Optimal control of Trajectory of reusable launcher in OpenMDAO/dymos

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1 Introduction

New challenges arise from the space exploration development of our ages bringing space technology and research to focus its attention on new, low-cost optimal solutions to satisfy these demands. In section 2 the dares of human Moon exploration are pointed out and a context is given for the main topic of the study: the optimal transfer between a Low Lunar Orbit and the lunar surface. The instruments developed to solve such complex problems are powerful algorithms through which an iterative solution is obtained after an appropriate transcription. The same approach has been used to optimize the trajectories presented in this study as described in chapter 3. Similar approaches have already been proposed by some authors as reported in section 2.1. Their results are taken as initial reference for the conducted investigations to validate some of the results summarized in chapter 4.

2 State of Art

Following the new trends in the Moon exploration different studies were recently carried out to exploit possible solutions for complex missions such as the transfer from the Lunar Orbital Platform Gateway to the lunar surface and back. Since a direct transfer is not feasible, the ascent and descent trajectories have to be split in two phases considering an intermediate Low Lunar Parking Orbit (LLO) where the spacecraft will be temporarily placed. Starting from the knowledge acquired during the Apollo program, several authors propose new optimal solutions for two and three-dimensional transfers to accomplish such a mission.

2.1 Theoretical background

Regarding the descent, Ramanan [8] and Remesh [9] study different solutions suitable for a robotic probe landing with different initial conditions and constraints. In both cases an initial circular parking orbit at 100 km altitude is considered while the descent trajectory is split in multiple steps: impulsive deorbit burn, Hohmann transfer and powered descent at constant thrust to reduce the energy of the probe matching the final constraints. The optimal solution touches the ground with a small angle respect to the local vertical, thus posing several safety concerns. This phenomenon is avoided adding a final vertical phase ensuring a safer touchdown. Regarding the ascent phase, Zhang [11] allows the thrust magnitude to vary from zero to a specified maximum value. Many solutions are computed for target orbits at different altitudes proving that the fuel consumption is minimum when an *on/off* control scheme is employed for the thrust magnitude. Moving further, Ma [5] proposes a constant thrust trajectory with a constraint to perform a vertical takeoff. Without this condition the fuel-optimal solution clears the ground with a small angle respect to the surface. In other papers [6, 7] the same author solves the optimal control problem for three-dimensional ascent trajectories.

2.2 Tools

An optimal control problem is usually solved through an indirect method that requires at first a complex algebraic manipulation of the equations of motion (EOMs). Moreover, the numerical solution has a small region of convergence but with a close enough initial guess an highly accurate solution is readily computed. To overcome those issues direct methods are used. The optimal control problem is transcribed into a Nonlinear Programming problem (NLP) and solved with an iterative routine. The states and controls are discretized in time and then approximated with an interpolating polynomial to fit the discrete data. High-Order Gauss-Lobatto and Radau Pseudospectral are the transcription methods most commonly used and thus implemented in different libraries. The open-source, Python-based dymos library [4] takes advantage of the OpenMDAO [3] framework to perform such a transcription and solves the resulting NLP problem wrapping an external gradient-based solver such as IPOPT [10].

3 Development

Throughout this work the optimal transfer trajectories are obtained as solutions of a continuous-time optimal control problem arisen from the equations of motion (EOMs) that describe the spacecraft dynamics under the restricted two-body problem assumption. A transcription method is then applied to convert the optimal control problem into the corresponding NLP problem which is then numerically solved with an appropriate iterative algorithm.

Since the main goal is to find the most fuel-efficient ascent trajectory the resulting objective function is given by:

Minimize:

$$J = -m(t_f) \tag{1}$$

3.1 2D and 3D model

In 2 dimensions the EOMs of a spacecraft subject to the gravitational pull of a central spherical body as well as its own thrust are expressed in polar coordinates. Moving from a 2D to a 3D scenario the spacecraft dynamics under the keplerian two-body assumption is described in cartesian coordinates respect to an intertial reference frame whose origin coincides with the center of the Moon, while the target orbit is specified through its five Classical Orbital Elements (COEs) a, e, i, Ω, ω. For the first case the boundary conditions at both endpoints of the trajectory are imposed on the state variables $\boldsymbol{x} = [r, \theta, u, v, m]$ while path constraints could be applied to both state and control variables which are $\boldsymbol{u} = [T, \alpha]$. In the second model the specific angular momentum and eccentricity vectors $\boldsymbol{h}, \boldsymbol{e}$ has to be preferred to impose the final boundaries. Those vectors can be computed both from the spacecraft final state vector and the target COEs as described by Curtis [2]. The initial conditions are imposed on the states $\boldsymbol{r}, \boldsymbol{v}, m$ while the thrust direction $\hat{u}$ is forced to have a unitary magnitude throughout the whole phase.

3.2 Python implementation

The Problem is built considering the OpenMDAO System and Driver class instances. By default, the optimizer used is IPOPT [10] coupled with the linear solver MA57 [1], selected as the most promising solver to obtain an accurate solution in a reasonable computational time. Objective function, boundary conditions and path constraints are readily set through the corresponding methods implemented by both classes Phase and Trajectory that inherit from the Group class. Before a solution can be actually computed an initial guess has to be provided either as a linear interpolation of the defined boundaries or computing a simplified solution. The optimal transfer trajectory is then found letting the optimizer to drive the collocation defects below the specified tolerance.

4 Results

The simulations performed using the above approach can be divided between 2D and 3D transfers. In the first case a circular LLO at 86.87 km altitude is set as the intermediate parking orbit to be targeted by the launcher. The solution obtained imposing a constant thrust results in a short time of flight and propellant fraction around 36.8% of the initial mass, while letting the thrust to vary from zero to its maximum the total time of flight increases by 8 times but the fuel fraction is reduced to 33.64%. Moving on, a path constraint for the ground clearance is applied to the last case to avoid any possible geographical feature on the lunar surface. Indeed, the optimal variable thrust solution without this expedient establish the launcher to perform a lift-off with a small angle wrt the lunar surface and run along it until the necessary velocity to achieve the target orbit is achieved and the engines switched-off. When applied this last constraint, the time of flight is rather reduced while the amount of propellant is increased up to 35.5% as expected. For the 3D transfers different simulations have been performed to validate the model implementation and analyze the possible results. For the first purpose a polar LLO is reached from the South Pole of the Moon performing a in-plane trajectory at constant thrust. The results resembles the equivalent 2D simulation thus validating the developed tools. In the second case an highly elliptical orbit with 60° of inclination is targeted from the same launch site. The results show an important out-of-plane manoeuvre that implies a longer time of flight and a consistent increase in the required propellant mass.

5 Conclusions

The above results show a great correspondence with the literature studies presented in chapter 2 thus validating the theoretical background and the methodologies employed in this work. Future developments are foreseen for both 2D and 3D implementations. The next steps for the 2D model is the implementation of a geographically constrained descent trajectory and a parametric study for different Isp and *thrust/weight* ratio for a variable thrust ascent trajectory. Moving to the 3D counterpart, the constant thrust assumption will be removed allowing the thrust to vary between T_{min} and T_{max} provided on a case-by-case basis. Additional control variables will be then added to avoid sudden changes in the launcher attitude rates and a descent transfer trajectory will be implemented. Finally, an high fidelity dynamic model will be developed to take into account the spherical harmonics of the Moon gravitational potential and the perturbations introduced by the Earth, the Sun and the other planets in the Solar System.

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