

Solutions for Chapter 1

Exercise 1.1

(a) $R = 5\text{k} + 10\text{k} = \boxed{15\text{k}\Omega}$

(b) $R = \frac{R_1 R_2}{R_1 + R_2} = \frac{5\text{k} \cdot 10\text{k}}{5\text{k} + 10\text{k}} = \boxed{3.33\text{k}\Omega}$

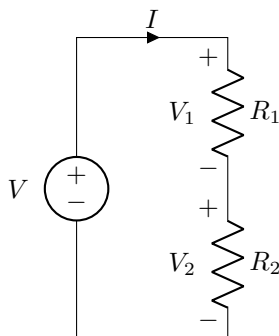
Exercise 1.2

$$P = IV = \left(\frac{V}{R}\right) V = \frac{(12\text{V})^2}{1\Omega} = \boxed{144\text{W}}$$

Exercise 1.3

Consider a simple series resistor circuit.

Figure 1: A basic series circuit.



By KVL and Ohm's law

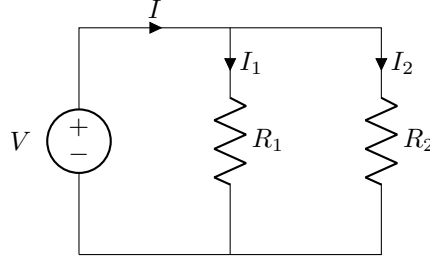
$$V = V_1 + V_2 = R_1 \cdot I + R_2 \cdot I = (R_1 + R_2) \cdot I = R \cdot I$$

where

$$\boxed{R = R_1 + R_2}$$

is the resistance of R_1 and R_2 in series. Now, consider a simple parallel resistor circuit.

Figure 2: A basic parallel circuit.



By KCL and Ohm's law

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \cdot V$$

solving for V as a function of I we get

$$V = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot I = \frac{R_1 R_2}{R_1 + R_2} \cdot I = R \cdot I$$

where

$$\boxed{R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}}$$

is the resistance of R_1 and R_2 in parallel.

Exercise 1.4

We know that the resistance R_{12} ¹ of two resistors R_1 and R_2 in parallel is given by

$$R_{12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Now, the resistance R_{123} of three resistors R_1 , R_2 and R_3 in parallel is equal to the resistance of two resistors R_{12} (the resistance between R_1 and R_2 in parallel) and R_3 in parallel, then

$$R_{123} = \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_3}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

We will prove by induction that the resistance $R_{1\dots n}$ of n resistances R_1, R_2, \dots, R_n in parallel is given by

$$R_{1\dots n} = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}}$$

First, it's trivial to show that with $n = 1$ the equality holds. Now, we will assume that the equality is satisfied for $n = k$, that is

$$R_{1\dots k} = \frac{1}{\sum_{i=1}^k \frac{1}{R_i}}$$

¹Here we have only assigned a name to the resistance in parallel between R_1 and R_2 .

Then, we must show that equality holds for $n = k + 1$. Thus, the resistance $R_{1\dots(k+1)}$ of $(k+1)$ resistances R_1, R_2, \dots, R_{k+1} in parallel is equal to the resistance of two resistors $R_{1\dots k}$ and R_{k+1} in parallel, then

$$R_{1\dots(k+1)} = \frac{1}{\frac{1}{R_{1\dots k}} + \frac{1}{R_{k+1}}} = \frac{1}{\sum_{i=1}^k \frac{1}{R_i} + \frac{1}{R_{k+1}}} = \frac{1}{\sum_{i=1}^{k+1} \frac{1}{R_i}}$$

where we have proved that equality holds for $n = k + 1$. Finally, the resistance of n resistors in parallel is given by

$$\mathbf{R_{1\dots n} = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}}$$

Exercise 1.5

Given that $P = \frac{V^2}{R}$, we know that the maximum voltage we can achieve is 15V and the smallest resistance we can have across the resistor in question is 1k Ω . Therefore, the maximum amount of power dissipated can be given by

$$P = \frac{V^2}{R} = \frac{(15\text{V})^2}{1\text{k}\Omega} = \mathbf{0.225\text{W}}$$

This is less than the 1/4W power rating.

Exercise 1.6

- (a) The total current required by New York City that will flow through the cable is $I = \frac{P}{V} = \frac{10^{10} \text{ W}}{115 \text{ V}} = 86.96 \text{ MA}$. Therefore, the total power lost per foot of cable can be calculated by:

$$P = I^2 R = (86.96 \times 10^6 \text{ A})^2 \times \left(5 \times 10^{-8} \frac{\Omega}{\text{ft}} \right) = \mathbf{3.78 \times 10^8 \frac{\text{W}}{\text{ft}}}$$

- (b) The length of cable over which all 10^{10} W will be lost is:

$$L = \frac{10^{10} \text{ W}}{3.78 \times 10^8 \frac{\text{W}}{\text{ft}}} = \mathbf{26.45 \text{ ft}}$$

- (c) To calculate the heat dissipated by the cable, we can use the Stefan-Boltzmann equation $T = \sqrt[4]{\frac{P}{A\sigma}}$, with A corresponding to the cylindrical surface area of the 26.45 foot long section of 1 foot diameter cable. Note that σ is given in cm^2 , so we will need to use consistent units.

$$A = \pi DL = \pi \times 30.48 \text{ cm} \times 806.196 \text{ cm} = 7.72 \times 10^4 \text{ cm}^2$$

Therefore,

$$T = \sqrt[4]{\frac{P}{A\sigma}} = \sqrt[4]{\frac{10^{10} \text{ W}}{7.72 \times 10^4 \text{ cm}^2 \times 6 \times 10^{-12} \frac{\text{W}}{\text{K}^4 \text{ cm}^2}}} = \mathbf{12,121 \text{ K}}$$

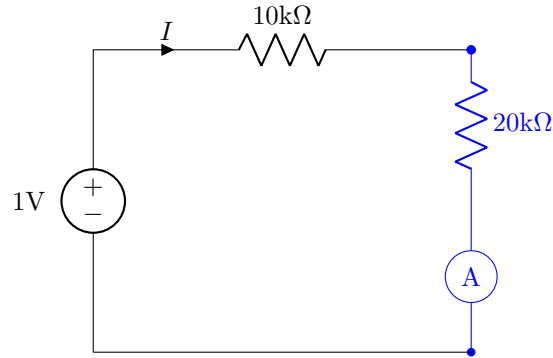
This is indeed a preposterous temperature, more than twice that at the surface of the Sun! The solution to this problem is that power should be transmitted along long distances at high voltage. This greatly reduces $I^2 R$ losses. For example, a typical high voltage line voltage is 115kV. At this voltage, the power loss per foot of cable is only 378 W per foot. Intuitively, we know that reducing current allows for lower power dissipation. We can deliver the same amount of power with a lower current by using a higher voltage.

Exercise 1.7

A $20,000\Omega/\text{V}$ meter read, on its 1V scale, puts an $20,000\Omega/\text{V} \cdot 1\text{V} = 20,000\Omega = 20\text{k}\Omega$ resistor in series with an ideal ammeter (ampere meter). Also, a voltage source with an internal resistance is equivalent to an ideal voltage source with its internal resistance in series.

(a) In the first question, we have the following circuit:

Figure 3: A voltage source with internal resistance and a $20,000\Omega/\text{V}$ meter read in its 1V scale.

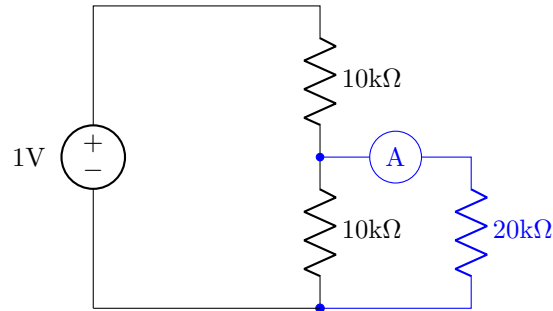


Then, we have that the current in the ideal ammeter and the voltage in the meter resistance are given by²

$$I = \frac{1\text{V}}{10\text{k}\Omega + 20\text{k}\Omega} = \boxed{0.0333\text{mA}} \quad \text{and} \quad V = 0.0333\text{mA} \cdot 20\text{k}\Omega = \boxed{0.666\text{V}}$$

(b) In the second question, we have the following circuit:

Figure 4: A $10\text{k}\Omega - 10\text{k}\Omega$ voltage divider and a $20,000\Omega/\text{V}$ meter read in its 1V scale.



Now, we can to obtain the Thévenin equivalent circuit of circuit in Figure 4 with

$$R_{\text{Th}} = \frac{10\text{k}\Omega \cdot 10\text{k}\Omega}{10\text{k}\Omega + 10\text{k}\Omega} = 5\text{k}\Omega$$

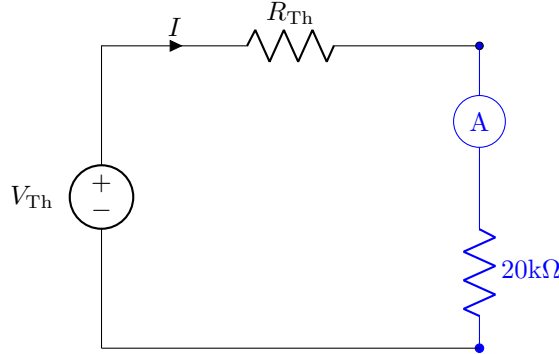
²When a meter only measures currents, it puts a resistance in series to measures the current through that resistance and internally converts that current into voltage to *measure voltages*.

and

$$V_{Th} = 1V \cdot \frac{10k\Omega}{10k\Omega + 10k\Omega} = 0.5V$$

Then, we have the following equivalent circuit:

Figure 5: Thévenin equivalent circuit of circuit in Figure 4.



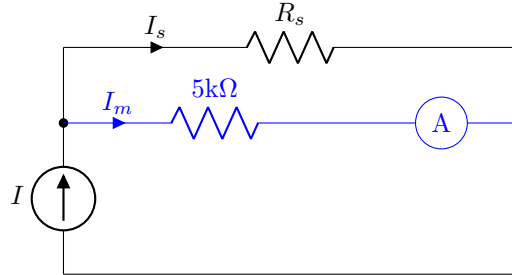
Finally, we have that the current in the ideal ammeter and the voltage in the meter resistance are given by

$$I = \frac{0.5V}{5k\Omega + 20k\Omega} = \boxed{0.02mA} \quad \text{and} \quad V = 0.02mA \cdot 20k\Omega = \boxed{0.4V}$$

Exercise 1.8

(a) In the first part, we have the following circuit:

Figure 6: $50\mu A$ ammeter with $5k\Omega$ internal resistance (shown in blue) in parallel with shunt resistor.



We want to measure I for 0-1 A, and the ideal ammeter measures up to $50\mu A$. To find what shunt resistance R_s allows us to do so, we set $I = 1A$ and $I_m = 50\mu A$. By KCL we know $I_s = 0.999950A$. To determine R_s , we still need to find the voltage across it. We can find this voltage by doing

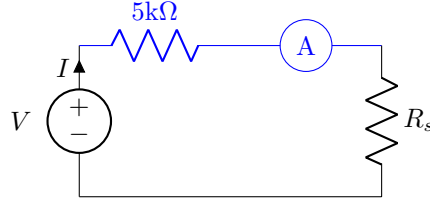
$$V = I_m R_m = 50\mu A \cdot 5k\Omega = 0.25V$$

Then we simply do

$$R_s = \frac{V}{I_s} = \frac{0.25V}{0.999950A} = \boxed{0.25\Omega}$$

(b) In the second part, we have the following circuit:

Figure 7: $50\mu\text{A}$ ammeter with $5\text{k}\Omega$ internal resistance (shown in blue) with a series resistor.



We want to measure V for 0-10 V, and the ideal ammeter measures up to $50\mu\text{A}$. To find the series resistance R_s , we set $V = 10\text{V}$ and $I = 50\mu\text{A}$. Then we solve

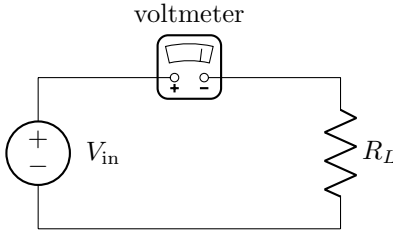
$$\frac{V}{I} = 5\text{k}\Omega + R_s$$

$$R_s = \frac{10\text{V}}{50\mu\text{A}} - 5\text{k}\Omega = \boxed{195\text{k}\Omega}$$

Exercise 1.9

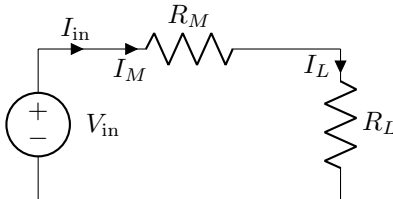
In order to measure resistance well above the range of your multimeter, you need to get creative. We will be using the multimeter in voltmeter mode. Lets start by connecting our DC voltage source, voltmeter, and the high-value resistor in series. (The reason for doing this will become clear later).

Figure 8: Connection of three components.



V_{in} is our test voltage, and R_L is our leakage resistance. We need to revise the model for our voltmeter.

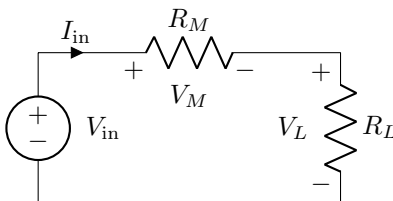
Figure 9: The voltmeter is now modeled as a resistor with value R_M .



The current flowing through our meter (modeled by the resistor R_M) is equal to the current flowing through the leakage resistor. This is also equal to the current supplied from our voltage source.

$$I_M = I_L = I_{in}$$

Figure 10: Voltage and current labels are added.



Notice: this test circuit is a **voltage divider**. When you use this technique, the voltmeter itself makes up half of the divider. The voltage across the leakage resistor cannot be measured directly, so we calculate it using Kirchhoff's Voltage Law by subtracting our voltmeter's reading from the voltage of our DC supply.

$$V_L = V_{in} - V_M$$

The current through the voltmeter's resistance is given by Ohm's Law.

$$I_M = \frac{V_M}{R_M}$$

The current through the leakage resistor is given by Ohm's Law.

$$I_L = \frac{V_L}{R_L}$$

We already determined that I_M and I_L are equal, so we can set the two previous expressions equal to each other.

$$I_M = I_L \Rightarrow \frac{V_M}{R_M} = \frac{V_L}{R_L}$$

We will rearrange the above equation to give an expression for R_L .

$$R_L = R_M \frac{V_L}{V_M}$$

Now we can substitute our first expression for V_L into the previous equation to eliminate V_M (the final unknown term).

$$R_L = R_M \frac{V_{in} - V_M}{V_M}$$

Rewriting the equation, the final result is

$$\boxed{R_L = R_M \left(\frac{V_{in}}{V_M} - 1 \right)}$$

To measure leakage current with a voltmeter, simply divide the meter's reading by the resistance of the meter. For example, if your $10 \text{ M}\Omega$ voltmeter measures 0.023 V , then $I_{leakage} = 23\text{mV}/10\text{M}\Omega = 2.3\text{nA}$. The accuracy of such a measurement depends both on the accuracy of the voltage measurement, and the tolerance of the meter's resistance.

Exercise 1.10

- (a) With two equal-value resistors, the output voltage is half the input voltage.

$$V_{out} = \frac{1}{2}V_{in} = \frac{30V}{2} = \boxed{15V}$$

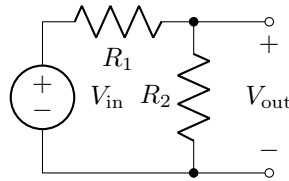
- (b) To treat R_2 and R_{load} as a single resistor, combine the two resistors which are in parallel to find that the combined (equivalent) resistance is $5k\Omega$. Now, we have a simple voltage divider with a $10k\Omega$ resistor in series with the $5k\Omega$ equivalent resistor. The output voltage is across this equivalent resistance. The output voltage is given by

$$V_{out} = V_{in} \frac{5k\Omega}{10k\Omega + 5k\Omega} = \frac{30V}{3} = \boxed{10V}$$

TODO: Add a diagram to make this clearer

- (c) We can redraw the voltage divider circuit to make the “port” clearer.

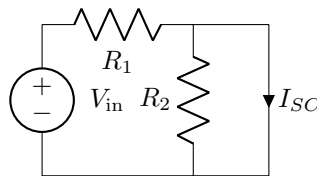
Figure 11: Voltage divider with port shown.



We can find V_{Th} by leaving the ports open (open circuit) and measuring V_{out} , the voltage across R_2 . This comes out to be half the input voltage when $R_1 = R_2$, so $V_{out} = 15V$. Thus $V_{Th} = \boxed{15V}$.

To find the Thévenin resistance, we need to find the short circuit current, I_{SC} . We short circuit the port and measure the current flowing through it.

Figure 12: Voltage divider with short circuit on the output.



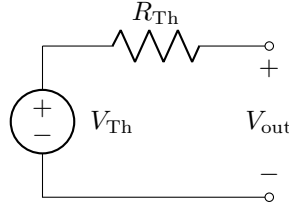
In this circuit, no current flows through R_2 , flowing through the short instead. Thus we have $I_{SC} = \frac{V_{in}}{R_1}$.

From this, we can find R_{Th} from $R_{Th} = \frac{V_{Th}}{I_{SC}}$. This gives us

$$R_{Th} = \frac{V_{Th}}{I_{SC}} = \frac{V_{Th}}{V_{in}/R_1} = \frac{15V}{30V/10k\Omega} = \boxed{5k\Omega}$$

The Thévenin equivalent circuit takes the form shown below.

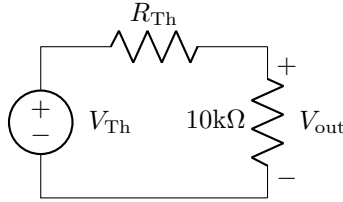
Figure 13: Thévenin equivalent circuit.



In terms of behavior at the ports, this circuit is equivalent to the circuit in Figure 11.

- (d) We connect the $10k\Omega$ load to the port of the Thévenin equivalent circuit in Figure 13 to get the following circuit.

Figure 14: Thévenin equivalent circuit with $10k\Omega$ load.



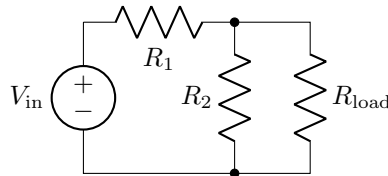
From here, we can find V_{out} , treating this circuit as a voltage divider.

$$V_{out} = \frac{10k\Omega}{R_{Th} + 10k\Omega} V_{Th} = \frac{10k\Omega}{5k\Omega + 10k\Omega} \cdot 15V = \boxed{10V}$$

This is the same answer we got in part (b).

- (e) To find the power dissipated in each resistor, we return to the original three-resistor circuit.

Figure 15: Original voltage divider with $10k\Omega$ load attached.



From part (d), we know that the output voltage is 10V and that this is the voltage across the load resistor. Since $P = IV = \frac{V^2}{R}$, we find that the power through R_{load} is

$$P_{load} = \frac{V^2}{R_{load}} = \frac{(10V)^2}{10k\Omega} = \boxed{10mW}$$

Similarly, we know that the power across R_2 is the same since the voltage across R_2 is the same as the voltage across R_{load} . Thus we have

$$P_2 = \boxed{10\text{mW}}$$

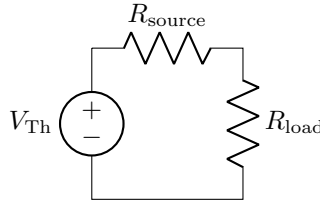
To find the power dissipated in R_1 , we first have to find the voltage across it. From Kirchoff's loop rule, we know that the voltage around any closed loop in the circuit must be zero. We can choose the loop going through the voltage source, R_1 , and R_2 . The voltage supplied by the source is 30V. The voltage dropped across R_2 is 10V as discussed before. Thus the voltage dropped across R_1 must be $30\text{V} - 10\text{V} = 20\text{V}$. Now we know the voltage across and the resistance of R_1 . We use the same formula as before to find the power dissipated.

$$P_1 = \frac{V^2}{R_1} = \frac{(20\text{V})^2}{10\text{k}\Omega} = \boxed{40\text{mW}}$$

Exercise 1.11

Consider the following Thévenin circuit where R_{source} is just another name for the Thévenin resistance, R_{Th} .

Figure 16: Standard Thévenin circuit with attached load.



We will first calculate the power dissipated in the load and then maximize it with calculus. We can find the power through a resistor using current and resistance since $P = IV = I(IR) = I^2R$. To find the total current flowing through the resistors, we find the equivalent resistance which is $R_{\text{source}} + R_{\text{load}}$. Thus the total current flowing is $I = \frac{V_{\text{Th}}}{R_{\text{source}} + R_{\text{load}}}$. The power dissipated in R_{load} is thus

$$P_{\text{load}} = I^2 R_{\text{load}} = \frac{V_{\text{Th}}^2 R_{\text{load}}}{(R_{\text{source}} + R_{\text{load}})^2}$$

To maximize this function, we take the derivative and set it equal to 0.

$$\begin{aligned} \frac{dP_{\text{load}}}{dR_{\text{load}}} &= V_{\text{Th}} \frac{(R_{\text{source}} + R_{\text{load}})^2 - 2R_{\text{load}}(R_{\text{source}} + R_{\text{load}})}{(R_{\text{source}} + R_{\text{load}})^4} = 0 \\ \implies R_{\text{source}} + R_{\text{load}} &= 2R_{\text{load}} \\ \implies R_{\text{source}} &= R_{\text{load}} \end{aligned}$$

Exercise 1.12

(a) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{3/20} = \boxed{1.413}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{3/10} = \boxed{1.995}$

(b) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{6/20} = \boxed{1.995}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{6/10} = \boxed{3.981}$

(c) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{10/20} = \boxed{3.162}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{10/10} = \boxed{10}$

(d) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{20/20} = \boxed{10}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{20/10} = \boxed{100}$

Exercise 1.13

There are two important facts to notice from Exercise 1.12:

1. An increase of 3dB corresponds to doubling the power
2. An increase of 10dB corresponds to 10 times the power.

Using these two facts, we can fill in the table. Start from 10dB. Fill in 7dB, 4dB, and 1dB using fact 1. Then fill in 11dB using fact 2. Then fill in 8dB, 5dB, and 2dB using fact 1 and approximating 3.125 as π .

dB	ratio(P/P_0)
0	1
1	$\boxed{1.25}$
2	$\boxed{\pi/2}$
3	2
4	$\boxed{2.5}$
5	$\boxed{3.125 \approx \pi}$
6	4
7	$\boxed{5}$
8	$\boxed{6.25}$
9	8
10	10
11	$\boxed{12.5}$

Exercise 1.14

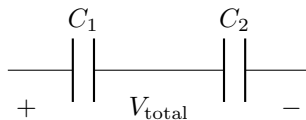
Recall the relationship between I , V , and C : $I = C \frac{dV}{dt}$. Now, we perform the integration:

$$\begin{aligned}
 \int dU &= \int_{t_0}^{t_1} V I dt \\
 U &= \int_{t_0}^{t_1} C V \frac{dV}{dt} dt \\
 &= C \int_0^{V_f} V dV \\
 U &= \frac{1}{2} C V_f^2
 \end{aligned}$$

Exercise 1.15

Consider the following two capacitors in series.

Figure 17: Two capacitors in series.



To prove the capacitance formula, we need to express the total capacitance of both of these capacitors in terms of the individual capacitances. From the definition of capacitance, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}}$$

Notice that V_{total} is the sum of the voltages across C_1 and C_2 . We can get each of these voltages using the definition of capacitance.

$$V_{\text{total}} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

The key observation now is that because the right plate of C_1 is connected to the left plate of C_2 , the charge stored on both plates must be of equal magnitude.³Therefore, we have $Q_1 = Q_2$. Let us call this charge stored Q (i.e. $Q = Q_1 = Q_2$). Now, we know that the total charge stored is also Q .⁴Therefore, we know that $Q_{\text{total}} = Q$. Now, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}} = \frac{Q}{Q_1/C_1 + Q_2/C_2} = \frac{Q}{Q/C_1 + Q/C_2} = \frac{1}{1/C_1 + 1/C_2}$$

Exercise 1.16

Equation 1.21 gives us the relationship between the time and the voltage (V_{out}) across the capacitor while charging. To find the rise time, subtract the time it takes to reach 10% of the final value from the time it takes to reach 90% of the final value.

$$\begin{aligned} V_{\text{out}} &= 0.1V_f = V_f(1 - e^{-t_1/RC}) \\ 0.1 &= 1 - e^{-t_1/RC} \\ t_1 &= -RC \ln(0.9) \end{aligned}$$

Similarly, we find that $t_2 = -RC \ln(0.1)$. Subtracting these two gives us

$$t_2 - t_1 = -RC(\ln(0.1) - \ln(0.9)) = 2.2RC$$

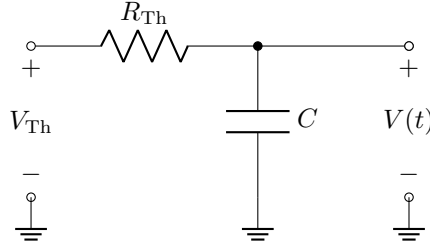
Exercise 1.17

The voltage divider on the left side of the circuit can be replaced with the Thévenin equivalent circuit found Exercise 1.10 (c). Recall that $V_{\text{Th}} = \frac{1}{2}V_{\text{in}}$ and $R_{\text{Th}} = 5\text{k}\Omega$. This gives us the following circuit.

³If this were not true, then there would be a net charge on these two plates and the wire between them. Because we assume that the capacitors started out with no net charge and there is no way for charge to leave the middle wire or the two plates it connects, this is impossible.

⁴If you are having trouble seeing this, suppose we apply a positive voltage to the left plate of C_1 relative to the right plate of C_2 . Suppose this causes the left plate of C_1 to charge to some charge q . We now must have a charge of $-q$ on the right plate of C_1 because q units of charge are now pushed onto the left plate of C_2 . Now the left of C_2 has q units of charge which causes a corresponding $-q$ charge on the right side of C_2 . Thus the overall total charge separated across these two capacitors is q .

Figure 18: Thévenin equivalent circuit to Figure 1.36 from the textbook.



Now we have a simple RC circuit which we can apply Equation 1.21 to. The voltage across the capacitor is given by

$$V(t) = V_{\text{final}}(1 - e^{-t/RC}) = V_{\text{Th}}(1 - e^{-t/R_{\text{Th}}C}) = \boxed{\frac{1}{2}V_{\text{in}}(1 - e^{-t/5 \times 10^{-4}})}$$

TODO: Add graph

Exercise 1.18

From the capacitor equation in the previous paragraph, we have

$$V(t) = (I/C)t = (1\text{mA}/1\mu\text{F})t = 10\text{V}$$

This gives us

$$\boxed{t = 0.01\text{s}}$$

Exercise 1.19

Suppose a current I is flowing through a loop of wire with cross-sectional area A . This induces a magnetic field B , and the flux Φ through the loop is

$$\Phi = BA$$

Now suppose the same current I flows through a wire coiled into n loops, each with the same cross-sectional area A . This induces a magnetic field of n times the strength, $B_n = nB$. Since each loop has area A , the total cross-sectional area of the coil can be considered $A_n = nA$. Then the magnetic flux through the coil is

$$\Phi_n = B_n A_n = n^2 BA = n^2 \Phi$$

Since inductance is defined as flux through a coil divided by current through the flux, we can see that $\Phi_n = n^2 \Phi$ implies $L \propto n^2$.

Exercise 1.20

We can use the formula for the full-wave rectifier ripple voltage to find the capacitance.

$$\frac{I_{\text{load}}}{2fC} = \Delta V \leq 0.1V_{\text{p-p}}$$

The maximum load current is 10mA and assuming a standard wall outlet frequency of 60 Hz, we have

$$C \geq \frac{10\text{mA}}{2 \times 60\text{Hz} \times 0.1\text{V}} = \boxed{833\mu\text{F}}$$

Now we need to find the AC input voltage. The peak voltage after rectification must be 10V (per the requirements). Since each phase of the AC signal must pass through 2 diode drops, we have to add this to find out what our AC peak-to-peak voltage must be. Thus we have

$$V_{\text{in,p-p}} = 10\text{V} + 2(0.6\text{V}) = \boxed{11.2\text{V}}$$

Exercise 1.30

V_{out} is simply the voltage at the output of an impedance voltage divider. We know that $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$. Thus we have

$$V_{\text{out}} = \frac{Z_C}{Z_R + Z_C} V_{\text{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{\text{in}} = \frac{1}{1 + j\omega RC} V_{\text{in}}$$

The magnitude of this expression can be found by multiplying by the complex conjugate and taking the square root.

$$\text{sqrt} V_{\text{out}} V_{\text{out}}^* = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} V_{\text{in}}$$