

PART III:CONVOLUTIONAL NEURAL NETWORKS

Discrete Convolution

1D:
[Spectra]

$$f(x) * g(x) = \sum_{k=-\infty}^{k=+\infty} f(k).g(k - x)$$

2D:
[Images]

$$f(x, y) * g(x, y) = \sum_{k=-\infty}^{k=+\infty} \sum_{l=-\infty}^{l=+\infty} f(k, l).g(x - k, y - l)$$

DISCRETE CONVOLUTION

1D:
[Spectra]

$$f(x) * g(x) = \sum_{k=-\infty}^{k=+\infty} f(k).g(k - x)$$

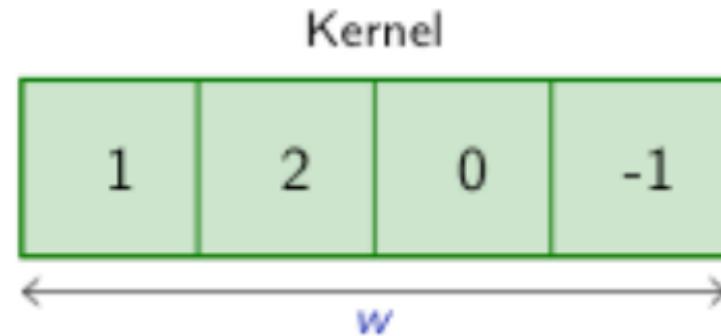
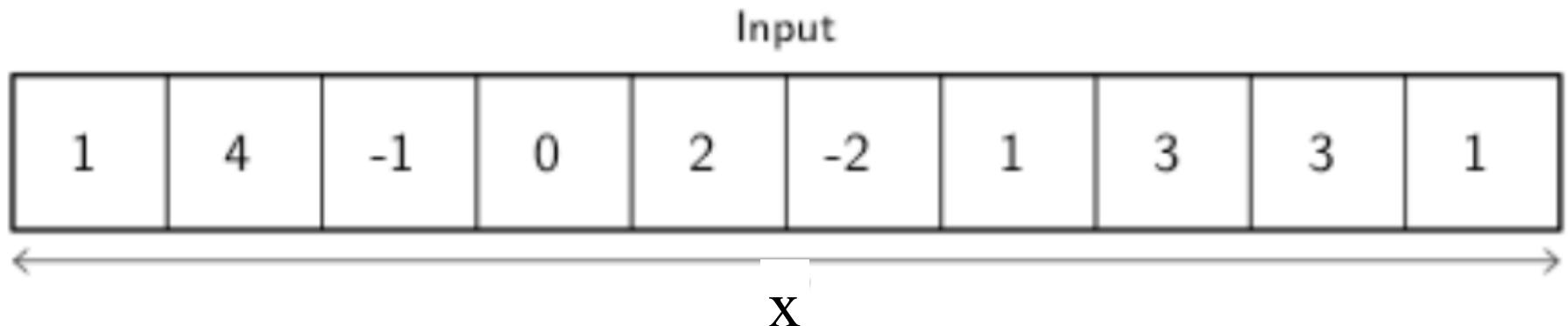
2D:
[Images]

$$f(x, y) * g(x, y) = \sum_{k=-\infty}^{k=+\infty} \sum_{l=-\infty}^{l=+\infty} f(k, l).g(x - k, y - l)$$

CONVOLUTION KERNEL

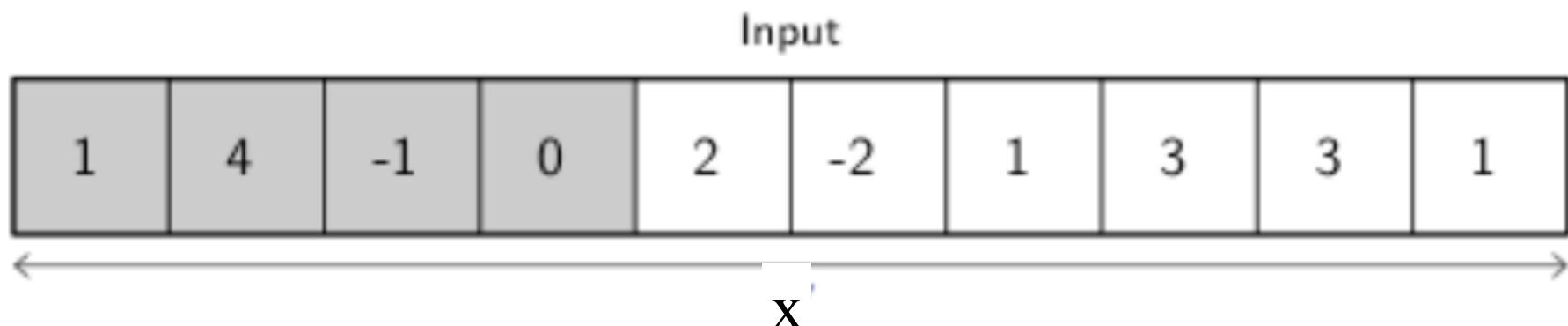
INPUT DATA

1-D CONVOLUTION



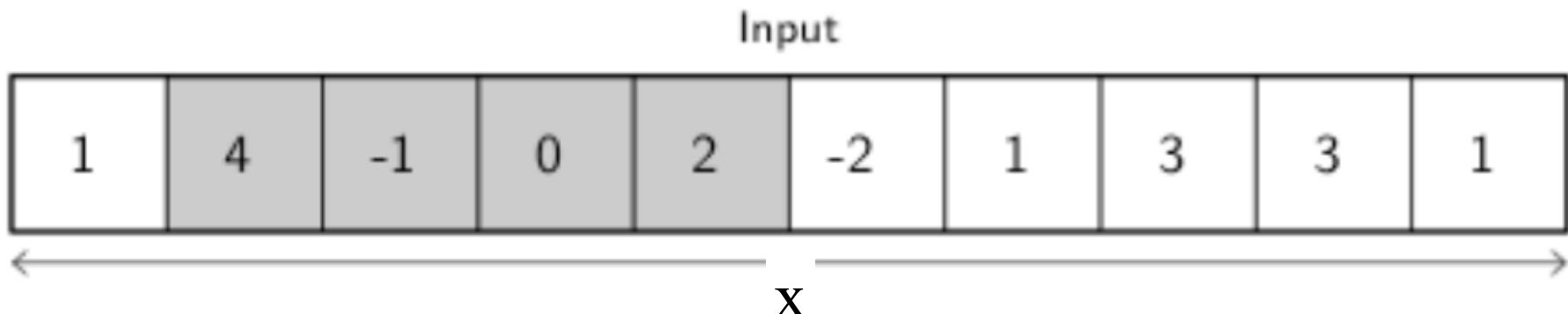
credit

1-D CONVOLUTION



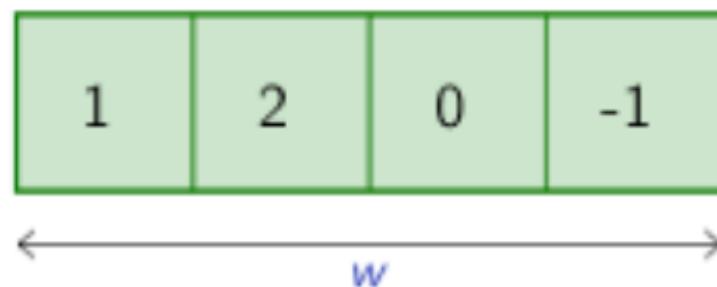
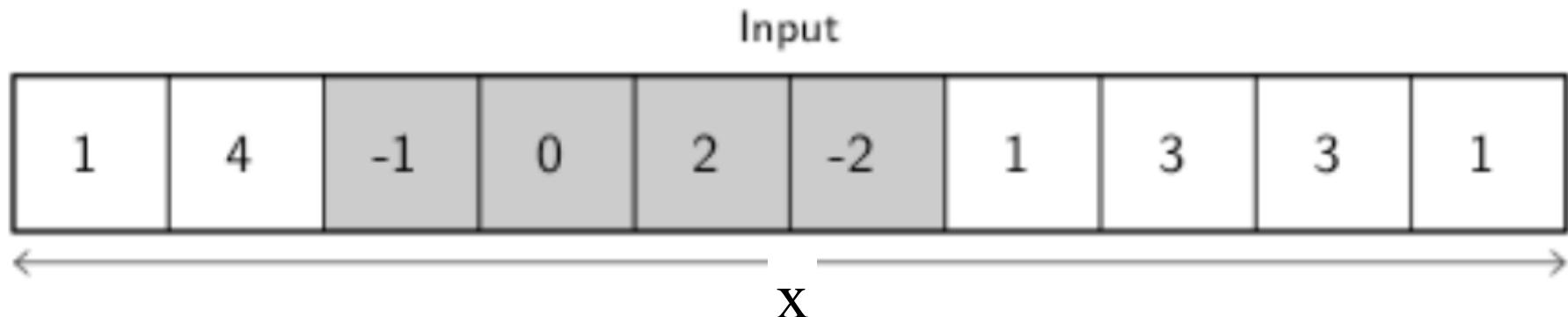
credit

1-D CONVOLUTION



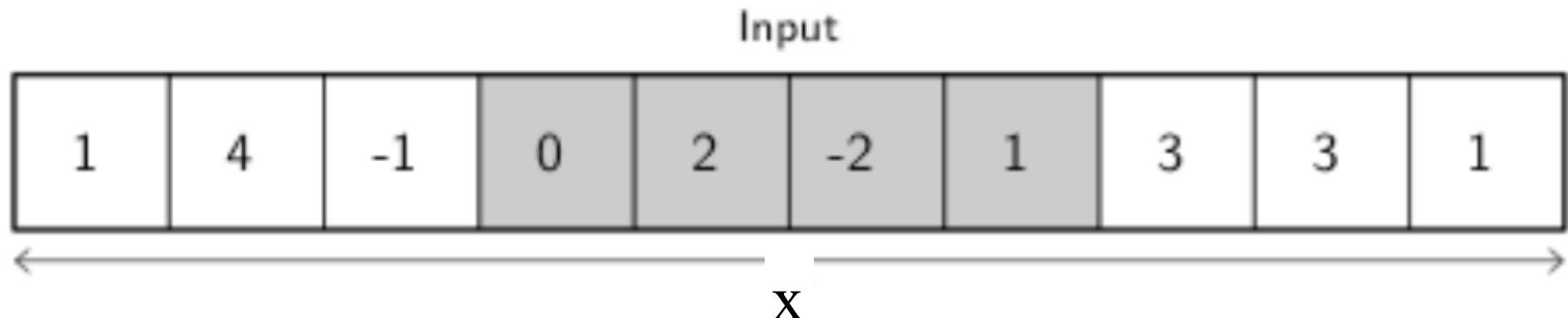
credit

1-D CONVOLUTION



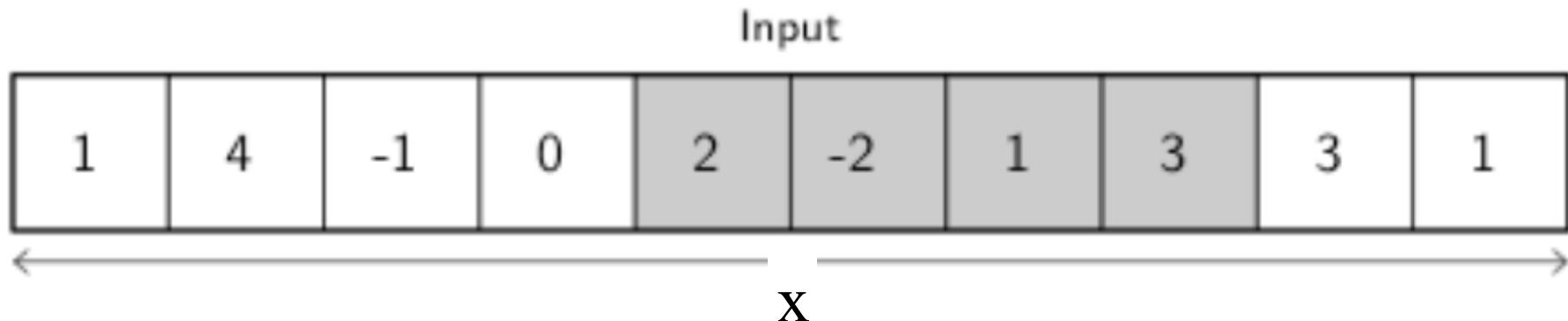
credit

1-D CONVOLUTION



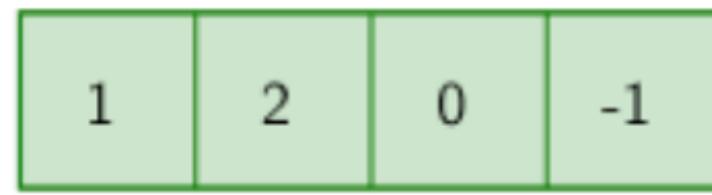
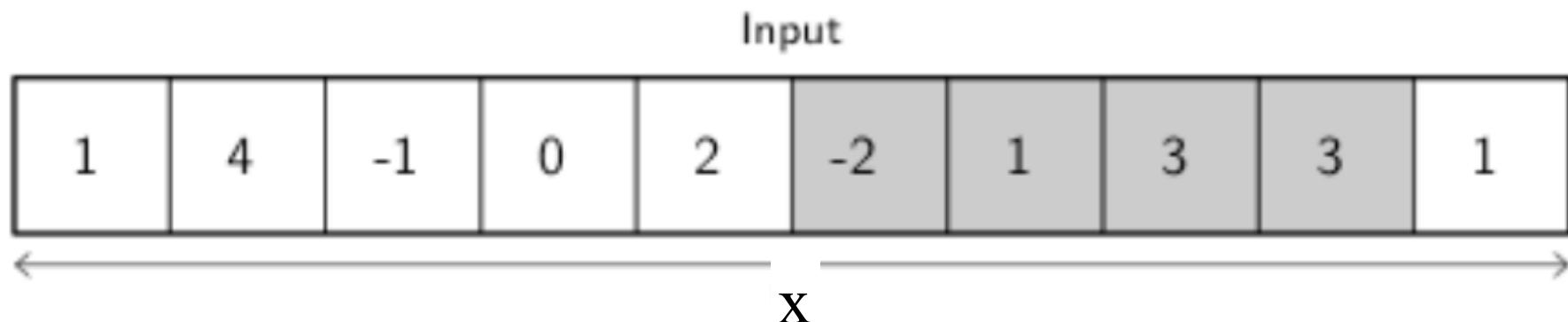
credit

1-D CONVOLUTION



credit

1-D CONVOLUTION



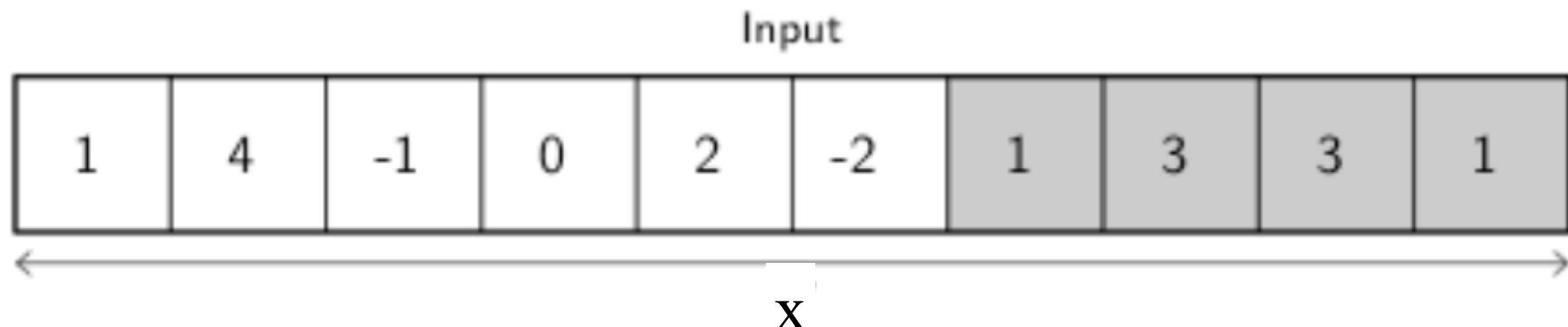
\longleftrightarrow

w



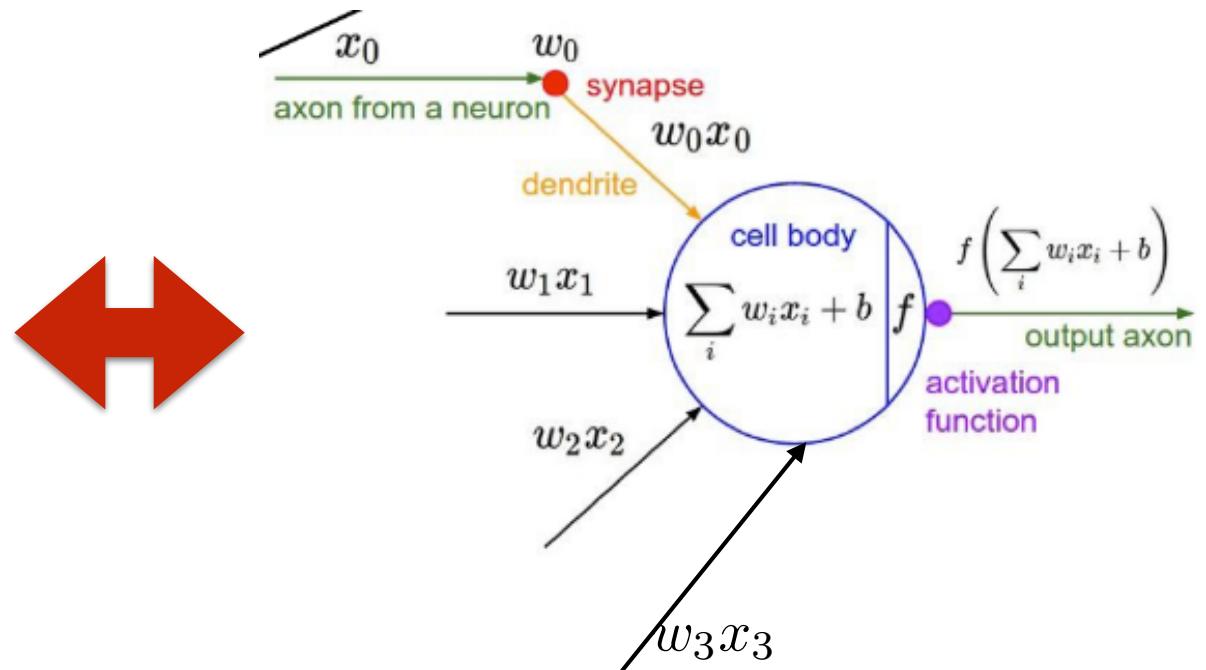
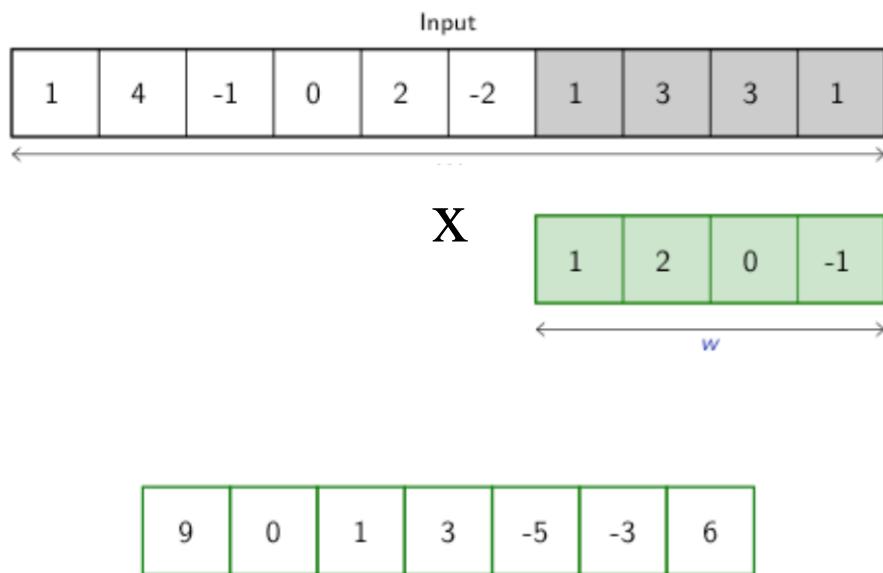
credit

1-D CONVOLUTION

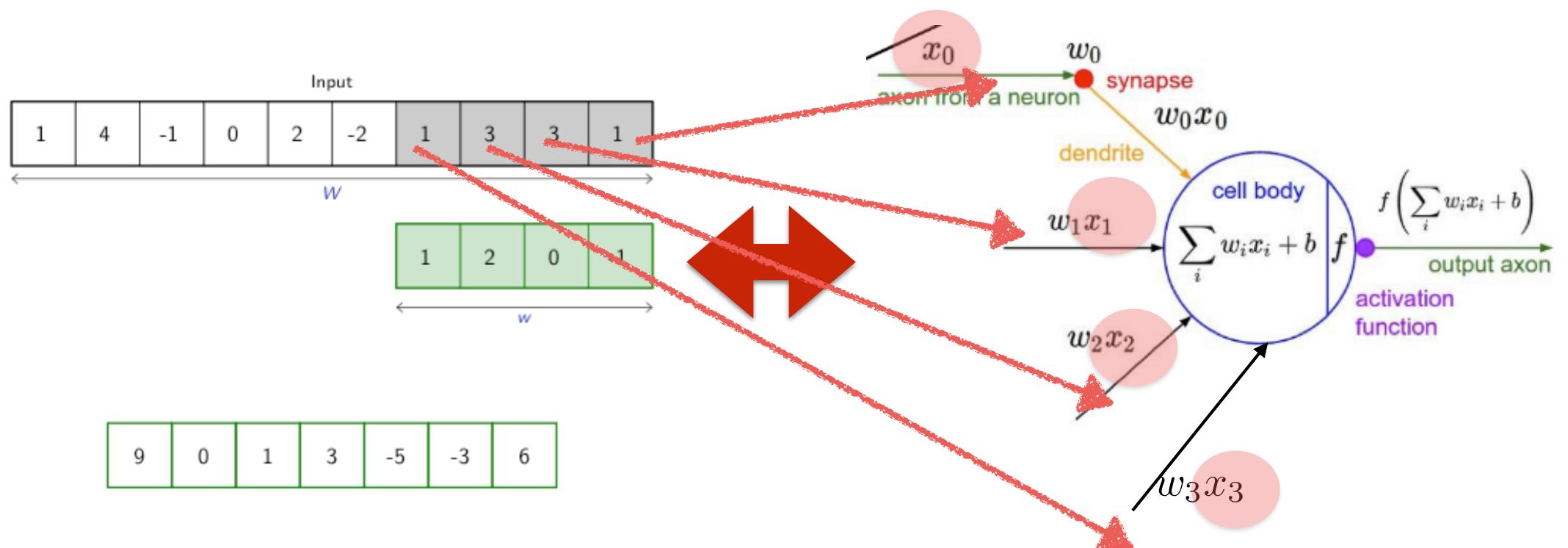


credit

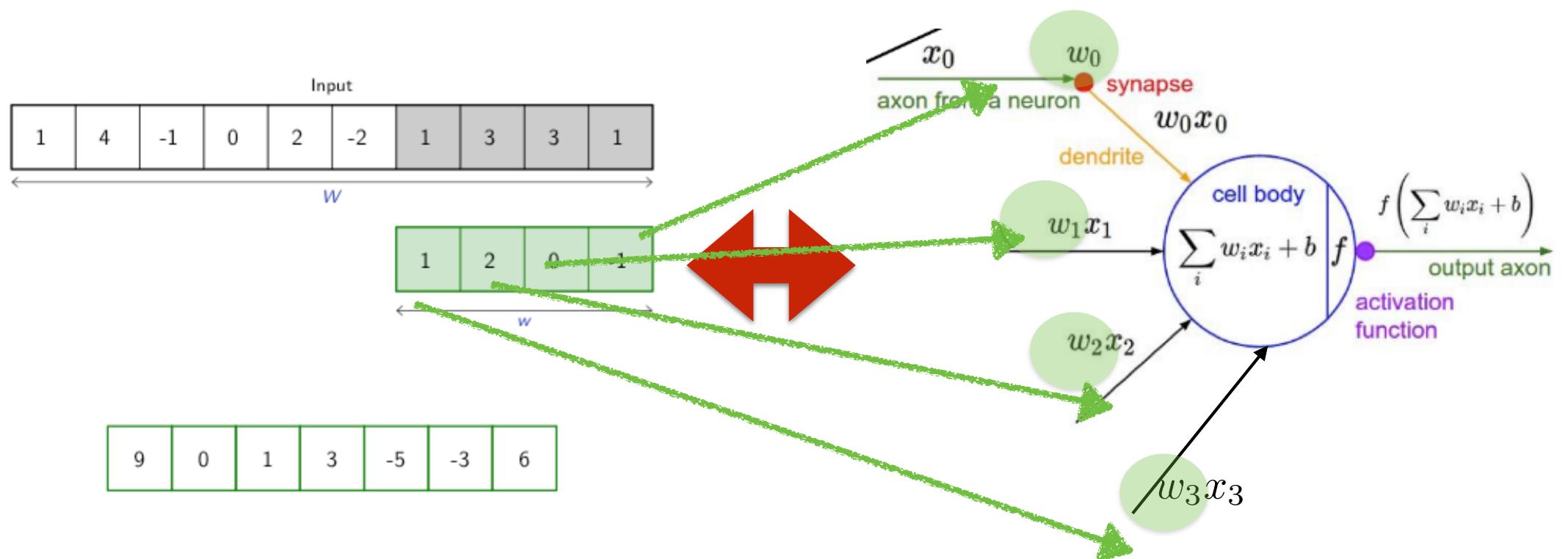
THE CONVOLUTION BUILDING BLOCK OPERATION IS EQUIVALENT TO A NEURON WITH AS MANY INPUTS AS KERNEL ELEMENTS AND WEIGHTS EQUAL TO THE KERNEL



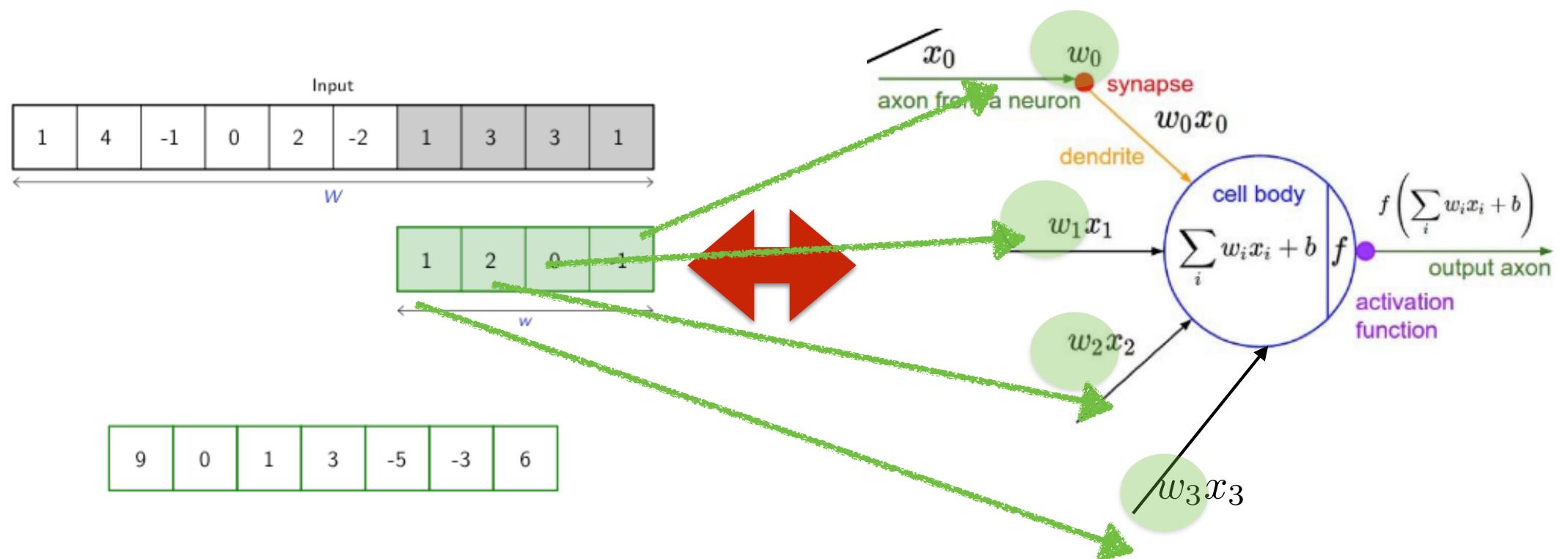
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THE CONVOLUTION BUILDING BLOCK OPERATION IS EQUIVALENT TO A NEURON WITH AS MANY INPUTS AS KERNEL ELEMENTS AND WEIGHTS EQUAL TO THE KERNEL



WITH THE ADVANTAGE THAT THE SAME WEIGHTS ARE APPLIED TO ALL THE SIGNAL: TRANSLATION INVARIANCE

2-D CONVOLUTION

SAME IDEA, BUT THE KERNEL IS NOW 2D

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

1.7	1.7	1.7
1.0	1.2	1.8
1.1	0.8	1.3

KERNEL

INPUT (IMAGE)

OUTPUT

Credit: animations from https://github.com/vdumoulin/conv_arithmetic

2-D CONVOLUTION

SAME IDEA, BUT THE KERNEL IS NOW 2D

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

1.7	1.7	1.7
1.0	1.2	1.8
1.1	0.8	1.3

IN THE EXAMPLE: EACH 3x3 REGION GENERATES AN OUTPUT

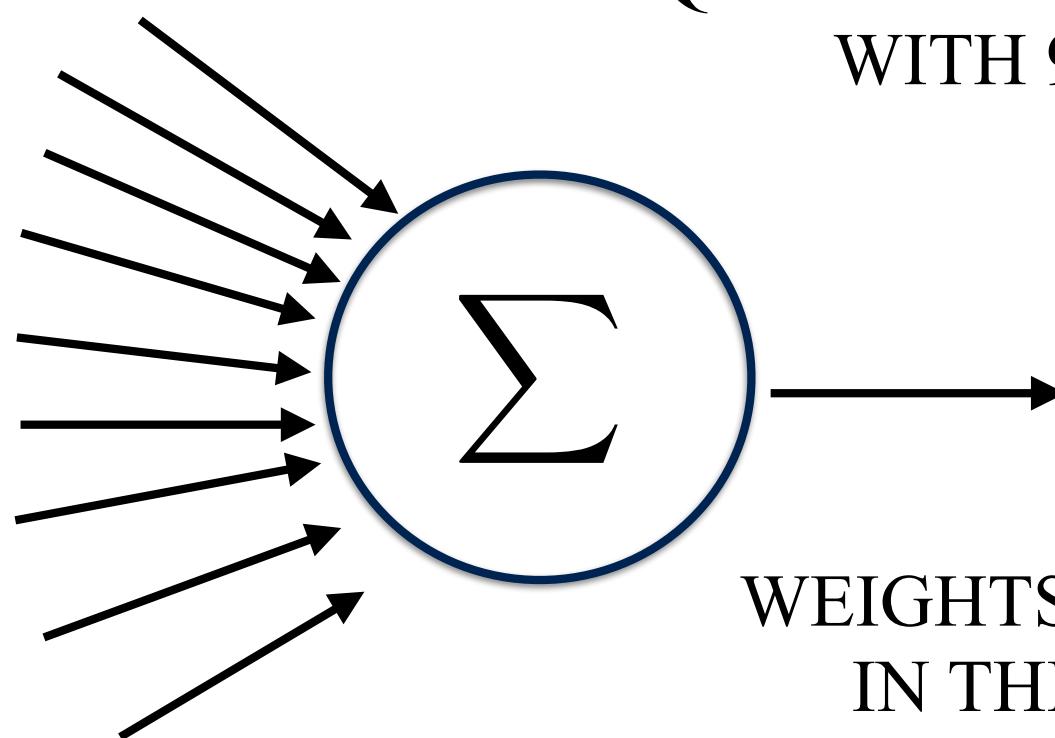
$$Size_{output} = Size_{input} - Size_{kernel} + 1$$

Credit: animations from https://github.com/vdumoulin/conv_arithmetic

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

1.7	1.7	1.7
1.0	1.2	1.8
1.1	0.8	1.3



EQUIVALENT TO A NEURON
WITH 9 INPUTS

WEIGHTS ARE CODED
IN THE KERNEL

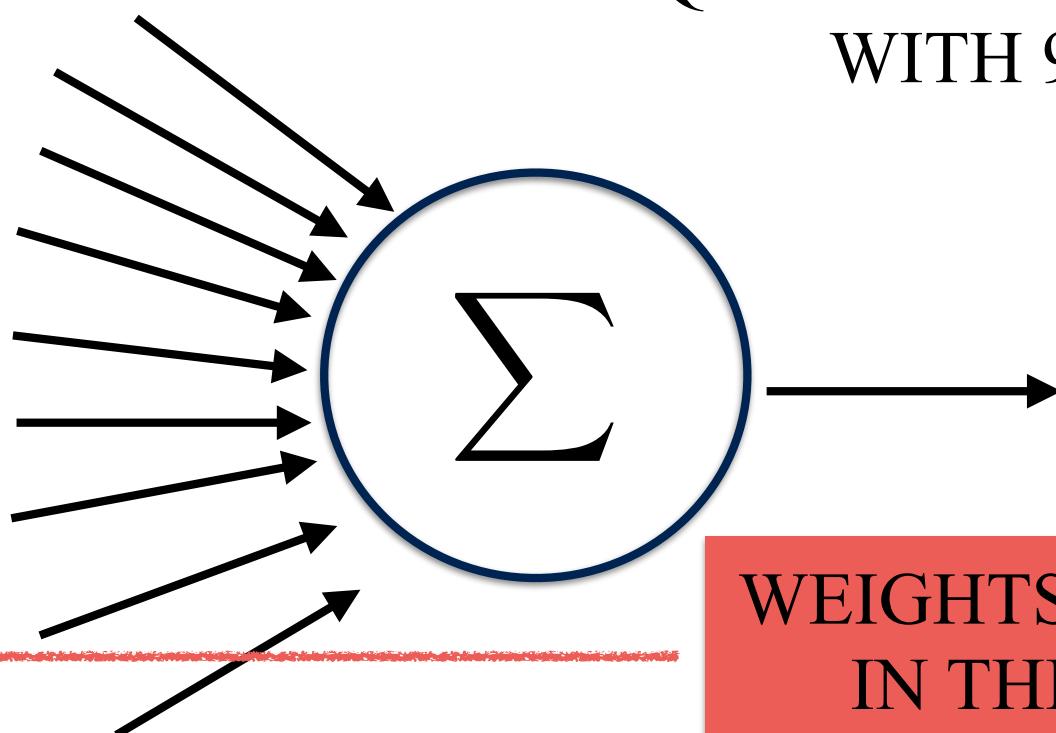
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

1.7	1.7	1.7
1.0	1.2	1.8
1.1	0.8	1.3



EQUIVALENT TO A NEURON
WITH 9 INPUTS



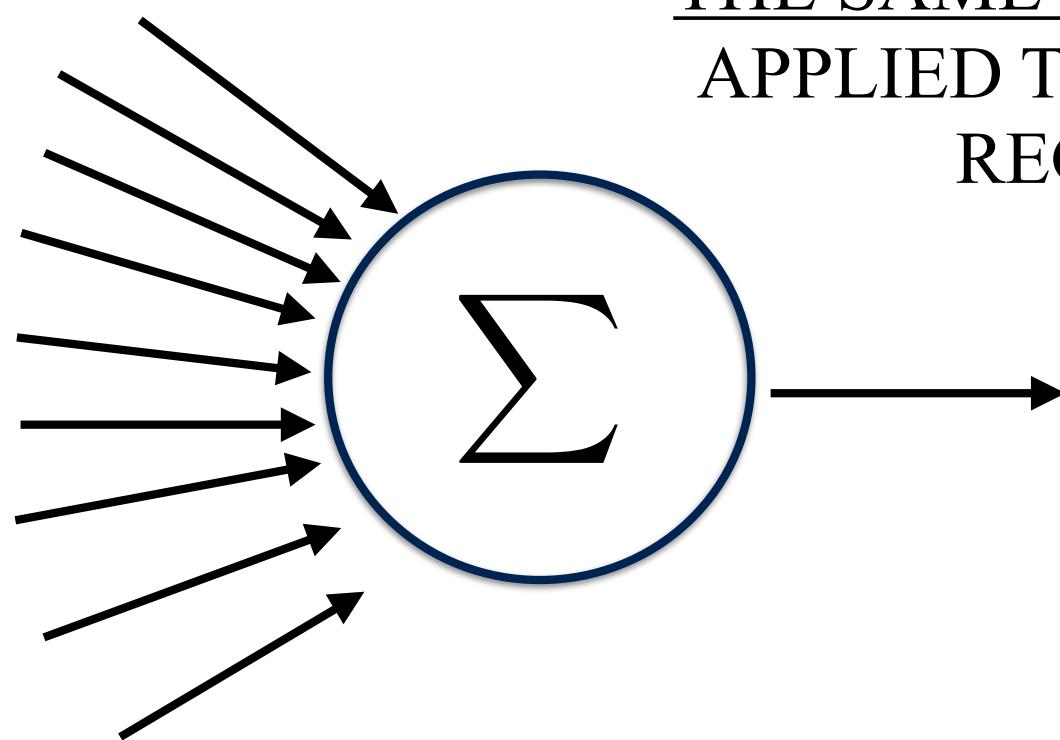
THIS IS WHAT
THE
NETWORK
LEARNS!

WEIGHTS ARE CODED
IN THE KERNEL

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

1.7	1.7	1.7
1.0	1.2	1.8
1.1	0.8	1.3



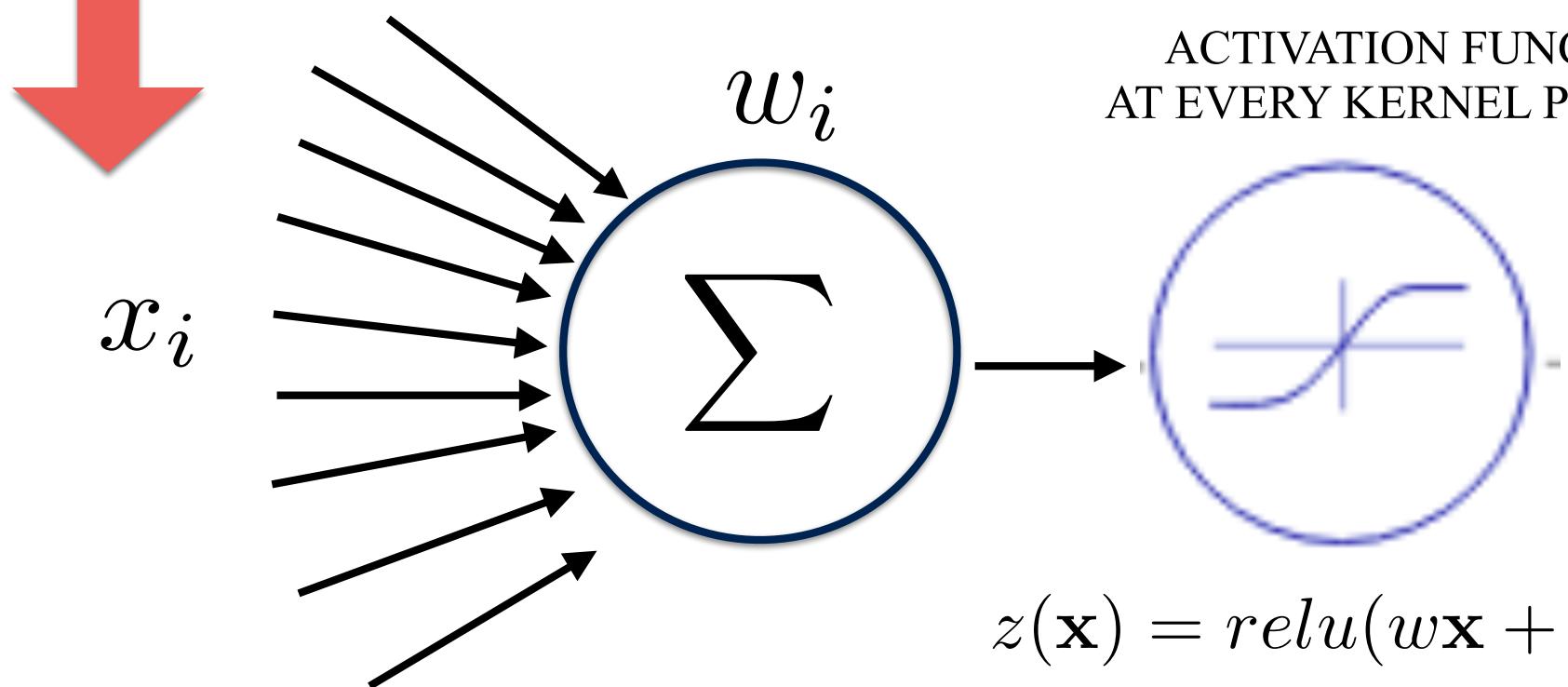
THE KEY IS AGAIN THAT
THE SAME WEIGHTS ARE
APPLIED TO ALL IMAGE
REGIONS

x_i

3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	1

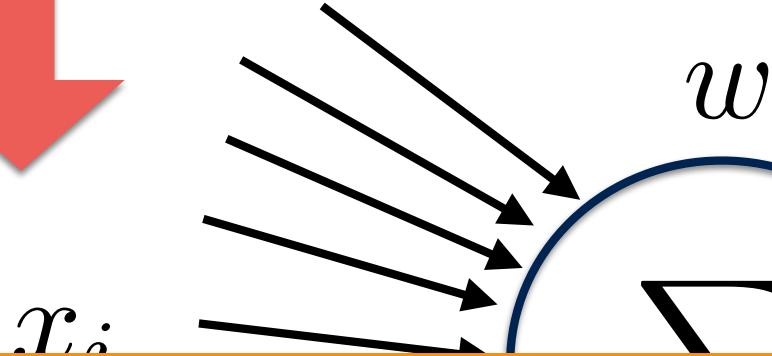
w_i
[weights]

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

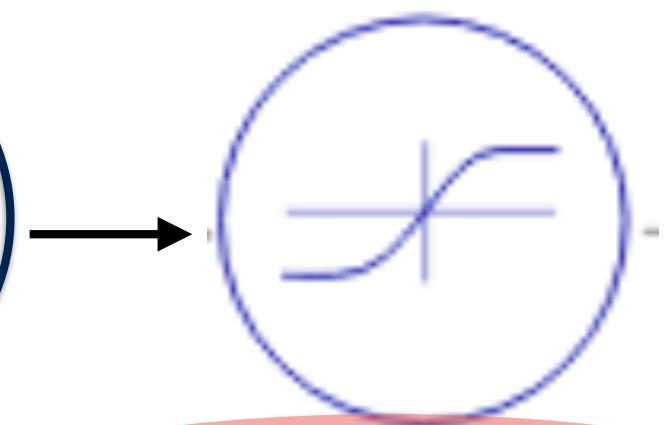


x_i		

w_i	
[weights]	



ACTIVATION FUNCTION
AT EVERY KERNEL POSITION

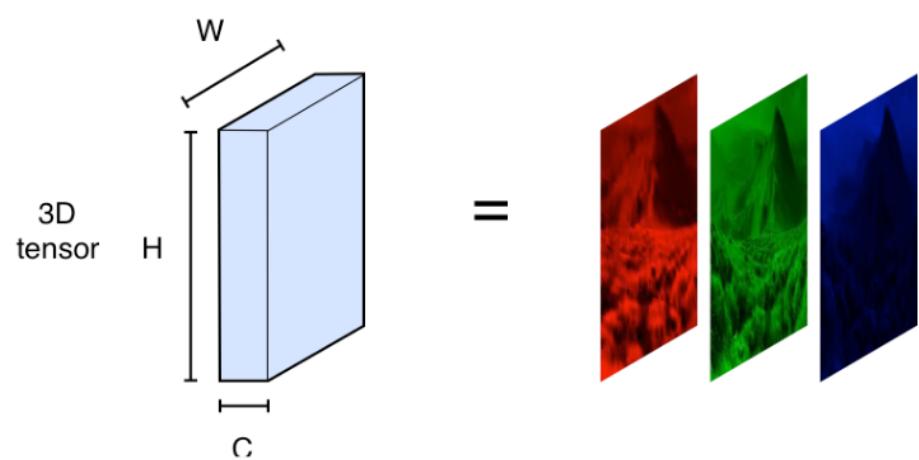
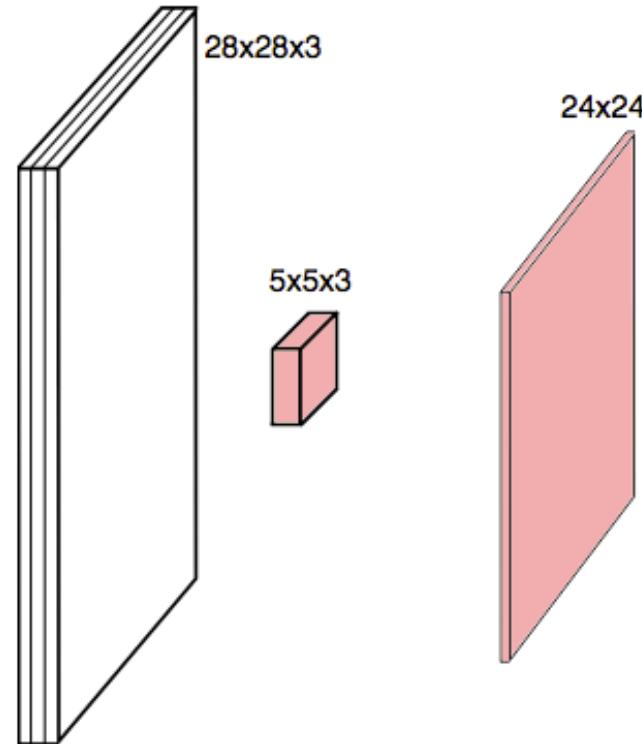


IN DEEP NETWORKS ReLU is the most commonly used activation function - see Vanishing Gradient Problem

$$z(\mathbf{x}) = \text{relu}(\mathbf{w}\mathbf{x} + b)$$

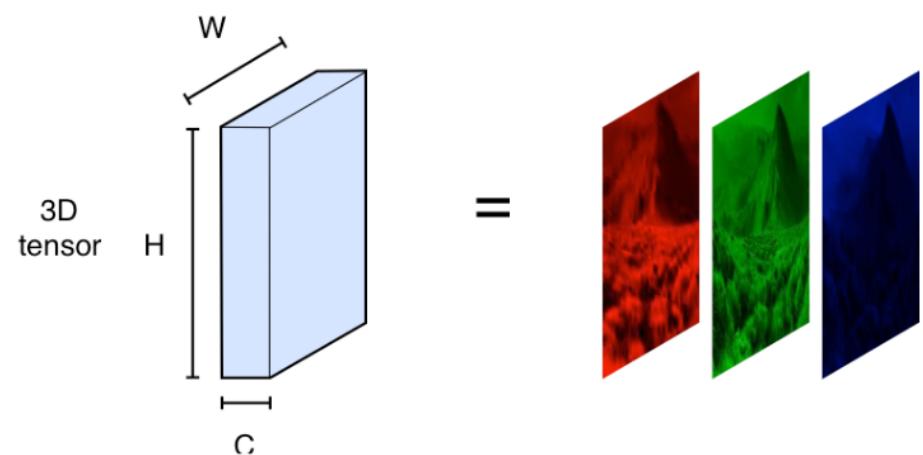
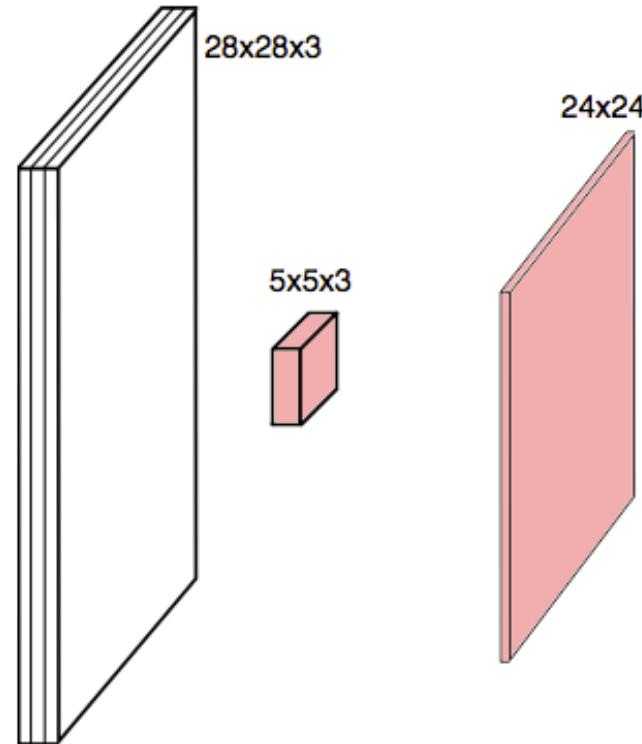
CONVOLUTIONS CAN ALSO BE COMPUTED ACROSS CHANNELS (OR COLORS)

A COLOR IMAGE IS A
TENSOR
OF SIZE height x width x
channels



CONVOLUTIONS CAN ALSO BE COMPUTED ACROSS CHANNELS (OR COLORS)

A COLOR IMAGE IS A
TENSOR
OF SIZE height x width x
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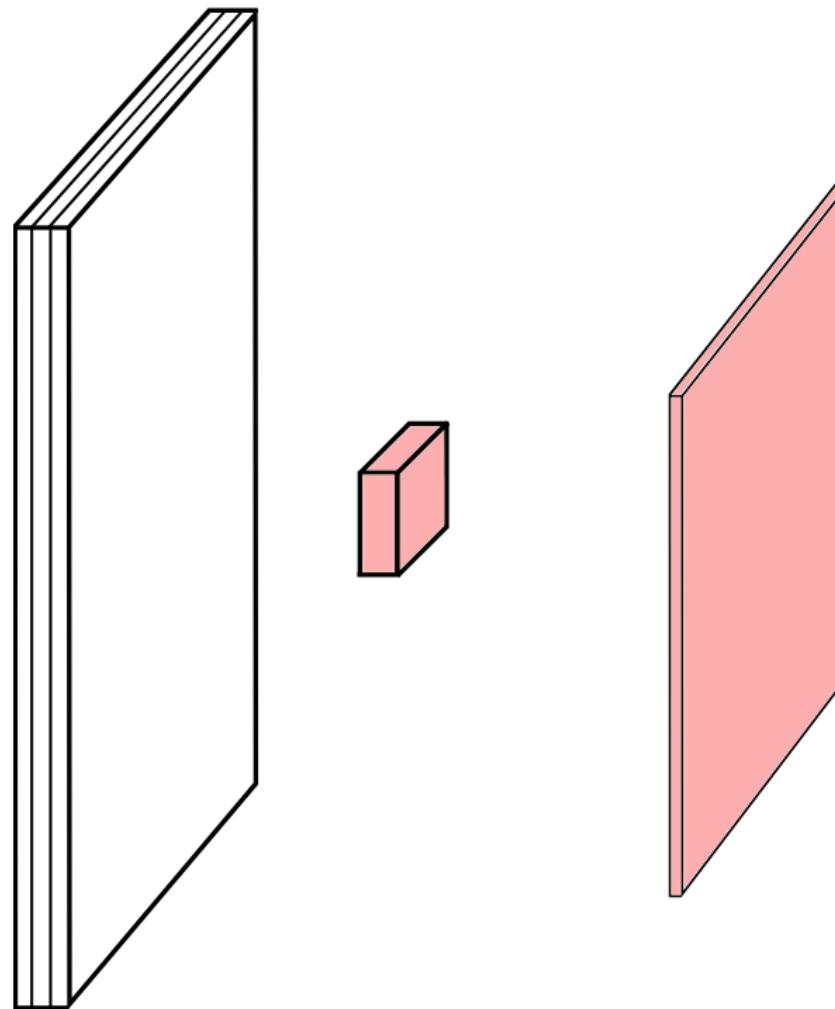


THEN THE KERNEL
HAS ALSO 3
CHANNELS

IN ASTRONOMY ...

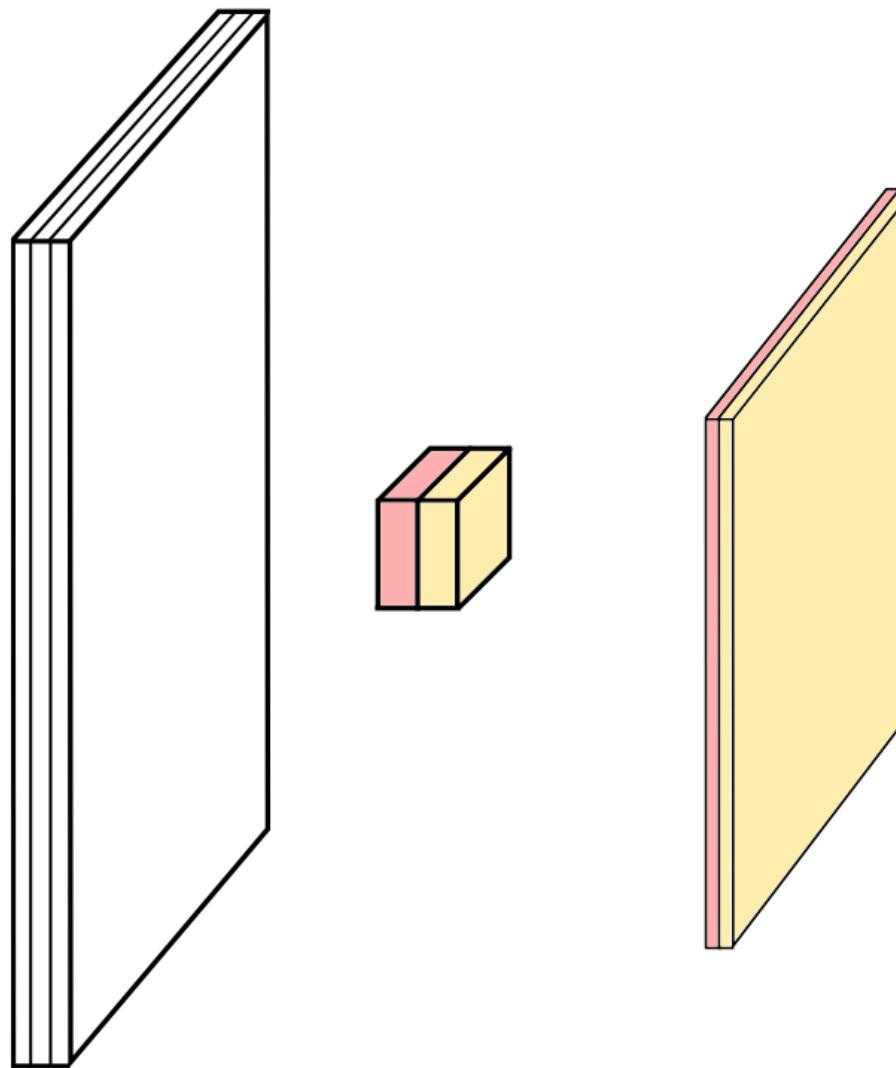
IT OPENS THE DOOR TO ANALYZE MULTIPLE
FILTERS SIMULTANEOUSLY

MULTIPLE CONVOLUTIONS WITH DIFFERENT KERNELS
CAN BE PERFORMED



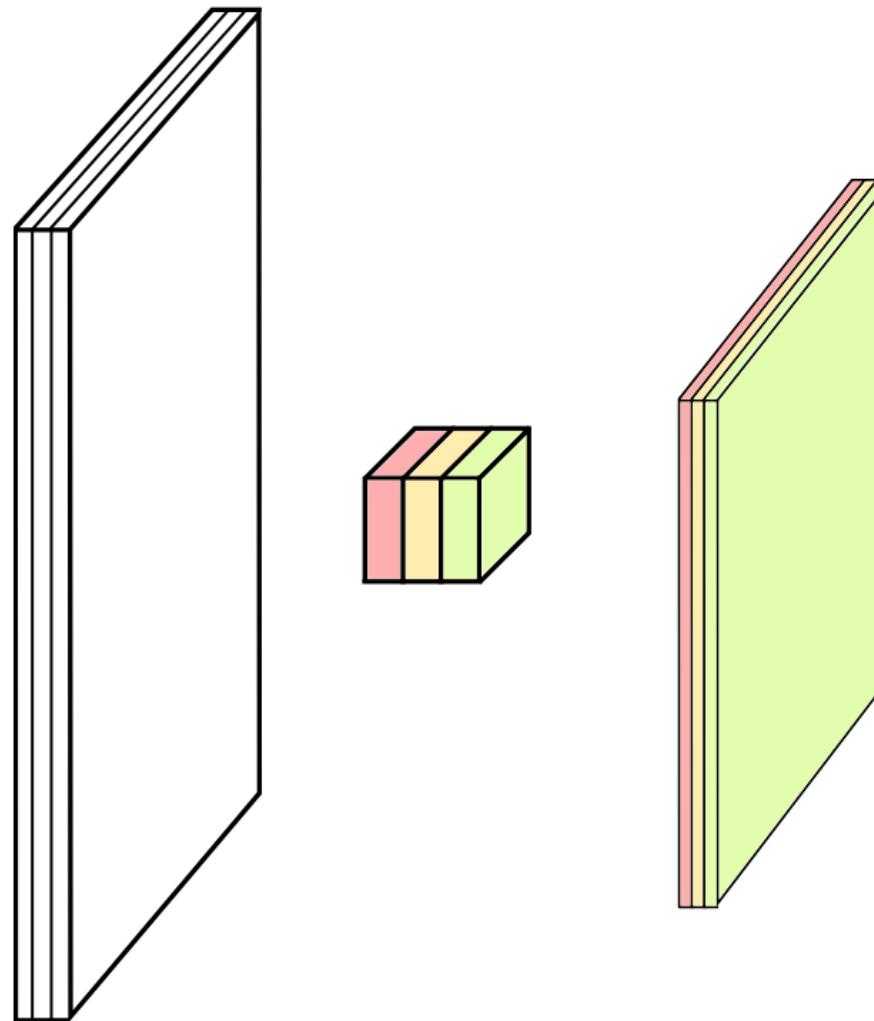
credit

MULTIPLE CONVOLUTIONS WITH DIFFERENT KERNELS CAN BE PERFORMED



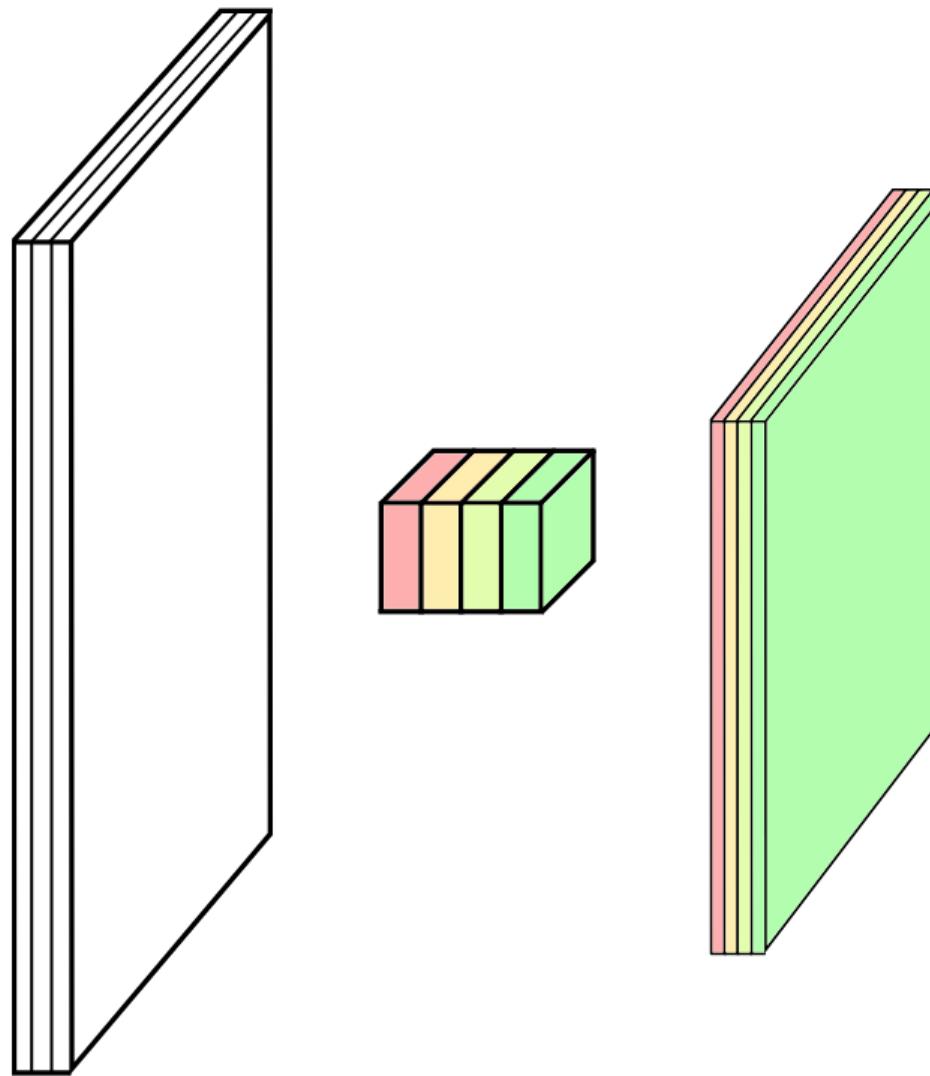
credit

MULTIPLE CONVOLUTIONS WITH DIFFERENT KERNELS CAN BE PERFORMED



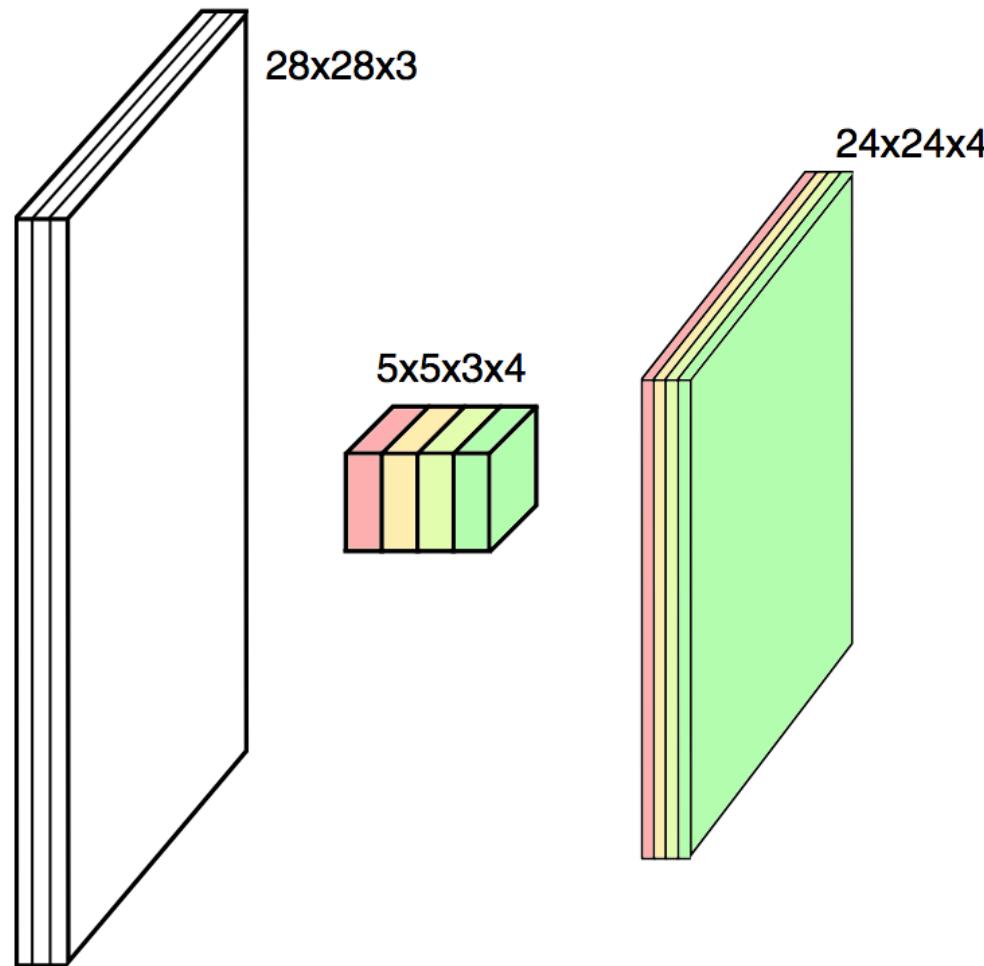
credit

MULTIPLE CONVOLUTIONS WITH DIFFERENT KERNELS CAN BE PERFORMED



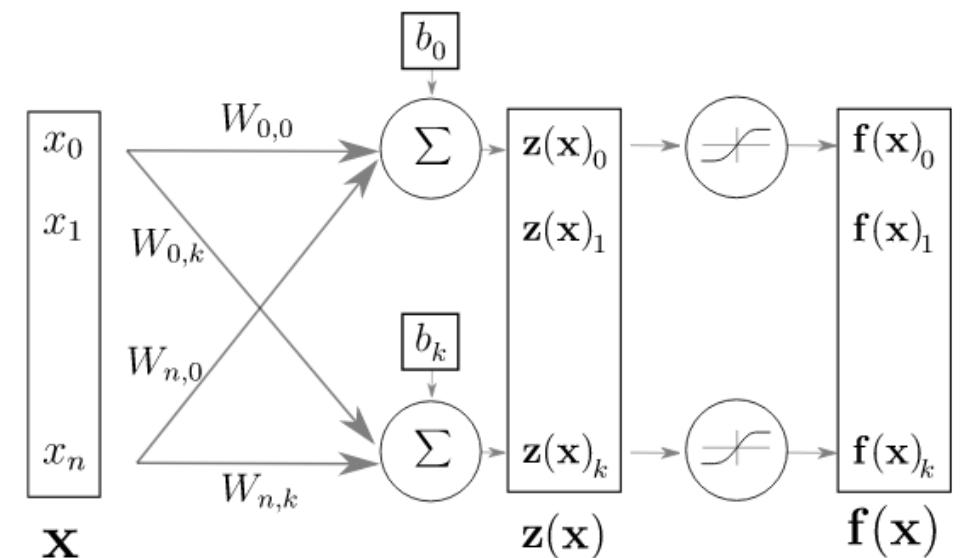
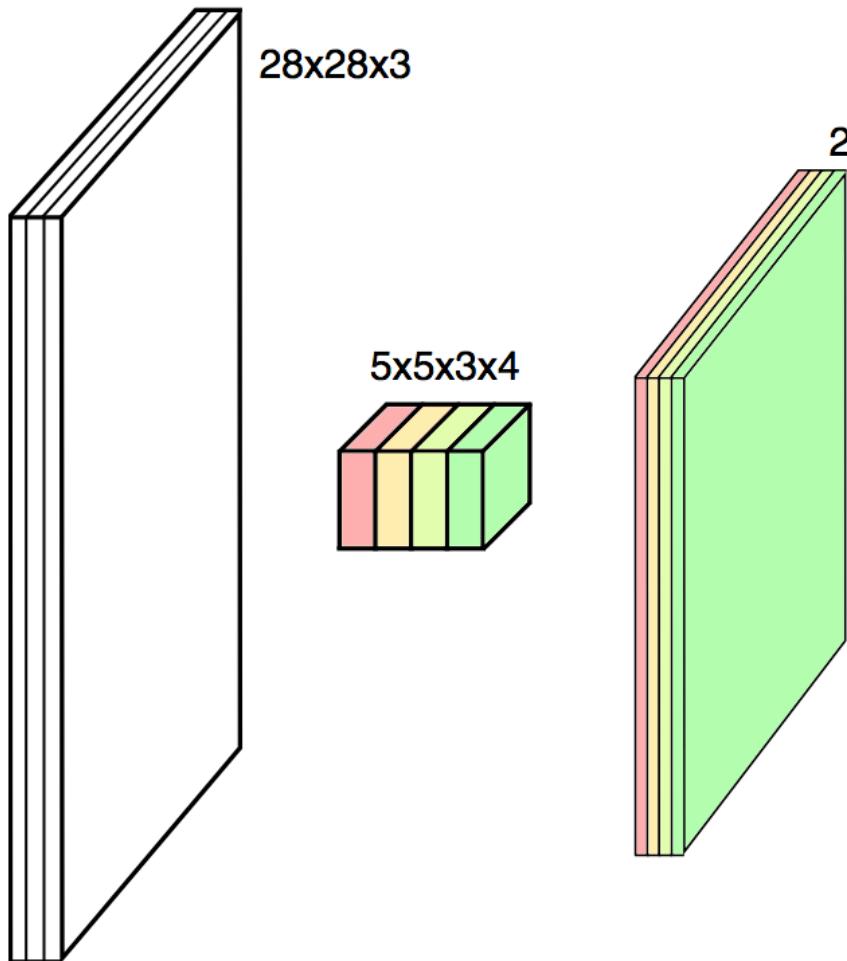
credit

MULTIPLE CONVOLUTIONS WITH DIFFERENT KERNELS CAN BE PERFORMED



credit

MULTIPLE CONVOLUTIONS WITH DIFFERENT KERNELS CAN BE PERFORMED

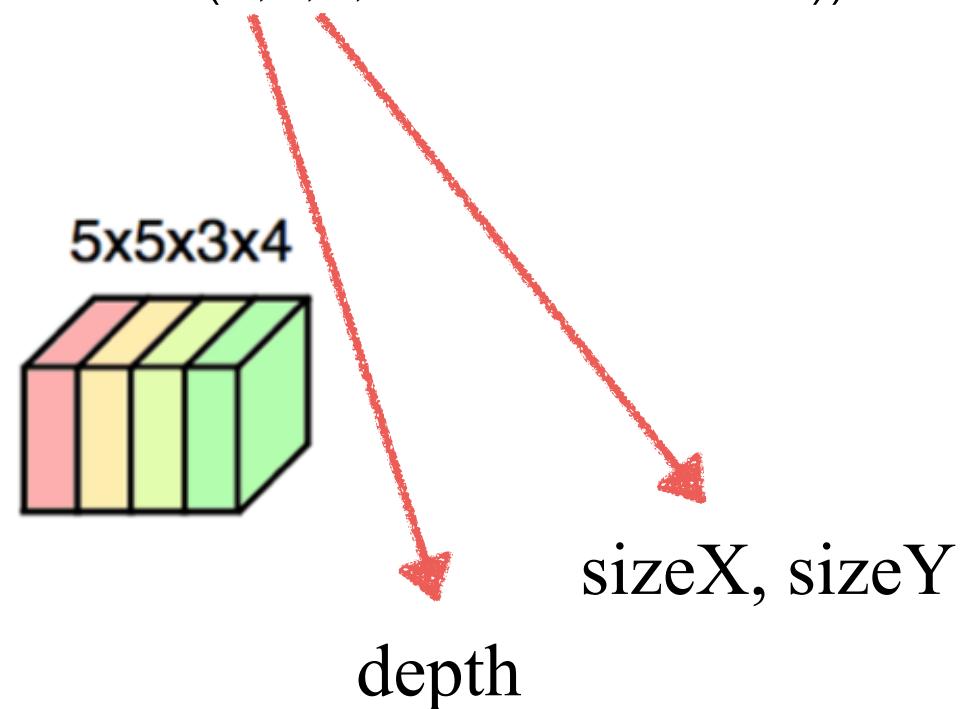


credit

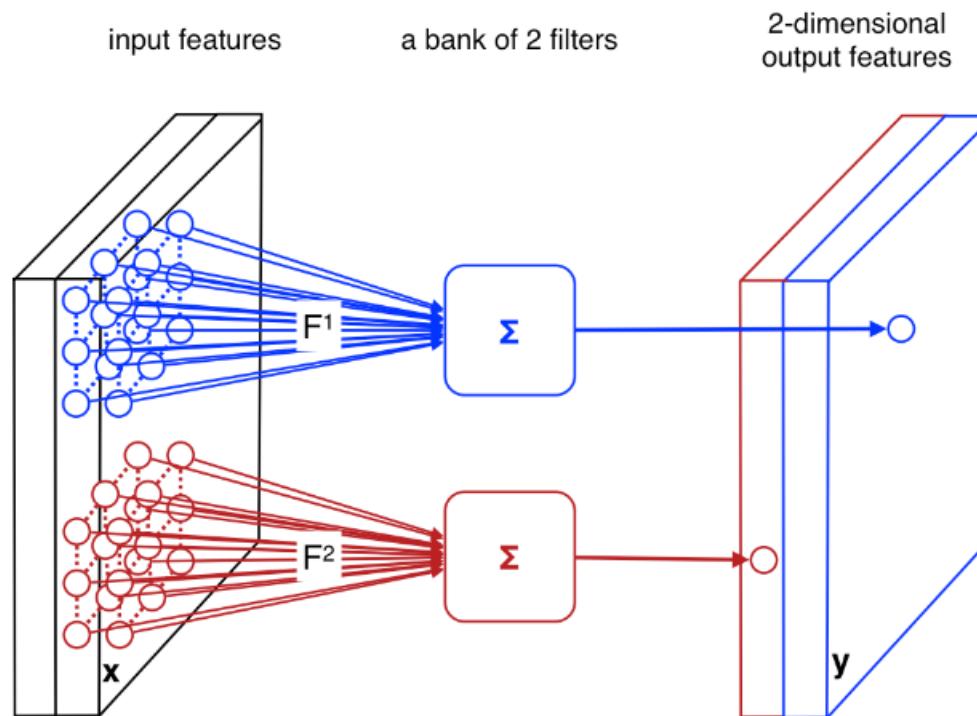
IN KERAS...

```
model = Sequential()
```

```
model.add(Convolution2D(4,5,5, activation="relu"))
```



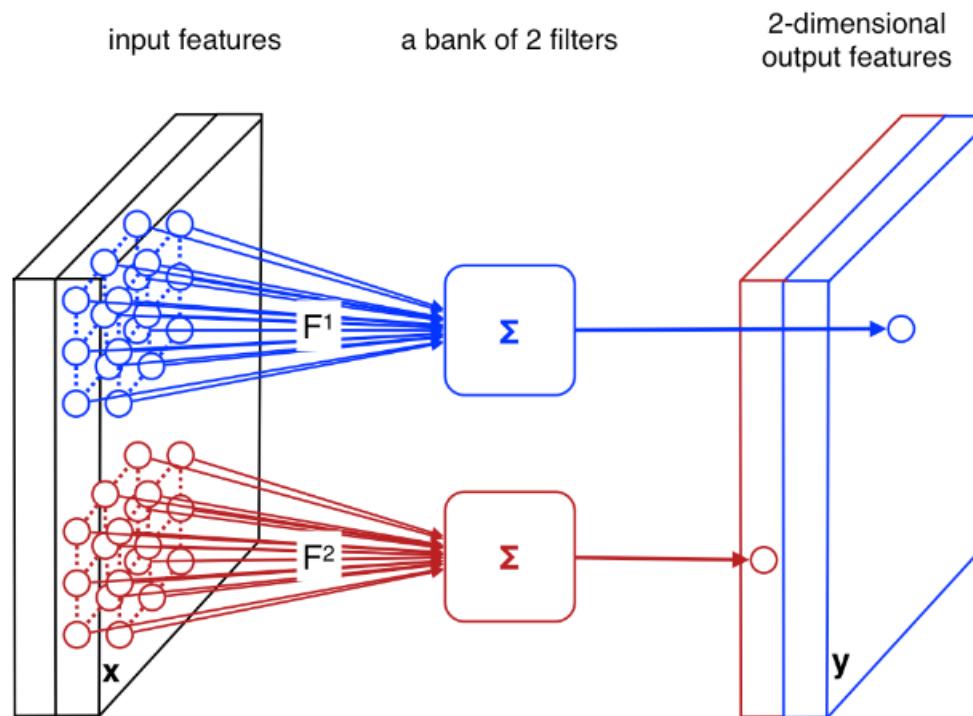
SINCE CONVOLUTIONS OUTPUT ONE SCALAR, THEY CAN BE SEEN AS AN INDIVIDUAL NEURON WITH A RECEPTIVE FIELD LIMITED TO THE KERNEL DIMENSIONS



Credit

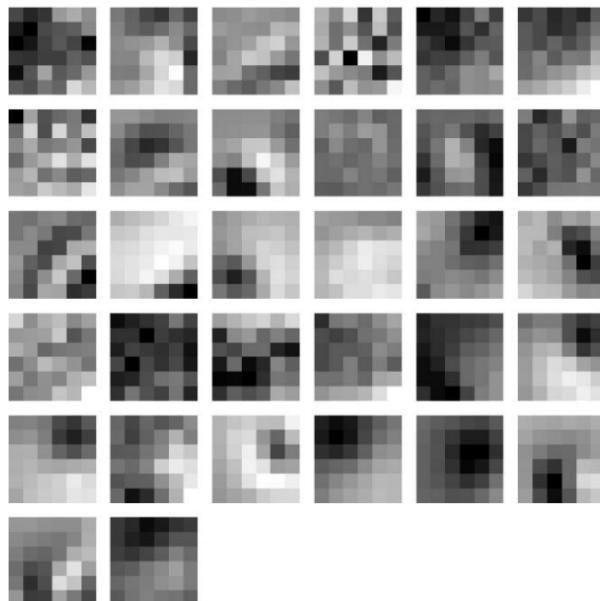
SINCE CONVOLUTIONS OUTPUT ONE SCALAR< THEY CAN BE SEEN AS AN INDIVIDUAL NEURON WITH A RECEPTIVE FIELD LIMITED TO THE KERNEL DIMENSIONS

THE SAME NEURON IS FIRED WITH DIFFERENT AREAS FROM THE INPUT

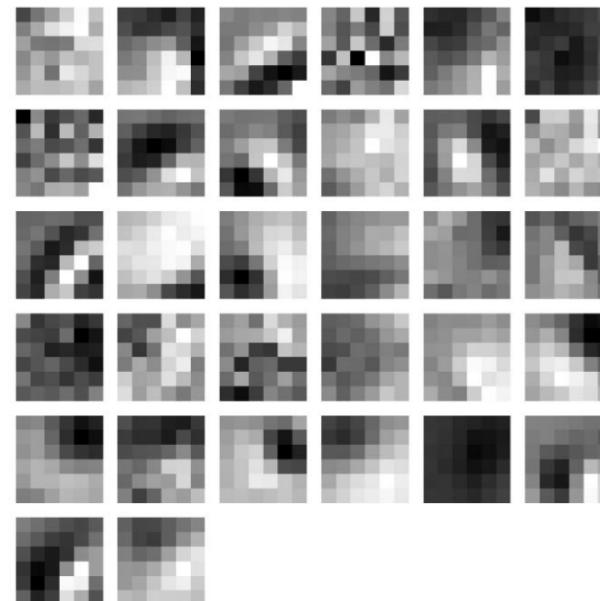


Credit

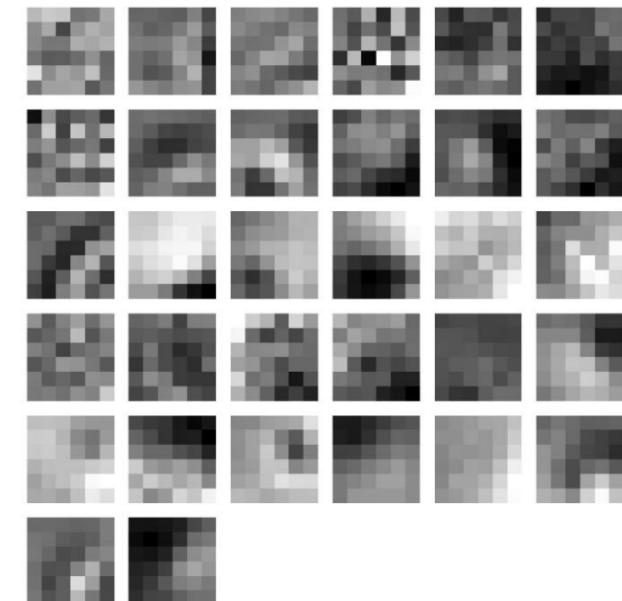
EXAMPLE OF 32 FILTERS LEARNED IN A CONVOLUTIONAL LAYER



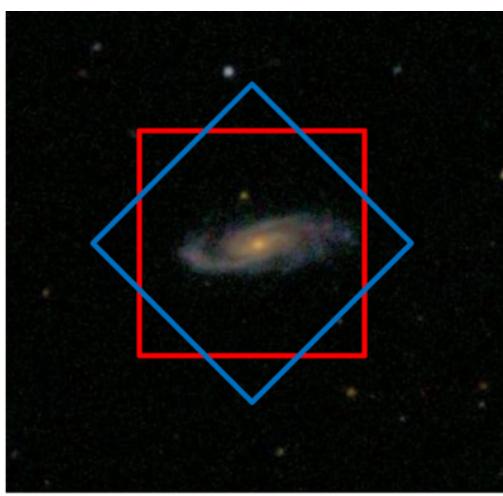
(a) red channel



(b) green channel

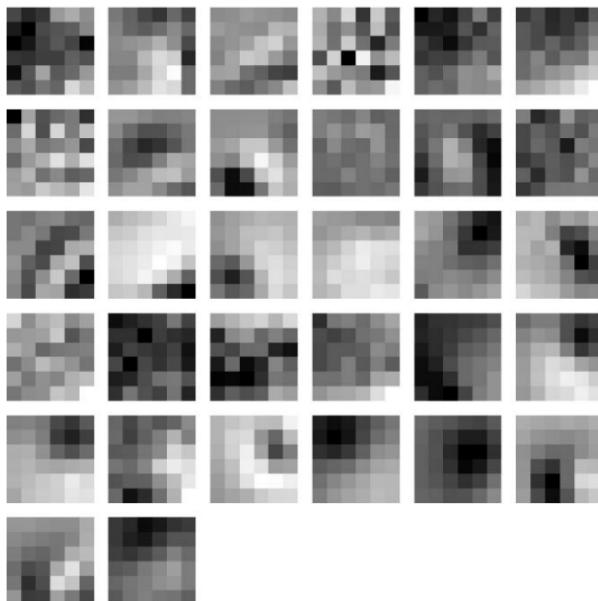


(c) blue channel

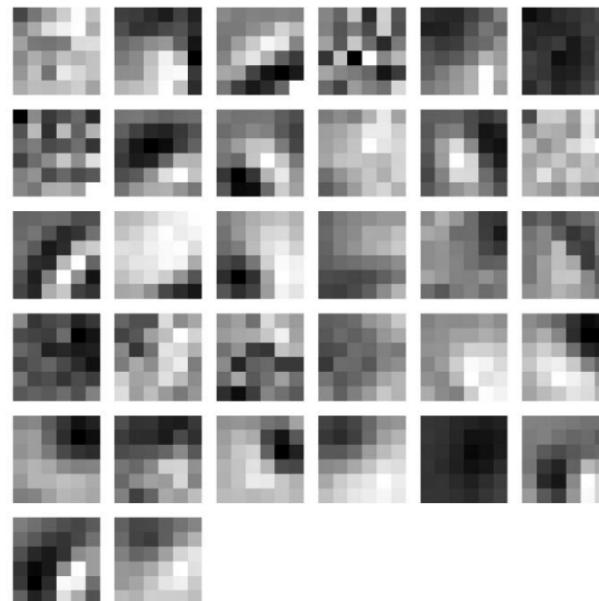


Dieleman+16

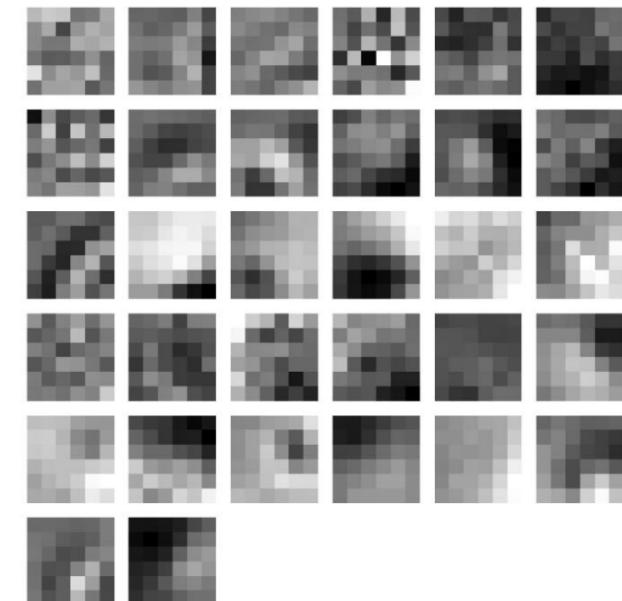
EXAMPLE OF 32 FILTERS LEARNED IN A CONVOLUTIONAL LAYER



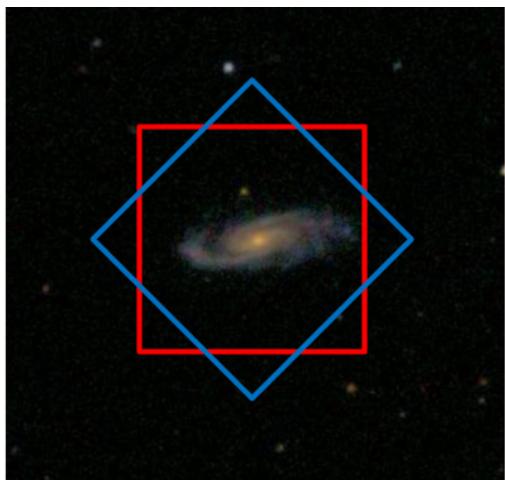
(a) red channel



(b) green channel



(c) blue channel



Dieleman+16

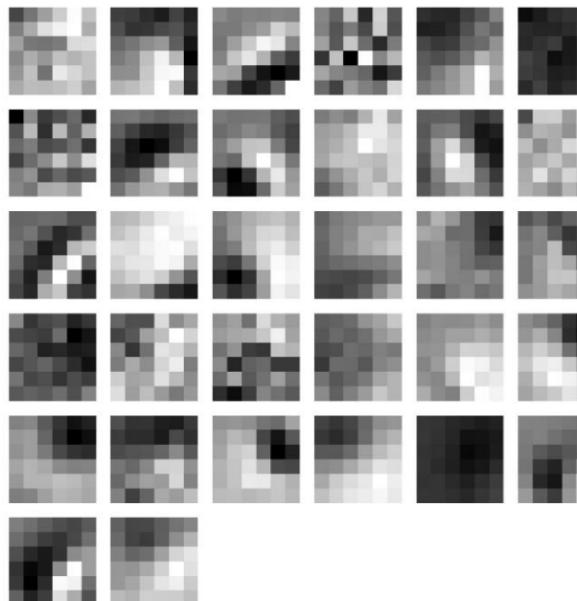
THESE ARE CALLED **FEATURE MAPS**

EXAMPLE OF 32 FILTERS APPLIED BY A CONVOLUTIONAL

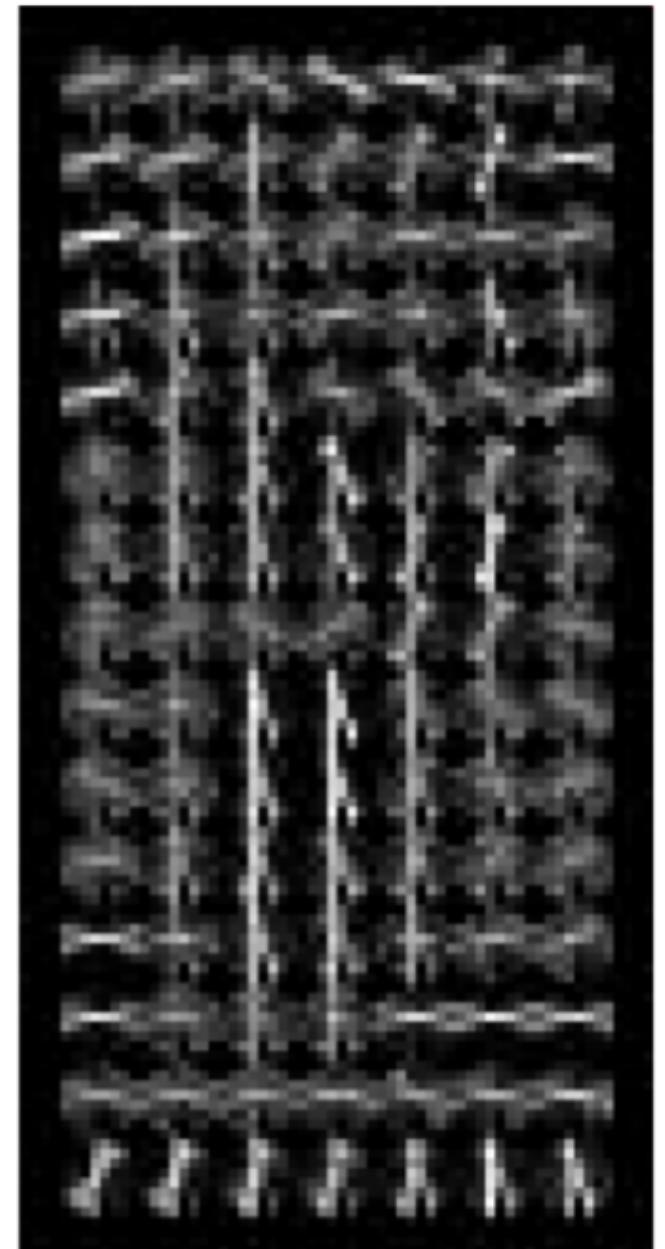
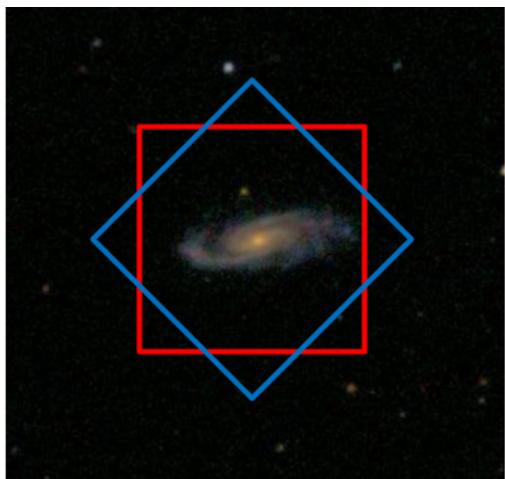


**REMEMBER
THE HoG ?**

(a) red channel



(b) green channel

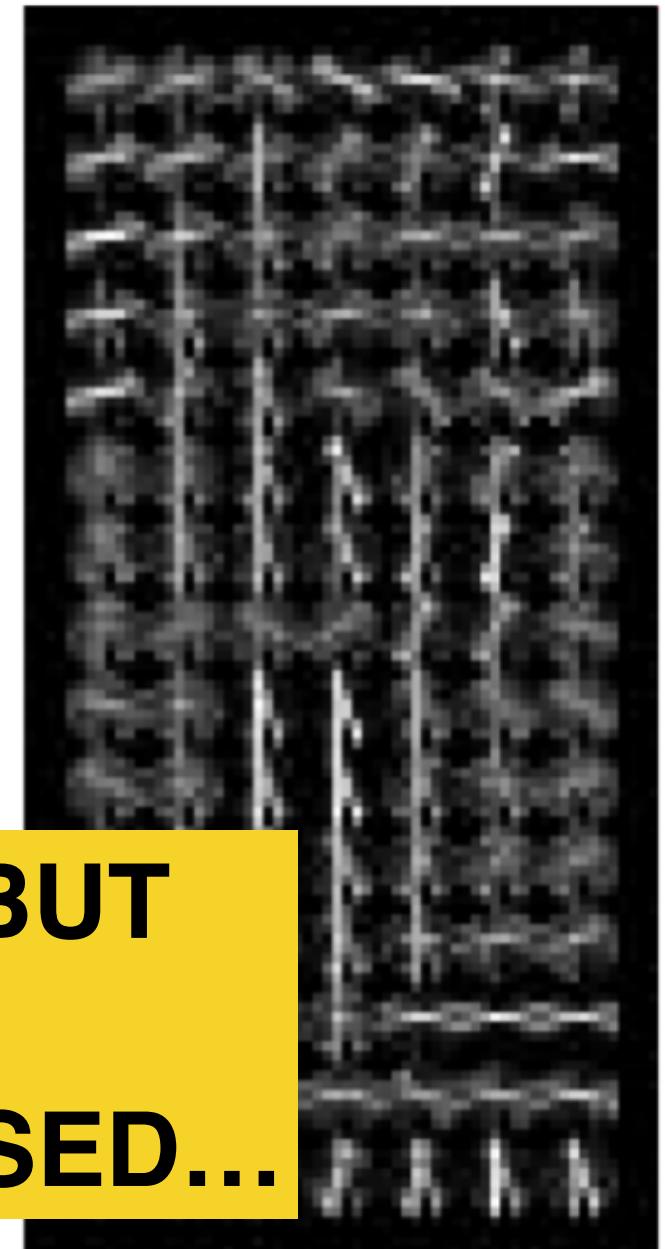
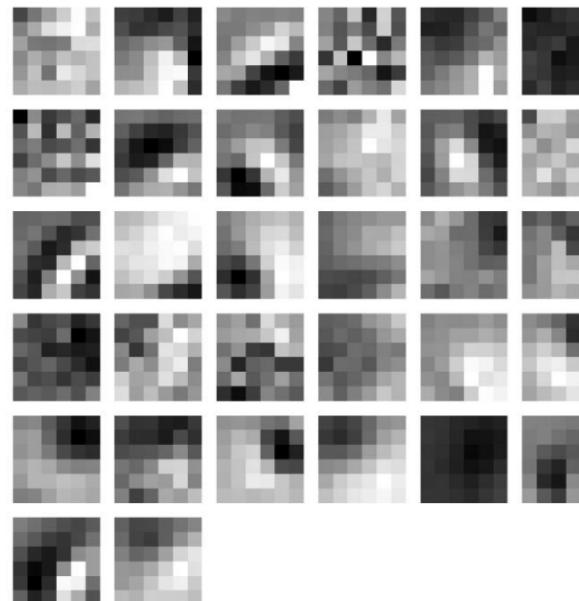


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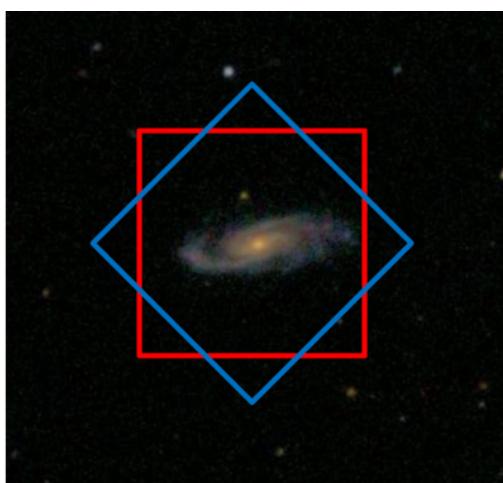
EXAMPLE OF 32 FILTERS APPLIED BY A CONVOLUTIONAL



**REMEMBER
THE HoG ?**



**SIMILAR, BUT
FULLY
UNSUPERVISED...**



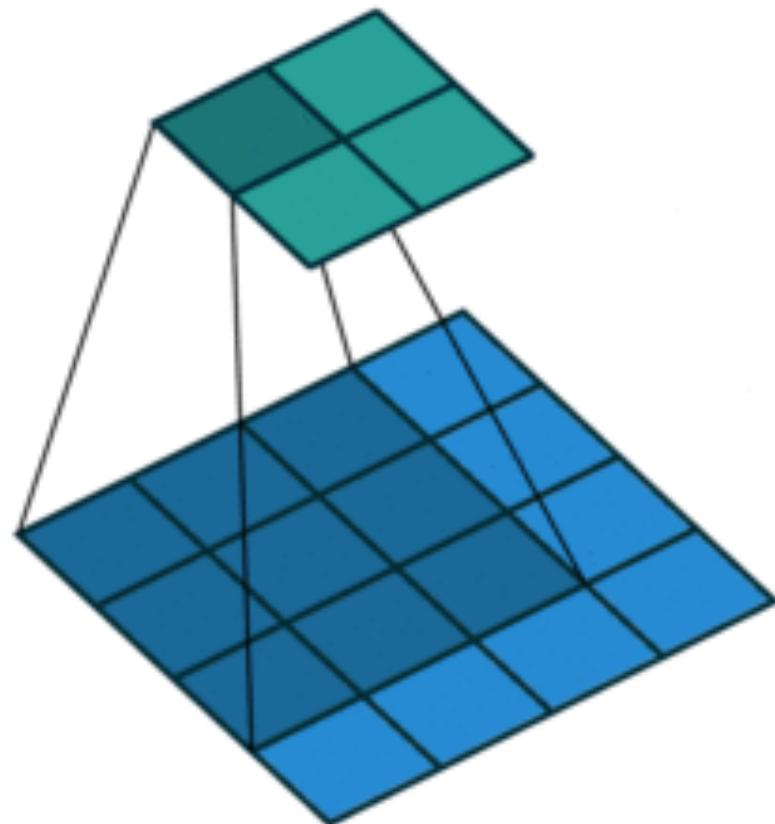
THESE ARE CALLED **FEATURE MAPS**

ESTIMATING SHAPES AND NUMBER OF PARAMETERS

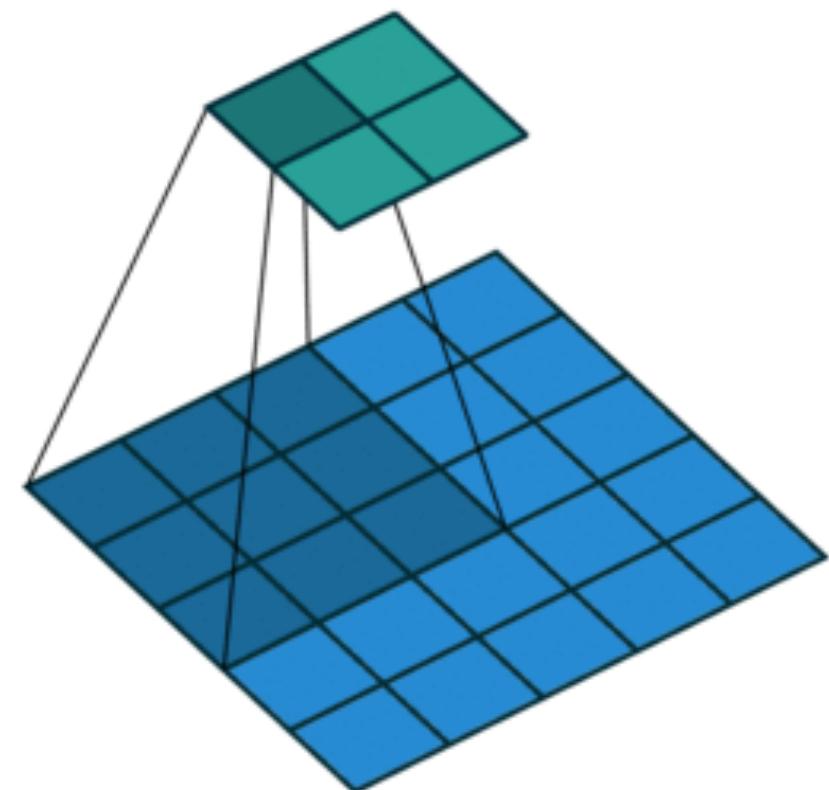
<u>KERNEL SHAPE:</u>	<u>PADDING:</u>	<u>STRIDES:</u>
(F, F, C^i, C^o)	P	S

OUTPUT SIZE: $W_0 = (W^i - F + 2P)/S + 1$

OPTIONS: STRIDES

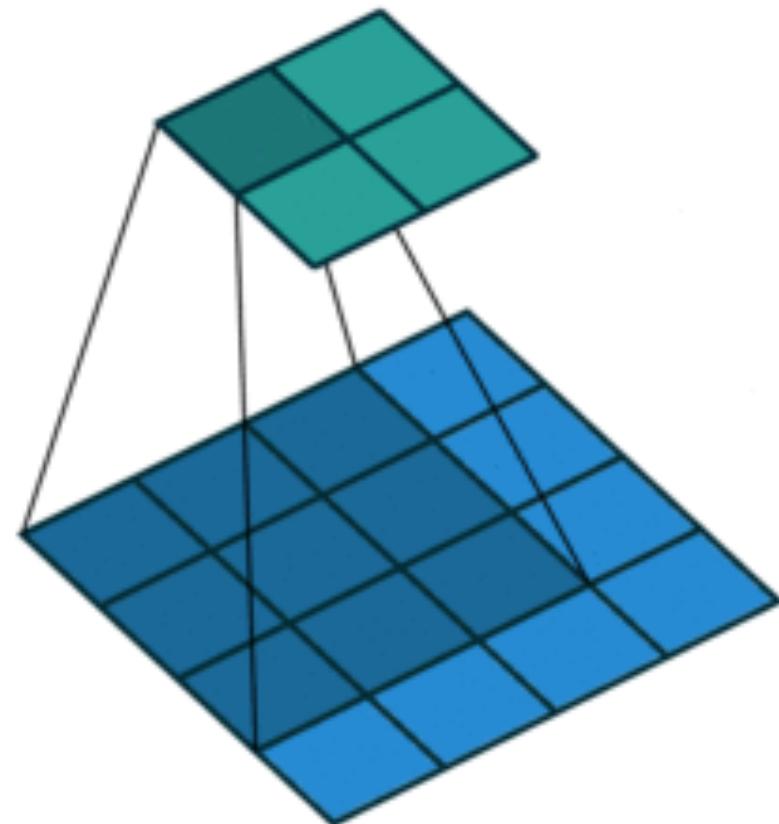


NO STRIDES

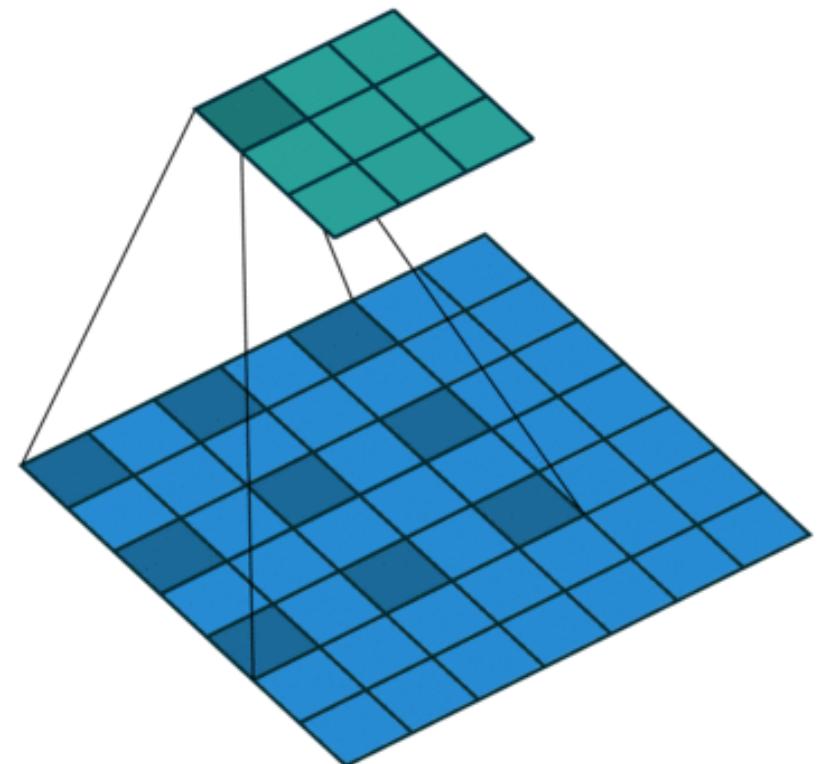


STRIDES

OPTIONS: DILATION

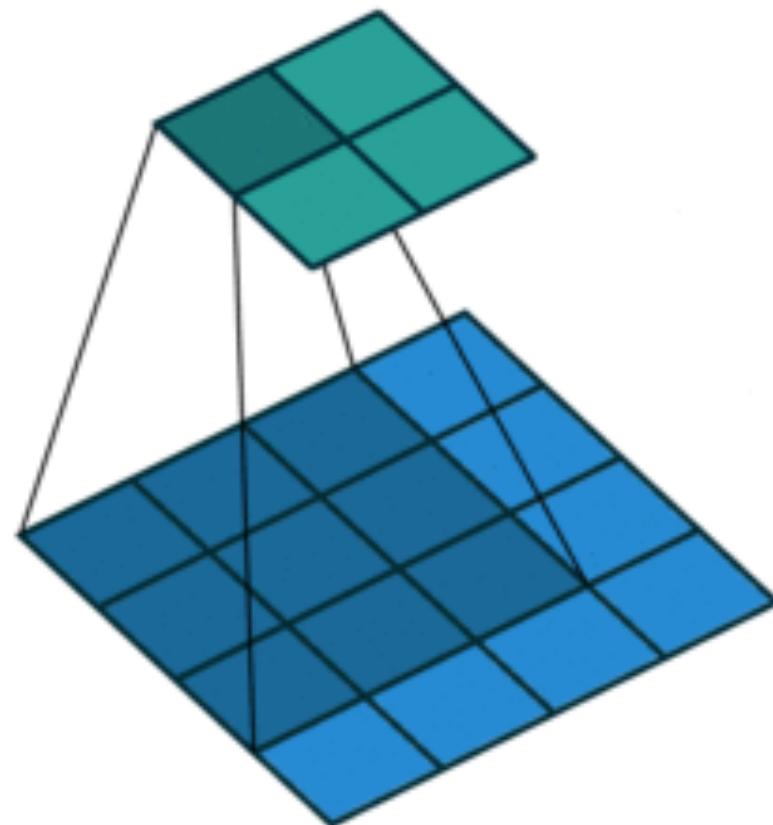


NO STRIDES

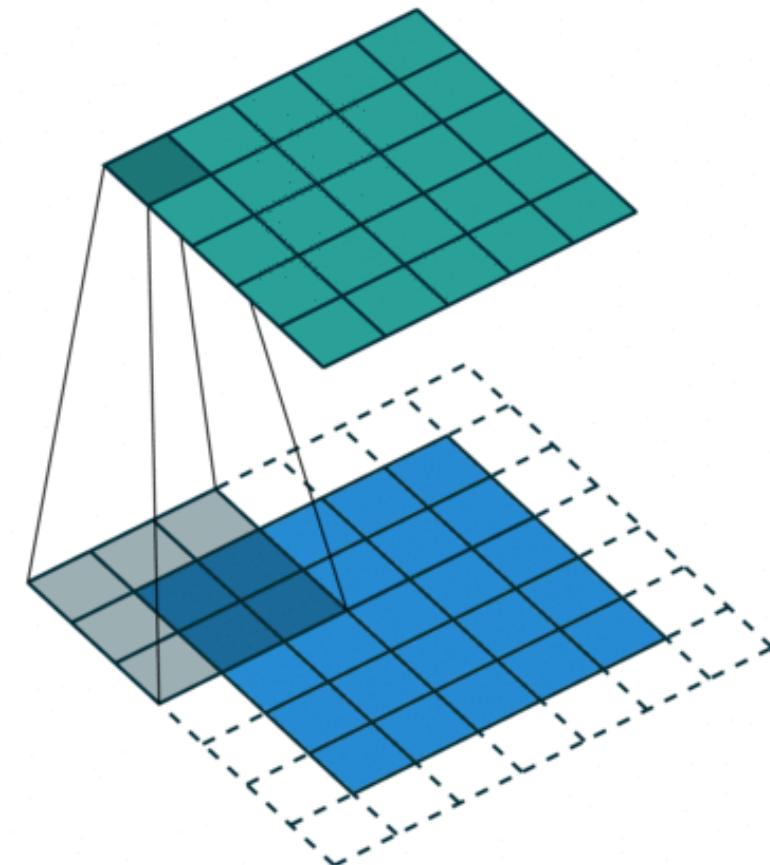


DILATION

OPTIONS: PADDING



NO STRIDES



PADDING

ESTIMATING SHAPES AND NUMBER OF PARAMETERS

<u>KERNEL SHAPE:</u>	<u>PADDING:</u>	<u>STRIDES:</u>
(F, F, C^i, C^o)	P	S

OUTPUT SIZE: $W_0 = (W^i - F + 2P)/S + 1$

NUMBER OF PARAMETERS: $(F \times F \times C^i + 1) \times C^o$

ESTIMATING SHAPES AND NUMBER OF PARAMETERS

KERNEL SHAPE:

$$(F, F, C^i, C^o)$$

PADDING:

$$P$$

STRIDES:

$$S$$

OUTPUT SIZE:

$$W_0 = (W^i - F + 2P)/S + 1$$

NUMBER OF PARAMETERS:

$$(F \times F \times C^i + 1) \times C^o$$

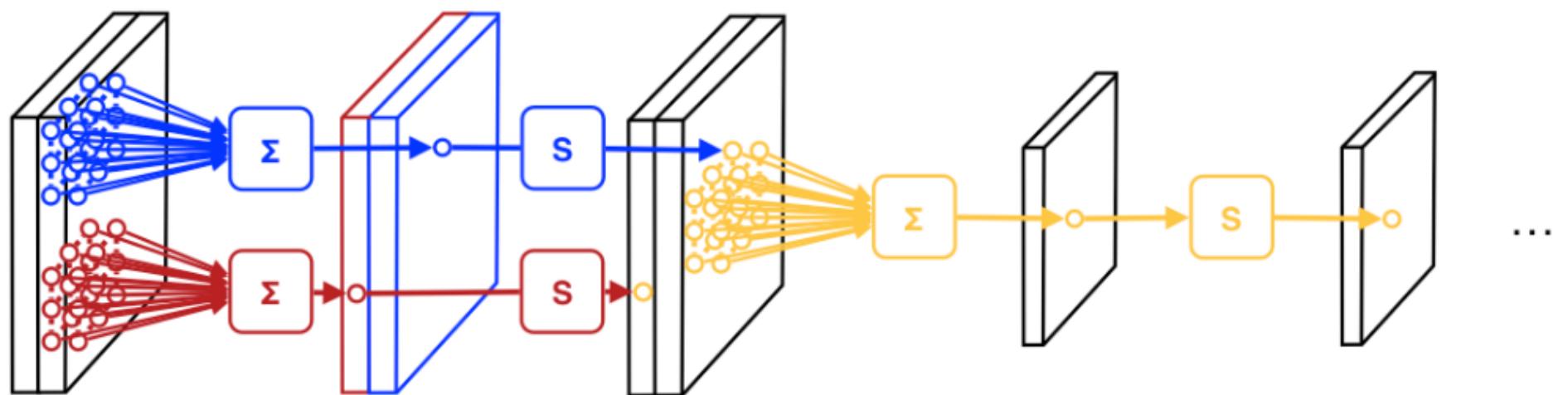


the number of parameters increases fast!

32 filters of 5*5 on a color image —> 2432 parameters to learn

DOWNSAMPLING

DOWNSAMPLING IS APPLIED TO REDUCE THE OVERALL SIZE OF TENSORS



POOLING

CONVOLUTIONS ARE OFTEN FOLLOWED BY AN OPERATION OF DOWNSAMPLING [POOLING]

VERY SIMPLE OPERATION - ONLY ONE OUT OF EVERY N PIXELS ARE KEPT

OFTEN MATCHED WITH AN INCREASE OF THE FEATURE CHANNELS

TYPES OF POOLING

SUM POOLING

$$y = \sum x_{uv}$$

SQUARE SUM POOLING

$$y = \sqrt{\sum x_{uv}^2}$$

MAX POOLING

$$y = \max(x_{uv})$$

TYPES OF POOLING

SUM POOLING

$$y = \sum x_{uv}$$

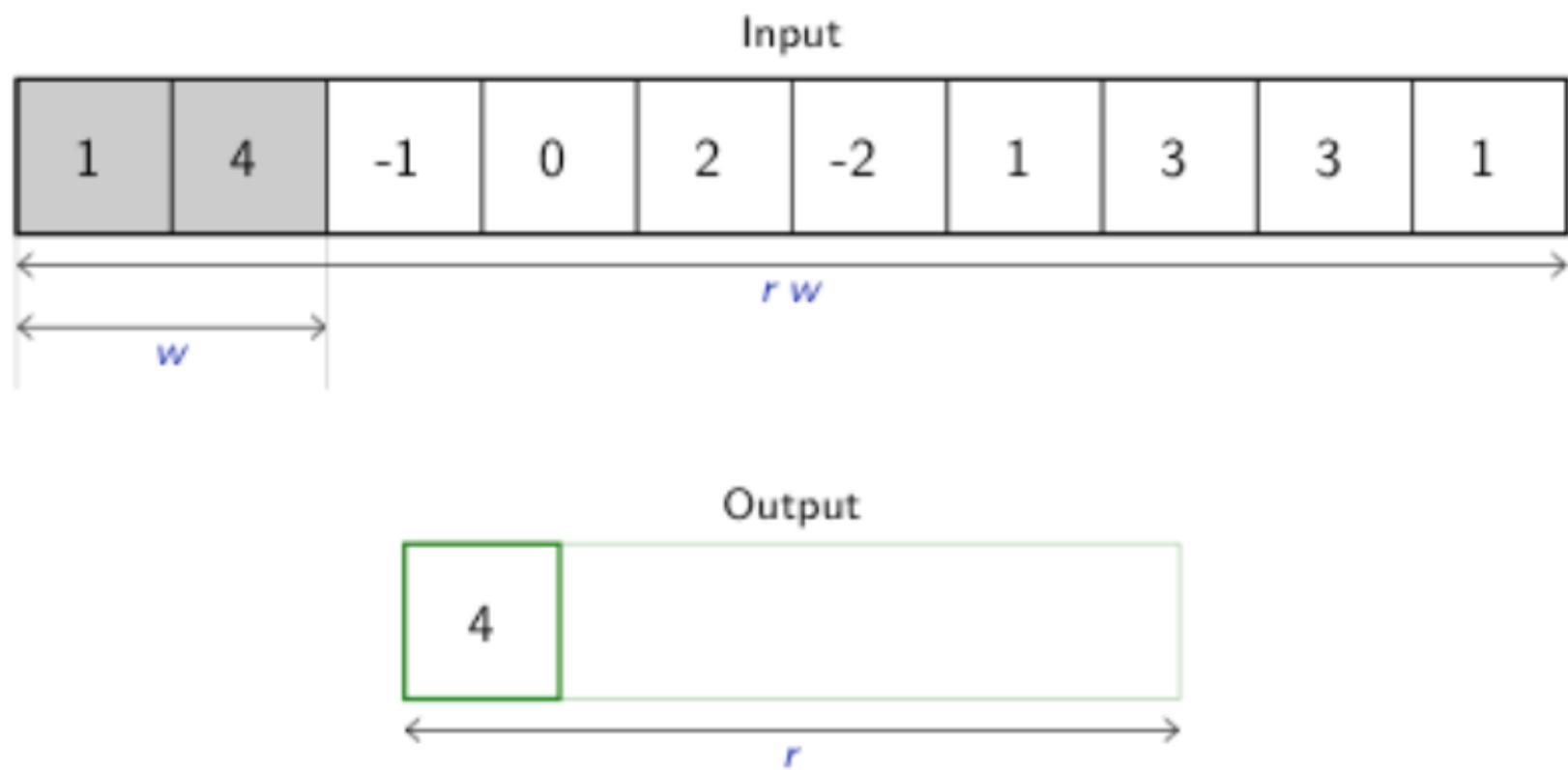
SQUARE SUM POOLING

$$y = \sqrt{\sum x_{uv}^2}$$

MAX POOLING

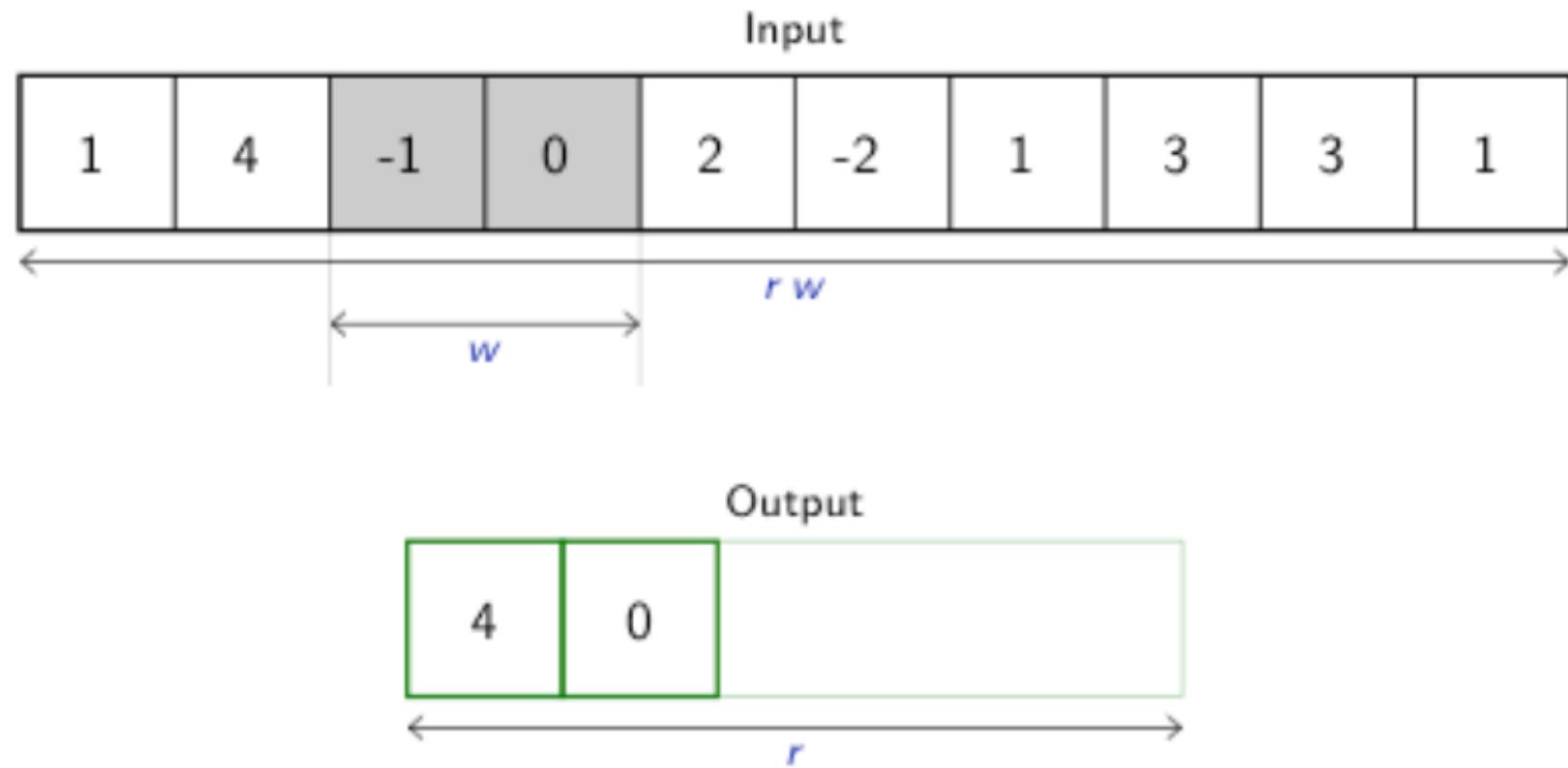
$$y = \max(x_{uv})$$

MAX POOLING 1D



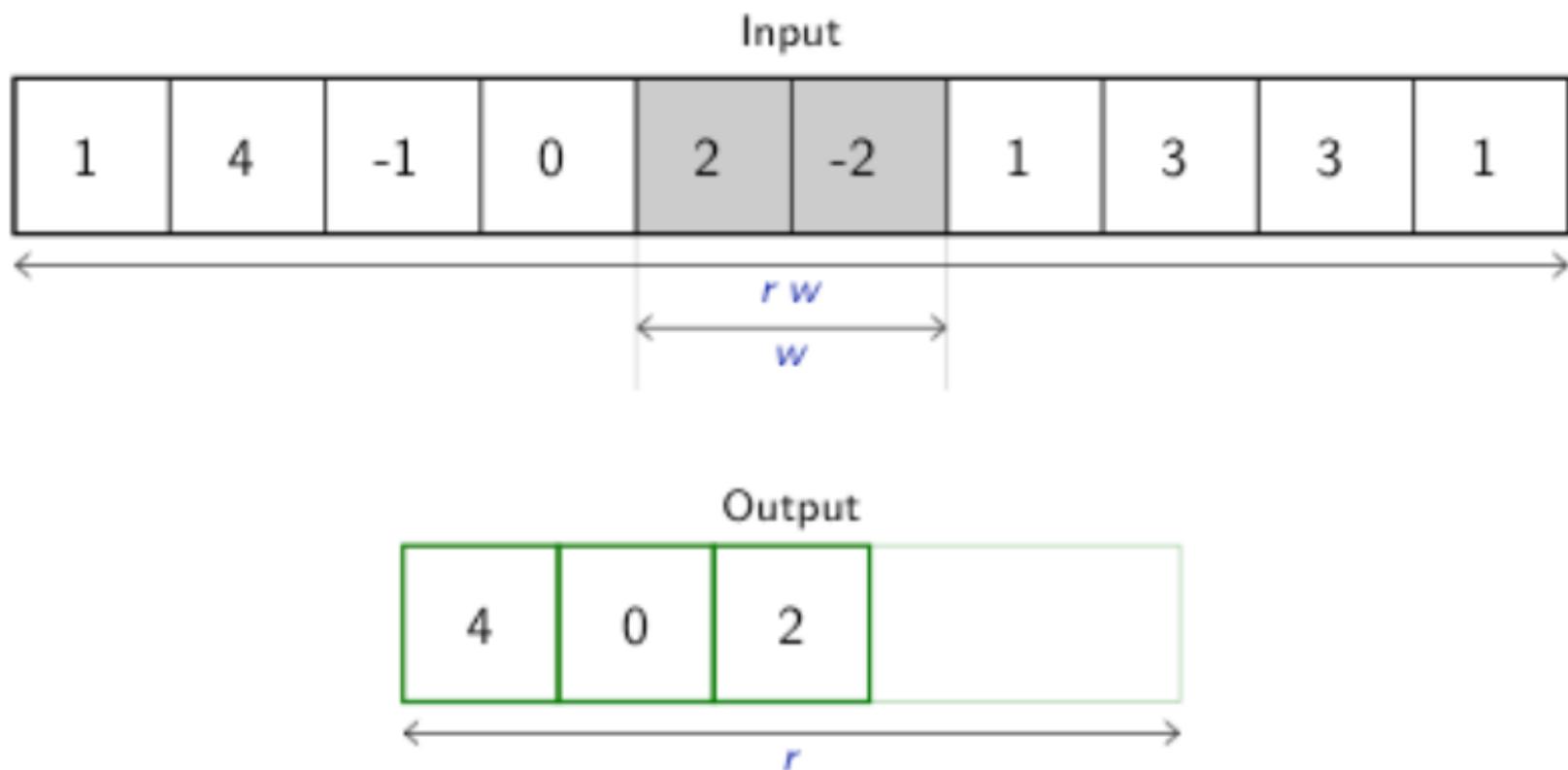
Credit: F. Fleuret

MAX POOLING 1D



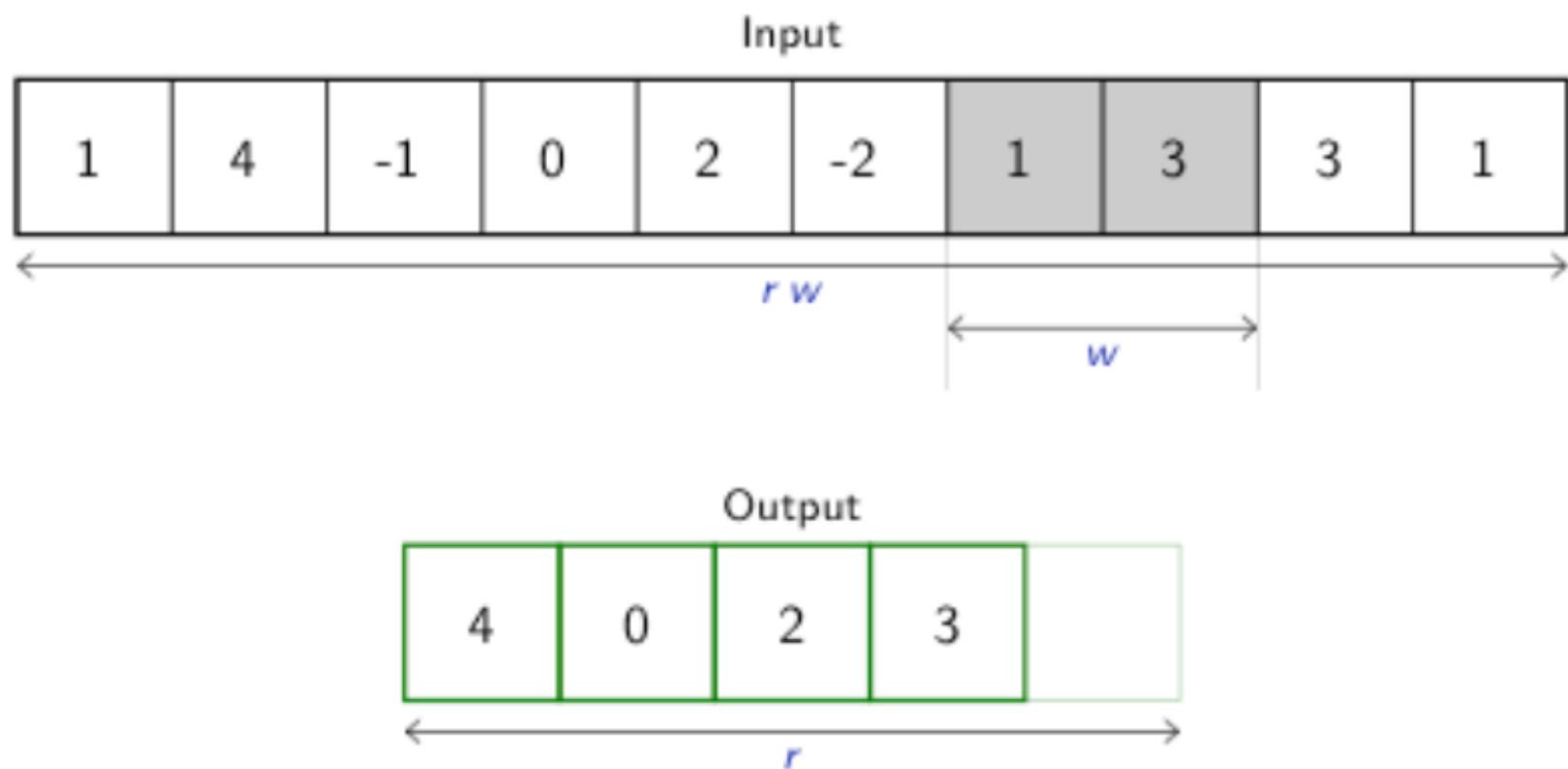
Credit: F. Fleuret

MAX POOLING 1D



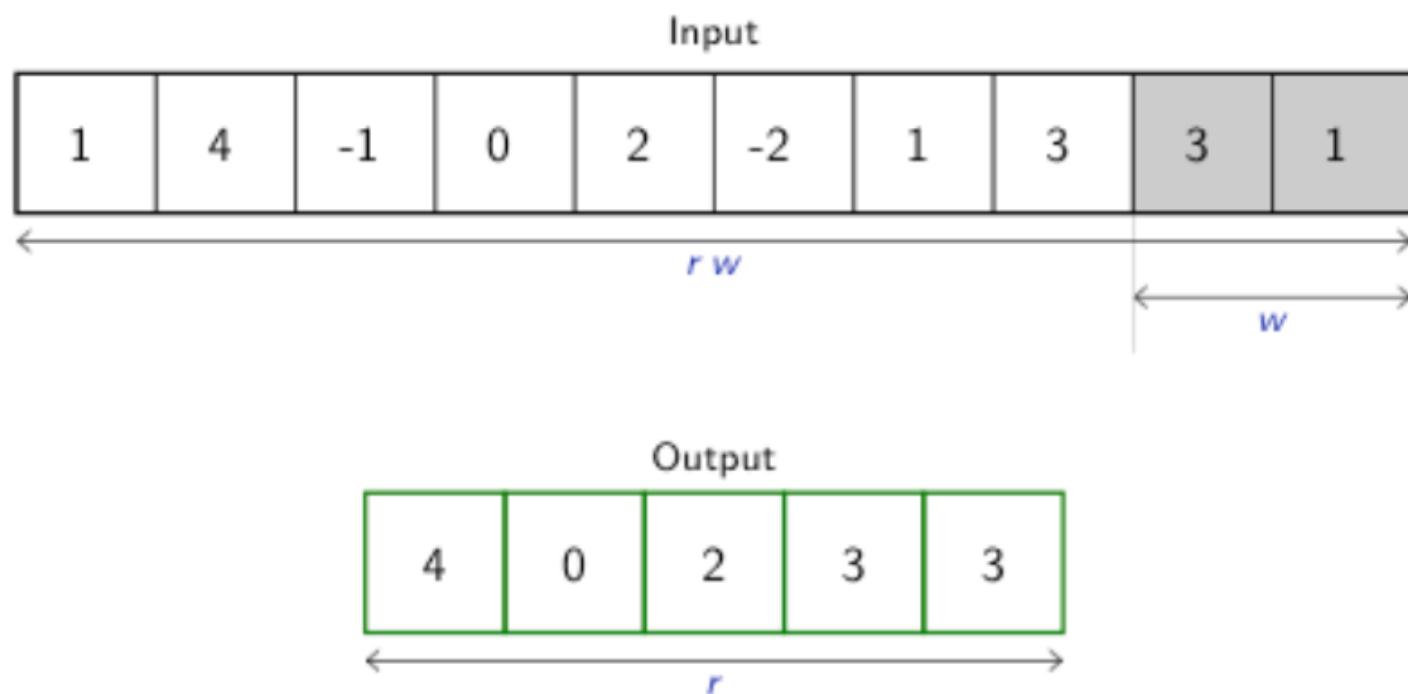
Credit: F. Fleuret

MAX POOLING 1D



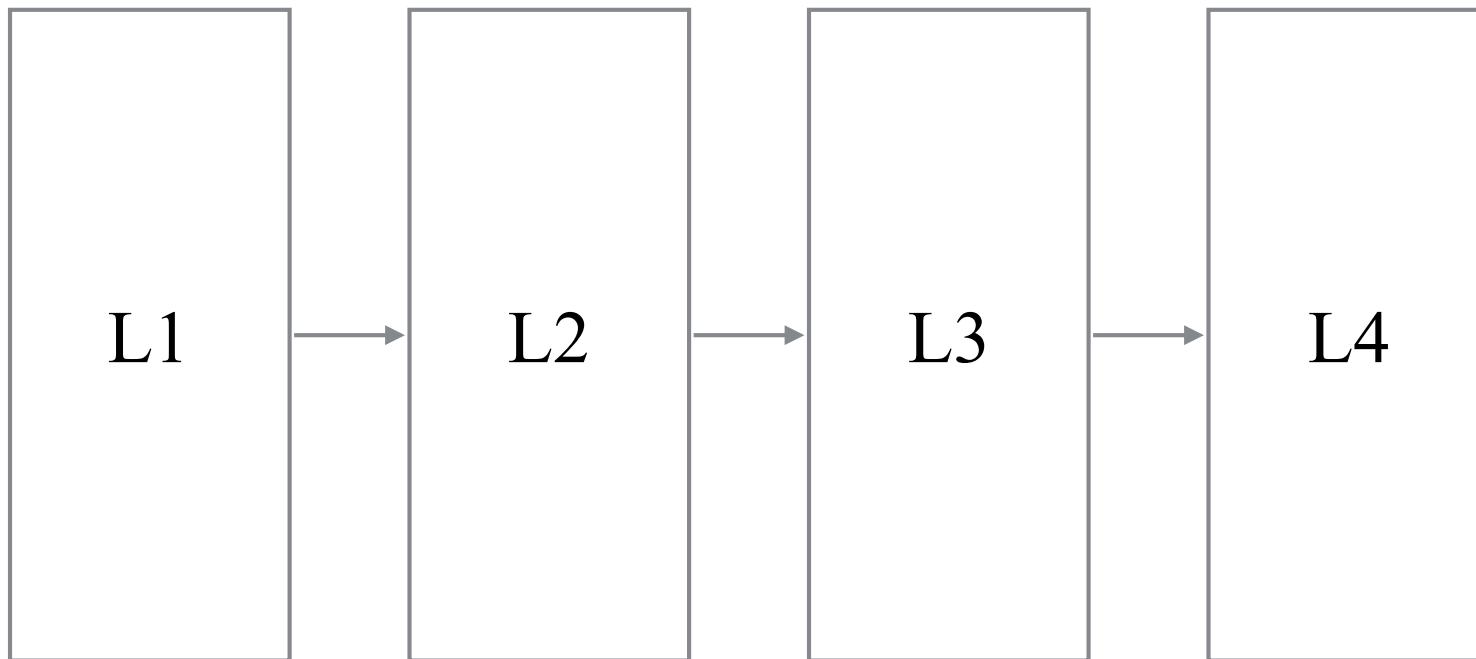
Credit: F. Fleuret

MAX POOLING 1D



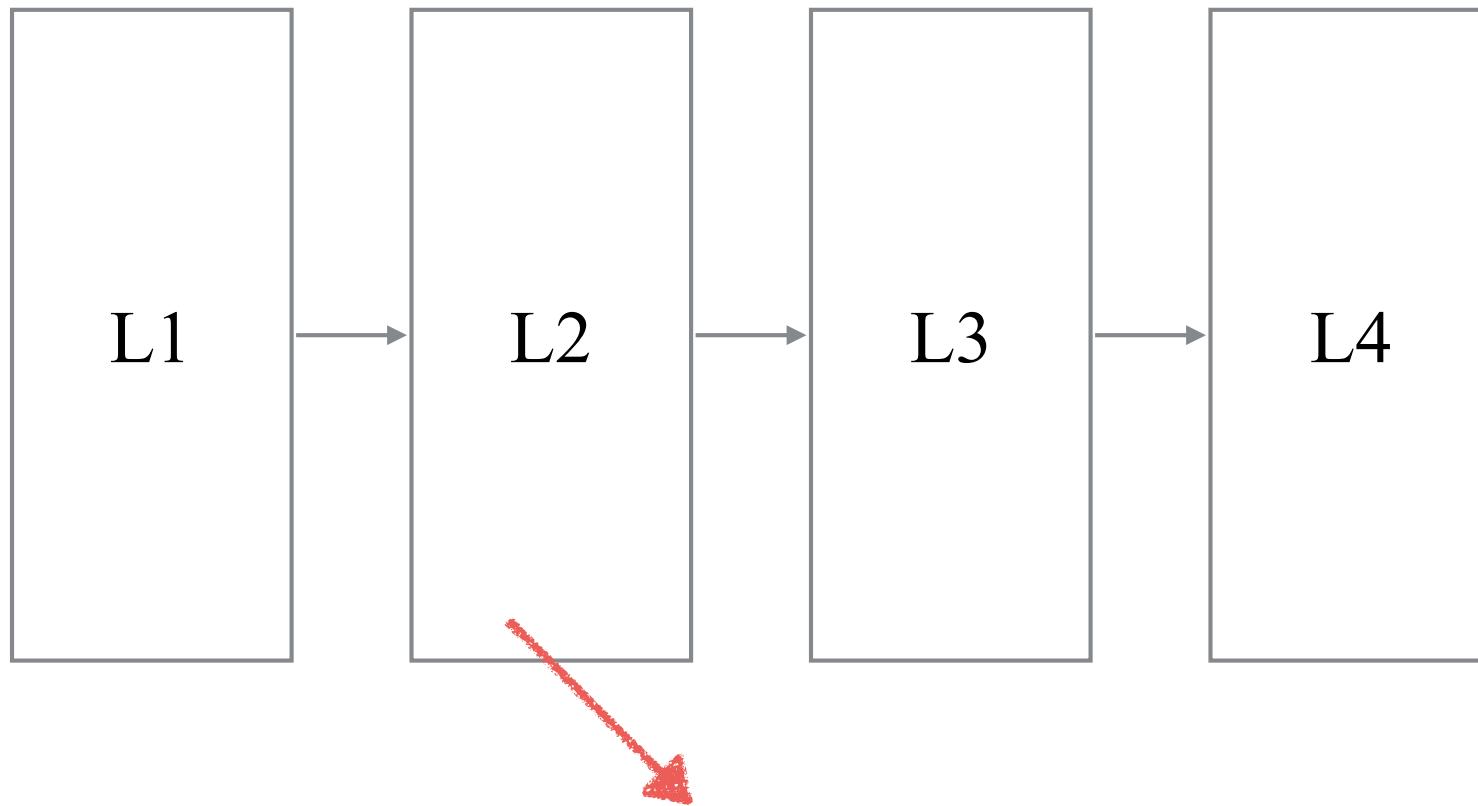
Credit: F. Fleuret

CONVNET OR CNN



A CONCATENATION OF MULTIPLE
CONVOLUTIONAL BLOCKS

CONVNET OR CNN



EACH BLOCK TYPICALLY MADE OF:

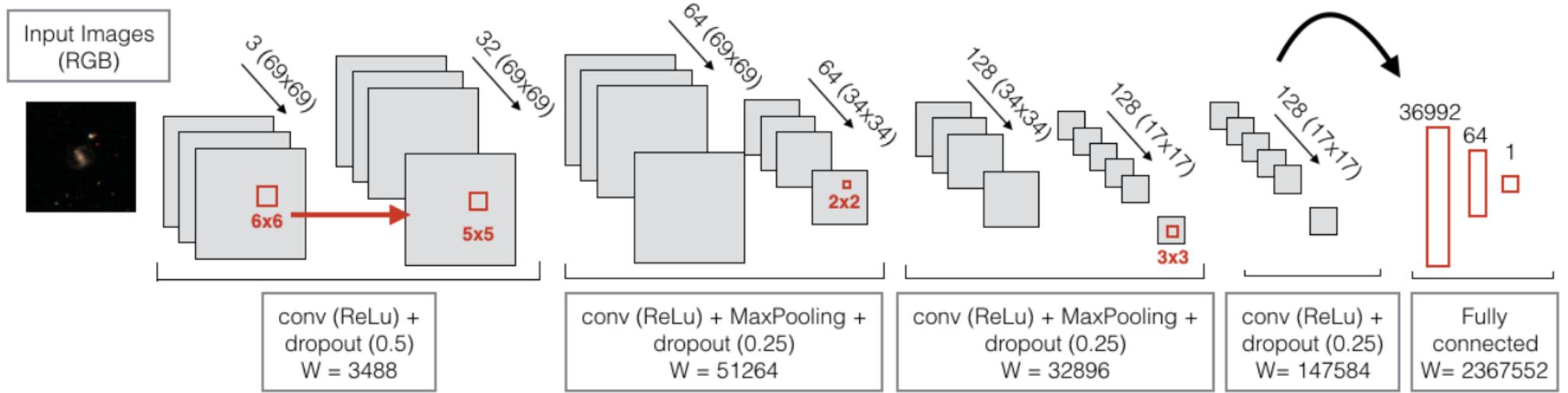
CONV

ACTIVATION

POOLING

(+dropout
for training)

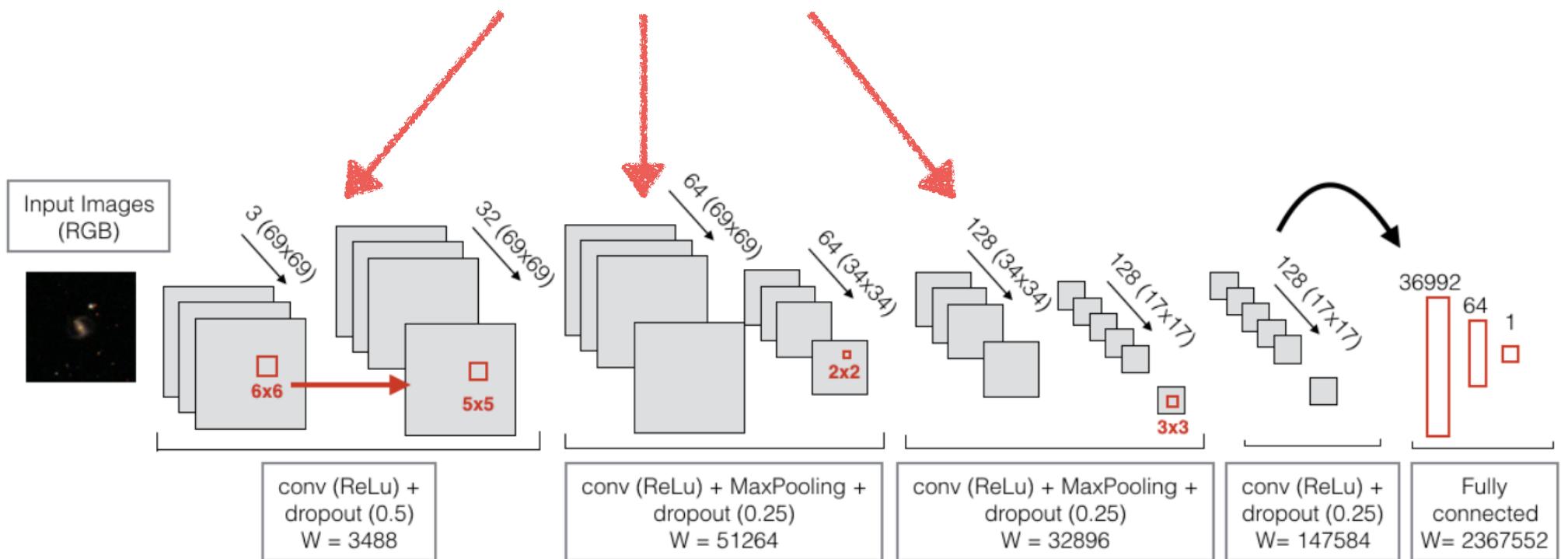
EXAMPLE OF VERY SIMPLE CNN



Dominguez-Sanchez+18

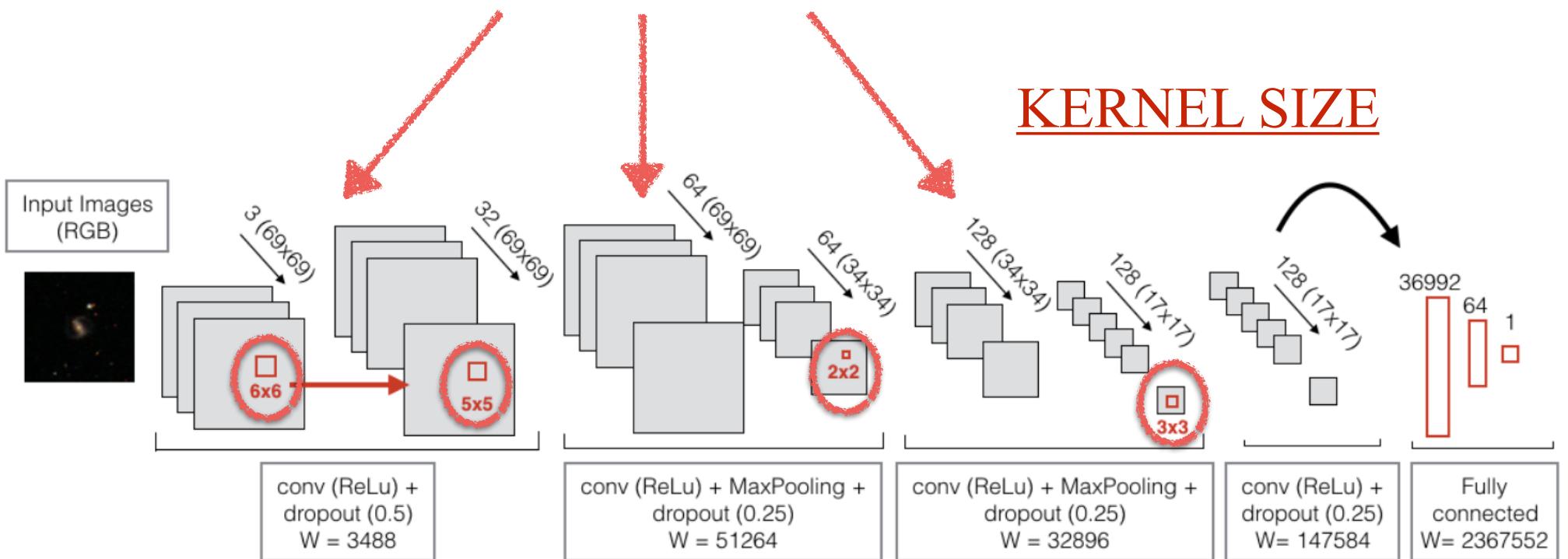
EXAMPLE OF VERY SIMPLE CNN

3 convolutional layers



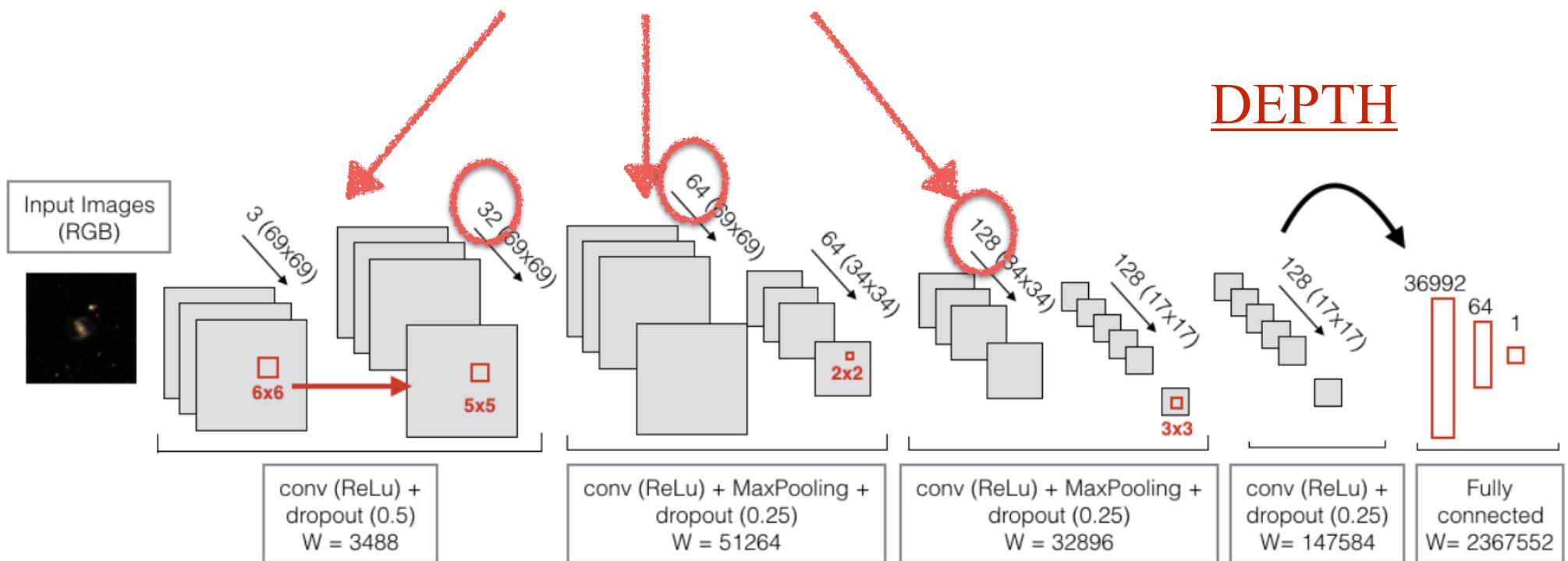
EXAMPLE OF VERY SIMPLE CNN

3 convolutional layers



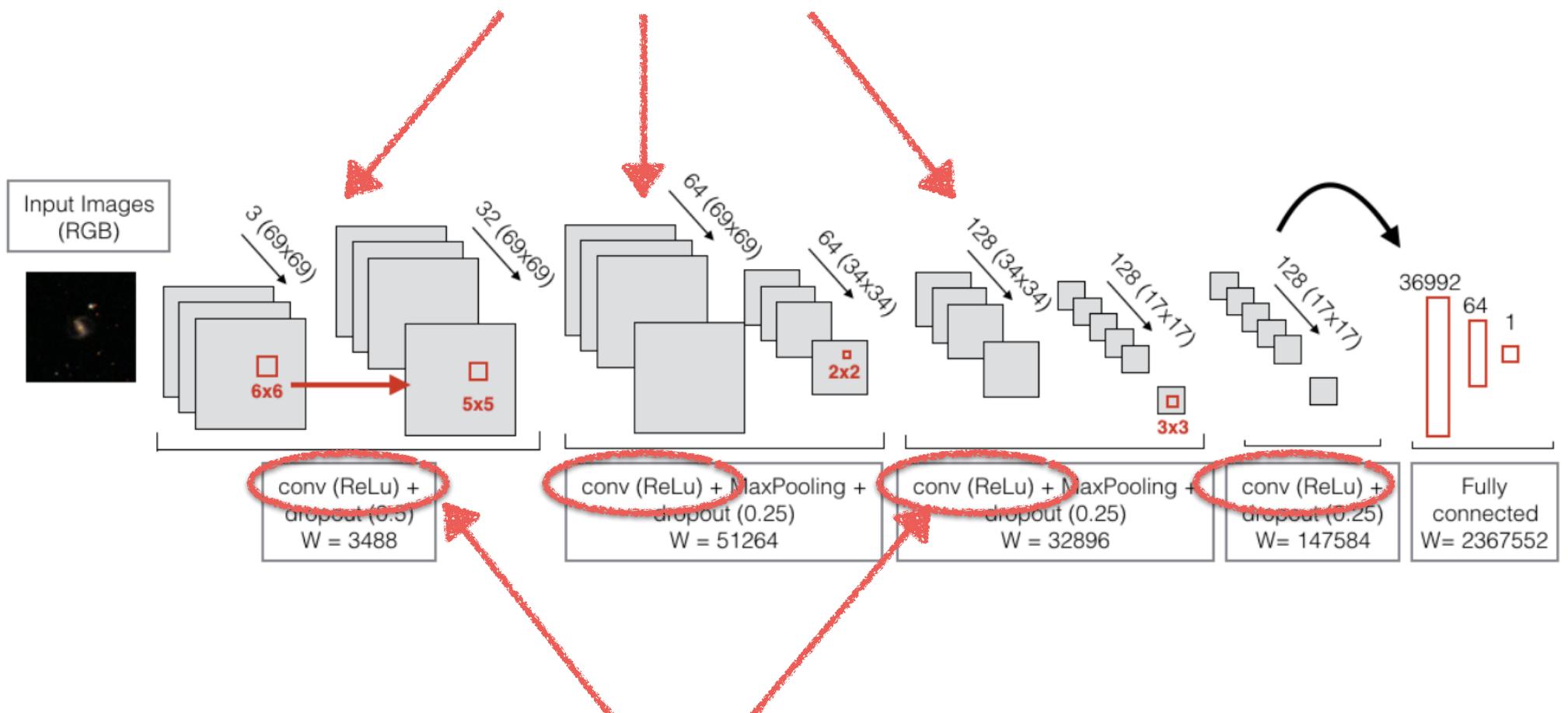
EXAMPLE OF VERY SIMPLE CNN

3 convolutional layers



EXAMPLE OF VERY SIMPLE CNN

3 convolutional layers

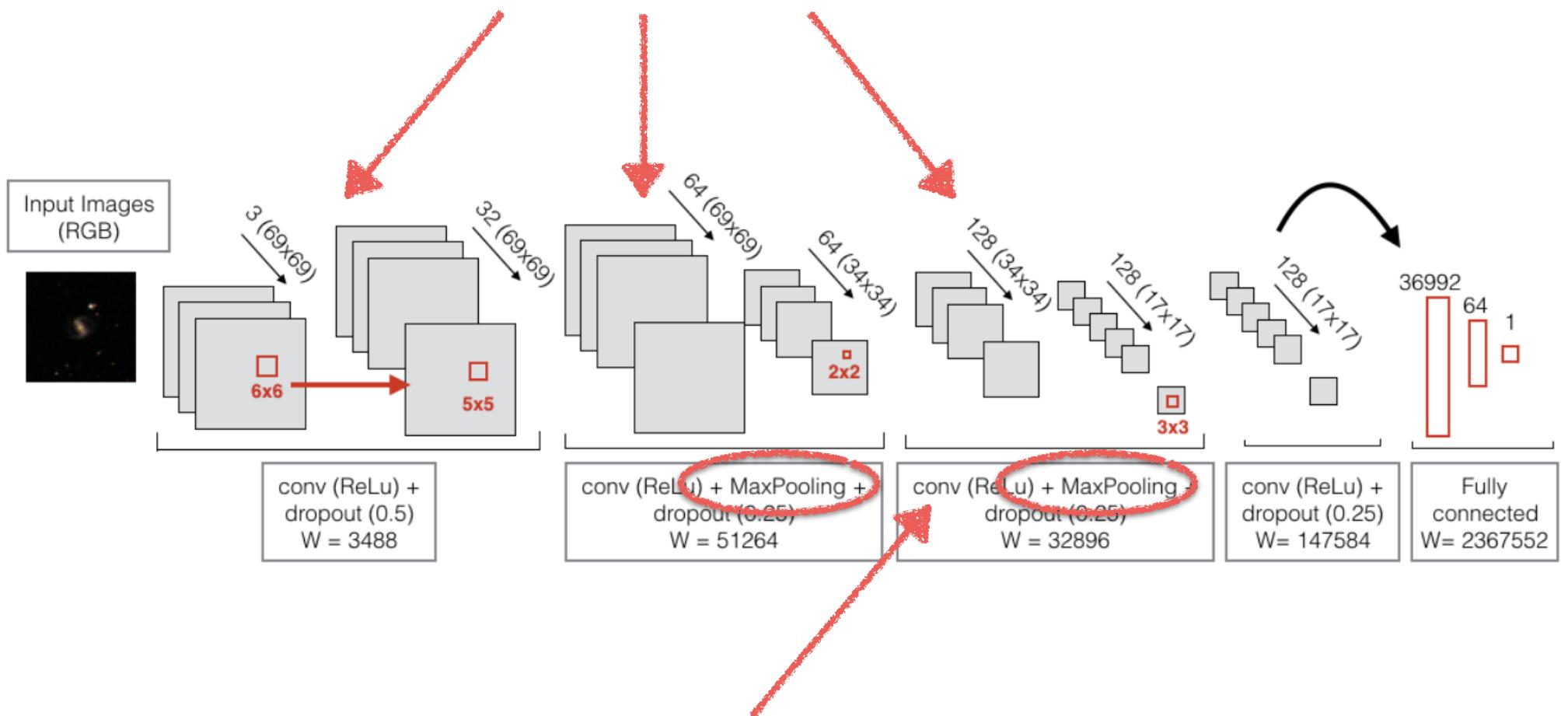


ReLU activation

Dominguez-Sanchez+18

EXAMPLE OF VERY SIMPLE CNN

3 convolutional layers



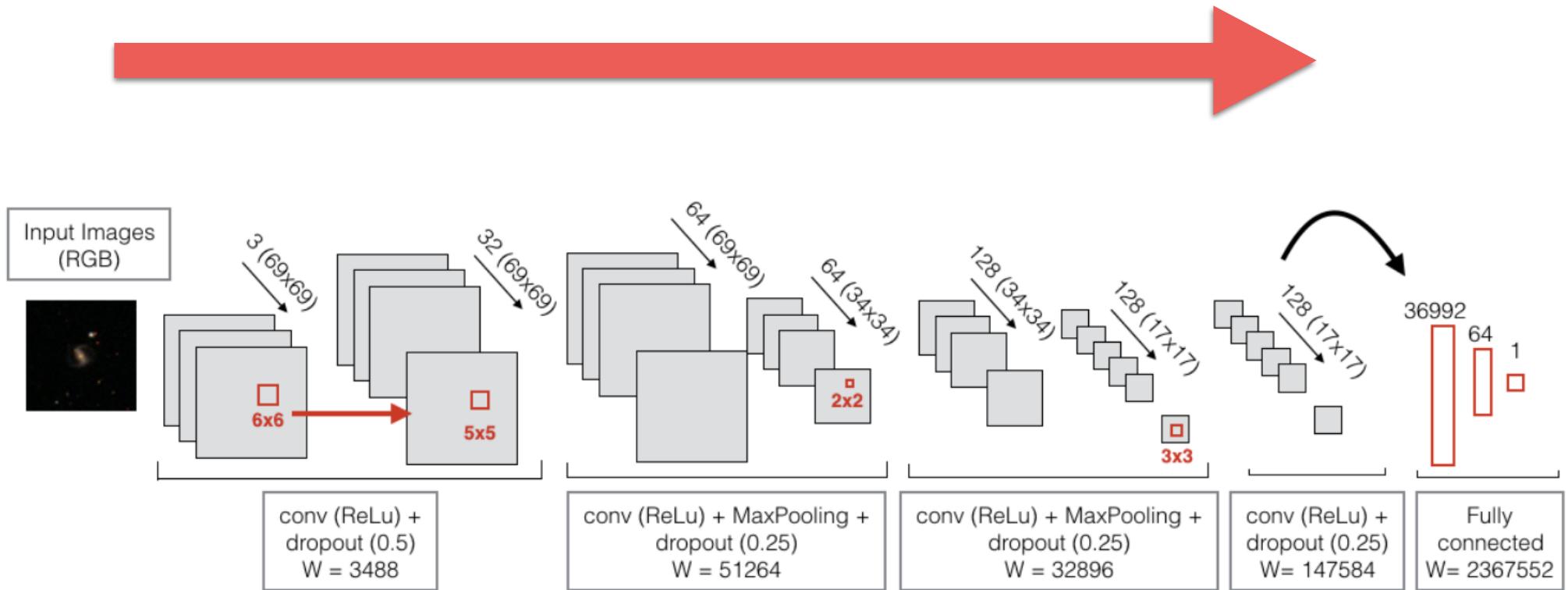
Pooling

Dominguez-Sanchez+18

EXAMPLE OF VERY SIMPLE CNN

OVERALL:

- decrease of tensor size
- increase of depth



IMPLEMENTATION IN KERAS

```
#===== Model definition=====

#Convolutional Layers

model = Sequential()
model.add(Convolution2D(32, 6,6, border_mode='same',
                       input_shape=(img_channels, img_rows, img_cols)))
model.add(Activation('relu'))
model.add(Dropout(0.5))

model.add(Convolution2D(64, 5, 5, border_mode='same'))
model.add(Activation('relu'))

model.add(MaxPooling2D(pool_size=(2, 2)))
model.add(Dropout(0.25))

model.add(Convolution2D(128, 2, 2, border_mode='same'))
model.add(Activation('relu'))

model.add(MaxPooling2D(pool_size=(2, 2)))
model.add(Dropout(0.25))

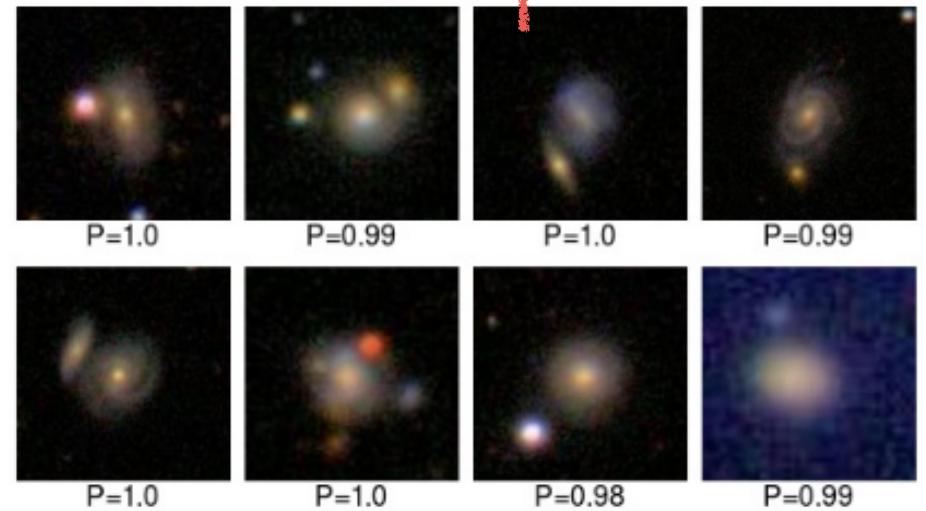
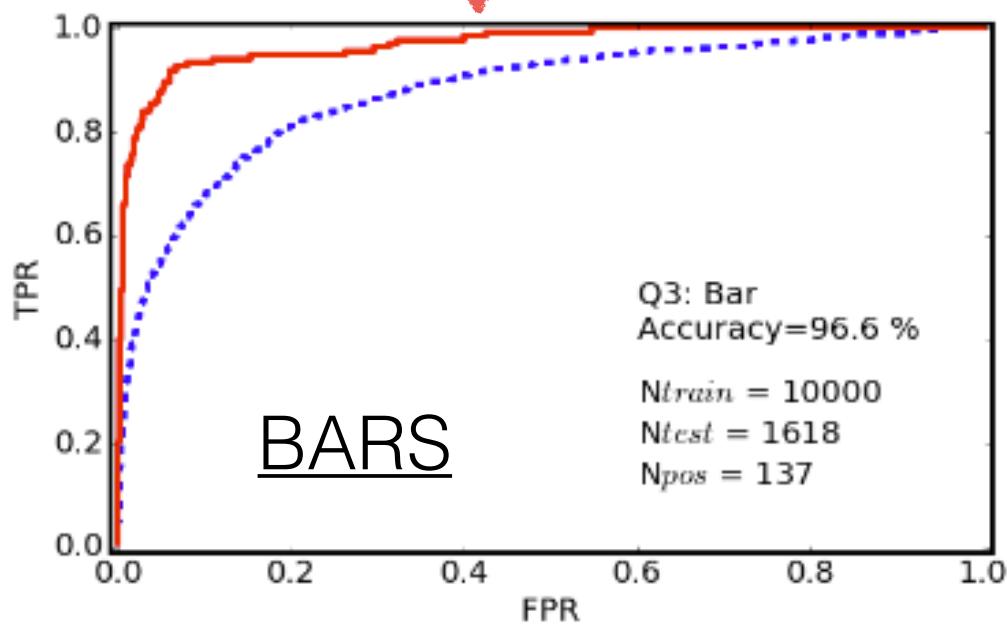
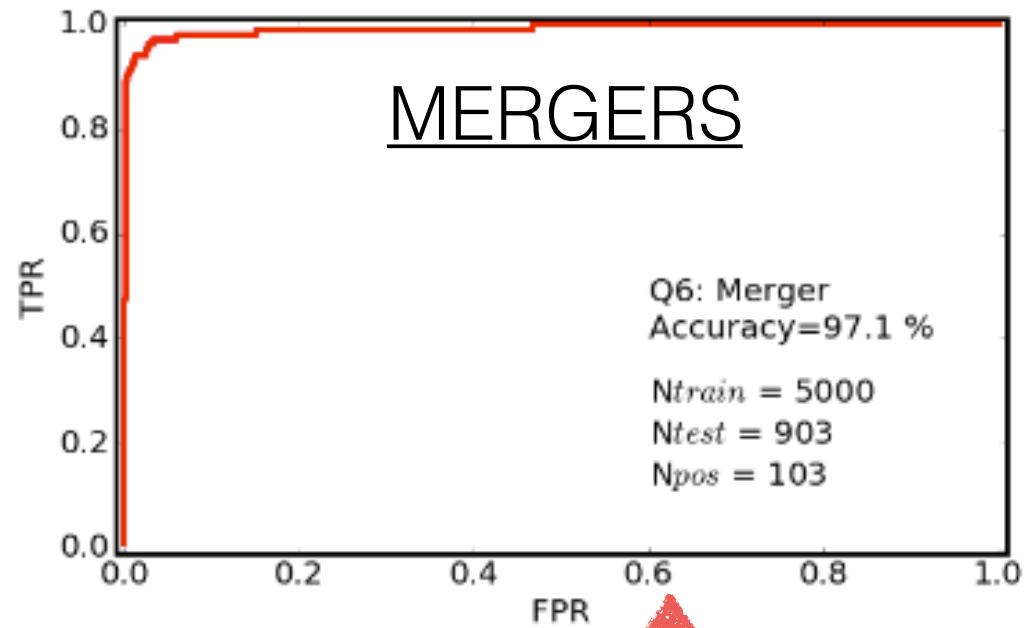
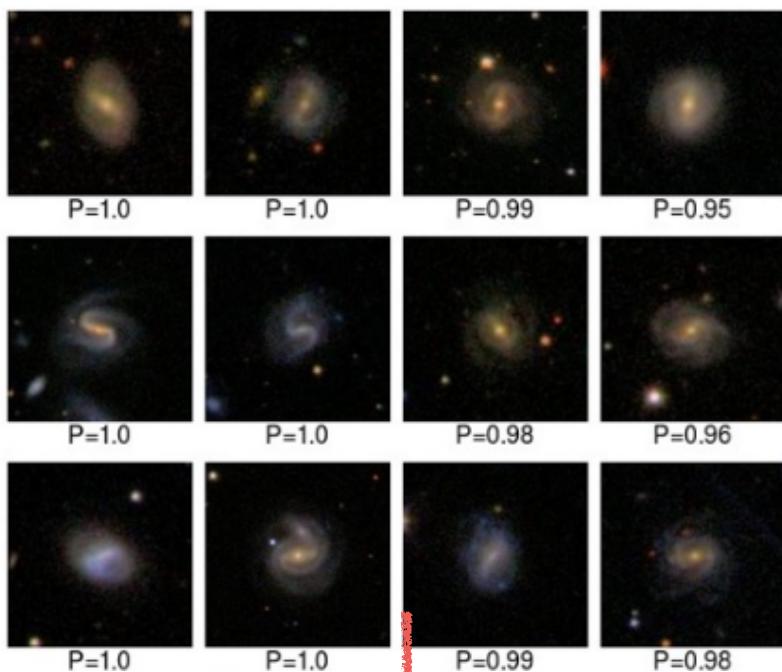
model.add(Convolution2D(128, 3, 3, border_mode='same'))
model.add(Activation('relu'))

model.add(Dropout(0.25))

#Fully Connected start here
#-----#
model.add(Flatten())
model.add(Dense(64, activation='relu'))
model.add(Dropout(.5))
model.add(Dense(1, init='uniform', activation='sigmoid'))

print("Compilation...")

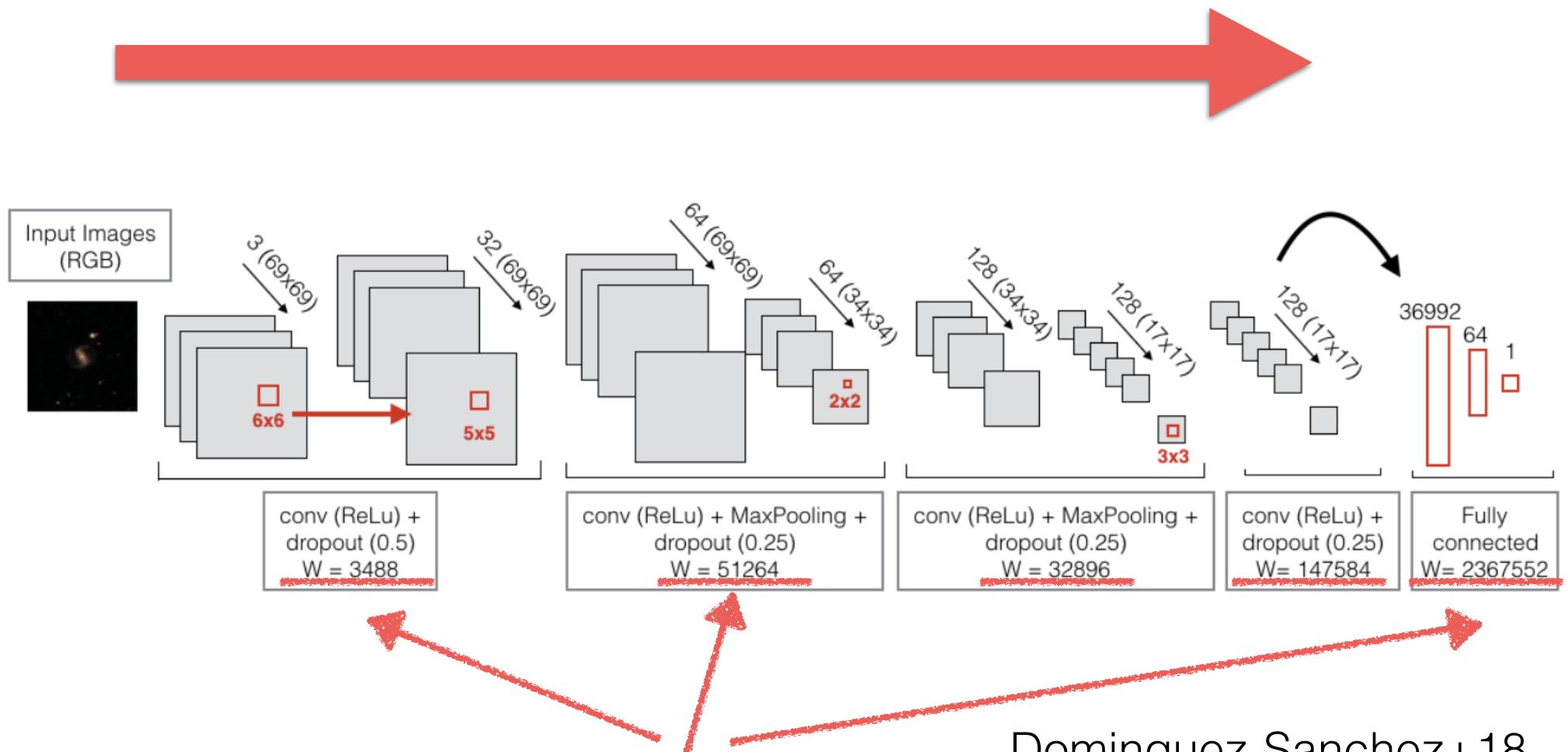
model.compile(loss='binary_crossentropy',optimizer='adam',metrics=['accuracy'])
```



EXAMPLE OF VERY SIMPLE CNN

OVERALL:

- decrease of tensor size
- increase of depth



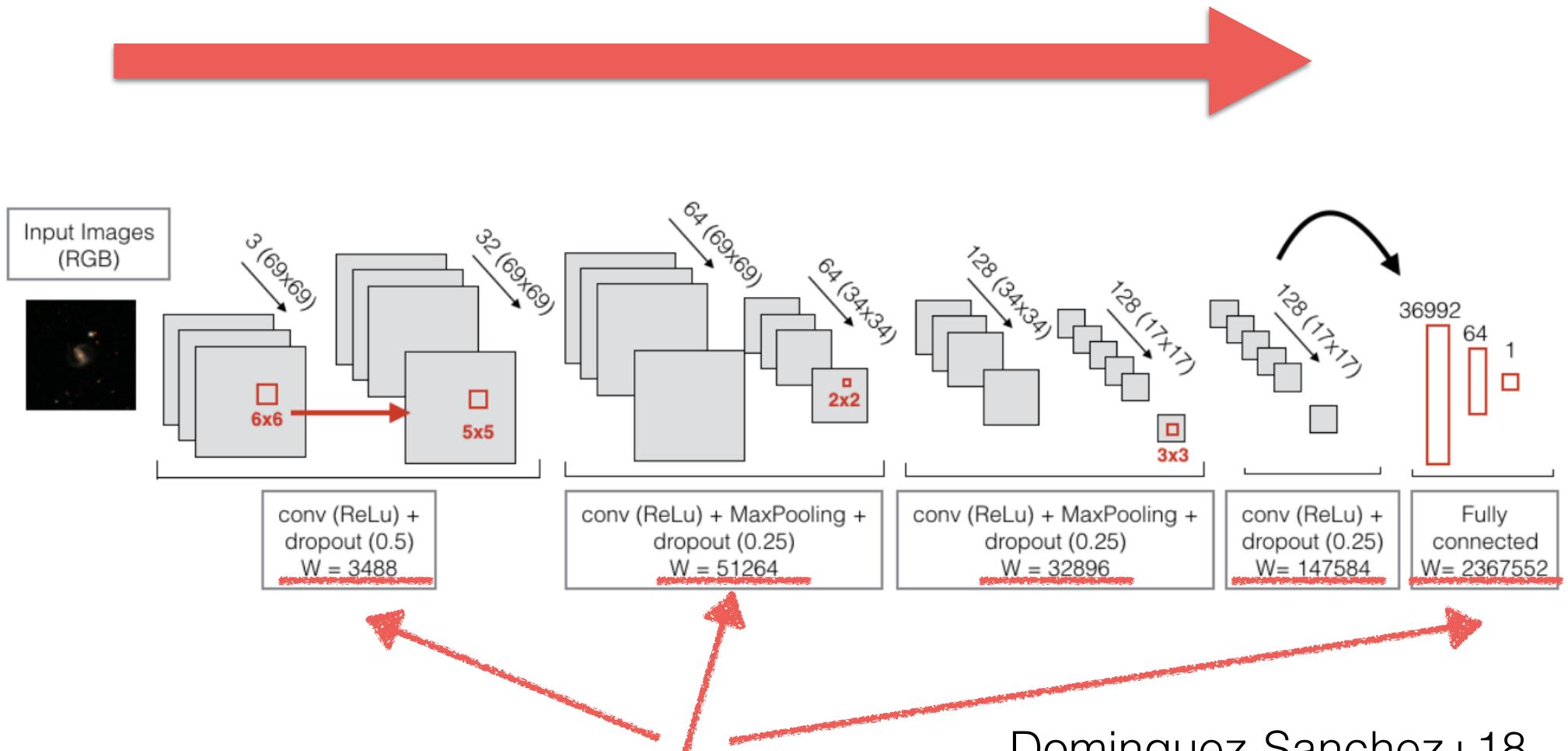
Number of parameters

Dominguez-Sanchez+18

EXAMPLE OF VERY SIMPLE CNN

OVERALL:

- decrease of tensor size
- increase of depth



2 million of parameters for this very simple network!

CHECKING THE NUMBER OF PARAMETERS / LAYERS WITH KERAS

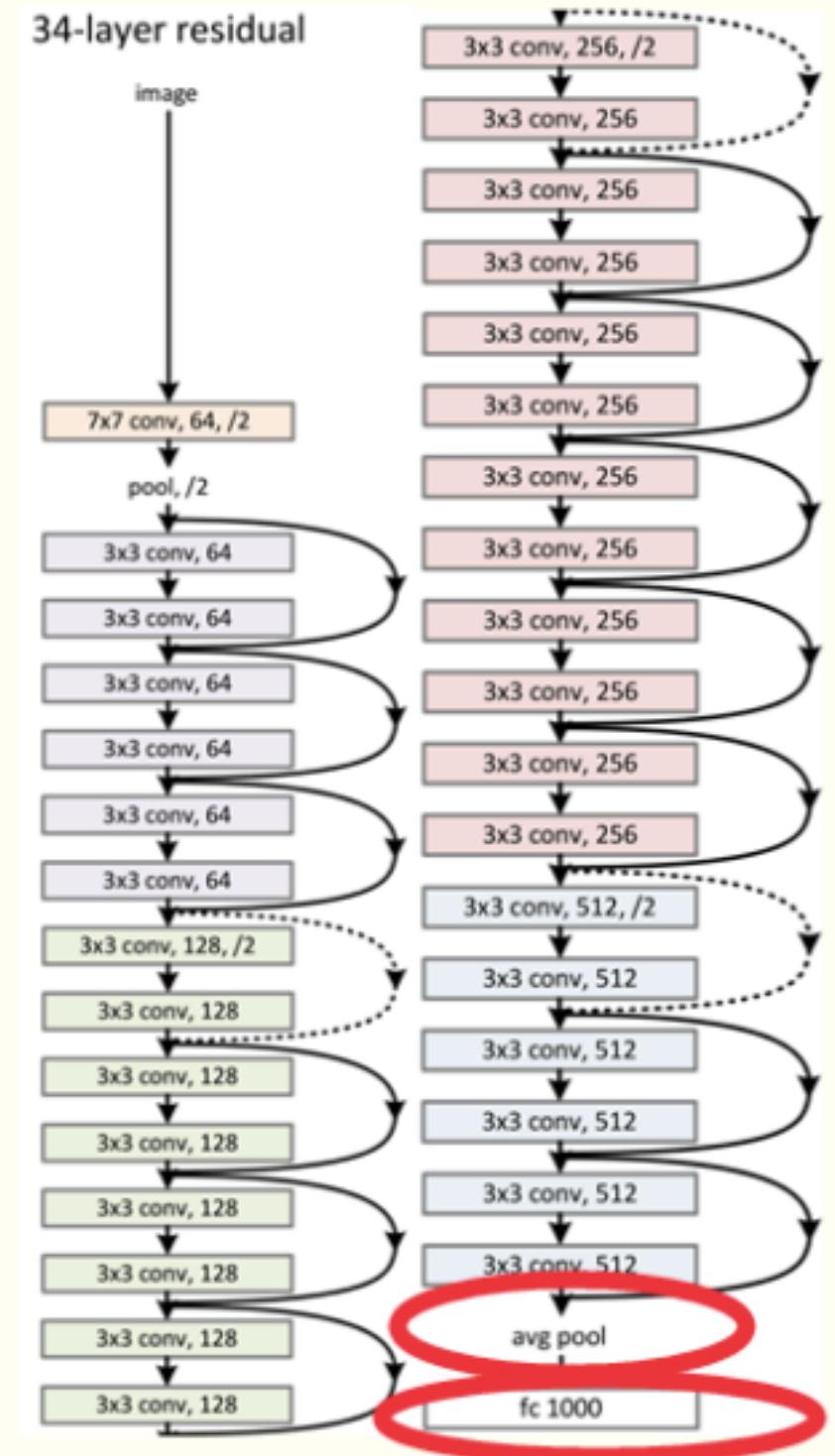
model.summary()



Layer (type)	Output Shape	Param #
<hr/>		
input_1 (InputLayer)	(None, 1, 16, 112, 112)	0
conv3d_1 (Conv3D)	(None, 16, 16, 112, 112)	448
batch_normalization_1 (Batch Normalization)	(None, 16, 16, 112, 112)	448
activation_1 (Activation)	(None, 16, 16, 112, 112)	0
max_pooling3d_1 (MaxPooling3)	(None, 16, 8, 56, 56)	0
conv3d_2 (Conv3D)	(None, 32, 8, 56, 56)	13856
batch_normalization_2 (Batch Normalization)	(None, 32, 8, 56, 56)	224
activation_2 (Activation)	(None, 32, 8, 56, 56)	0
max_pooling3d_2 (MaxPooling3)	(None, 32, 4, 28, 28)	0
conv3d_3 (Conv3D)	(None, 64, 4, 28, 28)	55360
batch_normalization_3 (Batch Normalization)	(None, 64, 4, 28, 28)	112
activation_3 (Activation)	(None, 64, 4, 28, 28)	0
max_pooling3d_3 (MaxPooling3)	(None, 64, 2, 14, 14)	0
activation_12 (Activation)	(None, 64, 2, 14, 14)	0
<hr/>		
Total params: 70,448		
Trainable params: 70,056		
Non-trainable params: 392		

IN THE REAL LIFE...

RESNET



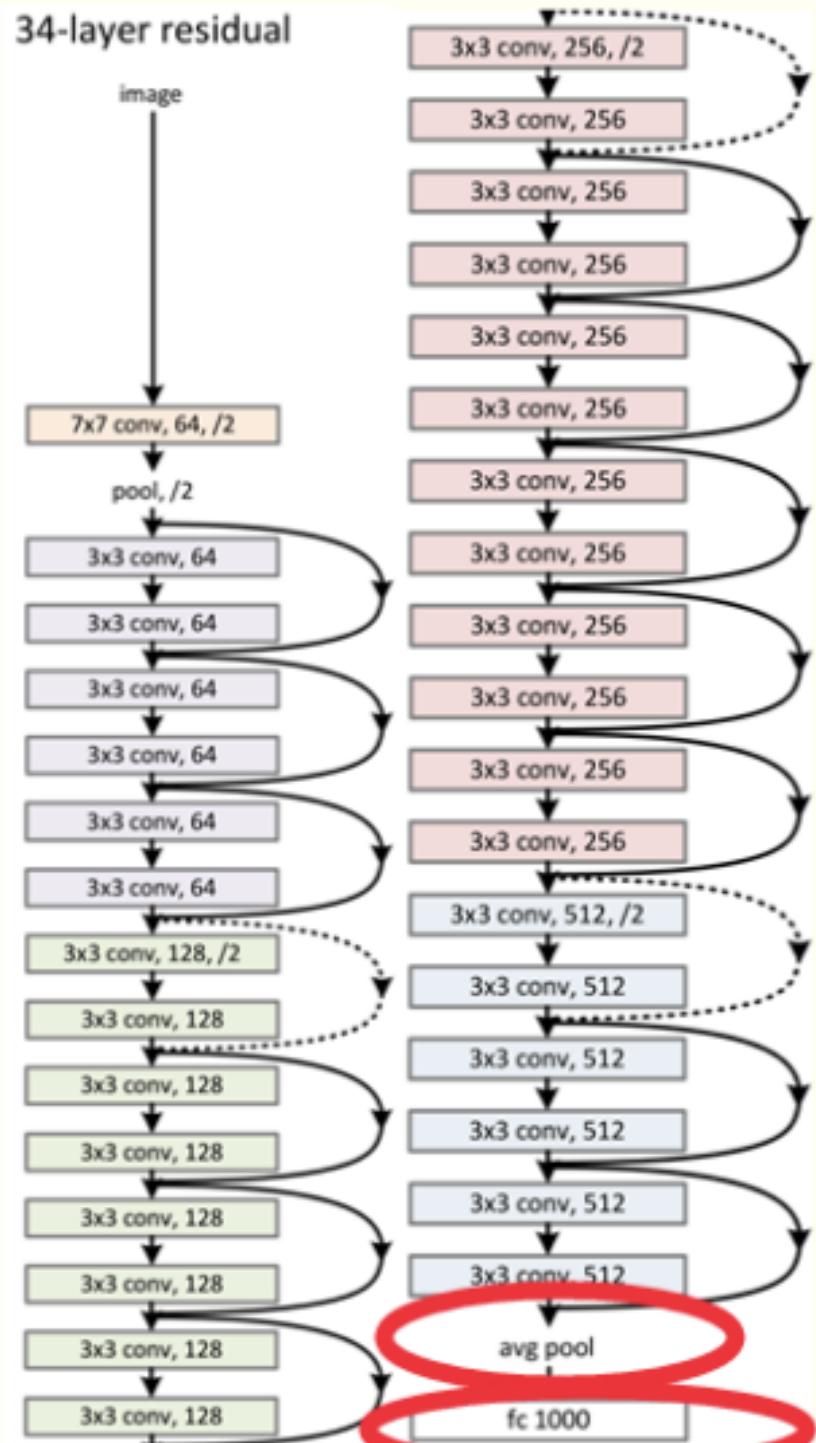
IN THE REAL LIFE...

RESNET

DO WE NEED TO GO
THIS DEEP FOR
ASTRONOMY
APPLICATIONS?

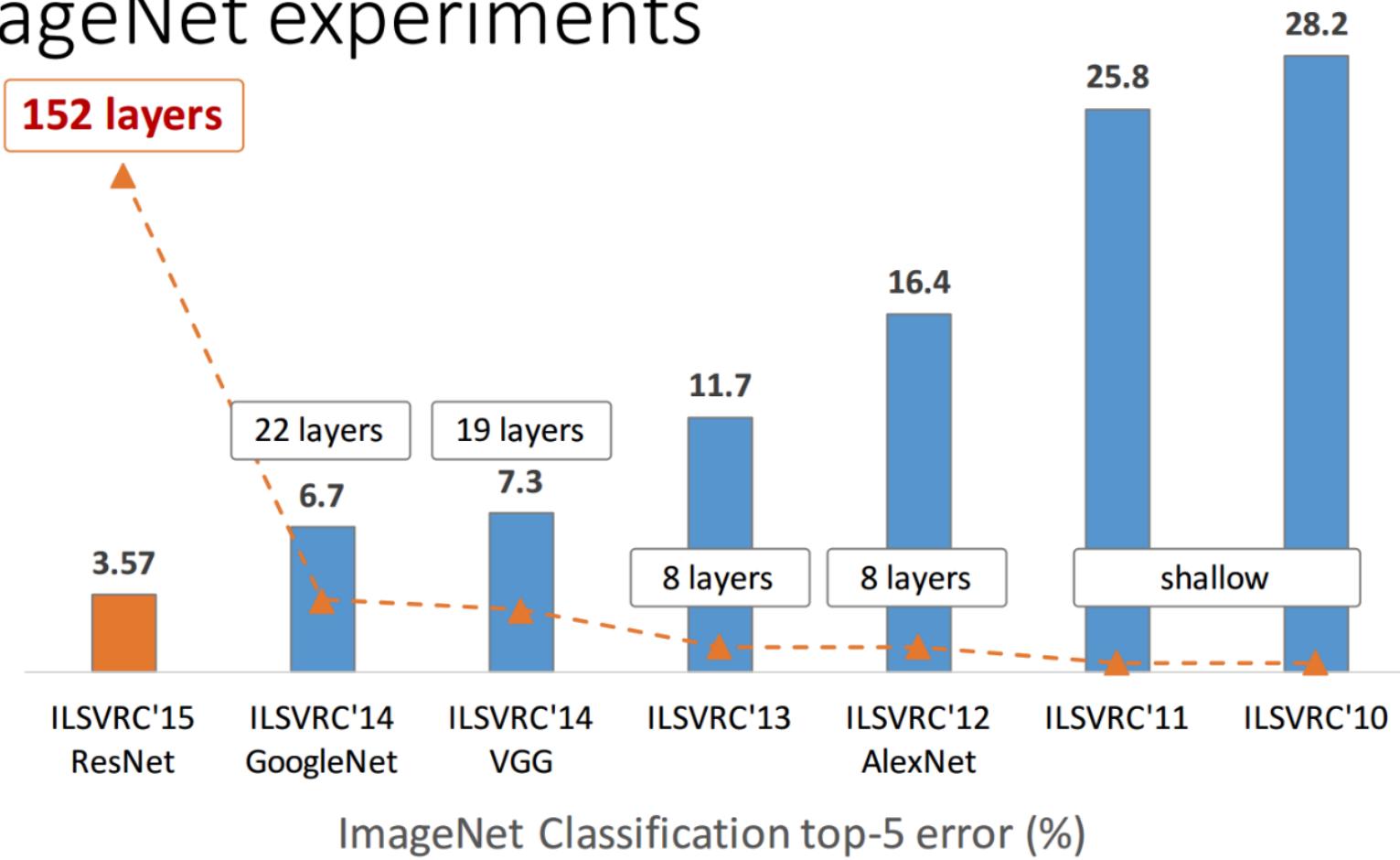
[34 layers - authors
explored up to 1202!]

34-layer residual



DEEPER TENDS TO BE BETTER...

ImageNet experiments



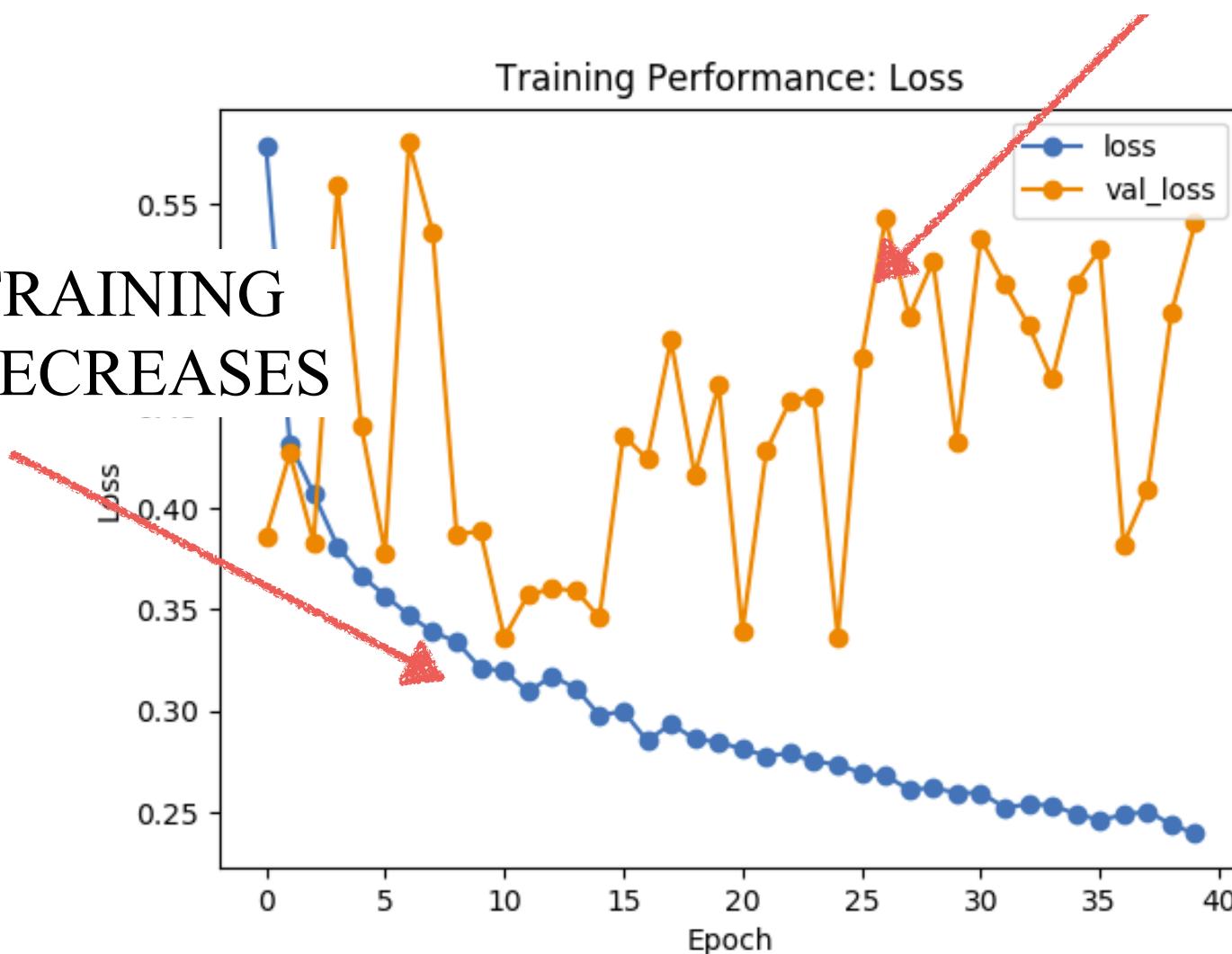
THE PROBLEMS OF GOING “TOO DEEP”

- DEEP NETWORKS ARE MORE DIFFICULT TO OPTIMIZE
- NEED MORE DATA - MORE SUBJECT TO OVERFITTING
- AND ALSO NEED MORE TIME ...

OVER-FITTING

THE TEST STAYS CONSTANT
OR INCREASES

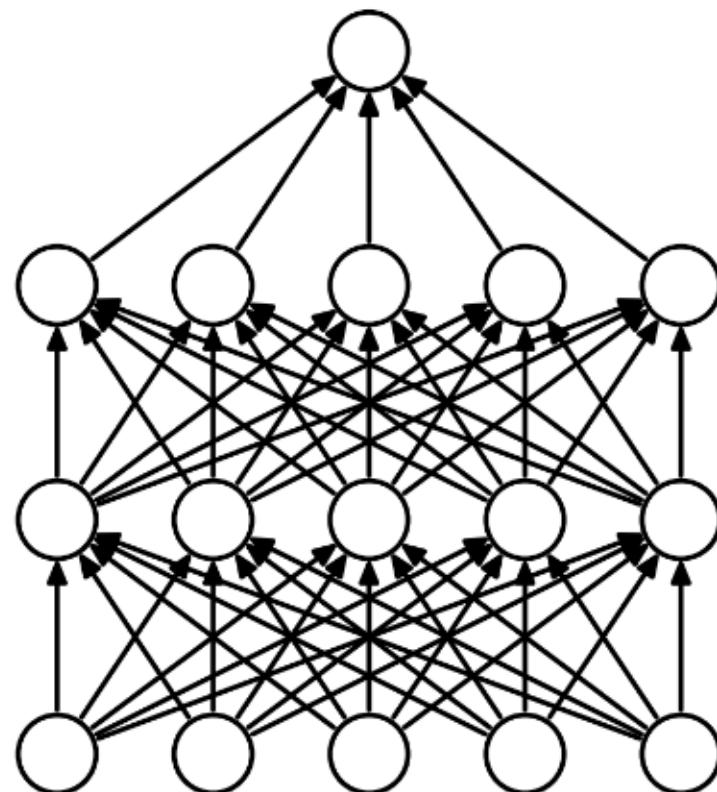
THE TRAINING
LOSS DECREASES



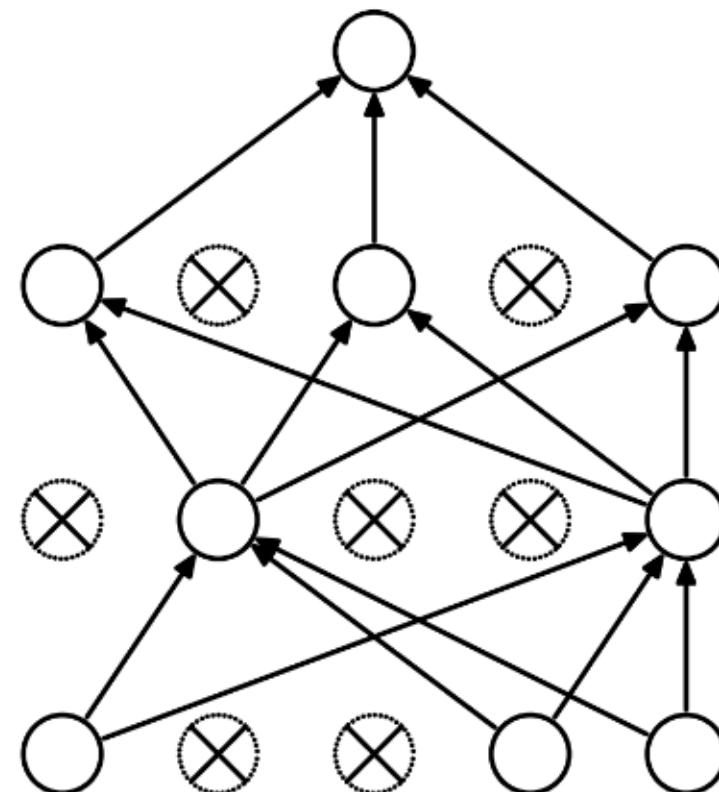
DROPOUT

[Hinton+12]

- THE IDEA IS TO REMOVE NEURONS RANDOMLY DURING THE TRAINING
- ALL NEURONS ARE PUT BACK DURING THE TEST PHASE



(a) Standard Neural Net



(b) After applying dropout.

DROPOUT

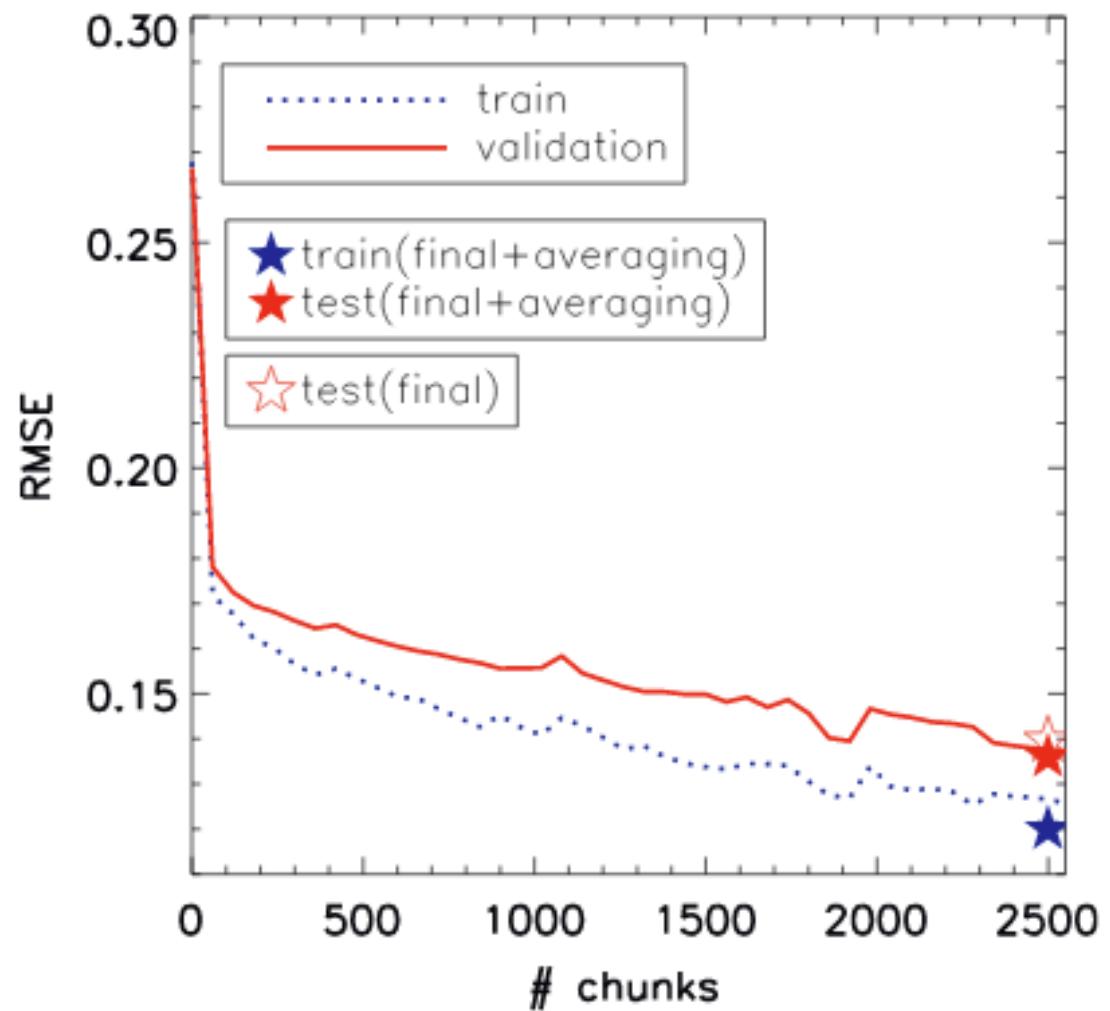
WHY DOES IT WORK?

1. SINCE NEURONS ARE REMOVED RANDOMLY, IT AVOIDS CO-ADAPTATION AMONG THEMSELVES

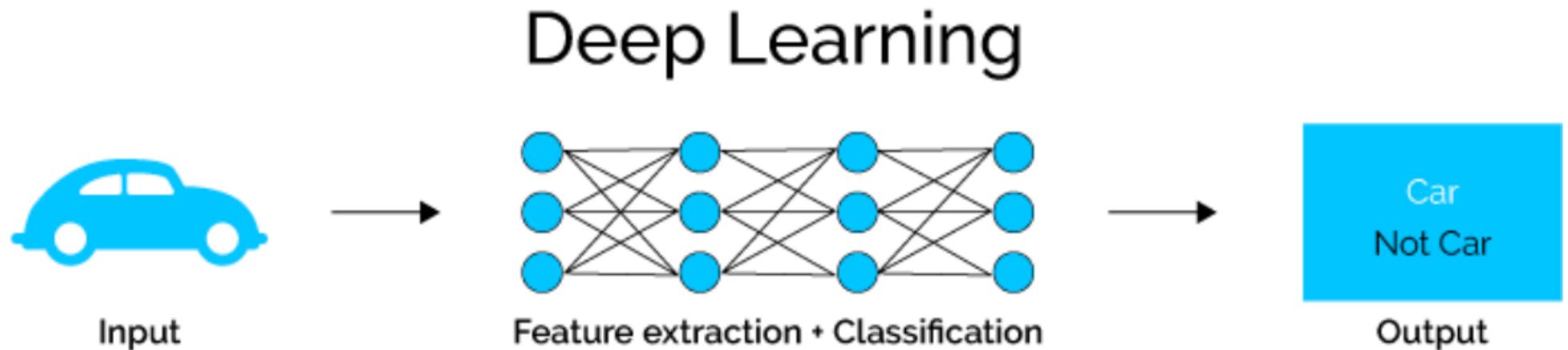
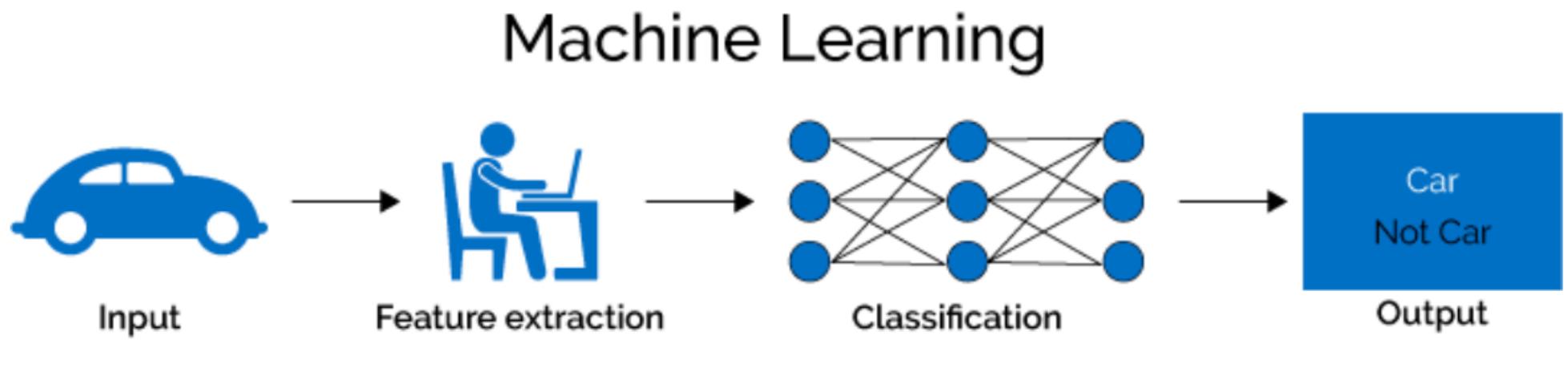
2. DIFFERENT SETS OF NEURONS WHICH ARE SWITCHED OFF, REPRESENT A DIFFERENT ARCHITECTURE AND ALL THESE DIFFERENT ARCHITECTURES ARE TRAINED IN PARALLEL. FOR N NEURONS ATTACHED TO DROPOUT, THE NUMBER OF SUBSET ARCHITECTURES FORMED IS 2^N . SO IT AMOUNTS TO PREDICTION BEING AVERAGED OVER THESE ENSEMBLES OF MODELS.

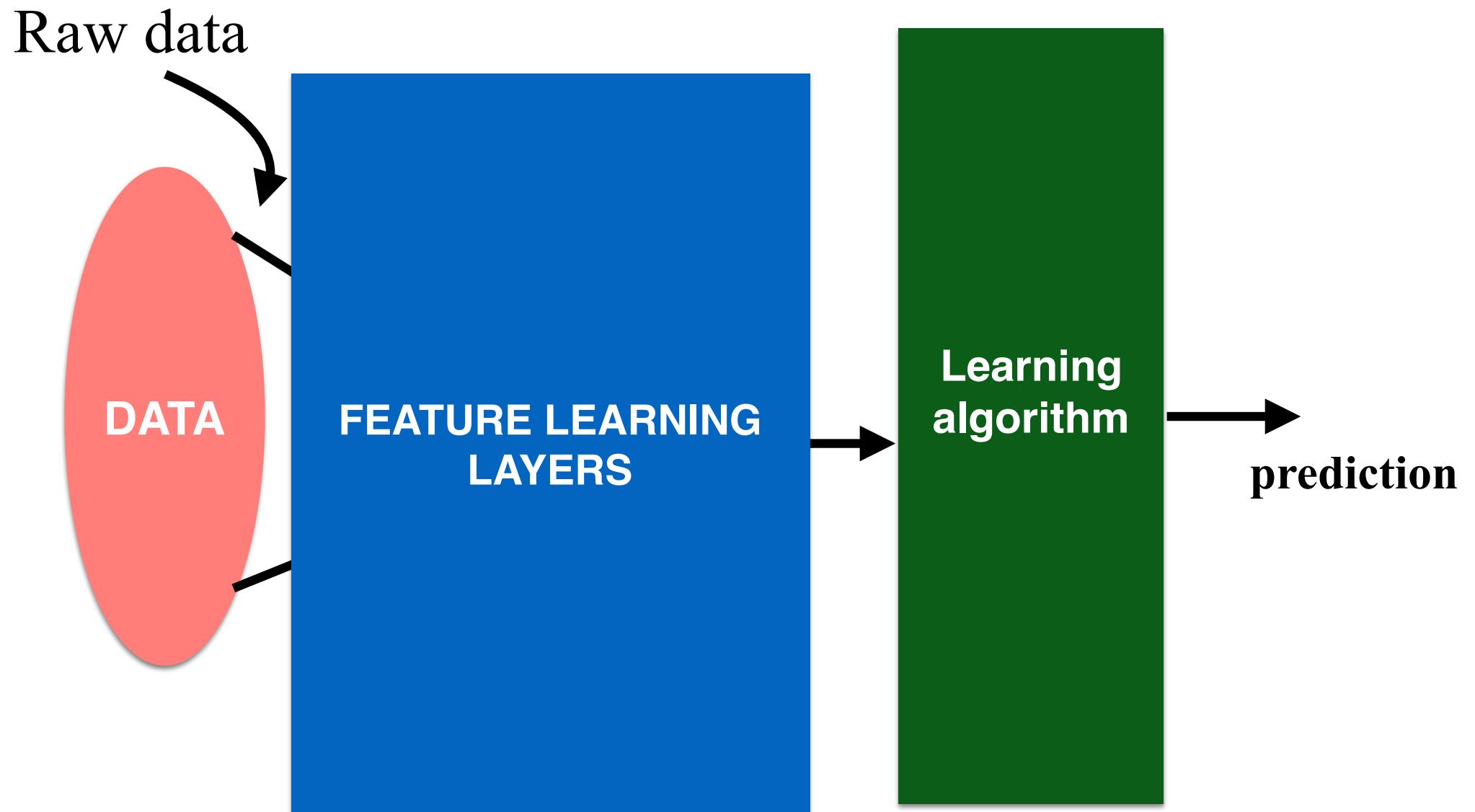
DROPOUT

WITH A LITTLE BIT
OF DROPOUT

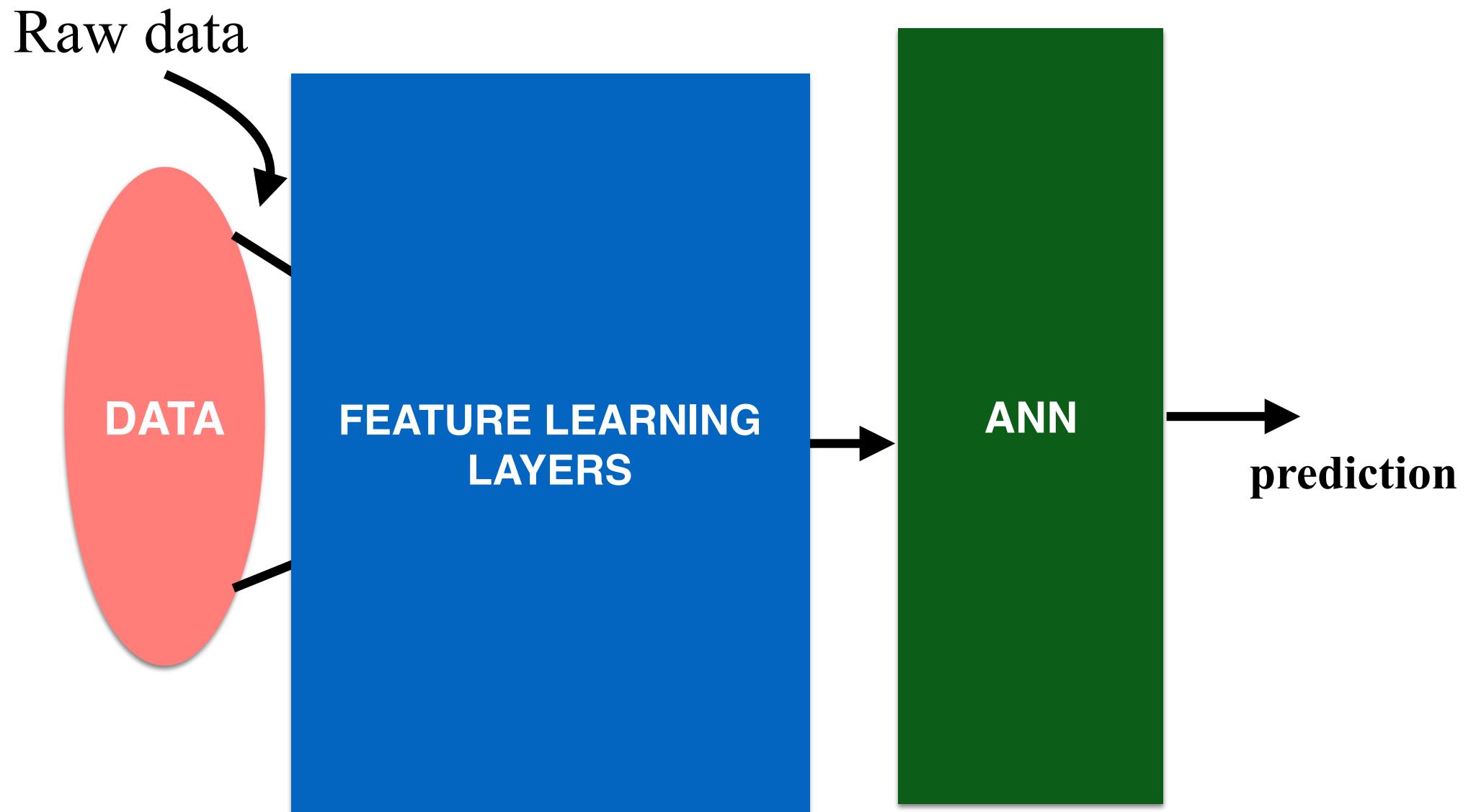


THIS IS A CHANGE OF PARADIGM!

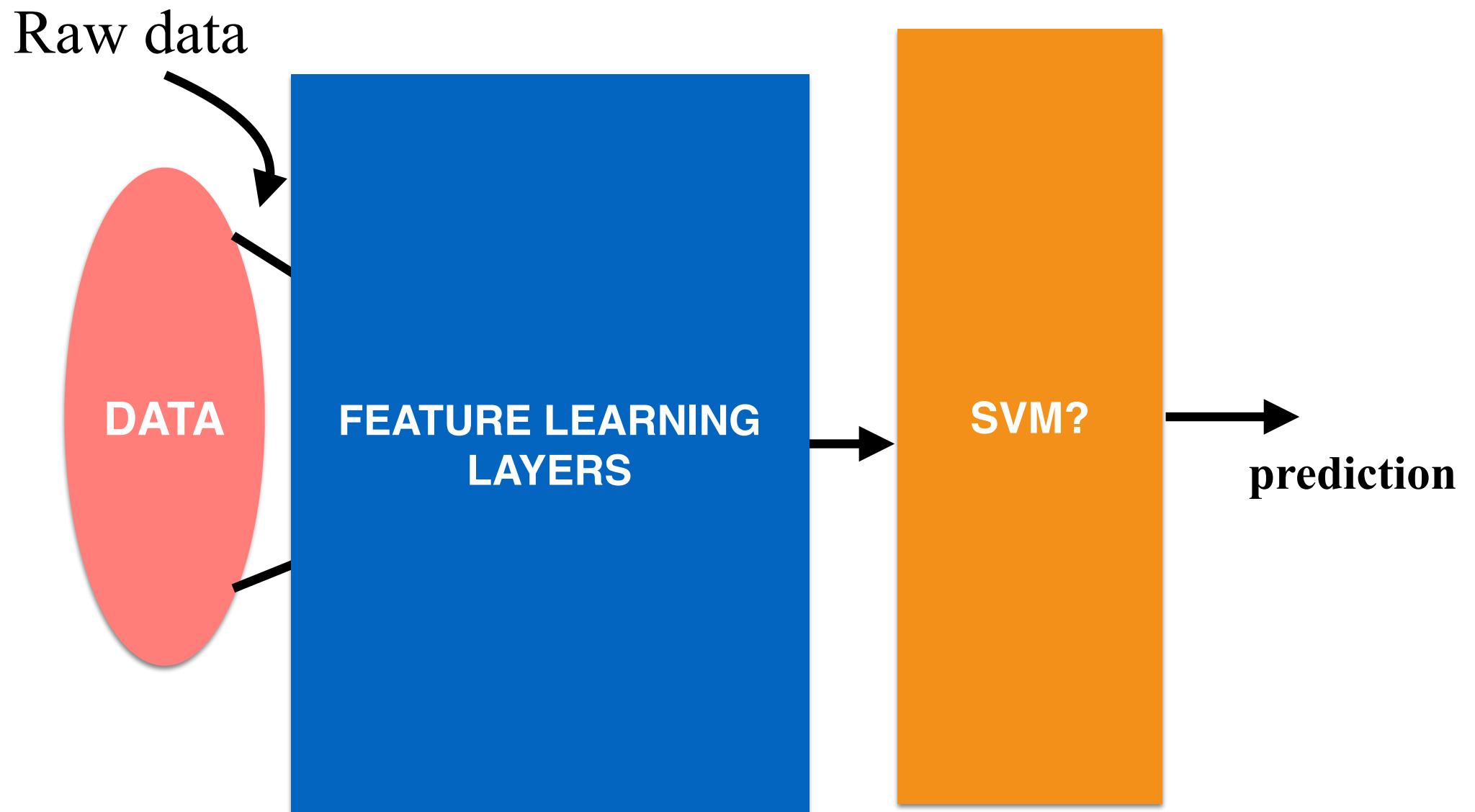




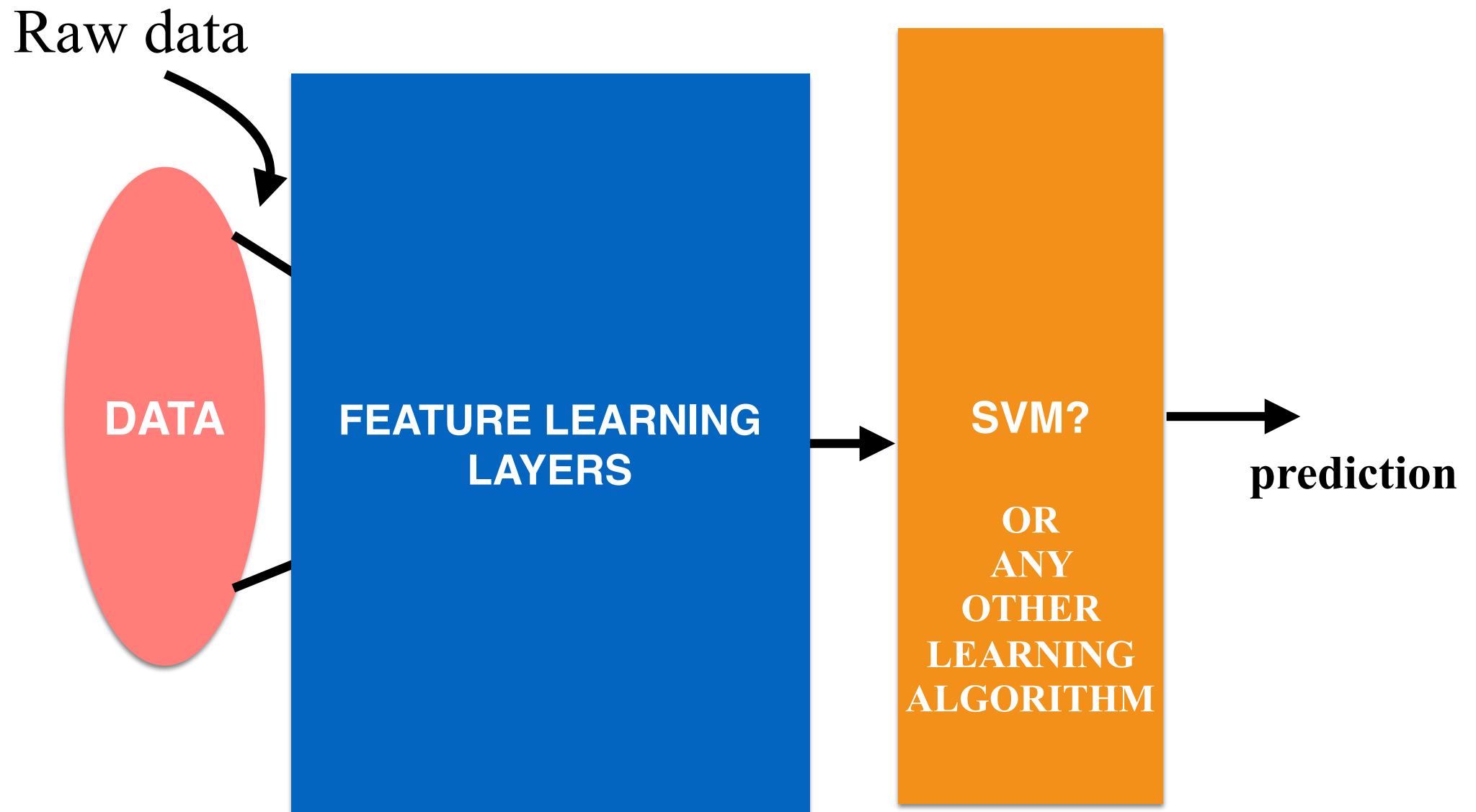
THE LEARNING ALGORITHM
CAN BE CHANGED



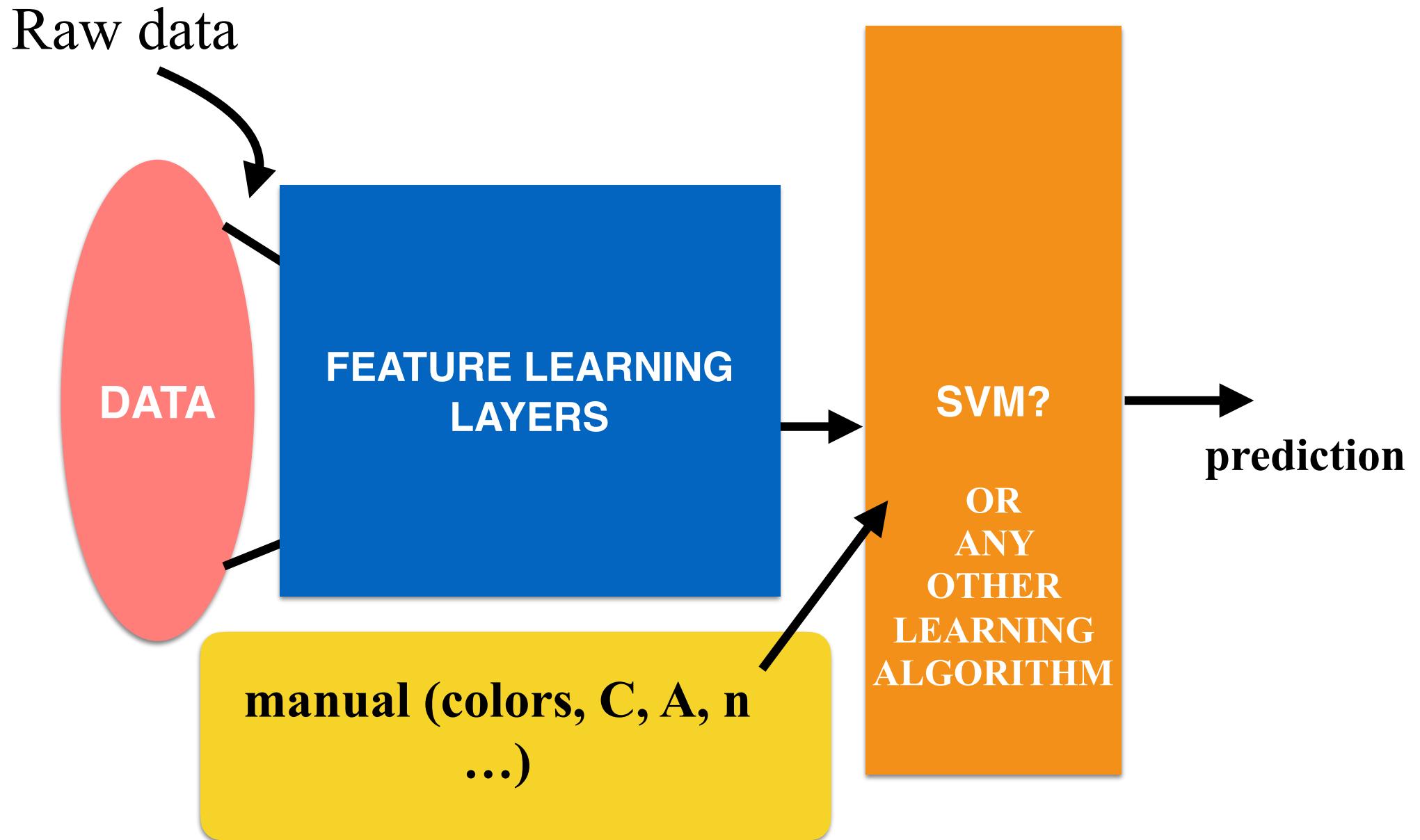
THE LEARNING ALGORITHM CAN BE CHANGED



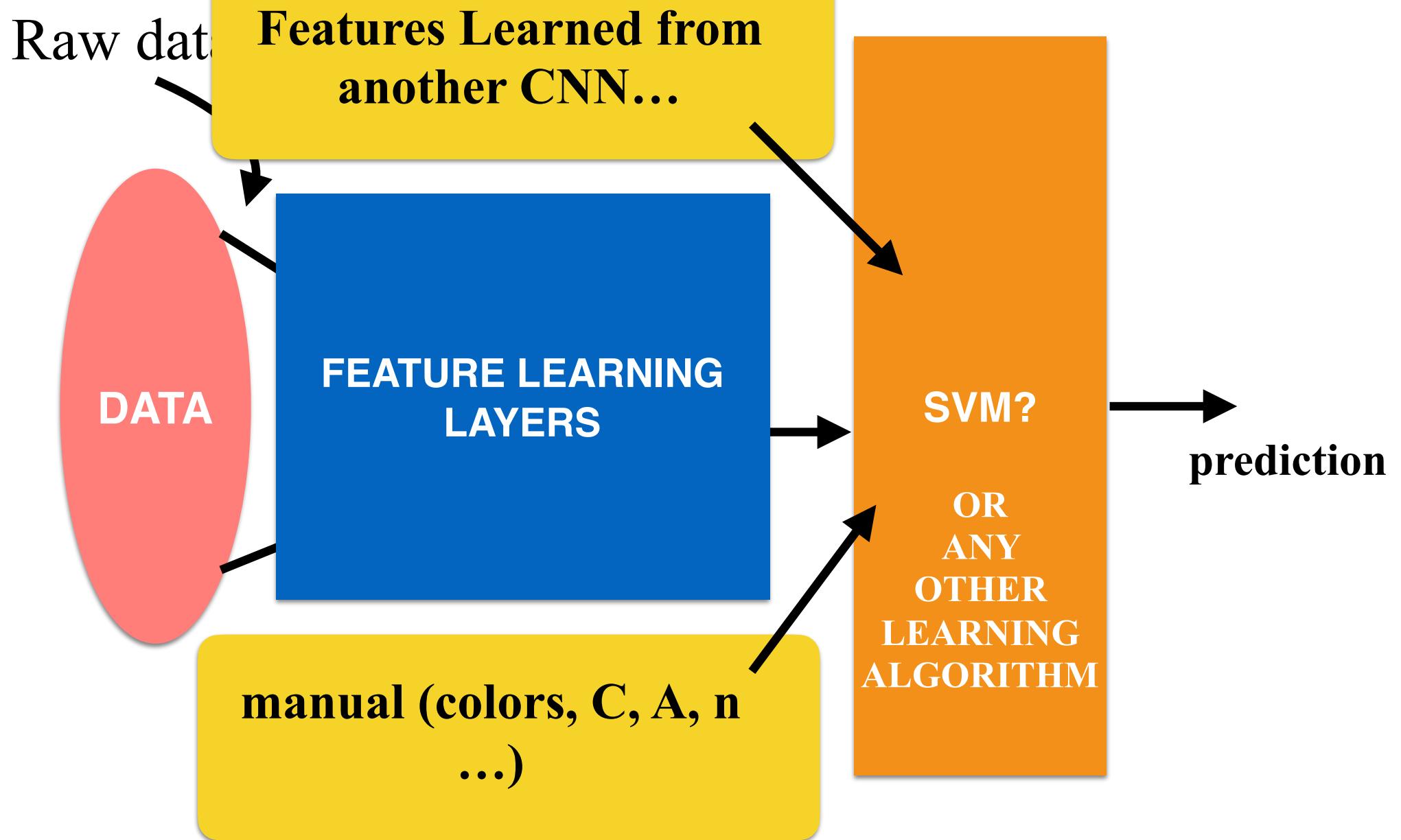
THE LEARNING ALGORITHM CAN BE CHANGED



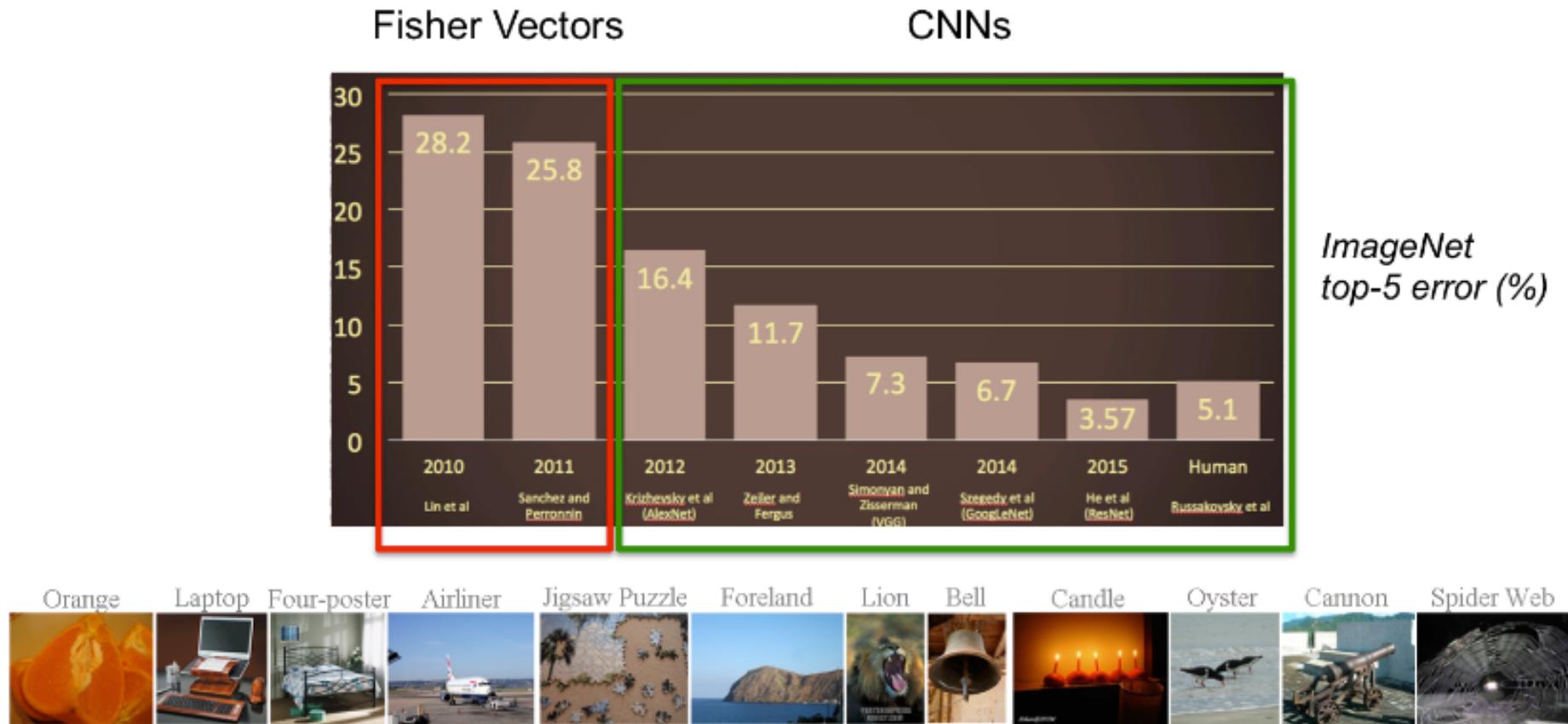
THE FEATURES CAN BE MANIPULATED OR COMBINED



THE FEATURES CAN
BE MANIPULATED OR COMBINED



THIS IS A CHANGE OF PARADIGM!



ALSO FOR GALAXY MORPHOLOGY

SVMs

CNNs

[HUERTAS-COMPANY+14]

AUTOMATIC

Late-Type

13

Early-Type

87

75

25

Early-Type



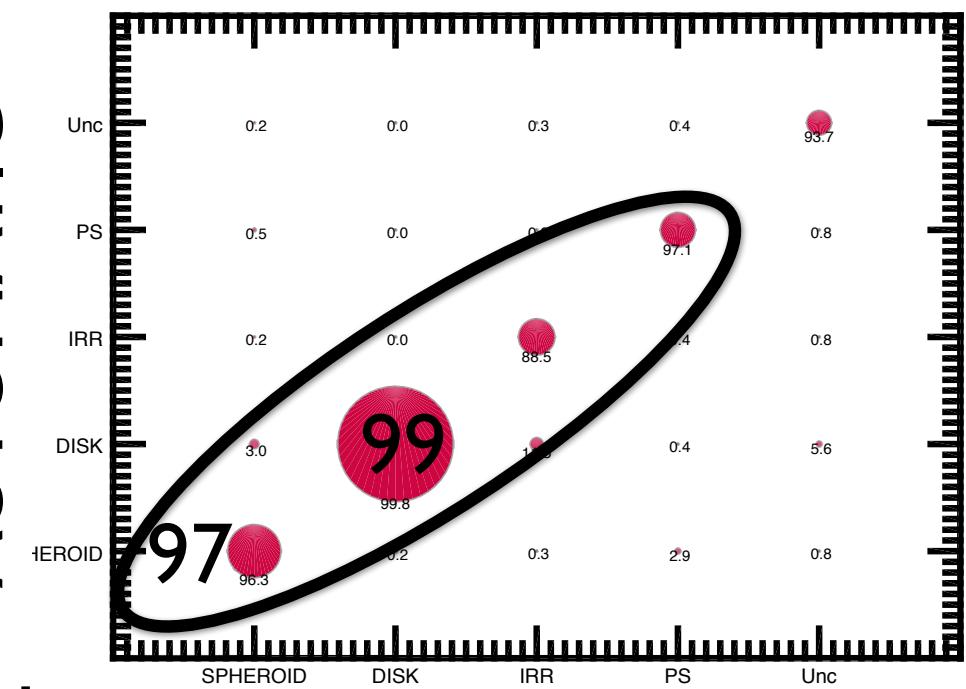
Late-Type



AUTOMATIC

[HUERTAS-COMPANY+15b]

VISUAL



VISUAL