exp(-t.^2);
X=foprier(x,w);
```

OUTPUT

```
X =
pi^(1/2)*exp(-w^2/4)
```

Task-2

Compute the inverse Fourier transform of X = exp(-1/4*w^2)*pi^(1/2) using the command for inverse Fourier transform and also verify your result using eq. (11.7). Give your results in following.

```
syms t w
X = exp(-1/4*w^2)*pi^(1/2);
ifoprier(X,w)
```

OUTPUT

```
ans =

(3991211251234741*exp(-w^2))/(2251799813685248*pi^(1/2))
```

Task-3

Compute the inverse Fourier transform of the function $X(\Omega) = 1/(1 + j\Omega)$ using command of Fourier and then take inverse of the resultant x(t) to produce again $X(\Omega)$.

Part 1)

```
syms t w
x = 1/1+(1i*w))
foprier(x,t)
```

OUTPUT

```
x =
1/(1 + w*-i)

ans =
   -pi*exp(t)*(sign(t) - 1)
```

Part 2)

```
syms f
w = 2*pi*f
syms t
x = -pi*exp(t)*(sign(t) - 1)
ifoprier(x,w)
```

OUTPUT

```
w =
2*pi*f
x =
-pi*exp(t)*(sign(t) - 1)
ans =
                            1/(pi*f*2i + 1)
```

Task-4

Let x(t) = 1, compute its Fourier transform to produce X(w) and then take inverse Fourier transform of X(w) to get back x(t), using commands of Fourier transform.

Part 1)

```
syms t w
x=1
X=foprier(x,w)
```

OUTPUT

```
x =
     1
X =
2*pi*dirac(w)
```

Part 2)

```
syms t w
x=2*pi*dirac(w)
x=ifoprier(x,t)
```

OUTPUT

```
x =
2*pi*dirac(w)
x =
    1
```

Task-5

Let x(t) = u(t), compute its Fourier transform, take inverse Fourier transform of the resultant signal to get back x(t).

Part 1)

```
t=0:1:5
syms w
x=t>=0
X=foprier(x,w)
```

OUTPUT

```
t =
0 1 2 3 4 5
x =
1×6 logical array
1 1 1 1 1 1
X =
[ 2*pi*dirac(w), 2*pi*dirac(w), 2*pi*dirac(w), 2*pi*dirac(w),
2*pi*dirac(w), 2*pi*dirac(w)]
```

Part 2)

```
syms t w
x=[ 2*pi*dirac(w), 2*pi*dirac(w), 2*pi*dirac(w), 2*pi*dirac(w),
2*pi*dirac(w), 2*pi*dirac(w)]
x=ifoprier(x,t)
```

OUTPUT

```
x =
[ 2*pi*dirac(w), 2*pi*dirac(w), 2*pi*dirac(w), 2*pi*dirac(w),
2*pi*dirac(w), 2*pi*dirac(w)]
x =
                              [ 1, 1, 1, 1, 1, 1]
```

Task-6

Let x(t) = δ(t), compute its Fourier transform, take inverse Fourier transform of the resultant signal and state whether it is possible to get back x(t) or not?

Part 1)

```
syms t
x=dirac(t)
X=foprier(x,w)
```

OUTPUT

```
x =
dirac(t)
X =
1
```

Part 2)

```
syms t
x=1
X=ifoprier(x,t)
```

OUTPUT

```
x =
1

X =
dirac(t)
```

Hence it is possible to get x(t) back

Task-7

Prove that $x(t) = \delta(t-2)$ and $X(\Omega) = e^{-j2\Omega}$, are Fourier transform pairs of each other.

Part 1)

```
syms t
x=dirac(t-2)
X=foprier(x,w)
```

OUTPUT

```
x =
dirac(t - 2)
X =
exp(-w*2i)
```

Part 2)

```
syms t w
x=exp(-w*2i)
X=ifoprier(x,t)
```

OUTPUT

```
x =
exp(-w*2i)
X =
dirac(t - 2)
```

Hence $x(t)$ **and** $X(\Omega)$ **are Fourier transform pairs of each other.**

Task-8

Prove that $x(t) = u(t-2)$ and $X(\Omega) = \exp(-2 * j * w) * (pi * \delta(w) - j/w)$, are Fourier transform pairs of each other.

Part 1)

```
syms w t x
a = heaviside(x-2);
A=foprier(a,w)
```

OUTPUT

```
A =
exp(-w*2i)*(pi*dirac(w) - 1i/w)
```

Part 2)

```
syms w t x
A=exp(-2*j*w)*(pi*dirac(w)-j/w);
a=ifoprier(A,w)
```

OUTPUT

```
a =
(pi + pi*sign(w - 2))/(2*pi)
```

Hence $x(t)$ **and** $\mathbf{X}(\Omega)$ **are Fourier transform pairs of each other.**

Critical Analysis / Conclusion

In this lab we learnt how to transform a continuous time domain signal into continuous frequency domain using the concept of Fourier Transform. we also learned how to take inverse Fourier Transform using MATLAB.

Lab Assessment		
Pre-Lab	**/1**	**/10**
In-Lab	**/5**	
Critical Analysis	**/4**	
Instructor Signature and Comments		