Slab-SVM for Implicit Surface Modelling

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Computational Machine Learning

Introduction

- How to make a model in a simulator?

1.2 Fluid–structure interaction in aeroelasticity

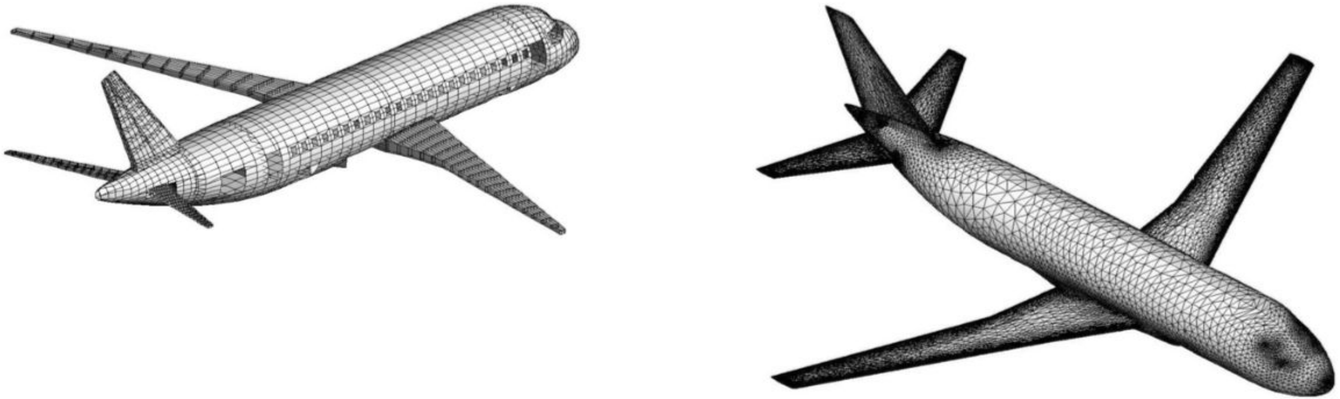


Fig. 1.3 The structural and aerodynamical model of a modern aircraft.

- Start with a point cloud

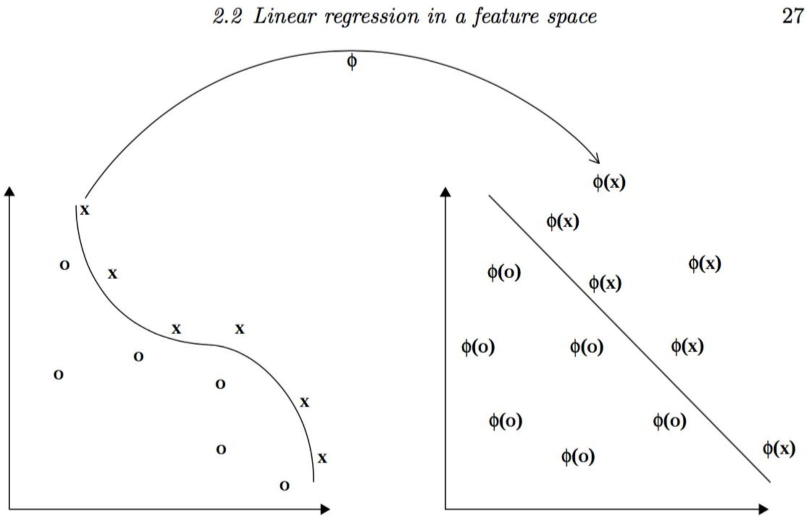
Holger Wendland. *Scattered Data Approximation*

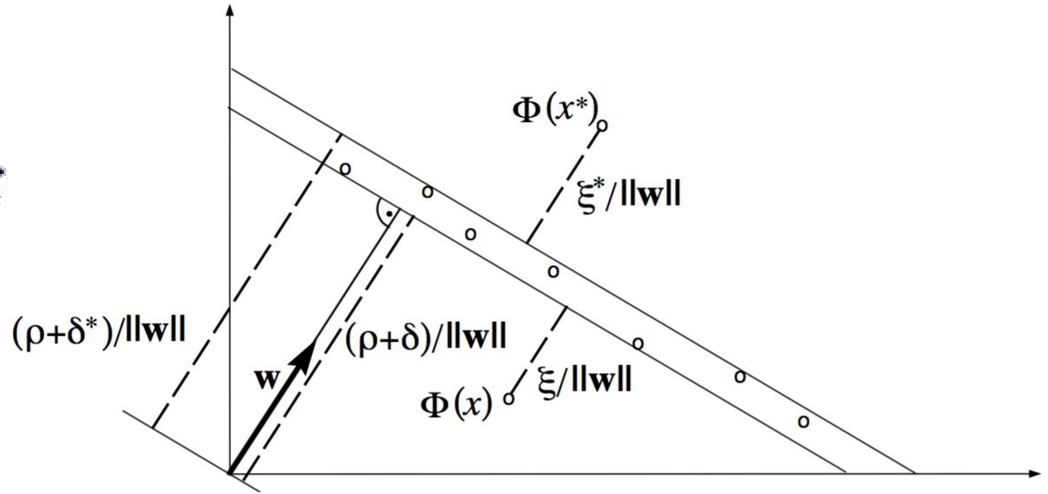
Problem Formulation

Let's pick a hyperplane (Gaussian kernel) that the points (nearly) go through.

Minimize norm of the function to find unique solution.

$$
\begin{aligned}
\underset{\mathbf{w}\in\mathcal{H},\boldsymbol{\xi}^{(*)}\in\mathbb{R}^m,\rho\in\mathbb{R}}{\text{minimize}} \quad & \frac{1}{2}\|\mathbf{w}\|^2 + \frac{1}{\nu m}\sum_i(\xi_i+\xi_i^*) - \rho \\
\text{subject to} \quad & \delta - \xi_i \le \langle \mathbf{w}, \Phi(x_i)\rangle - \rho \le \delta^* + \xi_i^* \\
\text{and} \quad & \xi_i^{(*)} \ge 0.
\end{aligned}
$$





Bernhard Scholkopf, Joachim Giesen, Simon Spalinger. Kernel Methods for Implicit Surface Modeling.

Quadratic Programming:

$$\min_{x \in R^d} \frac{1}{2} x^T Q x + c^T x$$

$$l^c \le Ax \le u^c,$$
$$l^x \le x \le u^x$$

Dual Problem:

$$\min_{\alpha \in R^d} \frac{1}{2} \alpha^T K \alpha + [-\delta \ldots -\delta] \alpha$$

$$1 \le [1 \ \ldots \ 1] \ \alpha \le 1,$$
$$\frac{-1}{vm} \le \alpha_i \le \frac{1}{vm}$$

Previous Work:

Datasets:

Issue:

Scaling ~ O(n^3)

From inverting the kernel matrix for Newton's Method (convergence ~ O(n ln(1/epsilon)))

- Optimizing Primal or Dual Problem

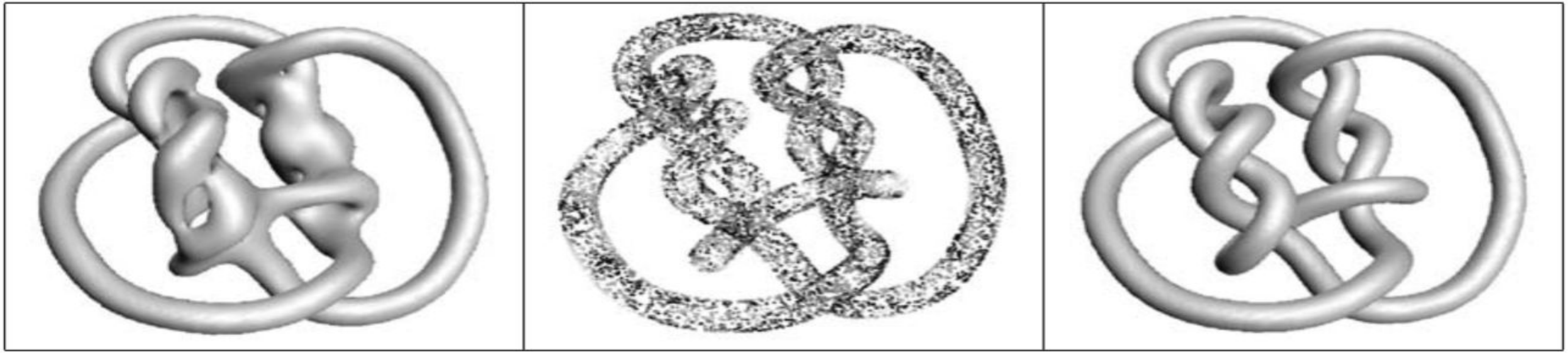
Bernhard Scholkopf, Joachim Giesen, Simon Spalinger. Kernel Methods for Implicit Surface Modeling.

Multi-Scale Approach

Epsilon Insensitivity:

- Randomly select small subset of points to train off.
- Add unseen points, measure error (residuals), and correct.

Not suited for sufficiently complex surface given some data size.



Bernhard Scholkopf, Joachim Giesen, Simon Spalinger. Kernel Methods for Implicit Surface Modeling.

Low Rank Kernel

Kernel Matrix ~ O(n^2) in size

Kernel Matrix Approximation ~ O(n k) in size, where k = significant eigenvalues

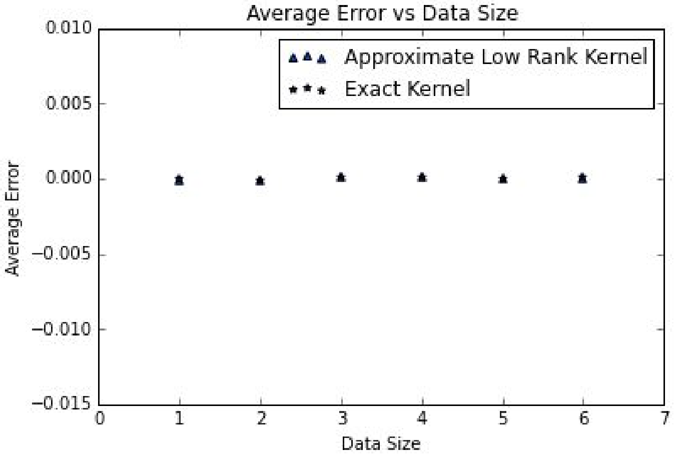
Decompose Kernel Matrix ~ O(n k^2)

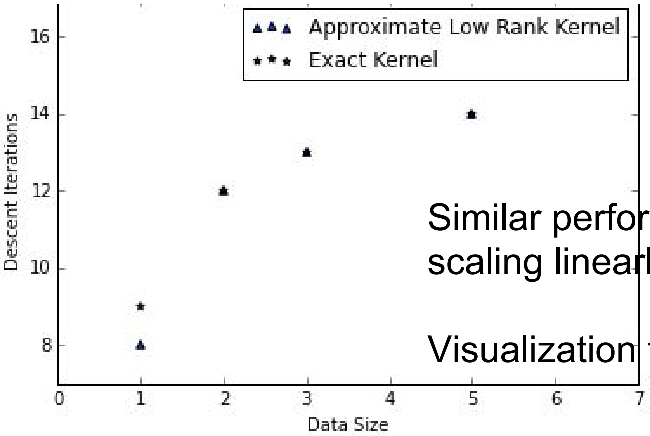
- Apply Incomplete Cholesky Factorization with Symmetric Pivotting
    - Only consider large eigenvalues.

Zero Eigenvalues give instability.

Shai Fine, Katya Scheinberg. Efficient SVM Training Using Low-Rank Kernel Representations.

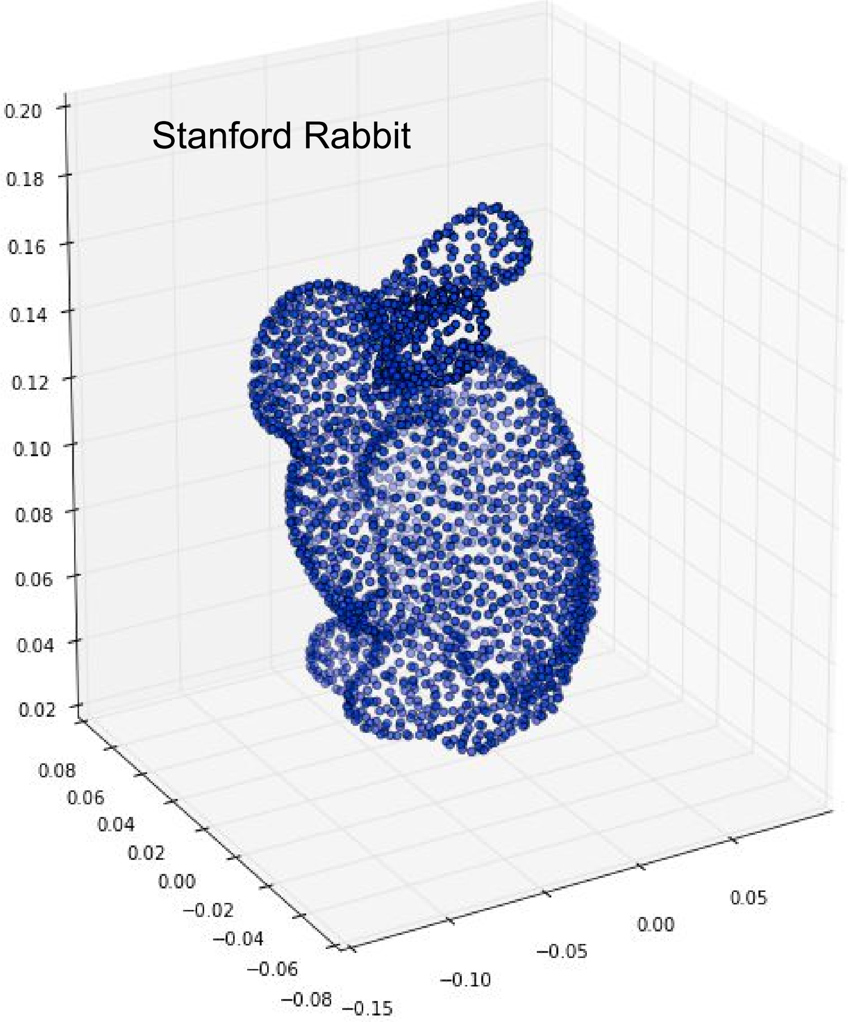
Experiments



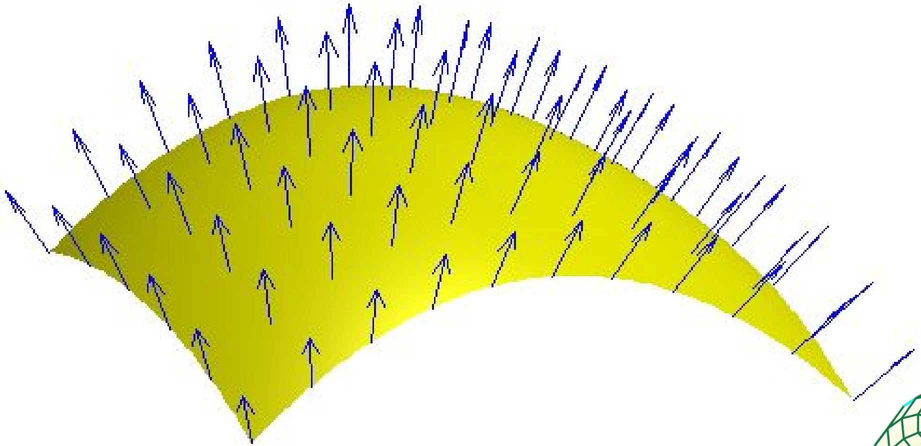


Similar performance yet scaling linearly.

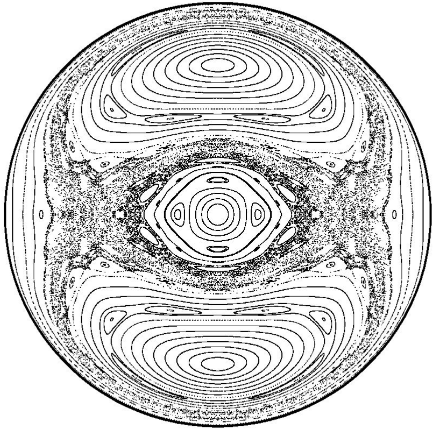
Visualization to come.

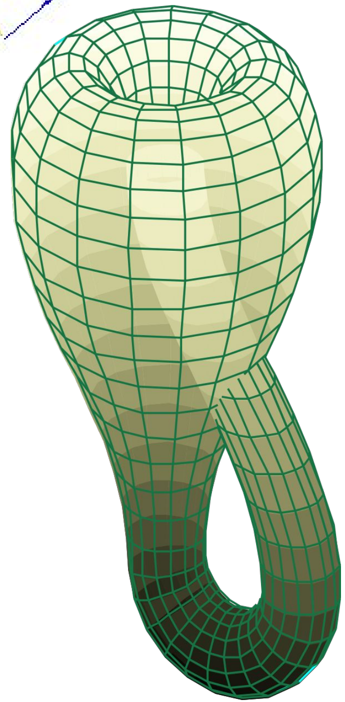


Future Work



- Numerical Stability
- Surface Normals
- Sharp Edges/Singularites
  - Gaussian radial basis function is always smooth
- Layered Surfaces
  - Magnetic Fluids





References

1. Bernhard Scholkopf, Joachim Giesen, Simon Spalinger. Kernel Methods for Implicit Surface Modeling.
2. Ali Rahimi, Ben Recht. Random Features for Large-Scale Kernel Machines.
3. Shai Fine, Katya Scheinberg. Efficient SVM Training Using Low-Rank Kernel Representations.
4. Holger Wendland. *Scattered Data Approximation.*
5. Francis R. Bach and Michael I. Jordan. Kernel Independent Component Analysis.