Signals & Systems

EEE-223

Lab # 05

Name	Muhammad Haris Irfan
Registration Number	FA18-BCE-090
Class	BCE-4A
Instructor's Name	Muhammad Bilal Qasim

LAB # 05

Study of Properties of Systems (Linearity, Causality, Memory, Stability and Time invariance)

Lab 05- Study of Properties of Systems (Linearity, Causality, Memory, Stability and Time invariance)

Pre-Lab Tasks

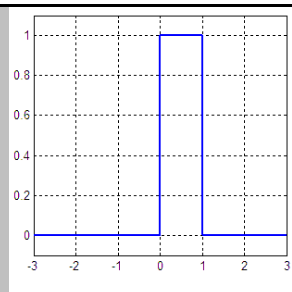
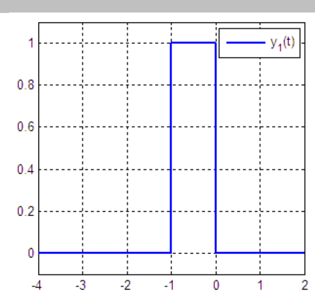
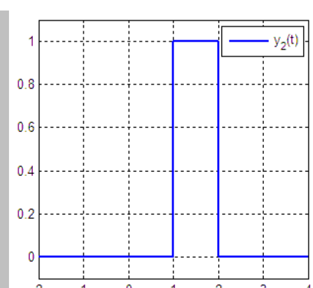
5.1 Properties of Systems:

5.1.1 Causal and Non-causal Systems:

A system is causal if the system output $y(t_0)$ at time $t = t_0$ does not depend on values of the input $x(t)$ for $t > t_0$. In other words, for any input signal $x(t)$, the corresponding output $y(t)$ depends upon the present and past values of $x(t)$. So, if the input to a causal system is zero for $t < t_0$, the output of the system is also zero for $t < t_0$. Correspondingly a discrete time system is causal if its output $y[n_0]$ at time $n = n_0$ depends only on the values of input signal $x[n]$ for $n \leq n_0$. All natural systems are causal.

Example:

Suppose the system S_1 is described by the I/O relationship $y(t) = x(t+1)$ while the I/O relationship of the system S_2 is given by $y(t) = x(t-1)$. Using the input signal $x(t) = u(t) - u(t-1)$ find out if the two systems are causal.

Commands	**Results**	**Comments**
t1=-3:0.1:0; **x1=zeros(size(t1));** **t2=0:0.1:1;** **x2=ones(size(t2));** **t3=1:0.1:3;** **x3=zeros(size(t3));** **t=[t1 t2 t3];** **x=[x1 x2 x3];** **plot(t,x,'linewidth',2),grid on** **ylim([-0.1 1.1]);** **legend('x(t)')**		Definition of the graph in the time interval $-3 \le t \le 3$ of the input signal $x(t) = u(t) - u(t-1)$
plot(t-1,x,'linewidth',2),grid on **ylim([-0.1 1.1]);** **legend('y_1(t)')**		The output of S_1 is given by $y(t) = x(t+1)$. The input signal $x(t)$ is zero for $t < 0$ but the output $y(t)$ is nonzero for $t < 0$, i.e., $y(t)$ depends upon the future values of $x(t)$; thus system S_1 is non-causal
plot(t+1,x,'linewidth',2),grid on **ylim([-0.1 1.1]);** **legend('y_2(t)')**		The output of S_2 is given by $y(t) = x(t-1)$. The output $y(t)$ is zero for $t < 1$, i.e., $y(t)$ depends only on the past values of $x(t)$; thus system S_2 is causal

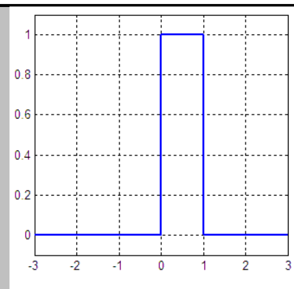
5.1.2 Static (Memory less) and Dynamic (with Memory) Systems:

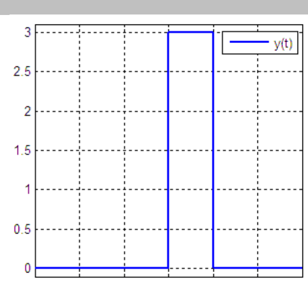
A system is static or memory less if for any input signal $x(t)$ or $x[n]$ the corresponding output $y(t)$ or $y[n]$ depends only on the value of the input signal at that time. A non-static system is called dynamic or dynamical.

Example:

Using the input signal $x(t)=u(t)-u(t-1)$ find out the systems described by the I/O relationship $y(t)=3x(t)$ and $y(t)=x(t)-x(t-1)$ are static or dynamic.

Commands	Results	Comments

`t1=-3:0.1:0;`
`x1=zeros(size(t1));`
`t2=0:0.1:1;`
`x2=ones(size(t2));`
`t3=1:0.1:3;`
`x3=zeros(size(t3));`
`t=[t1 t2 t3];`
`x=[x1 x2 x3];`
`plot(t,x,'linewidth',2),grid on`
`ylim([-0.1 1.1]);`
`legend('x(t)')`
		Definition of the graph in the time interval $-3 \le t \le 3$ of the input signal $x(t)=u(t)-u(t-1)$

`plot(t,3*x,'linewidth',2),grid on`
`ylim([-0.1 3.1]);`
`legend('y(t)')`
		The output of the system when I/O relationship $y(t)=3x(t)$ depends on the value of the input at the same time. Hence, it is a static (or memory less) system

In order to determine if the second system described by the I/O relationship $y(t)=x(t)-x(t-1)$ is static or dynamic, recall that $x(t)=u(t)-u(t-1)=1, 0 \le t \le 1;$ thus

$x(t-1)=u(t-1)-u(t-2)=1, 1\le t\le 2$ and so $y(t)=u(t)-u(t-2)=1, 0\le t\le 2$. The values of $y(t)$ depend on past values of $x(t)$ so system is dynamic.

5.1.3 Linear and Non-linear Systems:

Let $y(t)$ denote the response of the system S to an input signal $x(t)$, that is, $y(t)=S\{x(t)\}$. System S is linear if for any input signal s $x_1(t)$ and $x_2(t)$ and any scalar a_1 and a_2 the following relationship (equation 5.1) holds.

$$S\{a_1x_1(t)+a_2x_2(t)\}=a_1S\{x_1(t)\}+a_2S\{x_2(t)\}$$

In other words, the response of the linear system to an input that is a linear combination of two signals is the linear combination of the responses of the system to each one of these signals. The linearity property is generalized for any number of input signals, and this is often referred to as the principle of superposition. The linearity property is the combination of two other properties: the additivity property and the homogeneity property. A system S satisfies the additivity property if for any input signals $x_1(t)$ and $x_2(t)$

$$S\{x_1(t)+x_2(t)\}=S\{x_1(t)\}+S\{x_2(t)\}$$

While the homogeneity property implies that for any scalar a and any input signal $x(t)$,

$$S\{ax(t)\}=aS\{x(t)\}$$

Example:

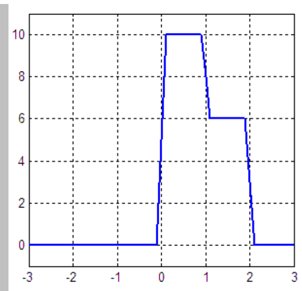
Let $x_1(t)=u(t)-u(t-1)$ and $x_2(t)=u(t)-u(t-2)$ be the input signals to the systems described by the I/O relationships $y(t)=2x(t)$ and $y(t)=x^2(t)$. Determine if the linearity property holds for these two systems.

To examine if the systems are linear, we use the scalars $a_1=2$ and $a_2=3$. The time interval considered is $-3\le t\le 3$.

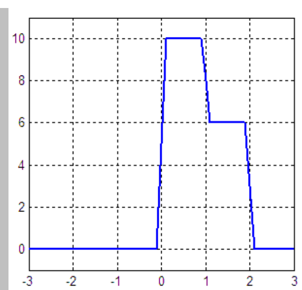
For the system described by the I/O relationship $y(t)=2x(t)$ the procedure is followed as

Commands	Results	Comments
t=-3:0.1:3; **x1=heaviside(t)-heaviside(t-1);** **x2=heaviside(t)-heaviside(t-2);**		Definition of the input signals $x_1(t)$ and $x_2(t)$

%computation of the left side of equation 5.1
 	
a1=2;
 	
a2=3;
 	
z=a1*x1+a2*x2;
		The expression $a_1x_1(t)+a_2x_2(t)$ is defined.

y=2*z; plot(t,y,'linewidth',2),grid on ylim([-1 11])		The left side of equation 5.1, namely, $S\{a_1x_1(t)+a_2x_2(t)\}$ is computed and the result is plotted.

%computation of the right side of equation 5.1
 	
z1=2*x1;
 	
z3=3*x2;
		Definition of $S\{x_1(t)\}$ and $S\{x_2(t)\}$.

y=a1*z1+a2*z2; plot(t,y,'linewidth',2),grid on ylim([-1 11])		The right side of equation 5.1, namely, $a_1S\{x_1(t)\}+a_2S\{x_2(t)\}$ is computed and the result is plotted.

The two graphs obtained are identical; hence the two sides of equation 5.1 are equal. Therefore the system described by the I/O relationship $y(t)=2x(t)$ is linear.

5.1.4 Time-Invariant and Time-Variant Systems:

A system is time invariant, if a tune shift in the input signal results in the same time shift in the output signal. In other words, if $y(t)$ is the response of a time-invariant system to an input signal $x(t)$, then the system response to the input signal $x(t-t_0)$ is $y(t-t_0)$. The mathematical expression (equation 5.2) is

$$y(t-t_0)=S\{x(t-t_0)\}$$

Equivalently, a discrete time system is time or (more appropriately) shift invariant if

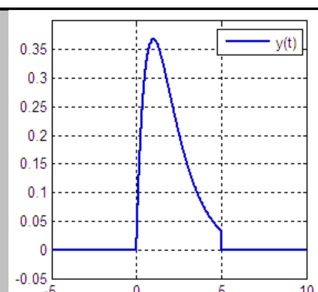
$y[n-n_0]=S\{x[n-n_0]\}$

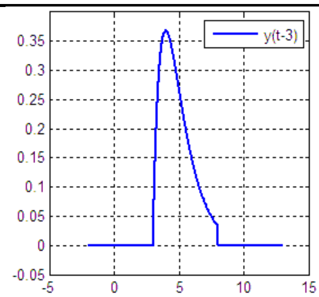
From the above equations, we conclude that is a system is time invariant, the amplitude of the output signal is the same independent of the time instance the input is applied. The difference is time shift in the input signal. A non-time invariant system is called time-varying or time-variant system.

Example:

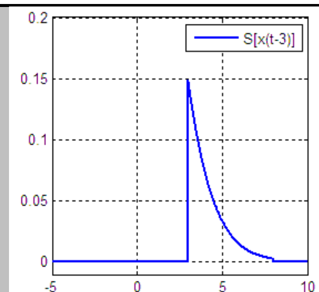
Suppose that response of a system S to an input signal $y(t)=te^{-t}x(t)$. Determine if the system is time invariant by using the input signal $x(t)=u(t)-u(t-5)$.

In order to determine if the system is time invariant, first we will have to compute and plot the system response $y(t)$ to the given input signal $x(t)=u(t)-u(t-5)$. Next, the computed output is shifted by 3 units to the right to represent the signal $y_1(t)=y(t-3)$. This corresponds to the left side of equation 5.2. As for the right side if equation 5.2, first the input signal $x(t)$ is shifted 3 units to the right in order to represent the signal $x(t-3)$. Next, the system response $y_2(t)=S\{x(t-3)\}$ is computed and plotted. If the two derived system responses are equal, the system under consideration is time invariant.

Commands	**Results**	**Comments**
t=-5:0.001:10; **p=heaviside(t)-heaviside(t-5);** **y=t.*exp(-t).*p;** **plot(t,y,'linewidth',2),grid on** **ylim([-0.05 0.4])** **legend('y(t)')**		The response $y(t)$ of the system to the input signal $x(t)=u(t)-u(t-5)$ is $y(t)=te^{-t}\left[u(t)-u(t-5)\right]=te^{-t}$, $0\le t\le 5$.

plot(t+3,y,'linewidth',2),grid on **ylim([-0.05 0.4])** **legend('y(t-3)')**		The output signal $y(t)$ is shifted 3 units to the right in order to obtain the signal $y_1(t) = y(t-3)$.

The input signal $x_2(t) = x(t-3)$ is given by $x_2(t) = u(t-3) - u(t-8)$ Thus the system response $y_2(t) = S\{x_2(t)\} = S\{x(t-3)\}$ is computed as $y_2(t) = te^{-t}\left[u(t-3) - u(t-8)\right]$.

Commands	**Results**
t=-5:0.001:10; **p=heaviside(t-3)-heaviside(t-8);** **y=t.*exp(-t).*p;** **plot(t,y,'linewidth',2),grid on** **ylim([-0.01 0.2])** **legend('S[x(t-3)]')**	

The two obtained graphs are not alike; thus the system described by the I/O relationship $y(t) = te^{-t}x(t)$ is time variant. A rule of thumb is that if the output of the system depends on time t outside of $x(t)$ the system is time variant.

5.1.5 Invertible and Non-Invertible Systems:

A system is invertible if the input signal $x(t)$ that is applied to the system can be derived from the system response $y(t)$. In other words, a system is invertible if the I/O relationship $y(t) = S\{x(t)\}$ is one to one, namely if different input values correspond to different output values.

Example:

Determine if the systems S_1 and S_2 described by the I/O relationships $y_1[n] = 3x[n]$ and $y_2[n] = x^2[n]$, respectively, are invertible. Consider the signal $x[n] = 2n, -2 \le n \le 2$ as the input signal.

Commands	**Results**	**Comments**

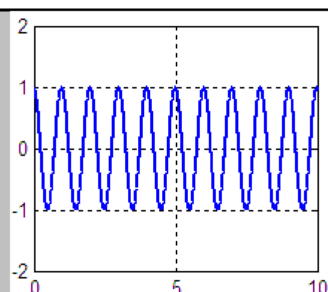
n=-2:2; **x=2*n;**	x= -4 -2 0 2 4	Input signal $x[n]$
y1=3*x;	y1= -12 -6 0 6 12	The output signal $y_1[n]=3x[n]$ of the system S_1
y2=x.^2;	y2= 16 4 0 4 16	The output signal $y_2[n]=x^2[n]$ of the system S_2

5.1.6 Stable and Unstable Systems:

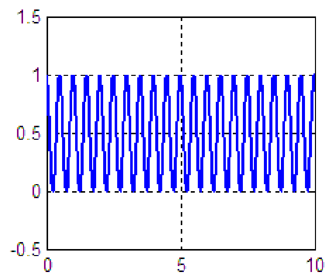
Stability is very important system property. The practical meaning of a stable system is that for a small applied input the system response is also small (does not diverge). A more formal definition is that a system is stable or bounded-input bounded-output (BIBO) stable if the system response to any bounded-input signal is bounded-output signal. The mathematical expression is as follows: Suppose that a positive number $M<\infty$ exists, such that $|x(t)|\le M$. The system is stable if $\forall t \in$ □ a positive number $N<\infty$ exists, such that $|y(t)|\le N$. A non-stable system is called unstable system.

Example:

Suppose that input signal $x(t)=\cos(2\pi t)$ is applied to two systems described by the I/O relationships $y_1(t)=x^2(t)$ and $y_2(t)=tx(t)$. Determine if the two systems are stable.

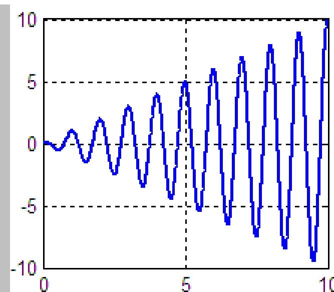
Commands	Results	Comments
1t=0:0.01:10; **x=cos(2*pi*t);** **plot(t,x,'linewidth',2), grid on** **ylim([-2 2])**		Definition and graph of $x(t)$. The input signal is bounded as $-1\le x(t)\le 1$, namely $x(t)$ is bounded by $M=1$ as $\|x(t)\|\le M$.

```
y1=x.^2;
plot(t,y1,'linewidth',2), grid on
ylim([-0.5 1.5])
```



Definition of graph of $y_1(t)$. The out signal $y_1(t)$ is bounded as $0 \le y_1(t) \le 1$, namely $y_1(t)$ is bounded by $N = 1$, as $|y_1(t)| \le N$. Hence the system described by the I/O relationship $y_1(t) = x^2(t)$ is BIBO stable.

```
y2=t.*x;
plot(t,y2,'linewidth',2), grid on
```



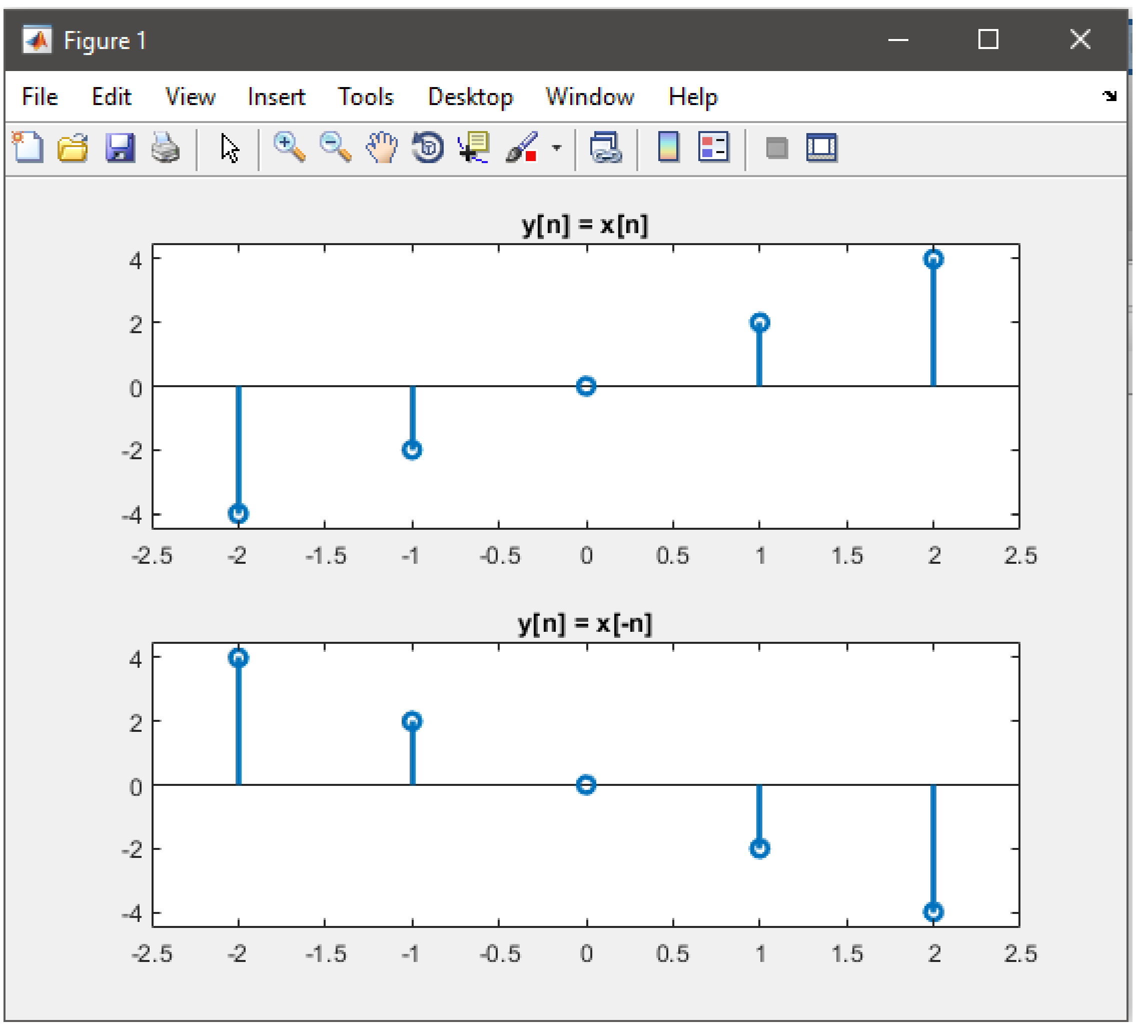
Definition of the graph of $y_2(t)$. The output signal $y_2(t)$ is not bounded as its amplitude is getting larger as time passes. Hence the system with I/O relationship $y_2(t) = tx(t)$ is not BIBO stable.

In-Lab Tasks

Task 01: Find out if the discrete-time system described by the I/O relationship $y[n] = x[-n]$ **is:**

a) Static or Dynamic (input signal $x[n] = 2n, -2 \le n \le 2$)
b) Causal or non-causal (input signal $x[n] = 2n, -2 \le n \le 2$)
c) Linear or non-linear (input signals $x_1[n] = 2n, -2 \le n \le 4, x_2[n] = n/3, -2 \le n \le 4, a_1 = 2 \, and \, a_2 = 3$)
d) Shift invariant or shift variant (input signal $x[n] = 2n, -2 \le n \le 4$ and shift $n_0 = 3$)

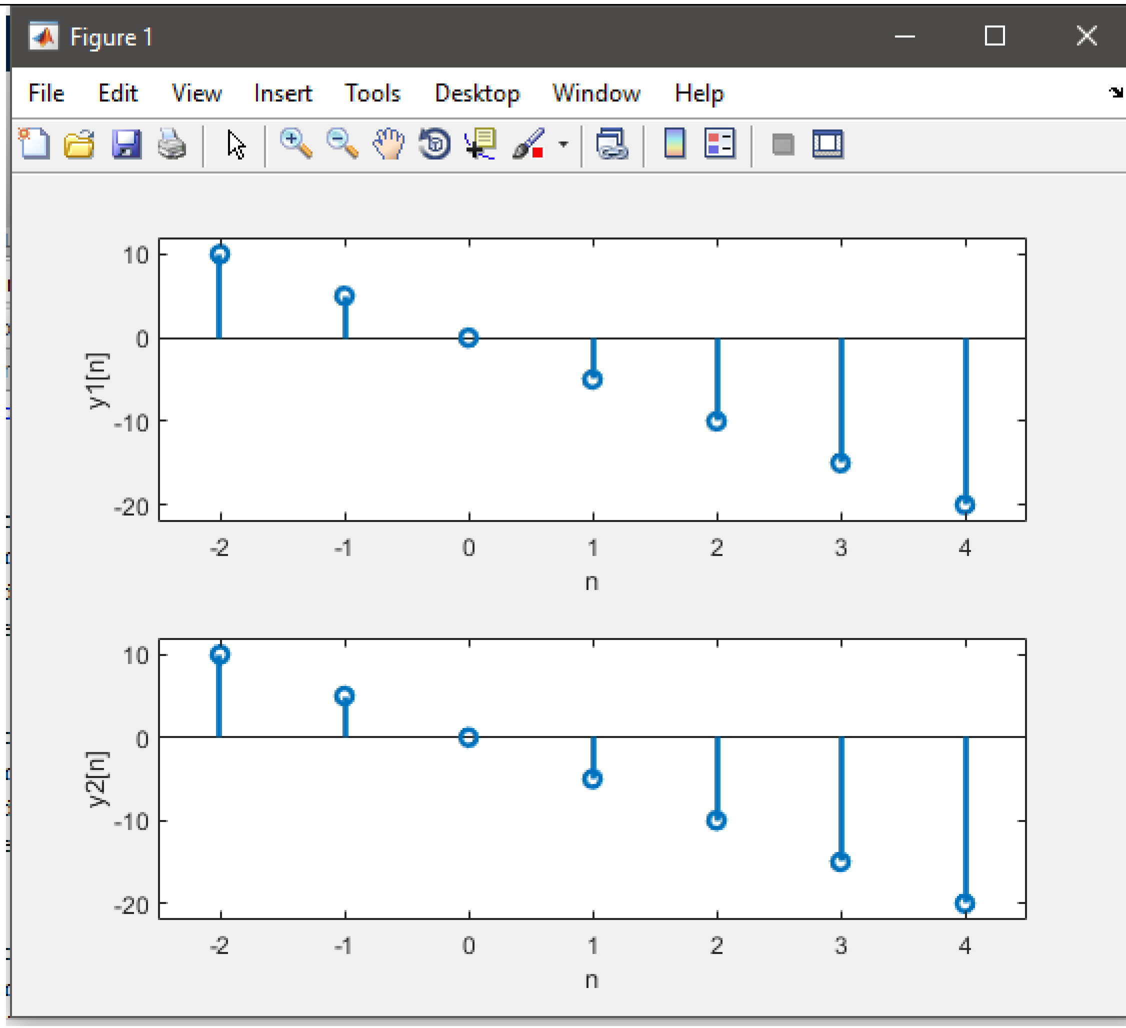
```
n = -2:2;
x = 2.*n;
subplot(2,1,1)
stem(n,x,'LineWidth',2)
xlim([-2.5,2.5])
ylim([-4.5,4.5])
title('y[n] = x[n]')

y = -x;
subplot(2,1,2)
stem(n,y,'LineWidth',2)
xlim([-2.5,2.5])
ylim([-4.5,4.5])
title('y[n] = x[-n]')
```



a) The system is dynamic.(with memory)

b) The system is Non-causal as the output depends on the future value of input.

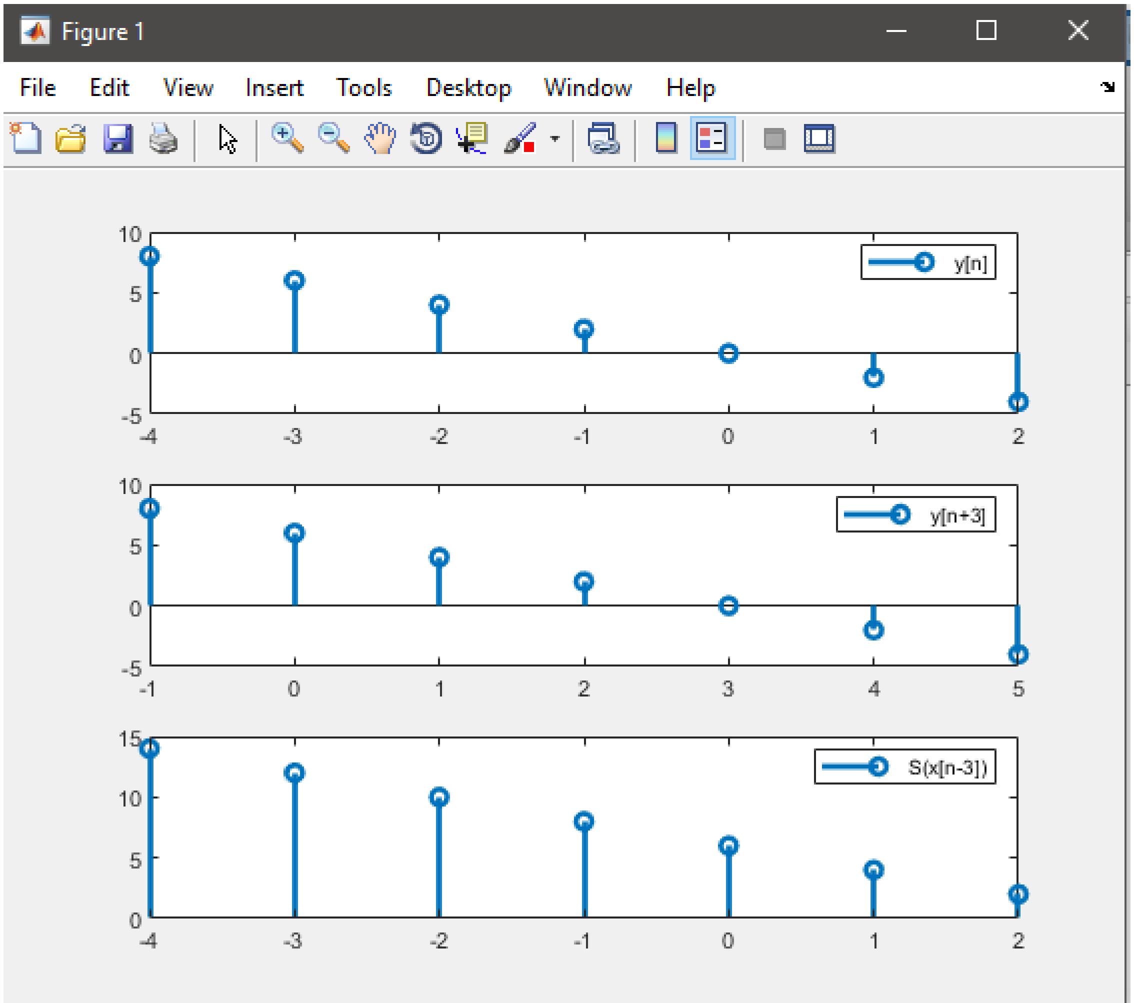
```
n = -2:4;
x1 = 2.*n;
x2 = n./3;
a1 = 2;
a2 = 3;
x = a1.*x1 + a2.*x2;
y = -x;
subplot(2,1,1)
stem(n,y,'LineWidth',2)
xlim([-2.5,4.5])
ylim([-22,12])
xlabel('n')
ylabel('y1[n]')

y1 = -x1;
y2 = -x2;
y = a1.*y1 + a2.*y2
subplot(2,1,2)
stem(n,y,'LineWidth',2)
xlim([-2.5,4.5])
ylim([-22,12])
xlabel('n')
ylabel('y2[n]')
```



c) As both the graphs are identical, hence the system is said to be linear.(as both the side of equation are equal)

```
n = -2:4;
x = 2.*n;
n0 = 3;
subplot(3,1,1)
stem(-n,x,'LineWidth',2)
legend('y[n]')
subplot(3,1,2)
stem(-(n-3),x,'LineWidth',2)
legend('y[n+3]')
x = 2.*(n+3);
```

```
subplot(3,1,3)
stem(-n,x,'LineWidth',2)
legend('S(x[n-3])')
```



d) as the graphs are not alike hence system is time variant

Task 02: Find out if the discrete-time system described by the I/O relationship $y[n] = x[1-2n]$

is:

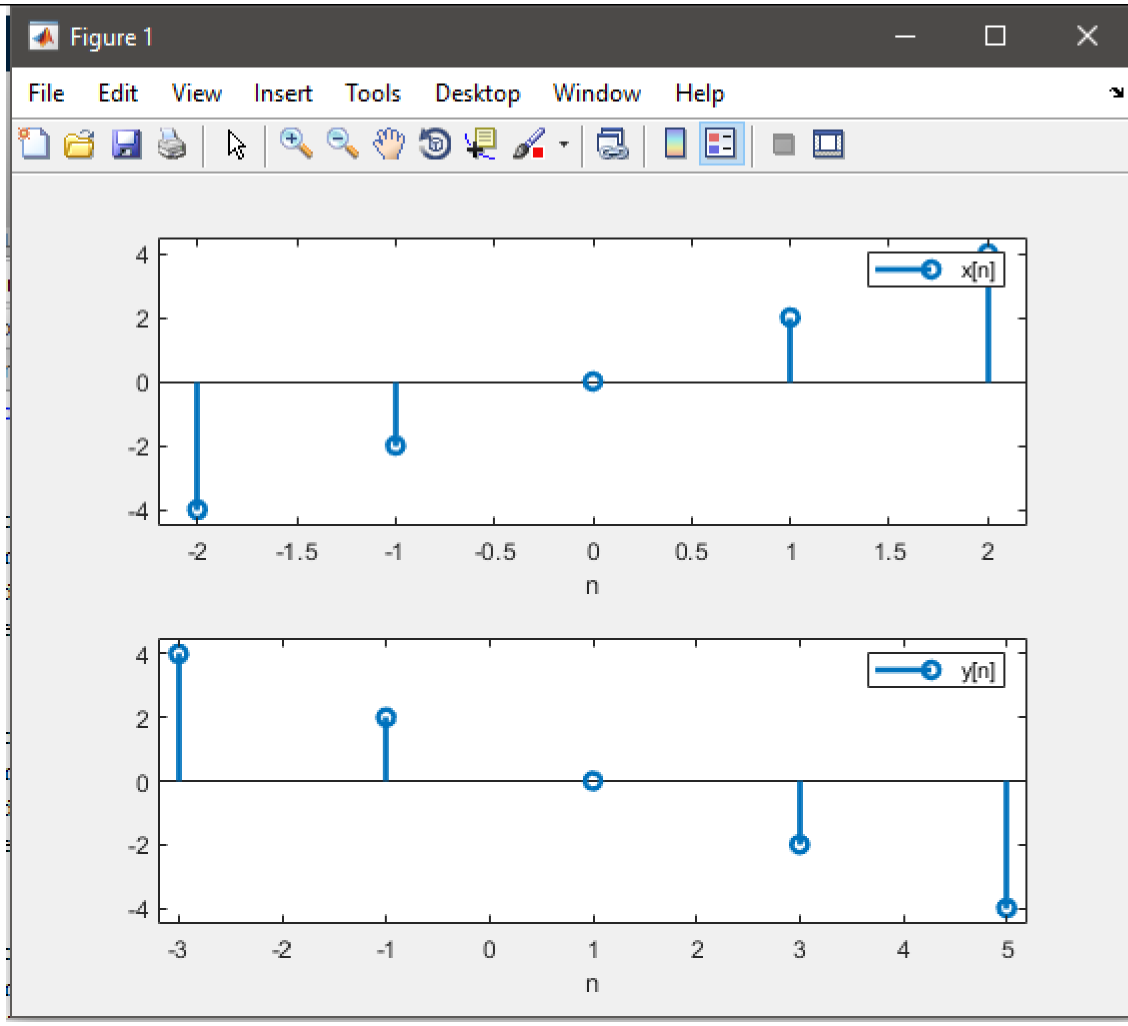
a) Static or Dynamic (input signal $x[n] = 2n, -2 \le n \le 2$)

b) Causal or non-causal (input signal $x[n] = 2n, -2 \le n \le 2$)

c) Linear or non-linear (input signals $x_1[n] = 2n, -2 \le n \le 4, x_2[n] = n/3, -2 \le n \le 4, a_1 = 2\, and\ a_2 = 3$)

d) Shift invariant or shift variant (input signal $x[n] = 2n, -2 \le n \le 4$ and shift $n_0 = 3$)

```
n = -2:2;
x = 2.*n;
subplot(2,1,1)
stem(n,x,'LineWidth',2)       %print x[n]
xlim([-2.2 2.2])
ylim([-4.5 4.5])
legend('x[n]')
xlabel('n')

y = x;
subplot(2,1,2)
stem(1-2.*n,y,'LineWidth',2) %print y[n]
xlim([-3.2 5.2])
ylim([-4.5 4.5])
legend('y[n]')
xlabel('n')
```

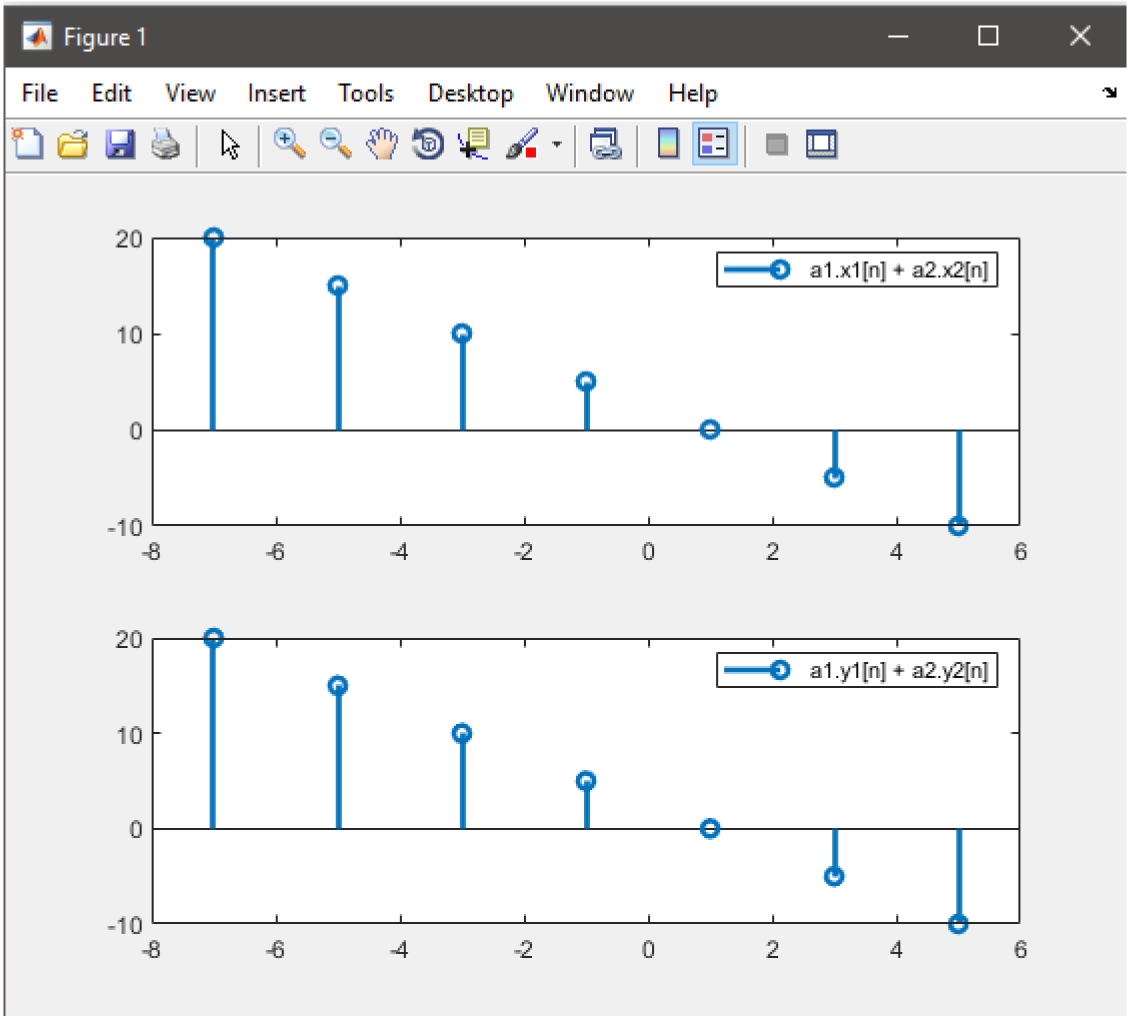


a) System is dynamic(with memory).

b) As the system depends on future as well as past values hence it is non causal.

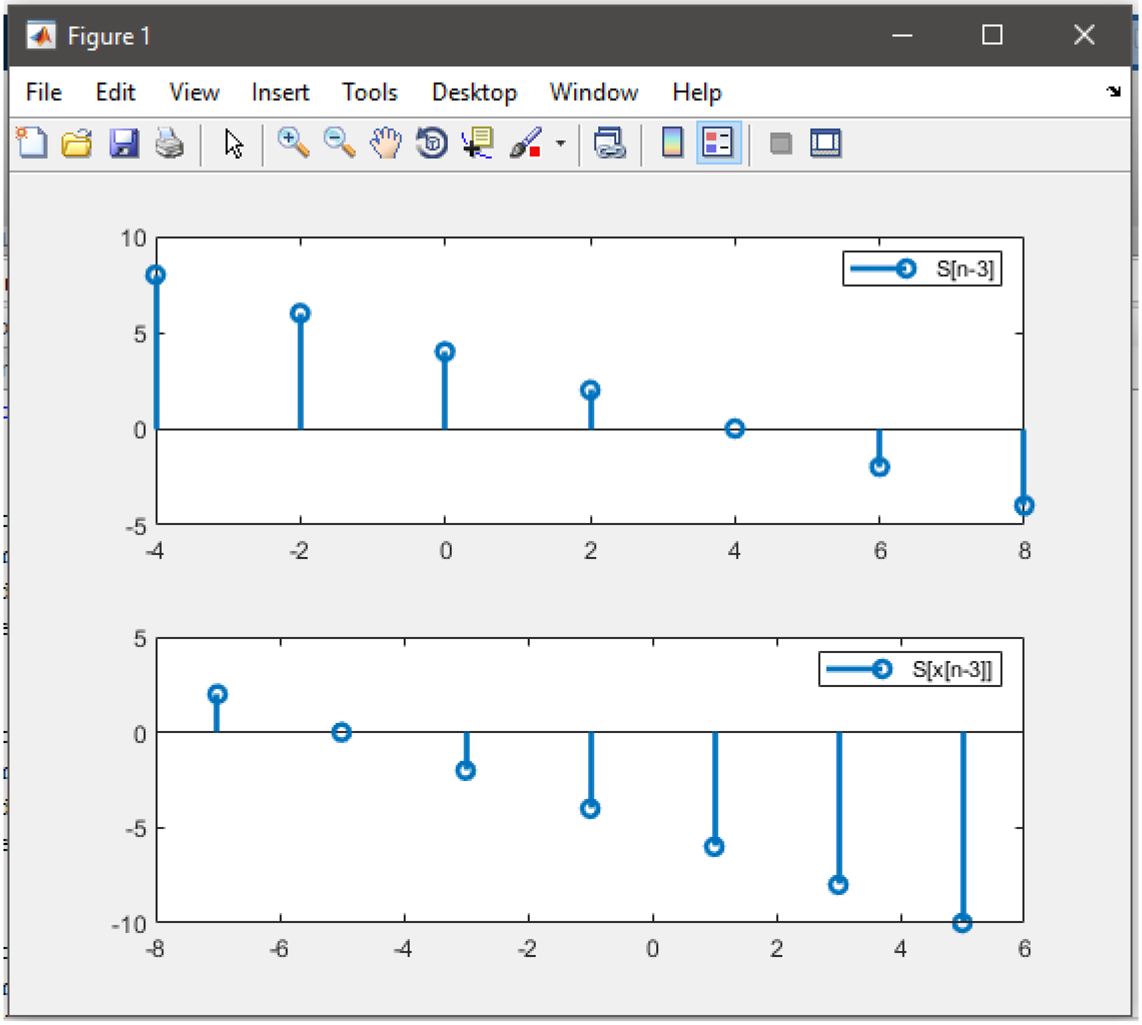
```
n = -2:4;
x1 = 2.*n;
x2 = n./3;
a1 = 2;
a2 = 3;
x = a1.*x1 + a2.*x2;
subplot(2,1,1)
stem(1-2.*n,x,'LineWidth',2)
legend('a1.x1[n] + a2.x2[n]')

legend('a1.x1[n] + a2.x2[n]')
subplot(2,1,2)
y1 = a1.*x1;
y2 = a2.*x2;
y = y1 + y2;
stem(1-2.*n,y,'LineWidth',2)
```

```
    legend('a1.y1[n] + a2.y2[n]')
```



c) As both the graphs are same, hence the system is linear.

```
n = -2:4;
x = 2.*n;
n0 = 3;
subplot(2,1,1)
stem(n0+1-2.*n,x,'LineWidth',2)
legend('S[n-3]')

y = 2.*(n-n0);
subplot(2,1,2)
stem(1-2.*n,y,'LineWidth',2)
legend('S[x[n-3]]')
```



d) As the graphs are not alike so the system is time variant.

Post-Lab Task

Critical Analysis / Conclusion

In this lab, we studied and practically implemented the concepts of Linearity, Causality, Memory, Stability and Time Variance. We completed various tasks and leant to differentiate the properties of these concepts using their graphs by plotting them on MATLAB.

<table>
<tr><th colspan="3">Lab Assessment</th></tr>
<tr><td>Pre-Lab</td><td>/1</td><td rowspan="3">/10</td></tr>
<tr><td>In-Lab</td><td>/5</td></tr>
<tr><td>Critical Analysis</td><td>/4</td></tr>
<tr><td colspan="3">Instructor Signature and Comments</td></tr>
</table>