Tema 2.3 Optimización II: Backpropagation

Miguel Ángel Martínez del Amor

Deep Learning

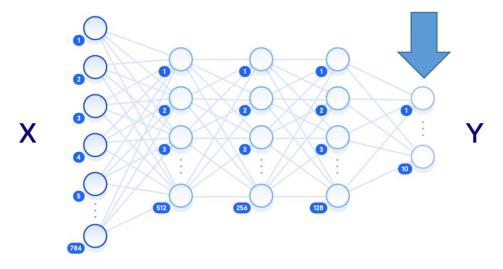
Departamento Ciencias de la Computación e Inteligencia ARtificial
Universidad de Sevilla

Contenido

- Propagación del gradiente
- Grafo computacional
- Vectorización

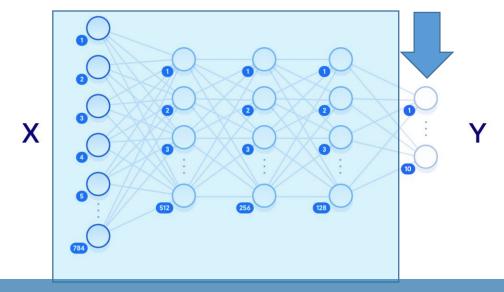
Descenso por gradiente

- Las actualizaciones se pueden aplicar directamente para actualizar pesos en modelos como regresión lineal y logística (perceptrón con función de activación sigmoide).
- ¿Cómo proceder con red neuronal multicapa?



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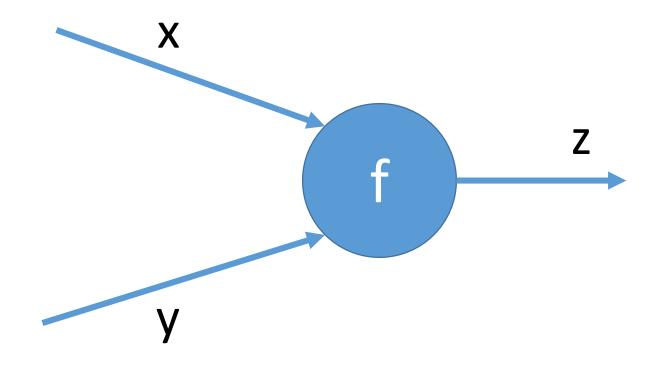
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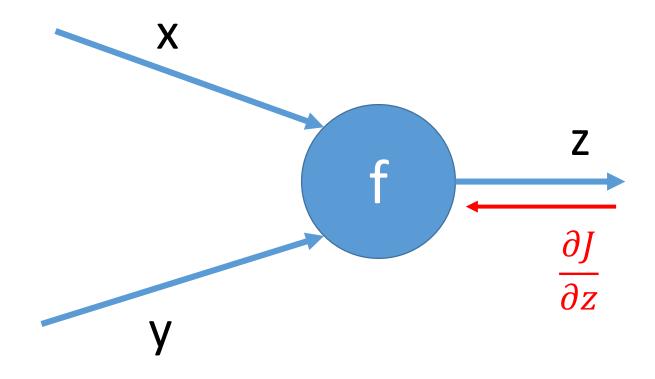
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- ¿Cómo proceder con red neuronal multicapa?
- En la capa de salida, usar la actualización de pesos con el gradiente sobre la función de coste.
- Problema: en las capas ocultas desconocemos los valores esperados.
- **Solución**: propagar los gradientes, algoritmo de retropropagación (backpropagation)

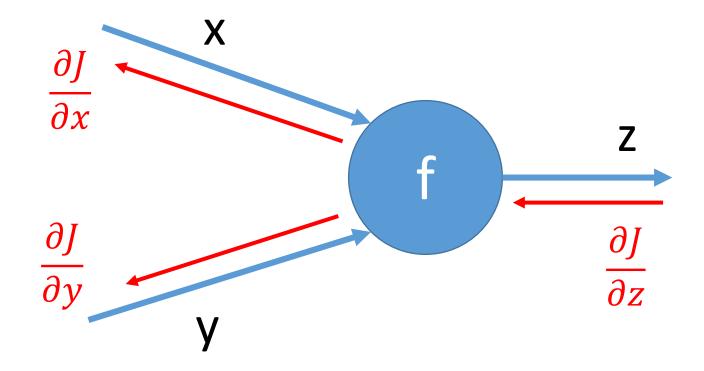
• Propagación hacia adelante (forward):



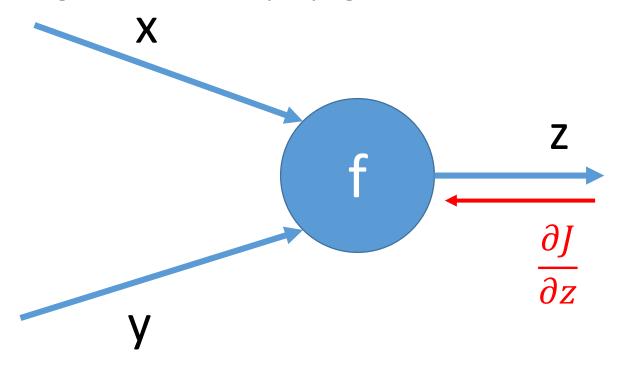
• Propagación hacia atrás (backward):



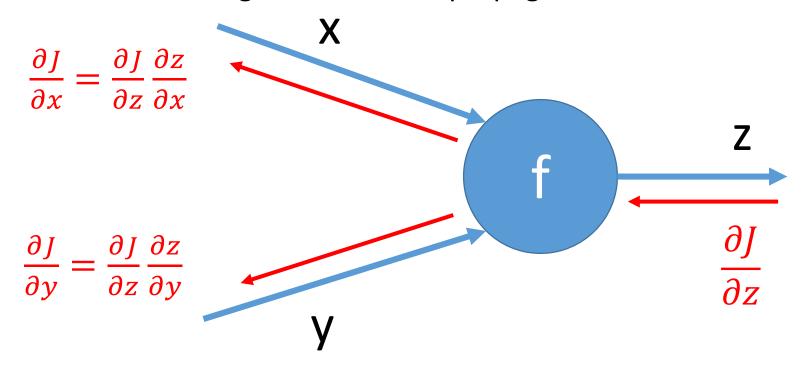
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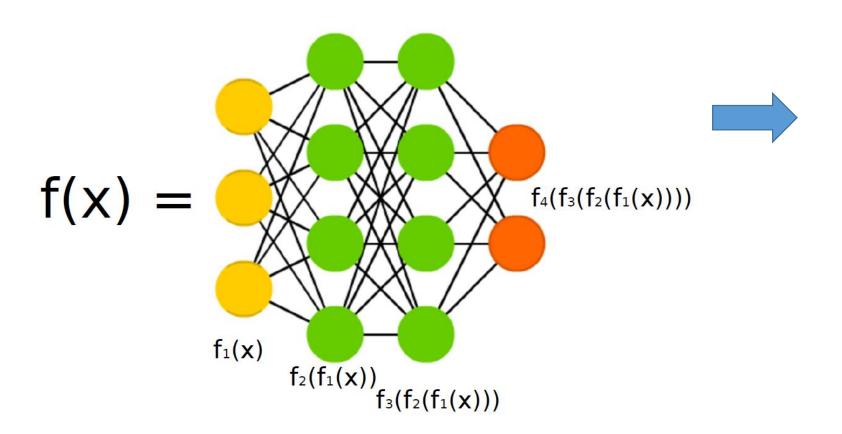


- Regla de la cadena:
 - Clave del algoritmo de backpropagation.

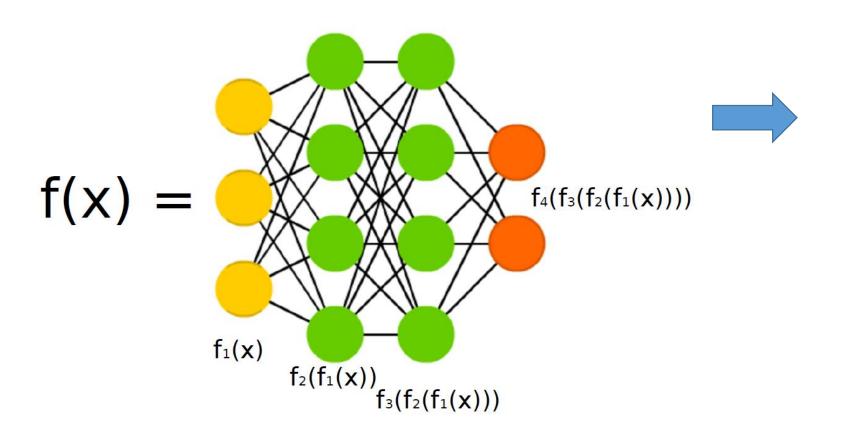


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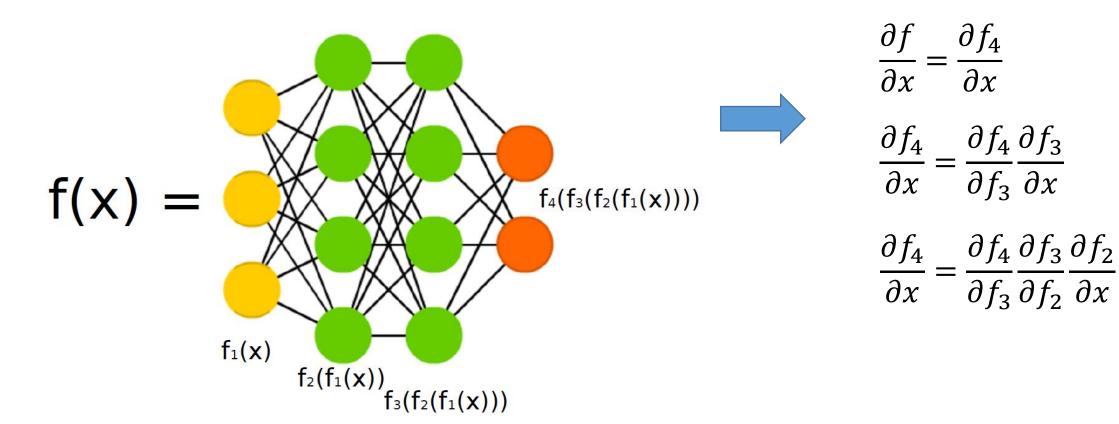


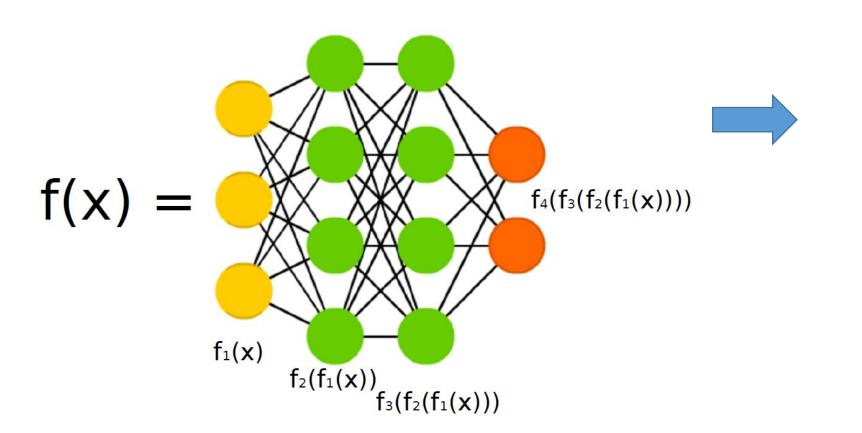
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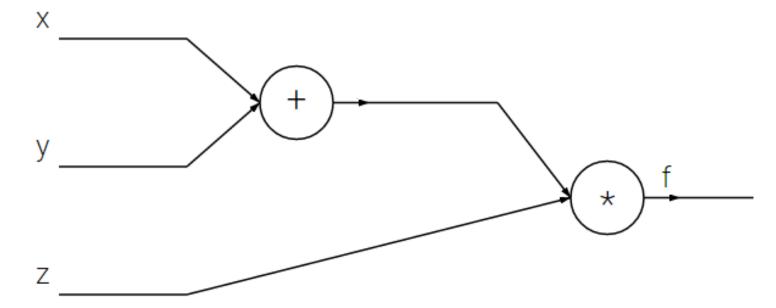
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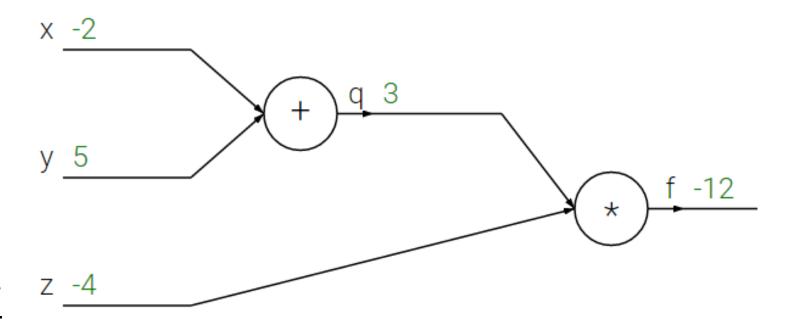
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- Sea x=-2, y=5, z=-4
- Sea q = (x + y)
- f = qz
- $\frac{\partial q}{\partial x} = 1$, $\frac{\partial q}{\partial y} = 1$
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- Buscamos: $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$



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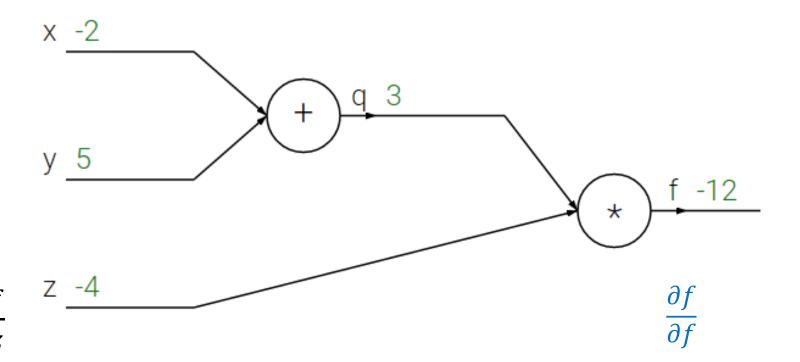
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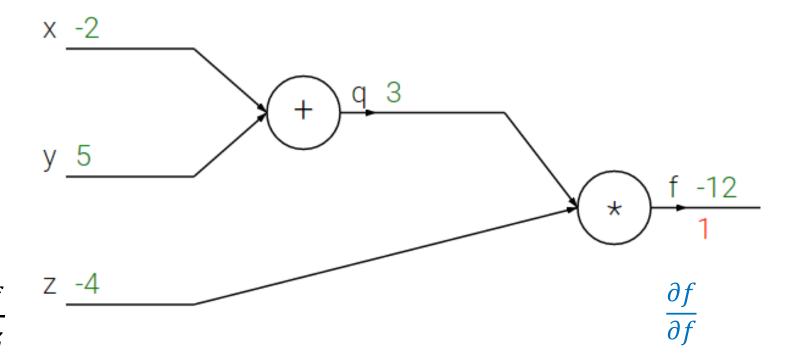
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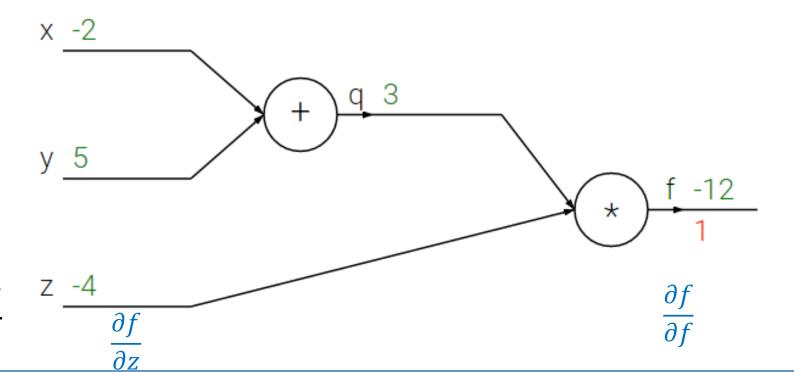
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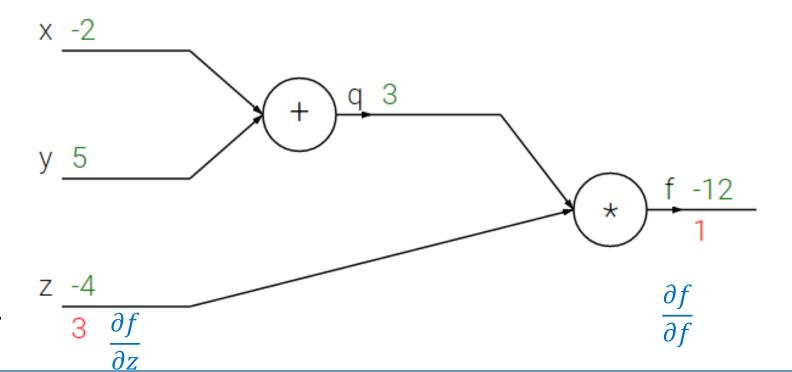
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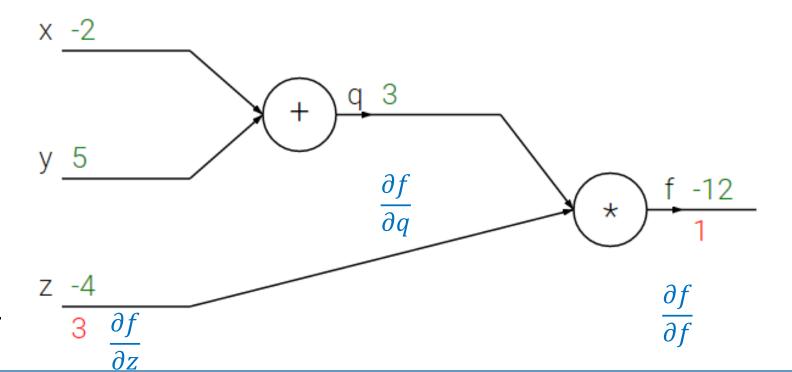
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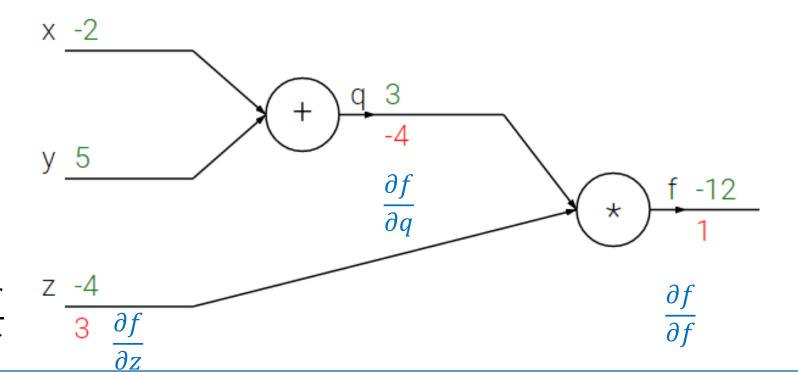
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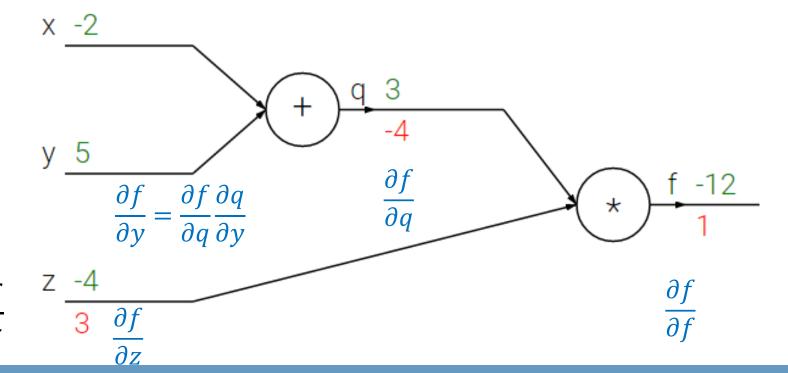
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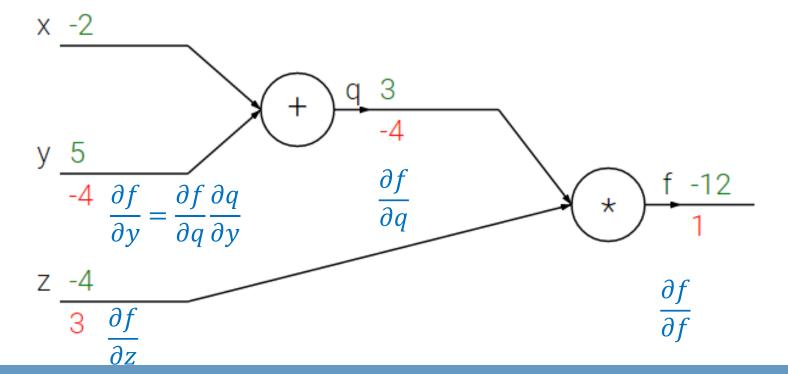
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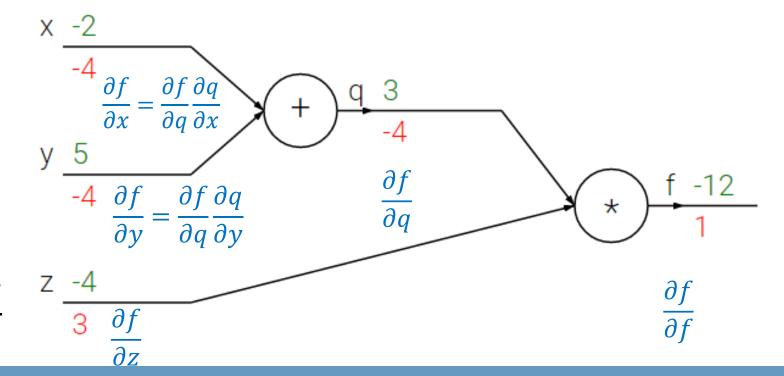
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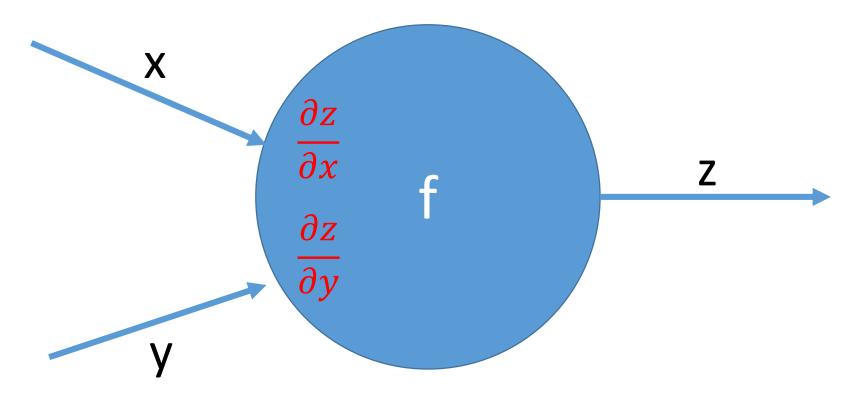
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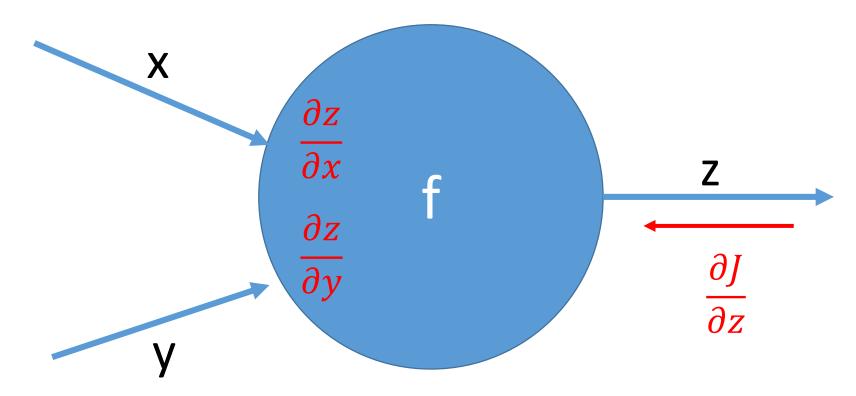
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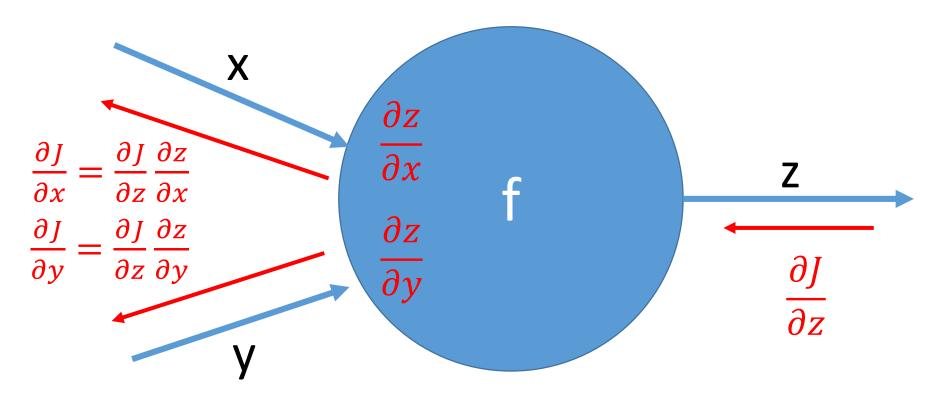
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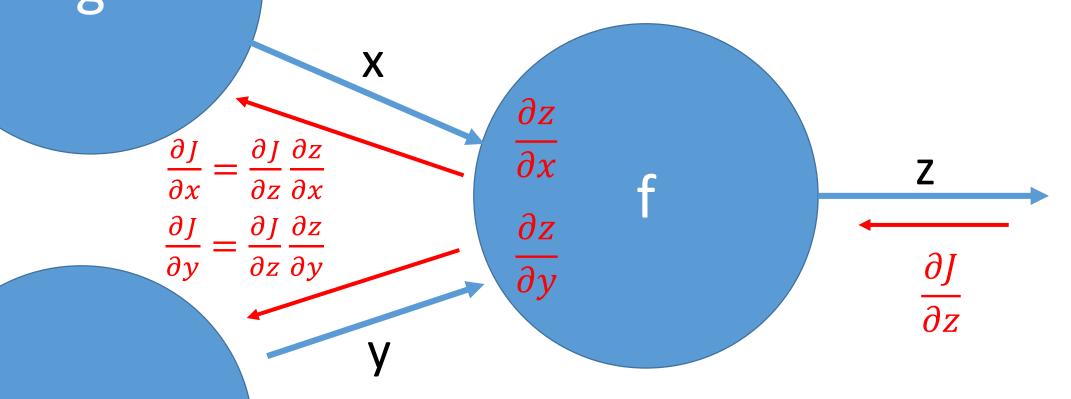
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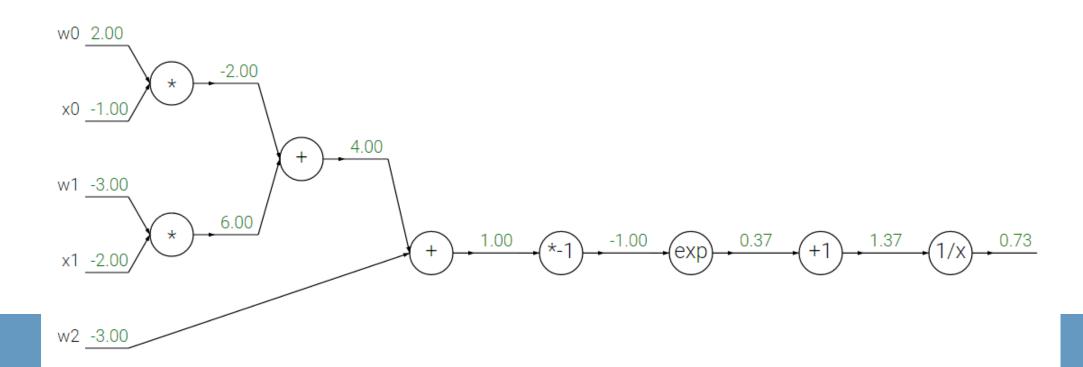


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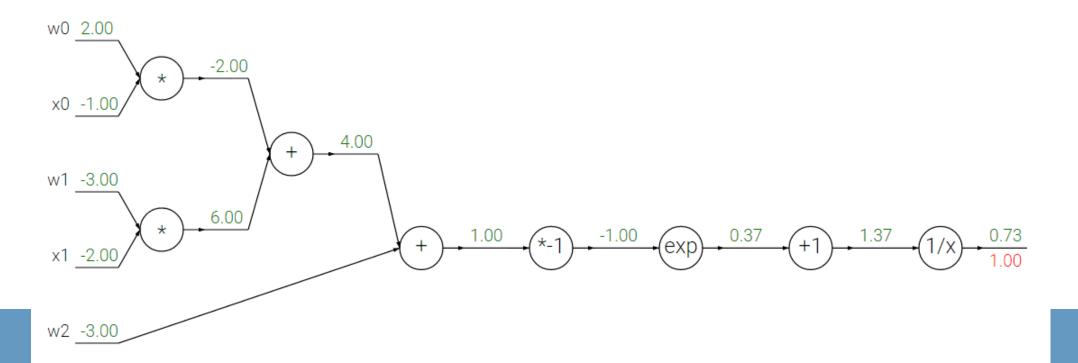


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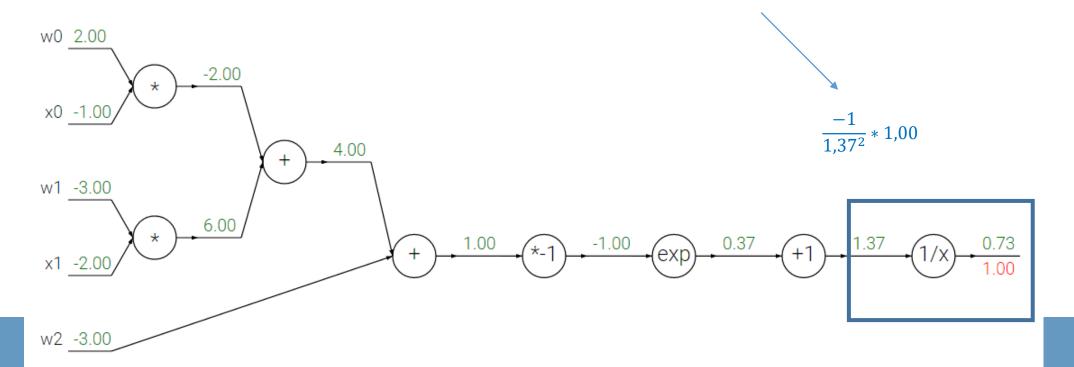




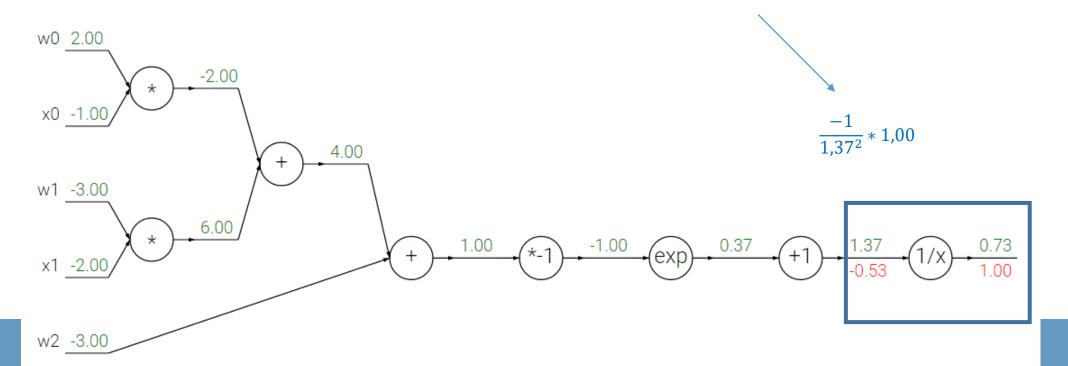
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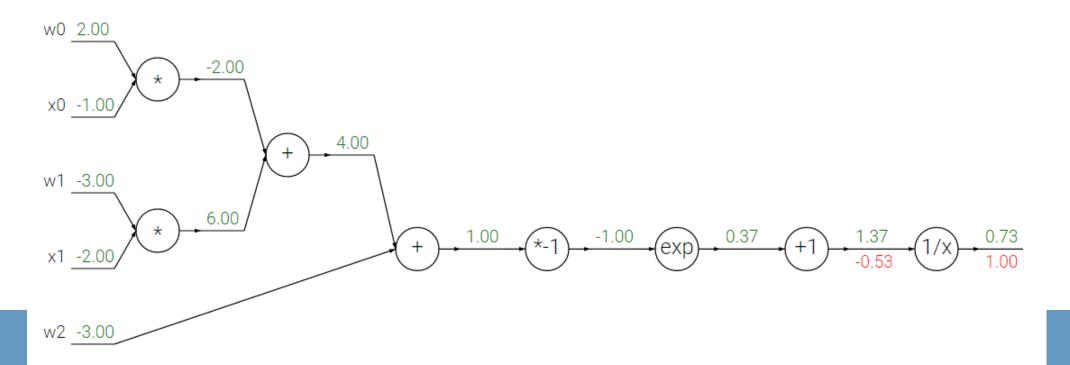
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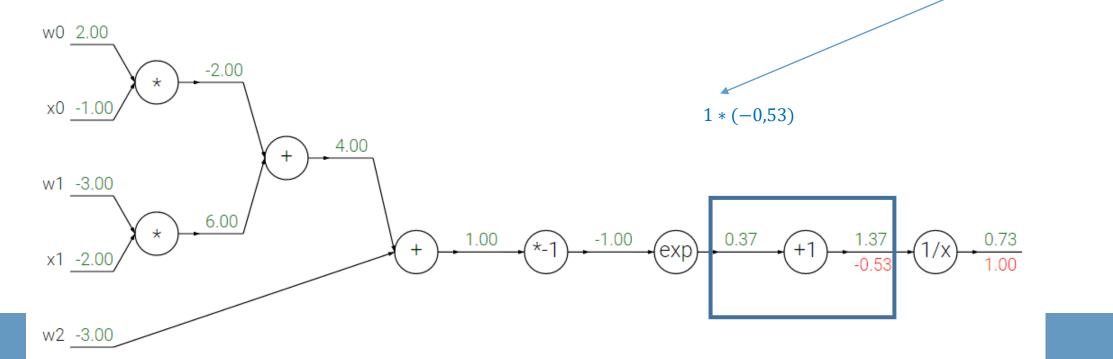
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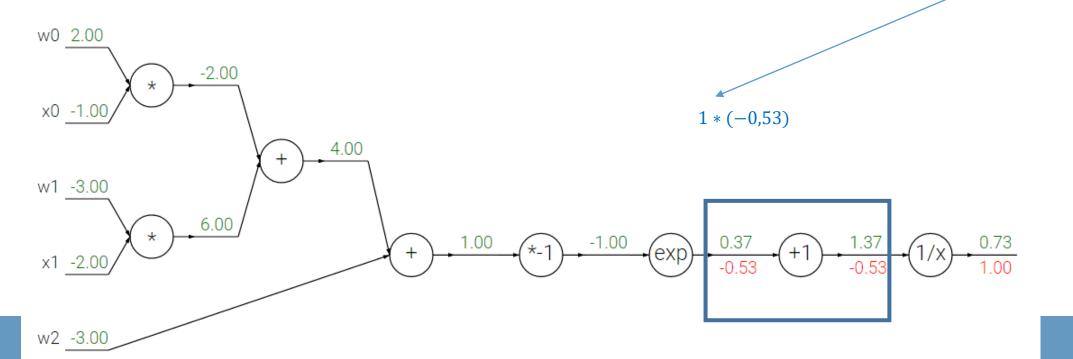
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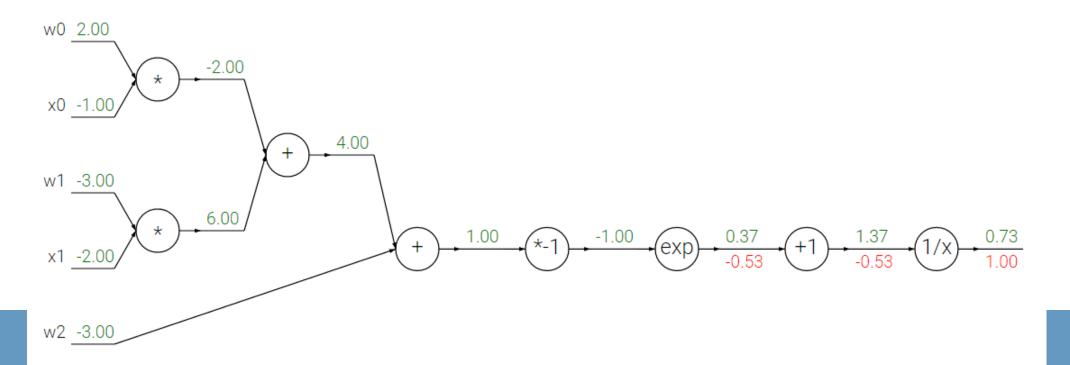
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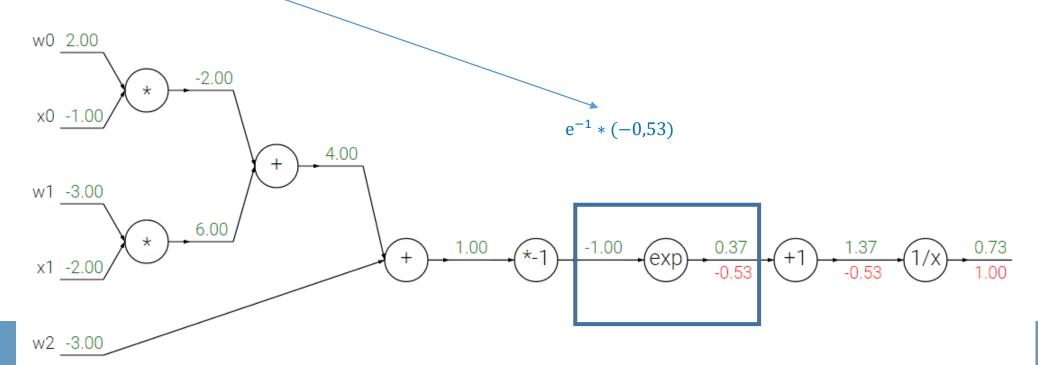
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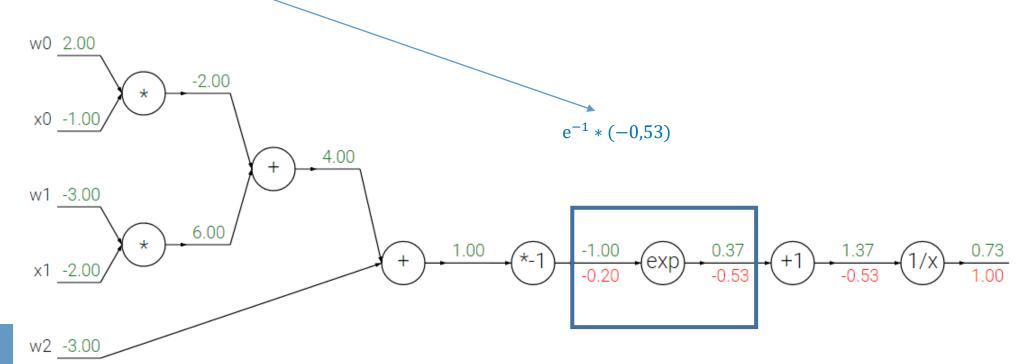
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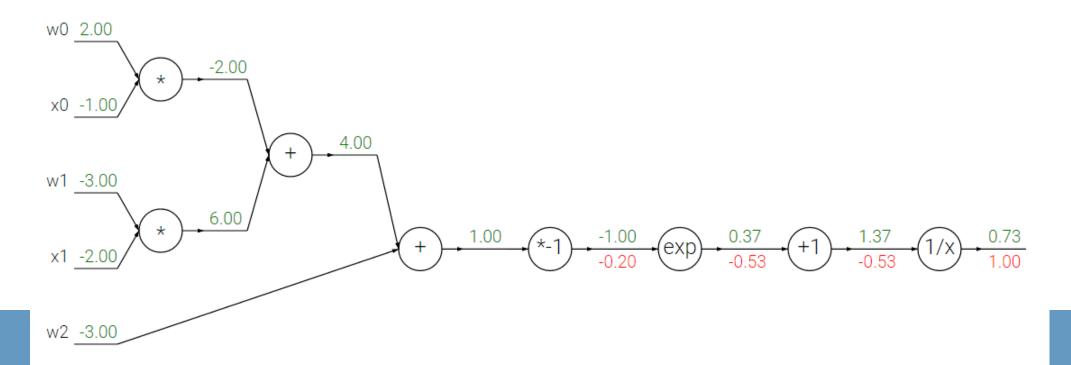
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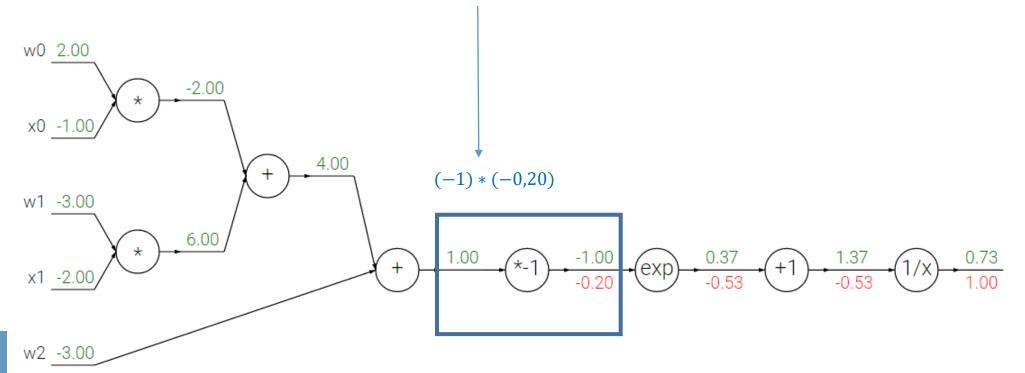
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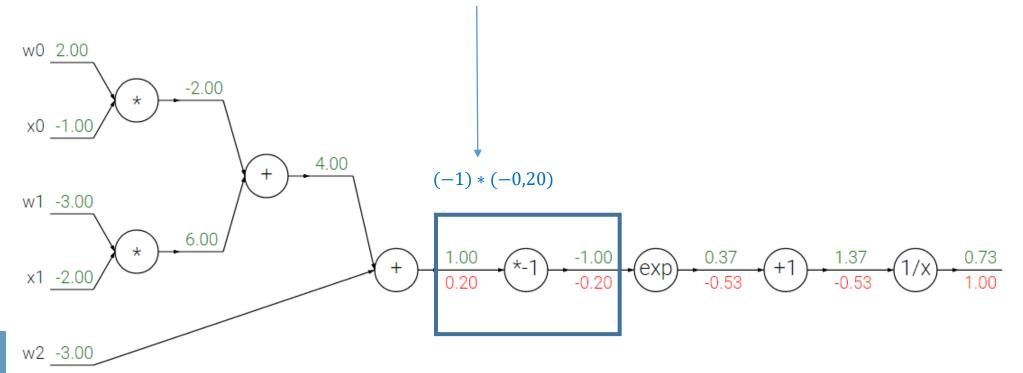
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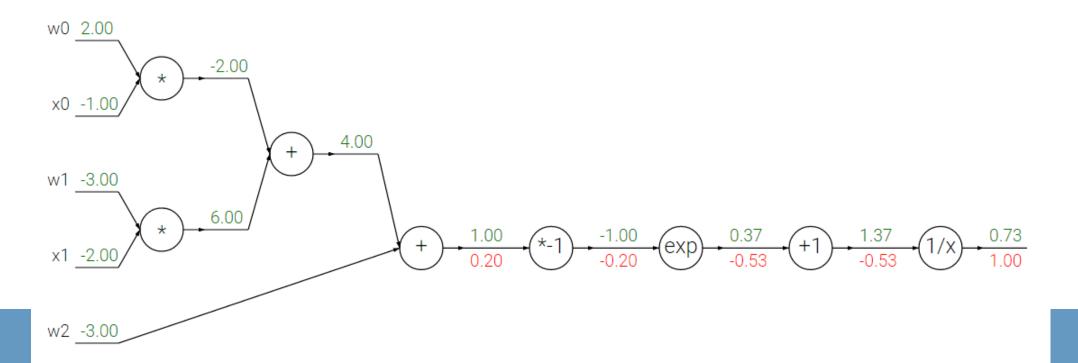
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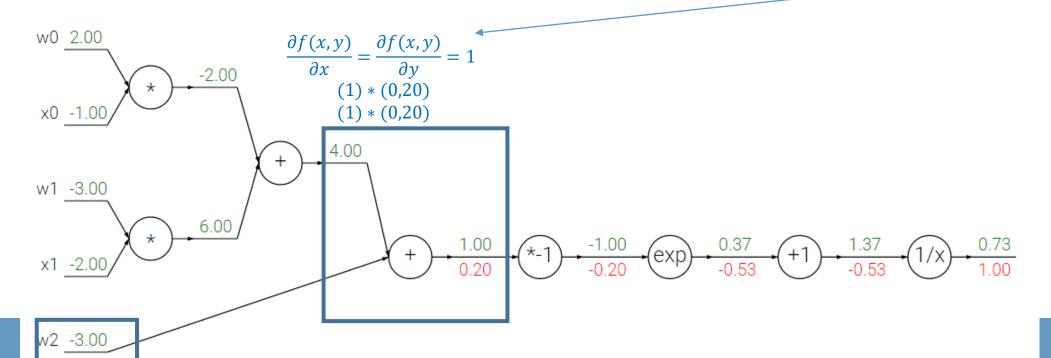
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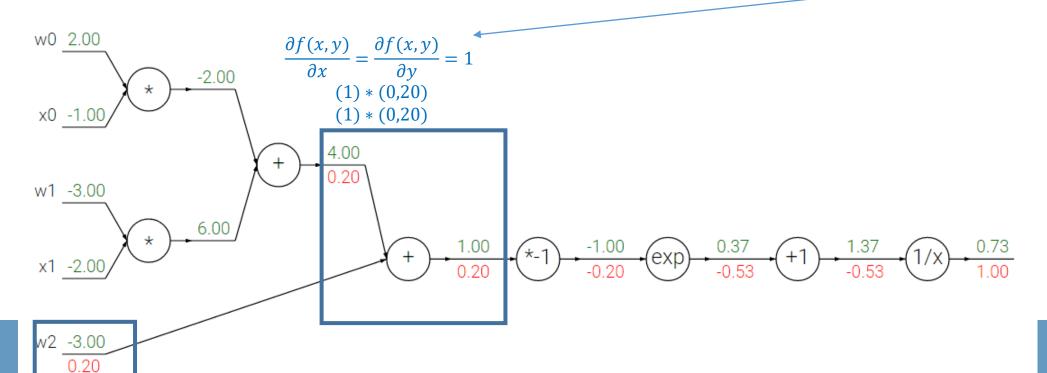
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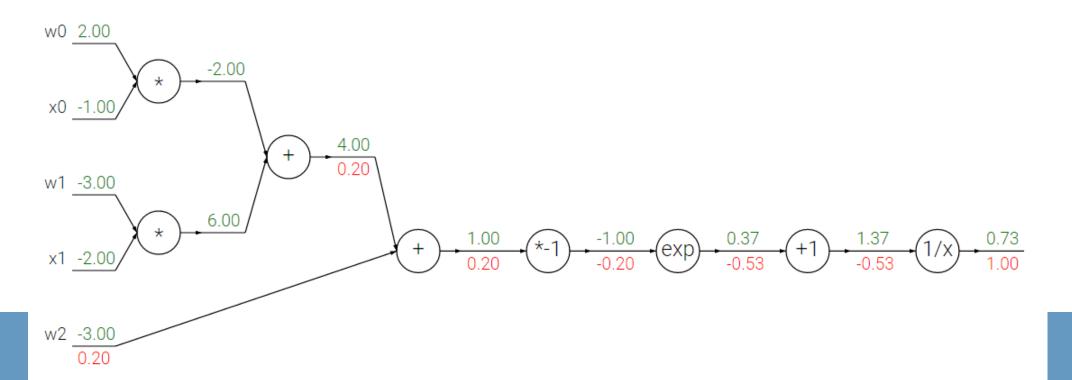
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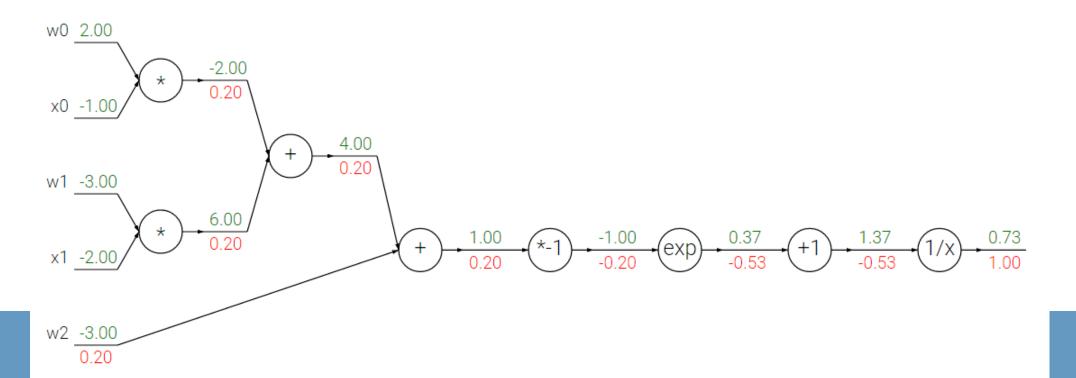
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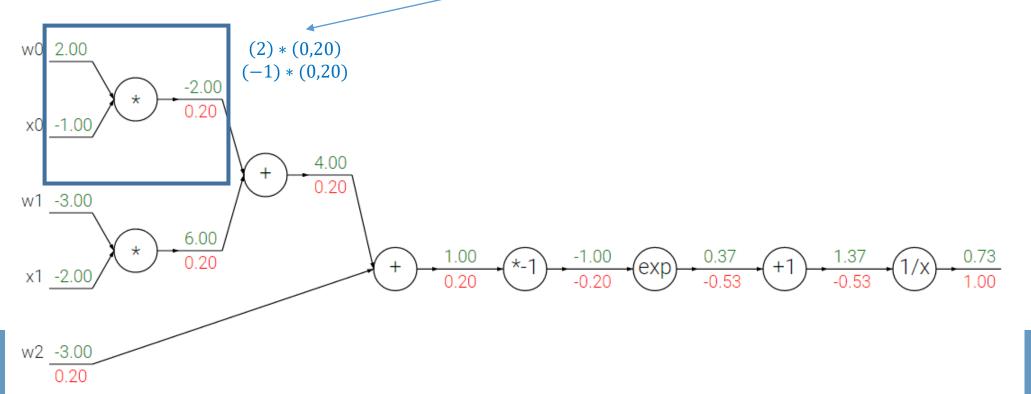
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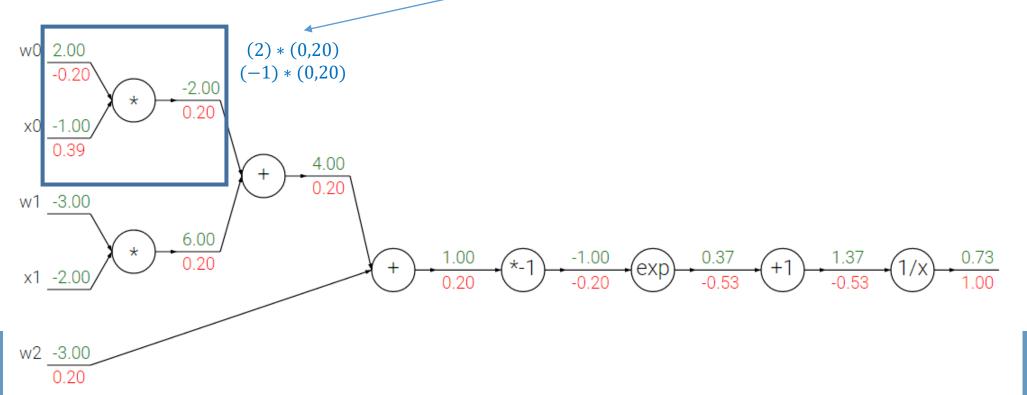
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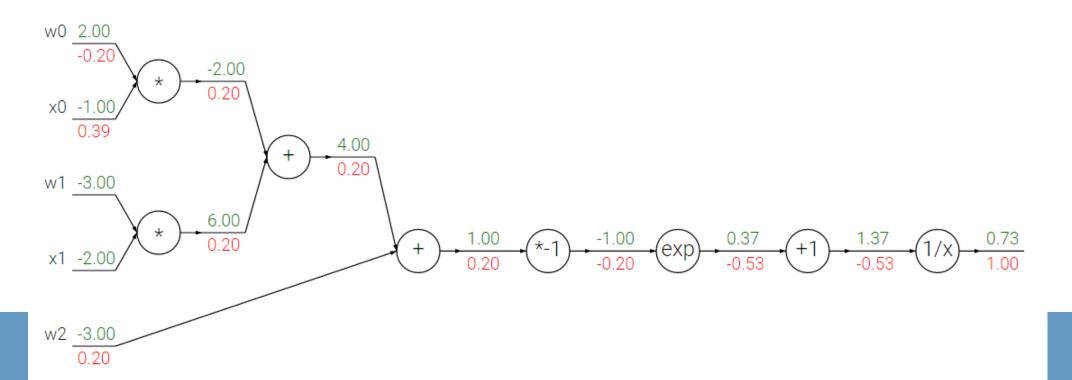
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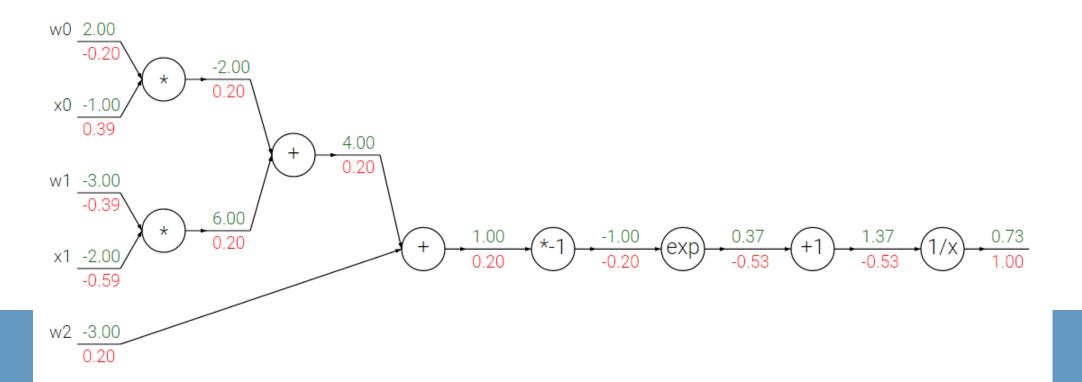
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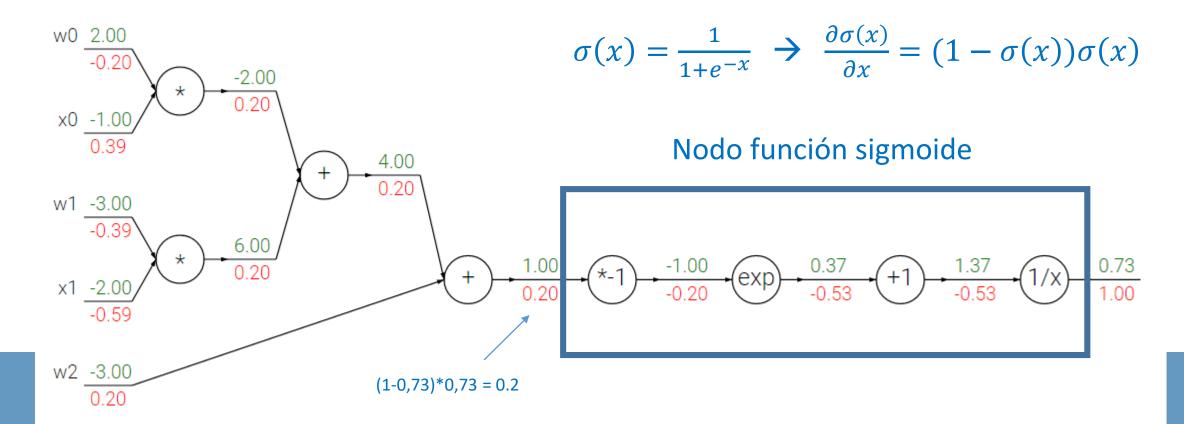
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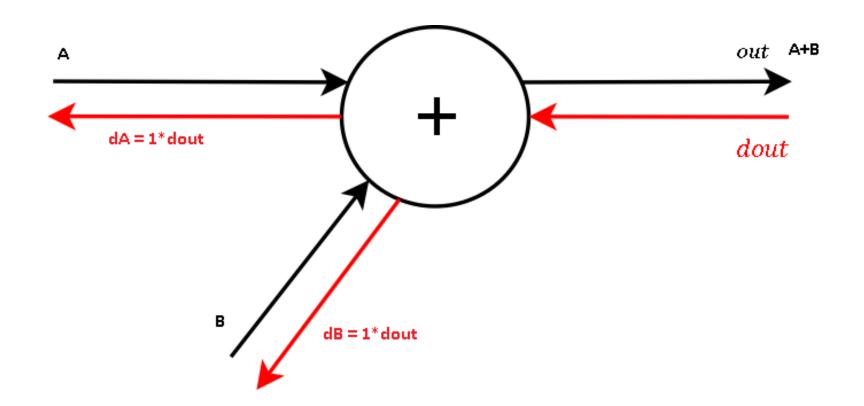
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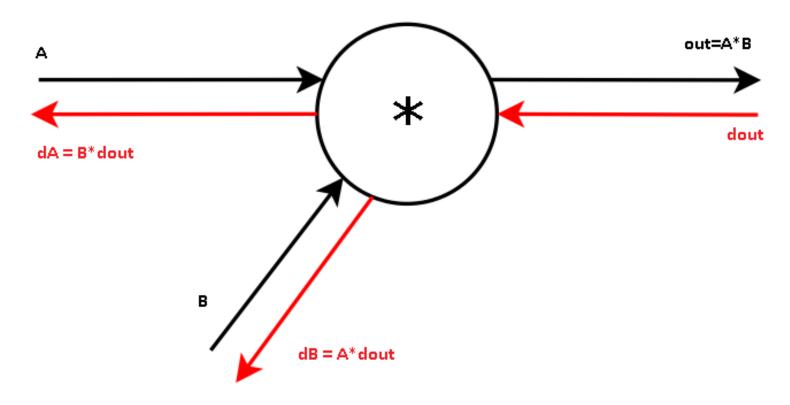
• Se pueden definir nodos para funciones conocidas para ahorrar pasos de computación.



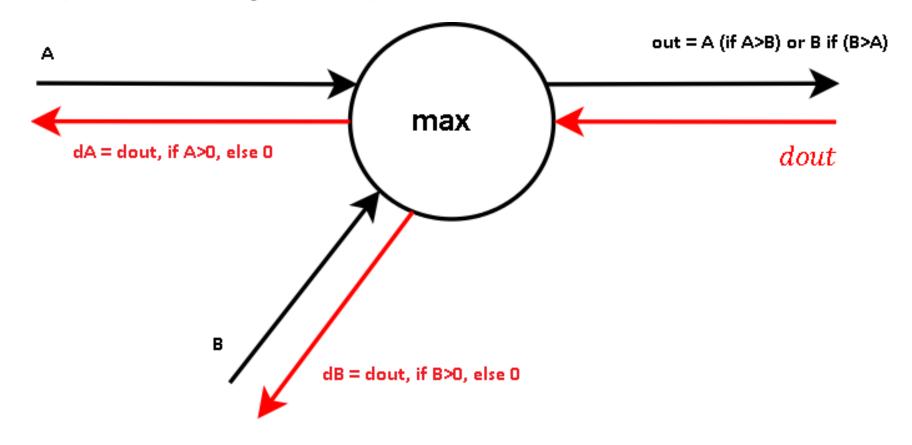
- Bloques básicos
 - Suma (distribuidor de gradiente)



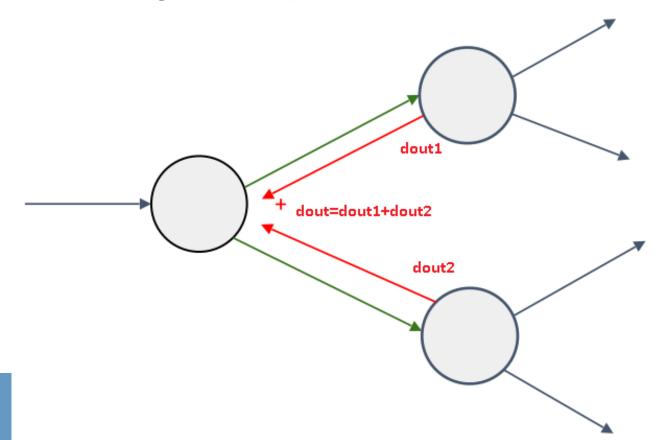
- Bloques básicos
 - Multiplicación (intercambiador de gradiente)



- Bloques básicos
 - Máximo (enrutador de gradiente)

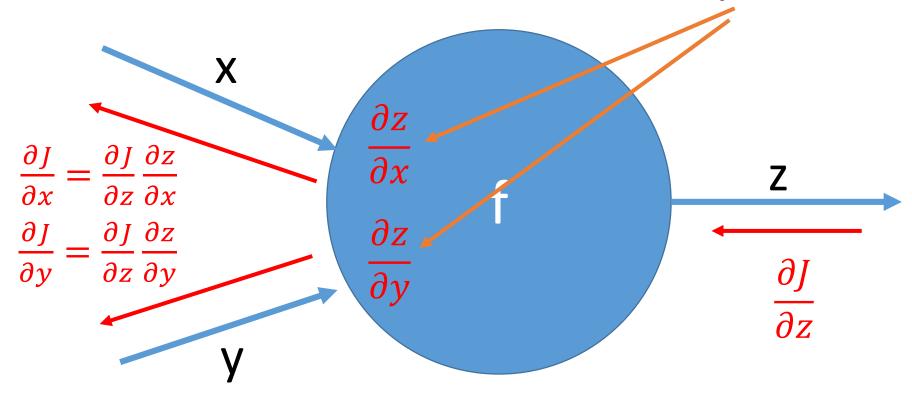


- Bloques básicos
 - Ramas (sumador de gradientes)



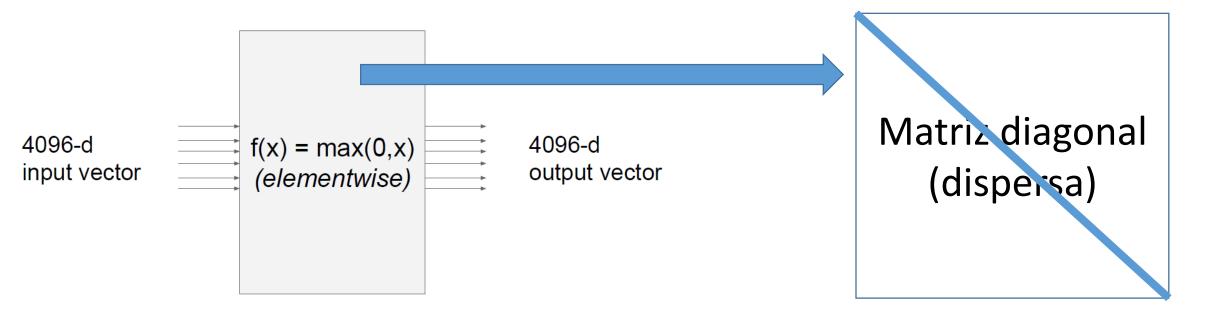
Propagación de gradiente con vectorización

• ¿Qué ocurre cuando las datos son vectores? Matrices jacobianas.



Propagación de gradiente con vectorización

• ¿Qué ocurre cuando las datos son vectores? Matrices jacobianas.



Recapitulación

- La base del algoritmo de backpropagation es la regla de la cadena.
- La **propagación del gradiente** es la clave para actualizar los pesos en las **capas ocultas**.
- Si vemos la red como un **grafo computacional**, podemos entender la intuitivamente cómo funciona el algoritmo.
- Es importante entender cómo se propagan los gradientes para evitar el **problema del desvanecimiento del gradiente**.