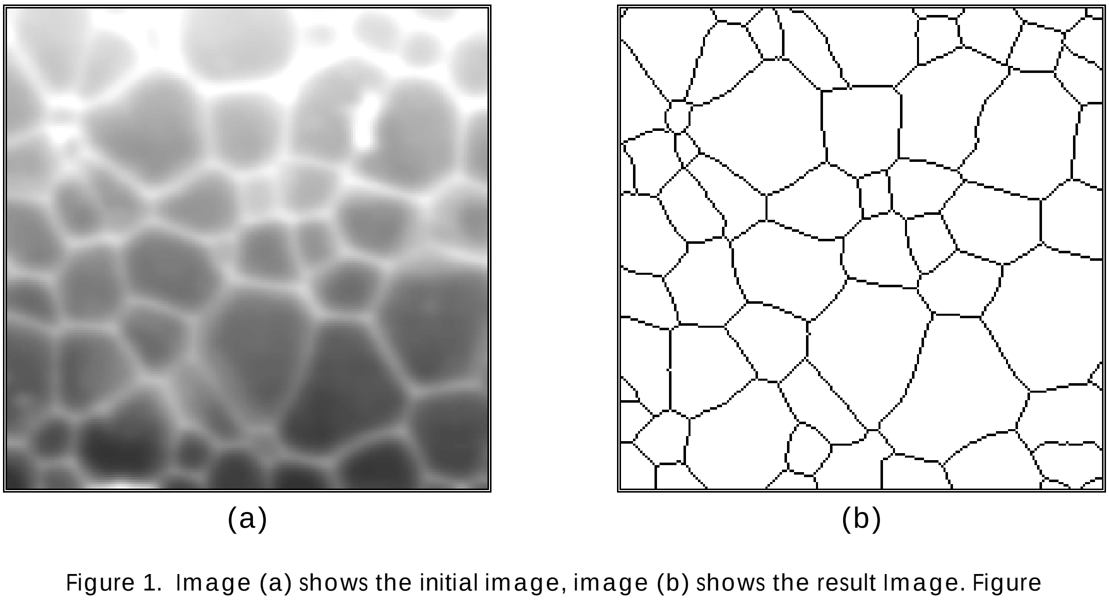
Watershed transformation

The watershed transformation was originally proposed by Digabel and Lantu_ejoul [1,2] and later improved by Beucher and Lantuéjoulin [3] in late of 70's as a tool for segmenting grayscale images. Nowadays it is used as an elemental step in many powerful segmentation procedures. The watershed constitutes one of the main concepts of Mathematical Morphology. The watershed transform can be classified as a region-based segmentation approach.

The intuitive idea underlying this method comes from geography. Since any grayscale image can be considered as topographic surface: we regard the intensity of a pixel as altitude of the point. Let us imagine the surface of this relief being immersed in still water, with holes created in local minima. Water fills up the dark areas "the basins" starting at these local minima. Where waters coming from different basins meet we will build dams. When the water level has reached the highest peak in the landscape, the process is stopped. As a result, the landscape is partitioned into regions or basins separated by dams, called watershed lines or simply watersheds.

Many different watershed algorithms have been proposed until today.



(a) (b)

Figure 1. Image (a) shows the initial image, image (b) shows the result Image. Figure 2 is an demonstrations of the basic idea applied on image (a). [5]

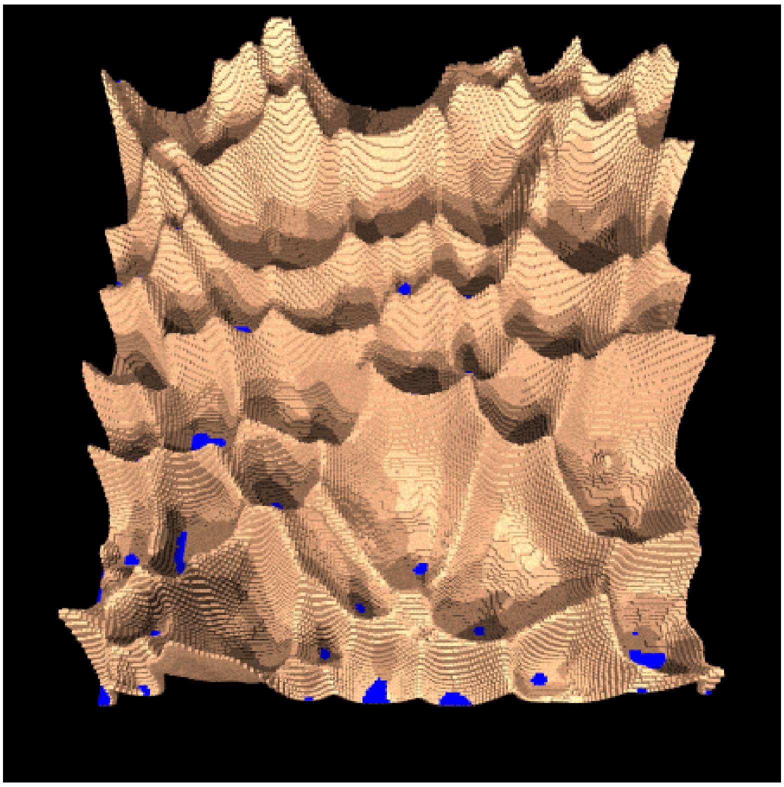


Figure 2. The demonstration of the algorithm on the image from Figure 1.

Watershed definitions.

Continues case.

A watershed definition for the continuous case can be based on distance functions. Assume that the image f is an element of the space C(D) of real twice continuously differentiable functions on a connected domain D with only isolated critical points. Then the topographical distance between points p and q in D is defined by:

$$T_f(p,q) = \inf_{\gamma} \int_{\gamma} \left\| \nabla f(\gamma(s)) \right\| ds$$

where the infimum is over all paths (smooth curves) inside D with (0) = p, (1) = q.

Let $f \in C(D)$ have minima $\{m_k\}_{k \in I}$, for some index set I. The catchment basin $CB(m_i)$ of a minimum m_i is defined as the set of points $x \in D$ which are topographically closer to m_i than to any other regional minimum m_j :

$$CB(m_i) = \{x \in D \quad | \forall j \in I \setminus \{\mathrm{i}\}: f(m_i) + Tf(x, m_i) < f(m_j) + Tf(x, m_j)\}$$

The watershed of f is the set of points which do not belong to any catchment basin:

$$Wshed(f) = D \cap \left(\bigcup_{i \in I} CB(m_i) \right)$$

So the watershed transform of f assigns labels to the points of D, such that

i. different catchment basins are uniquely labeled, and a

ii. special label W is assigned to all points of the watershed of f.

Discrete case (Flooding paradigm).

Meyer's flooding algorithm:

Label the regional minima with different colors
Repeat

- Select a pixel p, not colored, not watershed, adjacent to some colored pixels, and having the lowest possible gray level
- If p is adjacent to exactly one color then label p with this color

- If p is adjacent to more than one color then label p as watershed

Until no such pixel exists

Let (V,E) be a (undirected) graph and let X be a subset of V. We define a point x ($x \in X$) as a *W-destructible* for X if x is adjacent exactly one connected component of $\overline{X}$. We consider the image as a graph. The pixels are the vertices. The edges comes from the four neighbors of a pixels.

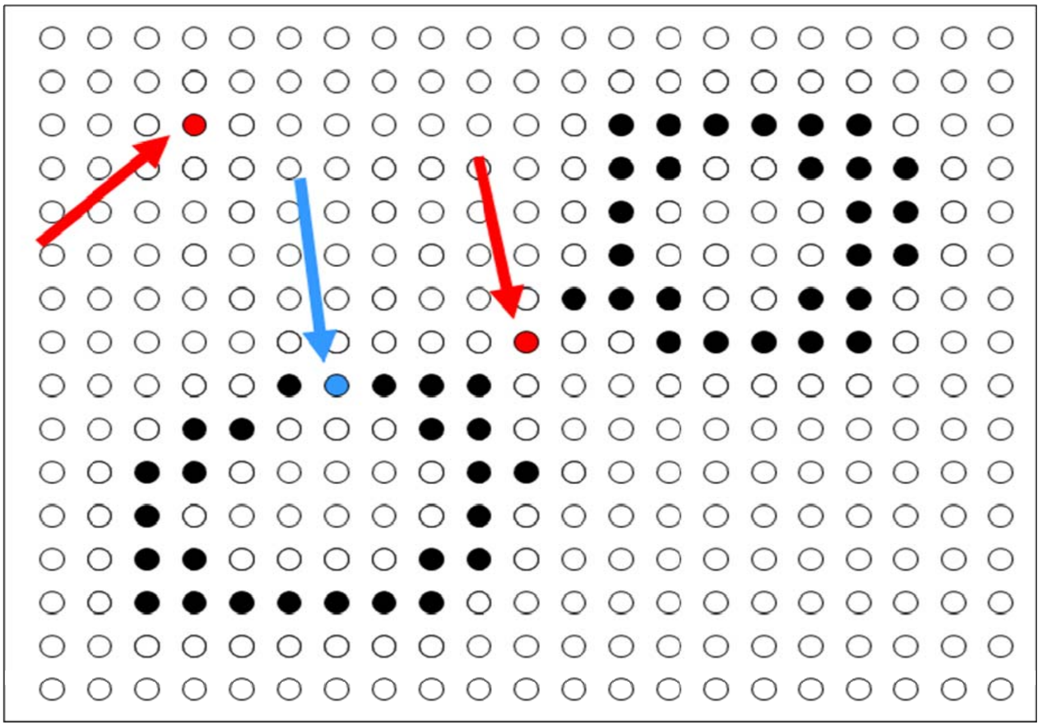


Figure 3. Illustration of W-destructible points. [6]

Let X and Y be two subsets of V. We say that Y is a W-thinning of X if Y may be obtained from X by iteratively removing W-destructible points.

Let X and Y be subsets of V. We say that Y is a watershed of X if Y is a W-thinning of X and if there is no W-destructible point for Y.

Let F and F’ be in F (V). We say that F’ is a thinning of F if F’ may be obtained from F by iteratively lowering destructible points (by 1).

Let F and F' be in F (V). We say that F' is a watershed of F if F' is a thinning of F and if there is no destructible point for F'.

Accuracy of watershed lines.

The result should be a close approximation of the continuous case. That is, the digital distances playing a role in the watershed calculation should approximate the Euclidean distance. Chamfer distances are an efficient way to achieve accurate watershed lines. The watershed method in its original form produces a severe over segmentation of the image, i.e., many small basins are produced due to many local minima in the input image. Several approaches exist to avoid this, such as markers or hierarchical watersheds. Parallelization of marker-based watershed algorithms has been studied also.

[1] **Digabel, H., and Lantuéjoul,** C. “Iterative algorithms”. In Actes du Second Symposium Européen d'Analyse Quantitative des Microstructur es en Sciences des Matériaux , Biologie et Médecine, Caen, 4-7 October 1977 (1978), J.-L. Chermant, Ed., Riederer Verlag, Stuttgart, pp. 85-99.

[2] **Lantuéjoul,** C. La squelettisation et son application aux mesures topologiques des mosaiques polycristallines. PhD thesis, Ecole des Mines, Paris, 1978.

[3] **Beucher, S., and Lantuéjoul**, C. “Use of watersheds in contour detection.” In Proc. International Workshop
on Image Processing, Real-Time Edge and Motion Detection/Estimation, Rennes, September (1979).

[4] **J.B.T.M. Roerdink, A. Meijster**, “The watershed transform: definitions, algorithms and parallelization strategies”, Fundamenta Informaticae, Vol. 41, pp. 187-228, 2000.

[5] **G. Bertrand**, "On topological watersheds", Journal of Mathematical Imaging and Vision, Vol. 22, No. 2-3, pp. 217-230, 2005.

[6] **M. Couprie and L. Najman and G. Bertrand,** "Quasi-linear algorithms for the topological watershed", Journal of Mathematical Imaging and Vision, Vol. 22, No. 2-3, pp. 231-249, 2005.