Signals & Systems

EEE-223

Lab # 12

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LAB#12

Open ended Lab

Lab 12- Open ended lab

Objectives

Objective of this lab is to encourage the analytical and problem-solving skills in a methodical way through literature survey, design and conduct of experiments, analysis and interpretation of experimental data and synthesis of information to derive the valid conclusions.

Pre-Lab

12.1 The Laplace Transform:

The Laplace transform expresses a signal in the complex frequency domain s (or s-domain); that is, a signal is described by a function F(s). Laplace transform is defined by the symbol L{.}; that is, one can write

$$F(s) = L\{f(t)\} \qquad 12.1$$

In order words, the Laplace transform of a function f(t) is a function F(s). An alternative way of writing (equation 12.1) is

$$f(t) \xrightarrow{L} F(s) \qquad 12.2$$

There are two available forms of Laplace transform. The first is two-sided (or bilateral) Laplace transform where the Laplace transform F(s) of a function f(t) is given by

$$F(s) = L\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt \qquad 12.3$$

The second form is one-sided (or unilateral) Laplace transform, which is described by the relationship

$$F(s) = L\{f(t)\} = \int_{0}^{\infty} f(t)e^{-st}\,dt \qquad 12.4$$

In order to return from the s-domain back to the time domain, the inverse Laplace transform is applied. The inverse Laplace transform is defined by the symbol $L^{-1}\{.\}$; that is, one can write

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}dt\ , \tag{12.5}$$

or alternatively

$$F(s) \xrightarrow{L^{-1}} f(t) \tag{12.6}$$

Example 1:

Compute the (unilateral) Laplace transform of the function $f(t) = e^{-t}$.

Commands	Results	Comments
syms t s		The command syms is used to define the symbolic variables t and s.
t	t=t	
s	s=s	The function $f(t) = e^{-t}$ is defined as symbolic expression and the (unilateral) Laplace transform is computed.
f=exp(-t);		
laplace(f,t)	ans=1/(1+s)	

Example 2:

Compute the inverse Laplace transform of the function $F(s) = \dfrac{1}{1+s}$.

Commands	Results	Comments

syms t s		The command syms is used to define the symbolic variables t and s.
F=1/(1+s);		The function $F(s)=\dfrac{1}{1+s}$ is defined as symbolic expression and the inverse Laplace transform is computed.
ilaplace(F,s)	ans=exp(-t)	

12.2 The *residue* Function

The Matlab *residue* function converts its argument function, given in rational form, to partial fraction form. The syntax is [R, P, K] = residue (B,A), where B is the vector containing the coefficients of the numerator polynomial and A is the vector of the coefficients of the denominator polynomial. Suppose that a signal X(s) is written in the rational form

$$X(s)=\frac{B(s)}{A(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

By defining the vector $B=[b_m,\ldots,b_0]$ and $A=[a_n,\ldots,a_0]$, and executing the command [R, P, K]=residue(B,A), the signal X(s) can be written in partial fraction form as

$$X(s)=\frac{r_1}{s-p_1}+\frac{r_2}{s-p_2}+\cdots+\frac{r_n}{s-p_n}+K$$

Where, $\begin{matrix} R=[r_1,r_2,\ldots,r_n] \\ P=[p_1,p_2,\ldots,p_n] \end{matrix}$,

and K is the residue of division between the two polynomials, K is null when m<n.

The command residue is illustrated in the following examples.

Example 3:

Express the signal transform $X(s)$ given below in partial fraction form

$$X(s)=\frac{s^2+3s+1}{s^3+5s^2+2s-8}$$

Commands	Results	Comments
num = [1 3 1]; **den = [1 5 2 -8];** **[R,P,K] = residue(num,den)** **syms s** **X=R(1)/(s-P(1))+R(2)/(s-P(2))+R(3)/(s-P(3))** **Pretty(X)**	R=0.5000 0.1667 0.3333 P=-4.0000 -2.0000 1.0000 K=[] The signal X(s) is written as $1/2\frac{1}{s+4}+1/6\frac{1}{s+2}+1/3\frac{1}{s-1}$	The signal X(s) is expanded in partial fraction form. Notice that K is empty; that is, there is no residue.

Example 4:

Express the signal transform $X(s)$ given below in partial fraction form

$$X(s)=\frac{s^2-3s+2}{s^2+4s+5}$$

In this case, the roots of the denominator polynomial are complex numbers. The complex roots do not really change anything in the followed procedure.

Commands	Results	Comments

num = [1 -3 2]; **den = [1 4 5];** **[R,P,K] = residue(num,den)**	 R=-3.50-5.50i P=-2.00+1.00i K=1	 -3.50+3.50i -2.00-1.00i	The signal X(s) is expressed in partial fraction form as $X(s)=\frac{-3.5+5.5j}{s+2-j}+\frac{-3.5+5.5j}{s+2+j}+1$

Task 01: Compute the unilateral Laplace transform of the function $f(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-2t}$. Also evaluate the inverse Laplace transform of your result.

```
syms t s
f = -1.25+3.5*t*exp(-2*t)+1.25*exp(-2*t);
l = laplace(f,s);
pretty(l);
il = ilaplace(l,t);
pretty(il);
```

OUTPUT:

```
    5          7                  5
--------- + ----------  -        ---
4 (s + 2)    2(s+2)^2            4 s

exp(-2 t) 5   t exp(-2 t) 7       5
----------- + -------------   -   -
     4            2               4
```

Task 02: Compute the unilateral Laplace transform of the function $f(t) = 1$.

```
syms t s
f = 1;
l = laplace(f,s);
pretty(l);
```

OUTPUT:

1

--

s

Task 03: Express in the partial fraction form the signal

$$X(s) = \frac{s^3 - 3s + 2}{s^2 + 4s + 5}$$

```
syms s t
num = [ 1 0 -3 2 ];
deno = [ 1 4 5 ];
[R P K] = residue(num,deno)
X = R(1)/(s-P(1)) + R(2)/(s-P(2)) + s*K(1) + K(2);
pretty(X);
```

OUTPUT:

```
    4 - 3i      4 + 3i
s + --------- + ---------- - 4
  s + 2 - i   s + 2 + 1i
```

Task 04: Express in the partial fraction form the signal

$$X(s) = \frac{s^2 + 5s + 4}{s^4 + 1}$$

Verify your result by computing the inverse Laplace transform from both forms (partial fraction and rational) and plot both of the results.

```
syms s t
num = [ 1 5 4 ];
deno = [ 1 0 0 0 1 ];
[R P K] = residue(num,deno);
X = R(1)/(s-P(1)) + R(2)/(s-P(2)) + R(3)/(s-P(3)) + R(4)/(s-
P(4)) ;
il = ilaplace(X,t)
ezplot(il);
title('ILT Through Partial Fraction Form');
syms s
figure();
rat = (s^2 + 5*s + 4)/(s^4 + 1);
il2 = ilaplace(rat,t);
ezplot(il2);
title('ILT Through Rational Form');
```

Inverse Laplace through Partial Fraction

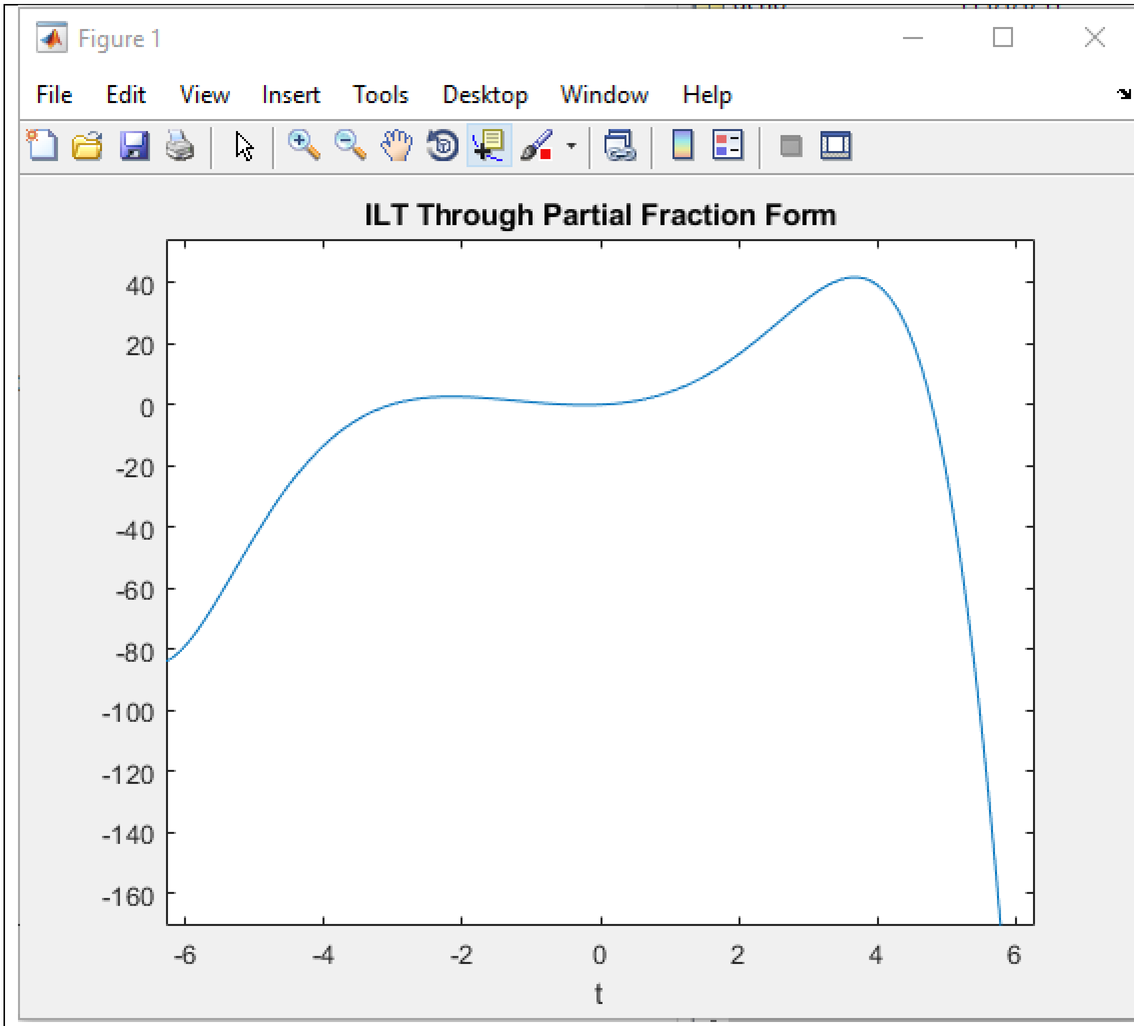
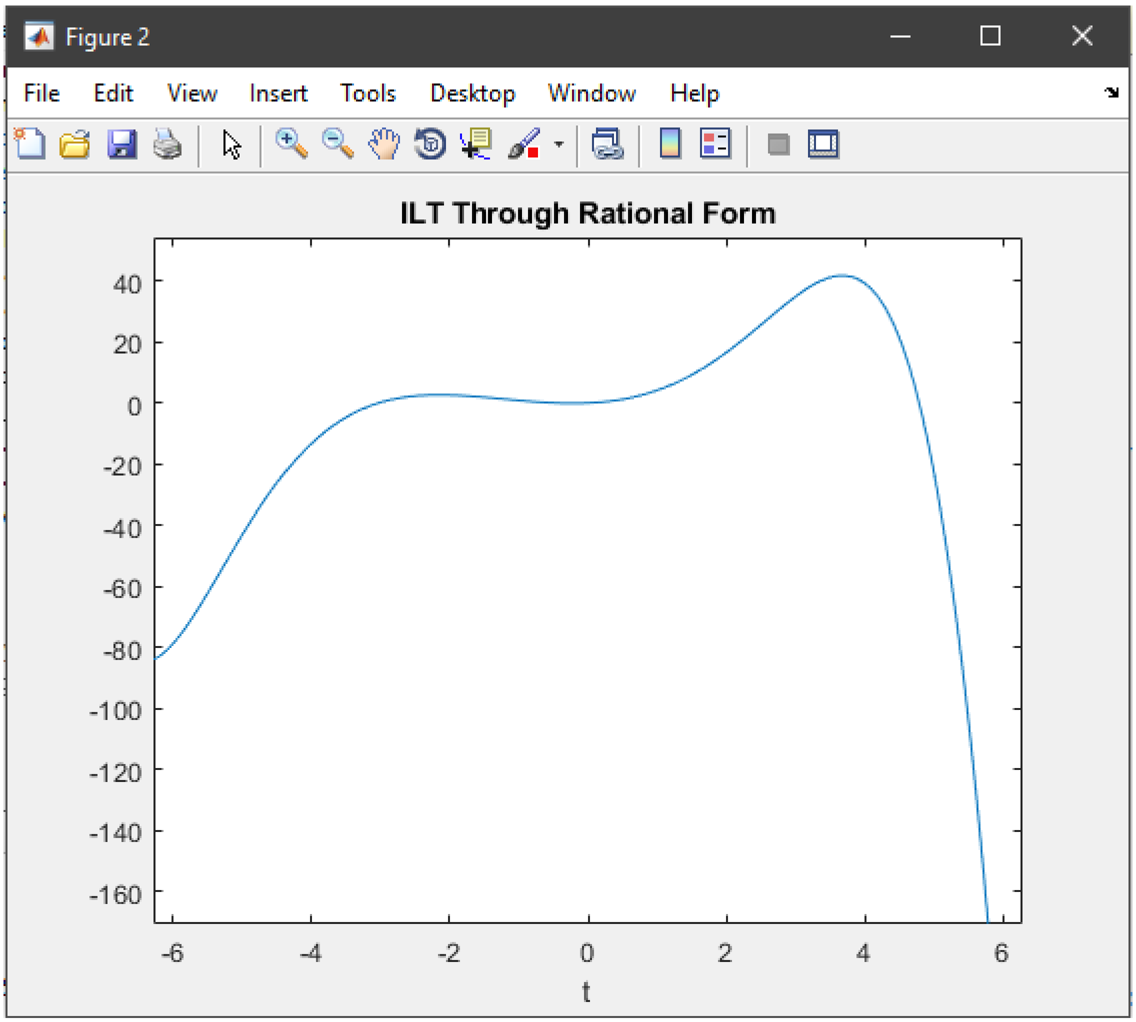


Figure 1
File Edit View Insert Tools Desktop Window Help
ILT Through Partial Fraction Form
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-6
-4
-2
0
2
4
6
t

Inverse Laplace through Rational Form



In-Lab Open ended Tasks

Task 01: Examine the network shown in figure 12.1 below. Assume the network is in steady state prior to $t = 0$.

I. **Plot the output current $i(t)$, for $t > 0$**
II. **Determine whether the system is stable or not?**

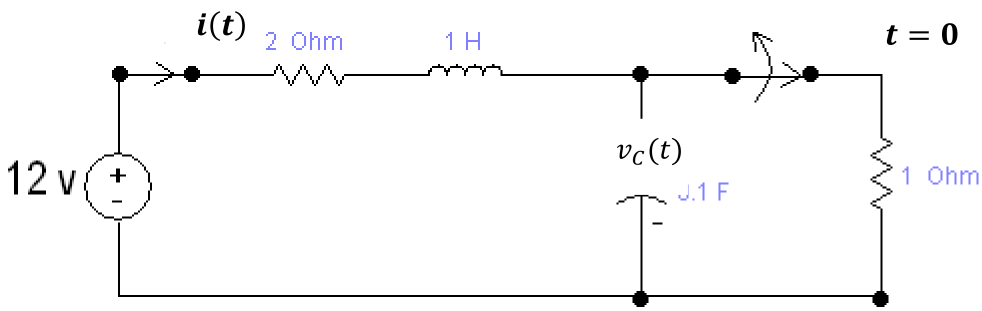
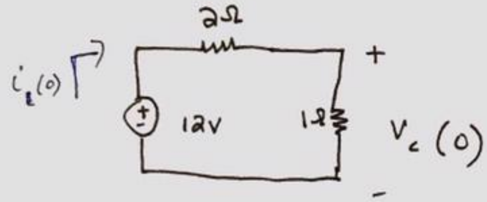


Figure 12.1

Task 1

for $t < 0$



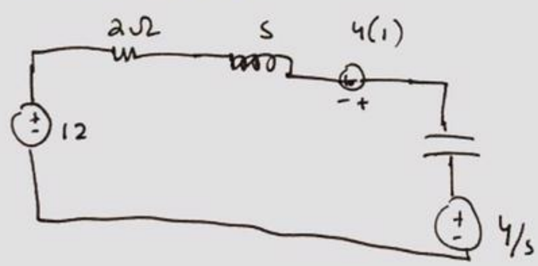
$V_c(0) = \frac{1}{3}(12)$ ∴ by voltage divider.

$$\boxed{V_c(0) = 4V}$$

$$i_L(0) = \frac{V}{R} = \frac{12}{3}$$

$$\boxed{i_L(0) = 4A}$$

for $t > 0$

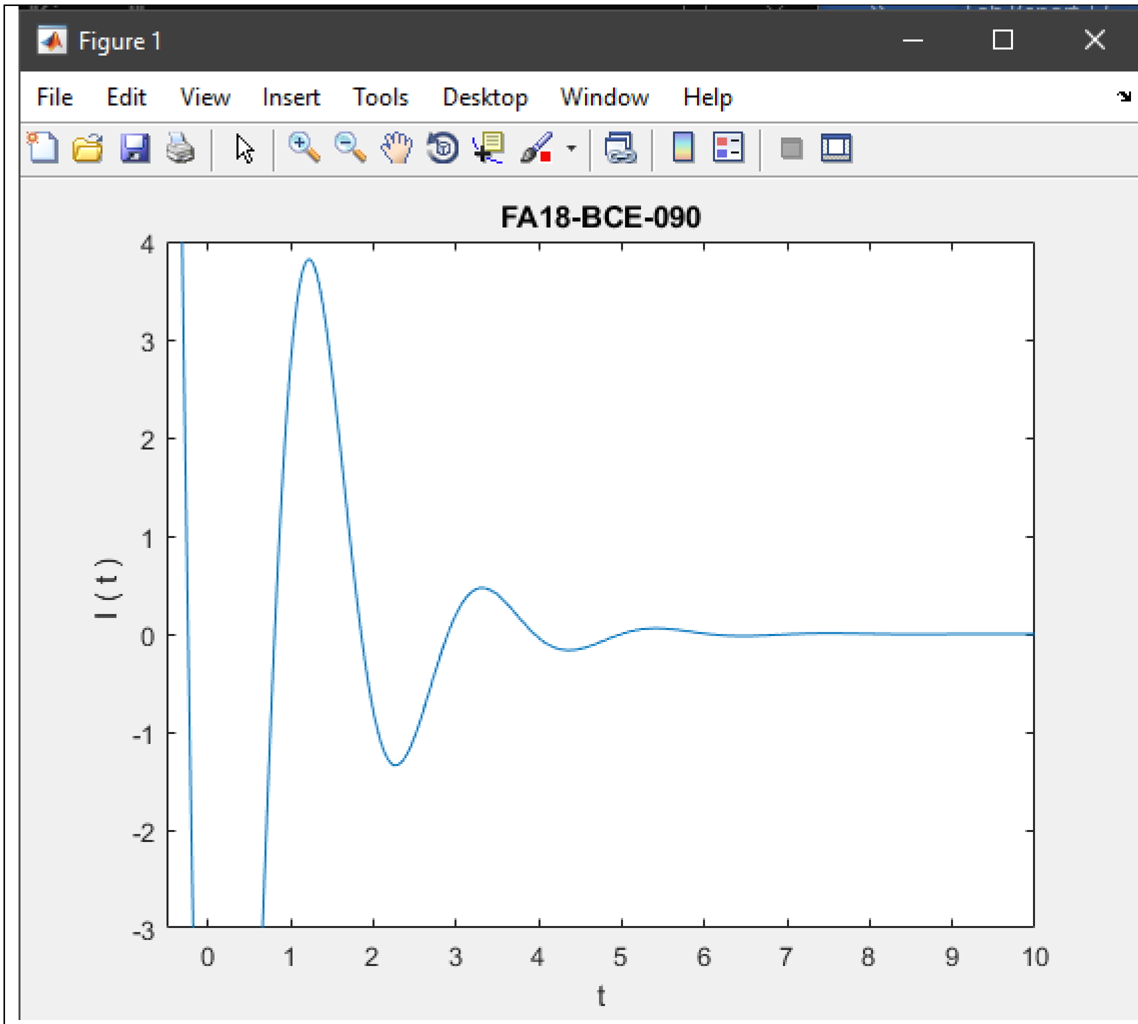


$$i(s) = \frac{12 + 4 - 4/s}{2 + s + 10/s} = \frac{4s - 1}{s + 2 + 10/s}$$

$$\boxed{i(s) = \frac{4s - 1}{s + 2 + 10/s}}$$

CODE:

```
syms t s
I=(4*s-1)/(2+s+10/s);
R = (2+s+10/s);
IL=ilaplace(I,t);
ezplot(IL,[-0.5 10]);
title('FA18-BCE-090');
ylim([-3 4]);
xlabel('t');
ylabel('I ( t ) ');
E=int(abs(I*R).^2,s,0,inf)
```



BIBO Unstable

```
E=int(abs(V).^2,s,-inf,inf)

E =

Inf
```

Task 02: For the circuit shown in figure 12.2, the input voltage is $V_i(t) = 10\cos(2t)\,u(t)$

I. **Plot the steady state output voltage $v_{oss}(t)$ for $t > 0$ assuming zero initial conditions.**

II. **Determine whether the system is stable or not?**

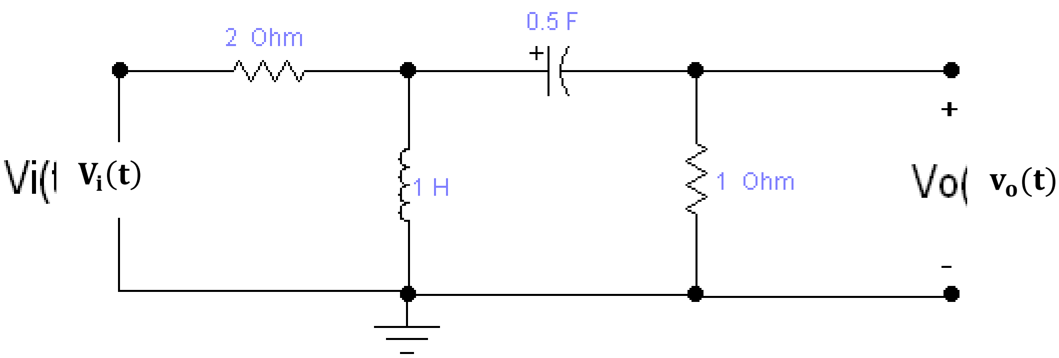
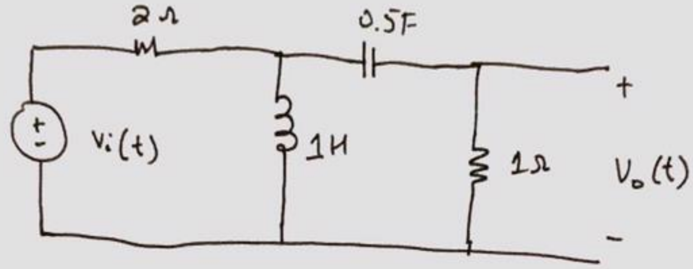
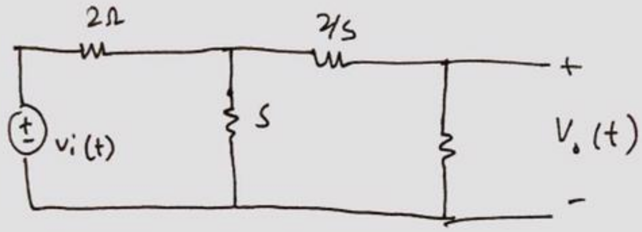


Figure 12.2

Task 2

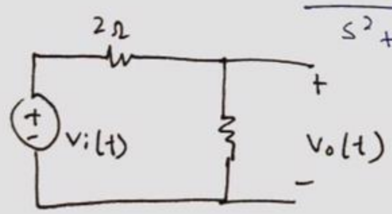
$v_i(t) = 10\cos(2t)u(t)$





$$1 + 2/s \parallel s \Rightarrow \frac{\frac{s+2}{s} \cdot s}{s + \frac{s+2}{2}} = \frac{s+2}{\frac{s^2+s+2}{s}}$$

$$= \frac{(s+2)s}{s^2+s+2}$$



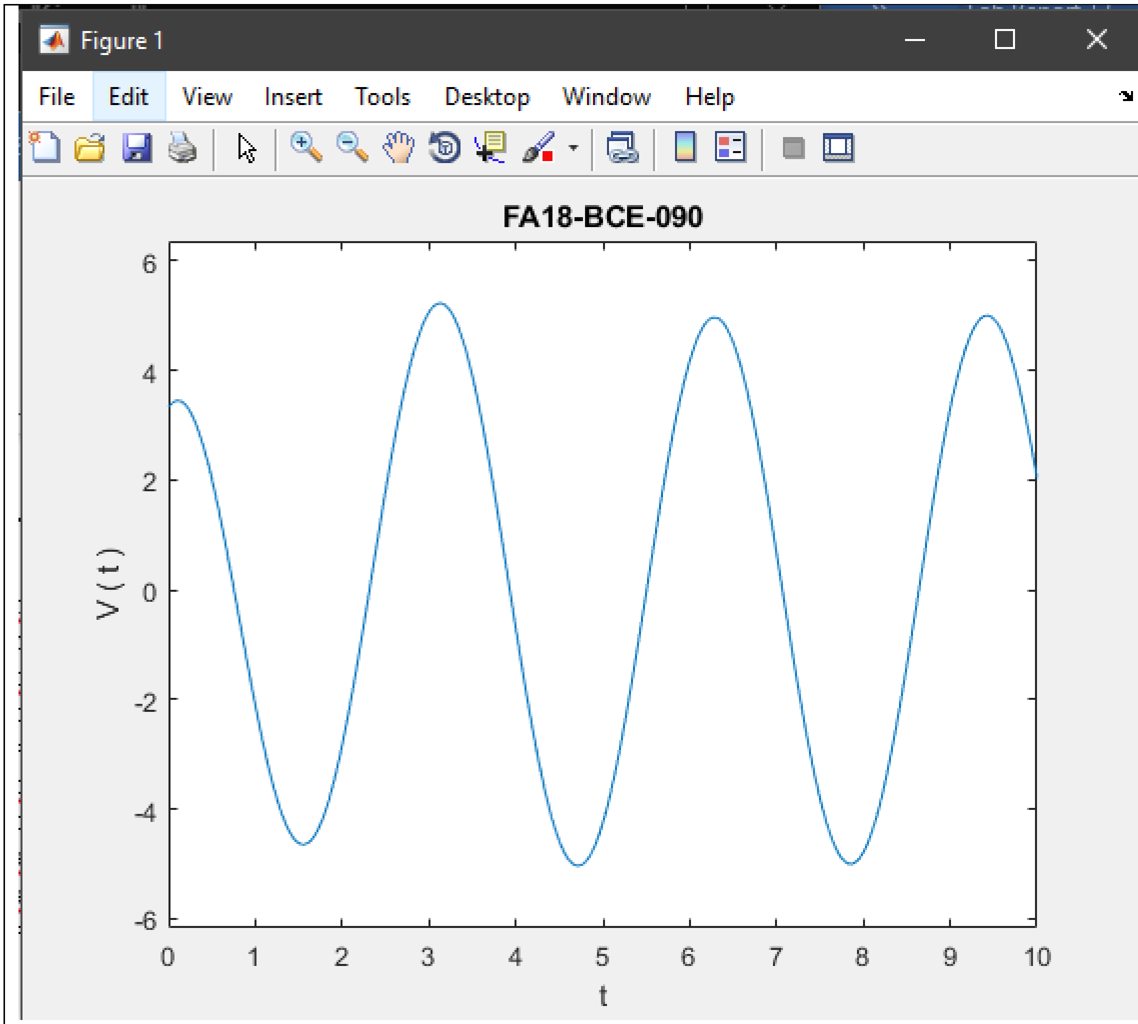
Applying Voltage divider :

$$V_o(t) = \frac{\frac{(s+2)s}{s^2+s+2}}{2 + \frac{(s+2)s}{s^2+s+2}} \times v_i(t)$$

$$\boxed{V_o(t) = \left(\frac{s(s+2)}{3s^2+4s+4}\right) \times V_i(t)}$$

CODE:

```
syms t s
Vin=10*cos(2*t)*heaviside(t);
lap_Vin=laplace(Vin,s);
output=lap_Vin*((s*(s+2))/(3*(s^2)+4*s+4));
Vo=ilaplace(output,t);
ezplot(Vo,[0 10]);
title('FA18-BCE-090');
xlabel('t');
ylabel('V ( t )');
e = int(abs(output).^2,s,-inf,inf)
```



BIBO Stable

```
e = int(abs(output).^2,s,-inf,inf)

e =

(25*pi*2^(1/2))/16
```

Critical Analysis / Conclusion

In this lab we learned about Laplace transformation as well as Inverse Laplace transformation. We used "laplace" and "ilaplace" commands in MATLAB to convert from time (t) domain to Laplace (s) domain or from Laplace (s) domain to time (t) domain, respectively. These commands were applied in RLC circuits as well. We also learned the residue function which converts a given rational function into partial fraction.