GR 6307
Public Economics and Development

Section 1
Tooling Up: Applied Welfare Analysis
or *How I Learned to Stop Worrying and Love the Envelope Theorem*

Michael Carlos Best

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What is it All For? Goals of Public Economics

This lecture borrows heavily from Nathan Hendren's notes. A big thank you to him.

- Often ask "What government policies do the most to improve citizens' welfare?" or "Which policies should the government use to achieve a policy goal?"
  - Should high earners face a higher marginal income tax rate?
  - How can we reduce child poverty?
  - How much access should informal workers have to social insurance programs?
- Want to be able to leverage ever-increasing supply of estimates of causal impacts of a wide range of policy changes
  - "Credibility Revolution" (Angrist & Pischke, 2010)
  - See also 2010 JEP (vol 24 issue 3) symposium on theory in development economics
- So how can we take all these causal estimates and translate them into statements about *desirability* of policy changes?
  - Need coherent framework to move from "Positive" to "Normative" analysis

Outline

Welfare

- Goal: What is the welfare impact of government policy changes?
- Environment:
  - Government chooses a vector of policies P that can affect constraints and preferences
  - Individuals maximize utility s.t. budget constraint $\Rightarrow$ indirect utility $v_i(P)$

$$v_i(P) = \max_{x \in B_i(P)} u_i(x; P)$$

  - Very general: Admits households & firms, GE, discrete choices, heterogeneity etc.
- Think about how this environment can incorporate
  - Public goods expenditures?
  - Taxes?
  - Uncertainty and dynamics $\left(u_i = \mathrm{E}\left[\sum_{t \geq 0} \beta^t v_{it}\right]\right)$?
  - Externalities?

Social Welfare

- Define social welfare function (Bergson-Samuelson SWF)

$$W(P) = \int_i \psi_i v_i(P)\, di$$

- ψ_i is individual i's **Pareto weight**
  - ψ_i can be a function (Saez & Stantcheva, 2016). What can it depend on?
- Nests special (classic) case:

$$W(P) = \int_i \omega(v_i)\, di$$

  by setting $\psi_i = \omega'(v_i)$
- This will work well for small policy changes. What can happen when the changes are large? (more on this later).

Social Preferences

- Define each individual's **social marginal utility of income** $\eta_i = \psi_i \lambda_i$, where λ_i is the *individual's* marginal utility of income.

$$\lambda_i = \frac{d}{dy_i} v_i = \frac{\partial u_i}{\partial c} \text{ (why?)}$$

- The ratio of two individuals' ηs: η_i / η_j corresponds to "Okun's Bucket" (Okun, 1976)
  - If $\eta_i / \eta_j = 2$ it means we are willing to take \$1 from j to give \$0.50 to i
- These social marginal utilities of income provide general representations of social preferences
  - What has been ruled out?

Impacts of Policy Changes on Social Welfare

- $W(P)$ gives us a measure we can use to evaluate government policy changes
- Consider a (small) policy change dp (e.g. an increase in a tax rate, a change in an eligibility threshold etc)
- The first-order effect of this on welfare is:

$$\frac{dW}{dp} = \int_i \psi_i \frac{dv_i}{dp} = \int_i \eta_i \underbrace{\frac{dv_i/dp}{\lambda_i}}_{i\text{'s WTP}} = \bar{\eta}_p \int_i WTP_i$$

- $\int_i WTP_i = \int_i \frac{dv_i}{dp}/\lambda_i$ is the sum of WTP of affected individuals out of their own income for the policy
- $\bar{\eta}_p = \int \eta_i \frac{WTP_i}{\int_i WTP_i}$ is the incidence-weighted average social marginal utility of income

The Government's Budget Constraint

- Most policies (the dp) are not budget neutral, so we need to think about where the money comes from / goes
  - e.g. increased tax rates
  - expanded eligibility for transfers
  - audits
  - switching from in-kind to cash transfers
- Usually in empirical work we don't need to specify how to satisfy the government's BC in order to achieve identification.
  - But this implies that the counterfactual used for identification is generally not feasible

The Government's Budget Constraint

- Envelope theorem: behavioral changes don't have 1st-order effects on individual welfare. Only mechanical effects. But behavioral changes do have effect on government's budget – the *Fiscal Externality.* How we account for these is at the heart of how we think about welfare effects

1. CBA + MCPF (Atkinson & Stern 1974; Kaplow 2011): Estimate Cost-Benefit ratio and adjust for DWL of taxation. Imagine closing budget by increasing linear tax rate (e.g. Heckman et al 2010)
2. MEB/MDWL (Auerbach & Hines 2002 handbook). Imagine that we can use individual-specific transfers to close budget.
3. MVPF (Mayshar 1990, Slemrod & Yitzhaki 1996, Hendren 2016, 2019): Imagine the budget is closed with some other policy (that you know the MVPF of).

Slemrod & Yitzhaki (1996)

- Antecedent to Hendren MVPF. Itself builds on Mayshar (1990)
- Incorporates compliance and administrative costs (more on this in a few weeks)
- Setup: Representative taxpayer with utility $u(X)$ and budget constraint $y = \sum_i q_i X_i$, $q_i = p_i + t_i$
- Define **Marginal Burden** of a tax reform dq_i

$$MB = \sum_i X_i dq_i$$

the income equivalent of the effects of the reform.

  - Thank you Roy's identity (envelope theorem!) again. (WTP?)

Slemrod & Yitzhaki (1996)

- Tax revenue in this economy is

$$R\left(t,p,y\right)=\sum_{i}t_{i}X_{i}\left(p+t,y\right)$$

- For reform to be revenue neutral we require

$$MR=\sum_{i}\frac{\partial R}{\partial t_{i}}dt_{i}=\sum_{i}MR_{i}dt_{i}=0$$

  where MR_i is the *total* change in tax revenue due to changing t_i
- Denoting the revenue effects $\delta_i = MR_i dt_i$ we can see that

$$MB=\sum_{i}\frac{X_{i}}{MR_{i}}\delta_{i}$$

  where revenue neutrality requires $\sum_i \delta_i = 0$

Slemrod & Yitzhaki (1996)

- The object X_i/MR_i is the **Marginal Efficiency Cost of Public Funds** (MECF). It's the cost to society of increasing tax revenue by a dollar through a change in the ith tax rate. It's the taxpayer's willingness to pay to avoid dt_i over the effect of the reform on the goverment's budget.
- Example: Revenue-neutral tax reform involving two taxes: $\delta_1 = -\delta_2$

$$MB = \left[\frac{X_1}{MR_1}\delta_1 + \frac{X_2}{MR_2}\delta_2\right] = (MECF_1 - MECF_2)\,\delta_1$$

- Now we can see what types of reforms increase welfare:
  - if $MECF_2 > MECF_1$ then you want $\delta_1 > 0$ (shift towards t_1)
  - if $MECF_2 < MECF_1$ then you want $\delta_1 < 0$ (shift towards t_2)
- Note that to implement this you only need X_1, X_2 (expenditures on the two goods) and MR_1, MR_2 the marginal change in tax revenue
- This can be applied to any part of the tax/transfer system, not just rates.

Slemrod & Yitzhaki (1996)

- Now incorporate avoidance/evasion, compliance costs and administrative costs.
- The tax base for t_1 is X_1, but that's not what is collected if we marginally increase t_1, we only collect MR_1:

$$X_i = \underbrace{(X_i - MR_i)}_{\text{leakage}} + MR_i$$

- What is the social value of these leaked dollars?
  - Depends on what the taxpayer is willing to give up to avoid paying that dollar.
  - Bounded from above by \$1. If it's avoidance, you'd pay up to a dollar to avoid that tax. If it's evasion, you'd bear up to \$1 of increased risk to evade. Doesn't really matter what the mechanism is, just the willingness to pay. Call the marginal cost γ

Slemrod & Yitzhaki (1996)

- Now add in marginal compliance costs to the taxpayer of C_i and marginal administrative costs to the government of A_i

$$MECF_i = \frac{\gamma\left(X_i - MR_i\right) + C_i + MR_i}{MR_i - A_i} = \frac{WTP}{\text{Effect on govt budget}}$$

- Can also add in distributional concerns. Now we want to weight the burdens on the taxpayers h by their social marginal utility of income:

$$MECF_i = \frac{\frac{\sum_h \eta_h x_{ih}}{\sum_h \eta_h}}{MR_i}$$

Hendren (2016) Hendren & Sprung-Keyser (2020)

- These concepts re-cast and shown to be empirically valuable in Hendren (2016) and Hendren & Sprung-Keyser (2020)
- But with a different name, the **Marginal Value of Public Funds (MVPF)**
- Write the government budget in very general terms

$$R(P) = g(P) - \int_i T(y_i; P)$$

  - $g(P)$ is government spending
  - $T(\cdot; P)$ is the tax schedule
  - y_i are taxable earnings
- Define $G = dR/dP$: the marginal impact of the policy change on the government budget

$$G = \frac{dg}{dp} - \int_i \left(\frac{\partial T}{\partial y_i} \frac{dy_i}{dp} + \frac{\partial T}{\partial p} \right)$$

  - This includes *all* fiscal externalities from all behavioral responses to the policy.

Hendren (2016) Hendren & Sprung-Keyser (2020)

- Define the **Marginal Value of Public Funds (MVPF)** of policy p as

$$MVPF_p = \frac{\int_i WTP_i}{G} = \frac{\text{Willingness to Pay}}{\text{Net Cost}}$$

- Thought experiment: \$1 of government spending on the policy delivers $\$MVPF$ of benefits to the policy's beneficiaries.
- And \$1 of spending delivers $\bar{\eta}_p MVPF_p$ in social welfare.
  - recall $\bar{\eta}_p = \int \eta_i \frac{WTP_i}{\int_i WTP_i}$ captures the distributional concerns: It's the incidence-weighted marginal social welfare weights.

Hendren (2016) Hendren & Sprung-Keyser (2020)

- With the MVPF we can try and construct portfolios of policy reforms that increase welfare
- Take two (non budget-neutral) policies: policy 1 and policy 2
- Combine into a budget-neutral policy dp that increases spending on policy 1 financed by decreasing spending on policy 2
- To first order, this is a "good" idea $(dW/dp > 0)$ if and only if

$$\bar{\eta}_p MVPF_p > 0 \iff \bar{\eta}_1 MVPF_1 - \bar{\eta}_2 MVPF_2 > 0$$

- The MVPF normalizes the net government spending on the two policies so we can compare them.
- Note that a lot of *normative* ("should") judgements are embeded in the $\bar{\eta}$s. $\Rightarrow$ more straightforward to make comparisons between policies that have similar $\bar{\eta}$s. i.e. policies that have similar sets of beneficiaries.

Hendren (2016) Hendren & Sprung-Keyser (2020)

- An example: What is the MVPF of a (small) reduction in the marginal income tax rate τ by $d\tau$?
  - τ is the marginal tax rate on earnings y (linear tax schedule for simplicity, but generalizes easily)
  - Average earnings in the population are $\mathsf{E}[y]$.
- Start with the denominator: Government revenue is

$$R = \tau \mathsf{E}[y]$$

- So a small tax cut has a marginal effect on government revenue of

$$G_{d\tau} = \frac{dR}{d\tau} = \mathsf{E}[y] + \tau \frac{d\mathsf{E}[y]}{d\tau} = \mathsf{E}[y](1+\varepsilon)$$

  where $\varepsilon \equiv \frac{\tau}{\mathsf{E}[y]} \frac{\partial \mathsf{E}[y]}{\partial \tau}$ is the elasticity of the tax base wrt the tax rate
  - Note ε captures the *causal* effect of the tax change on tax revenue.

Hendren (2016) Hendren & Sprung-Keyser (2020)

- Now turn to the numerator: WTP. Here is where envelope theorem is powerful
  - To first order, individuals don't value changes in their incomes (by EVT, since income chosen optimally)
  - So if you earn \$100 and taxes go from 25% to 24% you lose \$1 and your WTP to avoid that is \$1, regardless of how you change earnings (what about evasion?).

$$\frac{d}{d\tau}\frac{v_i}{\lambda_i} = y_i$$

- So, the average WTP for the change $d\tau$ is $\mathsf{E}\left[y\right]$ and the MVPF is

$$MVPF_{d\tau} = \frac{\mathsf{WTP}_{d\tau}}{G_{d\tau}} = \frac{\mathsf{E}\left[y\right]}{\mathsf{E}\left[y\right](1+\varepsilon)} = \frac{1}{1+\varepsilon}$$

- So the key thing we need to know is the causal effect of changing tax rates on government revenue. For every \$1 of a tax cut, by how much do individuals change their incomes (and all other taxable behavior).
- At home: Try thinking about how these expressions change if the tax cut is only for high earners above a threshold $\bar{y}$.

Hendren (2016) Hendren & Sprung-Keyser (2020)

- Extreme cases of the MVPF:
- What happens if $\varepsilon < -1$?
  - the policy now “pays for itself”: cutting taxes by $1 increased income by more than $1
  - This is the “Laffer” effect, we were beyond the Laffer rate.
- Define $MVPF = \infty$ when $WTP > 0$ and $G < 0$ (and $MVPF = -\infty$ when $WTP < 0$ and $G > 0$)
- Policies with $MVPF = \infty$ pay for themselves.
- $MVPF = \infty$ generalizes the Laffer idea that it is possible for a tax cut to pay for itself to other government policies
- Surprisingly many examples of $MVPF = \infty$ despite very few of them being taxes!

Outline

Welfare Concepts

Application of MVPF

Sufficient Statistics

Outline

Application of MVPF

Hendren & Sprung-Keyser (QJE 2020) *A Unified Welfare Analysis of Government Policies*

Hendren & Sprung-Keyser (2020)

- Hendren & Sprung-Keyser take the MVPF framework and apply it to 133 government policies (social insurance, education and job training, taxes and cash transfers, in-kind transfers...)
- You can see all of them (and more that other people have added) at www.policyimpacts.org
- To build sample, use survey and review articles
- Assess robustness to several assumptions
  - parameters of the program evaluation (discount rates, tax rates, etc.)
  - extrapolating observed effects
  - credibility of design (RCT/RD/DD; peer revived vs not...)
  - publication bias (Andrews & Kasy 2018)
  - missing causal estimates
- All the code is in GitHub

Hendren & Sprung-Keyser (2020)

- Example: What is the MVPF of admission to Florida International University?
- FIU had a minimum GPA threshold for admission $\Rightarrow$ Fuzzy RDD
- Zimmerman (2014) uses this to examine impact of FIU admission on earnings up to 14 years post admission

Hendren & Sprung-Keyser (2020)

- Impact of college attendance on earnings: Zimmerman (2014)

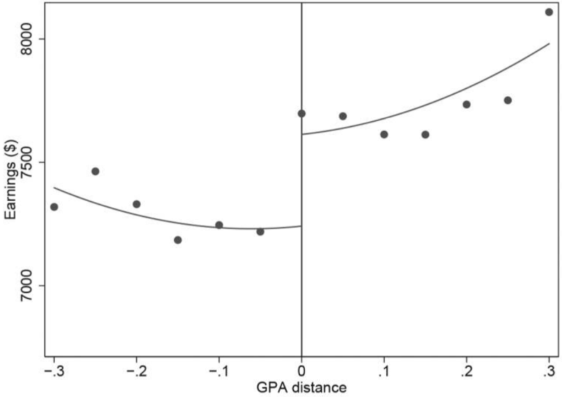
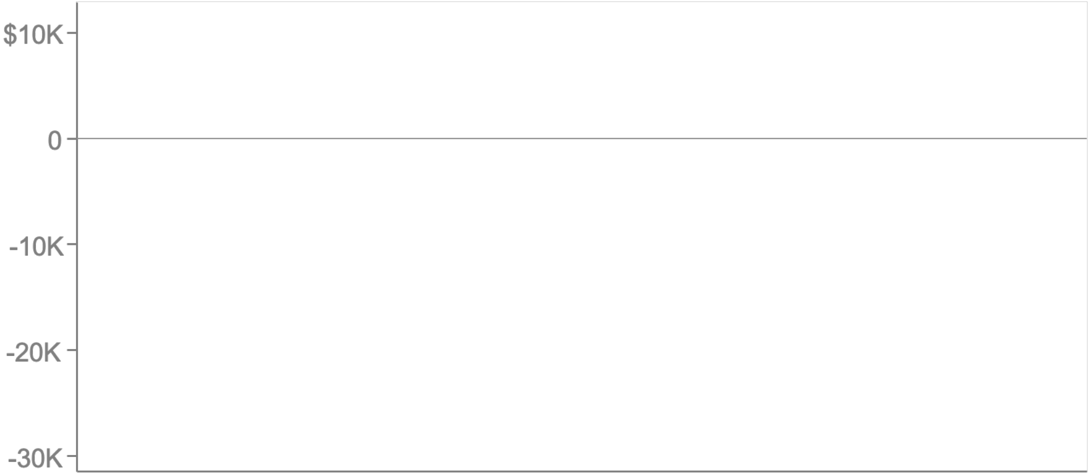


FIG. 8.—Quarterly earnings by distance from GPA cutoff. Lines are fitted values based on the main specification. Dots, shown every .05 grade points, are rolling averages of values within .05 grade points on either side that have the same value of the threshold-crossing dummy.

Hendren & Sprung-Keyser (2020)



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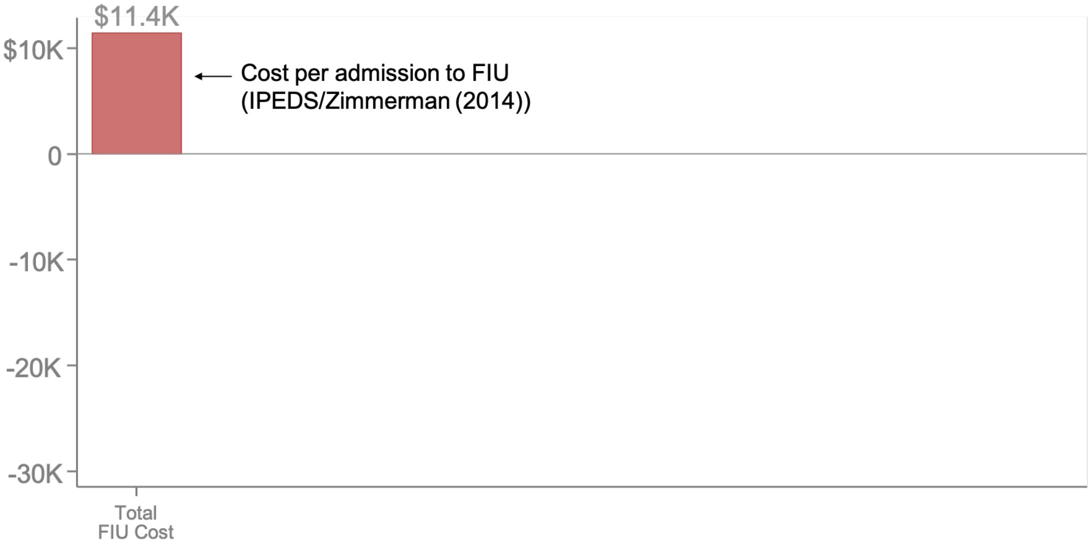
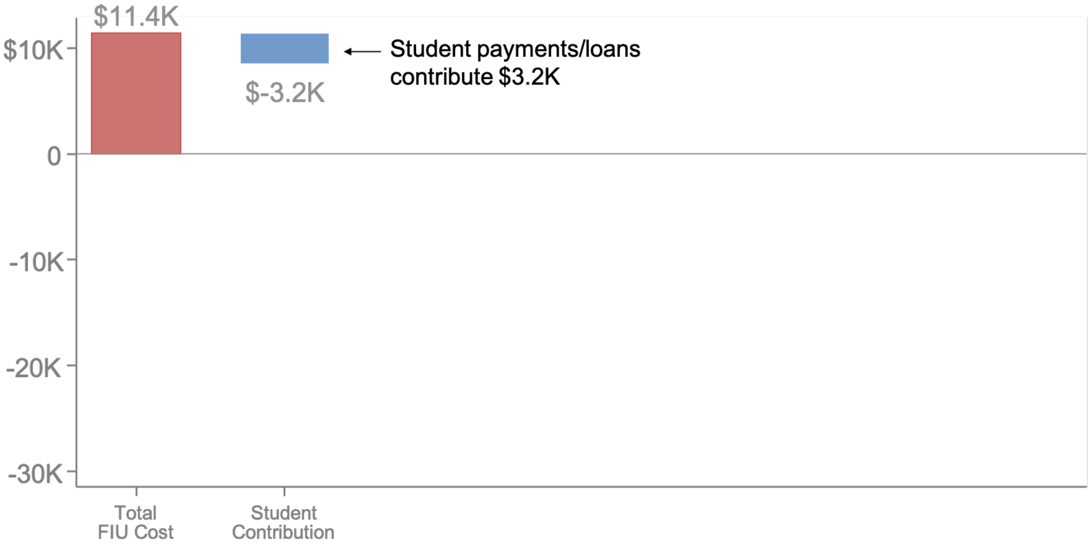


Hendren & Sprung-Keyser (2020)



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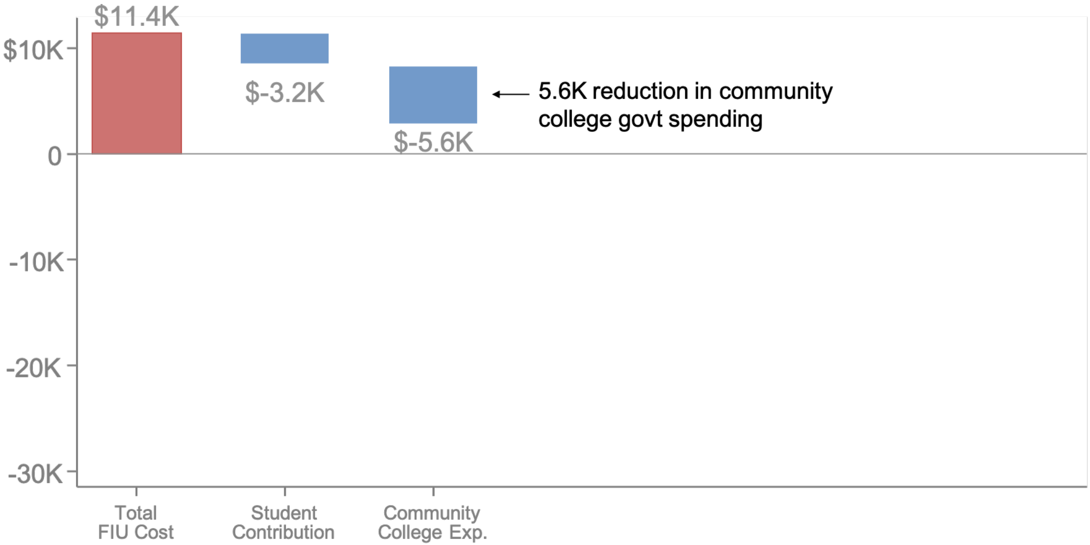

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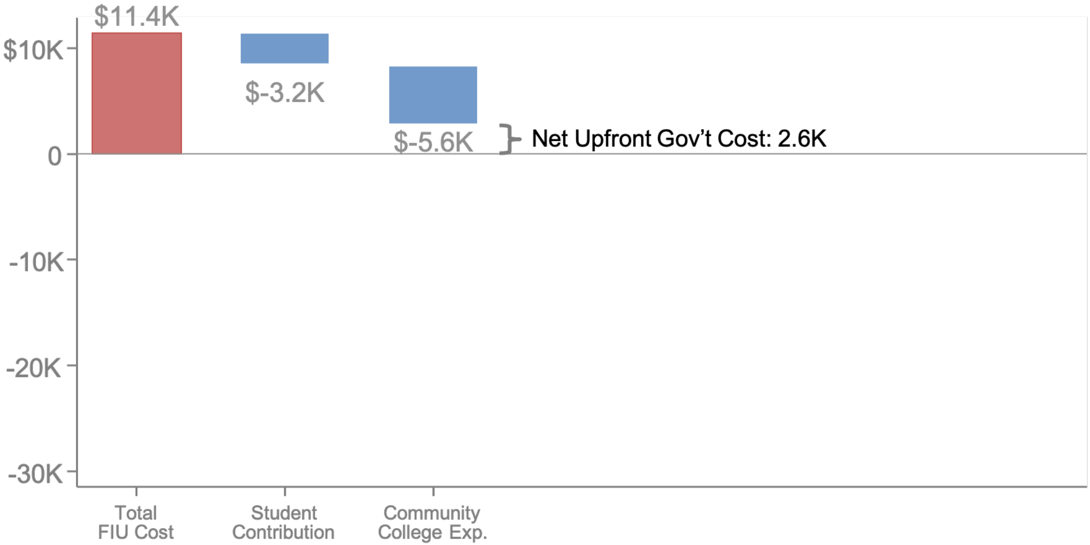

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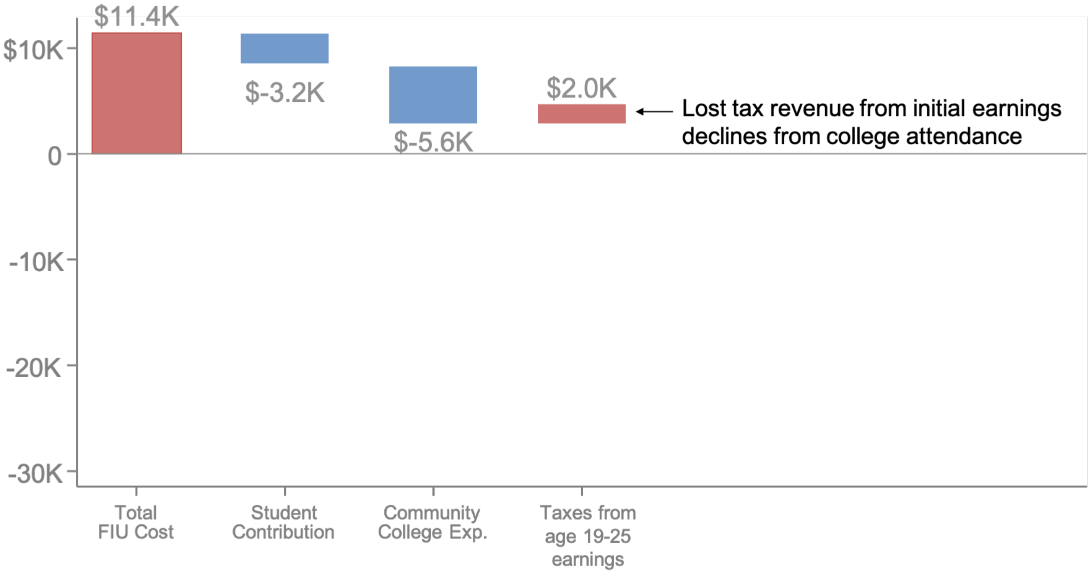
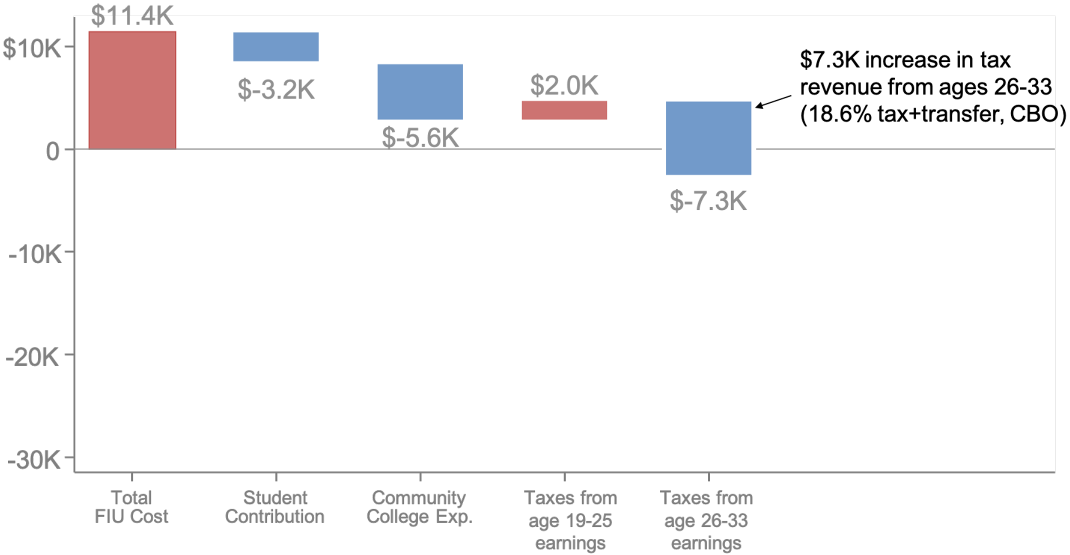


Hendren & Sprung-Keyser (2020)



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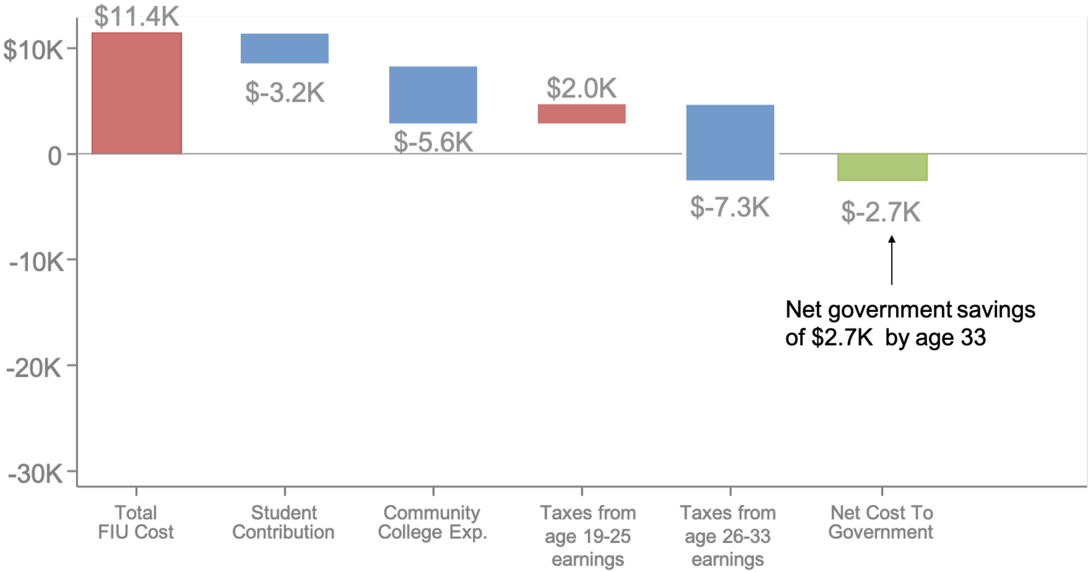

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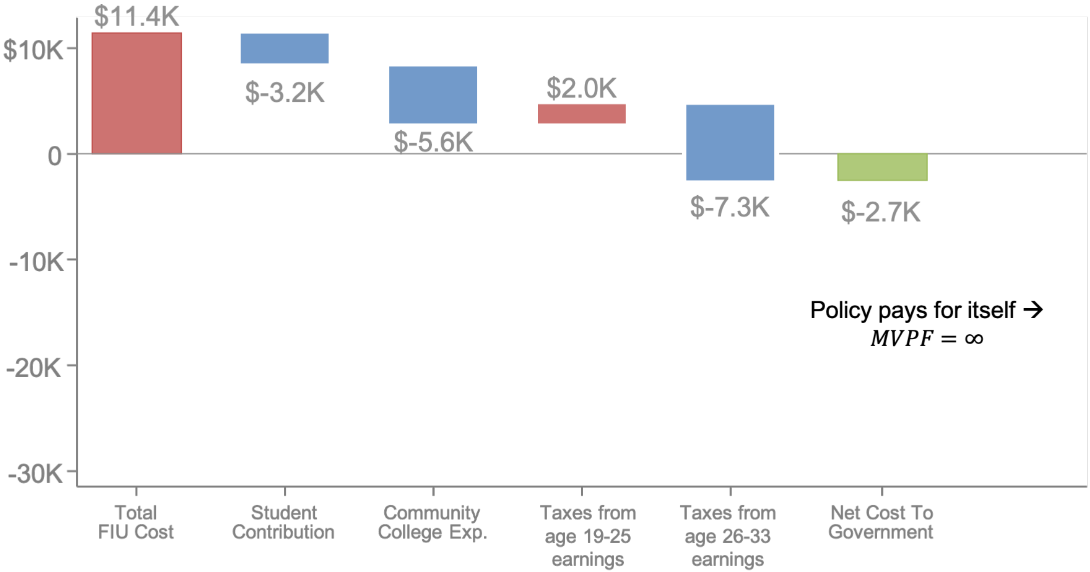

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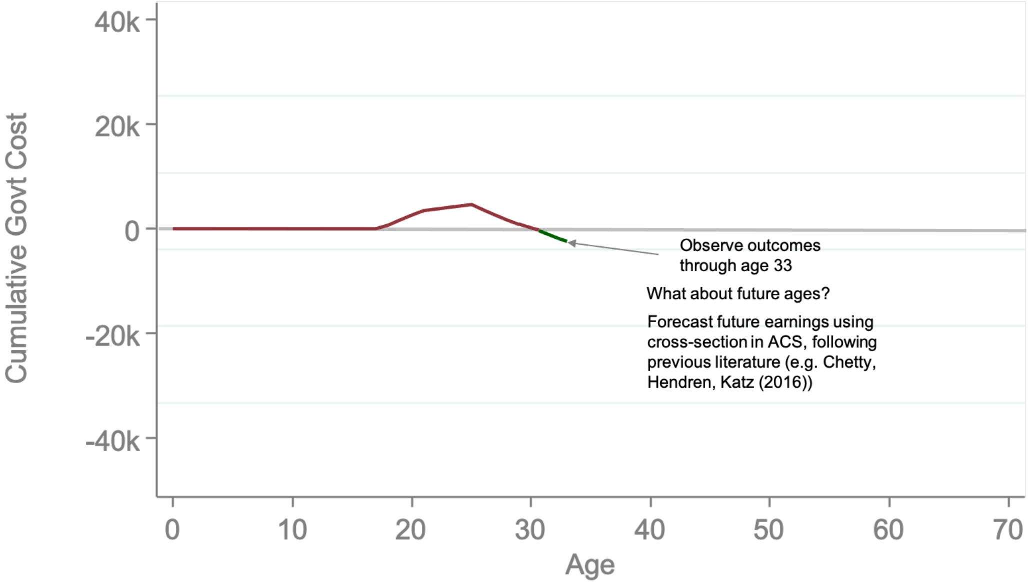

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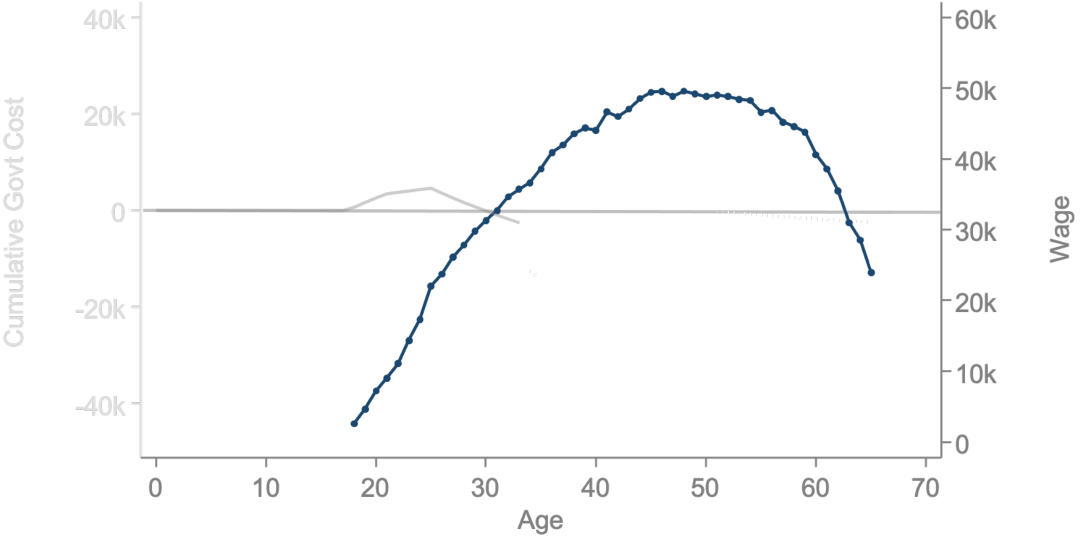

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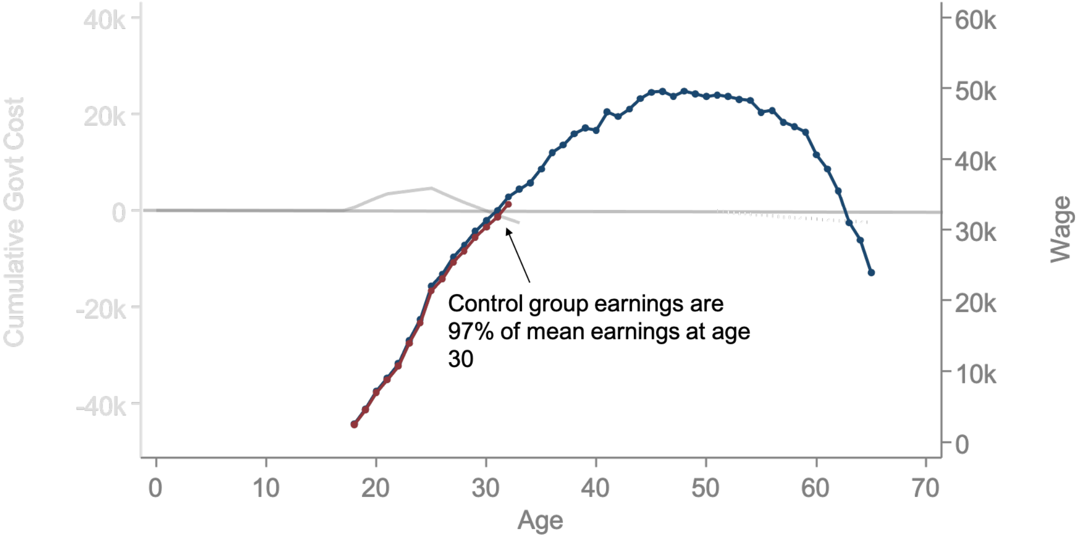

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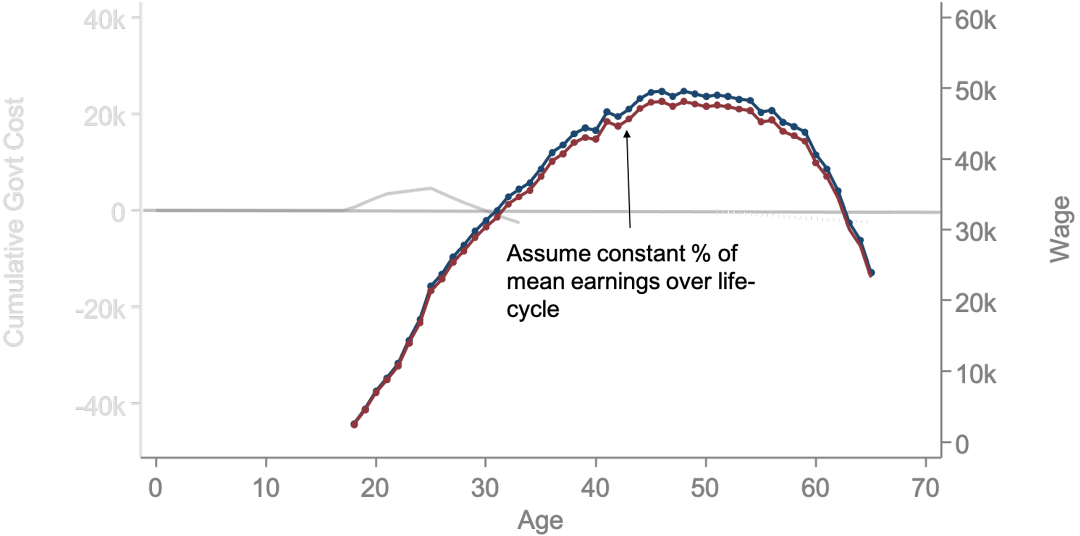

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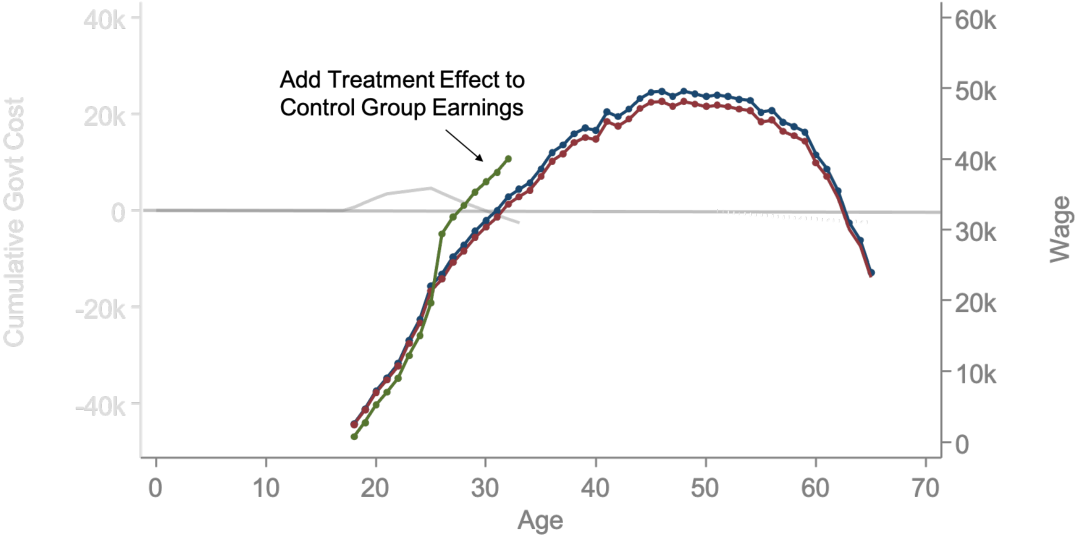

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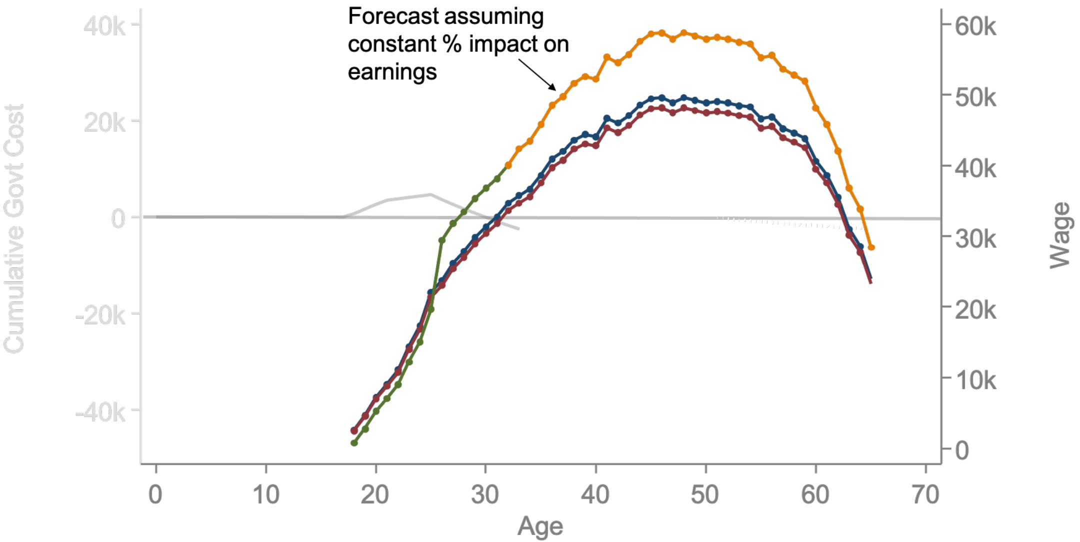

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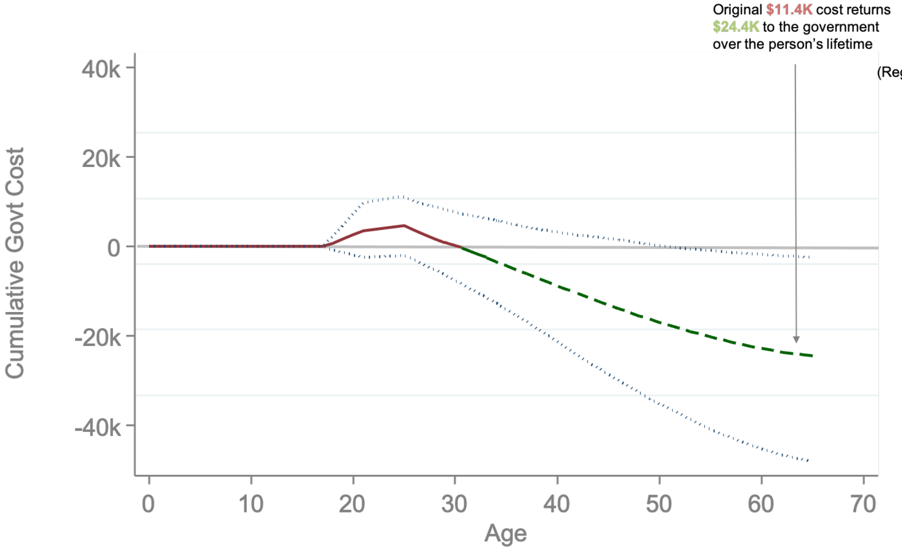
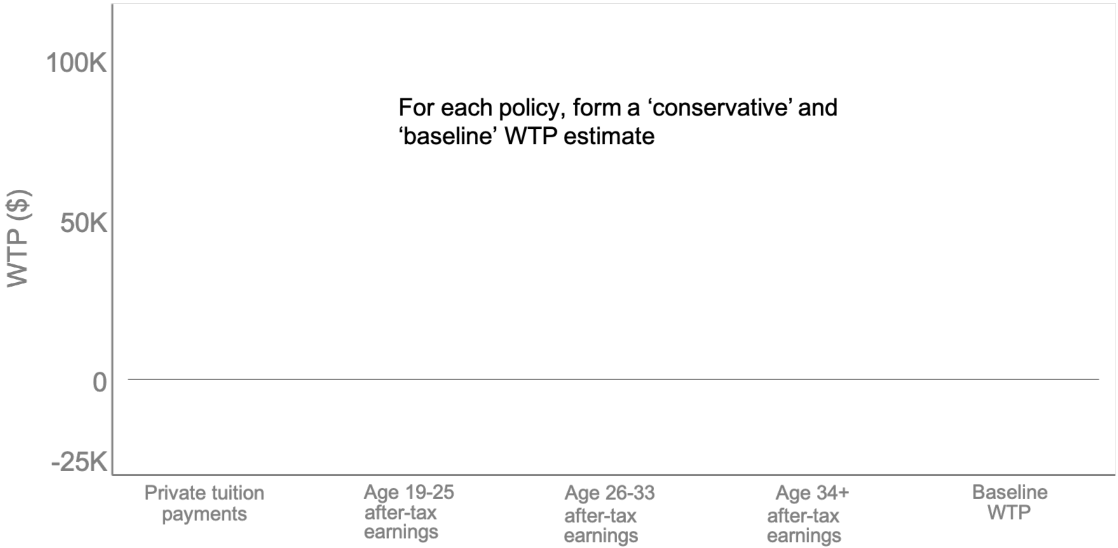


$$MVPF = \infty$$

(Regardless of WTP)

Hendren & Sprung-Keyser (2020)



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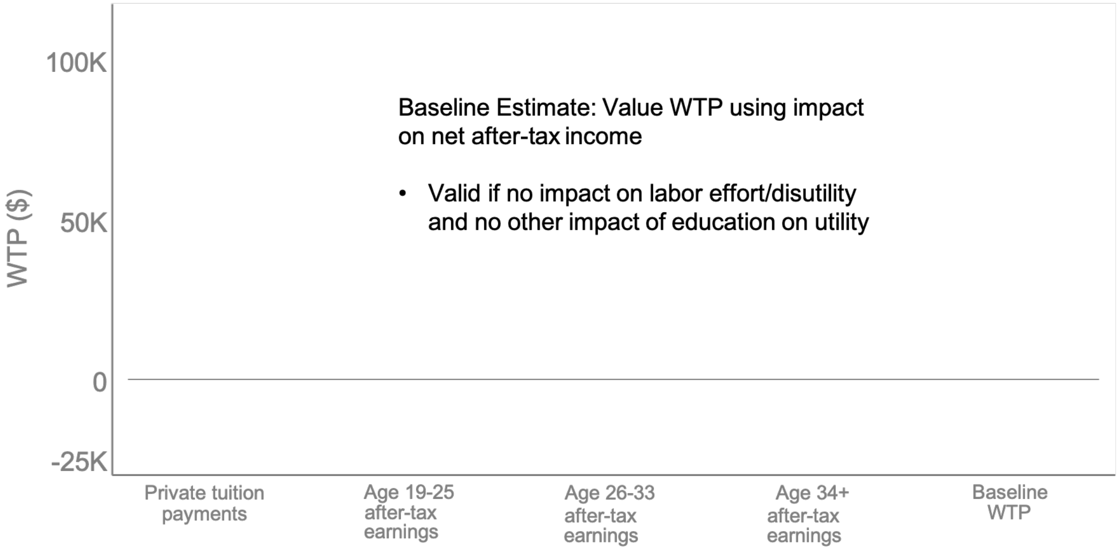

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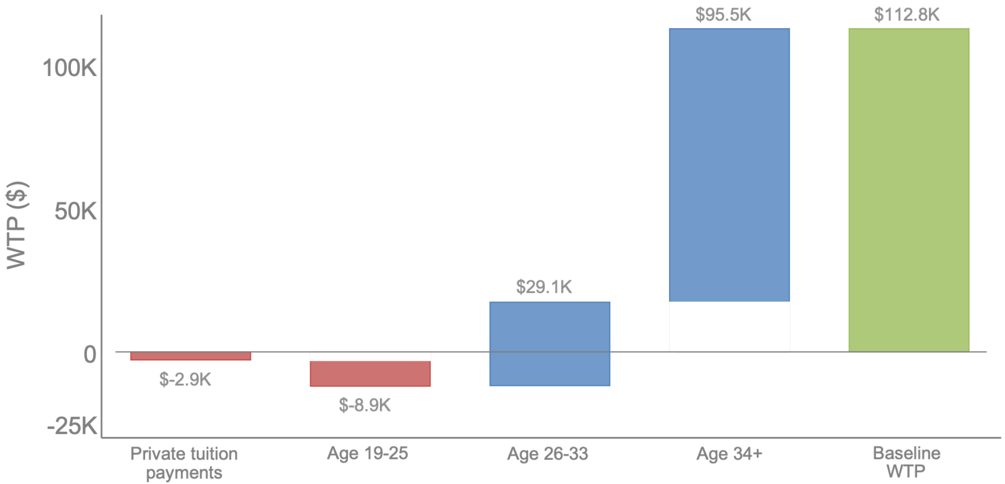
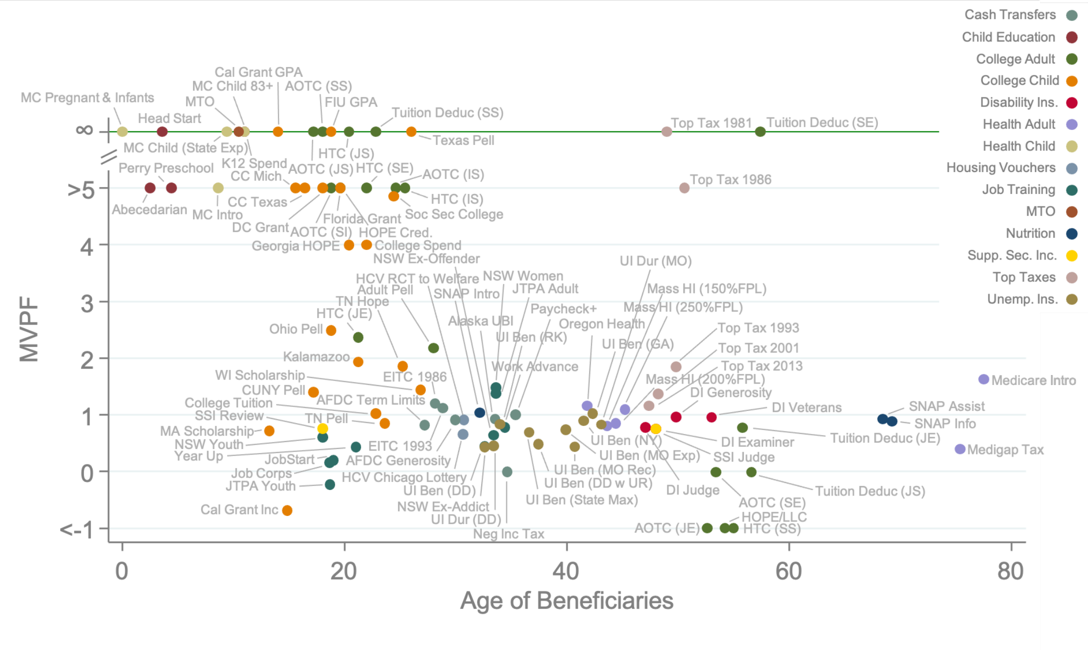
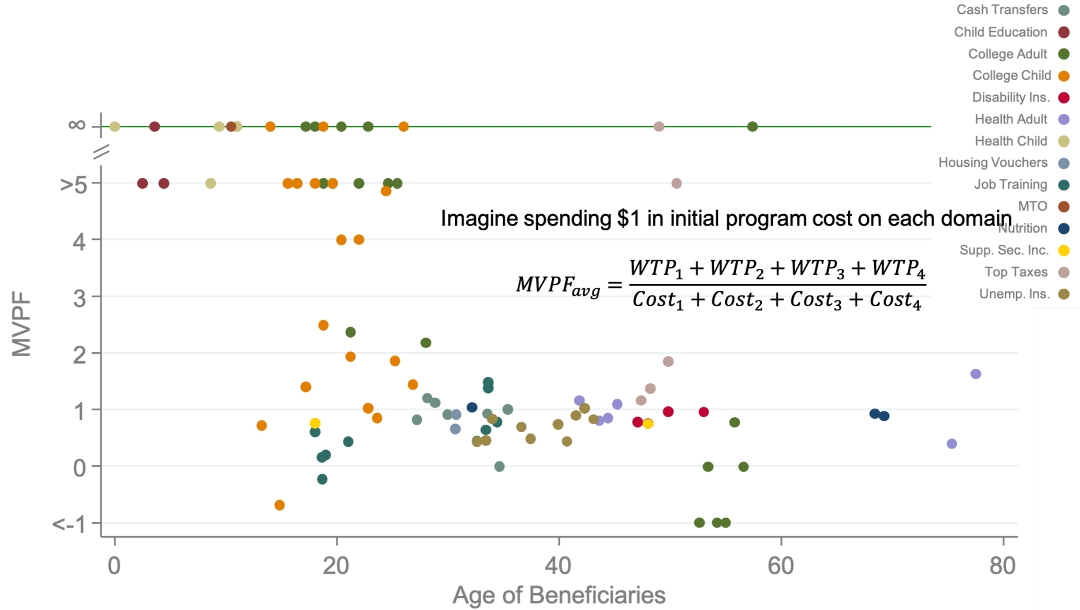




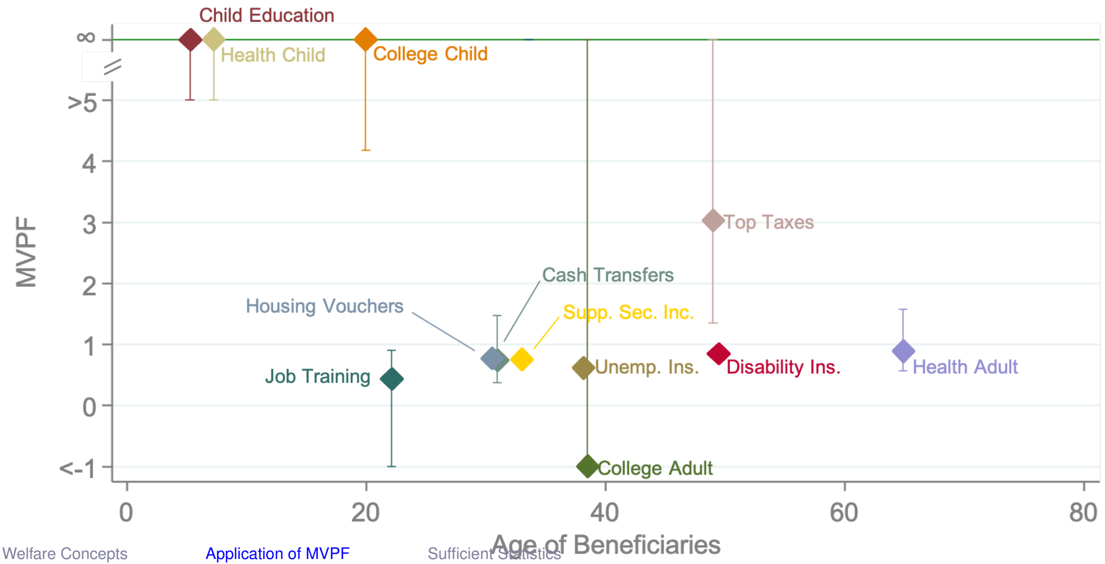
Cash Transfers
Child Education
College Adult
College Child
Disability Ins.
Health Adult
Health Child
Housing Vouchers
Job Training
MTO
Nutrition
Supp. Sec. Inc.
Top Taxes
Unemp. Ins.
MVPF
Age of Beneficiaries
∞
>5
4
3
2
1
0
<-1
0
20
40
60
80
MC Pregnant & Infants
Head Start
MC Child 83+
MTO
Cal Grant GPA
AOTC (SS)
FIU GPA
Tuition Deduc (SS)
Texas Pell
MC Child (State Exp)
HTC (JS)
Perry Preschool
K12 Spend
CC Mich
AOTC (JS)
HTC (SE)
AOTC (IS)
HTC (IS)
Soc Sec College
Abecedarian
MC Intro
CC Texas
DC Grant
Florida Grant
AOTC (SI)
HOPE Cred.
Georgia HOPE
College Spend
NSW Ex-Offender
Top Tax 1981
Tuition Deduc (SE)
Top Tax 1986
UI Dur (MO)
HCV RCT to Welfare
NSW Women
Adult Pell
SNAP Intro
JTPA Adult
Mass HI (150%FPL)
TN Hope
Paycheck+
Mass HI (250%FPL)
HTC (JE)
Ohio Pell
Alaska UBI
Oregon Health
UI Ben (RK)
UI Ben (GA)
Top Tax 1993
Top Tax 2001
Kalamazoo
Work Advance
Top Tax 2013
Mass HI (200%FPL)
WI Scholarship
EITC 1986
DI Generosity
CUNY Pell
AFDC Term Limits
College Tuition
DI Veterans
SSI Review
TN Pell
SNAP Assist
SNAP Info
Medicare Intro
MA Scholarship
NSW Youth
Year Up
EITC 1993
UI Ben (NY)
DI Examiner
Tuition Deduc (JE)
Medigap Tax
JobStart
AFDC Generosity
UI Ben (MO Exp)
SSI Judge
Job Corps
HCV Chicago Lottery
UI Ben (MO Rec)
JTPA Youth
UI Ben (DD)
UI Ben (DD w UR)
DI Judge
Tuition Deduc (JS)
NSW Ex-Addict
UI Ben (State Max)
AOTC (SE)
Cal Grant Inc
UI Dur (DD)
HOPE/LLC
Neg Inc Tax
AOTC (JE)
HTC (SS)



Imagine spending $1 in initial program cost on each domain

$$MVPF_{avg} = \frac{WTP_1 + WTP_2 + WTP_3 + WTP_4}{Cost_1 + Cost_2 + Cost_3 + Cost_4}$$

Hendren & Sprung-Keyser (2020)



Hendren & Sprung-Keyser (2020)

- What about other ways of doing welfare comparisons?
- Another (historically) popular approach is to calculate the Marginal Excess Burden (MEB) of each policy
- 2 ways you can think of the thought experiment:

1. How much extra revenue could the government get if the policy change is implemented but utility is held constant by using individual specific lump-sum transfers?
2. How much are individuals willing to pay for the policy if the government closes the budget constraint through taxes using individual-specific, non-distortionary transfers

- How different is this from the MVPF? It's all in the budget constraint!

Hendren & Sprung-Keyser (2020)

- Individuals face budget constraint $c \leq (1-\tau)\, y + t$ where t is lump sum transfer
- Consider a policy reform $d\tau^c$ that raises taxes by $d\tau$ and rebates lump sum dt

$$G_{d\tau^c} = \frac{dR}{d\tau^c} = \underbrace{\mathsf{E}[y] + \tau\frac{d\mathsf{E}[y]}{d\tau}}_{\text{tax rate cut}} - \underbrace{\mathsf{E}[y] - \tau\frac{d\mathsf{E}[y]}{dt}}_{\text{lump-sum}}$$

$$= \tau\left(\underbrace{\frac{d\mathsf{E}[y]}{d\tau}}_{\text{inc + subs eff.}} - \underbrace{\frac{d\mathsf{E}[y]}{dt}}_{\text{inc eff only}}\right)$$

- Normalize by WTP ($\mathsf{E}[y]$) and

$$MEB = \varepsilon^c$$

- ε^c is the **compensated** elasticity of tax revenue (only the substitution effect)

Hendren & Sprung-Keyser (2020)

- Two things make using the MEB hard:

1. Need the *compensated* elasticities. But typically the causal estimates that we have are of *uncompensated* effects. So we would need to estimate income effects (hard!)
2. Individual-specific transfers are not really feasible, that's how we got the whole field of optimal taxation!

- Nevertheless, if you can define and estimate the right quantities, you can compare the MEB across policies.

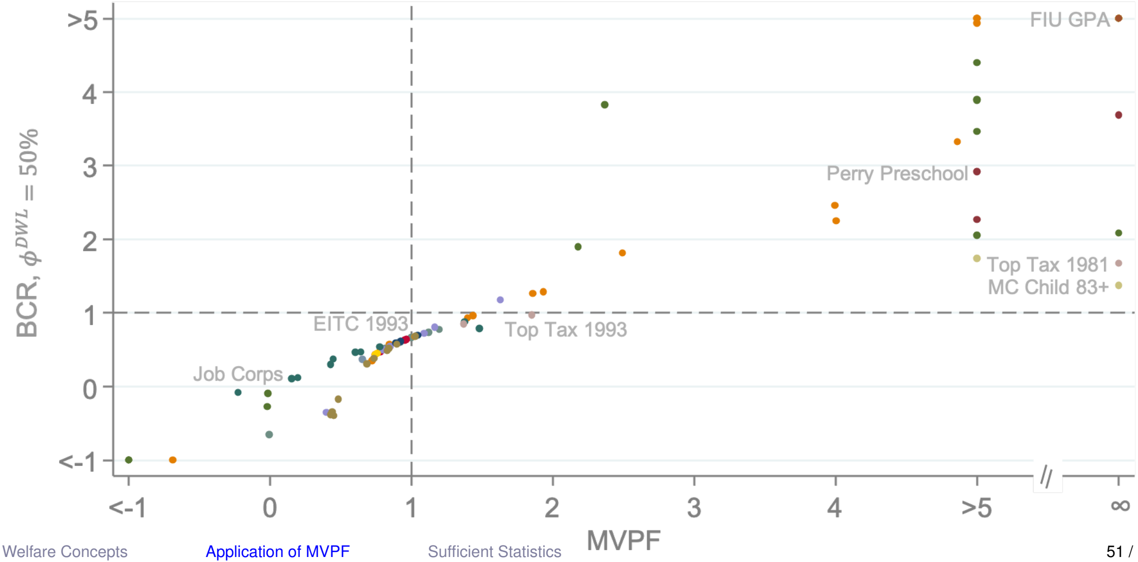
Hendren & Sprung-Keyser (2020)

- Another common approach is to estimate Benefit Cost Ratios (BCRs). Heckman et al 2010, much of regulatory policy.
- Compare the net benefits to the program cost:

$$BCR = \frac{\text{Social Benefits} - \text{Social Costs}}{\text{Program Costs}\,(1 + \phi^{DWL})}$$

- Multiply the cost in the denominator to account for excess burden of taxation
- Revenue effects appear in the numerator

Hendren & Sprung-Keyser (2020)



Outline

Outline

Sufficient Statistics

- Chetty (ARE 2009) *Sufficient Statistics as Bridge*
- Kleven (2019) *Sufficient Statistics Revisited*

Chetty (2009): 2 Competing Paradigms

- We can characterize 2 competing paradigms for policy evaluation & welfare analysis

1. **Structural:** specify a *complete* model, and estimate or calibrate the model's primitives.
2. **Reduced form:** prioritize clean *identification* of causal effects. Accept narrower scope of analysis.

- PRO structural / CON reduced form:
    1. Estimate statistics that are policy-invariant parameters of models.
    2. Can simulate effects of changes in policies on behavior and welfare.
- PRO reduced form / CON of structural approach:
    1. (quasi-)experimental research designs achieve compelling estimates of treatment effects
    2. Need to estimate all primitive parameters. Impossible to be compelling (selection, simultaneity, omitted variables etc)

Chetty (2009): A Bridge Between the 2

- Public Economics has pioneered an approach to compromise between the two: **Sufficient Statistics.**
- Setup:
  - Policy instrument t
  - Social welfare $W(t)$ (e.g. $\sum_h \gamma_h V_h(t)$)

$$\text{What is } \frac{dW(t)}{dt}??$$

- Structural approach:
  1. Write model with primitives $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$
  2. Derive
  $$\frac{dW(t)}{dt} = f(\boldsymbol{\omega})$$
  3. Estimate $\boldsymbol{\omega}$
  4. Calculate $dW(t)/dt$

Chetty (2009): A Bridge Between the 2

Primitives	Sufficient statistics	Welfare change
ω_1, ω_2, ..., ω_N	$\beta_1(t)$, $\beta_2(t)$	$\frac{dW}{dt}(t)$
ω = preferences, constraints	$\beta = f(\omega, t)$ $y = \beta_1 X_1 + \beta_2 X_2 + \varepsilon$	dW/dt used for policy analysis

Chetty (2009): Benefits

1. Simpler to estimate.
   1.1 Less data and variation needed to identify marginal treatment effects than full structural model
   1.2 Especially beneficial with heterogeneity and discrete choice (lots of primitives, still few MTEs)
2. Weaker assumptions and design-based empirical methods.
   2.1 more transparent and empirically credible estimates.
3. Can be implemented even when we're uncertain about what the right model is.

Chetty (2009): Costs

1. Each question requires its own sufficient-statistics formula
    1.1 e.g. unemployment benefit level vs duration of unemployment benefits; tax rate vs tax base etc.
    1.2 In some settings it might be hard to characterize the sufficient statistics formula.
2. More potential to be misapplied: A little bit of knowledge is a dangerous thing!
    2.1 One can draw policy conclusions from a sufficient-statistics formula without evaluating the validity of the model it is based on. Structural requires full estimation of the model so can only draw conclusions from models that fit the data.

Precedent: Harberger (1964)

- Remember Harberger's deadweight loss triangle?
- That's the first sufficient statistics formula!
- The sufficient statistic is the elasticity of equilibrium quantity of the taxed good wrt its after-tax price
- The structural primitives are the demand- and the supply-elasticities of all the goods in the economy

Precedent: Harberger (1964)

- Consider a static, general equilibrium model.
- An individual is endowed with Z units of numeraire good y (think of it as labor)
- Firms use the numeraire as input to production of J consumption goods $\boldsymbol{x} = (x_1, \ldots, x_J)$ with convex cost functions $c_j(x_j)$
- Total cost of production is $c(\boldsymbol{x}) = \sum_{j=1}^{J} c_j(x_j)$. Production is perfectly competitive.
- Government taxes good 1 at rate t. $\boldsymbol{p} = (p_1, \ldots, p_J)$ is the vector of (endogenous) pretax prices

Precedent: Harberger (1964)

- Consumer takes prices as given and maximizes quasi-linear utility:

$$\max_{\boldsymbol{x},y} u\left(x_1,\ldots,x_J\right)+y$$
$$\text{s.t. } \boldsymbol{p}\cdot\boldsymbol{x}+tx_1+y=Z$$

- Firms take prices as given and solve

$$\max_{\boldsymbol{x}} \boldsymbol{p}\cdot\boldsymbol{x}-c\left(\boldsymbol{x}\right)$$

- These two problems give us demand and supply of the J goods: $\boldsymbol{x}^D\left(\boldsymbol{p}\right)$ and $\boldsymbol{x}^S\left(\boldsymbol{p}\right)$
- Markets clear to close the model: $\boldsymbol{x}^D\left(\boldsymbol{p}\right)=\boldsymbol{x}^S\left(\boldsymbol{p}\right)$

Precedent: Harberger (1964)

- What is the welfare cost of the tax t? It's the loss of social surplus from transactions that fail to take place because of the tax.
- Conceptual experiment: what is the loss in welfare if we raise the tax rate and rebate the revenues lump sum to consumers?

$$W(t) = \underbrace{\left\{\max_{\boldsymbol{x}} u(\boldsymbol{x}) + Z - tx_1 - \boldsymbol{p}(t) \cdot \boldsymbol{x}\right\}}_{\text{consumer surplus } CS(\boldsymbol{x};t)} + \underbrace{\left\{\max_{\boldsymbol{x}} \boldsymbol{p}(t) \cdot \boldsymbol{x} - c(\boldsymbol{x})\right\}}_{\text{producer surplus } PS(\boldsymbol{x};t)} + \underbrace{tx_1}_{\text{tax revenue}}$$

- Note: consumers don't take account of change in size of rebate when choosing x_1: It is a "*fiscal externality*"

Precedent: Harberger (1964)

- So how can we estimate $dW(t)/dt$?

1. Estimate J good demand and supply system to get $u(\boldsymbol{x})$ and $c(\boldsymbol{x})$. The problem is simultaneity: To get the slope of the demand and supply curves, we need $2J$ instruments!
2. Harberger's simpler approach: Exploit the power of the envelope theorem. Consumers and producers are choosing $\boldsymbol{x}$ optimally so we can ignore behavioral responses $d\boldsymbol{x}/dt$ in the curly brackets:

$$\frac{dCS(\boldsymbol{x};t)}{dt} = \frac{\partial CS(\boldsymbol{x};t)}{\partial \boldsymbol{x}}\frac{d\boldsymbol{x}}{dt} + \frac{\partial CS(\boldsymbol{x};t)}{\partial t} = \frac{\partial CS(\boldsymbol{x};t)}{\partial t}$$

(and similarly for producer surplus)

Precedent: Harberger (1964)

- Let's demonstrate this for consumer surplus
- Consumer's FOCs are

$$\frac{\partial u(\boldsymbol{x})}{\partial x_1} = p_1 + t \quad \frac{\partial u(\boldsymbol{x})}{\partial x_j} = p_j,\ j = 2, \ldots, J$$

- Totally differentiating $CS(\boldsymbol{x}; t)$

$$\frac{dCS(\boldsymbol{x};t)}{dt} = \underbrace{\sum_{j=1}^{J} \left(\frac{\partial u(\boldsymbol{x})}{\partial x_j} - p_j \right) \frac{\partial x_j}{\partial t} - t \frac{\partial x_1}{\partial t}}_{\partial CS(\boldsymbol{x};t)/\partial \boldsymbol{x}\ \partial \boldsymbol{x}/\partial t} \underbrace{- \frac{\partial \boldsymbol{p}(t)}{\partial t} \cdot \boldsymbol{x} - x_1}_{\partial CS(\boldsymbol{x};t)/\partial t}$$

$$= -\frac{\partial \boldsymbol{p}(t)}{\partial t} \cdot \boldsymbol{x} - x_1 = \frac{\partial CS(\boldsymbol{x};t)}{\partial t}$$

Precedent: Harberger (1964)

- Using the power of the envelope theorem we have

$$\begin{aligned}
\frac{dW(t)}{dt} &= \frac{dCS(\boldsymbol{x};t)}{dt} + \frac{dPS(\boldsymbol{x};t)}{dt} + \frac{dtx_1}{dt} \\
&= \frac{\partial CS(\boldsymbol{x};t)}{\partial t} + \frac{\partial PS(\boldsymbol{x};t)}{\partial t} + \frac{dtx_1}{dt} \\
&= \left(-\frac{\partial \boldsymbol{p}(t)}{\partial t}\cdot \boldsymbol{x} - x_1\right) + \left(\frac{\partial \boldsymbol{p}(t)}{\partial t}\cdot \boldsymbol{x}\right) + \left(x_1 + t\frac{dx_1}{dt}\right) \\
&= t\frac{dx_1(t)}{dt}
\end{aligned}$$

- $dx_1(t)/dt$ is a **sufficient statistic** for the welfare loss

1. Taxes induce behavioral responses $d\boldsymbol{x}/dt$ but these have no first-order effects on welfare because households and firms are optimizing (envelope theorem)
2. Taxes induce changes in prices $d\boldsymbol{p}/dt$ but these have no first-order effects on welfare, they only redistribute surplus between producers and consumers

A General Cookbook

- Here's a general cookbook (we'll focus on a single agent in a static model, but easy to generalize)
- Step 1: Specify the structure of the model. What are the agent's choices and constraints?

$$\max_{x} U(\boldsymbol{x}) \quad s.t.\ G_1(\boldsymbol{x}, t, T), \ldots, G_M(\boldsymbol{x}, t, T)$$

  where $\boldsymbol{x} = (x_1, \ldots, x_J)$ are choices, t is "tax" on x_1, $T(t)$ is transfer in units of x_J
- Solution to this problem defines welfare as a function of the policy instrument

$$W(t) = \max_{x} U(\boldsymbol{x}) + \sum_{m=1}^{M} \lambda_m G_m(\boldsymbol{x}, t, T)$$

A General Cookbook

- Step 2: Express $dW(t)/dt$ in terms of multipliers:

$$\frac{dW(t)}{dt} = \sum_{m=1}^{M} \lambda_m \left\{ \frac{\partial G_m}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial G_m}{\partial t} \right\}$$

- We know $\partial T/\partial t$ from the government budget constraint, and can calculate $\partial G_m/\partial T$ and $\partial G_m/\partial t$. The key unknowns are the λ_ms.

A General Cookbook

- Step 3: Substitute Multipliers by marginal utilities. The agent's FOCs imply

$$\frac{\partial u\left(\boldsymbol{x}\right)}{\partial x_j} = -\sum_{m=1}^{M} \lambda_m \frac{\partial G_m}{\partial x_j}$$

- This maps the λs to the marginal utilities. Let's make an assumption on the structure of the constraints:

$$\frac{\partial G_m}{\partial t} = k_t\left(\boldsymbol{x}, t, T\right) \frac{\partial G_m}{\partial x_1} \; \forall m = 1, \ldots, M$$
$$\frac{\partial G_m}{\partial T} = -k_T\left(\boldsymbol{x}, t, T\right) \frac{\partial G_M}{\partial x_J} \; \forall m = 1, \ldots, M$$

A General Cookbook

- Now we have

$$
\begin{aligned}
\frac{dW(t)}{dt} &= \sum_{m=1}^{M} \lambda_m \left\{ \frac{\partial G_m}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial G_m}{\partial t} \right\} \\
&= \sum_{m=1}^{M} \lambda_m \left\{ -k_T(\boldsymbol{x}, t, T) \frac{\partial G_M}{\partial x_J} \frac{\partial T}{\partial t} + k_t(\boldsymbol{x}, t, T) \frac{\partial G_m}{\partial x_1} \right\} \\
&= -k_T \frac{\partial T}{\partial t} \sum_{m=1}^{M} \lambda_m \frac{\partial G_M}{\partial x_J} + k_t \sum_{m=1}^{M} \lambda_m \frac{\partial G_m}{\partial x_1} \\
&= k_T \frac{\partial T}{\partial t} u'(x_J(t)) - k_t u'(x_1(t))
\end{aligned}
$$

A General Cookbook

- Step 4: Recover the marginal utilities from observed choices.
- Sometimes we make assumptions about the marginal utilities (e.g. quasilinear utility in the Harberger example means that $u'(x_J) = 1$)
- In general, try and use the fact that the marginal utilites are inputs into observed choices and then recover them from how choices change in response to price/policy changes. e.g. in Harberger example, $u'(x_1) = p_1 + t$

A General Cookbook

- Step 5: Empirical implementation
- The work so far tells us which empirical objects we need to try and estimate:

$$\text{e.g. } \frac{dW(t)}{dt} = f\left(\frac{\partial x_1}{\partial t}, \frac{\partial x_1}{\partial Z}, t\right)$$

- Estimate these objects using policy/price changes
- Step 6: Model evaluation

1. Test predictions of the model that was needed to get the sufficient statistics model
2. Identify at least one set of structural parameters ω that is consistent with the model

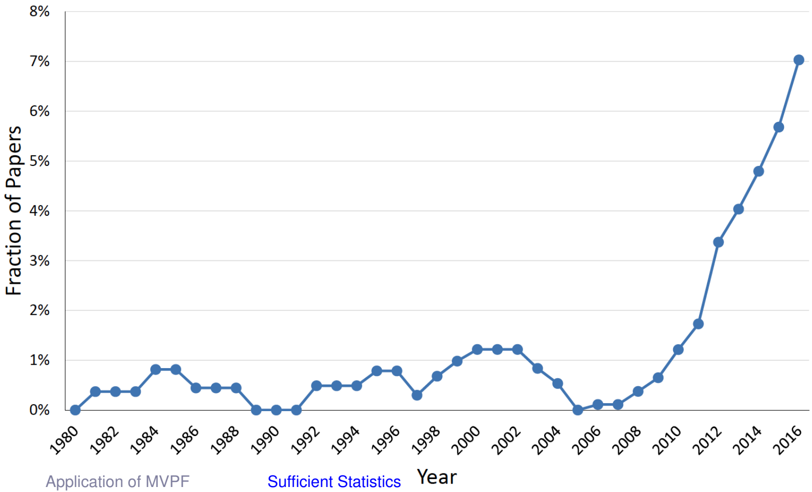
Outline

Sufficient Statistics

- Chetty (ARE 2009) *Sufficient Statistics as Bridge*
- Kleven (2019) *Sufficient Statistics Revisited*

Kleven (2019): Sufficient Statistics Approaches Have Become Popular

FIGURE 1: FRACTION OF NBER WORKING PAPERS IN PUBLIC ECONOMICS REFERRING TO THE SUFFICIENT STATISTICS APPROACH



Kleven (2019): Overview

- The sufficient statistics approach we just saw relies on 3 things

1. The reform being analyzed is small
2. There are no other distortions in the economy
3. Decisions about the structure of the environment that pin down which elasticities are sufficient in a particular setting

- This paper:

1. Provide a general framework to showcase how general sufficient statistics approach is
2. Show how it generalizes when we relax 1. and 2.
3. Show how quickly the estimation requirements go up!

Kleven (2019): General Model

- Continuum of individuals indexed by i. Discrete set of goods $j = 0, \ldots, J$

$$u^i\left(x_0^i, \ldots, x_J^i\right) = u^i\left(\boldsymbol{x}^i\right)$$

- Budget constraint (normalize pre-tax prices to 1)

$$\sum_{j=1}^{J} x_j^i + T\left(x_0^i, \ldots, x_J^i\right) = y^i$$

  where $T\left(\boldsymbol{x}\right)$ is a tax fundtion that need not be separable embodying all taxes and transfers.

- Assume that $T\left(x_0^i, \ldots, x_J^i\right)$ is piecewise linear and denote marginal tax rates $\partial T / \partial x_j^i \equiv \tau_j^i$. BC becomes

$$\sum_{j=0}^{J} \left(1 + \tau_j^i\right) x_j^i = Y^i$$

  where $Y^i = y^i + \sum_{j=0}^{J} \tau_j^i x_j^i - T\left(x_0^i, \ldots, x_J^i\right)$ is virtual income.

Kleven (2019): General Model

- Solution given by FOCs
$$\frac{\partial u^i}{\partial x^i_j} - \lambda^i \left(1+\tau^i_j\right) = 0 \; \forall j$$
- Yielding indirect utility
$$v^i\left(1+\tau^i_0, \ldots, 1+\tau^i_J, Y^i\right) = u^i\left(x^i_0\left(1+\tau^i_0, \ldots, 1+\tau^i_J, Y^i\right), \ldots \right.$$
$$\left. \ldots, x^i_J\left(1+\tau^i_0, \ldots, 1+\tau^i_J, Y^i\right)\right)$$
- Remember 1st year micro results:
$$\frac{\partial v^i}{\partial Y^i} = \lambda^i, \quad \underbrace{\frac{\partial v^i}{\partial\left(1+\tau^i_k\right)} = -\lambda^i x^i_k}_{\text{Roy's identity}} \quad \underbrace{\frac{\partial x^i_j}{\partial\left(1+\tau^i_k\right)} = \frac{\partial \tilde{x}^i_j}{\partial\left(1+\tau^i_k\right)} - x^i_k \frac{\partial x^i_j}{\partial Y^i}}_{\text{Slutsky decomposition}}$$

The Welfare Effect of Small Reforms

- Specify the tax policy as a function of treatment parameters θ: $T\left(x_0^i, \ldots, x_J^i, \theta\right)$, and $\tau\left(x_0^i, \ldots, x_J^i, \theta\right)$
- Consider a small reform $d\theta \approx 0$
- Money-metric measure of effect on individuals' utility:

$$
\begin{aligned}
\frac{dv^i}{d\theta} &= \sum_{j=0}^{J} \frac{\partial v^i}{\partial\left(1+\tau_j^i\right)} \frac{d\tau_j^i}{d\theta} + \frac{\partial v^i}{\partial Y^i} \frac{dY^i}{d\theta} \\
&= -\lambda^i \left(\sum_{j=0}^{J} x_j^i \frac{d\tau_j^i}{d\theta} + \frac{dY^i}{d\theta} \right) \\
&= -\lambda^i \frac{\partial T^i}{\partial \theta}
\end{aligned}
$$

  where the last equality uses $\frac{dY^i}{d\theta} = \sum_{j=0}^{J} x_j^i \frac{d\tau_j^i}{d\theta} - \frac{\partial T^i}{\partial \theta}$:
- The utility effect is equal to the mechanical revenue effect

The Welfare Effect of Small Reforms

- Now let's look at the social welfare effect. Define
$$W(\theta) = \int_i \omega^i v^i(\theta)\, di + \mu \int_i T^i(\theta)\, di$$
where ω^i is a Pareto weight on individual i and μ is the marginal value of government revenue.
- Differentiating
$$\frac{dW(\theta)/d\theta}{\mu} = \int_i \left[\frac{\omega^i}{\mu}\frac{dv^i}{d\theta} + \frac{dT^i}{d\theta}\right] di = \int_i \left[\frac{dT^i}{d\theta} - g^i \frac{\partial T^i}{\partial \theta}\right] di$$
$$= \int_i \left(\underbrace{\left[\frac{dT^i}{d\theta} - \frac{\partial T^i}{\partial \theta}\right]}_{\text{efficiency}} + \underbrace{\left(1 - g^i\right)\frac{\partial T^i}{\partial \theta}}_{\text{equity}} \right) di$$
where $g^i = \frac{\omega^i \lambda^i}{\mu}$ is each individual's social marginal welfare weight

The Welfare Effects of Small Reforms

- The efficiency term is the *fiscal externality*: behavioral changes reduce government revenue and reduce the potential transfer others can receive
- So what are the sufficient statistics? (and are they going to be externally valid?) Using $T\left(x_0^i, \ldots, x_J^i, \theta\right)$ the fiscal externality can be rewritten as

$$\begin{aligned}
\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} &= \int_i \sum_{j=0}^{J} \tau_j^i \left[\sum_{k=0}^{J} \frac{\partial x_j^i}{\partial\left(1+\tau_k^i\right)} \frac{d\tau_k^i}{d\theta} + \frac{\partial x_j^i}{\partial Y^i} \frac{dY^i}{d\theta}\right] di \\
&= \int_i \sum_{j=0}^{J} \tau_j^i \left[\sum_{k=0}^{J} \frac{\partial x_j^i}{\partial\left(1+\tau_k^i\right)} \frac{d\tau_k^i}{d\theta}\right. \\
&\qquad \left. + \frac{\partial x_j^i}{\partial Y^i} \left(\sum_{k=0}^{J} \frac{d\tau_k^i}{d\theta} x_k^i - \frac{\partial T^i}{\partial \theta}\right)\right] di
\end{aligned}$$

The Welfare Effect of Small Reforms

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \sum_{j=0}^{J} \left[\sum_{k=0}^{J} \tau_j^i x_j^i \varepsilon_{jk}^i \frac{d\tau_k^i/d\theta}{1+\tau_k^i} - \tau_j^i x_j^i \eta_j^i \frac{\partial T^i/\partial\theta}{Y^i}\right]$$

where $\varepsilon_{jk}^i \equiv \frac{\partial \tilde{x}_j^i}{\partial\left(1+\tau_k^i\right)} \frac{1+\tau_k^i}{x_j^i}$ is the Hicksian (compensated) price elasticity and $\eta_j^i \equiv \frac{\partial x_j^i}{\partial Y^i} \frac{Y^i}{x_j^i}$ is the income elasticity.

- Therefore, the sufficient statistics for evaluating the reform are $\left\{\varepsilon_{jk}^i, \eta_j^i\right\}_{\forall j,k,i}$
- Completely general given 1) small reform; and 2) no non-policy imperfections
- It is also a *general equilibrium* result, allowing for cross market effects etc.

But You Said That Sufficient Statistics was Simple!

- Of course, the problem is that J is potentially very large. So we require restrictions on either a) the tax policy space (what the $d\tau/d\theta$ and $\partial T/\partial\theta$ terms look like) or on behavioral responses (the εs and ηs)
- How can we get back to that nice simple Harberger equation?
- Assume

1. Utility is quasi-linear: $\Rightarrow \eta_j^i = 0 \; \forall j, i$
2. Only one good is taxed: $\tau_0 \neq 0; \; \tau_j = 0 \; \forall j = 1, \ldots, J$
3. The tax is linear: $T = \tau_0 x_0$

- Now that big equation collapses down to

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \bar{\varepsilon}_0 \frac{\tau_0}{1+\tau_0}\frac{d\tau_0}{d\theta}$$

where $\bar{\varepsilon}_0 = \int_i x_0^i \varepsilon_{00}^i di$, the demand-weighted average elasticity is the sufficient statistic

But You Said That Sufficient Statistics was Simple!

- A different way to do this is to assume quasi-linearity but that goods $0, \ldots, J_0$ are taxed at rate τ_0 while goods $J_0 + 1, \ldots, J$ are taxed at rate τ_1 $(= 0$ wlog$)$
- Now the sufficient statistic is $\bar{\varepsilon}_0 = \int_i \left[\sum_{j=0}^{J_0} \sum_{k=0}^{J_0} x_j^i \varepsilon_{jk}^i \right] di$ is the demand-weighted elasticity across goods $0, \ldots, J_0$ with respect to their tax rate τ_0. Still a single elasticity, but a different one.
- e.g. 1: Goods $0, \ldots, J_0$ are labor supply in different periods and the others are consumption in those periods. Now we can interpret $\bar{\varepsilon}_0$ as the elasticity of *lifetime* rather than contemporaneous earnings.
- e.g. 2: Goods $0, \ldots, J_0$ are multiple dimensions of labor supply (hours, effort, occupation, training etc). Now the elasticity is the elasticity of total labor income: the *elasticity of taxable income* (Feldstein, 1999)

Large Reforms

- When reforms are large, the small-reform sufficient statistics are the first-order Taylor approximation to the welfare effect.
- Can we do better? Consider a large reform to the tax policy $T\left(x_0^i, \ldots x_J^i, \theta\right)$ and its associated marginal tax rates $\tau_j^i(\theta)$. Define marginal tax rates as $\tau_j^i + \theta\Delta\tau_j^i$ where Δ_j^i are the reform-induced changes to the MTRs. Then we consider going from $\theta_0 = 0$ to $\theta_1 = 1$.
- The change in welfare is

$$\Delta W = W(1) - W(0) = \int_0^1 \frac{dW}{d\theta} d\theta$$

Large Reforms

- The efficiency effect of a large reform is

$$\left.\frac{\Delta W}{\mu_0}\right|_{g^i=1} = \int_0^1 \left.\frac{dW/d\theta}{\mu_0}\right|_{g^i=1} d\theta$$

  where

$$\frac{dW/d\theta}{\mu_0} \approx \int_i \sum_{j=0}^{J} \left[\sum_{k=0}^{J} \left(\tau_j^i + \theta \Delta t_j^i\right) x_j^i(\theta)\, \varepsilon_{jk}^i(\theta) \frac{\Delta \tau_k^i}{1+\tau_k^i + \theta \Delta \tau_k^i} \right.$$
$$\left. - \left(\tau_j^i + \theta \Delta \tau_j^i\right) x_j^i(\theta)\, \eta_j^i(\theta) \frac{\partial T^i/\partial\theta}{Y^i(\theta)} \right] di$$

- Small-reform expression gets it wrong because
  - wedges change over the reform path
  - elasticities change over the reform path

Large Reforms

- Let's simplify things a little

1. Instead of the full integral, consider the trapezoid approximation

$$\Delta W \approx \frac{1}{2}\left[\frac{dW\left(0\right)}{d\theta}+\frac{dW\left(1\right)}{d\theta}\right]$$

2. Assume quasi-linear utility and a single tax rate on taxed goods τ_0

- Now we get

$$\left.\frac{\Delta W}{\mu_0}\right|_{g^i=1} \approx \frac{1}{2}\left\{\bar{\varepsilon}_0\left(0\right)\frac{\tau_0}{1+\tau_0}\Delta\tau_0+\bar{\varepsilon}_0\left(1\right)\frac{\tau_0+\Delta\tau_0}{1+\tau_0+\Delta\tau_0}\Delta\tau_0\right\}$$

Large Reforms

- Now we can see what the correction term is we need to apply to the small-reform sufficient statistics formula:

$$\left.\frac{\Delta W}{\mu_0}\right|_{g^i=1} \approx \underbrace{\bar{\varepsilon}_0 \frac{\tau_0}{1+\tau_0} \Delta\tau_0}_{\text{Small reform formula}} + \frac{1}{2}\left\{\bar{\varepsilon}_0 \Delta\left[\frac{\tau_0}{1+\tau_0}\right] + \Delta\bar{\varepsilon}_0 \frac{\tau_0}{1+\tau_0} + \Delta\bar{\varepsilon}_0 \Delta\left[\frac{\tau_0}{1+\tau_0}\right]\right\} \Delta\tau_0$$

- Note that now the sufficient statistics are the elasticity $\bar{\varepsilon}_0$ and the elasticity change $\Delta\bar{\varepsilon}_0 = \bar{\varepsilon}_0(1) - \bar{\varepsilon}_0(0)$
- With iso-elastic preferences $(\Delta\bar{\varepsilon}_0 = 0)$

$$\left.\frac{\Delta W}{\mu_0}\right|_{g^i=1} \approx \bar{\varepsilon}_0 \left(\frac{\tau_0}{1+\tau_0} + \frac{1}{2}\Delta\left[\frac{\tau_0}{1+\tau_0}\right]\right) \Delta\tau_0$$

Non-Government Distortions

- What if there are other sources of wedges between private and social incentives?

$$u^i\left(x_0^i, \ldots, x_J^i; E_0^i, \ldots, E_J^i\right)$$

where $E_j^i \equiv \int_{\hat{i}} \phi_j^{i\hat{i}} x_j^{\hat{i}} d\hat{i}$ are externalities on individual i of consumption of good j

1. $\phi^{i\hat{i}} = 1$: Atmospheric externality, depends only on sum of consumption
2. $\phi^{i\hat{i}} = 0\ \forall i \neq \hat{i}$ and $\phi^{ii} = 1$: Internality. Gap between decision (takes E_j^i as given) and experienced utility
3. $\phi^{i\hat{i}} = -1\ \forall i \neq \hat{i}$ and $\phi^{ii} = 1$: Relative consumption concerns

Non-Government Distortions

- Effect on money-metric utility of a small reform:

$$\frac{dv^i/d\theta}{\lambda^i} = -\frac{\partial T^i}{\partial \theta} + \sum_{j=0}^{J} \frac{\partial v^i/\partial E^i_j}{\lambda^i} \frac{dE^i_j}{d\theta}$$

- The fiscal externality is still there, but there's also an externality effect
- The effect of small reforms on efficiency is

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \left[\frac{dT^i}{d\theta} - \frac{\partial T^i}{\partial \theta} + \sum_{j=0}^{J} \frac{\partial v^i/\partial E^i_j}{\lambda^i} \frac{dE_j}{d\theta} \right] di$$

Non-Government Distortions

- To go from this to a sufficient statistics formula, assume

1. $x_j^i = x_j^i\left(1+\tau_0^i, \ldots, 1+\tau_J^i, Y^i\right)$: Demand doesn't depend on the externalities
2. The externalities take the form $\phi^i = \phi_{I_j}^i \mathbf{1}\left\{\hat{i} = i\right\} + \phi_{E_j}^i$

- Now we get

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \sum_{j=0}^{J}\left[\left(\tau_j^i + \tau_{I_j}^i + \tau_{E_j}\right)\frac{dx_j^i}{d\theta}\right] di$$

$$= \int_i \left[\sum_{j=0}^{J}\sum_{k=0}^{J}\hat{\tau}^i x_j^i \varepsilon_{jk}^i \frac{d\tau_k^i/d\theta}{1+\tau_k^i} - \sum_{j=0}^{J}\hat{\tau}_j x_j^i \eta_j^i \frac{\partial T^i/\partial\theta}{Y^i}\right] di$$

where $\tau_{I_j}^i \equiv \frac{\partial v^i/\partial E_j^i}{\lambda^i}\phi_{I_j}^i$, $\tau_{E_j} \equiv \int_i \frac{\partial v^i/\partial E_j^i}{\lambda^i}\phi_{E_j}^i$, and $\hat{\tau}_j^i = \tau_j^i + \tau_{I_j}^i + \tau_{E_j}$

Non-Government Distortions

- If we specialize to the Harberger case, this becomes

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \bar{\varepsilon}_0 \frac{\hat{\tau}_0}{1+\tau_0} \frac{d\tau_0}{d\theta}$$

- The formula is almost the same.
- But we need to estimate $\hat{\tau}_0$ so the sufficient statistics are $\{\bar{\varepsilon}_0, \hat{\tau}_0\}$