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# Master of Aerospace Engineering Research Project

## Optimal control of Trajectory of reusable launcher in OpenMDAO/dymos

# S3 Progress Report

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Duration: 14 Months

# Contents

# 1 Goal of the project

This study is focused on the analysis of new ascent and descent trajectories from the Moon surface to different possible lunar orbits, such as Low Lunar Orbits (LLO) and Near Rectilinear Halo Orbits (NRHO). The objective is to provide a possible answer to the questions arisen after the renewed interest in the Moon exploration and *in-situ* resources exploitation, as demonstrated by the current development of the Lunar Orbital Platform-Gateway (LOP-G) and the Space Launch System (SLS).

The main objective is to determine the most fuel-efficient trajectory for a given final orbit suitable for a crewed reusable launcher, for which different constraints and safety requirements must be taken into account. The principal tools used to achieve the final goal are OpenMDAO and Dymos, two Python libraries that provide a *user-friendly* and powerful interface for solving optimal control problems through different numerical techniques.

# 2 Main achievements of the project during Semester 2

The second semester has been dedicated to an in-depth study of State-of-the-Art methodologies for the modellization and simulation of optimal, continuous-thrust transfer trajectories and the implementation of suitable Python routines for the computation of fuel-optimal, two-dimensional ascent and descent trajectories from the Moon surface to a specified LLO.

The problem has been formulated as a continuous-time optimal control problem in which the underlying dynamics describes the motion of a massless spacecraft subject to the gravitational pull of the Moon and the acceleration due to its own thrust. Appropriate boundary and path constraints guarantee the spacecraft departs from a predetermined state and inject on a specified orbit or lands smoothly on the Moon surface without violating any physical law or bound imposed on the engine performances. The objective is to minimise the propellant consumption or equivalently maximise the final spacecraft mass.

The infinite-dimension problem is then discretized through an High-Order Gauss-Lobatto [1] or Radau Pseudospectral [2] transcription method into a finite-dimension Non Linear Programming Problem (NLP) finally solved with gradient-based optimizers such as IPOPT [3, 4] and SNOPT [5]. The employed transcription methods are provided by the Python package dymos [6] while the interface between the Python code and the NLP solvers is guaranteed by the open-source libraries OpenMDAO [7] and pyOptSparse [8].

A first solution has been computed for constant thrust ascent trajectories from the Moon surface to a specified LLO. For this simple case a minimisation of the propellant consumption is equivalent to a minimisation of the corresponding time of flight and thus the optimal solutions are characterised by short transfer times and high flight path angles at departure. The previous solution has been improved allowing the thrust magnitude to vary from zero to its maximum thus obtaining an optimal solution characterised by a *bang-bang* control scheme and a lower fuel consumption. On the other side, the time of flight is significantly increased and the spacecraft leaves the surface with a shallow angle that poses several safety concerns due to rocks and geographical features that may be located near the defined launch site. To overcome this issue, an appropriate path constraint is added to the problem formulation to force a vertical take-off and impose a minimum safe altitude under which the spacecraft is not allowed to travel.

Regarding the descent, different approaches has been explored following the work presented by Ramanan [9] and Remesh [10]. The descent has been split into an initial Hohmann transfer from the specified LLO to a lower perigee altitude and a subsequent powered descent phase to decrease the spacecraft velocity until the final touch-down. A parametric study has been then conducted to determine the optimal perigee altitude that minimises the propellant consumption while satisfying the safety constraints required for a crewed vehicle. A vertical touch-down has been then guaranteed splitting the powered descent into an initial attitude-free phase followed by a vertical descent that starts from a specified altitude or last for a predetermined amount of time.

Finally, a new dynamic model has been implemented to compute three-dimensional ascent trajectories at constant thrust. Different simulations have been then carried out for both in-plane and out-of plane transfers obtaining in the first case results analogous to the corresponding two-dimensional solutions.

# 3 Work done from the beginning of Semester 3

As first task, the algorithms already employed to compute optimal ascent trajectories with variable thrust magnitude are adapted to the corresponding descent trajectories to improve the constant-thrust solutions previously obtained and removing the necessity of a multi-phase transfer composed by an initial ballistic phase and a subsequent powered descent. If only boundary conditions are imposed on the problem, the corresponding solutions present the same safety concerns on the ground clearance already described in chapter 1. Similarly as before, these issues are then overcome adding an appropriate path constraint that maintains the spacecraft above a specified minimum safe altitude until a vertical landing is performed on the designated landing site.

This research project fits into the Multidisciplinary Design and Optimisation (MDO) of a future reusable lunar lander for which it aims to compute the most fuel-efficient transfer trajectories given the engine characteristics as design parameters. The implemented algorithms constitute a single block in the whole optimization framework that comprises several disciplines such as spacecraft structures and rocket propulsion. As a consequence, at every Multidisciplinary Analysis (MDA) corresponding to a single iteration of the overall MDO the trajectory module has to be iteratively solved for the current values of engine specific impulse $Isp$ and thrust over weight ratio $twr$. Being this discipline particularly computationally expensive, its solutions represent a bottleneck that considerably reduces the overall MDO performances.

The computational effort can be alleviated substituting the solution of the optimal control problem with the evaluation of a surrogate model that approximates the required $\Delta v$ and propellant consumption for each possible value of $Isp$ and $twr$. These meta-models are built with a two-step process composed by an initial space sampling and a subsequent model training [11]. The initial sampling is required to compute several optimal solutions for different values of $Isp, twr$ that will constitute the bases for the model training. Full factorial or Latin Hypercube are the algorithms usually employed to obtain an effective sampling that minimise the model error. Regarding the surrogate, multiple solutions have been proposed to interpolate the sampled data across the whole search space. Throughout this work, all the meta-models are built using the sampling techniques and surrogate models available in the Python package SMT (Surrogate Modelling Toolbox) maintained by the MDOlab and Onera [12].

Finally, suitable dynamical models and numerical algorithms are implemented to compute and analyse optimal transfer trajectories from a circular LLO to a Moon-centred Highly Elliptical Orbit (HEO). These solutions represent a first planar approximation of an LLO to NRHO transfer that constitute the primary objective for the future project developments.

# 4 Remaining project issues

A first issue has been noticed regarding the smoothness of the computed surrogate models that lead to a difficult convergence of the MDO process. Looking at small scale, the numerical derivative computed during the optimization procedure present in fact an highly noisy component that prevent the optimiser from achieving convergence, forcing the solution to continuously oscillate around a particular value. This problem is currently under investigation and different interpolation methods are being tested to determine the one that is most adapted for the problem of interest.

Secondly, while analysing the solutions obtained for LLO to HEO transfers some discrepancies have been observed between the implicit solution and the explicit simulation obtained with a numerical integration of the Equations of Motion (EOMs) from the imposed initial conditions and the optimal control profile. This lack of accuracy is mainly related to the long propagation time that require an high number of discretization nodes to obtain a feasible solution that matches with the corresponding numerical integration. The problem is then overcome with a significant increase in the number of nodes in which the states and controls are discretized thus leading to an highly computationally expensive NLP solved with the ISAE supercomputer Rainman.

# 5 Last milestones of the project

To move from a two-dimensional approximation to an authentic representation of an LLO to NRHO transfer, the first step is the replacement of the Keplerian dynamics with a more accurate force model that takes into account the gravitational attraction of both the Earth and the Moon. A suitable framework is provided by the Circular Restricted Three-Body Problem (CR3BP) formulation, which provides the Equations of Motions (EOMs) of a massless spacecraft under the gravitational pull of two massive bodies supposed to be in a circular orbit about their common barycentre [13]. An important role in this context is played by the epoch at which the manoeuvre is initiated, since the departure and arrival orbit are conveniently defined in different reference frames whose transformation matrix depends on time. An LLO orbit is in fact easily described in a Lunar Centred Inertial (LCI) frame while an NRHO is intuitively visualised in a rotating frame centred in the Earth-Moon barycentre in which the two primaries remain fixed. Moving further, the three-dimensional model developed for constant thrust transfers from the lunar surface to an LLO will be extended to include both ascent and descent trajectories with constant or variable thrust as second step in the simulation of the complete transfer from the Lunar Orbital Platform-Gateway (LOP-G) to the Moon surface and back.

# 6 Task 1 - Surrogate models

## 6.1 Description of Work

The project is placed in the context of a future reusable launcher design, all the disciplines involved will be dynamically optimised within a multidisciplinary design and optimisation (MDO) framework. These disciplines involved in the complete model of the launcher can be represented as blocks with input and output data that are iterate until convergence is achieved. The Trajectory block is one of the most expensive components in terms of computational effort and represents a bottleneck for the global optimisation framework as described in chapter 1. To avoid this problem and speed up the iterative process related to the MDO, the results coming from the Trajectory block are provided by a surrogate model such as the ones implemented in the SMT [12] Python package. Fitted in this context, the algorithm returns the propellant consumption and the transfer time for every combination of design parameters $Isp, twr$ in the intervals $[250\ s, 500\ s]$ and $[1, 4]$ respectively. A limited number of actual solutions are computed taking couples of $Isp, twr$ values with a Latin Hypercube Distribution, the remaining cases are then found interpolating the previously obtained results.

## 6.2 Technical Progress: 2 months achievements

Using the SMT routines, two surrogate models have been computed for both ascent and descent trajectories from the Moon surface to a circular LLO at 100.0 $km$ altitude. The two versions differs for the initial sampling point as described below.

The first step in the model construction is the choice of the set of points where actual solutions will be computed solving the corresponding NLP problem, this stage is called sampling. The resulting model will be then influenced by this initial grid since its precision depends both on the total number of samples and their distribution in the overall search space. Two kinds of grids are taken in consideration:

- Latin Hypercube Sampling (LHS) that consists in a grid with an arbitrary high number of dimensions where each sample is the only one in each axis-aligned hyper-plane containing it.
- Full Factorial Sampling (FFS) that consists in a grid with an arbitrary high number of dimensions where samples are computed for all the possible combinations of all the axis-aligned hyper-planes.

The next step involves the interpolation of the sampling point in the full grid to find all the possible output values for the quantities of interest. This phase is called training and it is influenced by the previous choice of set points. In the case of a FFS grid, this last step is not necessary and a simple spline is used to display the final contour graph. There are several algorithms dedicated to this purpose whose suitability and accuracy depend on the type of sampling grid but also on the problem characteristics such as the number of design variables involved in the computation. The Kriging method (KRG) and the Second-order polynomial approximation (QP) implemented in SMT are finally selected since they minimise the error between a FFS grid and the corresponding LHS approximation.

In this study, $Isp$ and $twr$ are the design variables of interest varied in the model to compute propellant consumption and time of flight for a transfer trajectory between the Moon surface and a 100.0 $km$ altitude LLO and vice-versa. In particular, a surrogate model is built for each kind of transfer already modelled both for ascent and descent cases including constant thrust, variable thrust, variable thrust with constrained minimum safe altitude and variable thrust with vertical take off or landing. The resulting models for different sampling methods can be seen in figure 6.1 in the case of an ascent trajectory with variable thrust.

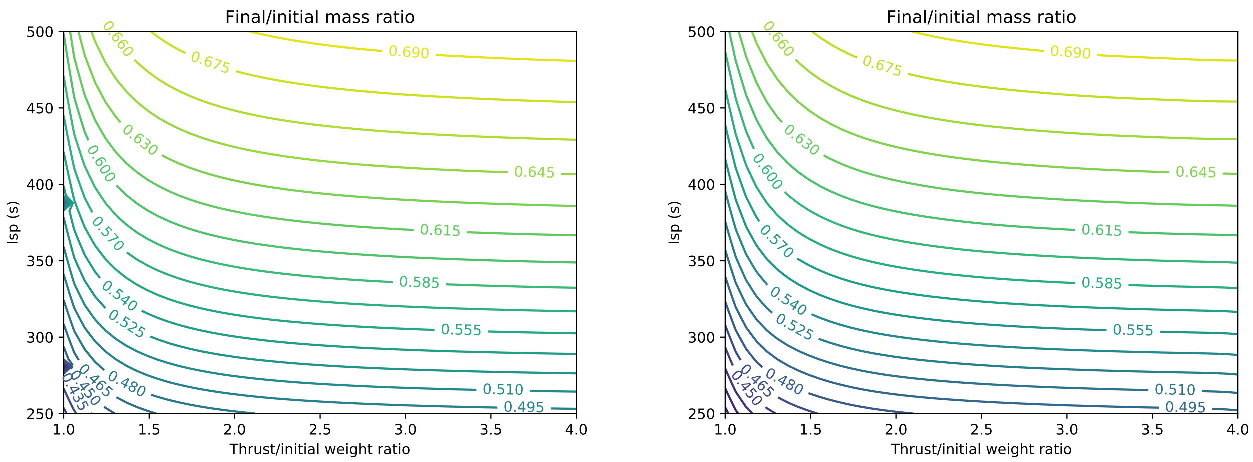


Figure 6.1: Surrogate models for an ascent trajectory with variable thrust using Full Factorial Sampling (left) and Latin Hypercube Sampling (right)

## 6.3 Planned work for the next 4 months

The integration of the surrogate models into the Trajectory block of the complete MDO shows that the interpolating methods chosen for the training procedure do not guarantee continuity of the design variables gradients, thus resulting in highly noisy functions for their numerical approximations. Since the optimiser requires the gradient to be continuous to achieve convergence, the computed models are unsuitable for the initial goal.

The future work will then focus on the computation of new surrogates using appropriate interpolating routines that ensure continuity in the variables gradients. OpenMDAO also provides a meta-model package that may be suitable for the purpose and included in the conducted work.

# 7 Task 2 - LLO to HEO transfers

## 7.1 Description of Work

The last task is the development of a simplified model to describe and simulate optimal transfer trajectories from LLOs to NRHOs. The dynamics is described in the restricted two-body problem approximation and the actual NRHO is substituted by an Highly Elliptical Orbit (HEO) characterised by the same orbital period and periapsis altitude. The aim is to compute a first approximation of the $\Delta v$ and propellant mass required to transfer the spacecraft from a circular orbit at very low altitude to an NRHO characterised by a periapsis passage close to the Moon surface but also a very high apoapsis altitude.

## 7.2 Technical Progress: 2 months achievements

A first model assumes the spacecraft to perform an impulsive injection burn at the apoapsis of the arrival HEO thus reducing the problem to the determination of a powered trajectory that allows the spacecraft to depart the initial LLO and inject into an intermediate coasting orbit whose apoapsis has the same altitude as the one of the targeted HEO. The optimal control problem assumes a constant thrust transfer with fixed initial conditions on the states and the opposite of the final spacecraft mass as the objective to minimise. The time of flight is treated as design variable while an appropriate boundary condition is imposed on the final node to guarantee the spacecraft is injected into a transfer trajectory with the specified apoapsis radius $r_a$. This constraint is expressed in terms of orbit radius $r$, radial and tangential velocities $u, v$ as follows:

$$\Psi(r, u, v, r_a) = r_a - \frac{r\left[\mu + \sqrt{(rv^2 - \mu)^2 + (ruv)^2}\,\right]}{2\mu - r(u^2 + v^2)} = 0 \tag{7.1}$$

Once this condition is satisfied, the spacecraft enters a ballistic phase until it reaches the required altitude where an instantaneous $\Delta v$ is imparted to inject in the target HEO.

A second model relaxes the previous assumption of an impulsive injection burn and models the whole transfer as a three phases trajectory with two powered phases separated by an intermediate ballistic arc. Proper constraints are then enforced at each transition to guarantee continuity in the state variables across subsequent phases.

In both cases the continuous-time optimal control problem is transcribed into a Non-Linear Programming Problem (NLP) using a Gauss-Lobatto transcription method with $3^{rd}$ order polynomials. For the first case, 200 segments are used to discretize the single powered phase while for the second simulations $200, 1200, 100$ segments are used for the first powered phase, the ballistic arc and the last finite burn. The high number of nodes used in the intermediate coasting phase is required to limit the final error due to the long propagation time, in the order of few days. The spacecraft departs from a circular LLO at 100.0 $km$ altitude and targets an HEO with periapsis and apoapsis radii of 3150.0 $km$ and 65227.4 $km$ respectively. The engine performances are given in terms of specific impulse $Isp$ and initial thrust over weight ratio $twr$ chosen equal to 450.0 $s$

and 2.1 for both simulations.

| | Single phase + impulsive burn | Three phases |
|---|---|---|
| Time of flight (days) | 3.189798 | 3.190680 |
| Propellant fraction | 0.139711 | 0.139711 |
| Departure $\Delta v$ $(m/s)$ | 645.073323 | 645.071546 |
| Insertion $\Delta v$ $(m/s)$ | 19.024167 | 19.024079 |

Table 7.1: Numerical Results for an LLO to HEO transfer with $Isp = 450.0$ $s$ and $twr = 2.1$

As reported in table 7.1, the results obtained for the two simulations are almost coincident and demonstrate the limited impact of the injection burn in the overall mass budget. As expected, since its magnitude is limited the corresponding losses are negligible and the finite burn can be approximated with an impulsive $\Delta v$ without loss of accuracy. On the other side, given the large $\Delta v$ required to leave the initial LLO, the finite thrust hypothesis has to be maintained to finally achieve the required level of accuracy. All numerical results presented in this section are validated with an explicit simulation of the transfer trajectory starting from the imposed initial conditions and the computed optimal control profile.

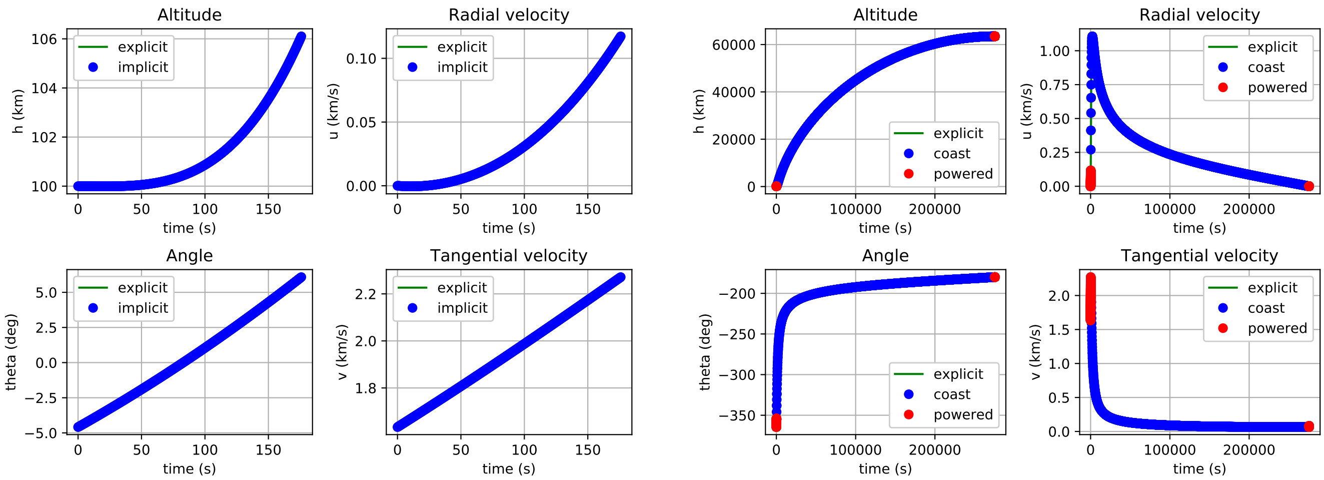


Figure 7.1: States profiles for the first powered phase (left) and the three-phases trajectory (right)

## 7.3 Planned work for the next 4 months

After a successful simulation of an LLO to HEO transfer, the next step will be the development of an appropriate dynamical model to describe the three-dimensional motion of the spacecraft in the cislunar realm and the design of optimal transfer trajectories between LLO and actual NRHO. State-of-the-art solutions will be initially reviewed before proceeding with the optimal control problem formulation and the algorithm implementation to finally optimise and simulate continuous thrust transfers in more complex dynamical frameworks.

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