Optimal control of trajectory of reusable launcher in OpenMDAO/dymos

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Outline

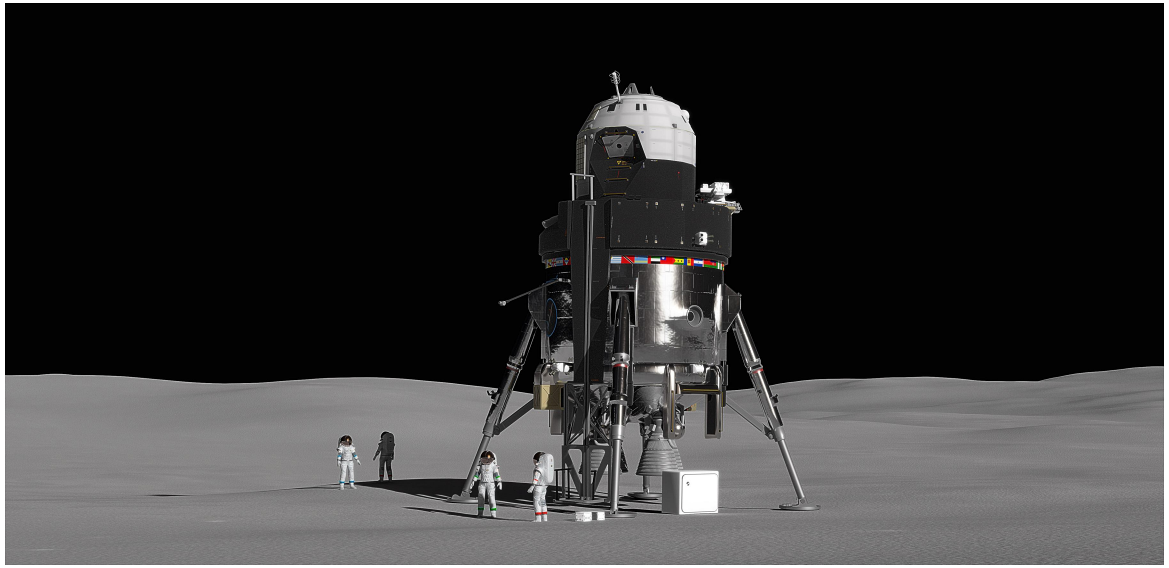
- *Context*
- *Optimal control problem*
- *State of the Art*
- *2D Ascent Trajectory*
- *2D Descent Trajectory*
- *3D Ascent Trajectory*
- *Conclusions & Future achievements*

Context

Moon and Mars Exploration

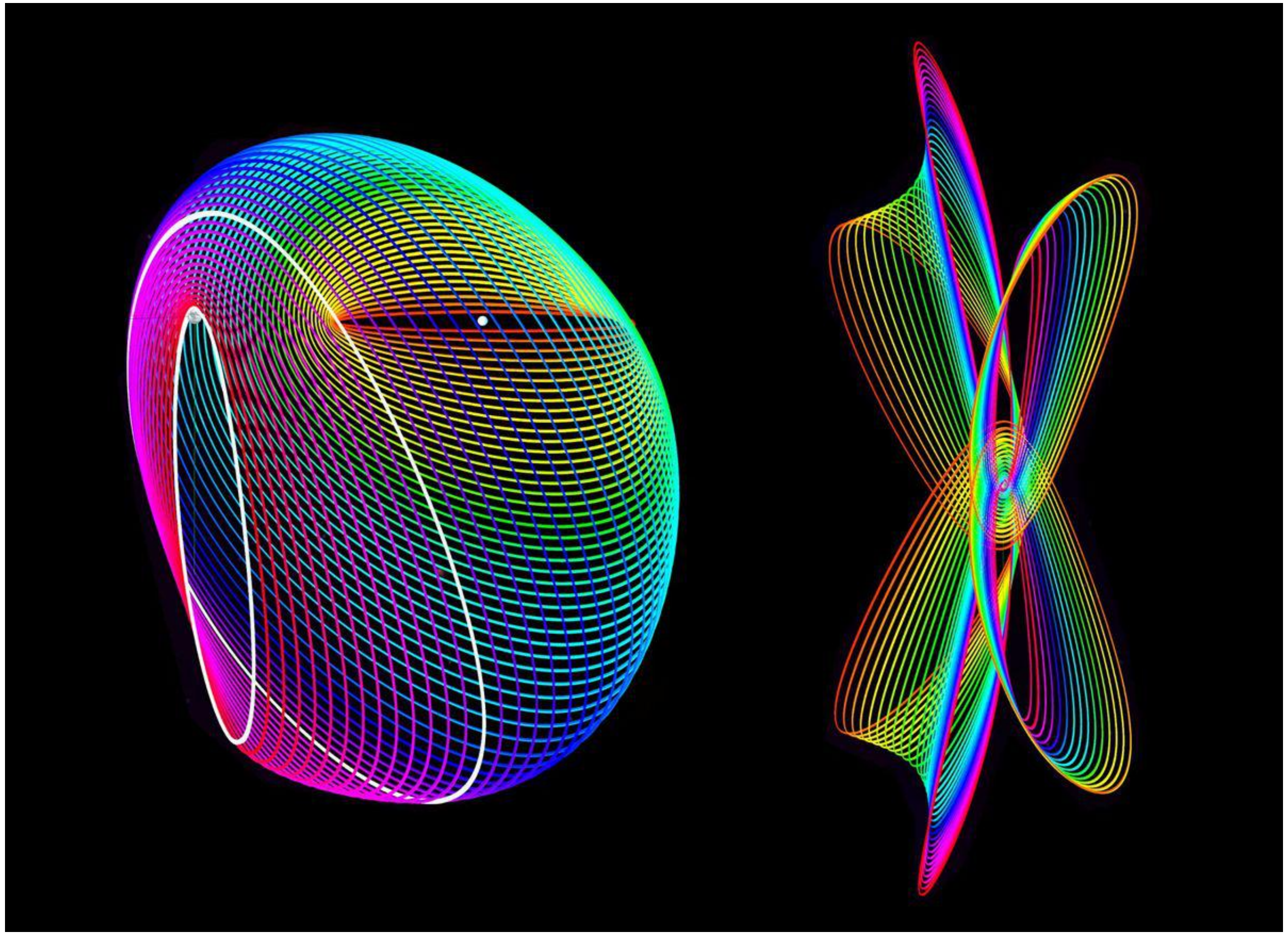
In-situ resources exploitation

Lunar Orbital Platform Gateway



credits: Lockheed Martin

HALO orbits



credits: Purdue University

Optimal control problem: formulation

Mathematical formulation of a continuous-time optimal control problem

Minimize:

$$J = \phi(t_f, \boldsymbol{x}_f) + \int_{t_0}^{t_f} L(t, \boldsymbol{x}, \boldsymbol{u})dt$$

Subject to:

$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})$	dynamics
$\boldsymbol{x}(t_0) = \boldsymbol{x}_0$	initial conditions
$\boldsymbol{u} \in U$	controls
$\boldsymbol{\Psi}(t_f, \boldsymbol{x}_f) = \boldsymbol{0}$	terminal constraints
$S(\boldsymbol{x}) \geq 0$	path constraints

Optimal control problem: solution

Solution of an optimal control problem - indirect methods

Calculus of variations

↓

Euler-Lagrange first-order optimality conditions
Pontryagin Minimum Principle

↓

Two-Points Boundary Value Problem (TPBVP)

Pros:

- **highly accurate solution**
- **low computational effort**

Cons:

- **heavy algebraic manipulation of EOMs**
- **small region of convergence**

Optimal control problem: solution

Solution of an optimal control problem - direct methods

Direct transcription

States and controls discretization

Non Linear Programming problem (NLP)

Iterative solution

Collocation defects driven to zero

Pros:
- **no algebraic manipulation of EOMs**
- **wider region of convergence**

Cons:
- **high number of decision variables**
- **high computational effort**

State of the Art: Tools

Direct transcription methods:
High-Order Gauss-Lobatto
Radau Pseudospectral

NLP solvers:
Gradient-based (SNOPT[1], IPOPT[2,3], fmincon[4])
Gradient-free (PSO, DE,SA)

Libraries:

- GPOPS-II[5] (MATLAB[4])
- OpenMDAO/dymos[6,7] (Python)

[1]*Gill et al., SIAM Review, 2005*
[2]*Wächter et al., Mathematical Programming, 2006*
[3]*HSL, http://www.hsl.rl.ac.uk/*
[4]*Matworks, https://www.mathworks.com/*
[5]*Patterson et al., ACM Trans. Math. Softw., 2014*
[6]*Gray et al., Structural and Multidisciplinary Optimization, 20197*
[7]*Hendricks et al., AIAA, 2017*

Implementation

Optimal control problem for spacecraft trajectory optimization

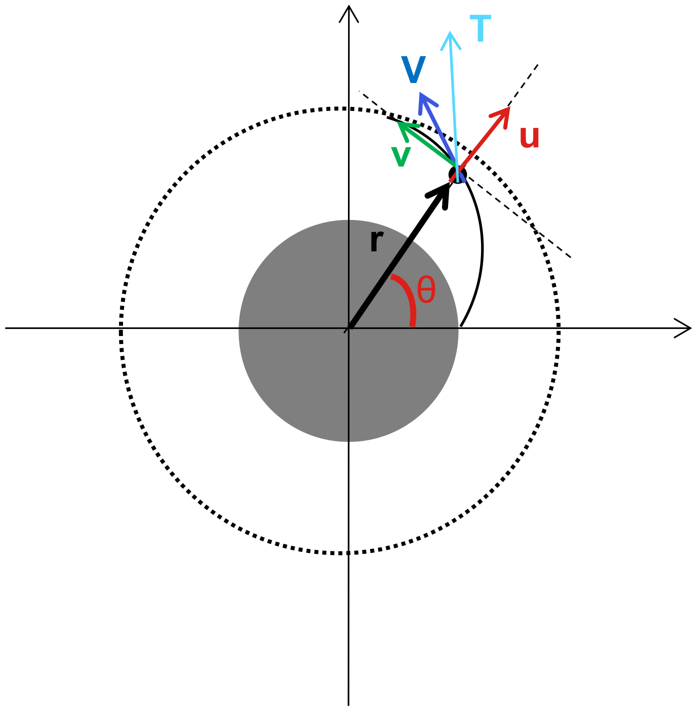
Direct transcription
High-Order Gauss-Lobatto, OpenMDAO/dymos library

NLP problem
IPOPT solver, linear solver MA57

Validation
IVP from initial conditions and optimal control profile, *simulate* method

2D Ascent Trajectory: EOMs

Equations of motions (EOMs):



$$\dot{r} = u$$

$$\dot{\theta} = \frac{v}{r}$$

$$\dot{u} = -\frac{\mu}{r^2} + \frac{v^2}{r} + \frac{T}{m}\sin\alpha$$

$$\dot{v} = -\frac{uv}{r} + \frac{T}{m}\cos\alpha$$

$$\dot{m} = -\frac{T}{Isp\, g_0}$$

legend:

r: radial distance

θ: swept angle

u: radial velocity

v: tangential velocity

m: mass

T: thrust magnitude

α: thrust direction

Isp: specific impulse

g_0: standard gravitational acceleration

μ: Moon standard gravitational parameter

2D Ascent: Optimal control problem

Minimize:

$$J = -m(t_f)$$

Subject to:

$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})$	dynamics
$\boldsymbol{x} = [r, \theta, u, v, m]$	state variables
$\boldsymbol{u} = [T, \alpha]$	controls
$\boldsymbol{x}(t_0) = \boldsymbol{x}_0$	initial conditions
$\boldsymbol{\Psi}(t_f, \boldsymbol{x}_f) = \boldsymbol{0}$	terminal constraints
$S(\boldsymbol{x}) \geq 0$	path constraints

Initial conditions $\boldsymbol{x}_0$ and terminal constraints $\Psi(t_f, \boldsymbol{x}_f)$:

$$\begin{cases} t_0 = 0 \\ r(t_0) = R \\ \theta(t_0) = 0 \\ u(t_0) = 0 \\ v(t_0) = 0 \\ m(t_0) = m_0 \end{cases} \qquad \begin{cases} t_f = free \\ r(t_f) = R + H \\ \theta(t_f) = free \\ u(t_f) = 0 \\ v(t_f) = \sqrt{\mu/(R+H)} \\ m(t_f) = free \end{cases}$$

Path constraints $S(\boldsymbol{x})$:

$$\begin{cases} r(t) > R \\ m(t) > 0 \\ T_{min} < T < T_{max} \end{cases}$$

legend:

R: Moon radius

H: target orbit altitude

m_0: initial spacecraft mass

T_{min}, T_{max}: minimum and maximum thrust

2D Ascent: Solutions

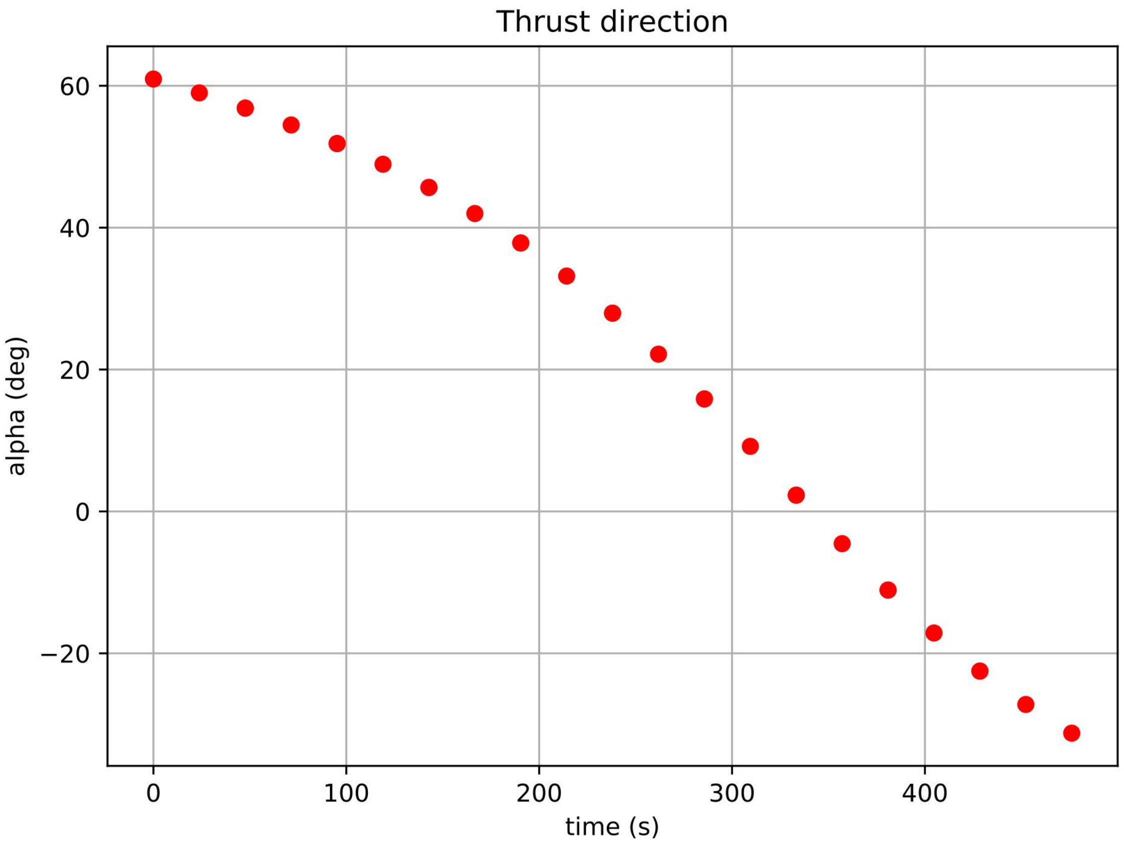
Common parameters:

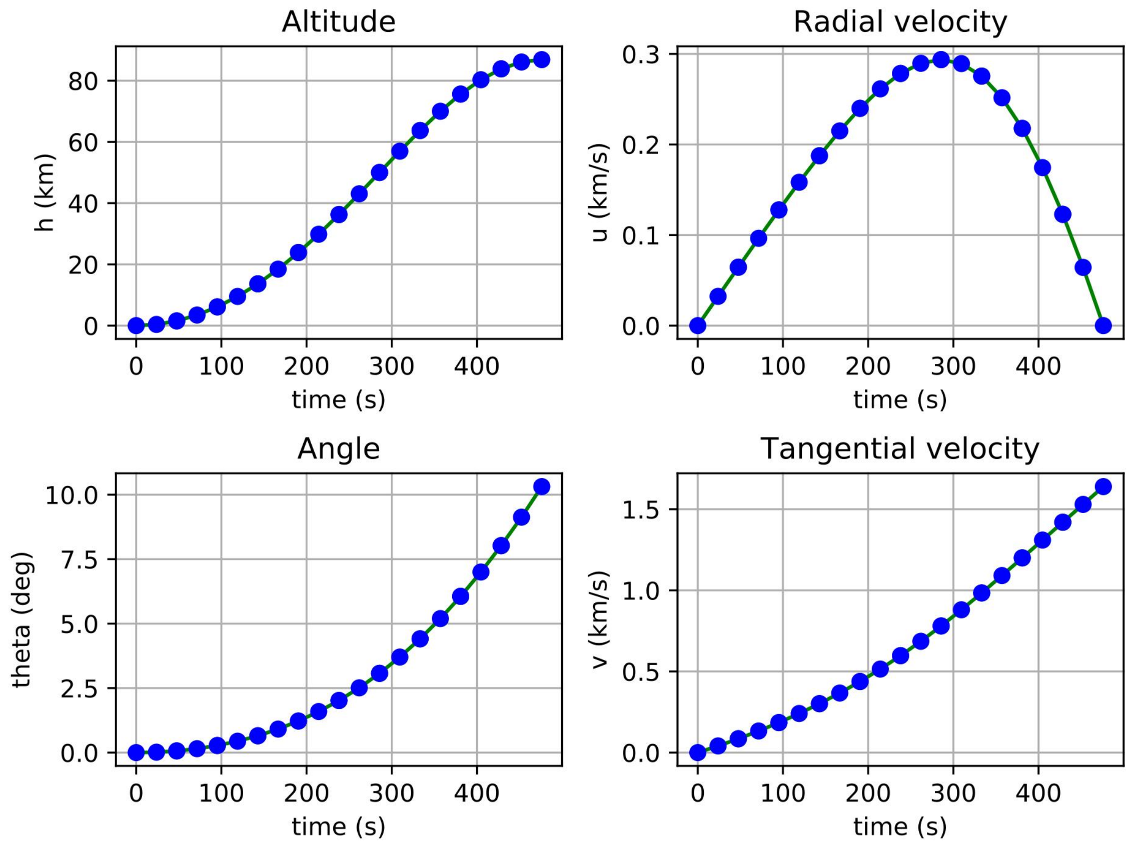
Target orbit altitude	86.87 km
Isp	450 s
Initial thrust/weight ratio	2.1

Proposed solutions:

Ascent Trajectory	**Time of Flight**	**Propellant consumption**
Constant Thrust	476.13 s	36.80%
Variable Thrust	3697.56 s	33.64%
Semianalytic initial guess	3803.26 s	32.81%
Variable Thrust, Minimum safe altitude	3367.77 s	35.50%

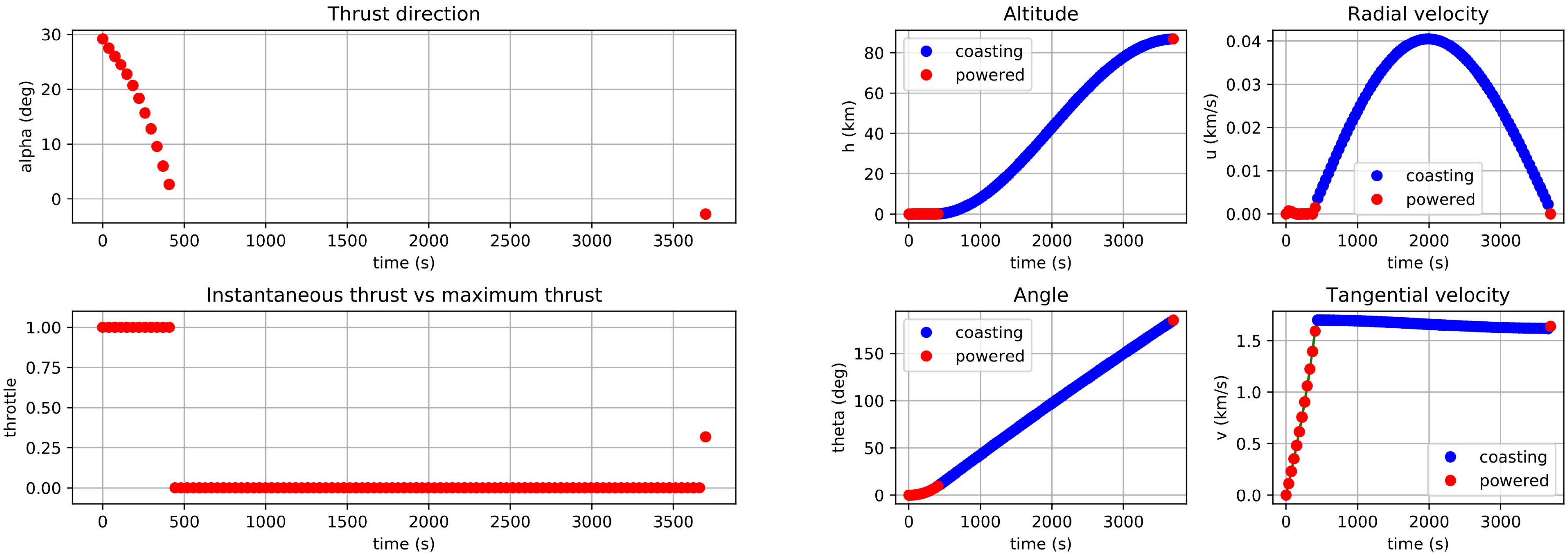
2D Ascent: Constant thrust





Time of Flight	Propellant consumption
476.13 s	36.80%

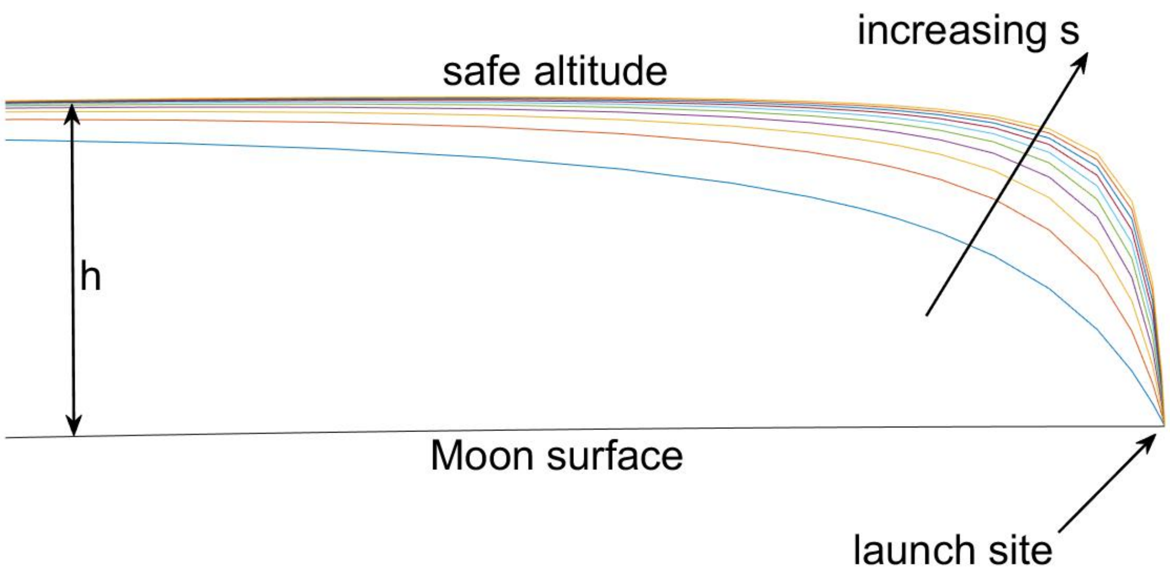
2D Ascent: Variable thrust



Time of Flight	Propellant consumption
3697.56 s	33.64%

2D Ascent: Minimum safe altitude

Path constraint to avoid lunar highlands:



$$r > R + \frac{hR\theta}{R\theta + h/s}$$

legend:

r: radial distance

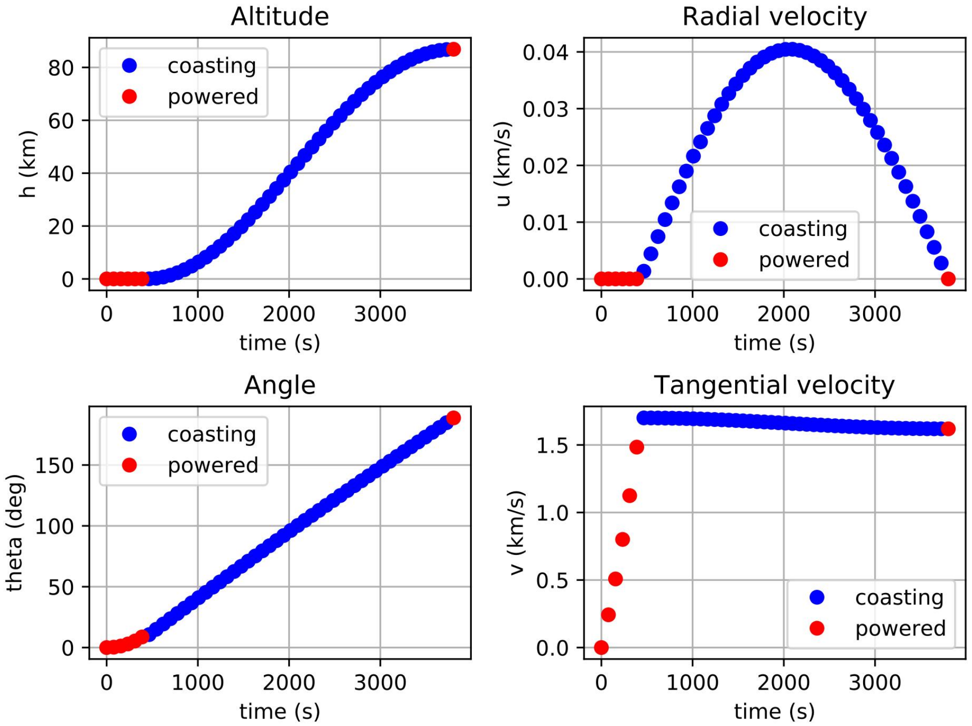
θ: swept angle

R: Moon radius

h: safe altitude

s: constraint shape parameter

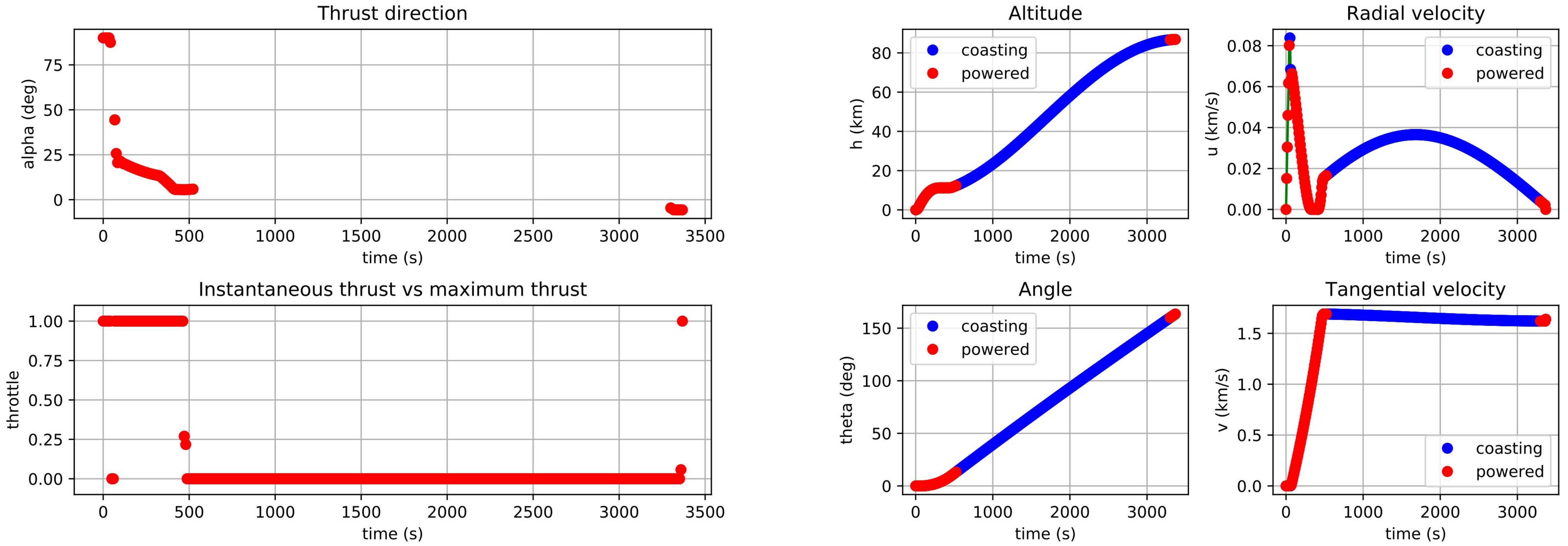
Seminanalytic initial guess:



Three-phases trajectory:

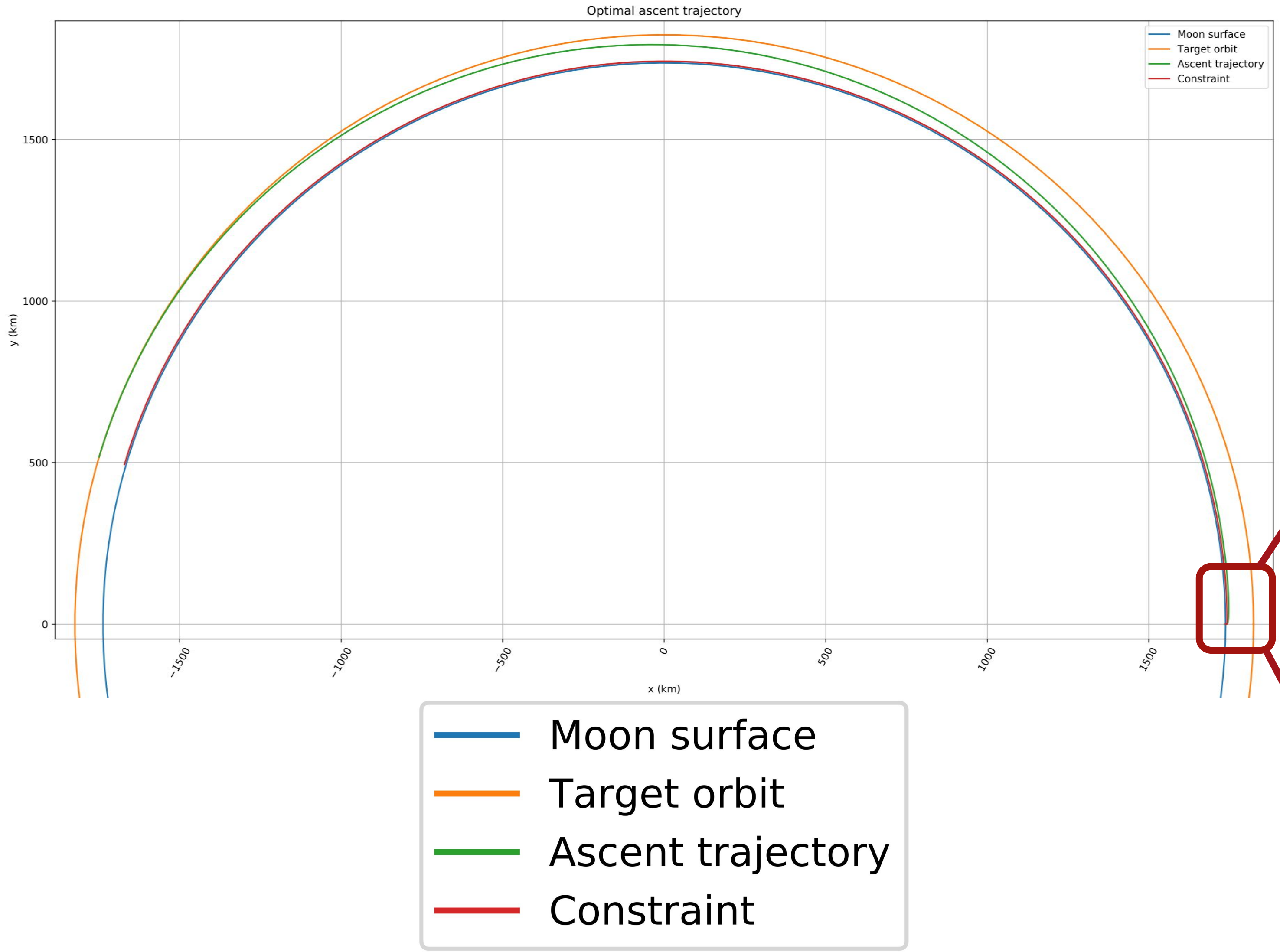
1. powered phase at constant radius r=R
2. Hohmann transfer
3. impulsive injection burn

2D Ascent: Minimum safe altitude

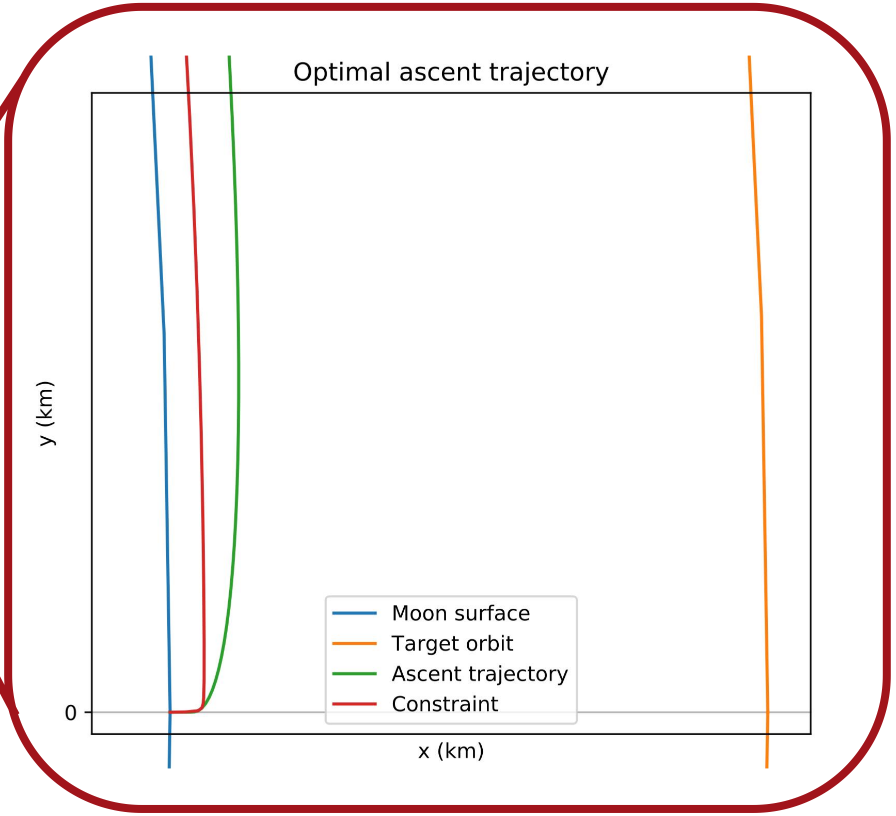


Ascent Trajectory	Time of Flight	Propellant consumption
Initial guess	3803.26 s	32.81%
Variable Thrust, Minimum safe altitude	3367.77 s	35.50%

2D Ascent: Minimum safe altitude



h	5.0 km
s	100



2D Descent: Optimal control problem

Minimize:

$$J = -m(t_f)$$

Subject to:

$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})$	dynamics
$\boldsymbol{x} = [r, \theta, u, v, m]$	state variables
$\boldsymbol{u} = [T, \alpha]$	controls
$\boldsymbol{x}(t_0) = \boldsymbol{x}_0$	initial conditions
$\boldsymbol{\Psi}(t_f, \boldsymbol{x}_f) = \boldsymbol{0}$	terminal constraints
$S(\boldsymbol{x}) \geq 0$	path constraints

Initial conditions $\boldsymbol{x}_0$ and terminal constraints $\Psi(t_f, \boldsymbol{x}_f)$:

$$\begin{cases} t_0 = 0 \\ r(t_0) = R + h_p \\ \theta(t_0) = 0 \\ u(t_0) = 0 \\ v(t_0) = v_p \\ m(t_0) = m_0 - \Delta m \end{cases} \qquad \begin{cases} t_f = free \\ r(t_f) = R \\ \theta(t_f) = free \\ u(t_f) = 0 \\ v(t_f) = 0 \\ m(t_f) = free \end{cases}$$

Path constraints to enforce vertical landing:

$$\begin{cases} v(r < R + h_v) = 0 \\ \alpha(r < R + h_v) = 90^\circ \end{cases}$$

legend:

R: Moon radius

h_p: Hohmann transfer periapsis altitude

v_p: Hohmann transfer periapsis velocity

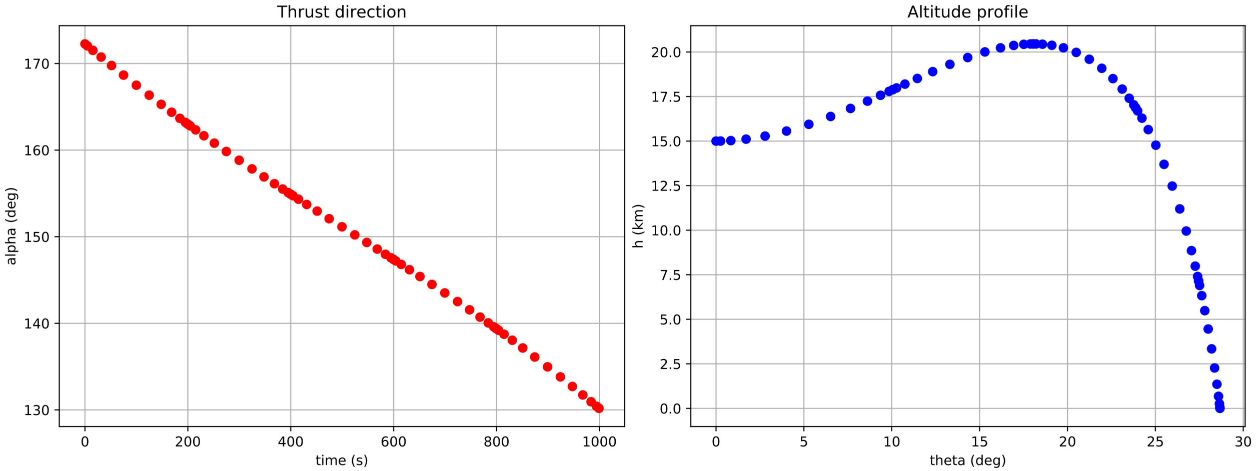
h_v: vertical landing initial altitude

m_0: initial spacecraft mass

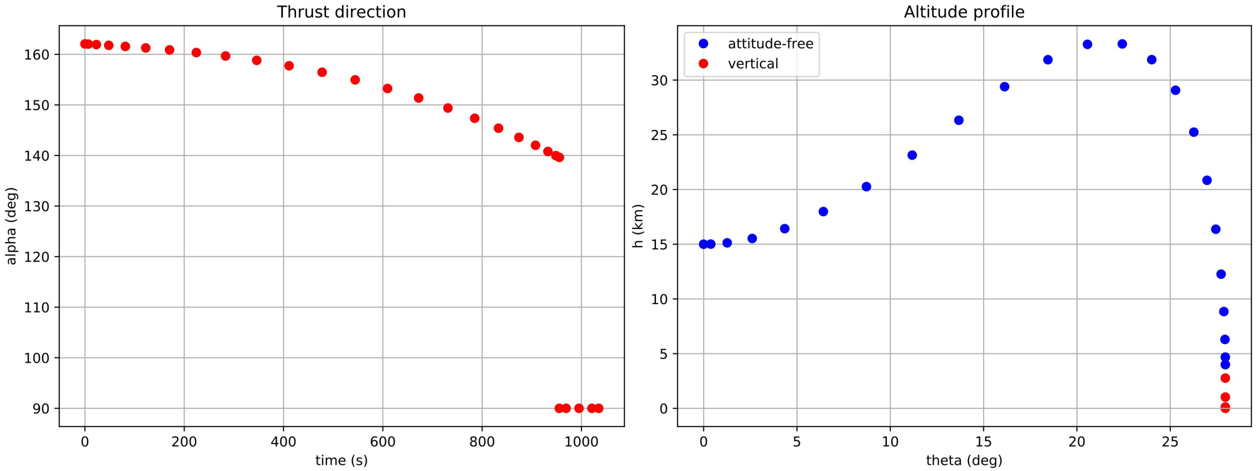
Δm: deorbit burn propellant mass

2D Descent Trajectory

Descent Trajectory without final constraints



Descent Trajectory with final constraints



Parking orbit altitude	100 km
Powered descent phase initial altitude	15 km
Vertical phase initial altitude	4 km

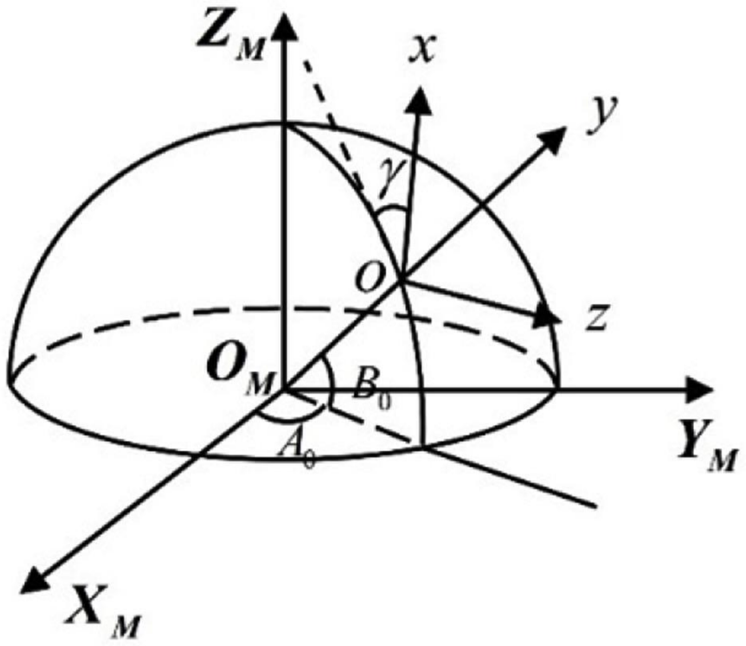
Isp	310 s
initial thrust/weight ratio	0.90

	Time of Flight	Propellant consumption
Free	999.05 s	48.51%
Vertical constrained	1034.77 s	50.24%

3D Ascent: EOMs

Equations of motion (EOMs):

$$\dot{\boldsymbol{r}} = \boldsymbol{v}$$



$$\dot{\boldsymbol{v}} = -\frac{\mu}{r^3}\boldsymbol{r} + \frac{T}{m}\hat{u}$$

$$\dot{m} = -\frac{T}{Isp\ g_0}$$

$$\boldsymbol{r} = [x, y, z]$$
$$\boldsymbol{v} = [v_x, v_y, v_z]$$
$$\hat{u} = [u_x, u_y, u_z]$$

legend:

$\boldsymbol{r}$: position vector

$\boldsymbol{v}$: velocity vector

m: mass

T: thrust magnitude

$\hat{u}$: thrust direction

Isp: specific impulse

g_0: standard gravitational acceleration

μ: Moon standard gravitational parameter

3D Ascent: Optimal control problem

Minimize:

$$J = -m(t_f)$$

Subject to:

$$\begin{aligned}
\dot{\boldsymbol{x}} &= \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}) && \text{dynamics} \\
\boldsymbol{x} &= [\boldsymbol{r}, \boldsymbol{v}, m] && \text{state variables} \\
\boldsymbol{u} &= [T, \hat{u}] && \text{controls} \\
\boldsymbol{x}(t_0) &= \boldsymbol{x}_0 && \text{initial conditions} \\
\boldsymbol{\Psi}(t_f, \boldsymbol{x}_f) &= \boldsymbol{0} && \text{terminal constraints} \\
S(\boldsymbol{x}) &\geq 0 && \text{path constraints}
\end{aligned}$$

Initial conditions $\boldsymbol{x}_0$ and terminal constraints $\Psi(t_f, \boldsymbol{x}_f)$:

$$\begin{cases} t_0 = 0 \\ \boldsymbol{r}(t_0) = \boldsymbol{r}_0 \\ \boldsymbol{v}(t_0) = \boldsymbol{0} \\ m(t_0) = m_0 \end{cases} \qquad \begin{cases} t_f = free \\ \boldsymbol{h}(t_f) - \boldsymbol{h}_{tgt} = \boldsymbol{0} \\ \boldsymbol{e}(t_f) - \boldsymbol{e}_{tgt} = \boldsymbol{0} \\ m(t_f) = free \end{cases}$$

Path constraints $S(\boldsymbol{x})$:

$$\begin{cases} \|\boldsymbol{r}(t)\| > R \\ m(t) > 0 \\ T_{min} < T < T_{max} \\ \|\hat{u}\| = 1 \end{cases}$$

legend:

$\boldsymbol{r}_0$: initial position vector

m_0: initial spacecraft mass

$\boldsymbol{h}$: specific angular momentum vector

$\boldsymbol{e}$: eccentricity vector

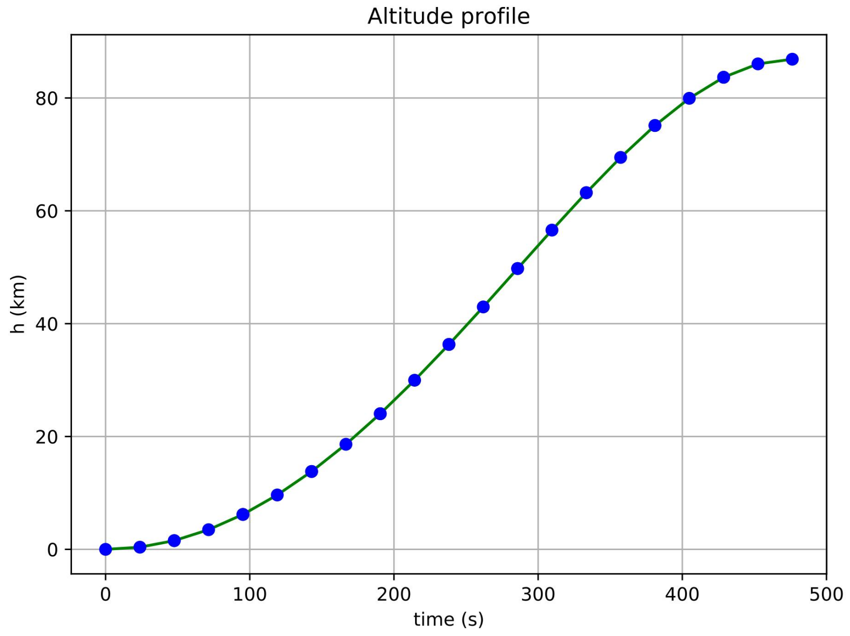
R: Moon radius

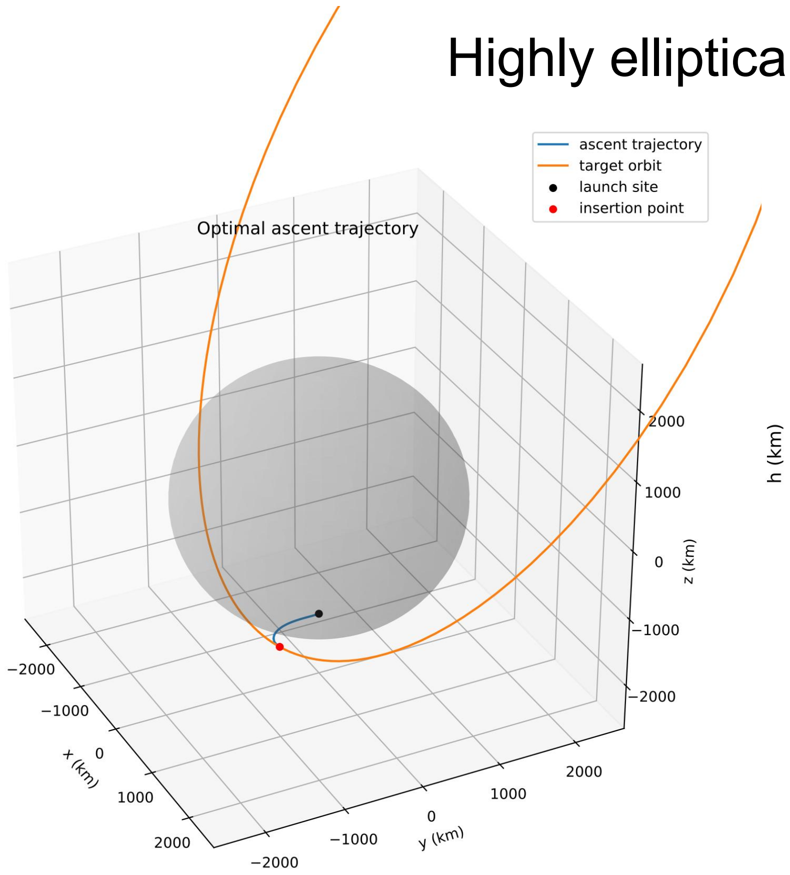
T_{min}, T_{max}: minimum and maximum thrust

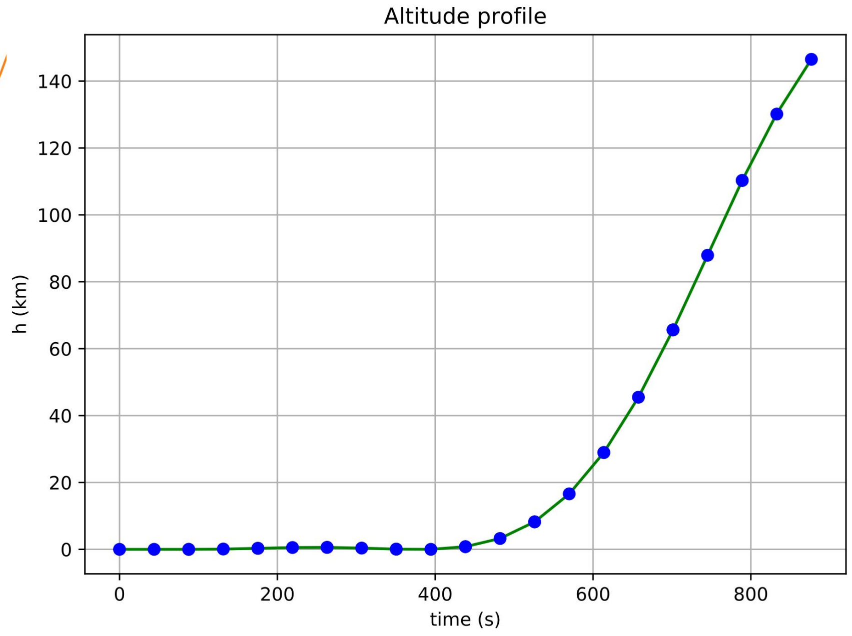
3D Ascent: Constant thrust

Polar orbit - coplanar transfer

Highly elliptical orbit - out-of-plane manoeuvre







Type of orbit	Eccentricity	Inclination	Argument of perilune	Semimajor axis	Time of flight	Propellant consumption
Polar	0	90°	-	1824.27 km	476.29 s	36.81%
Higly elliptical	0.7	60°	270°	6080.90 km	876.73 s	67.76%

Conclusions & Future achievements

3D ascent trajectory improvements:

- Variable thrust
- Limits on attitude rates
- Constraints on ground clearance

Other developments:

- 3D descent trajectory
- High fidelity gravity model
- Moon rotation

Main issues:

- Understanding OpenMDAO/dymos framework
- Inadequate NLP solvers (SLSQP)
- Poor State of the Art
- Difficulties while selecting correct transctiption
- High sensitivity to variables scaling

Selected bibliography

Benson, David. A Gauss Pseudospectral Transcription for Optimal Control. Massachusetts Institute of Technology, 2005, https://dspace.mit.edu/handle/1721.1/28919.

Find Minimum of Constrained Nonlinear Multivariable Function. https://www.mathworks.com/help/optim/ug/fmincon.html?s_tid=srchtitle.

Gill, Philip E., et al. ‘SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization’. *SIAM Review*, vol. 47, no. 1, Jan. 2005, pp. 99–131. doi:10.1137/S0036144504446096.

Gray, Justin S., et al. ‘OpenMDAO: An Open-Source Framework for Multidisciplinary Design, Analysis, and Optimization’. *Structural and Multidisciplinary Optimization*, vol. 59, no. 4, Apr. 2019, pp. 1075–104. doi:10.1007/s00158-019-02211-z.

Hendricks, Eric S., et al. ‘Simultaneous Propulsion System and Trajectory Optimization’. *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, American Institute of Aeronautics and Astronautics, 2017. doi:10.2514/6.2017-4435.

HSL, A Collection of Fortran Codes for Large Scale Scientific Computation. http://www.hsl.rl.ac.uk/.

Longuski, Jim, et al. *Optimal Control with Aerospace Applications*. Springer ; Published jointly by Microcosm Press, 2014.

Ma, Lin, et al. ‘Trajectory Optimization for Lunar Rover Performing Vertical Takeoff Vertical Landing Maneuvers in the Presence of Terrain’. *Acta Astronautica*, vol. 146, May 2018, pp. 289–99. doi:10.1016/j.actaastro.2018.03.013.

Patterson, Michael A., and Anil V. Rao. ‘GPOPS-II: A MATLAB Software for Solving Multiple-Phase Optimal Control Problems Using Hp-Adaptive Gaussian Quadrature Collocation Methods and Sparse Nonlinear Programming’. *ACM Transactions on Mathematical Software*, vol. 41, no. 1, Oct. 2014, pp. 1–37. doi:10.1145/2558904.

Ramanan, R. V., and Madan Lal. ‘Analysis of Optimal Strategies for Soft Landing on the Moon from Lunar Parking Orbits’. *Journal of Earth System Science*, vol. 114, no. 6, Dec. 2005, pp. 807–13. doi:10.1007/BF02715967.

Shirazi, Abolfazl, et al. ‘Spacecraft Trajectory Optimization: A Review of Models, Objectives, Approaches and Solutions’. *Progress in Aerospace Sciences*, vol. 102, Oct. 2018, pp. 76–98. doi:10.1016/j.paerosci.2018.07.007.

Topputo, F., and C. Zhang. ‘Survey of Direct Transcription for Low-Thrust Space Trajectory Optimization with Applications’. *Abstract and Applied Analysis*, vol. 2014, 2014, pp. 1–15. doi:10.1155/2014/851720.

Wächter, Andreas, and Lorenz T. Biegler. ‘On the Implementation of an Interior-Point Filter Line-Search Algorithm for Large-Scale Nonlinear Programming’. *Mathematical Programming*, vol. 106, no. 1, Mar. 2006, pp. 25–57. doi:10.1007/s10107-004-0559-y.

Thank you for your attention

Any question?

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