Modal logic - solutions

Michael Franke

1   Solutions

1.1   Ex 1

Translate the following sentences into epistemic modal logic. Use as your translation key: p "the earth is round", q "the moon is made of cheese", "a" Alex, and b "Bo."

(i) If Alex believes that the moon is made of cheese, then Alex also believes that the earth is not round.

$\Box_a q \rightarrow \Box_a \neg p$

(ii) Bo believes that Alex thinks it's possible that Bo believes that the moon is made of cheese.

$\Box_b \Diamond_a \Box_b q$

(iii) Alex considers it possible that, if the moon is made of cheese, then Bo believes it.

$\Diamond_a (q \rightarrow \Box_b q)$

(iv) Whatever Bo believes about whether the earth is round, Alex believes the same.

$(\Diamond_b p \leftrightarrow \Diamond_a p) \wedge (\Box_b p \leftrightarrow \Box_a p)$

(v) Alex is uncertain about whether Bo believes that Alex believes that the moon is made of cheese.

$\Diamond_a \Box_b q \wedge \Diamond_a \neg \Box_b q$

1.2 Ex 2

$V_{M,w_1}(\Box_a p) = 1$

$V_{M,w_1}(\Box_a p \land \Box_a \Diamond_a \neg p) = 1$

$V_{M,w_3}(\Box_a(p \land \neg q)) = 1$

$V_{M,w_4}(p \to \Box_b p) = 1$

a. $\Box_a p$ in $w_1 \rightsquigarrow 0$

b. 0

c. 0

d. 1